Grade 12
Applied Mathematics
Achievement Test

Student Booklet

January 2016

Manitoba

After the administration of this test, print copies of this resource will be
available for purchase from the Manitoba Learning Resource Centre
(formerly the Manitoba Text Book Bureau). Order online at
<www.mtbb.mb.ca>.

This resource will also be available on the Manitoba Education and
Advanced Learning website at <www.edu.gov.mb.ca/k12/assess/
archives/index.html>.

Websites are subject to change without notice.

Disponible en français.

Available in alternate formats upon request.

GRADE 12 APPLIED MATHEMATICS
ACHIEVEMENT TEST

DESCRIPTION

Total Possible Marks: 65   **Time: 3 hours**

Unit	Description	Marks
A	Relations and Functions	16
B	Probability	15
C	Financial Mathematics	14
D	Design and Measurement	9
E	Logical Reasoning	11

RESOURCES

You may use the following resources:

- *Formula Sheet* **(tear-out page at the back of this booklet)**

- one $8\frac{1}{2}'' \times 11''$ study sheet

- ruler

- graphing calculator, computer software, and/or app

- Internet access for tools such as applets or mortgage payment calculators

  Use of the Internet to communicate or access other content, including but not limited to course notes, definitions, or conceptual information is not permitted during the test.

<div style="border:1px solid black; text-align:center">

Electronic communication between students through phones, email, or file sharing during the test is strictly prohibited.

</div>

DIRECTIONS

Read all instructions on the test carefully.

If you need extra paper or you print out an answer to a question, let the teacher know. Indicate in the response space of the question that your answer is on a separate sheet.

Remember to

- indicate your input values by writing them in your booklet or printing a copy if using a technology tool
- include your booklet number and question number on additional pages (e.g., printouts) and attach them to the corresponding page in the booklet
- express answers in decimal and percentage form to **two decimal places** when rounding, unless otherwise indicated

  Example: $\dfrac{15}{29}$ = 0.52 or 51.72%

- state any assumptions you make

A "graphic organizer" is a visual representation of information. Examples include a tree diagram, a chart, a list, a Venn diagram, a truth table, Pascal's triangle, etc.


A clearly communicated answer

— is easily identified in the response space

— includes the parameters in the equation, and "$y =$", "sin", "ln", or "x", as applicable

— includes the units of measure, where applicable

— includes labels, units, and scales for the axes on graphs

— is expressed as an exact value or is appropriately rounded

Marks may be deducted for errors relating to any of the above.

STOP
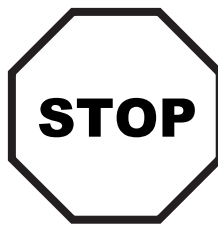
PLEASE WAIT UNTIL INSTRUCTED TO TURN THE PAGE.
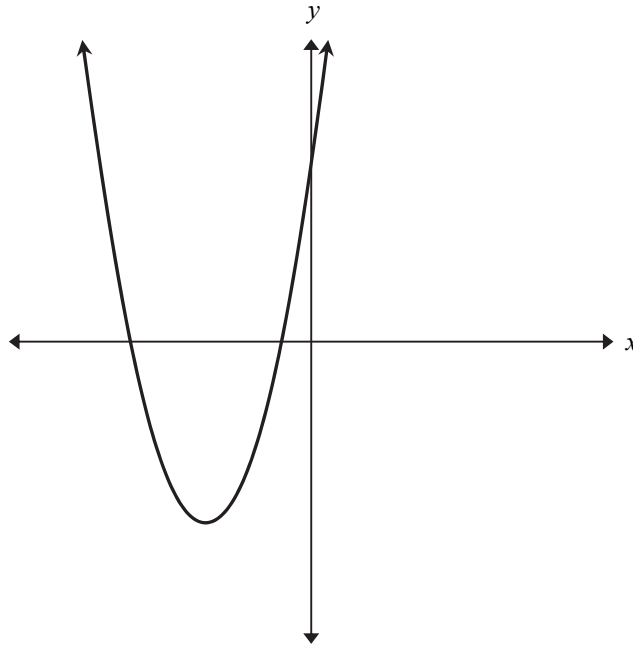
RELATIONS AND FUNCTIONS

Question 1	**Total: 1 mark**

Use the graph below to answer the following question and select the best answer.

Which equation does the graph represent?

A. $y = 2x^2 + 7x - 3$

B. $y = -2x^2 + 7x + 3$

C. $y = 2x^2 + 7x + 3$

D. $y = -2x^2 + 7x - 3$

Question 2

Total: 1 mark

Select the best answer.

Which of the following functions has an unrestricted range, $\{y \mid y \in \mathbf{R}\}$?

A. exponential

B. logarithmic

C. quadratic

D. sinusoidal

In a diving competition, Tracy's first dive can be modelled by the equation:

$$h = -4.90t^2 + 2.72t + 10$$

where t represents the dive time (in seconds)
and h represents the diver's height (in metres) above the water.

How much time does it take for Tracy to reach the water? Show your work.

Question 4   Total: 3 marks

A garden was treated to control pests. Every hour after treatment, there were half as many pests as there were the previous hour. After six (6) hours, 65 pests remained.

a) How many pests were there before treatment? Show your work.

(2 marks)

b) *Select the best answer.*

Which function best represents this situation?

(1 mark)

    A. linear

    B. quadratic

    C. cubic

    D. exponential

The illuminated area of the moon varies sinusoidally with time. A lunar cycle begins and ends with a new moon. A *new moon*, which is 0% illuminated, occurs on the 1st and the 29th day of the cycle. Midway through the cycle (15th day), a *full moon*, which is 100% illuminated, occurs.

106

a)   Determine the sinusoidal equation that models the relationship between the illuminated area of the moon and the lunar cycle. Show your work.

(2 marks)

107

b)   According to your equation in (a), what percentage of the moon is illuminated on the 18th day of the cycle?

(1 mark)

Bailey launched his remote control plane. He recorded the height of the plane at different times during the flight.

Time (s)	Height (ft.)
0	0
1	9
2	7
3	3
4	7
5	26

108

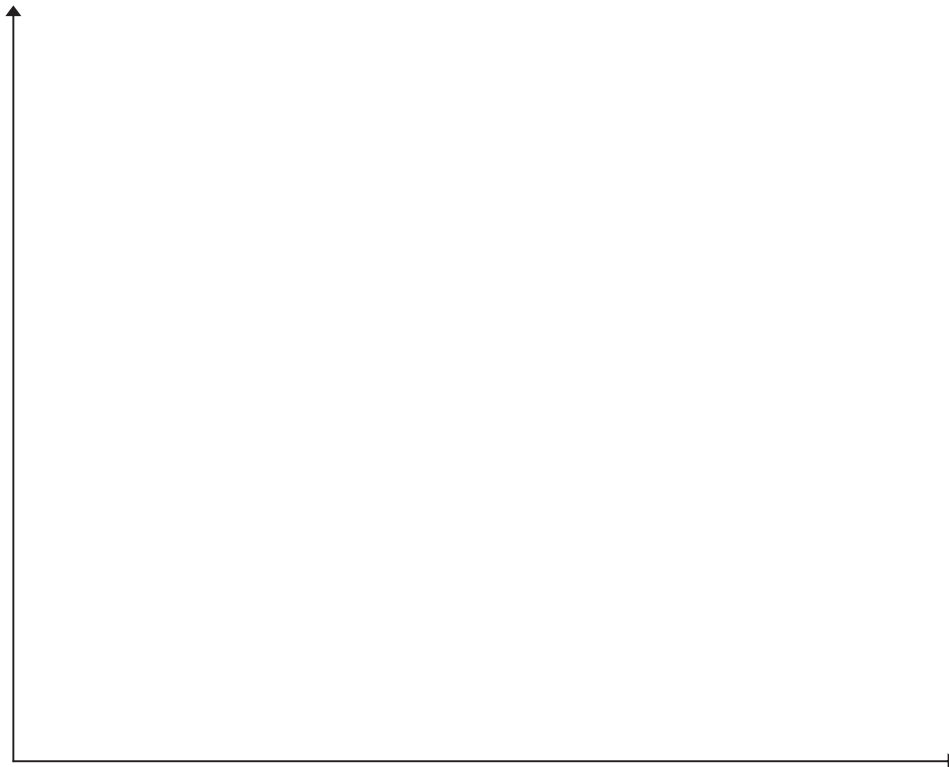a)   Determine the cubic regression equation that models this data.

(1 mark)

109

b)   Create a clearly labelled graph of the equation in (a).

(3 marks)

c) Using your equation in (a), determine how long it will take for the plane to reach a height of 100 ft.

(1 mark)

d) Provide one limitation of the domain.

(1 mark)

PROBABILITY

Question 7   Total: 1 mark

Select the best answer.

Which of the following values cannot describe the probability of an event?

A. 100%

B. $\dfrac{3}{4}$

C. 0

D. 1.2

Question 8   Total: 1 mark

Select the best answer.

You have a stack of ten cards numbered 11 to 20. What is the probability that a randomly drawn card is an odd number or a multiple of three?

A. 0.3

B. 0.5

C. 0.7

D. 0.8

Question 9   Total: 3 marks

Lena runs every day. If it is sunny, the probability she will run outside is 75%. If it is not sunny, the probability that she will run outside is 20%. A 45% probability of sunshine has been announced for tomorrow.

a)   Use a graphic organizer to show all the possible outcomes for this situation.

(1 mark)

b)   Determine the probability that Lena runs outside tomorrow. Show your work.

(2 marks)

Question 10   Total: 3 marks

a) Determine the number of different ways the letters of the word "CINCINNATI" can be arranged. Show your work.

(2 marks)

b) How many ways can the letters of the word "CINCINNATI" be arranged if the first letter must be T?

(1 mark)

Alec walks eight blocks from home to school every morning.

Home

N

Post office

School

118
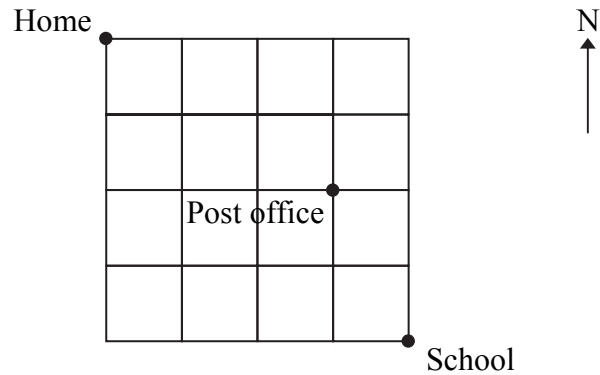
a)   If he only travels south and east, what is the total number of routes from home to school?

(1 mark)

119

b)   What is the probability that his route from home to school passes by the post office?
Show your work.

(2 marks)

Question 12   Total: 4 marks

There are 18 girls and 12 boys trying out for a debate team.

a) How many different teams of 4 members can be formed if there are no restrictions?

(1 mark)

b) How many different teams of 4 members can be formed if <u>exactly</u> two members must be girls?

(1 mark)

c) How many different teams of 4 members can be formed if <u>at least</u> two members must be girls? Show your work.

(2 marks)

FINANCIAL MATHEMATICS

Question 13   Total: 1 mark

Select the best answer.

Which of the following is an advantage of buying a house?

A.   no maintenance costs

B.   no property taxes

C.   no down payment required

D.   no restrictions on renovations

Question 14   Total: 1 mark

Select the best answer.

Approximately how many years will it take a $1000.00 investment to double its value at an interest rate of 3.60%, compounded annually?

A.   5

B.   7.2

C.   20

D.   50

Question 15   Total: 2 marks

Bruce is 24 years old. He graduated from college when he was 20 and since then he has worked full-time. He has also made some investments.

His financial advisor sends him an update on his investment portfolio.

Investment	Initial Investment	Gain/Loss
low-risk mutual fund	$2000	+ $100
guaranteed investment certificate	$6000	+ $220
bonds	$4000	+ $180

a)   What is the overall rate of return on Bruce's investments?

(1 mark)

b)   Do you think that Bruce's investment portfolio is appropriate for him at this stage of his life? Justify your answer, stating your assumptions.

(1 mark)

Question 16   Total: 2 marks

Pedro has a tax-free savings account (TFSA) with a balance of $5000.00. Interest is earned at a rate of 4.00%, compounded monthly.

If Pedro contributes $400.00 to the TFSA at the end of every month, how long will it take him to save $20 000.00? Show your work.

Question 17   Total: 3 marks

Kira purchases a sofa for $1015.87 (taxes included). The department store offers her a promotion of 0% interest with no payments for one year. If Kira does not pay the amount in full within one year, interest will be charged from the date of purchase at an annual rate of 28.80%, compounded monthly.

a)   If Kira does not make any payments, what will the department store bill her one year after the date of purchase? Show your work.

(2 marks)

b)   State a different compounding period such that the overall cost of the sofa is lower than if the annual interest rate were compounded monthly.

(1 mark)

Question 18   Total: 5 marks

Bill and Celine purchase a new home. They obtain a $375 000.00 mortgage amortized over 25 years with their credit union. The initial 5-year term of the mortgage requires monthly payments. Interest is calculated at a rate of 3.25%, compounded semi-annually.

130

a)   Calculate Bill and Celine's mortgage payment. Show your work.

(2 marks)

131

b)   How much will Bill and Celine owe at the end of their 5-year term?

(1 mark)

132

c)   Bill and Celine make an additional $10 000.00 payment on the principal at the end of the 5-year term. How much sooner will they pay off their mortgage if they keep the same payments and interest rate over the life of the mortgage? Show your work.

(2 marks)

DESIGN AND MEASUREMENT

Question 19	Total: 1 mark

Select the best answer.

The volume of a freezer is 0.46 m^3. This volume can also be expressed as:

A. 46 cm^3

B. 460 cm^3

C. 4600 cm^3

D. 460 000 cm^3

Sophie wants to install carpet in her bedroom. The 12 ft. by 9 ft. carpet she wants to install costs $32.50/yd^2, plus GST and PST.

Calculate the total cost of the carpet. Show your work.
(Note: GST = 5%, PST = 8%)
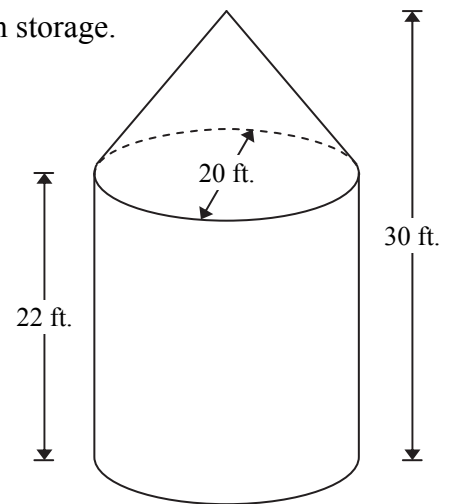
Question 21   **Total: 6 marks**

The Menard family has a grain farm. They own 5 bins for grain storage.

Each bin
- is composed of a cone and a cylinder
- is 30 feet tall (from the top of the cone to the bottom of the cylinder)
- has a diameter of 20 feet
- has a cylinder 22 feet tall

(Diagram is not drawn to scale.)

20 ft.

30 ft.

22 ft.

135

a)   Calculate the volume of one grain bin. Show your work.

(2 marks)

b) Mr. Menard decides to paint the exterior (sides and top) of the bins.

- Each bin needs 2 coats of paint.
- One can of paint covers 400 ft^2.
- Each can of paint costs $67.99, plus GST and PST.

Calculate the total cost of the paint for the 5 bins. Show your work.
(Note: GST = 5%, PST = 8%)

(4 marks)

LOGICAL REASONING

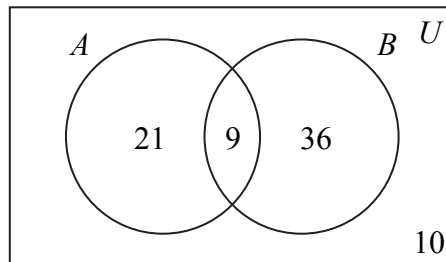**Question 22**   **Total: 1 mark**

Use the information below to answer the following question and select the best answer.

Students were surveyed about the technology they own.

A = owns a notebook computer
B = owns a tablet computer

Which of the following statements is true?

A. $n(A) = 21$

B. $n(B') = 36$

C. $n(A \cap B) = 9$

D. $n(A \cup B) = 76$

138

A class of 28 students was surveyed to discover the type of music to which they listen.

- 12 students said they listen to country
- 18 students said they listen to pop
- 2 students said they listen to neither country nor pop

Determine the number of students who listen only to pop. Show your work.

Question 24   Total: 2 marks

Consider the original statement:

> "If the end behaviour of a function extends from quadrant II to quadrant I,
> then the function is quadratic."

a)   Is the original statement true? If not, provide a counterexample.

(1 mark)

b)   Write the contrapositive of the original statement.

(1 mark)

Question 25   Total: 3 marks

Given the following sets:

$$U = \{1, 3, 5, 7, 9, 13, 17, 21\}$$
$$A = \{1, 9, 13, 21\}$$
$$B = \{1, 7, 9, 17, 21\}$$

141

a)   Illustrate these sets using a Venn diagram.

(1 mark)

142

b)   Determine $A' \cup B$.

(1 mark)

143

c)   Create a set C with 3 elements, where $C \subset B$.

(1 mark)

Question 26   Total: 3 marks

a)   Use the numbers 1, 2, 3, 4, 5, 6 only once to complete the chart.

(2 marks)

$$
\begin{array}{ccccccc}
\boxed{7} & + & \boxed{} & + & \boxed{9} & = & 22 \\
+ & & + & & + & & \\
\boxed{} & + & \boxed{} & + & \boxed{} & = & 7 \\
+ & & + & & + & & \\
\boxed{} & + & \boxed{8} & + & \boxed{} & = & 16 \\
\| & & \| & & \| & & \\
13 & & 18 & & 14 & &
\end{array}
$$

b)   What were the first two numbers you obtained? $\boxed{}$ and $\boxed{}$

Explain the strategy you used to obtain these numbers.

(1 mark)

END OF TEST

Formula Sheet: Applied Mathematics

Relations and Functions

$$y = ax + b$$

$$y = ax^2 + bx + c$$

$$y = ax^3 + bx^2 + cx + d$$

$$y = ab^x$$

$$y = a + b\ln(x)$$

$$y = a\log_b x$$

$$y = a\sin(bx + c) + d$$

$$y = a\cos(bx + c) + d$$

Financial Mathematics

$$t = \frac{72}{i}$$

$$I = Prt$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Net worth = Total assets − Total liabilities

$$\text{Debt-to-equity ratio (\%)} = \frac{(\text{Total liabilities} - \text{Mortgage})}{\text{Net worth}} \times 100$$

$$\text{Gross debt service ratio (\%)} = \frac{\left(\begin{array}{ccc}\text{Monthly} & \text{Monthly} & \text{Monthly}\\ \text{mortgage} + \text{property} + \text{heating}\\ \text{payment} & \text{taxes} & \text{costs}\end{array}\right)}{\text{Gross monthly income}} \times 100$$

$$\text{Rate of return (\%)} = \frac{\left(\begin{array}{cc}\text{Current value} & \text{Previous value}\\ \text{of portfolio} & - \text{ of portfolio}\end{array}\right)}{\text{Previous value of portfolio}} \times 100$$

Probability

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

$$_nP_r = \frac{n!}{(n - r)!}$$

$$_nC_r = \frac{n!}{r!(n - r)!}$$

Design and Measurement

Prism: Surface area $= Ph + 2B$

Volume $= Bh$

Pyramid: Surface area $= B + \dfrac{Ps}{2}$ (s = slant height)

Volume $= \dfrac{Bh}{3}$

Sphere: Surface area $= 4\pi r^2$

Volume $= \dfrac{4}{3}\pi r^3$

Cylinder: Surface area $= 2\pi rh + 2\pi r^2$

Volume $= \pi r^2 h$

Cone: Surface area $= \pi r^2 + \pi rs$

Volume $= \dfrac{\pi r^2 h}{3}$