
This resource is available in print and electronic formats.
ISBN: 978-0-7711-6156-8 (print)

1. Educational tests and measurements—Manitoba.
3. Mathematics—Examinations, questions, etc.
4. Mathematics—Study and teaching (Secondary)—Manitoba.
510.76

Manitoba Education and Advanced Learning
School Programs Division
Winnipeg, Manitoba, Canada

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This resource will also be available on the Manitoba Education and Advanced Learning website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

Available in alternate formats upon request.
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General Marking Instructions

Please ensure that
- the student booklet number matches the number on the Scoring Sheet
- only a pencil is used to complete the Scoring Sheet
- the final test mark is recorded on the Scoring Sheet
- the Scoring Sheet is complete and a copy has been made for school records

Do not make any marks in the student booklets. Booklets may be selected by Manitoba Education and Advanced Learning for sample marking.

Once marking is completed, please forward the Scoring Sheets to Manitoba Education and Advanced Learning using the envelope provided (for more information, see the administration manual).

Marking

Explanations for student errors for selected-response questions have been provided, if applicable.

To receive full marks for a question, a student’s response must be complete and correct. Partial marks may be awarded for an “appropriate strategy” with execution errors. An appropriate strategy is defined as one that is consistent with the learning outcomes and mathematical processes associated with the question and, if properly executed, would lead to the correct answer.

Some questions require a form of explanation or justification from students. Explanation or justification can be given through a labelled diagram, in words, by showing mathematical operations for answer verification, or by providing output from a technological tool. For this reason, appropriate flexibility is required when marking student responses.

Errors

Marks are deducted if conceptual or communication errors are committed.

Conceptual Errors

As a guiding principle, students should only be penalized once for each error committed in the context of a test question. For example, students may choose an inappropriate strategy for a question, but carry it through correctly and arrive at an incorrect answer. In such cases, students should be penalized for having selected an inappropriate strategy for the task at hand, but should be given credit for having arrived at an answer consistent with their choice of strategy.
Communication Errors

Communication errors are errors that are not related to the concepts and are tracked on the Scoring Sheet in a separate section. There will be a 0.5 mark deduction for each type of communication error committed, regardless of the number of errors committed for that type (see example on next page).

1  
\textbf{Notation}

- does not include braces when using set notation
- does not include a box when using a Venn diagram
- does not include one of the following in the equation: “y =”, “sin”, “ln”, or “x”, or writes parameters separately from the equation

2  
\textbf{Units}

- does not include the dollar sign for monetary values
- uses incorrect units of measure
- does not include the units in the final answer
- confuses square and cubic units (e.g., cm$^2$ instead of cm$^3$, or vice versa)
- does not include units with labels on a graph

3  
\textbf{Transcription/Transposition}

- makes a transcription error (inaccurate transferring of information)
- makes a transposition error (changing order of digits)

4  
\textbf{Final Answer}

- does not express monetary values to two decimal places
- does not include a percent sign
- does not identify the answer (e.g., TVM solver, Venn diagram)
- does not use a contextual variable when stating the domain or the range in set notation
- incorrectly states the final answer

5  
\textbf{Rounding}

- rounds incorrectly
- rounds too soon
- does not express the answer to the appropriate number of decimal places

6  
\textbf{Whole Units}

- does not use whole units for materials purchased in design and measurement questions
- does not use whole units in contextual questions involving discrete data (e.g., people)

The total mark deduction for communication errors for any student response is not to exceed the marks awarded for that response. For example, a student awarded one mark on a question is limited to two communication error deductions for that question.
Scoring

The marks allocated to questions are based on the concepts associated with the learning outcomes in the curriculum. For each question, shade in the circle on the Scoring Sheet that represents the mark awarded based on the concepts. A total of these marks will provide the preliminary mark.

The student’s final mark is determined by subtracting the communication errors from the preliminary mark.

Example:
A student has a preliminary mark of 46. The student committed one E1 error (0.5 mark deduction) and three E4 errors (0.5 mark deduction).

\[
\begin{array}{cccccc}
\text{Notation} & \text{Units} & \text{Transcription/Transposition} & \text{Final Answer} & \text{Rounding} & \text{Whole Units} \\
\end{array}
\]

<table>
<thead>
<tr>
<th>Communication Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preliminary Mark ( - ) (Number of error types ( \times 0.5 )) = Final Mark</td>
</tr>
<tr>
<td>46 ( - ) (2 ( \times 0.5 )) = 45</td>
</tr>
</tbody>
</table>

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If a Scoring Sheet is marked with “0” and/or “NR” only (e.g., student was present but did not attempt any questions) please document this on the Irregular Test Booklet Report.
Assistance

If any issue arises that cannot be resolved locally during marking, please call Manitoba Education and Advanced Learning at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the marking keys.

King Luu  
Assessment Consultant  
Grade 12 Applied Mathematics  
Telephone: 204-945-4035  
Toll-Free: 1-800-282-8069, ext. 4035  
Email: king.luu@gov.mb.ca
Marking Keys

Please note that this *Marking Guide* contains screen captures taken from a TI–84 Plus graphing calculator.
Use the graph below to answer the following question and select the best answer.

Which function does the graph represent?

A. \( y = 1(2)^x \)

B. \( y = \frac{1}{3}(2)^x \)

C. \( y = 1(3)^x \)

D. \( y = 1\left(\frac{1}{3}\right)^x \)
Which of the following statements is true?

A. The amplitude of the function is 2.
B. The period of the function is 4.
C. The equation of the midline is $y = 2$.
D. The domain of the function is $[2, 6]$. 
Question 3

Learning Outcome: 12A.R.3

Question Type: Constructed Response

A certain medication destroys diseased cells in a person’s body. The number of diseased cells decreases for a short time after a dose of the medication is administered and then increases again. This situation varies sinusoidally and is modelled by the following equation:

\[ c = 350 \sin(3.14t + 1.57) + 650 \]

where \( t \) represents the time (in weeks) and \( c \) represents the diseased cell count.

a) If the initial dose of medication is administered at \( t = 0 \), when is the second dose administered?

\((1 \text{ mark})\)

After 2 weeks.

b) What is the range of this function?

\((1 \text{ mark})\)

\[
\begin{align*}
\text{OR} & \quad [300, 1000] \\
\text{OR} & \quad \{300 \leq c \leq 1000\} \\
\text{OR} & \quad \text{The range is 300 to 1000.}
\end{align*}
\]

c) A patient claims to feel well when the diseased cell count is below 500. At \( t = 2.5 \) weeks, will the patient be feeling well? Show your work.

\((2 \text{ marks})\)

\[
\begin{align*}
\text{TRACE} & \quad t = 2.5 \\
c & = 651.67228...
\end{align*}
\]

No, the patient will not be feeling well since the number of diseased cells will be more than 500.

<table>
<thead>
<tr>
<th>Marking Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 \text{ mark for correct answer in (a)}</td>
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<tr>
<td>2 \text{ mark for correct answer in (b)}</td>
</tr>
<tr>
<td>3 \text{ mark for appropriate work in (c)}</td>
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<tr>
<td>4 \text{ mark for correct answer in (c)}</td>
</tr>
</tbody>
</table>
The Gateway Arch in St. Louis, in the United States, approximates a parabola. Steven learns that the arch is 192 m wide. (Diagram is not drawn to scale.)

At 2.5 m from his starting point, Steven measures the height of the arch to be 10 m.

(a) Determine the quadratic equation that models the shape of the arch. Show your work. Express all coefficients in the equation to a minimum of three (3) decimal places.

(2 marks)

\[
y = -0.021x^2 + 4.053x
\]
b) Calculate the arch’s maximum height.

\[ y = 194.53298... \]

The maximum height is 194.53 m.

*Using rounded values in (a), the answer is 195.56 m.*

**Marker Note(s):**

→ Regression equations may vary depending on the software used.

<table>
<thead>
<tr>
<th>Marking Key</th>
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<tbody>
<tr>
<td>1 mark for appropriate work in (a)</td>
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<tr>
<td>2 mark for consistent answer in (a)</td>
</tr>
<tr>
<td>3 mark for consistent answer in (b)</td>
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</tbody>
</table>
a) Determine the logarithmic regression equation that models this data.

\[ y = -35.84 + 20.62 \ln(x) \]

b) Create a clearly labelled graph of the equation in (a).
c) It is considered safe to operate a vehicle on lake ice if it is at least 30 cm thick. Use your equation in (a) to determine the first full day it will be safe to drive on the ice. Show your work.

(2 marks)

\[
\text{2nd TRACE} 5: \text{Intersect } y = 30 \\
x = 24.36
\]

It will be safe to drive on the ice on the 25th day.

Marker Note(s):
→ Regression equations may vary depending on the software used.

<table>
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<th>Marking Key</th>
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<tr>
<td>6</td>
</tr>
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PROBABILITY

Question 6  
Learning Outcome: 12A.P.1  
Question Type: Selected Response  

Select the best answer.

The odds in favour of Philip winning a badminton match are 5 : 4.

What is the probability that Philip will lose the match?

A. 0.20  
B. 0.44  
C. 0.56  
D. 0.80

Question 7  
Learning Outcome: 12A.P.3  
Question Type: Selected Response  

Select the best answer.

Elaine has a bag containing 5 red pens and 10 blue pens. She randomly picks two pens out of the bag (no replacement).

What is the probability that Elaine picked two red pens?

A. \( \frac{4}{45} \)  
B. \( \frac{2}{21} \)  
C. \( \frac{1}{9} \)  
D. \( \frac{1}{3} \)
Evan is arranging 8 different bicycles in a rack at his school.

a) How many ways can the bicycles be arranged in the rack?

(1 mark)

\[ 8! = 40320 \]

There are 40320 ways.

b) How many ways can the bicycles be arranged in the rack if 3 students want their bicycles placed together? Show your work.

(2 marks)

\[ \frac{3!}{\text{arrangements of 3 bicycles together}} \times \frac{6!}{\text{arrangements of groups}} = 4320 \]

There are 4320 ways.
Question 9  
Learning Outcome: 12A.P.6  
Question Type: Constructed Response  
Total: 2 marks

Rosalind wants to make a trail mix for a snack. She has 15 ingredients to choose from.

What is the total number of different mixes Rosalind can create containing 3, 4, or 5 ingredients? Show your work.

\[
\begin{align*}
\binom{15}{3} & \quad + \quad \binom{15}{4} & \quad + & \quad \binom{15}{5} \\
= 455 & \quad + \quad 1365 & \quad + & \quad 3003 \\
= 4823
\end{align*}
\]

There are 4823 different mixes.

Marking Key

<table>
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<tr>
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<th>Mark for appropriate work</th>
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<th>Mark for consistent answer</th>
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<tr>
<td>2</td>
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</table>
Question 10  

Total: 2 marks

Learning Outcome: 12A.P.1  

Question Type: Constructed Response

Given the following collection of objects:

```
■ ◊ △ □ △ □ △ □ ◊ ■
```

a) Calculate the probability of randomly choosing ■.

(1 mark)

**Original collection of objects:**

Probability of choosing ■ at random

\[
P(■) = \frac{4}{12} = \frac{1}{3}
\]

b) How can the collection be changed so that the probability of choosing ■ is exactly 40%?

(1 mark)

The probability can be increased by

- taking away 2 ◊ or △
- adding 2 ■ and 1 ◊ or △

\[
P(■) = \frac{4}{10} = 0.4 \text{ or } 40%
\]

OR

\[
P(■) = \frac{6}{15} = 0.4 \text{ or } 40%
\]

*Other answers are possible.*

**Marking Key**

<table>
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<tr>
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<th>1 mark for correct answer in (a)</th>
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<tbody>
<tr>
<td>2</td>
<td>1 mark for appropriate change to collection in (b)</td>
</tr>
</tbody>
</table>
Tim has a set of cards numbered 1 to 15. He randomly draws one card.

Consider the following events:

Event A: drawing a card that is a multiple of 2
Event B: drawing a card that is a multiple of 3

a) Are these events mutually exclusive? Justify your answer.

(1 mark)

These events are not mutually exclusive because there are two numbers (6 and 12) that are both a multiple of 2 and a multiple of 3.

b) What is the probability of drawing a numbered card that is a multiple of 2 or a multiple of 3? Show your work.

(2 marks)

\[ A = \{2, 4, 6, 8, 10, 12, 14\} \]
\[ B = \{3, 6, 9, 12, 15\} \]

\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]

\[ P(\text{multiple of 2 or multiple of 3}) = P(\text{multiple of 2}) + P(\text{multiple of 3}) - P(\text{both}) \]

\[ = \frac{7}{15} + \frac{5}{15} - \frac{2}{15} \]

\[ = \frac{10}{15} \]

\[ = \frac{2}{3} \]

The probability is \( \frac{2}{3} \), 0.67, or 66.67%.
You are asked to take a 3-question multiple-choice quiz. Each question has 4 possible answers, one of which is correct.

a) If you randomly pick an answer for each question, what is the probability that all 3 answers are wrong?

\[
\left( \frac{3}{4} \right) \left( \frac{3}{4} \right) \left( \frac{3}{4} \right) = \frac{27}{64}
\]

The probability is \( \frac{27}{64} \), 0.42, or 42.19%.

b) What is the probability of getting at least one of the questions correct?

\[
1 - \frac{27}{64} = \frac{37}{64}
\]

The probability is \( \frac{37}{64} \), 0.58, or 57.81%.

---

**Marking Key**

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<tbody>
<tr>
<td>1</td>
<td>1 mark for consistent answer in (b)</td>
</tr>
</tbody>
</table>
Student ID codes are made up using any two upper case letters of the alphabet followed by any two digits.

a) How many ID codes are possible?

\[
26 \times 26 \times 10 \times 10 = 67\,600
\]

67 600 codes are possible.

b) How many ID codes are possible given the following conditions?

- Repetition is not allowed.
- The letters “I” and “O” cannot be used.

\[
24 \times 23 \times 10 \times 9 = 49\,680
\]

49 680 codes are possible.
Which of the following graphs shows the amount of principal paid on each payment over a 25-year mortgage?

A. 

B. 

C. 

D. 

Select the best answer.
Question 15  Total: 1 mark

Learning Outcome: 12A.FM.2  Question Type: Selected Response

Select the best answer.

The price of a new car is $26 000 (taxes included). It depreciates at a rate of 20% per year. What is the approximate residual value of the car after 3 years?

A. $10 400
B. $13 300
C. $15 600
D. $20 800

Student Error

A: $26 000 – ($26 000 \times 0.20 \times 3)$
C: $26 000 \times (0.20 \times 3)$
D: $26 000 \times 0.80$

Question 16  Total: 1 mark

Learning Outcome: 12A.FM.1  Question Type: Selected Response

Use the following information to answer this question and select the best answer.

Scenario 1: A loan of $30 000.00 at an interest rate of 7.00%, compounded monthly for 1 year

Scenario 2: A loan of $30 000.00 at an interest rate of 7.00%, compounded _______ for 1 year

Assuming no payments are made, which compounding period frequency in Scenario 2 would result in less interest than in Scenario 1?

A. every two weeks
B. daily
C. semi-annually
D. weekly
Mr. and Mrs. Murthy have a total monthly gross income of $6000.00. They are interested in purchasing a house with a mortgage payment of $1300.00 per month, annual heating costs of $2100.00, and annual property taxes of $3675.00.

Calculate the gross debt service ratio (GDSR). Determine if a bank is likely to offer them a mortgage. Justify your answer.

\[
\text{GDSR} \% = \left( \frac{\text{Monthly mortgage payment} + \text{Monthly property taxes} + \text{Monthly heating costs}}{\text{Gross monthly income}} \right) \times 100
\]

\[
\text{GDSR} \% = \left( \frac{\$1300.00 + \$2100.00 + \$3675.00}{12} \right) \times 100 = 29.69\%
\]

Yes, the bank will likely offer them a mortgage since the GDSR is below 32%.
List two advantages of leasing a vehicle over buying.

- lower monthly payment
- can change vehicle at the end of the lease period
- do not have to worry about selling the vehicle

*Other answers are possible.*
Question 19

Learning Outcome: 12A.FM.2
Question Type: Constructed Response

Gabrielle wants to buy a cabin at Lac du Bonnet that costs $165 000.00. She has saved $25 000.00 that she will use as a down payment. The bank will give her a 15-year mortgage for the balance at 3.49%, compounded semi-annually.

a) What will Gabrielle’s payment be every two weeks? Show your work.

(2 marks)

b) If the cabin appreciates in value an average of 3.00% per year, calculate the appreciated value of the cabin after 10 years.

(1 mark)

Future value = ($165 000.00)(1.03)^{10}
= $221 746.20

The cabin will be valued at $221 746.20.

c) How much equity will Gabrielle have in the cabin after 10 years?

(1 mark)

bal(10 x 26) = $54 930.80
Equity = $221 746.20 − $54 930.80
= $166 815.40

She will have $166 815.40 in equity after 10 years.

OR

Down payment = $25 000.00
Σ Prn(1, 260) = $85 069.20

Appreciation = $165 000.00 × (1.03)^{10} − $165 000.00
= $56 746.20

Equity = $25 000.00 + $85 069.20 + $56 746.20
= $166 815.40

She will have $166 815.40 in equity after 10 years.

Marking Key

1. 1 mark for appropriate work in (a)
2. 1 mark for consistent answer in (a)
3. 1 mark for correct appreciated value in (b)
4. 1 mark for consistent equity in (c)
At the age of 30, Alfred began investing $350.00 monthly into an investment account at an interest rate of 7.00%, compounded monthly. When he turned 45, the interest rate on this investment decreased to 5.00%, compounded monthly. Alfred plans to continue making monthly investments until he retires at 58.

a) How much money will Alfred have in his account when he retires? Show your work.

(3 marks)

To age of 45:

\[
\begin{align*}
N &= 180 \\
I &= 7 \\
PMT &= -350 \\
FV &= 110936.8039 \\
P/Y &= 12 \\
C/Y &= 12 \\
PMT &= \text{END BEGIN}
\end{align*}
\]

From age of 45 to 58:

\[
\begin{align*}
N &= 156 \\
I &= 5 \\
PMT &= -110936.8039 \\
PMT &= 350 \\
FV &= 288905.4889 \\
P/Y &= 12 \\
C/Y &= 12 \\
PMT &= \text{END BEGIN}
\end{align*}
\]

Alfred will have $288,905.49 at retirement.

b) Alfred withdraws $2000.00 per month from his account after he retires. If the interest rate remains at 5.00%, how many months can he withdraw $2000.00?

(1 mark)

\[
\begin{align*}
N &= 221.5045054 \\
I &= 5 \\
PMT &= -288905.49 \\
PMT &= 2000 \\
FV &= 0 \\
P/Y &= 12 \\
C/Y &= 12 \\
PMT &= \text{END BEGIN}
\end{align*}
\]

He can make withdrawals for 221 months.
c) Alfred’s sister, Marianne, retires at the age of 60. Her portfolio is valued at $200 000.00, earning 5.00%, compounded monthly. If Marianne wants the money to last until she is 85 years old, what is the maximum she can withdraw each month? Show your work.

(2 marks)

Marianne can withdraw a maximum of $1169.18 each month.

Marking Key

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<tbody>
<tr>
<td>1</td>
<td>1 mark for appropriate work in (a)</td>
</tr>
<tr>
<td>2</td>
<td>1 mark for correct value of investment at age 45 in (a)</td>
</tr>
<tr>
<td>3</td>
<td>1 mark for consistent value of investment at age 58 in (a)</td>
</tr>
<tr>
<td>4</td>
<td>1 mark for consistent answer in (b)</td>
</tr>
<tr>
<td>5</td>
<td>1 mark for appropriate work in (c)</td>
</tr>
<tr>
<td>6</td>
<td>1 mark for consistent answer in (c)</td>
</tr>
</tbody>
</table>
THIS PAGE WAS INTENTIONALLY LEFT BLANK.
Sheena is wrapping boxes using a roll of wrapping paper with a total area of 3.5 m$^2$. Each box has a 20 cm width, 20 cm length, and 40 cm height. If Sheena uses 20% extra paper on each box, how many boxes can she wrap? Show your work.

\[
S.A = Ph + 2B
\]
\[
= (4 \times 0.20 \text{ m})(0.40 \text{ m}) + 2(0.20 \text{ m})^2
\]
\[
= 0.40 \text{ m}^2
\]

extra paper = 0.40 m$^2 \times 0.20$
\[
= 0.08 \text{ m}^2
\]

total for 1 box = 0.48 m$^2$

\[
\frac{3.5 \text{ m}^2}{0.48 \text{ m}^2} = 7.29
\]

⇒ 7 boxes

She can wrap 7 boxes.

**Marking Key**

1. 1 mark for correct area of paper needed to wrap one box
2. 1 mark for consistent number of boxes wrapped
A traditional plains tipi is built using a rectangular canvas that is twice as long as it is wide. The diagram below shows how a semicircle would be cut out of a canvas with dimensions of 30 ft. (length) and 15 ft. (width). (Diagram is not drawn to scale.)

![Diagram of a tipi with a semicircle cut out.](image)

**a)** Calculate how much canvas is left over after the semicircle has been cut out.

(2 marks)

Area of rectangular piece = 30 ft. × 15 ft. = 450 ft²

Area of semicircle = \( \frac{\pi (15 \text{ ft})^2}{2} \) = 353.43 ft²

Leftover canvas = 450 ft² − 353.43 ft²

= 96.57 ft²
b) You would like to construct a tipi with both dimensions three times greater than those for the tipi in (a). Given that canvas is only sold in a rectangular shape, how much will the canvas cost if the price of canvas is $7.39/ft², plus taxes? (Note: GST = 5%, PST = 8%)

(2 marks)

Area of rectangular canvas needed for the tipi:
90 ft. × 45 ft. = 4050 ft²  OR  450 ft² × 3² = 4050 ft²

Cost of canvas: 4050 ft² × $7.39/ft² = $29,929.50

$1496.48 (GST)
+ $2394.36 (PST)
---
$33,820.34

The canvas will cost $33,820.34.

Marking Key

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<thead>
<tr>
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<tbody>
<tr>
<td>1</td>
<td>1 mark for consistent answer in (a)</td>
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<tr>
<td>2</td>
<td>1 mark for correct area of larger canvas in (b)</td>
</tr>
<tr>
<td>3</td>
<td>1 mark for consistent answer with taxes included in (b)</td>
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</table>
LOGICAL REASONING

Question 23

Learning Outcome: 12A.L.3

Total: 2 marks

Question Type: Constructed Response

Complete the truth table.

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<tbody>
<tr>
<td>$p$</td>
<td>$q$</td>
<td>$\sim q$ (not $q$)</td>
</tr>
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<td>True</td>
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<td>False</td>
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<tr>
<td>False</td>
<td>False</td>
<td>True</td>
</tr>
</tbody>
</table>

Marking Key

1. 1 mark for correct $\sim q$ column
2. 1 mark for consistent $p \Rightarrow \sim q$ column
There are 160 Grade 12 students attending a high school.

This semester, the science teacher noted that there are

- 30 students in Biology
- 54 students in Chemistry
- 43 students in Physics
- 11 students in both Biology and Physics
- 8 students in both Biology and Chemistry
- 9 students in both Chemistry and Physics
- 3 students taking all three courses

a) Use a Venn diagram to represent this situation.

\[(3 \text{ marks})\]

\begin{center}
\begin{tikzpicture}
\node at (0,0) [below] {Chemistry};
\node at (0,3) [below] {Physics};
\node at (-2.5,1.5) [below] {Biology};
\node at (1,0) {14};
\node at (1,3) {26};
\node at (2,4) {58};
\node at (0,2) {8};
\node at (-1,1) {5};
\node at (0,1) {3};
\node at (1,2) {40};
\node at (1,0) {6};
\end{tikzpicture}
\end{center}

b) How many students are taking Biology or Chemistry?

\[(1 \text{ mark})\]

\[14 + 5 + 3 + 8 + 6 + 40 = 76\]

There are 76 students.
Paula is trying to solve the following puzzle. Each $2 \times 2$ box must contain the digits 1 to 4 only once. As well, each column and each row must contain the digits 1 to 4 only once.

Paula indicates that the ★ must be a “4”. Explain why she is incorrect and solve the puzzle.

\[
\begin{array}{ccc}
1 & 4 & \star \\
3 & 2 & 1 & 4 \\
2 & 3 & 4 & 1 \\
4 & 1 & 2 & 3 \\
\end{array}
\]

Paula’s solution is not possible since the “4” must go in the 2nd spot on the top row. Therefore, the ★ must be a “3”.

OR

Paula’s solution is not possible since the “4” must go in the 3rd or 4th spot in the 2nd row. Therefore, the ★ cannot be a “4” because it is in the same $2 \times 2$ box.

Other explanations are possible.

**Marking Key**

1. 1 mark for appropriate explanation
2. 1 mark for correctly solving the puzzle
Exemplars may contain screen captures taken from software or Internet pages.
Question 3

A certain medication destroys diseased cells in a person’s body. The number of diseased cells decreases for a short time after a dose of the medication is administered and then increases again. This situation varies sinusoidally and is modelled by the following equation:

\[ c = 350 \sin (3.14t + 1.57) + 650 \]

where \( t \) represents the time (in weeks) and \( c \) represents the diseased cell count.

(a) If the initial dose of medication is administered at \( t = 0 \), when is the second dose administered?

(1 mark)

\[ 2 \text{ weeks} \]

(b) What is the range of this function?

(1 mark)

\[ 1000 \geq Y \geq 300 \]

(c) A patient claims to feel well when the diseased cell count is below 500. At \( t = 2.5 \) weeks, will the patient be feeling well? Show your work.

(2 marks)

\[ \text{No, the cell count is above 500} \]
**Exemplar 1 (continued)**

\[ y = 350 \sin (3.14x + 1.57) + 650 \]

**4 marks:**

1. \( \rightarrow \) 1 mark for correct answer in (a)
2. \( \rightarrow \) 1 mark for correct answer in (b)
3. \( \rightarrow \) 1 mark for appropriate work in (c)
4. \( \rightarrow \) 1 mark for correct answer in (c)

\( \rightarrow \) does not include braces when using set notation
\( \rightarrow \) does not use a contextual variable when stating the domain or the range in set notation
A certain medication destroys diseased cells in a person’s body. The number of diseased cells decreases for a short time after a dose of the medication is administered and then increases again. This situation varies sinusoidally and is modelled by the following equation:

\[ c = 350 \sin (3.14t + 1.57) + 650 \]

where \( t \) represents the time (in weeks) and \( c \) represents the diseased cell count.

a) If the initial dose of medication is administered at \( t = 0 \), when is the second dose administered?

(1 mark)

\[ t = 2 \]

b) What is the range of this function?

(1 mark)

\( (300, 1000) \)

c) A patient claims to feel well when the diseased cell count is below 500. At \( t = 2.5 \) weeks, will the patient be feeling well? Show your work.

(2 marks)

At the point 2.5 the patient will not be feeling well because the cell count is above 500, it's at 651.67

3 marks:

1 \( \rightarrow \) 1 mark for correct answer in (a)
2 \( \rightarrow \) 1 mark for appropriate work in (c)
3 \( \rightarrow \) 1 mark for correct answer in (c)
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Exemplar 1

Question 4

The Gateway Arch in St. Louis, in the United States, approximates a parabola. Steven learns that the arch is 192 m wide. (Diagram is not drawn to scale.)

At 2.5 m from his starting point, Steven measures the height of the arch to be 10 m.

a) Determine the quadratic equation that models the shape of the arch. Show your work. Express all coefficients in the equation to a minimum of three (3) decimal places.

(2 marks)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2.5</td>
<td>10</td>
</tr>
<tr>
<td>189.5</td>
<td>10</td>
</tr>
</tbody>
</table>

\[
y = a(x^2 - 2.5) = 189.5
\]

\[
y = -0.02111x^2 + 4.05277x - (2.01 \times 10^{-10})
\]

x = distance from starting point
y = height
Exemplar 1 (continued)

b) Calculate the arch’s maximum height.

(1 mark)

\[ x = 95.99 \quad \quad y = 194.52 \text{ m} \]
Exemplar 2

**Question 4**

The Gateway Arch in St. Louis, in the United States, approximates a parabola. Steven learns that the arch is 192 m wide. (Diagram is not drawn to scale.)

At 2.5 m from his starting point, Steven measures the height of the arch to be 10 m.

**a)** Determine the quadratic equation that models the shape of the arch. Show your work. Express all coefficients in the equation to a minimum of three (3) decimal places.

(2 marks)

\[ y = ax^2 + bx + c \]

\[ y = -2.5x^2 + 10x + 192 \]
Exemplar 2 (continued)

b) Calculate the arch’s maximum height.

(1 mark)

maximum = 202 m

Graphed it and traced it
Exemplar 1

Question 5  

The ice thickness on Lake Mathitoba is measured weekly every winter. The data for one season is shown in the table below:

<table>
<thead>
<tr>
<th>Time (day)</th>
<th>7</th>
<th>14</th>
<th>21</th>
<th>28</th>
<th>35</th>
<th>42</th>
<th>49</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ice thickness (cm)</td>
<td>5.2</td>
<td>17.4</td>
<td>26.8</td>
<td>32.4</td>
<td>37.1</td>
<td>42.6</td>
<td>44.3</td>
</tr>
</tbody>
</table>

a) Determine the logarithmic regression equation that models this data.

\[ y = a + b \ln x \]

\[ y = -35.8 + 20.6 \ln x \]

b) Create a clearly labelled graph of the equation in (a).

(3 marks)
Exemplar 1 (continued)

c) It is considered safe to operate a vehicle on lake ice if it is at least 30 cm thick. Use your equation in (a) to determine the first full day it will be safe to drive on the ice. Show your work.

(2 marks)

\[ y = -35.8 + 20.6 \ln x \]

\[ y = -35.8 + 20.6 \ln (25) = 30.5 \]

4 marks:

1 → 1 mark for correct equation in (a)
2 → 1 mark for communicating the context of the graph with appropriate title and/or labels in (b)
3 → 1 mark for using an appropriate domain and range (i.e., window settings/grid range) for the context of the question in (b)
5 → 1 mark for appropriate work in (c)

6 → does not express the answer to the appropriate number of decimal places
Exemplar 2

**Question 5**

The ice thickness on Lake Mathitoba is measured weekly every winter. The data for one season is shown in the table below:

<table>
<thead>
<tr>
<th>Time (day)</th>
<th>7</th>
<th>14</th>
<th>21</th>
<th>28</th>
<th>35</th>
<th>42</th>
<th>49</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ice thickness (cm)</td>
<td>5.2</td>
<td>17.4</td>
<td>26.8</td>
<td>32.4</td>
<td>37.1</td>
<td>42.6</td>
<td>44.3</td>
</tr>
</tbody>
</table>

**a)** Determine the logarithmic regression equation that models this data.

\[
y = a \times + b \\
y = 0.91x + 3.97
\]

(1 mark)

**b)** Create a clearly labelled graph of the equation in (a).

(3 marks)
Exemplar 2 (continued)

c) It is considered safe to operate a vehicle on lake ice if it is at least 30 cm thick. Use your equation in (a) to determine the first full day it will be safe to drive on the ice. Show your work.

(2 marks)

\[ y_1 = 0.91x + 3.97 \]
\[ y_2 = 30 \]

2nd \( \Rightarrow \) Trace \( \Rightarrow \) Intersect \( x = 28.66 \)

\( y = 30 \)

On day 29 it will be safe.
The ice thickness on Lake Mathitoba is measured weekly every winter. The data for one season is shown in the table below:

<table>
<thead>
<tr>
<th>Time (day)</th>
<th>7</th>
<th>14</th>
<th>21</th>
<th>28</th>
<th>35</th>
<th>42</th>
<th>49</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ice thickness (cm)</td>
<td>5.2</td>
<td>17.4</td>
<td>26.8</td>
<td>32.4</td>
<td>37.1</td>
<td>42.6</td>
<td>44.3</td>
</tr>
</tbody>
</table>

a) Determine the logarithmic regression equation that models this data.

\[ y = a + b \ln(x) \]

\[ y = 35.84 + 20.62 \ln(x) \]

b) Create a clearly labelled graph of the equation in (a).
c) It is considered safe to operate a vehicle on lake ice if it is at least 30 cm thick. Use your equation in (a) to determine the first full day it will be safe to drive on the ice. Show your work.

\[ y = 30 \rightarrow 2\text{nd} \rightarrow \text{trace} \rightarrow \text{intersect} \rightarrow x = 24.358667 \]

The first day it will be safe to drive on the ice is day 24.

5 marks:

1 → 1 mark for correct equation in (a)
2 → 1 mark for communicating the context of the graph with appropriate title and/or labels in (b)
3 → 1 mark for an appropriate shape that illustrates key characteristics of the function (e.g., maximum, minimum, asymptotes, intercepts) in (b)
4 → 1 mark for appropriate work in (c)
5 → 1 mark for consistent answer in (c)
6 → rounds incorrectly
Evan is arranging 8 different bicycles in a rack at his school.

a) How many ways can the bicycles be arranged in the rack?

\[ 8! = 40,320 \]  

(1 mark)

b) How many ways can the bicycles be arranged in the rack if 3 students want their bicycles placed together? Show your work.

\[ 5! \times 3! = 720 \]  

(2 marks)

2 marks:

1 → 1 mark for correct answer in (a)
3 → 1 mark for consistent answer in (b)
Evan is arranging 8 different bicycles in a rack at his school.

a) How many ways can the bicycles be arranged in the rack?

(1 mark)

\[ 8! = 40320 \]

b) How many ways can the bicycles be arranged in the rack if 3 students want their bicycles placed together? Show your work.

(2 marks)

\[ \frac{8!}{3!} = 6720 \]
Rosalind wants to make a trail mix for a snack. She has 15 ingredients to choose from.

What is the total number of different mixes Rosalind can create containing 3, 4, or 5 ingredients? Show your work.

\[
\begin{align*}
15 \binom{4}{} &= 32760 \\
15 \binom{3}{} &= 2730 \\
15 \binom{5}{} &= 360360 \\
32760 + 2730 + 360360 &= 395850
\end{align*}
\]

Rosalind can create a mix 395,850 ways.

1 mark:

2 → 1 mark for consistent answer
Rosalind wants to make a trail mix for a snack. She has 15 ingredients to choose from.

What is the total number of different mixes Rosalind can create containing 3, 4, or 5 ingredients? Show your work.

\[
\begin{align*}
\binom{15}{3} &= 455 \\
\binom{15}{4} &= 1365 \\
\binom{15}{5} &= 3003
\end{align*}
\]

1 mark: 1 mark for appropriate work
Exemplar 1

Question 10  

Given the following collection of objects:

[Diagram of objects]

a) Calculate the probability of randomly choosing ■.

(1 mark)

\[ \frac{4}{12} = 0.33 \% \]

b) How can the collection be changed so that the probability of choosing ■ is exactly 40%?

(1 mark)

"can add another ■ so \( \frac{5}{12} = 0.42 \% \)."

1 mark:

1 → 1 mark for correct answer in (a)
23 → incorrectly states the final answer
Exemplar 2

Question 10

Total: 2 marks

Given the following collection of objects:

■ ⬤ △ △ ■ △ ■ △ △ ⬤ ⬤ ■

a) Calculate the probability of randomly choosing ■.

(1 mark)

\[ \frac{9}{12} = .333 \]

33.33%.

b) How can the collection be changed so that the probability of choosing ■ is exactly 40%?

(1 mark)

Remove 2 of the other objects.

\[ \frac{4}{10} = .4 \]

40.00%.

2 marks:

1 → 1 mark for correct answer in (a)

2 → 1 mark for appropriate change to collection in (b)
Exemplar 1

Question 11  
Total: 3 marks

Tim has a set of cards numbered 1 to 15. He randomly draws one card.

Consider the following events:

**Event A:** drawing a card that is a multiple of 2

**Event B:** drawing a card that is a multiple of 3

a) Are these events mutually exclusive? Justify your answer.

(1 mark)

b) What is the probability of drawing a numbered card that is a multiple of 2 or a multiple of 3? Show your work.

(2 marks)
Exemplar 2

Question 11

Tim has a set of cards numbered 1 to 15. He randomly draws one card.

Consider the following events:

**Event A:** drawing a card that is a multiple of 2

**Event B:** drawing a card that is a multiple of 3

a) Are these events mutually exclusive? Justify your answer.

\(1\) mark

No because Tim can have a card that is a multiple of 2 and a multiple of 3.

b) What is the probability of drawing a numbered card that is a multiple of 2 or a multiple of 3? Show your work.

\(2\) marks

\[
P(A \text{ or } B) = P(0.47) + P(0.33) - P(0.13) = P(0.67)
\]

\[
67\%
\]

3 marks:

1 → 1 mark for appropriate justification in (a)
2 → 1 mark for appropriate work in (b)
3 → 1 mark for consistent answer in (b)

E5 → does not express the answer to the appropriate number of decimal places
You are asked to take a 3-question multiple-choice quiz. Each question has 4 possible answers, one of which is correct.

a) If you randomly pick an answer for each question, what is the probability that all 3 answers are wrong?

\[
\frac{4}{\text{any answer}} \times \frac{4}{\text{any answer}} \times \frac{4}{\text{any answer}} = 64 \text{ ways to answer}
\]

\[
\frac{3}{\text{incorrect}} \times \frac{3}{\text{incorrect}} \times \frac{3}{\text{incorrect}} = 27
\]

\[
P(\text{all incorrect}) = \frac{27}{64}
\]

(1 mark)

b) What is the probability of getting at least one of the questions correct?

\[
\frac{1}{\text{correct}} \times \frac{3}{\text{incorrect}} \times \frac{3}{\text{incorrect}} = 9
\]

\[
\frac{1}{\text{correct}} \times \frac{1}{\text{correct}} \times \frac{3}{\text{incorrect}} = 3
\]

\[
\frac{1}{\text{correct}} \times \frac{1}{\text{correct}} \times \frac{1}{\text{correct}} = 1
\]

\[
P(\text{at least 1 correct}) = \frac{13}{64}
\]

(1 mark)

1 mark:

\(\Rightarrow\) 1 mark for correct answer in (a)
Exemplar 2

Question 12  Total: 2 marks

You are asked to take a 3-question multiple-choice quiz. Each question has 4 possible answers, one of which is correct.

a) If you randomly pick an answer for each question, what is the probability that all 3 answers are wrong?

\[ \text{Q1} = \frac{1}{4} = 0.25 \]
\[ \text{Q2} = \frac{1}{4} = 0.25 \]
\[ \text{Q3} = \frac{1}{4} = 0.25 \]

(1 mark)

\[ 0.75 \times 100 \]

\[ 75\% \text{ chance of you getting all 3 incorrect.} \]

b) What is the probability of getting at least one of the questions correct?

(1 mark)

\[ \frac{3}{12} = 0.25 \times 100 = 25\% \]

There is a 25\% chance you get 1 question right

1 mark:

\[ \checkmark \rightarrow 1 \text{ mark for consistent answer in (b) } \]
Exemplar 1

<table>
<thead>
<tr>
<th>Question 13</th>
<th>Total: 2 marks</th>
</tr>
</thead>
</table>

Student ID codes are made up using any two upper case letters of the alphabet followed by any two digits.

a) How many ID codes are possible?

   (1 mark)

\[
\begin{align*}
26 \times 26 & = 676 \\
& + 81,796 \\
& = 88,462
\end{align*}
\]

b) How many ID codes are possible given the following conditions?
   - Repetition is not allowed.
   - The letters “I” and “O” cannot be used.

   (1 mark)

\[
\begin{align*}
24 \times 23 & = 552 \\
& + 60,720 \\
& = 61,272
\end{align*}
\]

1 mark: $\rightarrow$ 1 mark for consistent answer in (b)
Exemplar 2

Question 13

Student ID codes are made up using any two upper case letters of the alphabet followed by any two digits.

a) How many ID codes are possible?

(1 mark)

\[
\frac{26}{\text{any letter}} \times \frac{26}{\text{any letter}} \times \frac{10}{\text{any digit}} \times \frac{10}{\text{any digit}} = 67600 \text{ codes}
\]

b) How many ID codes are possible given the following conditions?

- Repetition is not allowed.
- The letters “I” and “O” cannot be used.

(1 mark)

\[
\frac{24}{\text{any letter, but no I, O}} \times \frac{23}{\text{any letter, but no I, O}} \times \frac{10}{\text{any digit}} \times \frac{10}{\text{any digit}} = 55200 \text{ codes}
\]

1 mark:

\[\checkmark \rightarrow 1 \text{ mark for correct answer in (a)}\]
Mr. and Mrs. Murthy have a total monthly gross income of $6000.00. They are interested in purchasing a house with a mortgage payment of $1300.00 per month, annual heating costs of $2100.00, and annual property taxes of $3675.00.

Calculate the gross debt service ratio (GDSR). Determine if a bank is likely to offer them a mortgage. Justify your answer.

\[
\text{GDSR} = \frac{\text{monthly mortgage payment} + \text{monthly property taxes} + \text{monthly heating costs}}{\text{Gross monthly income}} \times 100
\]

\[
= \frac{1300 + 175 + 306.25}{6000} \times 100
\]

\[
= 29.69\%
\]

Yes, the bank will offer them a mortgage.
Mr. and Mrs. Murthy have a total monthly gross income of $6000.00. They are interested in purchasing a house with a mortgage payment of $1300.00 per month, annual heating costs of $2100.00, and annual property taxes of $3675.00.

Calculate the gross debt service ratio (GDSR). Determine if a bank is likely to offer them a mortgage. Justify your answer.

\[
\left( \frac{\text{Monthly property tax} + \text{monthly heating} + \text{monthly mortgage}}{\text{Gross household income}} \right) \times 100
\]

\[
\left( \frac{3675.00 + 2100.00 + 1300.00}{6000.00} \right) \times 100 = 117.92
\]

NO, 117.92 is more than 32%.

1 mark:

- 1 mark for appropriate justification by making reference to 32%
- does not include a percent sign
Exemplar 1

<table>
<thead>
<tr>
<th>Question 18</th>
<th>Total: 2 marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>List two advantages of leasing a vehicle over buying.</td>
<td></td>
</tr>
</tbody>
</table>

- lower monthly payments
- don't have to worry about repairs

**1 mark:**

① → 1 mark for correct first advantage
List two advantages of leasing a vehicle over buying.

- **Less initial money required for purchase**
- **Line of credit to show responsible debt paying habits**
Exemplar 1

**Question 19**

Gabrielle wants to buy a cabin at Lac du Bonnet that costs $165 000.00. She has saved $25 000.00 that she will use as a down payment. The bank will give her a 15-year mortgage for the balance at 3.49%, compounded semi-annually.

a) What will Gabrielle’s payment be every two weeks? Show your work.

\[ \text{TVM Solver} \]
\[ N = 120 \]
\[ I = 3.99 \]
\[ PV = 140000 \]
\[ PMT = \approx 427.54 \] 

Her payment will be $427.54.

b) If the cabin appreciates in value an average of 3.00% per year, calculate the appreciated value of the cabin after 10 years.

\[ 165000 \times 1.03^{10} = \$221746.20 \]

c) How much equity will Gabrielle have in the cabin after 10 years?

\[ \$221746.20 - \$165000.00 = \$56746.20 \]

2 marks:

- 1 mark for consistent answer in (a)
- 1 mark for correct appreciated value in (b)
Gabrielle wants to buy a cabin at Lac du Bonnet that costs $165 000.00. She has saved $25 000.00 that she will use as a down payment. The bank will give her a 15-year mortgage for the balance at 3.49%, compounded semi-annually.

a) What will Gabrielle’s payment be every two weeks? Show your work.

\[
\text{TVM}
\begin{array}{c|cc|c}
N & 15 \times 104 = 1560 \\
I & 3.49 \\
PV & 140000 \rightarrow \boxed{1560} \\
PMT & -115.06 \\
FV & 0 \\
P/Y & 104 \\
C/Y & 2 \\
\end{array}
\]

Her payment every 2 weeks will be $115.06.

b) If the cabin appreciates in value an average of 3.00% per year, calculate the appreciated value of the cabin after 10 years.

\[
$165\,000 \times 1.03^{10} = $221\,746.20
\]

c) How much equity will Gabrielle have in the cabin after 10 years?

\[
\text{TVM}
\begin{array}{c|cc|c}
N & 10 \times 104 = 1040 \\
I & 3.49 \\
PV & 14000 \rightarrow \boxed{1040} \\
PMT & -115.06 \\
FV & -54930.80 \\
P/Y & 104 \\
C/Y & 2 \\
\end{array}
\]

$221\,746.20 - $54930.80 = $166\,815.40

3 marks:
- 1 mark for consistent answer in (a)
- 1 mark for correct appreciated value in (b)
- 1 mark for consistent equity in (c)
- makes a transcription error (inaccurate transferring of information)
At the age of 30, Alfred began investing $350.00 monthly into an investment account at an interest rate of 7.00%, compounded monthly. When he turned 45, the interest rate on this investment decreased to 5.00%, compounded monthly. Alfred plans to continue making monthly investments until he retires at 58.

a) How much money will Alfred have in his account when he retires? Show your work.

\[
\begin{array}{llll}
30 - 45 & 45 - 58 \\
N = 180 & N = 156 \\
I = 7 & I = 5 \\
Pv = 0 & Pv = 0 \\
Pmt = -350 & Pmt = -350 \\
Fv = 110,936.80 & Fv = 76688.28 \\
P/y = 12 & P/y = 12 \\
C/y = 12 & C/y = 12 \\
\end{array}
\]

He will have $187,625.08 in his account when he retires.

b) Alfred withdraws $2000.00 per month from his account after he retires. If the interest rate remains at 5.00%, how many months can he withdraw $2000.00?

\[
\frac{187,625.08}{2000} = 93.81
\]

94 months
Exemplar 1 (continued)

c) Alfred’s sister, Marianne, retires at the age of 60. Her portfolio is valued at $200 000.00, earning 5.00%, compounded monthly. If Marianne wants the money to last until she is 85 years old, what is the maximum she can withdraw each month? Show your work.

(2 marks)

\[ N = 300 \]
\[ I = 5 \]
\[ P = 0 \]
\[ PMT = -33.584674 \]
\[ FV = 200\,000 \]
\[ P/Y = 12 \]
\[ C/Y = 12 \]

The most she can take out is $33.58 per month.

3 marks:

1. → 1 mark for appropriate work in (a)
2. → 1 mark for correct value of investment at age 45 in (a)
3. → 1 mark for consistent answer in (c)
4. → rounds incorrectly
Exemplar 2

Question 20

At the age of 30, Alfred began investing $350.00 monthly into an investment account at an interest rate of 7.00%, compounded monthly. When he turned 45, the interest rate on this investment decreased to 5.00%, compounded monthly. Alfred plans to continue making monthly investments until he retires at 58.

a) How much money will Alfred have in his account when he retires? Show your work.

(3 marks)

\[
\begin{align*}
30 - 45 &= \#110936.80 \\
45 - 58 &= \#76688.29 \\
\hline
\#110936.80 &\quad + \quad \#76688.29 \\
\hline
\#187625.09
\end{align*}
\]

\[\therefore \text{Alfred's account will have } \#187625.09 \text{ by the time he retires}\]

b) Alfred withdraws $2000.00 per month from his account after he retires. If the interest rate remains at 5.00%, how many months can he withdraw $2000.00?

(1 mark)

119.04 months

(3)
c) Alfred’s sister, Marianne, retires at the age of 60. Her portfolio is valued at $200 000.00, earning 5.00%, compounded monthly. If Marianne wants the money to last until she is 85 years old, what is the maximum she can withdraw each month? Show your work.

(2 marks)

She can withdraw a maximum of $1109.18
Sheena is wrapping boxes using a roll of wrapping paper with a total area of 3.5 m². Each box has a 20 cm width, 20 cm length, and 40 cm height. If Sheena uses 20% extra paper on each box, how many boxes can she wrap? Show your work.

\[
\begin{align*}
(20 \times 2) + (20 \times 2) + (40 \times 2) &= 160\text{cm} = 1.6\text{m} \\
1.6 \times 1.2 &= 1.92\text{m} \\
3.5\text{m}^2 / 1.92\text{m}^2 &= 1.82 \text{ boxes she will be able to wrap}
\end{align*}
\]

So, one whole box

1 mark:

- 1 mark for consistent number of boxes wrapped

2
Sheena is wrapping boxes using a roll of wrapping paper with a total area of 3.5 m². Each box has a 20 cm width, 20 cm length, and 40 cm height. If Sheena uses 20% extra paper on each box, how many boxes can she wrap? Show your work.

\[
\begin{align*}
3.5 \text{ m}^2 \\
350 \text{ cm}^2 \\
4 \times (40 \times 20) = 3200 \text{ cm}^2 \\
2 \times (20 \times 20) = 800 \text{ cm}^2 \\
\frac{4000 \text{ cm}^2 \times 1.2}{4800 \text{ cm}^2} \\
\end{align*}
\]

**1 mark:**

- 1 mark for correct area of paper needed to wrap one box
A traditional plains tipi is built using a rectangular canvas that is twice as long as it is wide. The diagram below shows how a semicircle would be cut out of a canvas with dimensions of 30 ft. (length) and 15 ft. (width). (Diagram is not drawn to scale.)

a) Calculate how much canvas is left over after the semicircle has been cut out.

(2 marks)

\[ r = 15 \]

\[ A = \frac{\pi r^2}{2} \]

\[ A = \frac{\pi \times 15^2}{2} \]

\[ A = 353.43 \text{ ft}^2 \]
Exemplar 1 (continued)

b) You would like to construct a tipi with both dimensions three times greater than those for the tipi in (a). Given that canvas is only sold in a rectangular shape, how much will the canvas cost if the price of canvas is $7.39/ft^2, plus taxes? (Note: GST = 5%, PST = 8%)

(2 marks)

\[ 45 \times 90 = 4,050 \text{ ft}^2 \]

\[ 4,050 \times 7.39 = 29,929.50 \]

\[ 29,929.50 \times 1.13 = 33,820.335 \]

3 marks:

1 → 1 mark for appropriate work in (a)
2 → 1 mark for correct area of larger canvas in (b)
3 → 1 mark for consistent answer with taxes included in (b)
4 → does not express monetary values to two decimal places
Exemplar 2

Question 22

A traditional plains tipi is built using a rectangular canvas that is twice as long as it is wide. The diagram below shows how a semicircle would be cut out of a canvas with dimensions of 30 ft. (length) and 15 ft. (width). (Diagram is not drawn to scale.)

![Diagram of a tipi with a semicircle cut out]

a) Calculate how much canvas is left over after the semicircle has been cut out.

(2 marks)

Area of rectangle = \( l \times w \)

\[ 30 \times 15 = 450 \text{ ft}^2 \]

Area of circle = \( \pi r^2 \)

\[ \pi (15)^2 \div 2 = 353.42 \]

\[ 450 - 353.42 = 96.58 \text{ ft}^2 \]
Exemplar 2 (continued)

b) You would like to construct a tipi with both dimensions three times greater than those for the tipi in (a). Given that canvas is only sold in a rectangular shape, how much will the canvas cost if the price of canvas is $7.39/ft^2, plus taxes? (Note: GST = 5%, PST = 8%)

(2 marks)

\[90 \times 45 = 4050\]

It will cost $548.03

3 marks:

① → 1 mark for appropriate work in (a)
② → 1 mark for consistent answer in (a)
③ → 1 mark for correct area of larger canvas in (b)
⑤ → rounds incorrectly
Complete the truth table.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$q$</td>
<td>$\sim q$ (not $q$)</td>
<td>$p \Rightarrow \sim q$</td>
<td></td>
</tr>
<tr>
<td>True</td>
<td>True</td>
<td>F</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>True</td>
<td>False</td>
<td>T</td>
<td>T</td>
<td></td>
</tr>
<tr>
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<td>True</td>
<td>T</td>
<td>T</td>
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<tr>
<td>False</td>
<td>False</td>
<td>F</td>
<td>T</td>
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</tr>
</tbody>
</table>

1 mark:
$\Rightarrow$ 1 mark for consistent $p \Rightarrow \sim q$ column
Exemplar 1

Question 24

Total: 4 marks

There are 160 Grade 12 students attending a high school.

This semester, the science teacher noted that there are

- 30 students in Biology
- 54 students in Chemistry
- 43 students in Physics
- 11 students in both Biology and Physics
- 8 students in both Biology and Chemistry
- 9 students in both Chemistry and Physics
- 3 students taking all three courses

a) Use a Venn diagram to represent this situation.

(3 marks)

b) How many students are taking Biology or Chemistry?

(1 mark)

\[ 30 + 54 = 84 \]
There are 160 Grade 12 students attending a high school.

This semester, the science teacher noted that there are
- 30 students in Biology
- 54 students in Chemistry
- 43 students in Physics
- 11 students in both Biology and Physics
- 8 students in both Biology and Chemistry
- 9 students in both Chemistry and Physics
- 3 students taking all three courses

a) Use a Venn diagram to represent this situation.

b) How many students are taking Biology or Chemistry?

2 marks:
1 → 1 mark for correctly calculating and placing the number of students taking two or more science courses in (a)
2 → 1 mark for consistent number of students taking exactly one science course in (a)
3 → does not include a box when using a Venn diagram
Exemplar 1

Question 25

Paula is trying to solve the following puzzle. Each $2 \times 2$ box must contain the digits 1 to 4 only once. As well, each column and each row must contain the digits 1 to 4 only once.

Paula indicates that the ★ must be a “4”. Explain why she is incorrect and solve the puzzle.

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<tr>
<th></th>
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<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>4</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

★ could either be a 4 or 3.

but □ has to be the opposite

0 marks:
→ no criteria met
Paula is trying to solve the following puzzle. Each $2 \times 2$ box must contain the digits 1 to 4 only once. As well, each column and each row must contain the digits 1 to 4 only once.

Paula indicates that the $\star$ must be a “4”. Explain why she is incorrect and solve the puzzle.

It must be a 3 because there is only 2 spots in the box. The 3 is over riding 1 spot leaving the other spot the only spot left for the 3 in that box. So the first must go in the other box.
Appendices
# Appendix A:
# Table of Questions by Unit and Learning Outcome

<table>
<thead>
<tr>
<th>Unit</th>
<th>Question</th>
<th>Type</th>
<th>Learning Outcome</th>
<th>Mark</th>
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<td>1</td>
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<td>1</td>
</tr>
<tr>
<td>A</td>
<td>3</td>
<td>CR</td>
<td>12A.R.3</td>
<td>4</td>
</tr>
<tr>
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<td>4</td>
<td>CR</td>
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<td>CR</td>
<td>12A.R.2</td>
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<td><strong>Total = 15</strong></td>
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<td><strong>Total = 17</strong></td>
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<tr>
<td>D</td>
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<td>12A.D.1</td>
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<td>22</td>
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</table>

**Legend for Units:**
- A: Relations and Functions
- B: Probability
- C: Financial Mathematics
- D: Design and Measurement
- E: Logical Reasoning

**Legend for Question Types:**
- SR: Selected Response
- CR: Constructed Response
Appendix B: Irregularities in Provincial Tests

A Guide for Local Marking

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an Irregular Test Booklet Report should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all “NR”) or only incorrect responses (“0”)

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an Irregular Test Booklet Report.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an Irregular Test Booklet Report documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the department along with the test materials.
Irregular Test Booklet Report

Test: _____________________________________________________________

Date marked:  _____________________________________________________

Booklet No.:  _____________________________________________________

Problem(s) noted:  _________________________________________________

Question(s) affected:  _____________________________________________

Action taken or rationale for assigning marks:  ________________________

______________________________________________________________

______________________________________________________________

______________________________________________________________

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