

Grade 12
Applied Mathematics
Achievement Test

Student Booklet

June 2013

Manitoba 

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GRADE 12 APPLIED MATHEMATICS ACHIEVEMENT TEST

DESCRIPTION:

Total Possible Marks: 60

Time: 3 hours

Unit	Description	Marks
A	Relations and Functions	16
B	Probability	14
C	Financial Mathematics	15
D	Design and Measurement	8
E	Logical Reasoning	7

Formula Sheet: Applied Mathematics

Relations and Functions	Financial Mathematics
$y = ax^2 + bx + c$ $y = ax^3 + bx^2 + cx + d$ $y = ab^x$ $y = a + b \ln(x)$ $y = a \sin(bx + c) + d$ $y = a \cos(bx + c) + d$ $\text{Period} = \frac{2\pi}{b} \quad \text{or} \quad \frac{1}{\text{Frequency}}$	$I = Prt$ $A = P \left(1 + \frac{r}{n} \right)^{nt}$ <p>Debt to equity ratio = $\frac{(\text{Total liabilities} - \text{Mortgage})}{\text{Net worth}} \times 100$</p> <p>Gross debt service ratio = $\frac{\left(\begin{array}{l} \text{Monthly mortgage} \\ \text{payment} \end{array} + \begin{array}{l} \text{Monthly property} \\ \text{taxes} \end{array} + \begin{array}{l} \text{Monthly heating} \\ \text{costs} \end{array} \right)}{\text{Gross monthly income}} \times 100$</p> <p>Average rate of return = $\frac{\left(\begin{array}{l} \text{Current value} \\ \text{of portfolio} \end{array} - \begin{array}{l} \text{Previous value} \\ \text{of portfolio} \end{array} \right)}{\text{Previous value of portfolio}} \times 100$</p>
Probability	Design and Measurement
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $P(A \text{ and } B) = P(A) \times P(B)$ ${}_n P_r = \frac{n!}{(n-r)!}$ ${}_n C_r = \frac{n!}{r!(n-r)!}$	<p>Prism: Surface area = $Ph + 2B$ Volume = Bh</p> <p>Pyramid: Surface area = $B + \frac{Pl}{2}$ (l = slant height) Volume = $\frac{Bh}{3}$</p> <p>Sphere: Surface area = $4\pi r^2$ Volume = $\frac{4}{3}\pi r^3$</p> <p>Cylinder: Surface area = $2\pi rh + 2\pi r^2$ Volume = $\pi r^2 h$</p>

TEST RESOURCES AND DIRECTIONS:

- You may consult your $8\frac{1}{2}'' \times 11''$ study sheet during the test.
- You may use a ruler, a graphing calculator, and computer software. You may also have access to the Internet for tools such as applets or a mortgage payment calculator. **Use of the Internet to access course notes, find definitions, or search for conceptual information about the course is prohibited during the test.**
- For short-answer and long-answer questions, you may print out diagrams from the computer or your calculator where applicable. Indicate your booklet number and the question number on the printouts. Remain seated and your teacher will distribute these printouts to you. Indicate in the response space of the question that the answer is on a printed sheet and staple it to the page.
- If you need more space to answer a question, extra paper may be provided by your teacher. Write your booklet number and the question number on any extra paper used and staple it into the booklet where your answer begins. Indicate in the response space of the question that the answer is on a separate sheet.
- Provide clear explanations or justifications where applicable. This can be done through labelled diagrams, in words, by showing mathematical operations to verify your answer, or by referring to a calculator or software program.
 - If you refer to a calculator program, indicate all your input values.
 - If you refer to a software program or a website, indicate all your input values and print or copy the screen showing the answers.
 - If you refer to a spreadsheet, print a copy of the answers.
- A graphic organizer is a visual representation of information. Examples include a tree diagram, a chart, a list, a Venn diagram, a truth table, Pascal's triangle, etc.
- Round your final answers to two decimal places unless otherwise indicated.
- An answer without any work shown will be considered incomplete.
- Always state your assumptions.

The following errors may result in a 0.5 mark deduction:

- not including one of the following in the equation: “ $y =$ ”, “sin”, “ln”, or “ x ”, or writing parameters separately from the equation
- not including the units in the final answer
- not including one of the following on the graph: labels for the axes, units for the axes, or scales for the axes
- not stating or incorrectly stating the final answer
- rounding too soon or rounding incorrectly
- not using whole units appropriately

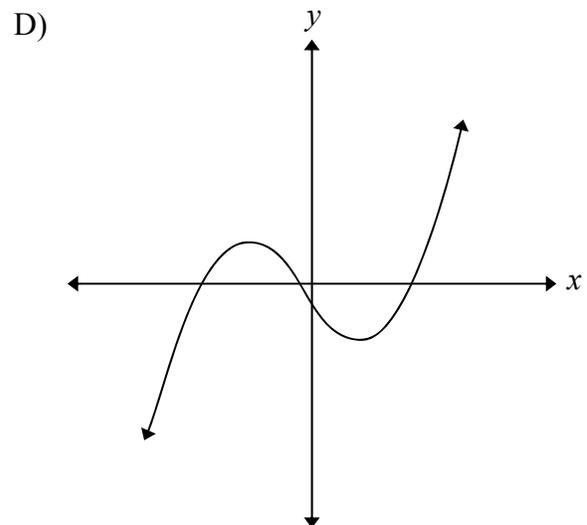
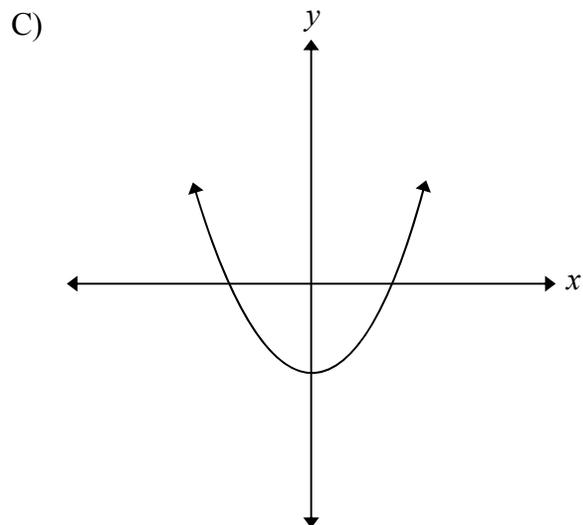
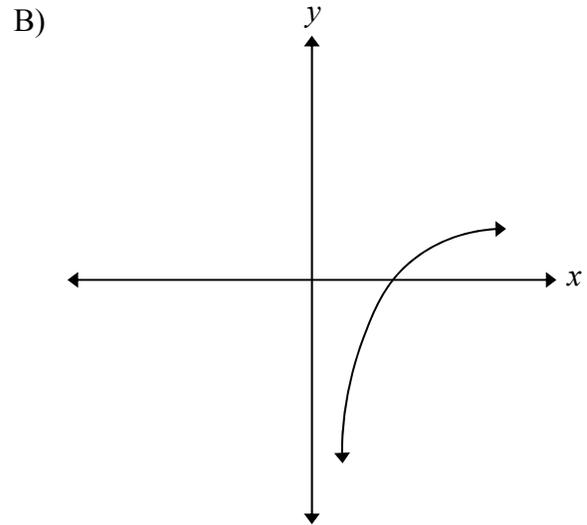
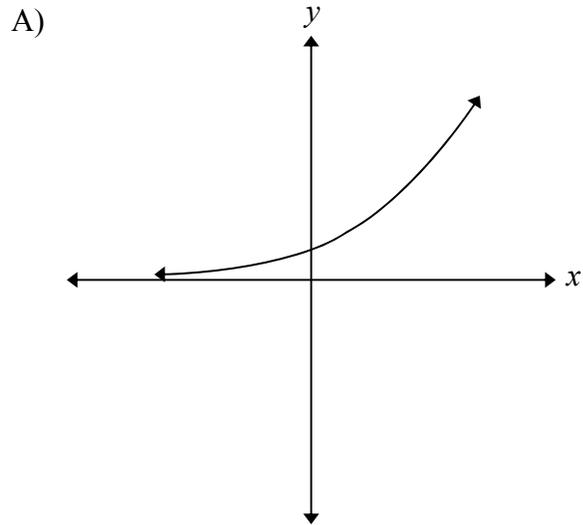
RELATIONS AND FUNCTIONS

Question No. 1

Total: 1 mark

101

Circle the graph below which best represents a cubic function.



Question No. 2

Total: 2 marks

Given the following function which represents the change in a town's population with respect to time:

$$y = 1000(1.05)^x$$

Referring to the town, explain the meaning of:

a) "1000"

(1 mark)

102

b) "1.05"

(1 mark)

103

Question No. 3

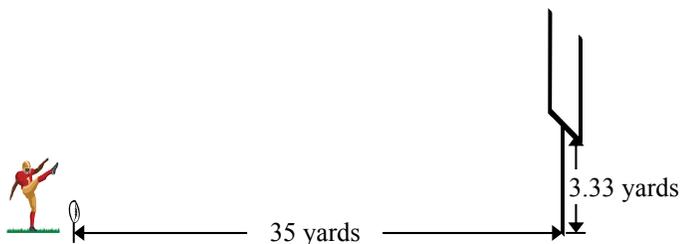
Total: 2 marks

104

A football player wants to kick a football so it will go over a crossbar that is 35 yards away and 3.33 yards high. (Diagram is not drawn to scale.)

The horizontal distance (d , in yards) and the height (h , in yards) that the football travels are represented by the following equation:

$$h = -0.04d^2 + 1.51d$$



How far above or below the crossbar will the football travel? Show your work.

Question No. 4

Total: 3 marks

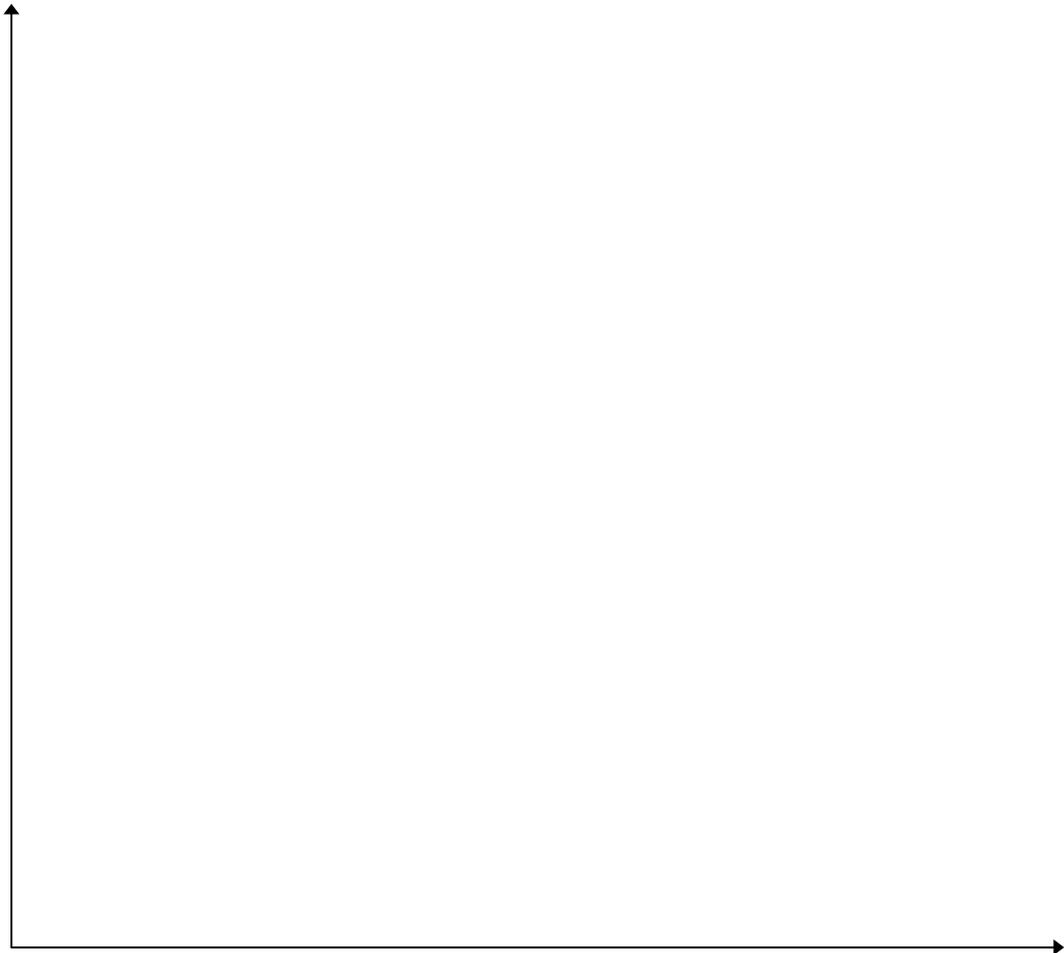
105

The mass of a steel ball varies with respect to its diameter.

diameter (mm)	0	5	10	15	20
mass (g)	0	2	10	32	80

Determine the cubic equation that best represents the data. Sketch a clearly labelled graph of the equation.

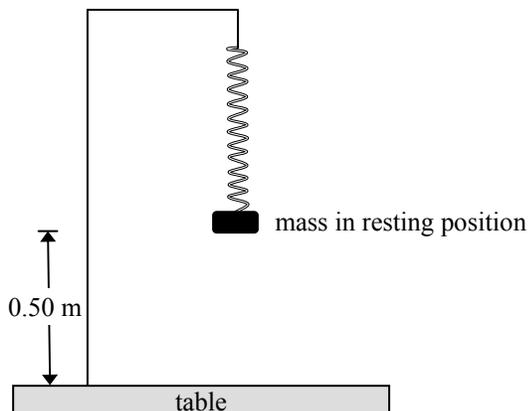
cubic equation: _____



Question No. 5

Total: 3 marks

A mass is suspended by a spring and is in a resting position 0.50 metres above a table.



The mass is pulled down 0.40 metres and is then released. The following information is obtained:

- It takes 1.20 seconds for the mass to return to its lowest position.
- The mass reaches a maximum height of 0.90 metres.

a) Determine the sinusoidal equation that best represents the distance of the mass with respect to the table as a function of time since it was released. Show your work.

(2 marks)

106

b) When will the mass be 0.75 metres above the table for the first time?

(1 mark)

107

Question No. 6

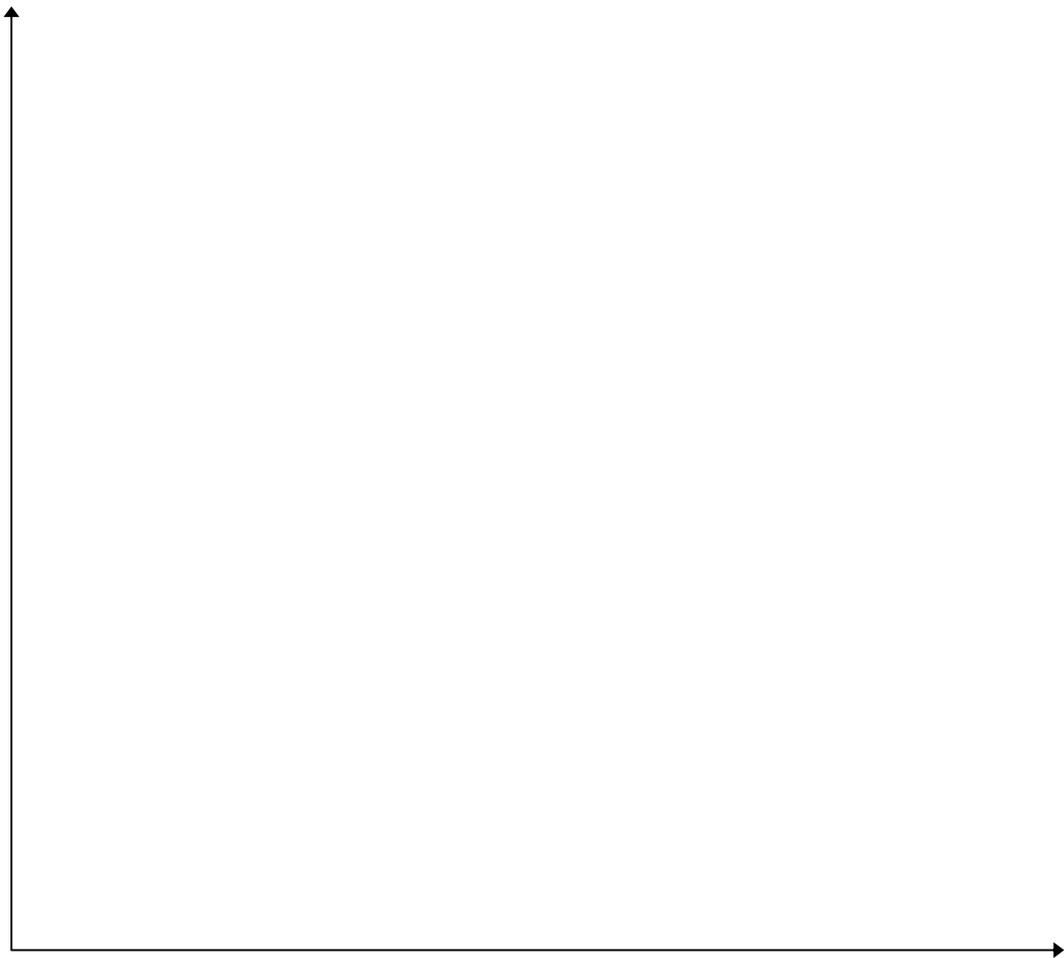
Total: 5 marks

It becomes easier and easier to see the headlights of an oncoming car the closer that it gets. The distance (d , in metres) between the car and an observer can be described as a function of the intensity (I , in lumens) of the headlight brightness:

$$d = 350 - 72 \ln(I)$$

- a) Sketch a clearly labelled graph of the equation.

(2 marks)



108

- b) Determine the distance to an oncoming car if the intensity of its headlights is 75 lumens.
(1 mark)

- c) What is the maximum intensity of the headlights? Justify your answer.
(2 marks)

PROBABILITY

Question No. 7**Total: 1 mark**

Licence plates in Ontario contain 4 upper case letters followed by 3 digits, with repetition allowed. Circle the maximum possible number of licence plates that begin with the letters: MMBA, MANI, or BNTP.

111

- A) 2160
- B) 2880
- C) 3000
- D) 4000

Question No. 8**Total: 1 mark**

A bag contains 6 white marbles, 8 blue marbles, 2 yellow marbles, and 4 green marbles. What are the odds in favour of selecting a white marble?

112

- A) 6:14
- B) 6:20
- C) 14:6
- D) 20:6

Question No. 9

Total: 1 mark

113

A cookie jar contains 10 chocolate chip cookies, 12 double chocolate cookies, and 15 oatmeal cookies. Allison says that the odds against selecting a cookie with chocolate are 15 to 37. Ryan says that the odds against are 15 to 22. Who is correct? Explain your answer.

Question No. 10

Total: 2 marks

Describe a situation containing mutually exclusive events. Explain why the events are mutually exclusive.

114

Question No. 11

Total: 4 marks

An organization consisting of 15 women and 19 men must create a 10-person committee.

- a) How many committees can be created that will include 4 women and 6 men? Show your work.

115

(2 marks)

- b) If a 10-person committee is randomly selected, what is the probability that the committee will include 4 women and 6 men? Show your work.

116

(2 marks)

Question No. 12

Total: 5 marks

You have been asked to create a four-character password for your computer using:

- the 26 upper case letters of the alphabet (A, B, C, ...)
- the 26 lower case letters of the alphabet (a, b, c, ...)
- the digits from 0 to 9
- the symbols:

~ ! @ # \$ % ^ & *

- a) How many different four-character passwords are possible if any of the letters, digits, or symbols can be used for each character if repetition is allowed?

117

(1 mark)

- b) How many different four-character passwords are possible if repetition is not allowed?

118

(1 mark)

- c) How many four-character passwords begin with a letter and end with a digit if repetition is allowed?

119

(1 mark)

- d) How many different four-character passwords containing at least one symbol are possible if repetition is allowed? Show your work.

(2 marks)

FINANCIAL MATHEMATICS

Question No. 13**Total: 1 mark**

Brigitte invests \$5000.00 at an interest rate of 6% for 5 years. Circle the compounding period below that would maximize the rate of return on the investment.

121

- A) daily
- B) monthly
- C) quarterly
- D) semi-annually

Question No. 14**Total: 1 mark**

Circle the asset below which is most likely to depreciate in value.

122

- A) rare coin collection
- B) classic car
- C) house
- D) computer

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Question No. 15

Total: 3 marks

Mr. Chang is 64 and plans to retire next year. His portfolio includes the following investments:

- \$50 000.00 in mutual funds
- \$100 000.00 in stocks
- \$20 000.00 in guaranteed investment certificates (GICs)

- a) Does this portfolio include an appropriate level of risk for Mr. Chang at this stage in his life? Explain your answer.

123

(1 mark)

- b) Mr. Chang's investments had the following returns last year: mutual funds increased by 12.00%, stocks decreased by 4.00%, and GICs had an annual interest rate of 3.00%. Calculate the average rate of return for the year for this portfolio. Show your work.

(2 marks)

Type of investment	Principal (\$)	Return (\$)	End of the year (\$)
mutual funds	50 000.00		
stocks	100 000.00		
GICs	20 000.00		
Total:			

Question No. 16

Total: 5 marks

Amar dreams of retiring at the age of 55. He had planned on starting to save for his retirement at the age of 50, but his financial advisor does not agree. He recommends that Amar starts to save sooner.

- a) If Amar invests \$1000.00 on his 25th birthday and contributes \$200.00 every month to an account that earns 8.00% compounded monthly, what will be the value of the investment on his 55th birthday? Show your work.

125

(2 marks)

- b) If Amar invests \$1000.00 on his 50th birthday, how much will he have to contribute every month to match the final value of the investment in (a)? Assume that the interest rate and the compounding periods are the same.

126

(1 mark)

- c) Calculate the difference between Amar's total contribution in (a) and in (b)? Show your work.

(2 marks)

Question No. 17

Total: 5 marks

The Reimers have purchased a house valued at \$250 000.00 and have made a down payment of \$25 000.00.

- a) Calculate their monthly mortgage payment if they obtain a mortgage amortized over 15 years at an interest rate of 5.50% compounded semi-annually. Show your work.

(2 marks)

128

- b) How much equity will the Reimers have in their house after 5 years if the value of the house appreciates at a rate of 2.00% per year? Show your work.

(3 marks)

DESIGN AND MEASUREMENT

Question No. 18

Total: 2 marks

A cake mix will produce 230 cubic inches of batter. You are using cylinder-shaped baking cups that have a diameter of 3 inches and a depth of 2 inches for the batter. How many cupcakes will you be able to make? Show your work.

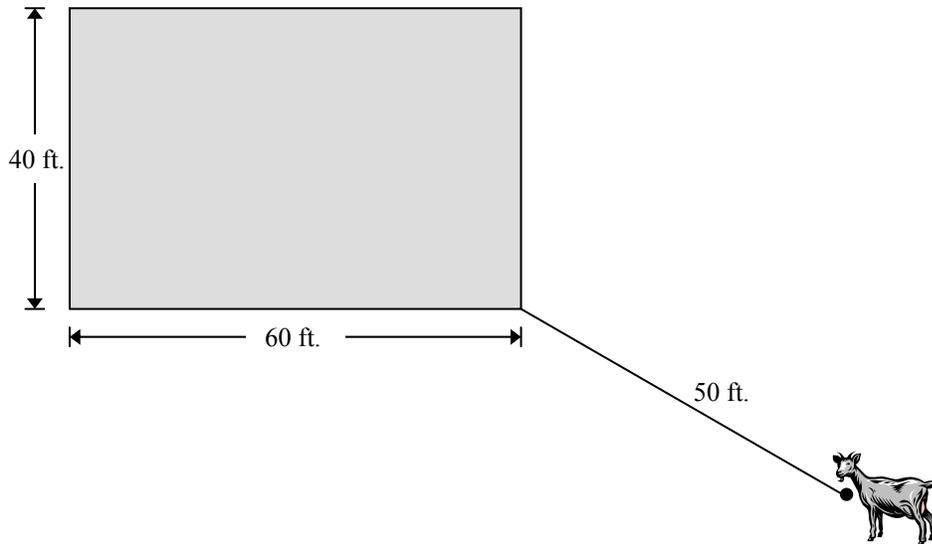
130

Question No. 19

Total: 2 marks

131

A goat is tied to the corner of a barn with a 50-foot rope. The barn measures 60 feet by 40 feet. Calculate the total area outside of the barn that is available to the goat. Show your work.



Question No. 20**Total: 4 marks**

132

The Manitoba Beach Volleyball Association has asked you to design a souvenir beach ball according to the following information:

- The beach ball must have a volume between 1 and 3 cubic feet.
- The plastic material costs \$0.15 per ft^2 .
- Labour and other materials cost \$1.25 per beach ball.
- The Association wants to make a profit of 80% of the cost of making each beach ball.

Based on your design, what is the minimum selling price for each souvenir beach ball? Show your work.

LOGICAL REASONING

Question No. 21

Total: 1 mark

133

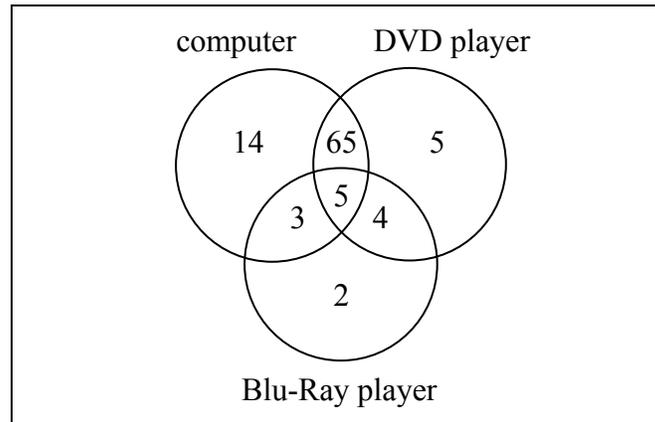
Given the statement: “If the temperature outside is below -40°C , then schools will be closed.”
Circle the contrapositive below.

- A) “If schools are closed, then the temperature outside is below -40°C .”
- B) “If schools are not closed, then the temperature outside is not below -40°C .”
- C) “If the temperature outside is not below -40°C , then schools will not be closed.”
- D) “Schools will be closed if and only if the temperature outside is below -40°C .”

Question No. 22

Total: 2 marks

A sample of 100 families was surveyed regarding the electronic devices they have in their homes. The Venn diagram below shows the number of families that have a computer, a DVD player, or a Blu-Ray player.



134

a) How many families have all three electronic devices in their homes?

(1 mark)

135

b) How many families do not have any of these electronic devices in their homes?

(1 mark)

Question No. 23

Total: 2 marks

Marc wrote the statement: “An isosceles triangle is equilateral.”

- a) Rewrite the statement in “if-then” form.

(1 mark)

136

- b) Provide a counter-example to show that the “if-then” statement in (a) is false.

(1 mark)

137

Question No. 24

Total: 2 marks

The following students attend the same school and participate in the extracurricular activities as indicated below.

The basketball team consists of:

$$B = \{\text{Jacquie, Lisa, Mangu, Maya, Nora, Sabrina}\}$$

The student tutoring group consists of:

$$T = \{\text{Jacquie, Mangu, Paul, Sabrina, Sam, Simon}\}$$

The volleyball team consists of:

$$V = \{\text{Nick, Paul, Pieter, Quinton, Sam, Simon}\}$$

a) Identify the two sets from above that are disjoint.

(1 mark)

138

b) Determine $B \cap T$.

(1 mark)

139

END OF TEST

**NO MARKS WILL BE AWARDED
FOR WORK DONE ON THIS PAGE.**

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