Grade 12
Applied Mathematics
Achievement Test

Student Booklet

January 2017

Manitoba

After the administration of this test, print copies of this resource will be
available for purchase from the Manitoba Learning Resource Centre
(formerly the Manitoba Text Book Bureau). Order online at
www.mtbb.mb.ca.

This resource will also be available on the Manitoba Education and
Training website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

While the department is committed to making its publications as accessible as
possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

GRADE 12 APPLIED MATHEMATICS
ACHIEVEMENT TEST

DESCRIPTION

Total Possible Marks: 61   **Time: 3 hours**

Unit	Description	Marks
A	Relations and Functions	15
B	Probability	15
C	Financial Mathematics	17
D	Design and Measurement	9
E	Logical Reasoning	5

RESOURCES

You may use the following resources:

- *Formula Sheet* **(tear-out page at the back of this booklet)**

- one 8½″ × 11″ study sheet

- ruler

- graphing calculator, computer software, and/or app

- Internet access for tools such as applets or mortgage payment calculators

   Use of the Internet to communicate or access other content, including but not limited to course notes, definitions, or conceptual information is not permitted during the test.

> **Electronic communication between students through phones, email, or file sharing during the test is strictly prohibited.**

DIRECTIONS

Read all instructions on the test carefully.

If you need extra paper or you print out an answer to a question, let the teacher know. Indicate in the response space of the question that your answer is on a separate sheet.

Remember to

- indicate your input values by writing them in your booklet or printing a copy if using a technology tool

- include your booklet number and question number on additional pages (e.g., printouts) and attach them to the corresponding page in the booklet

- express your answers in decimal and percentage form to the nearest hundredth (two decimal places) when rounding, unless otherwise indicated

 Example: $\dfrac{15}{29}$ = 0.52 or 51.72%

- state any assumptions you make

A "graphic organizer" is a visual representation of information. Examples include a tree diagram, a chart, a list, a Venn diagram, a truth table, Pascal's triangle, etc.

A clearly communicated answer

— is easily identified in the response space

— includes the parameters in the equation, and "$y =$", "sin", "ln", or "x", as applicable

— includes the units of measure, where applicable

— includes labels, units, and scales for the axes on graphs

— is expressed as an exact value or is appropriately rounded

Marks may be deducted for errors relating to any of the above.

STOP
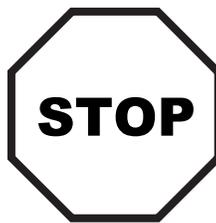
PLEASE WAIT UNTIL INSTRUCTED TO TURN THE PAGE.
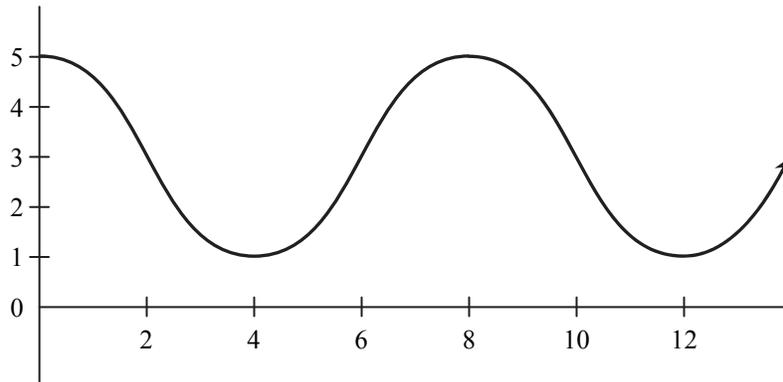
RELATIONS AND FUNCTIONS

Question 1   **Total: 1 mark**

Use the graph below to answer the following question and select the best answer.   101

The median of the graph is

A. 2

B. 3

C. 5

D. 8

Question 2   Total: 2 marks

Yang kicks a soccer ball off the ground. The height of the ball is tracked over time. The ball reaches a maximum height of 20 m at 2.1 seconds.

State the domain and range of the quadratic function that models the height of the soccer ball from when it is kicked until it hits the ground.
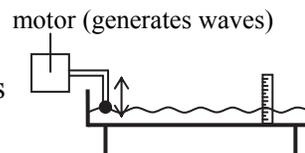
Domain: _____

Range: _____

Question 3   Total: 4 marks

Stephanie experiments with the frequency of waves in a ripple tank during her Physics class. At the beginning of the experiment, she places a ruler into the tank and measures the median height of the water to be 10 cm. The water then rises to a maximum height of 13 cm at 1 second.

motor (generates waves)

103

a) Determine the sinusoidal regression equation that models the relationship between the height of the water and time. Show your work.

(2 marks)

104

b) Determine the height of the water at 15 seconds.

(1 mark)

105

c) Stephanie changes the settings of the motor to generate waves more quickly. Write an equation that can model this change if all other conditions remain the same.

(1 mark)

Question 4   Total: 2 marks

The average price of an electronic device is based on its memory capacity. The following equation models this relationship:

$$P = -24.22 + 15.15 \ln c$$

where P represents the average price in dollars
and c represents the memory capacity in gigabytes (GB).

106

a)   What is the average price of a 256 GB device?

(1 mark)

107

b)   Provide one limitation of the equation that models this relationship.

(1 mark)

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Question 5   Total: 6 marks

Pam drives her car over a nail. As a result, one of the tires on her car starts to lose air pressure. Her tire pressure sensor provides the following data:

t (h)	0	1	2	3	6	10
P (psi)	40	32	26	21	10	5

where P represents the air pressure (in psi)
and t represents the time (in hours).

108

a)   Create a clearly labelled graph by plotting the given data. Draw a curve of best fit.

(3 marks)

109

b)   Determine the regression equation that best models the data in this situation.

(1 mark)

c)   The car becomes unsafe to drive when the tire pressure is 14 psi or under. How many hours can Pam safely drive her car after driving over the nail?

Express your final answer to the nearest hundredth (two decimal places). Show your work.

(2 marks)

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PROBABILITY

Question 6   Total: 1 mark

Select the best answer.

Which of the following expressions represents the number of ways to create a 4-digit passcode for your phone using the digits 0 to 9, if repetition is allowed?

A. $10 \times 9 \times 8 \times 7$

B. 10^4

C. $_{10}C_4$

D. $_{10}P_4$

Question 7   Total: 1 mark

Select the best answer.

Scott can choose from 8 toppings to make a pizza. How many pizzas can be made with 3 different toppings?

A. 6

B. 56

C. 336

D. 6720

Question 8   **Total: 3 marks**

Joseph has 20 cards; 4 cards of each of the following colours: red, blue, yellow, green, and purple.

113

a)   What is the probability of randomly drawing 2 red cards in a row if the first card is replaced before drawing the second card?

(1 mark)

114

b)   What is the probability of randomly drawing 2 red cards in a row if the first card is not replaced before drawing the second card?

(1 mark)

c) Explain which part, (a) or (b), is an example of dependent events.

(1 mark)

116

If 4 coins are tossed at the same time, what is the probability that they will land as either all heads or as all tails? Show your work.

Question 10   Total: 3 marks

Last year, it was reported that 50.30% of Mathmatica's population was 40 years of age or older.

That same year, 65.74% of people 40 years of age or older saw a doctor and 60.09% of people younger than 40 years of age saw a doctor.

If Mathmatica's population last year was 1 265 400, determine how many people in Mathmatica did not see a doctor. Show your work.

Question 11   Total: 2 marks

The probability that Louise will go out for dinner tonight is 0.4. The probability that she will watch a movie is 0.7. The probability she will do neither is 0.2.

118

a)   Draw a Venn diagram to represent this situation.

(1 mark)

119

b)   Determine the probability that Louise does only one of these activities.

(1 mark)

Question 12   Total: 3 marks

A coach randomly selects 5 players from a team of 18 to line up for a shot on goal.

120

a)   How many different 5-player arrangements are possible?

(1 mark)

121

b)   If Dustin and Andrew are 2 of the 18 players, what is the probability that Dustin will shoot first and Andrew will shoot second? Show your work. Express your answer as a fraction or round to the nearest thousandth (three decimal places).
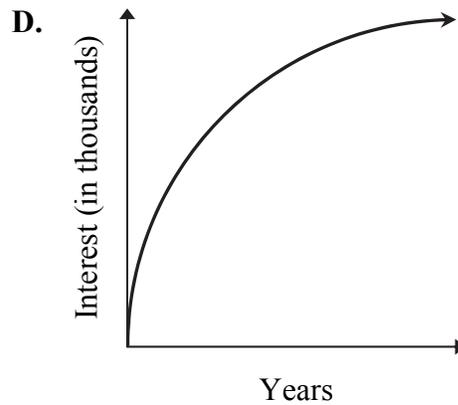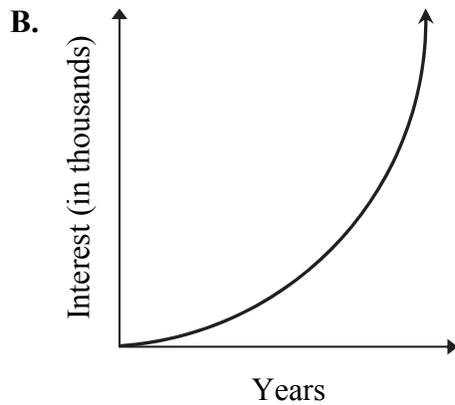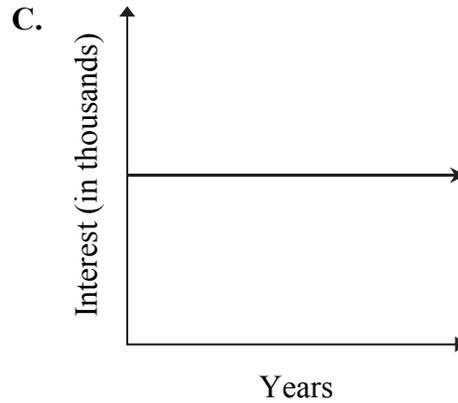
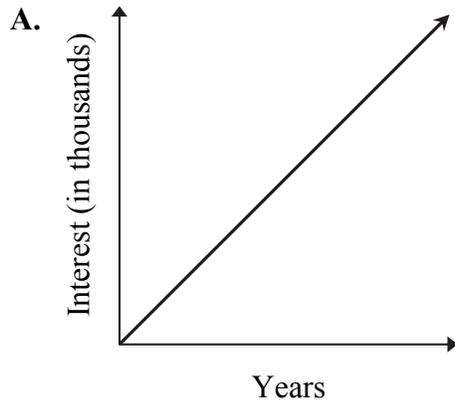*(2 marks)*

FINANCIAL MATHEMATICS

Question 13   **Total: 1 mark**

Select the best answer.

Which of the following graphs shows the total amount of interest paid over the course of a 25-year mortgage?

A.

C.

B.

D.

Question 14   **Total: 1 mark**

Select the best answer.

According to the Rule of 72, a reasonable estimate for the time it would take to double an investment of $24 000.00 at an interest rate of 6.00%, compounded monthly is

A. 3 years

B. 4 years

C. 12 years

D. 18 years

124

Imani is going to buy a car. She can afford monthly payments of $600.00. The dealer offers two financing options:

Option 1: financing over 60 months at a rate of 0.90% compounded monthly

Option 2: financing over 60 months at a rate of 2.90% compounded monthly with an instant rebate of $3000.00 at the time of purchase

Which option allows Imani to purchase a more expensive car? Show your work.

Question 16   **Total: 2 marks**

Your friend has $10 000.00 and is considering an investment in stocks, a guaranteed investment certificate (GIC), or rare collectibles.

Choose one of the three investments mentioned above and indicate one advantage and one disadvantage for your choice.

Question 17   Total: 4 marks

When she turned 25, Alexa began investing $400.00 monthly into a mutual fund account producing average returns of 6.00%, compounded monthly. Alexa will stop contributing when she retires at age 55.

126

a) How much money will her investment be worth at retirement? Show your work.

(2 marks)

127

b) Alexa will withdraw $2500.00 per month from her account after retiring. If the average return rate stays the same, how old will she be when the account balance is zero? Show your work.

(2 marks)

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Shirley and Cameron have just moved to Brandon and are considering the two following housing options:

Option 1: House for purchase

- purchase price of $249 000.00
- down payment of $50 000.00 required
- 25-year mortgage at an interest rate of 3.00%, compounded semi-annually

Option 2: Apartment for rent

- monthly payments of $1300.00
- monthly parking fees of $60.00

128

a) What would be Shirley and Cameron's monthly mortgage payment with Option 1? Show your work.

(2 marks)

b) What will be the total amount paid for each option at the end of 10 years?

(2 marks)

c) State one advantage of renting the apartment.

(1 mark)

Question 19   **Total: 2 marks**

Eleni is placing cups onto a 7 in. by 10 in. tray. Each cup has a circumference of 10 in.

Determine the maximum number of cups she can place on the tray. Show your work.

Question 20   **Total: 4 marks**

You want to build a garden.

- The garden is square.
- The side length is between 8 ft. and 10 ft.
- The garden is enclosed using boards stacked two boards high.
- There are 12 metal supports used to connect the structure.
- The garden is filled with soil to a depth of 15 in.

(Diagram is not drawn to scale.)

Below is the price list for materials:

Item	Cost (plus GST and PST)
metal supports	$2.00 each
boards	$2.50/linear foot
soil	$12.00/yd^3

side length: _____ ft.

15 in.
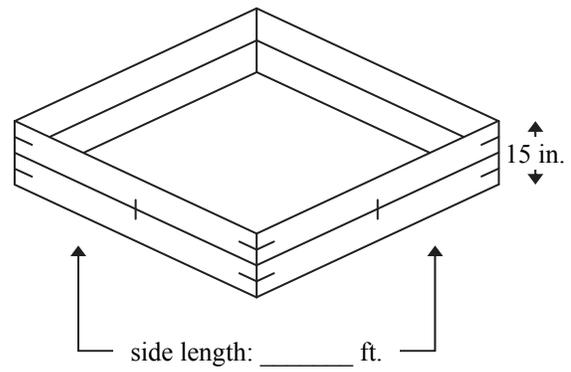
Determine the total cost of building your garden. All items must be purchased in whole units. Show your work. (Note: GST = 5%, PST = 8%)
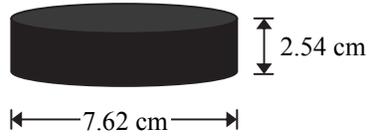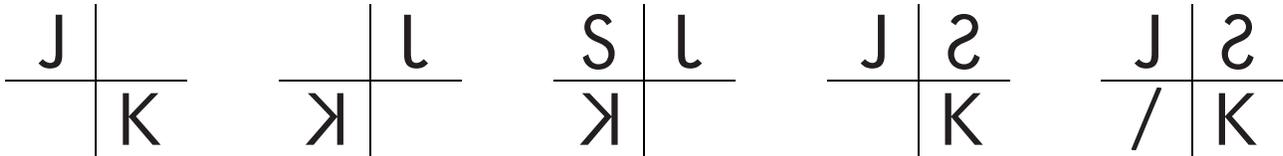
Question 21   Total: 3 marks

133

a)   What is the volume of a rubber hockey puck? (Diagram is not drawn to scale.)

(1 mark)

2.54 cm

7.62 cm

134

b)   How many pucks can be produced for $1000.00 if it costs $0.24 to print a logo on each puck and rubber costs $0.003 6 per cm^3?
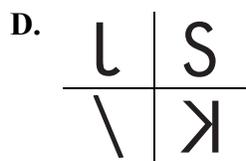
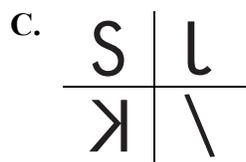*(2 marks)*

Question 22   **Total: 1 mark**

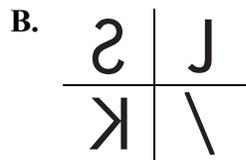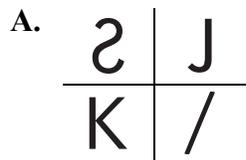*Select the best answer.*

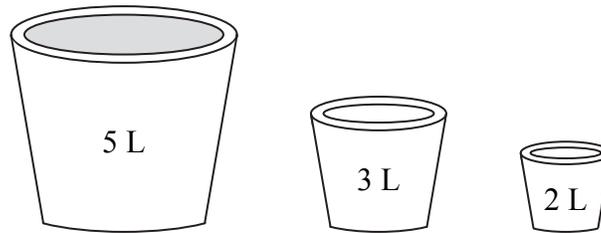Given the following pattern:

Which of the following continues this pattern?

A.

B.

C.

D.

You have 3 pails with volumes of 2 L, 3 L, and 5 L.

The 5 L pail is filled with water. The other pails are empty.

Using only these pails, explain how to measure out exactly 4 L of water into the 5 L pail.

Question 24   Total: 2 marks

Consider the original statement:

"If students are in Grade 12, then they will graduate this June."

a) Write the converse of the statement.

(1 mark)

b) Determine if a biconditional statement is possible. If yes, write the biconditional statement. If not, provide a counterexample.

(1 mark)

END OF TEST

**NO MARKS WILL BE AWARDED
FOR WORK DONE ON THIS PAGE.**

Formula Sheet: Applied Mathematics

Relations and Functions	Financial Mathematics
$y = ax + b$	$t = \dfrac{72}{i}$
$y = ax^2 + bx + c$	$I = Prt$
$y = ax^3 + bx^2 + cx + d$	$A = P\left(1 + \dfrac{r}{n}\right)^{nt}$
$y = ab^x$	Net worth = Total assets − Total liabilities
$y = a + b\ln(x)$	$\dfrac{\text{Debt-to-equity}}{\text{ratio (\%)}} = \dfrac{(\text{Total liabilities − Mortgage})}{\text{Net worth}} \times 100$
$y = a\log_b x$	
$y = a\sin(bx + c) + d$	$\dfrac{\text{Gross debt}}{\text{service}}{\text{ratio (\%)}} = \dfrac{\left(\begin{array}{ccc}\text{Monthly} & \text{Monthly} & \text{Monthly} \\ \text{mortgage} + \text{property} + \text{heating} \\ \text{payment} & \text{taxes} & \text{costs}\end{array}\right)}{\text{Gross monthly income}} \times 100$
$y = a\cos(bx + c) + d$	
	$\dfrac{\text{Rate of}}{\text{return (\%)}} = \dfrac{\left(\begin{array}{cc}\text{Current value} & \text{Previous value} \\ \text{of portfolio} & \text{of portfolio}\end{array}\right)}{\text{Previous value of portfolio}} \times 100$

Probability	Design and Measurement
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$	Prism: Surface area = $Ph + 2B$
$P(A \text{ and } B) = P(A) \times P(B)$	Volume = Bh
$P(A \text{ and } B) = P(A) \times P(B\mid A)$	Pyramid: Surface area = $B + \dfrac{Ps}{2}$ (s = slant height)
$_nP_r = \dfrac{n!}{(n-r)!}$	Volume = $\dfrac{Bh}{3}$
$_nC_r = \dfrac{n!}{r!(n-r)!}$	Sphere: Surface area = $4\pi r^2$
	Volume = $\dfrac{4}{3}\pi r^3$
	Cylinder: Surface area = $2\pi rh + 2\pi r^2$
	Volume = $\pi r^2 h$
	Cone: Surface area = $\pi r^2 + \pi rs$
	Volume = $\dfrac{\pi r^2 h}{3}$