Grade 12
Applied Mathematics
Achievement Test

Marking Guide

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This resource will also be available on the Manitoba Education and Training website at www.edu.gov.mb.ca/k12/assess/archives/index.html.
Websites are subject to change without notice.

Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.
Available in alternate formats upon request.
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General Marking Instructions

Please ensure that
• the student booklet number matches the number on the Scoring Sheet
• only a pencil is used to complete the Scoring Sheet
• the final test mark is recorded on the Scoring Sheet
• the Scoring Sheet is complete and a copy has been made for school records

Do not make any marks in the student booklets. Booklets may be selected by Manitoba Education and Training for sample marking.

Once marking is completed, please forward the Scoring Sheets to Manitoba Education and Training using the envelope provided (for more information, see the administration manual).

Marking

Explanations for student errors for selected-response questions have been provided, if applicable.

To receive full marks for a question, a student’s response must be complete and correct. Partial marks may be awarded for an “appropriate strategy” with execution errors. An appropriate strategy is defined as one that is consistent with the learning outcomes and mathematical processes associated with the question and, if properly executed, would lead to the correct answer.

Some questions require a form of explanation or justification from students. Explanation or justification can be given through a labelled diagram, in words, by showing mathematical operations for answer verification, or by providing output from a technological tool. For this reason, appropriate flexibility is required when marking student responses.

Errors

Marks are deducted if conceptual or communication errors are committed.

Conceptual Errors

As a guiding principle, students should only be penalized once for each error committed in the context of a test question. For example, students may choose an inappropriate strategy for a question, but carry it through correctly and arrive at an incorrect answer. In such cases, students should be penalized for having selected an inappropriate strategy for the task at hand, but should be given credit for having arrived at an answer consistent with their choice of strategy.
Communication Errors

Communication errors are errors that are not related to the concepts and are tracked on the Scoring Sheet in a separate section. There will be a 0.5 mark deduction for each type of communication error committed, regardless of the number of errors committed for that type (see example on next page).

\[ \text{Notation} \]

- does not include braces when using set notation
- does not include a box when using a Venn diagram
- does not include one of the following in the equation: “\( y = \)”, “\( \sin \)”, “\( \ln \)”, or “\( x \)”, or writes parameters separately from the equation

\[ \text{Units} \]

- does not include the dollar sign for monetary values
- uses incorrect units of measure
- does not include the units in the final answer
- confuses square and cubic units (e.g., cm² instead of cm³, or vice versa)
- does not include units with labels on a graph

\[ \text{Transcription/Transposition} \]

- makes a transcription error (inaccurate transferring of information)
- makes a transposition error (changing order of digits)

\[ \text{Final Answer} \]

- does not express monetary values to two decimal places
- does not include a percent sign
- does not identify the answer (e.g., TVM solver, Venn diagram)
- does not use a contextual variable when stating the domain or the range in set notation
- incorrectly states the final answer

\[ \text{Rounding} \]

- rounds incorrectly
- rounds too soon
- does not express the answer to the appropriate number of decimal places

\[ \text{Whole Units} \]

- does not use whole units for materials purchased in design and measurement questions
- does not use whole units in contextual questions involving discrete data (e.g., people)

When a given response includes multiple types of communication errors, deductions are indicated in the order in which the errors occur in the response. No communication errors are recorded for work that has not been awarded marks. The total deduction may not exceed the marks awarded.
Scoring

The marks allocated to questions are based on the concepts associated with the learning outcomes in the curriculum. For each question, shade in the circle on the Scoring Sheet that represents the mark awarded based on the concepts. A total of these marks will provide the preliminary mark.

The student’s final mark is determined by subtracting the communication errors from the preliminary mark.

Example:
A student has a preliminary mark of 46. The student committed one E1 error (0.5 mark deduction) and three E4 errors (0.5 mark deduction).

\[
\text{Communication Errors} = \text{Preliminary Mark} - (\text{Number of error types} \times 0.5)
\]

\[
= 46 - (2 \times 0.5)
= 45
\]

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If a Scoring Sheet is marked with “0” and/or “NR” only (e.g., student was present but did not attempt any questions) please document this on the Irregular Test Booklet Report.
Assistance

If any issue arises that cannot be resolved locally during marking, please call Manitoba Education and Training at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the marking keys.

Allison Potter
Assessment Consultant
Grade 12 Applied Mathematics
Telephone: 204-945-3411
Toll-Free: 1-800-282-8069, ext. 3411
Email: allison.potter@gov.mb.ca
Marking Keys

Please note that this *Marking Guide* contains screen captures taken from a TI–84 Plus graphing calculator.
Select the answer that best completes the statement.

The end behaviour of a cubic function can extend from:

A. quadrant I to quadrant II
B. quadrant II to quadrant I
C. **quadrant II to quadrant IV**
D. quadrant III to quadrant IV
Which equation does the graph represent?

**A.** \( y = 6 \sin(4x) + 5 \)

**B.** \( y = 12 \sin(4x) + 5 \)

**C.** \( y = -6 \sin(4x) + 5 \)

**D.** \( y = -12 \sin(4x) + 5 \)
Jordan is making an open-top box using an 8 in. by 10 in. sheet of cardboard. He plans to cut squares of length $x$ from the corners and fold up the sides. The function that represents the volume of the box, $V$, can be modelled by the equation:

$$V = (x)(10 - 2x)(8 - 2x)$$

Determine the maximum volume of the box. Show your work.

$$\text{CALC } 4: \text{ maximum } (1.472..., 52.513... )$$

$$y = 52.51$$

The maximum volume is 52.51 in$^3$. 

**Marking Key**

1 mark for appropriate work

1 mark for consistent answer
When jumping rope, the centre of the rope reaches a maximum height of 1.90 m at 0.38 s and a minimum height of 0.08 m at 0.88 s.

For the sinusoidal function that models the height at the centre of the rope as a function of time,

a) determine the period.

(1 mark)

\[
\text{half of the period } = 0.88 - 0.38 = 0.50 \\
\therefore \text{ period } = 0.50 \times 2 = 1.00
\]

The period is 1.00 s.

b) determine the median height.

(1 mark)

\[
\text{median height } = \frac{\text{maximum height} + \text{minimum height}}{2} \\
= \frac{1.90 + 0.08}{2} \\
= \frac{1.98}{2} \\
= 0.99
\]

The median height is 0.99 m.

---

**Marking Key**

1 mark for correct period in (a)

1 mark for correct median height in (b)
A microbiologist is studying bacterial growth over 2 full days. She notices that the bacterial count doubles every 3 hours. The initial bacterial count is 6.

a) Determine an exponential equation that represents this situation. Show your work.

(2 marks)

\[
x = 0 \quad 6 \\
3 \quad 12 \\
6 \quad 24 \\
9 \quad 48
\]

\[
y = 6 (1.26)^x
\]

\[
y = 6 \left(2^{\frac{x}{3}}\right)
\]

b) How much time does it take until the bacterial count is 1000? Show your work.

(2 marks)

\[
Y_2 = 1000 \\
\text{CALC 5: intersect (22.142..., 1000)} \\
x = 22.14
\]

It takes 22.14 hours for the bacterial count to reach 1000.

c) State the domain for this situation.

(1 mark)

\[
\{0 \leq x \leq 48\}
\]

\[
\text{OR}
\]

\[
[0, 48]
\]

The time is greater than or equal to 0 and less than or equal to 48 hours.

**Marking Key**

1. 1 mark for appropriate work in (a)
2. 1 mark for consistent answer in (a)
3. 1 mark for appropriate work in (b)
4. 1 mark for consistent answer in (b)
5. 1 mark for correct domain in (c)
A small town gets cellphone service. The number of users increases as a function of time as shown in the table below.

<table>
<thead>
<tr>
<th>Time (weeks)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of users</td>
<td>202</td>
<td>319</td>
<td>480</td>
<td>656</td>
<td>743</td>
<td>821</td>
</tr>
</tbody>
</table>

a) Create a clearly labelled graph by plotting the given data. Draw the curve of best fit.

(3 marks)

b) Determine the regression equation that best models the data in this situation.

(1 mark)

\[ y = 189.40 + 216.26 \ln(x) \]
c) According to your equation in (b), how many users did the town have at 15 weeks?

(1 mark)

\[ \text{CALC 1 : value } x = 15, y = 775.03 \]

The town had 775 users.

OR

\[ x = 15 \]
\[ y = 189.40 + 216.26 \ln(15) \]
\[ = 775.03 \]

The town had 775 users.

Marker Note(s):
→ Award mark 3 with an 5 communication error for a correct graph with one incorrect data point.
→ Deduct an 6 communication error if the points are plotted but the curve of best fit is not drawn.

Marking Key

<table>
<thead>
<tr>
<th></th>
<th>1 mark for communicating the context of the graph with appropriate title and/or labels in (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1 mark for using an appropriate domain and range (i.e., window settings/grid range) for the context of the question in (a)</td>
</tr>
<tr>
<td>3</td>
<td>1 mark for correctly plotting the data in (a)</td>
</tr>
<tr>
<td>4</td>
<td>1 mark for correct equation in (b)</td>
</tr>
<tr>
<td>5</td>
<td>1 mark for consistent answer in (c)</td>
</tr>
</tbody>
</table>
**PROBABILITY**

**Question 7**

**Total: 1 mark**

**Learning Outcome: 12A.P.1**

**Question Type: Selected Response**

Select the best answer.

The probability of rain tomorrow is \( \frac{5}{8} \). What are the odds against rain tomorrow?

A. 3:8

B. 5:8

C. 3:5

D. 5:3

**Question 8**

**Total: 2 marks**

**Learning Outcome: 12A.P.2**

**Question Type: Constructed Response**

A survey of 25 students found that the probability of a student having a cat or a dog is 72%. Of the 25 students, 44% said they have a cat and 64% said they have a dog.

How many students have both a cat and a dog? Show your work.

\[
P(\text{cat} \cup \text{dog}) = P(\text{cat}) + P(\text{dog}) - P(\text{cat} \cap \text{dog})
\]

\[
72\% = 44\% + 64\% - P(\text{cat} \cap \text{dog})
\]

\[
36\% = P(\text{cat} \cap \text{dog})
\]

\[
0.36 \times 25 = 9
\]

Nine students have both a cat and a dog.

**Marking Key**

1. 1 mark for appropriate work
2. 1 mark for consistent answer
Question 9

Learning Outcomes: 12A.P.4, 12A.P.5  Question Type: Constructed Response

How many different routes are there from A to B if you only go east and south?
Show your work.

There are 240 different routes.

OR

\[
\begin{align*}
\text{EESS} & \quad \text{ES} & \quad \text{EEESSS} \\
& \quad \frac{4!}{2!2!} & \times & \quad \frac{6!}{3!3!} \\
& = 6 \times 2 \times 20 & & = 240
\end{align*}
\]

There are 240 different routes.

Marking Key

1 mark for appropriate work
1 mark for consistent answer
Andrei and Sergei are playing soccer. They each take a shot on goal. Andrei shoots first and Sergei shoots second. The probability of Andrei scoring is 0.70 and the probability of Sergei scoring is 0.60.

a) Use a graphic organizer to show all the possible outcomes for this situation.

(1 mark)

b) What is the probability that at least one of them will score? Show your work.

(2 marks) 

\[ P(\text{at least one scores}) = P(\text{both score}) + P(\text{Andrei scores, Sergei does not score}) + P(\text{Sergei scores, Andrei does not score}) \]

\[ = (0.70)(0.60) + (0.70)(0.40) + (0.60)(0.30) \]

\[ = 0.88 \]

The probability that at least one of them will score is 0.88 or 88%.

OR

\[ P(\text{at least one scores}) = 1 - P(\text{neither scores}) \]

\[ = 1 - (0.30)(0.40) \]

\[ = 0.88 \]

The probability that at least one of them will score is 0.88 or 88%.

Marking Key:

1 mark for correct graphic organizer in (a)
1 mark for appropriate work in (b)
1 mark for consistent answer in (b)
An ultimate frisbee team consists of 7 players. A team is formed by randomly choosing players from a group of 8 men and 7 women.

a) Determine the number of ways 7 players can be chosen to form a team.

\[ _{15}C_7 = 6435 \]

There are 6435 ways.

b) Determine the probability that the team has exactly 3 women. Show your work.

\[ P(\text{exactly 3 women}) = \frac{{_3C_3 \times _8C_4}}{{_{15}C_7}} = \frac{2450}{6435} = 0.3807 \]

The probability is \( \frac{490}{1287} \), 0.38, or 38.07%.

**Marking Key**

<table>
<thead>
<tr>
<th></th>
<th>1 mark for correct answer in (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1 mark for appropriate work in (b)</td>
</tr>
<tr>
<td>3</td>
<td>1 mark for consistent answer in (b)</td>
</tr>
</tbody>
</table>
While playing a game, Cally pulls out the following letters from a bag containing some vowels (A, E, I, O, U) and some consonants:

X A A U B L A

a) How many different arrangements can be made using all of the letters above? Show your work.

(2 marks)

\[
\frac{7!}{3!} = 840
\]

There are 840 different arrangements.

b) If Cally places all the vowels together and all the consonants together, how many different arrangements can be made using all of the letters above? Show your work.

(2 marks)

\[
\frac{4 \times 3 \times 2 \times 1}{3!} \times \frac{3 \times 2 \times 1}{2} = 48
\]

There are 48 different arrangements.

OR

\[
4 \times _3P_3 \times _2P_2 = 48
\]

There are 48 different arrangements.

Marking Key

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 mark for appropriate work in (a)</td>
</tr>
<tr>
<td>2</td>
<td>1 mark for consistent answer in (a)</td>
</tr>
<tr>
<td>3</td>
<td>1 mark for appropriate work in (b)</td>
</tr>
<tr>
<td>4</td>
<td>1 mark for consistent answer in (b)</td>
</tr>
</tbody>
</table>
Question 13  Total: 1 mark
Learning Outcome: 12A.FM.1  Question Type: Selected Response

Select the best answer.

Xavier needs a loan to purchase a car. Which of the following options would result in the least interest paid?

A loan with an interest rate of:

A. 5.50%, compounded daily

B. 5.50%, compounded weekly

C. 5.50%, compounded monthly

D. 5.50%, compounded annually

Question 14  Total: 1 mark
Learning Outcome: 12A.FM.1  Question Type: Selected Response

Select the best answer.

Shalini invests $25 000.00 in a Canada Savings Bond that earns a simple interest rate of 1.90%. Determine the value of the investment at the end of 5 years.

A. $2375.00

B. **$27 375.00**

C. $27 489.41

D. $237 500.00

**Student Error**
A: simple interest only
C: future value with interest rate compounded monthly
D: simple interest only without changing rate to a decimal
The Bashir family wants to buy a house. They can afford monthly payments of $1325.00. The bank offers them an interest rate of 3.25%, compounded semi-annually over 15 years or 25 years.

a) Determine the maximum amount they can borrow if the mortgage is amortized over 15 years. Show your work.

(2 marks)

The maximum amount is $188,851.29.

b) Determine the maximum amount they can borrow if the mortgage is amortized over 25 years.

(1 mark)

The maximum amount is $272,540.68.

c) Give one reason why the Bashir family would choose to buy the less expensive house.

(1 mark)

The Bashir family would choose to buy the less expensive house since they would own their house sooner.

Other answers are possible.

Marking Key

<table>
<thead>
<tr>
<th></th>
<th>1 mark for appropriate work in (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1 mark for consistent answer in (a)</td>
</tr>
<tr>
<td>3</td>
<td>1 mark for consistent answer in (b)</td>
</tr>
<tr>
<td>4</td>
<td>1 mark for appropriate reason in (c)</td>
</tr>
</tbody>
</table>
Question 16

You are presented with two different investment plans:

**Plan A:** Invest $1000.00 every year for 10 years at an interest rate of 6.00%, compounded monthly.

**Plan B:** Invest a lump sum of $10 000.00 at an interest rate of 6.00%, compounded monthly for 10 years.

a) Determine the value of each investment plan after 10 years. Show your work.

(3 marks)

<table>
<thead>
<tr>
<th>Plan A</th>
<th>Plan B</th>
</tr>
</thead>
<tbody>
<tr>
<td>N=10</td>
<td>N=10</td>
</tr>
<tr>
<td>I%=6</td>
<td>I%=6</td>
</tr>
<tr>
<td>FV=0</td>
<td>FV=10000</td>
</tr>
<tr>
<td>PMT=-1000</td>
<td>PMT=0</td>
</tr>
<tr>
<td>P/V=1</td>
<td>P/V=0</td>
</tr>
<tr>
<td>C/V=12</td>
<td>C/V=12</td>
</tr>
<tr>
<td>PMT:END BEGIN</td>
<td>PMT:END BEGIN</td>
</tr>
</tbody>
</table>

The value for Plan A is $13 285.11 while it is $18 193.97 for Plan B.

b) Calculate the total amount of interest earned for each plan.

(1 mark)

<table>
<thead>
<tr>
<th>Plan A</th>
<th>Plan B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$13 285.11</td>
<td>$18 193.97</td>
</tr>
<tr>
<td>$13 285.11 - $10 000.00</td>
<td>$18 193.97 - $10 000.00</td>
</tr>
<tr>
<td>$3285.11</td>
<td>$8193.97</td>
</tr>
</tbody>
</table>

c) Which plan would you choose? Explain.

(1 mark)

Plan A does not need a large upfront investment. OR Plan B earns more interest.

*Other answers are possible.*

Marker Note(s):
→ The value of Plan A is $14 104.51 if payments are made at the beginning of the month.

<table>
<thead>
<tr>
<th>Marking Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
</tbody>
</table>
Question 17

Learning Outcomes: 12A.FM.1, 12A.FM.2

The Connors are purchasing a condominium that costs $190 000.00. They have $25 000.00 saved for a down payment and will finance the balance at an interest rate of 5.50% for 20 years, compounded semi-annually.

a) Determine their monthly mortgage payment. Show your work.

(2 marks)

\[
\begin{align*}
N &= 240 \\
I &= 5.5 \\
P &= 165000 \\
PMT &= 1129.25 \\
v &= 12 \\
c &= 2 \\
PMT &= \text{EN} \text{ BEGIN}
\end{align*}
\]

Their monthly mortgage payment will be $1129.25.

b) There are condominium fees of $300.00 per month in addition to the down payment and mortgage payments. Calculate the total amount they will have paid after 5 years. Show your work.

(2 marks)

\[60(1129.25 + 300) + 25000 = 110755.00\]

They will have paid $110 755.00.

Marking Key

<table>
<thead>
<tr>
<th>Mark</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 mark for appropriate work in (a)</td>
</tr>
<tr>
<td>2</td>
<td>1 mark for consistent answer in (a)</td>
</tr>
<tr>
<td>3</td>
<td>1 mark for appropriate work in (b)</td>
</tr>
<tr>
<td>4</td>
<td>1 mark for consistent answer in (b)</td>
</tr>
</tbody>
</table>
A spherical balloon has a diameter of 30 cm. Air is added to the balloon until its surface area increases by 500 cm$^2$. What is the new diameter?

Surface area = $4\pi r^2$

$= 4\pi (15 \text{ cm})^2$

$= 2827.43 \text{ cm}^2$

New surface area = 2827.43 cm$^2$ + 500 cm$^2$

$= 3327.43 \text{ cm}^2$

$3327.43 \text{ cm}^2 = 4\pi r^2$

$r = \sqrt{\frac{3327.43 \text{ cm}^2}{4\pi}}$

$r = 16.27 \text{ cm}$

$d = 2r$

$d = 32.54 \text{ cm}$

The new diameter of the balloon is 32.54 cm.

Marking Key

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 mark for correct new surface area</td>
</tr>
<tr>
<td>2</td>
<td>1 mark for consistent r value</td>
</tr>
<tr>
<td>3</td>
<td>1 mark for consistent answer</td>
</tr>
</tbody>
</table>
Students are building birdhouses as a class project. The birdhouses are built to the specifications shown in the diagram below. (Diagram is not drawn to scale.)

Identify all of the faces of one birdhouse. Use the graphing paper on the next page to illustrate the number of birdhouses the students can build from a 4 ft. × 3 ft. sheet of plywood.

<table>
<thead>
<tr>
<th>Face</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>front</td>
<td>14 in.</td>
</tr>
<tr>
<td>back</td>
<td>14 in.</td>
</tr>
<tr>
<td>base</td>
<td>14 in.</td>
</tr>
<tr>
<td>roof</td>
<td>14 in.</td>
</tr>
<tr>
<td>sides</td>
<td>6 in. × 2</td>
</tr>
</tbody>
</table>
Question 19 continued

The students can build a maximum of 3 birdhouse(s).

Other arrangements are possible.

Marker Note(s):
→ Award mark 3 if the number of birdhouses is reflected in the surface area calculations.

Marking Key

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
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</tbody>
</table>

1 mark for correctly identifying all of the faces of one birdhouse
2 mark for appropriate work using the graphing paper
3 mark for correct answer
LOGICAL REASONING

Question 20

Total: 1 mark

Learning Outcome: 12A.L.3

Question Type: Selected Response

Select the best answer.

Which of the following statements is not biconditional?

A. If you invest at 6%, then your investment will double in about 12 years.

B. If a quadrilateral is a square, then it is a rectangle.

C. If $x + y = 3$, then $y = 3 - x$.

D. If a glass is half empty, then it is half full.

Question 21

Total: 1 mark

Learning Outcome: 12A.L.1

Question Type: Constructed Response

Complete the pattern in the lower-right quadrant.

Marking Key

1 mark for correct answer
A student is shown 5 sprockets and a chain. The student predicts that when sprocket A is turned clockwise the chain will go downwards. Is he correct? Explain.

He is incorrect.

OR

He is incorrect.

A, C, and E move clockwise while B and D move counter-clockwise. The chain is between D and E so it moves upwards.

Marking Key

|   | 1 mark for correct explanation |

Applied Mathematics: Marking Guide (June 2017) 27
Consider the conditional statement below:

“If the probability of snow tomorrow is \( \frac{4}{5} \),
then the odds in favour of snow tomorrow are 4:1.”

a) Write the converse of the conditional statement.

(1 mark)

“If the odds in favour of snow tomorrow are 4:1,
then the probability of snow tomorrow is \( \frac{4}{5} \).”

b) Determine if a biconditional statement is possible. If yes, write the biconditional statement. If not, provide a counterexample.

(1 mark)

Yes, a biconditional statement is possible.

“The probability of snow tomorrow is \( \frac{4}{5} \)
if and only if the odds in favour of snow tomorrow are 4:1.”

Marker Note(s):
→ Award mark ② if the student writes “iff” instead of “if and only if.”

<table>
<thead>
<tr>
<th>Marking Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>① 1 mark for correct converse statement in (a)</td>
</tr>
<tr>
<td>② 1 mark for correct biconditional statement in (b)</td>
</tr>
</tbody>
</table>
A survey of 95 families found that:

- 54 families have a car
- 46 families have a truck
- 28 families have a motorcycle
- 24 families have only a truck
- 10 families have a car and a truck
- 8 families have only a car and a motorcycle
- 5 families have all three

a) How many families do not have any vehicle? Show your work.

\[ 3 \text{ marks} \]

There are 2 families that do not have any vehicle.

b) How many families have at least two vehicles?

\[ 1 \text{ mark} \]

\[ 5 + 8 + 5 + 12 = 30 \]

There are 30 families that have at least two vehicles.

**Marking Key**

1. 1 mark for correctly calculating the numbers in the overlap regions in (a)
2. 1 mark for consistent number of families that have only one vehicle in (a)
3. 1 mark for consistent number of families that do not have any vehicle in (a)
4. 1 mark for consistent answer in (b)
Exemplars

Exemplars may contain screen captures taken from software or Internet pages.
Exemplar 1

Question 3  

Determine the maximum volume of the box. Show your work.

\[ V = (x)(10 - 2x)(8 - 2x) = (x)(80 - 20x - 16x + 4x^2) \\
= 4x^3 - 16x^2 - 24x + 80 \\
= 4x^3 - 36x^2 + 80 \]

\[ V' = 12x^2 - 72x = 12x(x - 6) \]

\[ V'' = 24x - 72 \]

\[ V''(x = 3) = 24(3) - 72 = 72 - 72 = 0 \]

\[ V''(x = 5) = 24(5) - 72 = 120 - 72 = 48 \]

\[ V''(x = 7) = 24(7) - 72 = 168 - 72 = 96 \]

\[ V(3) = 4(3)^3 - 36(3)^2 + 80 = 36 - 108 + 80 = 8 \text{ in}^3 \]

\[ V(5) = 4(5)^3 - 36(5)^2 + 80 = 250 - 750 + 80 = -420 \text{ in}^3 \]

\[ V(7) = 4(7)^3 - 36(7)^2 + 80 = 980 - 1890 + 80 = -830 \text{ in}^3 \]

\[ \boxed{2\text{nd} \text{ Trace} \quad 4:\text{max} = 80 \text{ in}^3} \]

1 mark:
\[ \rightarrow 1 \text{ mark for consistent answer} \]
Exemplar 2

Question 3

Jordan is making an open-top box using an 8 in. by 10 in. sheet of cardboard. He plans to cut squares of length $x$ from the corners and fold up the sides. The function that represents the volume of the box, $V$, can be modelled by the equation:

$$V = (x)(10 - 2x)(8 - 2x)$$

Determine the maximum volume of the box. Show your work.

$$V = x(10-2x)(8-2x)$$
$$= 2(10-2x)(8-2x)$$
$$= 2(10-4)(8-4)$$
$$= 2(6)(4)$$
$$= 48 \text{ in}^3$$
Exemplar 1

Question 4

When jumping rope, the centre of the rope reaches a maximum height of 1.90 m at 0.38 s and a minimum height of 0.08 m at 0.88 s.

For the sinusoidal function that models the height at the centre of the rope as a function of time,

a) determine the period.

(1 mark)

\[ \frac{0.88 + 0.38}{2} \times 2 = 1.64 \text{ seconds} \]

b) determine the median height.

(1 mark)

\[ \frac{1.9 + 0.08}{2} = 0.99 \]

1 mark:

② → 1 mark for correct median height in (b)
③ → does not include the units in the final answer
Exemplar 2

Question 4 Total: 2 marks

When jumping rope, the centre of the rope reaches a maximum height of 1.90 m at 0.38 s and a minimum height of 0.08 m at 0.88 s.

For the sinusoidal function that models the height at the centre of the rope as a function of time,

a) determine the period.

(1 mark)

\[ 0.88 - 0.38 = 0.5 \quad 0.5 \times 2 = 1 \]

\[ \therefore \text{ period } = 1 \text{ second} \]

b) determine the median height.

(1 mark)

\[ 1.90 - 0.08 = \frac{1.82}{2} = 0.91 \quad \text{ median height } = 0.91 \text{ m} \]

1 mark:

1 → 1 mark for correct period in (a)
A microbiologist is studying bacterial growth over 2 full days. She notices that the bacterial count doubles every 3 hours. The initial bacterial count is 6.

a) Determine an exponential equation that represents this situation. Show your work.

\begin{align*}
\begin{array}{c|c}
L_1 & L_2 \\
1 & 6 \\
3 & 12 \\
6 & 24 \\
9 & 48 \\
12 & 96 \\
\end{array}
\cdot \text{STAT} \\
\cdot \text{CALC} - \text{EXPREG} \\
\cdot \text{CALCULATE}
\end{align*}

\begin{align*}
y &= a \cdot b^x \\
y &= 5.21 \cdot 1.28^x \\
\end{align*}

(2 marks)

b) How much time does it take until the bacterial count is 1000? Show your work.

\begin{align*}
y_2 &= 1000 \\
\text{2nd} \; \text{trace} \; \text{intersect} \\
y &= 1000 \; \; \; x = 21.34317
\end{align*}

So it will take 21.34 hours until the bacterial count is 1000.

(2 marks)

c) State the domain for this situation.

(1 mark)

\[ \text{Domain} = \{ x \in \mathbb{R} \} \]

3 marks:

- ② → 1 mark for consistent answer in (a)
- ③ → 1 mark for appropriate work in (b)
- ④ → 1 mark for consistent answer in (b)
Exemplar 2

Question 5

A microbiologist is studying bacterial growth over 2 full days. She notices that the bacterial count doubles every 3 hours. The initial bacterial count is 6.

a) Determine an exponential equation that represents this situation. Show your work.

\[ y = 6(2^x) \]

(2 marks)

b) How much time does it take until the bacterial count is 1000? Show your work.

\[ y_1 = 6 \cdot 2^x \]
\[ y_2 = 1000 \]

\[ x = 7.38 \text{ hours} \]

(2 marks)

c) State the domain for this situation.

(1 mark)

\[ [0, +\infty) \]

3 marks:

① → 1 mark for consistent answer in (a)
② → 1 mark for appropriate work in (b)
③ → 1 mark for consistent answer in (b)
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Exemplar 1

Question 6  
Total: 5 marks

A small town gets cellphone service. The number of users increases as a function of time as shown in the table below.

<table>
<thead>
<tr>
<th>Time (weeks)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of users</td>
<td>202</td>
<td>319</td>
<td>480</td>
<td>656</td>
<td>743</td>
<td>821</td>
</tr>
</tbody>
</table>

a) Create a clearly labelled graph by plotting the given data. Draw the curve of best fit.

(3 marks)

b) Determine the regression equation that best models the data in this situation.

(1 mark)

\[
y = ax + b \\
y = 31.10 + 293.19
\]
Exemplar 1 (continued)

c) According to your equation in (b), how many users did the town have at 15 weeks?

(1 mark)

\[ x = 15, \quad y = 759.74. \]
Exemplar 2

Question 6

A small town gets cellphone service. The number of users increases as a function of time as shown in the table below.

<table>
<thead>
<tr>
<th>Time (weeks)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of users</td>
<td>202</td>
<td>319</td>
<td>480</td>
<td>656</td>
<td>743</td>
<td>821</td>
</tr>
</tbody>
</table>

a) Create a clearly labelled graph by plotting the given data. Draw the curve of best fit.

(3 marks)

b) Determine the regression equation that best models the data in this situation.

(1 mark)

\[ y = -2.34x^2 + 79.5x + 159.74 \]
Exemplar 2 (continued)

c) According to your equation in (b), how many users did the town have at 15 weeks?

(1 mark)

\[ 824.95 \]

So 824.
Exemplar 1

Question 8  

A survey of 25 students found that the probability of a student having a cat or a dog is 72%. Of the 25 students, 44% said they have a cat and 64% said they have a dog.

How many students have both a cat and a dog? Show your work.

\[ 44 + 64 = 108 \]
\[ 108 - 100 = 8 \]
\[ 25 \times 0.08 = 2 \]

2 students have a cat and a dog.
Exemplar 2

Question 8  
Total: 2 marks

A survey of 25 students found that the probability of a student having a cat or a dog is 72%. Of the 25 students, 44% said they have a cat and 64% said they have a dog.

How many students have both a cat and a dog? Show your work.

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B)
\]

\[
P(72\%) = P(44\%) + P(64\%) - P(\text{?})
\]

0 marks:  
→ no criteria met
Exemplar 1

Question 9

Total: 2 marks

How many different routes are there from A to B if you only go east and south?
Show your work.

\[12 \times 2 \times 36 = 864\]

0 marks:
→ no criteria met
Exemplar 2

Question 9  Total: 2 marks

How many different routes are there from A to B if you only go east and south?
Show your work.

\[
\frac{4!}{2^2} \times 2! \times \frac{9!}{3! \times 2!}
\]

1 mark:

\[\bigstar \rightarrow 1 \text{ mark for consistent answer} \]
Exemplar 1

Question 10

Andrei and Sergei are playing soccer. They each take a shot on goal. Andrei shoots first and Sergei shoots second. The probability of Andrei scoring is 0.70 and the probability of Sergei scoring is 0.60.

a) Use a graphic organizer to show all the possible outcomes for this situation.

(1 mark)

b) What is the probability that at least one of them will score? Show your work.

(2 marks)

\[ (.6 \times .7) + (.7 \times .4) + (.6 \times .3) \]

\[ = 88\% \]

2 marks:
② → 1 mark for appropriate work in (b)
③ → 1 mark for consistent answer in (b)
Andrei and Sergei are playing soccer. They each take a shot on goal. Andrei shoots first and Sergei shoots second. The probability of Andrei scoring is 0.70 and the probability of Sergei scoring is 0.60.

a) Use a graphic organizer to show all the possible outcomes for this situation.

(b) What is the probability that at least one of them will score? Show your work.

\[ 0.35 + 0.30 = 0.65 \]

65%: chance at least one will score.
Exemplar 1

Question 11  
Total: 3 marks

An ultimate frisbee team consists of 7 players. A team is formed by randomly choosing players from a group of 8 men and 7 women.

a) Determine the number of ways 7 players can be chosen to form a team.

\( \binom{15}{14} \binom{13}{12} \binom{11}{10} \binom{9}{9} \)

\[ = \frac{32432400}{32432400} = 1 \]

(1 mark)

b) Determine the probability that the team has exactly 3 women. Show your work.

\( \frac{7}{15} \frac{6}{14} \frac{5}{13} \frac{8}{12} \frac{7}{11} \frac{6}{10} \frac{5}{9} \)

\[ = \frac{352800}{32432400} = 0.01 \]

(2 marks)

2 marks:

\( \Rightarrow 1 \) mark for appropriate work in (b)

\( \Rightarrow 1 \) mark for consistent answer in (b)
Exemplar 2

Question 11

An ultimate frisbee team consists of 7 players. A team is formed by randomly choosing players from a group of 8 men and 7 women.

a) Determine the number of ways 7 players can be chosen to form a team.

(1 mark)

\[ \binom{15}{7} = 3432 \]

b) Determine the probability that the team has exactly 3 women. Show your work.

(2 marks)

\[ P(\text{exactly 3 women}) = \frac{\binom{8}{4} \cdot \binom{7}{3}}{\binom{15}{7}} \]

\[ = 0.7138 \ldots \]

\[ = 71.38 \]

2 marks:

\[ \Rightarrow 1 \text{ mark for appropriate work in (b)} \]

\[ \Rightarrow 1 \text{ mark for consistent answer in (b)} \]

\[ \Rightarrow \text{ does not include a percent sign} \]

\[ \Rightarrow \text{ rounds incorrectly} \]
Exemplar 1

Question 12

While playing a game, Cally pulls out the following letters from a bag containing some vowels (A, E, I, O, U) and some consonants:

X A A U B L A

a) How many different arrangements can be made using all of the letters above? Show your work.

(2 marks)

\[ \text{7 P} = 5040 \]

There are 5040 ways for Cally to arrange these letters.

b) If Cally places all the vowels together and all the consonants together, how many different arrangements can be made using all of the letters above? Show your work.

(2 marks)

\[ \frac{4 \times 3 \times 2 \times 1 \times 3 \times 2 \times 1}{\times \times \times \times} = 144 \]

If Cally places all vowels together and all consonants together, she could arrange the blocks 144 ways.

2 marks:

① 1 mark for consistent answer in (a)
② 1 mark for consistent answer in (b)
Exemplar 2

Question 12

While playing a game, Cally pulls out the following letters from a bag containing some vowels (A, E, I, O, U) and some consonants:

X A A U B L A

a) How many different arrangements can be made using all of the letters above? Show your work.

(2 marks)

\[
\frac{7!}{3!} = 840
\]

840 different ways to rearrange the letters

b) If Cally places all the vowels together and all the consonants together, how many different arrangements can be made using all of the letters above? Show your work.

(2 marks)

\[
\frac{4!}{3!} = 4
\]

4 different ways to rearrange the vowels

3 different ways to rearrange the consonants

(4)(3!) = 24

24 different ways to rearrange the vowels and consonants

3 marks:

1 \rightarrow 1 mark for appropriate work in (a)
2 \rightarrow 1 mark for consistent answer in (a)
3 \rightarrow 1 mark for consistent answer in (b)
Exemplar 1

Question 15  
Total: 4 marks

The Bashir family wants to buy a house. They can afford monthly payments of $1325.00. The bank offers them an interest rate of 3.25%, compounded semi-annually over 15 years or 25 years.

a) Determine the maximum amount they can borrow if the mortgage is amortized over 15 years. Show your work.

\[ \begin{align*} 
N &= 15 \times 12 = 180 \\
I &= 3.25 \\
P &= 0 \\
PMT &= 1325 \\
F &= 0 ? \\
\frac{P}{Y} &= 12 \\
\frac{C}{Y} &= 2 \\
\text{Answer: } &\$306,292.75
\end{align*} \]

(2 marks)

b) Determine the maximum amount they can borrow if the mortgage is amortized over 25 years.

\[ \begin{align*} 
N &= 25 \times 12 = 300 \\
I &= 3.25 \\
P &= 0 \\
PMT &= 1325 \\
F &= ? \\
\frac{P}{Y} &= 12 \\
\frac{C}{Y} &= 2 \\
\text{Answer: } &\$610,181.81
\end{align*} \]

(1 mark)

c) Give one reason why the Bashir family would choose to buy the less expensive house.

(1 mark)

\[ \text{It's cheaper.} \]

2 marks:

1 mark for consistent answer in (a)
1 mark for consistent answer in (b)
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## Exemplar 2

**Question 15**

<table>
<thead>
<tr>
<th>Question</th>
<th>Total: 4 marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>Determine the maximum amount they can borrow if the mortgage is amortized over 15 years. Show your work. (2 marks)</td>
</tr>
<tr>
<td>b)</td>
<td>Determine the maximum amount they can borrow if the mortgage is amortized over 25 years. (1 mark)</td>
</tr>
<tr>
<td>c)</td>
<td>Give one reason why the Bashir family would choose to buy the less expensive house. (1 mark)</td>
</tr>
</tbody>
</table>

The Bashir family wants to buy a house. They can afford monthly payments of $1325.00. The bank offers them an interest rate of 3.25%, compounded semi-annually over 15 years or 25 years.

a) Determine the maximum amount they can borrow if the mortgage is amortized over 15 years. Show your work.

(2 marks)

b) Determine the maximum amount they can borrow if the mortgage is amortized over 25 years.

(1 mark)

c) Give one reason why the Bashir family would choose to buy the less expensive house.

(1 mark) It’s 10 years less of payments they’ll have to make.
Exemplar 2 (continued)

15 year

Transaction Type
- Investment
- Retirement Plan
- Loan

Payment Frequency (per year)
- 1
- 2
- 4
- 12
- 24
- 26
- 365

Compound Frequency (per year)
- 1
- 2
- 4
- 12
- 24
- 26
- 365

Financial Details
- Initial Loan Amount: 189359.33
- Final Loan Balance: 0.00
- Monthly Payment: 1325.00
- Interest Rate (%): 3.25
- Years: 15
- Make Payment at: Start or End of Period

Financial Summary
- Loan Principal Paid: 189359.33
- Interest Charged: 49140.67
- Total Loan Payment: 238500.00

Max amount is $189359.33

25 year

Transaction Type
- Investment
- Retirement Plan
- Loan

Payment Frequency (per year)
- 1
- 2
- 4
- 12
- 24
- 26
- 365

Compound Frequency (per year)
- 1
- 2
- 4
- 12
- 24
- 26
- 365

Financial Details
- Initial Loan Amount: 273273.86
- Final Loan Balance: 0.00
- Monthly Payment: 1325.00
- Interest Rate (%): 3.25
- Years: 25
- Make Payment at: Start or End of Period

Financial Summary
- Loan Principal Paid: 273273.86
- Interest Charged: 124226.14
- Total Loan Payment: 397500.00

Max amount is $273273.86

4 marks:
- 1 → 1 mark for appropriate work in (a)
- 2 → 1 mark for consistent answer in (a)
- 3 → 1 mark for consistent answer in (b)
- 4 → 1 mark for appropriate reason in (c)
Exemplar 1

Question 16  

You are presented with two different investment plans:

**Plan A:** Invest $1000.00 every year for 10 years at an interest rate of 6.00%, compounded monthly.

**Plan B:** Invest a lump sum of $10 000.00 at an interest rate of 6.00%, compounded monthly for 10 years.

a) Determine the value of each investment plan after 10 years. Show your work.

(3 marks)

\[
\begin{align*}
\text{Plan A:} & \quad FV = 1000 \times \left( 1 + \frac{0.06}{12} \right)^{12 \times 10} = 13285.11 \\
\text{Plan B:} & \quad FV = 10000 \times \left( 1 + \frac{0.06}{1} \right)^{1 \times 10} = 132851.4
\end{align*}
\]

b) Calculate the total amount of interest earned for each plan.

(1 mark)

\[
\begin{align*}
\text{Plan A:} & \quad T.I = 13285.11 - 1000 = 12285.11 \\
\text{Plan B:} & \quad T.I = 132851.4 - 10000 = 122851.4
\end{align*}
\]

c) Which plan would you choose? Explain.

(1 mark) Plan B in my opinion is better, as they invested $10,000 each year, they received an additional $10,000 on their final value compared to Plan A where they invested $1000 only each year and so they only received an additional $1000 for their final value.

2 marks:

1 → 1 mark for appropriate work for Plan A in (a)
2 → 1 mark for consistent answer for Plan A in (a)
Exemplar 2

Question 16

Total: 5 marks

You are presented with two different investment plans:

**Plan A:** Invest $1000.00 every year for 10 years at an interest rate of 6.00%, compounded monthly.

**Plan B:** Invest a lump sum of $10 000.00 at an interest rate of 6.00%, compounded monthly for 10 years.

a) Determine the value of each investment plan after 10 years. Show your work.

(3 marks)

**Plan A**

\[
\begin{align*}
N &= 12 \\
FV &= 16864.94 \\
PV &= 0 \\
mt &= -1000 \\
P/Y &= 1 \\
C/Y &= 1
\end{align*}
\]

**Plan B**

\[
A = P(1+i)^n
\]

\[
\begin{align*}
\frac{6.00}{12} &= .5 \\
A &= 10000(1 + .005)^{120} = 18193.97
\end{align*}
\]

b) Calculate the total amount of interest earned for each plan.

(1 mark)

**Plan A**

\[
16864.94 - 10000 = 6864.94
\]

**Plan B**

\[
18193.97 - 10000 = 8193.97
\]

c) Which plan would you choose? Explain.

(1 mark)

Plan A is better because eventually it will make more interest than plan B. The principal is increasing each year and so is the compound.

2 marks:

1 mark for consistent answer for Plan B in (a)
1 mark for consistent interest calculations in (b)

→ does not include the dollar sign for monetary values
Exemplar 1

Question 17 Total: 4 marks

The Connors are purchasing a condominium that costs $190 000.00. They have $25 000.00 saved for a down payment and will finance the balance at an interest rate of 5.50% for 20 years, compounded semi-annually.

a) Determine their monthly mortgage payment. Show your work.

(2 marks)

\[
\text{TVM} \quad \text{L} \rightarrow \ 1129.25 \\
N = 240 \\
I = 5.5 \\
Pv = 165000 \\
pMt = 1129.25 \\
Fv = 0 \\
P/Y = 12 \\
C/Y = 2
\]

b) There are condominium fees of $300.00 per month in addition to the down payment and mortgage payments. Calculate the total amount they will have paid after 5 years. Show your work.

(2 marks)

\[300 \times 12 \times 5 = \$18000.00\]
The Connors are purchasing a condominium that costs $190 000.00. They have $25 000.00 saved for a down payment and will finance the balance at an interest rate of 5.50% for 20 years, compounded semi-annually.

a) Determine their monthly mortgage payment. Show your work.

(2 marks)

\[
\begin{align*}
N &= 240 \\
I\% &= 5.50 \\
PV &= 190 000.00 - 25 000.00 \\
Pmt &= ? \quad $1129.25 \\
FV &= 0 \\
p &= 12 \\
c &= 2
\end{align*}
\]

b) There are condominium fees of $300.00 per month in addition to the down payment and mortgage payments. Calculate the total amount they will have paid after 5 years. Show your work.

(2 marks)

\[
\begin{align*}
300 \times 12 &= 3600 \\
3600 \times 5 &= 18000 \\
1129.25 \times 12 &= 13551 \\
13551 \times 5 &= 67755
\end{align*}
\]

*total amount paid after 5 yrs. = $85755.00*

3 marks:

1. 1 mark for appropriate work in (a)
2. 1 mark for consistent answer in (a)
3. 1 mark for consistent answer in (b)
A spherical balloon has a diameter of 30 cm. Air is added to the balloon until its surface area increases by 500 cm$^2$. What is the new diameter?

\[
\begin{align*}
4\pi r^2 &= 500 \\
4\pi (15)^2 &= 2827.43 \text{ cm}^2 \\
2827.43 + 500 &= 3327.43 \text{ cm}^2 \\
\frac{3327.43}{4\pi} &= r^2 \\
\sqrt{2613.3} &= r \\
r &= 51.12 \text{ cm} \\
\text{d} &= 2r = 102.24 \text{ cm} \\
\therefore \text{ the new diameter is 102.24 cm}
\end{align*}
\]
A spherical balloon has a diameter of 30 cm. Air is added to the balloon until its surface area increases by 500 cm². What is the new diameter?

\[ SA = 4\pi r^2 \]

\[ SA = 4\pi (15)^2 \]

\[ SA = 2827.4 \] (30)

\[ SA = 3327.43 \]

\[ 3327.43 = 4\pi r^2 \]

\[ r = 16.27 \]

The new diameter is 32.54.

3 marks:

1. 1 mark for correct new surface area
2. 1 mark for consistent \( r \) value
3. 1 mark for consistent answer

\[ \text{does not include the units in the final answer} \]
Exemplar 1

Question 19  Total: 3 marks

Students are building birdhouses as a class project. The birdhouses are built to the specifications shown in the diagram below. (Diagram is not drawn to scale.)

Identify all of the faces of one birdhouse. Use the graphing paper on the next page to illustrate the number of birdhouses the students can build from a 4 ft. × 3 ft. sheet of plywood.

<table>
<thead>
<tr>
<th>Face</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>14 × 8</td>
</tr>
<tr>
<td>back</td>
<td>8 × 14</td>
</tr>
<tr>
<td>front</td>
<td>6 × 14</td>
</tr>
<tr>
<td>sides</td>
<td>16 × 6</td>
</tr>
</tbody>
</table>
The students can build a maximum of 3 birdhouse(s).
Students are building birdhouses as a class project. The birdhouses are built to the specifications shown in the diagram below. (Diagram is not drawn to scale.)

Identify all of the faces of one birdhouse. Use the graphing paper on the next page to illustrate the number of birdhouses the students can build from a 4 ft. × 3 ft. sheet of plywood.

<table>
<thead>
<tr>
<th>Face</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>roof</td>
<td>14 in x 8 in</td>
</tr>
<tr>
<td>ground</td>
<td>14 in x 6 in</td>
</tr>
<tr>
<td>front wall</td>
<td>14 in x 6 in</td>
</tr>
<tr>
<td>back wall</td>
<td>14 in x 8 in</td>
</tr>
<tr>
<td>side wall</td>
<td>6 in x 6 in</td>
</tr>
<tr>
<td>Top Corner</td>
<td>2 in x 6 in</td>
</tr>
<tr>
<td>Side wall</td>
<td>6 in x 6 in</td>
</tr>
</tbody>
</table>
Exemplar 2 (continued)

The students can build a maximum of 3 birdhouse(s).

3 marks:
1 → 1 mark for correctly identifying all of the faces of one birdhouse
2 → 1 mark for appropriate work using the graphing paper
3 → 1 mark for correct answer
Complete the pattern in the lower-right quadrant.

0 marks: → no criteria met
A student is shown 5 sprockets and a chain. The student predicts that when sprocket A is turned clockwise the chain will go downwards. Is he correct? Explain.
A student is shown 5 sprockets and a chain. The student predicts that when sprocket A is turned clockwise the chain will go downwards. Is he correct? Explain.

He's wrong, when he turns sprocket A as handle, it will ultimately make D pull the chain upwards.

0 marks: no criteria met
Exemplar 1

Question 23

Consider the conditional statement below:

“If the probability of snow tomorrow is \(\frac{4}{5}\),
then the odds in favour of snow tomorrow are 4:1.”

a) Write the converse of the conditional statement.

(1 mark)

If the odds in fav. are 4:1 then the prob. of snow is \(\frac{4}{5}\)

b) Determine if a biconditional statement is possible. If yes, write the biconditional statement. If not, provide a counterexample.

(1 mark)

Yes

the probability of snow is \(\frac{4}{5}\) if
the odds in favour are 4:1

1 mark:

\(\square\) 1 mark for correct converse statement in (a)
Exemplar 2

Consider the conditional statement below:

“If the probability of snow tomorrow is \( \frac{4}{5} \), then the odds in favour of snow tomorrow are 4:1.”

a) Write the converse of the conditional statement.

(1 mark)

The odds for snow tomorrow are 4:1, then the probability of snow is \( \frac{4}{5} \) for tomorrow.

b) Determine if a biconditional statement is possible. If yes, write the biconditional statement. If not, provide a counterexample.

(1 mark)

Yes.

"The odds against snow is 60\% and the probability of snow is 90\%."
Exemplar 1

Question 24

A survey of 95 families found that:

- 54 families have a car
- 46 families have a truck
- 28 families have a motorcycle
- 24 families have only a truck
- 10 families have a car and a truck
- 8 families have only a car and a motorcycle
- 5 families have all three

a) How many families do not have any vehicle? Show your work.

(3 marks)

\[ C = 10 + 8 + 5 \]
\[ = 23 \]
\[ 54 - 23 \]
\[ = 31 \]
\[ m - 5 + 8 + 7 \]
\[ = 20 \]
\[ 20 - 20 \]
\[ = 8 \]
\[ T - 10 + 5 + 24 \]
\[ = 39 \]
\[ 46 - 39 \]
\[ = 7 \]

b) How many families have at least two vehicles?

(1 mark)

\[ 7 + 10 + 5 + 8 \]
\[ = 30 \]

30 families have at least 2 vehicles.

3 marks:

2 → 1 mark for consistent number of families that have only one vehicle in (a)
3 → 1 mark for consistent number of families that do not have any vehicle in (a)
4 → 1 mark for consistent answer in (b)
A survey of 95 families found that:

- 54 families have a car
- 46 families have a truck
- 28 families have a motorcycle
- 24 families have only a truck
- 10 families have a car and a truck
- 8 families have only a car and a motorcycle
- 5 families have all three

a) How many families do not have any vehicle? Show your work.

b) How many families have at least two vehicles?

2 marks:

1 mark for correctly calculating the numbers in the overlap regions in (a)
1 mark for consistent answer in (b)

Does not include a box when using a Venn diagram.
Appendices
## Appendix A:
### Table of Questions by Unit and Learning Outcome

<table>
<thead>
<tr>
<th>Unit</th>
<th>Question</th>
<th>Type</th>
<th>Learning Outcome</th>
<th>Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>SR</td>
<td>12A.R.1</td>
<td>1</td>
</tr>
<tr>
<td>A</td>
<td>2</td>
<td>SR</td>
<td>12A.R.3</td>
<td>1</td>
</tr>
<tr>
<td>A</td>
<td>3</td>
<td>CR</td>
<td>12A.R.1</td>
<td>2</td>
</tr>
<tr>
<td>A</td>
<td>4</td>
<td>CR</td>
<td>12A.R.3</td>
<td>2</td>
</tr>
<tr>
<td>A</td>
<td>5</td>
<td>CR</td>
<td>12A.R.2</td>
<td>5</td>
</tr>
<tr>
<td>A</td>
<td>6</td>
<td>CR</td>
<td>12A.R.2</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>Total = 16</strong></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>7</td>
<td>SR</td>
<td>12A.P.1</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>8</td>
<td>CR</td>
<td>12A.P.2</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>9</td>
<td>CR</td>
<td>12A.P.4, 12A.P.5</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>10</td>
<td>CR</td>
<td>12A.P.2, 12A.P.3</td>
<td>3</td>
</tr>
<tr>
<td>B</td>
<td>11</td>
<td>CR</td>
<td>12A.P.6</td>
<td>3</td>
</tr>
<tr>
<td>B</td>
<td>12</td>
<td>CR</td>
<td>12A.P.4, 12A.P.5</td>
<td>4</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td><strong>Total = 15</strong></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>13</td>
<td>SR</td>
<td>12A.FM.1</td>
<td>1</td>
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<tr>
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<td>SR</td>
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</tr>
<tr>
<td>C</td>
<td>15</td>
<td>CR</td>
<td>12A.FM.1, 12A.FM.2</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>16</td>
<td>CR</td>
<td>12A.FM.3</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>17</td>
<td>CR</td>
<td>12A.FM.1, 12A.FM.2</td>
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<tr>
<td></td>
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<td></td>
<td><strong>Total = 15</strong></td>
<td></td>
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<tr>
<td>D</td>
<td>18</td>
<td>CR</td>
<td>12A.D.1</td>
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<tr>
<td>D</td>
<td>19</td>
<td>CR</td>
<td>12A.D.1</td>
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<td></td>
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<td></td>
<td><strong>Total = 6</strong></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>20</td>
<td>SR</td>
<td>12A.L.3</td>
<td>1</td>
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<tr>
<td>E</td>
<td>21</td>
<td>CR</td>
<td>12A.L.1</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>22</td>
<td>CR</td>
<td>12A.L.1</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>23</td>
<td>CR</td>
<td>12A.L.3, 12A.P.1</td>
<td>2</td>
</tr>
<tr>
<td>E</td>
<td>24</td>
<td>CR</td>
<td>12A.L.2</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>Total = 9</strong></td>
<td></td>
</tr>
</tbody>
</table>

### Legend for Units:
- A: Relations and Functions
- B: Probability
- C: Financial Mathematics
- D: Design and Measurement
- E: Logical Reasoning

### Legend for Question Types:
- SR: Selected Response
- CR: Constructed Response
Appendix B:  
Irregularities in Provincial Tests  
A Guide for Local Marking

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an Irregular Test Booklet Report should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all “NR”) or only incorrect responses (“0”)

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an Irregular Test Booklet Report.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an Irregular Test Booklet Report documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the department along with the test materials.
Irregular Test Booklet Report

Test: ________________________________________________________________

Date marked: ___________________________________________________________________

Booklet No.: __________________________________________________________________

Problem(s) noted: _____________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________

Question(s) affected: _________________________________________________________
______________________________________________________________________________
______________________________________________________________________________

Action taken or rationale for assigning marks: ________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________