GRADE 12 APPLIED MATHEMATICS
ACHIEVEMENT TEST

DESCRIPTION:

Total Possible Marks: 60

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<th>Description</th>
<th>Marks</th>
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<td>A</td>
<td>Relations and Functions</td>
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<td>B</td>
<td>Probability</td>
<td>14</td>
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<td>C</td>
<td>Financial Mathematics</td>
<td>15</td>
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<td>D</td>
<td>Design and Measurement</td>
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<td>E</td>
<td>Logical Reasoning</td>
<td>7</td>
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Time: 3 hours
### Relations and Functions

- \( y = ax^2 + bx + c \)
- \( y = ax^3 + bx^2 + cx + d \)
- \( y = ab^x \)
- \( y = a + b \ln(x) \)
- \( y = a \sin(bx + c) + d \)
- \( y = a \cos(bx + c) + d \)

\[ \text{Period} = \frac{2\pi}{b} \text{ or } \frac{1}{\text{Frequency}} \]

### Financial Mathematics

- \( I = Prt \)
- \( A = P \left(1 + \frac{r}{n}\right)^n \)

\[ \text{Debt to equity ratio} = \frac{(\text{Total liabilities} - \text{Mortgage})}{\text{Net worth}} \times 100 \]

\[ \text{Gross debt service ratio} = \frac{\left(Monthly \text{ mortgage} + \text{property} + \text{heating payment} \right)}{\left(\text{taxes costs} \right) \times 100} \]

\[ \text{Average rate of return} = \frac{\left(\text{Current value} - \text{Previous value of portfolio} \right)}{\text{Previous value of portfolio}} \times 100 \]

### Probability

- \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \)
- \( P(A \text{ and } B) = P(A) \times P(B) \)

\[ n^P_r = \frac{n!}{(n-r)!} \]

\[ n^C_r = \frac{n!}{r!(n-r)!} \]

### Design and Measurement

**Prism:**
- Surface area: \( Ph + 2B \)
- Volume: \( Bh \)

**Pyramid:**
- Surface area: \( B + \frac{Pl}{2} \) \((l = \text{slant height})\)
- Volume: \( \frac{Bh}{3} \)

**Sphere:**
- Surface area: \( 4\pi r^2 \)
- Volume: \( \frac{4}{3} \pi r^3 \)

**Cylinder:**
- Surface area: \( 2\pi rh + 2\pi r^2 \)
- Volume: \( \pi r^2 h \)
TEST RESOURCES AND DIRECTIONS:

- You may consult your 8½” × 11” study sheet during the test.
- You may use a ruler, a graphing calculator, and computer software. You may also have access to the Internet for tools such as applets or a mortgage payment calculator. Use of the Internet to access course notes, find definitions, or search for conceptual information about the course is prohibited during the test.
- For short-answer and long-answer questions, you may print out diagrams from the computer or your calculator where applicable. Indicate your booklet number and the question number on the printouts. Remain seated and your teacher will distribute these printouts to you. Indicate in the response space of the question that the answer is on a printed sheet and staple it to the page.
- If you need more space to answer a question, extra paper may be provided by your teacher. Write your booklet number and the question number on any extra paper used and staple it into the booklet where your answer begins. Indicate in the response space of the question that the answer is on a separate sheet.
- Provide clear explanations or justifications where applicable. This can be done through labelled diagrams, in words, by showing mathematical operations to verify your answer, or by referring to a calculator or software program.
  — If you refer to a calculator program, indicate all your input values.
  — If you refer to a software program or a website, indicate all your input values and print or copy the screen showing the answers.
  — If you refer to a spreadsheet, print a copy of the answers.
- A graphic organizer is a visual representation of information. Examples include a tree diagram, a chart, a list, a Venn diagram, a truth table, Pascal’s triangle, etc.
- Round your final answers to two decimal places unless otherwise indicated.
- An answer without any work shown will be considered incomplete.
- Always state your assumptions.

The following errors may result in a 0.5 mark deduction:

- not including one of the following in the equation: “y =”, “sin”, “ln”, or “x”, or writing parameters separately from the equation
- not including the units in the final answer
- not including one of the following on the graph: labels for the axes, units for the axes, or scales for the axes
- not stating or incorrectly stating the final answer
- rounding too soon or rounding incorrectly
- not using whole units appropriately
RELATIONS AND FUNCTIONS

Question No. 1

Circle the graph below which best represents a cubic function.

A)  

B)  

C)  

D)  

Total: 1 mark
Question No. 2  

Given the following function which represents the change in a town’s population with respect to time:

\[ y = 1000(1.05)^x \]

Referring to the town, explain the meaning of:

a) “1000”  

(1 mark)

b) “1.05”  

(1 mark)
A football player wants to kick a football so it will go over a crossbar that is 35 yards away and 3.33 yards high. (Diagram is not drawn to scale.)

The horizontal distance \((d, \text{ in yards})\) and the height \((h, \text{ in yards})\) that the football travels are represented by the following equation:

\[
h = -0.04d^2 + 1.51d
\]

How far above or below the crossbar will the football travel? Show your work.
The mass of a steel ball varies with respect to its diameter.

<table>
<thead>
<tr>
<th>diameter (mm)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass (g)</td>
<td>0</td>
<td>2</td>
<td>10</td>
<td>32</td>
<td>80</td>
</tr>
</tbody>
</table>

Determine the cubic equation that best represents the data. Sketch a clearly labelled graph of the equation.

cubic equation: __________________________________________________________
A mass is suspended by a spring and is in a resting position 0.50 metres above a table. 

The mass is pulled down 0.40 metres and is then released. The following information is obtained:

- It takes 1.20 seconds for the mass to return to its lowest position.
- The mass reaches a maximum height of 0.90 metres.

a) Determine the sinusoidal equation that best represents the distance of the mass with respect to the table as a function of time since it was released. Show your work. 

(2 marks)

b) When will the mass be 0.75 metres above the table for the first time?

(1 mark)
Question No. 6  

It becomes easier and easier to see the headlights of an oncoming car the closer that it gets. The distance \((d, \text{ in metres})\) between the car and an observer can be described as a function of the intensity \((I, \text{ in lumens})\) of the headlight brightness:

\[
d = 350 - 72 \ln(I)
\]

a) Sketch a clearly labelled graph of the equation.

(2 marks)
b) Determine the distance to an oncoming car if the intensity of its headlights is 75 lumens. 

(1 mark)

c) What is the maximum intensity of the headlights? Justify your answer. 

(2 marks)
PROBABILITY

Question No. 7  
Total: 1 mark

Licence plates in Ontario contain 4 upper case letters followed by 3 digits, with repetition allowed. Circle the maximum possible number of licence plates that begin with the letters: MMBA, MANI, or BNTP.

A) 2160
B) 2880
C) 3000
D) 4000

Question No. 8  
Total: 1 mark

A bag contains 6 white marbles, 8 blue marbles, 2 yellow marbles, and 4 green marbles. What are the odds in favour of selecting a white marble?

A) 6 : 14
B) 6 : 20
C) 14 : 6
D) 20 : 6
A cookie jar contains 10 chocolate chip cookies, 12 double chocolate cookies, and 15 oatmeal cookies. Allison says that the odds against selecting a cookie with chocolate are 15 to 37. Ryan says that the odds against are 15 to 22. Who is correct? Explain your answer.
Describe a situation containing mutually exclusive events. Explain why the events are mutually exclusive.
An organization consisting of 15 women and 19 men must create a 10-person committee.

a) How many committees can be created that will include 4 women and 6 men? Show your work.
   
   (2 marks)

b) If a 10-person committee is randomly selected, what is the probability that the committee will include 4 women and 6 men? Show your work.
   
   (2 marks)
You have been asked to create a four-character password for your computer using:

- the 26 upper case letters of the alphabet (A, B, C, …)
- the 26 lower case letters of the alphabet (a, b, c, …)
- the digits from 0 to 9
- the symbols: ~ ! @ # $ % ^ & *

a) How many different four-character passwords are possible if any of the letters, digits, or symbols can be used for each character if repetition is allowed?

(1 mark)

b) How many different four-character passwords are possible if repetition is not allowed?

(1 mark)

c) How many four-character passwords begin with a letter and end with a digit if repetition is allowed?

(1 mark)
d) How many different four-character passwords containing at least one symbol are possible if repetition is allowed? Show your work.

(2 marks)
FINANCIAL MATHEMATICS

Question No. 13  Total: 1 mark

Brigitte invests $5000.00 at an interest rate of 6% for 5 years. Circle the compounding period below that would maximize the rate of return on the investment.

A) daily

B) monthly

C) quarterly

D) semi-annually

Question No. 14  Total: 1 mark

Circle the asset below which is most likely to depreciate in value.

A) rare coin collection

B) classic car

C) house

D) computer
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Mr. Chang is 64 and plans to retire next year. His portfolio includes the following investments:

- $50 000.00 in mutual funds
- $100 000.00 in stocks
- $20 000.00 in guaranteed investment certificates (GICs)

a) Does this portfolio include an appropriate level of risk for Mr. Chang at this stage in his life? Explain your answer.

*(1 mark)*
b) Mr. Chang’s investments had the following returns last year: mutual funds increased by 12.00%, stocks decreased by 4.00%, and GICs had an annual interest rate of 3.00%. Calculate the average rate of return for the year for this portfolio. Show your work.

(2 marks)

<table>
<thead>
<tr>
<th>Type of investment</th>
<th>Principal ($)</th>
<th>Return ($)</th>
<th>End of the year ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mutual funds</td>
<td>50 000.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>stocks</td>
<td>100 000.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GICs</td>
<td>20 000.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Amar dreams of retiring at the age of 55. He had planned on starting to save for his retirement at the age of 50, but his financial advisor does not agree. He recommends that Amar starts to save sooner.

a) If Amar invests $1000.00 on his 25th birthday and contributes $200.00 every month to an account that earns 8.00% compounded monthly, what will be the value of the investment on his 55th birthday? Show your work.

(2 marks)

b) If Amar invests $1000.00 on his 50th birthday, how much will he have to contribute every month to match the final value of the investment in (a)? Assume that the interest rate and the compounding periods are the same.

(1 mark)
c) Calculate the difference between Amar’s total contribution in (a) and in (b)? Show your work.

(2 marks)
Question No. 17

The Reimers have purchased a house valued at $250,000.00 and have made a down payment of $25,000.00.

a) Calculate their monthly mortgage payment if they obtain a mortgage amortized over 15 years at an interest rate of 5.50% compounded semi-annually. Show your work.

(2 marks)
b) How much equity will the Reimers have in their house after 5 years if the value of the house appreciates at a rate of 2.00% per year? Show your work.

(3 marks)
A cake mix will produce 230 cubic inches of batter. You are using cylinder-shaped baking cups that have a diameter of 3 inches and a depth of 2 inches for the batter. How many cupcakes will you be able to make? Show your work.
A goat is tied to the corner of a barn with a 50-foot rope. The barn measures 60 feet by 40 feet. Calculate the total area outside of the barn that is available to the goat. Show your work.
The Manitoba Beach Volleyball Association has asked you to design a souvenir beach ball according to the following information:

- The beach ball must have a volume between 1 and 3 cubic feet.
- The plastic material costs $0.15 per ft².
- Labour and other materials cost $1.25 per beach ball.
- The Association wants to make a profit of 80% of the cost of making each beach ball.

Based on your design, what is the minimum selling price for each souvenir beach ball? Show your work.
Given the statement: “If the temperature outside is below –40°C, then schools will be closed.”

Circle the contrapositive below.

A) “If schools are closed, then the temperature outside is below –40°C.”

B) “If schools are not closed, then the temperature outside is not below –40°C.”

C) “If the temperature outside is not below –40°C, then schools will not be closed.”

D) “Schools will be closed if and only if the temperature outside is below –40°C.”
A sample of 100 families was surveyed regarding the electronic devices they have in their homes. The Venn diagram below shows the number of families that have a computer, a DVD player, or a Blu-Ray player.

![Venn Diagram]

a) How many families have all three electronic devices in their homes?

(1 mark)

b) How many families do not have any of these electronic devices in their homes?

(1 mark)
Marc wrote the statement: “An isosceles triangle is equilateral.”

a) Rewrite the statement in “if-then” form.

(1 mark)

b) Provide a counter-example to show that the “if-then” statement in (a) is false.

(1 mark)
The following students attend the same school and participate in the extracurricular activities as indicated below.

The basketball team consists of:
\[ B = \{ \text{Jacquie, Lisa, Mangu, Maya, Nora, Sabrina} \} \]

The student tutoring group consists of:
\[ T = \{ \text{Jacquie, Mangu, Paul, Sabrina, Sam, Simon} \} \]

The volleyball team consists of:
\[ V = \{ \text{Nick, Paul, Pieter, Quinton, Sam, Simon} \} \]

a) Identify the two sets from above that are disjoint.

(1 mark)

b) Determine \( B \cap T \).

(1 mark)
NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.
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