Grade 12
Applied Mathematics
Achievement Test

Student Booklet

June 2016

Manitoba

After the administration of this test, print copies of this resource will be
available for purchase from the Manitoba Learning Resource Centre
(formerly the Manitoba Text Book Bureau). Order online at
www.mtbb.mb.ca.

This resource will also be available on the Manitoba Education and
Advanced Learning website at www.edu.gov.mb.ca/k12/assess/
archives/index.html.

Websites are subject to change without notice.

Disponible en français.

Available in alternate formats upon request.

GRADE 12 APPLIED MATHEMATICS
ACHIEVEMENT TEST

DESCRIPTION

Total Possible Marks: 62   **Time: 3 hours**

Unit	Description	Marks
A	Relations and Functions	15
B	Probability	16
C	Financial Mathematics	17
D	Design and Measurement	6
E	Logical Reasoning	8

RESOURCES

You may use the following resources:

- *Formula Sheet* **(tear-out page at the back of this booklet)**
- one 8½″ × 11″ study sheet
- ruler
- graphing calculator, computer software, and/or app
- Internet access for tools such as applets or mortgage payment calculators

   Use of the Internet to communicate or access other content, including but not limited to course notes, definitions, or conceptual information is not permitted during the test.

> **Electronic communication between students through phones, email, or file sharing during the test is strictly prohibited.**

DIRECTIONS

Read all instructions on the test carefully.

If you need extra paper or you print out an answer to a question, let the teacher know. Indicate in the response space of the question that your answer is on a separate sheet.

Remember to

- indicate your input values by writing them in your booklet or printing a copy if using a technology tool

- include your booklet number and question number on additional pages (e.g., printouts) and attach them to the corresponding page in the booklet

- express answers in decimal and percentage form to **two decimal places** when rounding, unless otherwise indicated

  Example: $\dfrac{15}{29} = 0.52$ or 51.72%

- state any assumptions you make

A "graphic organizer" is a visual representation of information. Examples include a tree diagram, a chart, a list, a Venn diagram, a truth table, Pascal's triangle, etc.

A clearly communicated answer

— is easily identified in the response space

— includes the parameters in the equation, and "$y =$", "sin", "ln", or "x", as applicable

— includes the units of measure, where applicable

— includes labels, units, and scales for the axes on graphs

— is expressed as an exact value or is appropriately rounded

Marks may be deducted for errors relating to any of the above.

STOP
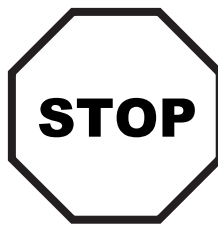
PLEASE WAIT UNTIL INSTRUCTED TO TURN THE PAGE.

RELATIONS AND FUNCTIONS

Question 1	**Total: 1 mark**

Use the graph below to answer the following question and select the best answer.
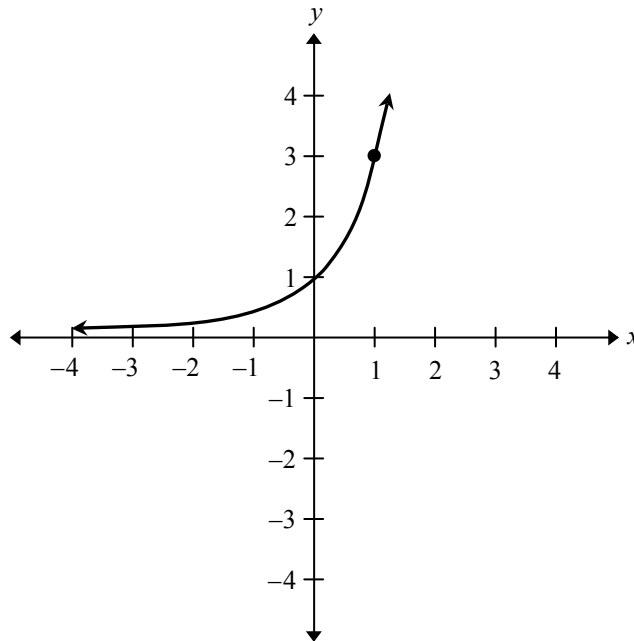
Which function does the graph represent?

A. $y = 1(2)^x$

B. $y = \dfrac{1}{3}(2)^x$

C. $y = 1(3)^x$

D. $y = 1\left(\dfrac{1}{3}\right)^x$

Use the graph below to answer the following question and select the best answer.   102
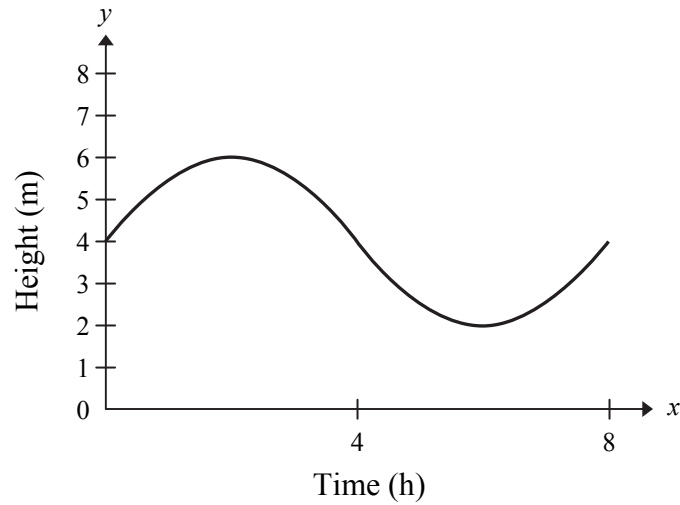
Which of the following statements is true?

A. The amplitude of the function is 2.

B. The period of the function is 4.

C. The equation of the midline is $y = 2$.

D. The domain of the function is $[2, 6]$.

Question 3   Total: 4 marks

A certain medication destroys diseased cells in a person's body. The number of diseased cells decreases for a short time after a dose of the medication is administered and then increases again. This situation varies sinusoidally and is modelled by the following equation:

$$c = 350 \sin(3.14t + 1.57) + 650$$

where t represents the time (in weeks) and c represents the diseased cell count.
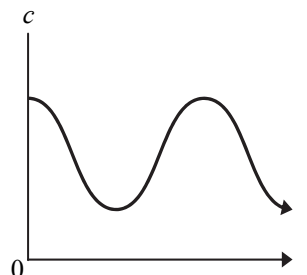
103

a)   If the initial dose of medication is administered at $t = 0$, when is the second dose administered?

(1 mark)

104

b)   What is the range of this function?

(1 mark)

105

c)   A patient claims to feel well when the diseased cell count is below 500. At $t = 2.5$ weeks, will the patient be feeling well? Show your work.

(2 marks)

The Gateway Arch in St. Louis, in the United States, approximates a parabola. Steven learns that the arch is 192 m wide. (Diagram is not drawn to scale.)

At 2.5 m from his starting point, Steven measures the height of the arch to be 10 m.

106
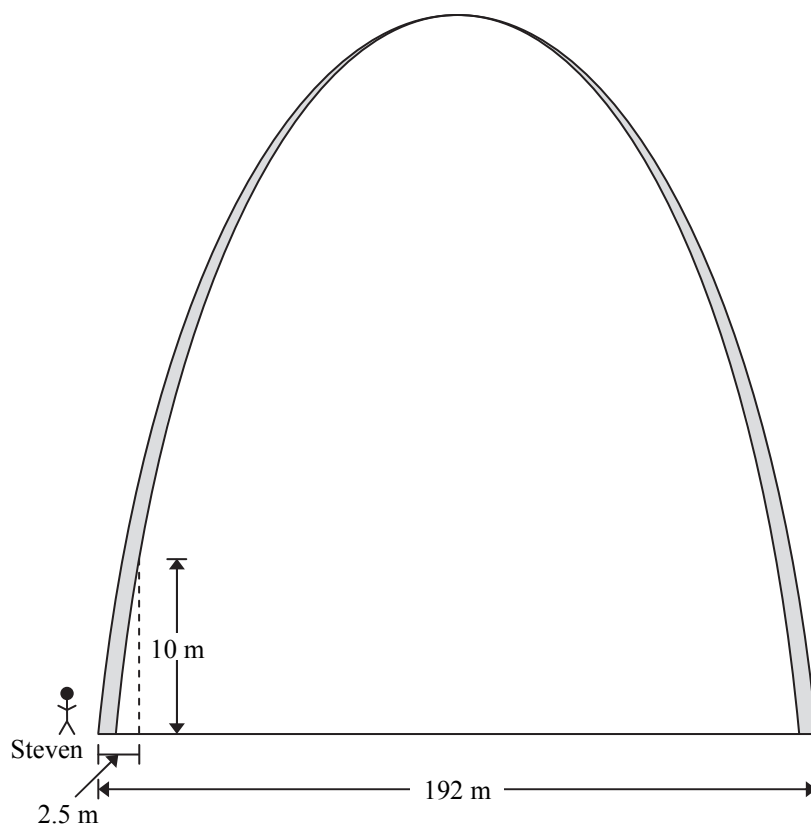
a)   Determine the quadratic equation that models the shape of the arch. Show your work. Express all coefficients in the equation to a minimum of three (3) decimal places.

(2 marks)

b)   Calculate the arch's maximum height.

(1 mark)

Question 5   Total: 6 marks

The ice thickness on Lake Mathitoba is measured weekly every winter. The data for one season is shown in the table below:

Time (day)	7	14	21	28	35	42	49
Ice thickness (cm)	5.2	17.4	26.8	32.4	37.1	42.6	44.3

a)   Determine the logarithmic regression equation that models this data.

(1 mark)

b)   Create a clearly labelled graph of the equation in (a).

(3 marks)

c) It is considered safe to operate a vehicle on lake ice if it is at least 30 cm thick. Use your equation in (a) to determine the first full day it will be safe to drive on the ice. Show your work.

(2 marks)

PROBABILITY

Question 6 — Total: 1 mark

Select the best answer.

The odds in favour of Philip winning a badminton match are $5:4$.

What is the probability that Philip will lose the match?

A. 0.20

B. 0.44

C. 0.56

D. 0.80

Question 7 — Total: 1 mark

Select the best answer.

Elaine has a bag containing 5 red pens and 10 blue pens. She randomly picks two pens out of the bag (no replacement).

What is the probability that Elaine picked two red pens?

A. $\dfrac{4}{45}$

B. $\dfrac{2}{21}$

C. $\dfrac{1}{9}$

D. $\dfrac{1}{3}$

Question 8   Total: 3 marks

Evan is arranging 8 different bicycles in a rack at his school.

113

a) How many ways can the bicycles be arranged in the rack?

(1 mark)

114

b) How many ways can the bicycles be arranged in the rack if 3 students want their bicycles placed together? Show your work.

(2 marks)

115

Rosalind wants to make a trail mix for a snack. She has 15 ingredients to choose from.

What is the total number of different mixes Rosalind can create containing 3, 4, or 5 ingredients? Show your work.

Question 10   Total: 2 marks

Given the following collection of objects:

$$\blacksquare \diamondsuit \triangle \triangle \blacksquare \triangle \blacksquare \triangle \triangle \diamondsuit \diamondsuit \blacksquare$$

116

a)   Calculate the probability of randomly choosing $\blacksquare$.

(1 mark)

117

b)   How can the collection be changed so that the probability of choosing $\blacksquare$ is exactly 40%?

(1 mark)

Question 11   Total: 3 marks

Tim has a set of cards numbered 1 to 15. He randomly draws one card.

Consider the following events:

Event A: drawing a card that is a multiple of 2

Event B: drawing a card that is a multiple of 3

a) Are these events mutually exclusive? Justify your answer.

(1 mark)

118

b) What is the probability of drawing a numbered card that is a multiple of 2 or a multiple of 3? Show your work.

(2 marks)

119

Question 12   Total: 2 marks

You are asked to take a 3-question multiple-choice quiz. Each question has 4 possible answers, one of which is correct.

120

a) If you randomly pick an answer for each question, what is the probability that all 3 answers are wrong?

(1 mark)

121

b) What is the probability of getting at least one of the questions correct?

(1 mark)

Question 13   Total: 2 marks

Student ID codes are made up using any two upper case letters of the alphabet followed by any two digits.

a) How many ID codes are possible?

(1 mark)

b) How many ID codes are possible given the following conditions?
    • Repetition is not allowed.
    • The letters "I" and "O" cannot be used.

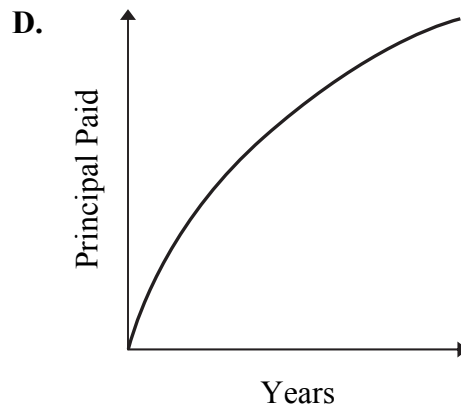*(1 mark)*

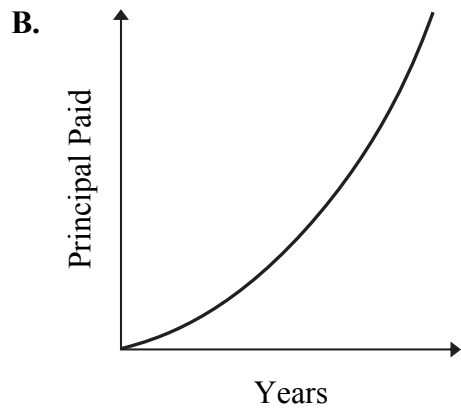# FINANCIAL MATHEMATICS

Question 14   Total: 1 mark

Select the best answer.

Which of the following graphs shows the amount of principal paid on each payment over a 25-year mortgage?

A.

Principal Paid (y-axis) vs Years (x-axis): straight line increasing from origin.

C.

Principal Paid (y-axis) vs Years (x-axis): horizontal line.

B.

Principal Paid (y-axis) vs Years (x-axis): upward curve (concave up) increasing.

D.

Principal Paid (y-axis) vs Years (x-axis): curve increasing and leveling off (concave down).
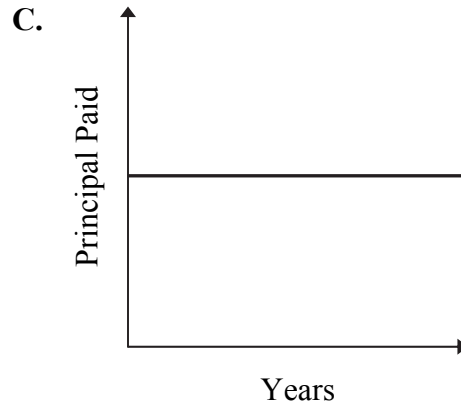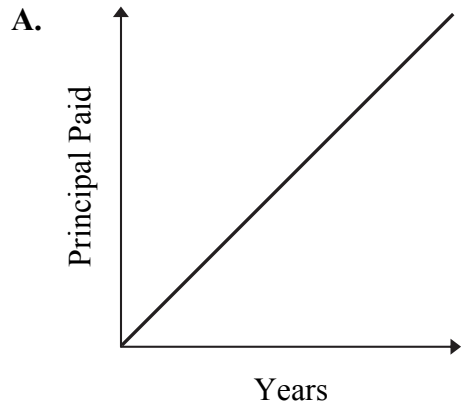
Question 15 — Total: 1 mark

Select the best answer.

The price of a new car is $26 000 (taxes included). It depreciates at a rate of 20% per year. What is the approximate residual value of the car after 3 years?

A. $10 400

B. $13 300

C. $15 600

D. $20 800

Question 16 — Total: 1 mark

Use the following information to answer this question and select the best answer.

Scenario 1: A loan of $30 000.00 at an interest rate of 7.00%, compounded monthly for 1 year

Scenario 2: A loan of $30 000.00 at an interest rate of 7.00%, compounded _____ for 1 year

Assuming no payments are made, which compounding period frequency in Scenario 2 would result in less interest than in Scenario 1?

A. every two weeks

B. daily

C. semi-annually

D. weekly

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Mr. and Mrs. Murthy have a total monthly gross income of $6000.00. They are interested in purchasing a house with a mortgage payment of $1300.00 per month, annual heating costs of $2100.00, and annual property taxes of $3675.00.

Calculate the gross debt service ratio (GDSR). Determine if a bank is likely to offer them a mortgage. Justify your answer.

Question 18   Total: 2 marks

List two advantages of leasing a vehicle over buying.

Question 19   Total: 4 marks

Gabrielle wants to buy a cabin at Lac du Bonnet that costs $165 000.00. She has saved
$25 000.00 that she will use as a down payment. The bank will give her a 15-year mortgage
for the balance at 3.49%, compounded semi-annually.

a)   What will Gabrielle's payment be every two weeks? Show your work.

(2 marks)

b)   If the cabin appreciates in value an average of 3.00% per year, calculate the appreciated
     value of the cabin after 10 years.

(1 mark)

c)   How much equity will Gabrielle have in the cabin after 10 years?

(1 mark)

Question 20   **Total: 6 marks**

At the age of 30, Alfred began investing $350.00 monthly into an investment account at an
interest rate of 7.00%, compounded monthly. When he turned 45, the interest rate on this
investment decreased to 5.00%, compounded monthly. Alfred plans to continue making monthly
investments until he retires at 58.

132

a)   How much money will Alfred have in his account when he retires? Show your work.

(3 marks)

133

b)   Alfred withdraws $2000.00 per month from his account after he retires. If the interest rate
     remains at 5.00%, how many months can he withdraw $2000.00?

(1 mark)

c)   Alfred's sister, Marianne, retires at the age of 60. Her portfolio is valued at $200 000.00, earning 5.00%, compounded monthly. If Marianne wants the money to last until she is 85 years old, what is the maximum she can withdraw each month? Show your work.

(2 marks)

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# DESIGN AND MEASUREMENT

Question 21	**Total: 2 marks**

Sheena is wrapping boxes using a roll of wrapping paper with a total area of 3.5 m^2. Each box has a 20 cm width, 20 cm length, and 40 cm height. If Sheena uses 20% extra paper on each box, how many boxes can she wrap? Show your work.

A traditional plains tipi is built using a rectangular canvas that is twice as long as it is wide. The diagram below shows how a semicircle would be cut out of a canvas with dimensions of 30 ft. (length) and 15 ft. (width). (Diagram is not drawn to scale.)

15 ft.

30 ft.

136
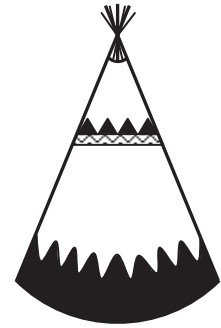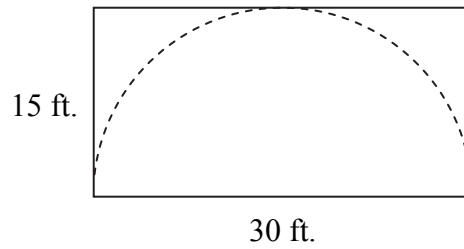
a)   Calculate how much canvas is left over after the semicircle has been cut out.

(2 marks)

b) You would like to construct a tipi with both dimensions three times greater than those for the tipi in (a). Given that canvas is only sold in a rectangular shape, how much will the canvas cost if the price of canvas is $7.39/ft^2, plus taxes? (Note: GST = 5%, PST = 8%)

(2 marks)

LOGICAL REASONING

Question 23   Total: 2 marks

Complete the truth table.

p	q	$\sim q$ (not q)	$p \Rightarrow \sim q$
True	True		
True	False		
False	True		
False	False		

Question 24   Total: 4 marks

There are 160 Grade 12 students attending a high school.

This semester, the science teacher noted that there are

- 30 students in Biology
- 54 students in Chemistry
- 43 students in Physics
- 11 students in both Biology and Physics
- 8 students in both Biology and Chemistry
- 9 students in both Chemistry and Physics
- 3 students taking all three courses

139

a) Use a Venn diagram to represent this situation.

(3 marks)

140

b) How many students are taking Biology or Chemistry?

(1 mark)

Paula is trying to solve the following puzzle. Each 2×2 box must contain the digits 1 to 4 only once. As well, each column and each row must contain the digits 1 to 4 only once.

Paula indicates that the ★ must be a "4". Explain why she is incorrect and solve the puzzle.

1		★	2
3	2		
			1
4			

END OF TEST

**NO MARKS WILL BE AWARDED
FOR WORK DONE ON THIS PAGE.**

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FOR WORK DONE ON THIS PAGE.**

Formula Sheet: Applied Mathematics

Relations and Functions	**Financial Mathematics**

Relations and Functions

$$y = ax + b$$

$$y = ax^2 + bx + c$$

$$y = ax^3 + bx^2 + cx + d$$

$$y = ab^x$$

$$y = a + b\ln(x)$$

$$y = a\log_b x$$

$$y = a\sin(bx + c) + d$$

$$y = a\cos(bx + c) + d$$

Financial Mathematics

$$t = \frac{72}{i}$$

$$I = Prt$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Net worth = Total assets − Total liabilities

$$\text{Debt-to-equity ratio (\%)} = \frac{(\text{Total liabilities} - \text{Mortgage})}{\text{Net worth}} \times 100$$

$$\text{Gross debt service ratio (\%)} = \frac{\left(\begin{array}{ccc}\text{Monthly} & \text{Monthly} & \text{Monthly} \\ \text{mortgage} + \text{property} + \text{heating} \\ \text{payment} & \text{taxes} & \text{costs}\end{array}\right)}{\text{Gross monthly income}} \times 100$$

$$\text{Rate of return (\%)} = \frac{\left(\begin{array}{cc}\text{Current value} & \text{Previous value} \\ \text{of portfolio} & - & \text{of portfolio}\end{array}\right)}{\text{Previous value of portfolio}} \times 100$$

Probability

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

$$_nP_r = \frac{n!}{(n-r)!}$$

$$_nC_r = \frac{n!}{r!(n-r)!}$$

Design and Measurement

Prism: Surface area $= Ph + 2B$

Volume $= Bh$

Pyramid: Surface area $= B + \dfrac{Ps}{2}$ (s = slant height)

Volume $= \dfrac{Bh}{3}$

Sphere: Surface area $= 4\pi r^2$

Volume $= \dfrac{4}{3}\pi r^3$

Cylinder: Surface area $= 2\pi rh + 2\pi r^2$

Volume $= \pi r^2 h$

Cone: Surface area $= \pi r^2 + \pi rs$

Volume $= \dfrac{\pi r^2 h}{3}$