Grade 12
Applied Mathematics
Achievement Test

Student Booklet

January 2019

Manitoba

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While the department is committed to making its publications as accessible as
possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

GRADE 12 APPLIED MATHEMATICS
ACHIEVEMENT TEST

DESCRIPTION

Total Possible Marks: 64   **Time: 3 hours**

Unit	Marks
Relations and Functions	14
Probability	17
Financial Mathematics	16
Design and Measurement	9
Logical Reasoning	8

DIRECTIONS

Remember to

- indicate your input values by writing them in your booklet or printing a copy if using a technology tool

- state any assumptions you make

- express your answers in decimal and percentage form to at least the nearest hundredth (two decimal places) when rounding, except for monetary values or when otherwise indicated

   Example: $\dfrac{15}{29} = 0.52$ or 51.72%

Note: Rounding too soon in your solution may result in an inaccurate final answer for which full marks will not be awarded.

A "graphic organizer" is a visual representation of information. Examples include a tree diagram, a chart, a list, a Venn diagram, a truth table, Pascal's triangle, etc.

A clearly communicated answer

— is easily identified in the response space

— includes the parameters in the equation, and "$y =$", "sin", "ln", or "x", as applicable

— includes the units of measure, where applicable

— includes labels, units, scales for the axes on graphs, and key characteristics of functions (e.g., maximum, minimum, intercepts, and appropriate shape)

— is expressed as an exact value or is appropriately rounded

Marks may be deducted for errors relating to any of the above.

**Electronic communication between students through phones,
email, or file sharing during the test is strictly prohibited.**

RELATIONS AND FUNCTIONS

Question 1   **Total: 1 mark**

Select the best answer.

Which of the following types of functions could have a range of $\{y \mid -2 \le y \le 12,\ y \in \mathbb{R}\}$?

A) exponential

B) sinusoidal

C) cubic

D) logarithmic

Question 2   Total: 1 mark

Select the best answer.

Which of the following functions has a domain of $(0, \infty)$?

A) $y = 3^x$

B) $y = -3x^2 + 18x - 18$

C) $y = 3x^3 + 2x^2 + x$

D) $y = 3\ln x$

Question 3 **Total: 1 mark**

The power generated by a wind turbine can be modelled by the equation:

$$P = 2.06w^3 + 0.56w^2 - 3.38w + 5$$

where P represents the power in kilowatts (kW)
and w represents the wind speed in metres per second (m/s).

The wind turbine starts to generate electricity when the power is 5000 kW or greater.

What wind speed is required for the turbine to start generating electricity?

Question 4   Total: 4 marks

One cup of coffee contains 94 mg of caffeine. Once consumed, the amount of caffeine in the human body is reduced by 16% every hour.

a) Determine the exponential equation that models the remaining amount of caffeine in the body as a function of time in hours. Show your work.

(2 marks)

b) Using your equation in (a), determine the remaining amount of caffeine in the body 19 hours after consuming one cup of coffee.

(1 mark)

c) If one cup of espresso coffee, containing 125 mg of caffeine, was consumed instead of the cup of coffee, describe how your equation in (a) would change.
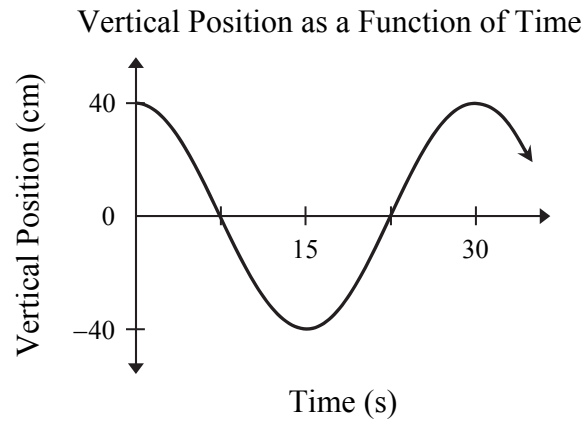
(1 mark)

Question 5   Total: 3 marks

A bridge is designed to move up and down with the wind. The graph below represents the vertical position of a point at the top of the bridge on a windy day.

Vertical Position as a Function of Time

107

a)   Determine a regression equation that represents this situation. Show your work.

(2 marks)

Time (s)					
Vertical Position (cm)					

108

b)   Determine how many times the point at the top of the bridge is at its maximum height during the first 60 seconds.

(1 mark)

During a science class experiment, Karlie stands on a ladder and throws a ball up in the air. Her partner records the time from when the ball leaves her hand until it hits the ground. She determines that the equation representing the height of the ball as a function of time is:

$$h(t) = -4.9t^2 + 10t + 4$$

where h represents the height in metres
and t represents the time in seconds.

109

a)   Create a clearly labelled graph of the equation given the context of this question.

(3 marks)

b) Determine the amount of time the ball is 4 metres or higher from the ground during this experiment.

(1 mark)

PROBABILITY

Question 7   Total: 1 mark

Select the best answer.

A building has 8 doors that can be used to either enter or exit.

How many ways can Cindy enter and exit the building if she must enter through one door and exit through another?

A) 15

B) 16

C) 56

D) 64

Question 8 — Total: 2 marks

There are 2 principals and 5 teachers lining up in a row for a photograph that will appear in the local paper.

Determine the number of ways they can line up in a row if a principal must stand on each end. Show your work.

Question 9   Total: 3 marks

The probability that Brian participates in a study group before his test is 0.70. If he participates in a study group, the probability that he will get an A on his test is 0.80. If he does not participate in a study group, the probability that he will get an A on his test is 0.40.

113

a) Use a graphic organizer to show all possible outcomes for this situation.

(1 mark)

114

b) Determine the probability that Brian will get an A on his test. Show your work.

(2 marks)

Question 10   **Total: 2 marks**

Each housing area in Manitoba is assigned a postal code.

- Each postal code is made up of 3 letters and 3 digits that alternate.
- The postal code must begin with the letter R.
- The letters D, F, I, O, Q, U are not used.
- Repetition is allowed.

Determine the number of postal codes that can be created in Manitoba. Show your work.

Question 11   Total: 3 marks

From his home, Rasik has to take either a bus or a train downtown and then take a different bus or train to work. He has the following transportation options:

- 5 different bus routes or 3 different train routes from home to downtown
- 4 different bus routes or 2 different train routes from downtown to work
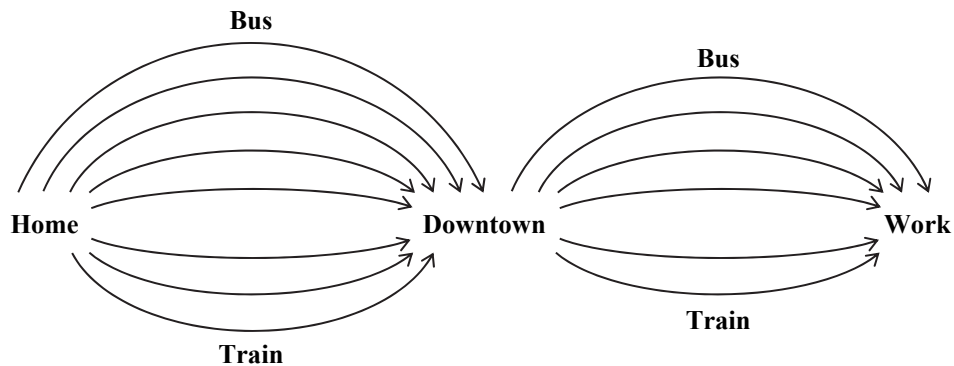
a) Determine the number of routes Rasik can take from home to work.

(1 mark)

b) Determine the number of routes Rasik can take from home to work by only taking the bus.

(1 mark)

c) If Rasik randomly chooses his routes, what are the odds that he takes only a bus from home to work?

(1 mark)

Question 12   **Total: 2 marks**

At an amusement park, there are 11 adults and 17 teenagers wanting to go on a boat ride. The boat has a 9-passenger capacity.

If there are at most 2 adult passengers, determine the number of ways 9 passengers can be chosen to go on the boat ride. Show your work.

Question 13   **Total: 4 marks**

Of the 41 students in Grade 12,

- 21 students are in drama club
- 17 students are in chess club
- 14 students are in environmental club
- 8 students are in chess club and drama club
- 10 students are in drama club and environmental club
- 3 students are in chess club and environmental club only
- 2 students are in all three clubs

120

a) Draw a Venn diagram to represent this situation.

(3 marks)

121

b) What is the probability that a randomly selected student is not in any of these clubs?

(1 mark)

FINANCIAL MATHEMATICS

Question 14 Total: 1 mark

Select the best answer.

Eric wants to borrow money from his bank.

Which of the following compounding periods results in the least amount of interest paid on the loan?

A) monthly

B) semi-annually

C) quarterly

D) daily

123

Jeannette's new job involves a lot of driving. She wants to lease a new vehicle. Her friend Simon is trying to convince her not to lease.

State two reasons that Simon could use to convince her not to lease.

Question 16   Total: 5 marks

Himesh wants to retire in 35 years. After meeting with his financial advisor, he determines that his portfolio will consist of the following two investments:

Investment 1: $15 000.00 in a mutual fund that earns an interest rate of 6.50%, compounded monthly.

Investment 2: regular biweekly deposits of $180.00 in a tax-free savings account (TFSA), with an interest rate of 3.75%, compounded biweekly. (Assume the initial value of the TFSA is 0.)

124

a) What will be the value of each investment when Himesh retires? Show your work.

(4 marks)

125

b) Himesh's goal is to have $500 000.00 in his portfolio by the time he retires. Determine if he will meet his goal. Justify your answer by using your investment values in (a).

(1 mark)

Question 17   Total: 5 marks

Kaia purchased a house for $400 000.00. At the time of purchase, she made a down payment of $100 000.00. The remaining balance of the mortgage was financed at an interest rate of 4.30%, compounded semi-annually, over 25 years.

a) How much is Kaia's monthly mortgage payment? Show your work.

(2 marks)

b) Kaia's house appreciates in value at an average rate of 2.00% per year. What will the value of her house be after 15 years?

(1 mark)

c) How much equity will Kaia have in the house after 15 years? Show your work.

(2 marks)

Question 18   Total: 3 marks

Dimitri owns a house with a mortgage of $175 000.00. He has $165 000.00 in other loans and his net worth is $300 000.00.

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a)   Determine his debt-to-equity ratio.

(1 mark)

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b)   Determine his total assets.

(1 mark)

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c)   Dimitri wants to borrow $10 000.00 to take his family on a vacation. Using his debt-to-equity ratio in (a), explain if the bank will lend him the money.

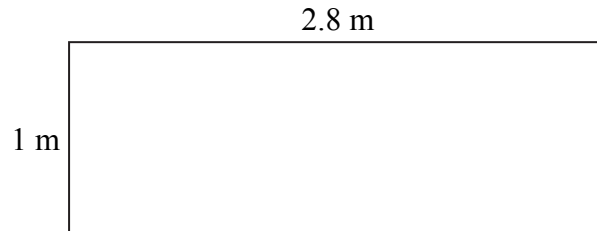*(1 mark)*

DESIGN AND MEASUREMENT

Question 19   **Total: 1 mark**

Select the best answer.

Maala needs to shovel a 1 m × 2.8 m walkway after a 30 cm snowfall. The maximum amount of snow each full shovel can remove is 0.04 m³ per scoop.

2.8 m

1 m

Assuming each scoop contains the maximum amount of snow, how many scoops will it take for her to remove all the snow from the walkway?

A) 21

B) 70

C) 336

D) 2100

Question 20   **Total: 2 marks**

Hugo creates a scale model of a planet for art class. He wants to paint it.

- The radius of the model is 20 cm.
- He has one bottle of paint that will cover 12 000 cm^2.
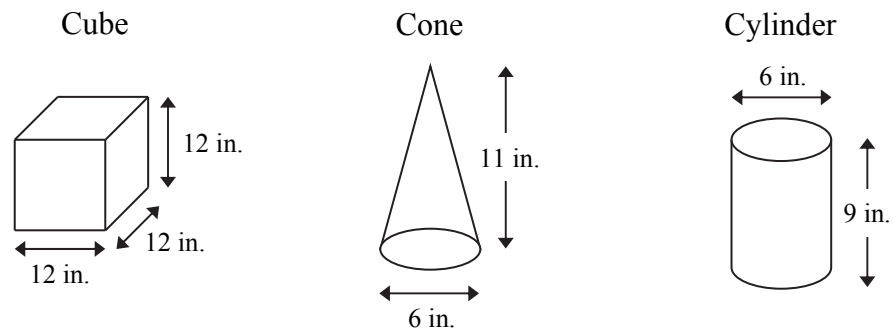- It is assumed that each coat applied requires the same amount of paint.

How many full coats of paint can be applied using the one bottle? Show your work.

You are entered in a sandcastle building contest. Pails in the following shapes are available for use:

Cube   Cone   Cylinder

- Cube: 12 in. (height), 12 in., 12 in.
- Cone: 11 in. (height), 6 in. (diameter)
- Cylinder: 6 in. (diameter), 9 in. (height)

Shapes are not drawn to scale.

Your sandcastle design must:

- be built on a 2 ft. × 1 ft. area
- use each shape at least once
- include a minimum of 5 shapes
- be no more than 3 shapes high

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a) Draw or describe a sandcastle that meets the above criteria.

(2 marks)

b)   Calculate the volume of sand required, in cubic feet, to build your sandcastle.
Show your work.

(3 marks)

c)   If the organizers of the contest purchased sand for $1.11/ft^3$, what is the cost of the sand needed to build your design? Assume whole units do not need to be used since the sand has already been purchased.

(1 mark)

LOGICAL REASONING

Question 22   **Total: 2 marks**

An online app awards 0 points, 3 points, or 7 points for performing tasks. Once your score adds up to 25 points or more, you pass the level.

State the scores that are not possible at any time during any level. Show your work using the list of numbers below.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Scores that are not possible: _____

Question 23   Total: 2 marks

The universal set of living things includes:

- the set of all birds (B)
- the set of all plants (P)
- the set of all owls (O)

- the set of all dogs (D)
- the set of all mammals (M)
- the set of all roses (R)

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a) Give one example of two disjoint subsets.

(1 mark)

139

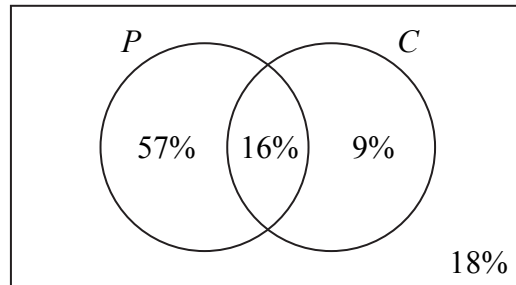b) Using set notation, give one example of a set that is a subset of another.

(1 mark)

Question 24   Total: 2 marks

A movie theatre employee tracked the number of people who purchased popcorn (P) and chocolate (C) in one month. The results are given below.

a)   What percentage of people purchased popcorn?

(1 mark)

b)   Describe what $(P \cup C)'$ represents in this situation.

(1 mark)

Question 25   **Total: 2 marks**

Given the following conditional statement:

> "If you live in Manitoba,
> then you live in Canada."

142

a) Identify what type of statement is logically equivalent to the above conditional statement.

(1 mark)

Select the best answer.

    A) converse

    B) inverse

    C) contrapositive

143

b) Write the statement you identified in (a).

(1 mark)

END OF TEST

**NO MARKS WILL BE AWARDED
FOR WORK DONE ON THIS PAGE.**

Formula Sheet: Applied Mathematics

RELATIONS AND FUNCTIONS	FINANCIAL MATHEMATICS
$y = ax + b$ $y = ax^2 + bx + c$ $y = ax^3 + bx^2 + cx + d$ $y = ab^x$ $y = a + b\ln(x)$ $y = a\sin(bx + c) + d$	$t = \dfrac{72}{i}$ $I = Prt$ $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$ Net worth = Total assets − Total liabilities

PROBABILITY

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A \cap B) = P(A) \times P(B|A)$$

$$_nP_r = \frac{n!}{(n-r)!}$$

$$_nC_r = \frac{n!}{r!(n-r)!}$$

$$\text{Debt-to-equity ratio (\%)} = \frac{(\text{Total liabilities} - \text{Mortgage})}{\text{Net worth}} \times 100$$

$$\text{Gross debt service ratio (\%)} = \frac{\left(\begin{array}{ccc}\text{Monthly} & \text{Monthly} & \text{Monthly}\\ \text{mortgage} + \text{property} + \text{heating}\\ \text{payment} & \text{taxes} & \text{costs}\end{array}\right)}{\text{Gross monthly income}} \times 100$$

$$\text{Rate of return (\%)} = \frac{\left(\begin{array}{cc}\text{Current value} & \text{Previous value}\\ \text{of portfolio} & \text{of portfolio}\end{array}\right)}{\text{Previous value of portfolio}} \times 100$$

DESIGN AND MEASUREMENT

Prism: Surface area $= Ph + 2B$

    Volume $= Bh$

Pyramid: Surface area $= B + \dfrac{1}{2}Ps$

    Volume $= \dfrac{1}{3}Bh$

Cube: Surface area $= 6l^2$

    Volume $= l^3$

Rectangular prism: Surface area $= 2lw + 2lh + 2wh$

    Volume $= lwh$

Triangular prism: Surface area $= bh + l(a + b + h)$

    Volume $= \dfrac{1}{2}bhl$

Square-based pyramid: Surface area $= b^2 + 2bs$

    Volume $= \dfrac{1}{3}b^2h$

Sphere: Surface area $= 4\pi r^2$

    Volume $= \dfrac{4}{3}\pi r^3$

Cylinder: Surface area $= 2\pi r^2 + 2\pi rh$

    Volume $= \pi r^2 h$

Cone: Surface area $= \pi r^2 + \pi rs$

    Volume $= \dfrac{1}{3}\pi r^2 h$