Grade 12
Applied Mathematics
Achievement Test

Marking Guide

June 2014

ISBN: 978-0-7711-5602-1

1. Educational tests and measurements—Manitoba.
3. Mathematics—Examinations, questions, etc.
4. Mathematics—Study and teaching (Secondary)—Manitoba.

Manitoba Education and Advanced Learning
School Programs Division
Winnipeg, Manitoba, Canada

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This resource will also be available on the Manitoba Education and Advanced Learning website at <www.edu.gov.mb.ca/k12/assess/archives/index.html>.

Websites are subject to change without notice.

Disponible en français.

Available in alternate formats upon request.
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General Marking Instructions

Please do not make any marks in the student booklets. If a student booklet is selected for sample marking, departmental staff will have to remove any marks in the booklet.

Please ensure that

- the student booklet number matches the number on the Scoring Sheet
- only a pencil is used to complete the Scoring Sheet
- the final test mark is recorded on the Scoring Sheet
- the Scoring Sheet is complete and a copy has been made for school records

Once marking is completed, please forward the Scoring Sheets to Manitoba Education and Advanced Learning in the envelope provided (for more information, see the administration manual).

Marking the Questions

Explanations for common errors for multiple-choice questions have been provided, if applicable.

To receive full marks for a question, a student’s response must be complete and correct. Partial marks may be awarded for an “appropriate strategy” with execution errors. An appropriate strategy is defined as one that is consistent with the learning outcomes and mathematical processes associated with the question and, if properly executed, would lead to the correct answer.

Some questions require a form of explanation or justification from students. Depending on the student’s learning style, the explanation or justification can be given through a labelled diagram, in words, by showing mathematical operations for answer verification, or by referring to a software or calculator program. For this reason, appropriate flexibility is required when marking student responses.

Student Errors

As a guiding principle, students should only be penalized once for each error committed in the context of a test question. For example, students may choose an inappropriate strategy for a question, but carry it through correctly and arrive at an incorrect answer. In such cases, students should be penalized for having selected an inappropriate strategy for the task at hand, but should be given credit for having arrived at an answer consistent with their choice of strategy.
Communication Errors

The marks allocated to questions are primarily based on the concepts associated with the learning outcomes in the curriculum. For each question, shade in the circle on the Scoring Sheet that represents the mark awarded based on the concepts. A total of these marks will provide the preliminary mark.

Errors that are not related to the concepts are called “Communication Errors” and these will be indicated on the Scoring Sheet in a separate section (see example below). There will be a 0.5 mark deduction for each type of communication error committed, regardless of the number of errors committed for a certain type (i.e., committing a second error for any type will not further affect a student’s mark).

The total mark deduction for communication errors for any student response is not to exceed the marks given for that response. When multiple communication errors are made in a given response, any deductions are to be indicated in the order in which the errors occur in the response, without exceeding the given marks.

There is a maximum deduction of 3.5 marks for communication errors.

The student’s final mark is determined by subtracting the communication errors from the preliminary mark.

Example:
A student has a preliminary mark of 46. The student committed two E1 errors (0.5 mark deduction) and three E4 errors (0.5 mark deduction).

\[
\text{Preliminary Mark} - \left( 0.5 \times \# \text{ of error types for a maximum deduction of 3.5 marks} \right) = \text{Final Mark}
\]

\[
46 - (0.5 \times 2) = 45
\]
**Irregularities in Provincial Tests**

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix C provides examples of such irregularities as well as procedures to follow to report irregularities.

If a *Scoring Sheet* is marked with “0” and/or “NR” only (e.g., student was present but did not attempt any questions) please document this on the *Irregular Test Booklet Report*.

**Assistance**

If any issue arises that cannot be resolved locally during marking, please call Manitoba Education and Advanced Learning at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the marking keys.

King Luu  
Assessment Consultant  
Grade 12 Applied Mathematics  
Telephone: 204-945-4035  
Toll-Free: 1-800-282-8069, ext. 4035  
Email: king.luu@gov.mb.ca
Marking Keys

Please note that this Marking Guide contains screen captures taken from a TI–83 Plus graphing calculator.
RELATIONS AND FUNCTIONS

Question 1  Total: 1 mark
Learning Outcome: 12A.R.2  Question Type: Multiple Choice

Select the equation below that is best represented by the following graph.

A. \( y = -4.70 \sin x \)
B. \( y = -1.00x^3 - 4.70x^2 + 5.00x \)
C. \( y = 5.00 + 4.70 \ln x \)
D. \( y = 4.70x^2 + 1.00x + 5.00 \)
When inflating a balloon, the volume of air in the balloon can be modelled by the equation:

\[ V = 0.02c^3 - 0.73c^2 + 11.30c - 12.79 \]

where \( V \) represents the volume (cm\(^3\)) of air in the balloon and \( c \) represents the circumference (cm) of the balloon.

How much air would need to be blown into the balloon so that it has a circumference of 60 cm? Show your work.

\(2\text{nd TRACE} 1: \text{value} \ x = 60, \ y = 2357.21\)

The balloon would need 2357.21 cm\(^3\) of air.

**OR**

\[ V = 0.02(60)^3 - 0.73(60)^2 + 11.30(60) - 12.79 \]

\[ = 2357.21 \text{ cm}^3 \]

The balloon would need 2357.21 cm\(^3\) of air.

**Marking Key**

<table>
<thead>
<tr>
<th></th>
<th>1 mark for appropriate work</th>
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<tbody>
<tr>
<td>2</td>
<td>1 mark for correct answer</td>
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</table>
Question 3  
Total: 2 marks

Learning Outcome: 12A.R.2  
Question Type: Short Answer

Cobalt-60 is an isotope used in medical imaging. It decays naturally over time according to the equation:

\[ t = 35.01 - 7.60 \ln P \]

where \( t \) represents the time in years  
and \( P \) represents the percentage of the original material that is still radioactive.

State the domain and the range of the logarithmic function in the context of this situation.

**Domain:**
- \( \{0 < P \leq 100\} \)
- \( \{P \mid 0 < P \leq 100, \ P \in \mathbb{R}\} \)
- \( (0, 100) \)
- \( P \) is greater than 0 and less than or equal to 100.

**Range:**
- \( \{t \geq 0\} \)
- \( \{t \mid t \geq 0, \ t \in \mathbb{R}\} \)
- \( [0, \infty) \)
- \( t \) is greater than or equal to 0.

**Marking Key**

<table>
<thead>
<tr>
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<th>1 mark for correct domain</th>
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</thead>
<tbody>
<tr>
<td>①</td>
<td>1 mark for correct range</td>
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</tbody>
</table>
A store owner wants to increase his profits. Suppose that his operating costs and his earnings are modelled by the following equations:

\[
\begin{align*}
\text{costs:} & \quad y = 0.04x^2 + 44.00x + 1500.00 \\
\text{earnings:} & \quad y = 1.25^x
\end{align*}
\]

where \( x \) represents the time, in hours, that the store is open per week and \( y \) represents the operating costs or earnings, in dollars.

a) Create a clearly labelled graph of both equations on the axes below.

(2 marks)
b) Using a graphing calculator or graphing software, determine the minimum number of hours that the store should stay open in order to make a profit (earnings are greater than the cost). Explain how you arrived at your answer. State your answer to one decimal place.

(2 marks)

$\text{2nd TRACE} 5: \text{intersect } x = 36.084, y = 3139.777$

The store should stay open for a minimum of 36.1 hours.

Marker Note(s):
→ An appropriate shape includes a scale relevant to the context of the question and key characteristics of the function (e.g., maximum, minimum, asymptotes, and intercepts).
The average monthly temperatures for one year in Snow Lake are shown in the table below.

<table>
<thead>
<tr>
<th>Month</th>
<th>Average Monthly Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>−20.2</td>
</tr>
<tr>
<td>February</td>
<td>−15.1</td>
</tr>
<tr>
<td>March</td>
<td>−8.0</td>
</tr>
<tr>
<td>April</td>
<td>1.9</td>
</tr>
<tr>
<td>May</td>
<td>9.6</td>
</tr>
<tr>
<td>June</td>
<td>15.8</td>
</tr>
<tr>
<td>July</td>
<td>18.6</td>
</tr>
<tr>
<td>August</td>
<td>17.3</td>
</tr>
<tr>
<td>September</td>
<td>10.0</td>
</tr>
<tr>
<td>October</td>
<td>3.2</td>
</tr>
<tr>
<td>November</td>
<td>−8.1</td>
</tr>
<tr>
<td>December</td>
<td>−17.6</td>
</tr>
</tbody>
</table>

a) Determine the sinusoidal equation that best represents this data.

(1 mark)

\[ y = 20.16 \sin (0.47x - 1.75) - 1.44 \]
b) Using your sinusoidal equation in (a), calculate the length of time that the average monthly temperature was at or above 10°C. Show your work.

\[ \text{(2 marks)} \]

Result shown:

\[ 2^\text{nd} \text{ TRACE 5: intersect (4.962 5..., 10); (9.047 3..., 10)} \]

Time:

\[ \text{time} = 9.047 3 - 4.962 5 = 4.084 8 \]

\[ \therefore 4.08 \text{ months} \]

The average monthly temperature was at or above 10°C for 4.08 months of the year.

Marker Note(s):

→ Regression equations may vary depending on the software used.
→ No mark deduction in (b) if student used equation in (a) and obtained a value between 4.09 and 4.12 months.

<table>
<thead>
<tr>
<th>Marking Key</th>
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<tbody>
<tr>
<td>1 mark for correct sinusoidal equation in (a)</td>
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<tr>
<td>1 mark for appropriate work in (b)</td>
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<tr>
<td>1 mark for correct answer in (b)</td>
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</tbody>
</table>
A water well has a pump that can initially extract 300 gallons of water per day. The water level in the well begins to drop according to the function:

\[ W = 300 \left( \frac{4}{5} \right)^{\frac{d}{10}} \]

where \( W \) represents the volume of water, in gallons, extracted daily and \( d \) represents the number of days after the water level begins to drop.

a) Determine the volume of water extracted on the 100th day after the water level begins to drop. Show your work.

\[
\text{2nd TRACE 1: value } x = 100, \ y = 32.21
\]

The volume of water extracted on the 100th day will be 32.21 gallons.

OR

\[
W = 300 \left( \frac{4}{5} \right)^{100} = 32.21
\]

The volume of water extracted on the 100th day will be 32.21 gallons.

b) On what day will the pump first extract less than 75 gallons of water per day? Show your work.

\[
Y_2 = 75
\]

\[
\text{2nd TRACE 5: intersect } x = 62.13, \ y = 75
\]

The pump will extract less than 75 gallons of water on the 63rd day.

<table>
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<td>1 mark for correct answer in (a)</td>
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<tr>
<td>1 mark for appropriate work in (b)</td>
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<td>1 mark for correct answer in (b)</td>
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</table>
PROBABILITY

Question 7
Learning Outcome: 12A.P.5
Question Type: Multiple Choice
Total: 1 mark

How many different ways can all 7 letters of the word “OAKBANK” be arranged?

A. 210
B. 1260
C. 2520
D. 5040

Common Errors
A: \(\frac{7!}{4!}\)
B: \(\frac{7!}{2!}\)
C: \(7!\)
D: \(7!\)

Question 8
Learning Outcome: 12A.P.3
Question Type: Short Answer
Total: 1 mark

Brien states that taking a driver’s education course and passing the road test on the first attempt are dependent events. Explain why Brien is correct.

These are dependent events since taking the driver’s education course affects the probability of passing the road test.

Other answers are possible.

Marking Key

1 mark for appropriate explanation
Question 9

Determine the number of paths you can use to go from point A to point B if you can only move south or east. Show your work.

There are 360 ways to go from point A to point B.

Marker Note(s):
→ Allow one addition error without any mark deduction.

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<tr>
<td>1</td>
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John has 24 coins in his piggy bank and 6 of them are quarters. He reaches into his piggy bank and pulls out a coin at random.

a) Determine the probability that the coin will be a quarter.

\[ P(\text{quarter}) = \frac{6}{24} \text{ or } 0.25 \text{ or } 25\% \]

b) Determine the odds against the coin being a quarter.

\[ 18:6 \text{ or } 3:1 \]
Question 11  
Learning Outcomes: 12A.P.4, 12A.P.5  
Question Type: Short Answer  

A group of 6 friends is going to a concert. How many different ways can they sit in a row if Jasmin and Leena must sit beside each other? Show your work.

\[ 5! \times 2! = 240 \]

There are 240 ways.

Marking Key

<table>
<thead>
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<tbody>
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<td>1 mark for correct answer</td>
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</table>
Among a group of students, 65% will attend a hockey game, 55% will go out for supper, and 30% will attend a hockey game and go out for supper.

Determine the percentage of students who will neither attend a hockey game nor go out for supper. Show your work.

10% will do neither.

OR

\[
1 - P(A \text{ or } B) = 1 - \left[ P(A) + P(B) - P(A \text{ and } B) \right] \\
= 1 - (0.65 + 0.55 - 0.30) \\
= 1 - 0.90 \\
= 0.10 \\
\therefore 10\% \text{ will do neither.}
\]

Marking Key

<table>
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<td>1 mark for correct answer</td>
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</table>
A fisherman knows that the probability of catching a fish depends on the weather. If it is raining, the probability of catching a fish is 30%. If it is not raining, the probability of catching a fish is 10%. During an average fishing season, it rains 20% of the time.

Determine the probability that the fisherman will catch a fish on any given day. Show your work.

\[
P(\text{catch fish on any given day}) = (0.20)(0.30) + (0.80)(0.10)
\]
\[
= 0.06 + 0.08
\]
\[
= 0.14 \text{ or } 14\%
\]

The probability will be 0.14 or 14%.

Marking Key

1 mark for appropriate work

1 mark for correct answer
A school’s drama club includes 14 members: 8 boys and 6 girls. Four members are selected to attend a workshop.

a) How many possible groups of 4 members can be selected if there are no restrictions?

(1 mark)

\[ \binom{14}{4} = 1001 \]

There are 1001 possible groups.

b) How many possible groups of 4 members can be selected if at least one boy must be in the group? Show your work.

(2 marks)

Total number of possible groups – number of groups with no boys

\[ \binom{14}{4} - \left( \binom{8}{0} \times \binom{6}{4} \right) \]

\[ = 1001 - 15 \]

\[ = 986 \]

There are 986 possible groups.

OR

1 boy: \[ \binom{8}{1} \times \binom{6}{3} \]

\[ = 160 \]

2 boys: \[ \binom{8}{2} \times \binom{6}{2} \]

\[ = 420 \]

3 boys: \[ \binom{8}{3} \times \binom{6}{1} \]

\[ = 336 \]

4 boys: \[ \binom{8}{4} \times \binom{6}{0} \]

\[ = 70 \]

\[ 160 + 420 + 336 + 70 = 986 \]

There are 986 possible groups.

Marking Key

1 mark for correct answer in (a)
1 mark for appropriate work in (b)
1 mark for correct answer in (b)
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FINANCIAL MATHEMATICS

Question 15
Learning Outcome: 12A.FM.3
Question Type: Multiple Choice
Total: 1 mark

Zoe made an investment of $5000.00. Ten (10) years later, the investment was worth $6917.11. What was the annual interest rate, if the interest was compounded monthly?

A. 3.25%
B. 3.30%
C. 3.83%
D. 39.59%

Common Errors
B: C/Y = 1
C: $1917.11 ∈$5000.00 × 1 / 10 × 100
D: P/Y = 12

Question 16
Learning Outcome: 12A.FM.2
Question Type: Multiple Choice
Total: 1 mark

Which of the following assets is most likely to appreciate in value?

A. car
B. house
C. computer
D. television
Marko wants a new sports car. He wonders whether buying or leasing the car would be the better option.

**Option 1: Buying**
- purchase price of $30 000.00 (taxes included)
- payments every two weeks
- down payment of $5000.00
- interest rate of 3.00%, compounded every two weeks

**Option 2: Leasing**
- monthly payments of $300.00 (taxes included) for 5 years
- residual value of $15 000.00 (taxes included)

a) If Marko chooses Option 1 and wants to pay off the car over a five-year period, how much would his payment be every two weeks? Show your work.

(2 marks)

His payment will be $207.20.
Question 17 continued

b) Calculate the total cost of Option 2 if Marko purchases the car for its residual value at the end of the lease.

(1 mark)

Total cost = \((300.00 \times 5 \times 12) + 15000.00\)

= \$33000.00


c) Which option should Marko choose? Explain your reasoning.

(1 mark)

\((207.20 \times 130) + 5000.00 = 31936.00\)

Marko should choose Option 1 because the total cost is lower.

OR

Marko should choose Option 2 because he would have lower monthly payments.

Other answers are possible.

Marker Note(s):
→ A maximum of 1 error is allowed in the input values of a financial template in (a) (award the mark for appropriate work, but not the mark for correct answer).

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<td>3</td>
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<td>4</td>
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</table>
Mr. Van Wyck’s assets are worth $650,000.00. The mortgage on his house is $250,000.00 and he owes $130,000.00 in total on his credit lines and credit cards.

a) Calculate Mr. Van Wyck’s net worth.

\[
\text{Net worth} = 650,000.00 - 250,000.00 - 130,000.00 = 270,000.00
\]

His net worth is $270,000.00.

b) Calculate Mr. Van Wyck’s debt to equity ratio. Based on your answer, do you think the bank will lend him money? Explain.

\[
\text{Debt to equity ratio} = \frac{380,000.00 - 250,000.00}{270,000.00} \times 100 = 48.15\%
\]

Yes, the bank will lend him money since his debt to equity ratio is under 50%.

Marking Key

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<tr>
<th>Mark</th>
<th>Correctness</th>
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<tbody>
<tr>
<td>1</td>
<td>1 mark for correct net worth in (a)</td>
</tr>
<tr>
<td>2</td>
<td>1 mark for correct debt to equity ratio in (b)</td>
</tr>
<tr>
<td>3</td>
<td>1 mark for correct explanation in (b)</td>
</tr>
</tbody>
</table>
Francis makes a one-time investment of $12,000.00 in a registered retirement savings plan at 5.00%, compounded semi-annually. He plans to withdraw the money when he retires in 30 years.

a) Determine the value of the investment when Francis retires. Show your work.

(2 marks)

The value of the investment is $52,797.48.

b) Calculate his rate of return over the 30 years. Show your work.

(2 marks)

$52,797.48 – $12,000.00 = $40,797.48

rate of return = \( \frac{\$40,797.48}{\$12,000.00} \times 100 = 339.98\% \)

His rate of return is 339.98%.

Marker Note(s):
→ A maximum of 1 error is allowed in the input values of a financial template in (a) (award the mark for appropriate work, but not the mark for correct answer).

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<tr>
<td>1 mark for appropriate work in (a)</td>
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<tr>
<td>2 mark for correct answer in (a)</td>
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<tr>
<td>3 mark for appropriate work in (b)</td>
</tr>
<tr>
<td>4 mark for correct answer in (b)</td>
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</tbody>
</table>
Therese and Alphonse purchased a house valued at $354 000.00. They made a $60 000.00 down payment and obtained a mortgage amortized over 25 years at an interest rate of 4.75%, compounded semi-annually.

a) Determine Therese and Alphonse’s monthly mortgage payment. Show your work.

(2 marks)

\[
\begin{align*}
N &= 300 \\
I &= 4.75 \% \\
P &= 294 000 \\
PMT &= -1 668.32 \\
P/Y &= 12 \\
C/Y &= 2 \\
PMT &= \text{BEGIN}
\end{align*}
\]

Their monthly mortgage payment is $1668.32.

b) What will be the balance owing on the mortgage after 5 years?

(1 mark)

\[
\text{bal(60)} = 259 178.21
\]

The balance owing on the mortgage after 5 years is $259 178.21.
c) After the initial 5-year period, Therese and Alphonse renegotiate their mortgage. The bank offers them an interest rate of 2.25%, compounded semi-annually. If their monthly payment remains the same, how much sooner will they be able to pay off their mortgage? Show your work.

(2 marks)

They will be able to pay off their mortgage 56 months (4 years and 8 months) sooner.

Marker Note(s):
→ A maximum of 1 error is allowed in the input values of a financial template in (a) and (c) (award the mark for appropriate work, but not the mark for correct answer).

<table>
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<th>Marking Key</th>
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<tr>
<td>1 mark for appropriate work in (a)</td>
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<tr>
<td>1 mark for correct answer in (a)</td>
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<tr>
<td>1 mark for correct balance owing in (b)</td>
</tr>
<tr>
<td>1 mark for appropriate work in (c)</td>
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<tr>
<td>1 mark for correct answer in (c)</td>
</tr>
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</table>
Philippa wants to cover her dining room floor with linoleum. The floor measures 14 ft. \times 12 \text{ ft.} The linoleum costs $13.99 per square yard and must be purchased in whole units.

What will be the total cost for the flooring, including taxes? Show your work. (Note: GST = 5%, PST = 8%)

Area:
14 \text{ ft.} \times 12 \text{ ft.} = 168 \text{ ft}^2

\frac{168 \text{ ft}^2}{9 \text{ ft}^2/\text{yd}^2} = 18.67 \text{ yd}^2

19 \text{ yd}^2 \times $13.99/\text{yd}^2 = $265.81

$13.29 \text{ (GST)}

+ $21.26 \text{ (PST)}

$300.36

The total cost will be $300.36.

**Marker Note(s):**
→ If student combined taxes and multiplied by 13%, award mark 2 for $300.37 as the final answer.

<table>
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</table>
Mackenzie Construction was awarded the contract to build gravel shoulders along the highway between Wabowden and Thompson. (Diagram is not drawn to scale.)

The gravel shoulders will be
- along a 22 mile segment of the highway
- on both sides of the highway
- 10 feet wide
- 20 inches deep

Note: 1 mile = 5280 feet

How many truckloads of gravel will be needed for the project if a truck holds 20 cubic yards of gravel? Show your work.

length: 22 mi. × 5280 ft./mi. = 116 160 ft.
depth: 20 in. × \(\frac{1 \text{ ft.}}{12 \text{ in.}}\) = \(\frac{20}{12}\) ft.

volume = length × width × depth

\[= \left(116 160 \text{ ft.} \times \frac{20}{12} \text{ ft.} \times 10 \text{ ft.}\right) \times 2 \text{ shoulders}\]

\[= 3 872 000 \text{ ft}^3\]

\[\frac{3 872 000 \text{ ft}^3 \times \frac{1 \text{ yd}^3}{27 \text{ ft}^3}}{143 407.41 \text{ yd}^3}\]

\[143 407.41 \text{ yd}^3 \times \frac{1 \text{ truckload}}{20 \text{ yd}^3} = 7170.37 \text{ truckloads}\]

7171 truckloads will be needed.

**Marker Note(s):**
→ *Award full marks (without 🌟) if student leaves final answer as 7170.37 truckloads.*

### Marking Key

<table>
<thead>
<tr>
<th></th>
<th>Marking Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 mark for appropriate work calculating volume of gravel</td>
</tr>
<tr>
<td>2</td>
<td>1 mark for correct volume of gravel</td>
</tr>
<tr>
<td>3</td>
<td>1 mark for correct conversion from cubic feet to cubic yards</td>
</tr>
<tr>
<td>4</td>
<td>1 mark for correct number of truckloads</td>
</tr>
</tbody>
</table>
LOGICAL REASONING

Question 23  Total: 1 mark

Learning Outcome: 12A.L.3  Question Type: Multiple Choice

Select the statement below which best completes the following truth table.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$q$</td>
<td></td>
</tr>
<tr>
<td>True</td>
<td>True</td>
<td>True</td>
</tr>
<tr>
<td>True</td>
<td>False</td>
<td>False</td>
</tr>
<tr>
<td>False</td>
<td>True</td>
<td>False</td>
</tr>
<tr>
<td>False</td>
<td>False</td>
<td>False</td>
</tr>
</tbody>
</table>

A. $p \land q$

B. $p \lor q$

C. $p \Rightarrow q$

D. $p \Leftrightarrow q$

Common Errors
B: $p$ or $q$
C: if $p$, then $q$
D: $p$ if and only if $q$
Given the following situation:

- the universal set $U = \{\text{positive integers less than 10}\}$
- $A = \{2, 3, 4, 5, 6\}$
- $B = \{\text{even numbers of } U\}$

Determine $A \cap B$.

$$A \cap B = \{2, 4, 6\}$$

Marking Key

|   | 1 mark for correct answer |
Given the statement: “If I live in Winnipeg, then I live in Manitoba.”

a) Write the inverse of the given statement.

(1 mark)
If I do not live in Winnipeg, then I do not live in Manitoba.

b) Is the given statement biconditional? Explain.

(1 mark)
No, the statement is not biconditional because I can live in Manitoba but not live in Winnipeg.

c) Write the contrapositive of the given statement.

(1 mark)
If I do not live in Manitoba, then I do not live in Winnipeg.

<table>
<thead>
<tr>
<th>Marking Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>
Exemplars
### Exemplar 1

**Question 2**

<table>
<thead>
<tr>
<th>Total: 2 marks</th>
</tr>
</thead>
</table>

When inflating a balloon, the volume of air in the balloon can be modelled by the equation:

\[
V = 0.02c^3 - 0.73c^2 + 11.30c - 12.79
\]

where \( V \) represents the volume (cm\(^3\)) of air in the balloon and \( c \) represents the circumference (cm) of the balloon.

How much air would need to be blown into the balloon so that it has a circumference of 60 cm? Show your work.

\[
\begin{align*}
V &= 0.02c^3 - 0.73c^2 + 11.30c - 12.79 \\
V &= 0.02(60)^3 - 0.73(60)^2 + 11.30(60) - 12.79 \\
V &= 2357.21 \text{cm}^3
\end{align*}
\]

The balloon would need to be filled with 2357.21 cm\(^3\) of air to have a circumference of 60 cm.

---

2 marks:

1. 1 mark for appropriate work
2. 1 mark for correct answer
Exemplar 2

Question 2

When inflating a balloon, the volume of air in the balloon can be modelled by the equation:

\[ V = 0.02c^3 - 0.73c^2 + 11.30c - 12.79 \]

where \( V \) represents the volume (cm\(^3\)) of air in the balloon and \( c \) represents the circumference (cm) of the balloon.

How much air would need to be blown into the balloon so that it has a circumference of 60 cm? Show your work.

when I graph the function and put in a horizontal line in y = 0. Then I put 2nd trace to find the intersection point. when y = 60 x = 15.01

1 mark:

- \( \checkmark \) → 1 mark for correct answer
- \( \checkmark \checkmark \) → 0.5 mark deduction (if applicable) for not including the units in the final answer
- \( \checkmark \checkmark \checkmark \) → 0.5 mark deduction (if applicable) for not stating or incorrectly stating the final answer
Cobalt-60 is an isotope used in medical imaging. It decays naturally over time according to the equation:

\[ t = 35.01 - 7.60 \ln P \]

where \( t \) represents the time in years

and \( P \) represents the percentage of the original material that is still radioactive.

State the domain and the range of the logarithmic function in the context of this situation.

**Domain:** \( 0 < P \leq 100 \)

**Range:** \( 0.011 \leq t \leq 35.01 \)

**1 mark:**

① → 1 mark for correct domain

② → 0.5 mark deduction (if applicable) for not stating or incorrectly stating the final answer
Cobalt-60 is an isotope used in medical imaging. It decays naturally over time according to the equation:

\[ t = 35.01 - 7.60 \ln P \]

where \( t \) represents the time in years
and \( P \) represents the percentage of the original material that is still radioactive.

State the domain and the range of the logarithmic function in the context of this situation.

**Domain:** \( \{0 < x \leq 100\} \)

**Range:** \( \{y \geq 0\} \)

<table>
<thead>
<tr>
<th>2 marks:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 → 1 mark for correct domain</td>
</tr>
<tr>
<td>2 → 1 mark for correct range</td>
</tr>
<tr>
<td>( \text{Mark} ) → 0.5 mark deduction (if applicable) for not stating or incorrectly stating the final answer</td>
</tr>
</tbody>
</table>
THIS PAGE WAS INTENTIONALLY LEFT BLANK.
A store owner wants to increase his profits. Suppose that his operating costs and his earnings are modelled by the following equations:

costs: \[ y = 0.04x^2 + 44.00x + 1500.00 \]

earnings: \[ y = 1.25^x \]

where \( x \) represents the time, in hours, that the store is open per week and \( y \) represents the operating costs or earnings, in dollars.

a) Create a clearly labelled graph of both equations on the axes below.

(2 marks)
Exemplar 1 (continued)

b) Using a graphing calculator or graphing software, determine the minimum number of hours that the store should stay open in order to make a profit (earnings are greater than the cost). Explain how you arrived at your answer. State your answer to one decimal place.

(2 marks) The intersect value happens at \((36, 3139)\) 
*Not exact intercept so the intersect should be where the Cost and Earnings are equal. This means that 37 hours is the minimum number of hours it has to be open so they make a profit.

\[
\begin{align*}
E &= 1.25^{36.083979} \\
C &= 0.04(36.083979)^2 + 44(36.083979) + 1500 \\
E &= 3139.78 \\
C &= 3139.78 \\
E &= C \\
E &= 1.25^{37} \\
C &= 0.04(37)^2 + 44(37) + 1500 \\
C &= 3182.76 \\
E &= 3851.86
\end{align*}
\]

\(E > C\)

4 marks:

1. 1 mark for correct graph with appropriate shapes in (a)
2. 1 mark for including: labels for the axes, units for the axes, and scales for the axes in (a)
3. 1 mark for appropriate work in (b)
4. 1 mark for correct answer in (b)

- 0.5 mark deduction (if applicable) for not including one of the following on the graph: labels for the axes, units for the axes, or scales for the axes
- 0.5 mark deduction (if applicable) for rounding too soon or rounding incorrectly
A store owner wants to increase his profits. Suppose that his operating costs and his earnings are modelled by the following equations:

\[ \text{costs: } y = 0.04x^2 + 44.00x + 1500.00 \]
\[ \text{earnings: } y = 1.25^x \]

where \( x \) represents the time, in hours, that the store is open per week and \( y \) represents the operating costs or earnings, in dollars.

a) Create a clearly labelled graph of both equations on the axes below. (2 marks)
Exemplar 2 (continued)

b) Using a graphing calculator or graphing software, determine the minimum number of hours that the store should stay open in order to make a profit (earnings are greater than the cost). Explain how you arrived at your answer. State your answer to one decimal place.

(2 marks)

To find the minimum number of hours that the store should stay open I have to find the intersection of C and E (Graph → 2nd → Trace → 5 → Press Enter on 1st curve → Press Enter on 2nd curve → Press Enter in spot where you think intersection is).

Intersection:
\[ x = 36.083979 \]
\[ y = 313.978 \]

Minimum number of hours is 36.0

3 marks:
- 1 mark for including: labels for the axes, units for the axes, and scales for the axes in (a)
- 1 mark for appropriate work in (b)
- 1 mark for correct answer in (b)
- 0.5 mark deduction (if applicable) for rounding too soon or rounding incorrectly
Exemplar 1

Question 5

The average monthly temperatures for one year in Snow Lake are shown in the table below.

<table>
<thead>
<tr>
<th>Month</th>
<th>Average Monthly Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>–20.2</td>
</tr>
<tr>
<td>February</td>
<td>–15.1</td>
</tr>
<tr>
<td>March</td>
<td>–8.0</td>
</tr>
<tr>
<td>April</td>
<td>1.9</td>
</tr>
<tr>
<td>May</td>
<td>9.6</td>
</tr>
<tr>
<td>June</td>
<td>15.8</td>
</tr>
<tr>
<td>July</td>
<td>18.6</td>
</tr>
<tr>
<td>August</td>
<td>17.3</td>
</tr>
<tr>
<td>September</td>
<td>10.0</td>
</tr>
<tr>
<td>October</td>
<td>3.2</td>
</tr>
<tr>
<td>November</td>
<td>–8.1</td>
</tr>
<tr>
<td>December</td>
<td>–17.6</td>
</tr>
</tbody>
</table>

a) Determine the sinusoidal equation that best represents this data.

(1 mark)

\[ 20.16 \times \sin (0.47 x - 1.75) + 1.44 \]
Exemplar 1 (continued)

b) Using your sinusoidal equation in (a), calculate the length of time that the average monthly temperature was at or above 10°C. Show your work.

(2 marks)

\[ 4.96 \text{ to 9.05} \]

\[ 4.09 \text{ months} \]

3 marks:

1 → 1 mark for correct sinusoidal equation in (a)
2 → 1 mark for appropriate work in (b)
3 → 1 mark for correct answer in (b)

51 → 0.5 mark deduction (if applicable) for not including one of the following in the equation: “y =”, “sin”, “ln”, or “x”, or for writing parameters separately from the equation
59 → 0.5 mark deduction (if applicable) for making a transcription or transposition error
Exemplar 2

Question 5

The average monthly temperatures for one year in Snow Lake are shown in the table below.

<table>
<thead>
<tr>
<th>Month</th>
<th>Average Monthly Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>–20.2</td>
</tr>
<tr>
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<td>–15.1</td>
</tr>
<tr>
<td>March</td>
<td>–8.0</td>
</tr>
<tr>
<td>April</td>
<td>1.9</td>
</tr>
<tr>
<td>May</td>
<td>9.6</td>
</tr>
<tr>
<td>June</td>
<td>15.8</td>
</tr>
<tr>
<td>July</td>
<td>18.6</td>
</tr>
<tr>
<td>August</td>
<td>17.3</td>
</tr>
<tr>
<td>September</td>
<td>10.0</td>
</tr>
<tr>
<td>October</td>
<td>3.2</td>
</tr>
<tr>
<td>November</td>
<td>–8.1</td>
</tr>
<tr>
<td>December</td>
<td>–17.6</td>
</tr>
</tbody>
</table>

a) Determine the sinusoidal equation that best represents this data.

(1 mark)

\[ y = 20.16 \sin (0.47 + 4.54) - 1.44 \]
Exemplar 2 (continued)

b) Using your sinusoidal equation in (a), calculate the length of time that the average monthly temperature was at or above 10°C. Show your work.

(2 marks)

\[
\text{9.048 - 4.9629 = 4.0851}
\]

:: The length of time that the average monthly temp is at or above 10°C

is 4.08 months.

3 marks:

1. 1 mark for correct sinusoidal equation in (a)
2. 1 mark for appropriate work in (b)
3. 1 mark for correct answer in (b)

4. 0.5 mark deduction (if applicable) for not including one of the following in the equation: “y =”, “sin”, “ln”, or “x”, or for writing parameters separately from the equation

5. 0.5 mark deduction (if applicable) for rounding too soon or rounding incorrectly
THIS PAGE WAS INTENTIONALLY LEFT BLANK.
A water well has a pump that can initially extract 300 gallons of water per day. The water level in the well begins to drop according to the function:

\[ W = 300 \left(\frac{4}{5}\right)^{\frac{d}{10}} \]

where \( W \) represents the volume of water, in gallons, extracted daily and \( d \) represents the number of days after the water level begins to drop.

a) Determine the volume of water extracted on the 100th day after the water level begins to drop. Show your work.

\( \text{(2 marks)} \)

\( \text{Answer on printout} \)

b) On what day will the pump first extract less than 75 gallons of water per day? Show your work.

\( \text{(2 marks)} \)

\( \text{Answer on printout} \)
Exemplar 1 (continued)

The volume of water extracted on the 100th day after the water level begins to drop is 32.21 gallons.
Exemplar 1 (continued)

4 marks:

① → 1 mark for appropriate work in (a)
② → 1 mark for correct answer in (a)
③ → 1 mark for appropriate work in (b)
④ → 1 mark for correct answer in (b)
⑤ → 0.5 mark deduction (if applicable) for not stating or incorrectly stating the final answer
Exemplar 2

Question 6

A water well has a pump that can initially extract 300 gallons of water per day. The water level in the well begins to drop according to the function:

\[ W = 300 \left( \frac{4}{5} \right)^d \]

where \( W \) represents the volume of water, in gallons, extracted daily and \( d \) represents the number of days after the water level begins to drop.

a) Determine the volume of water extracted on the 100th day after the water level begins to drop. Show your work.

\(2\) marks

b) On what day will the pump first extract less than 75 gallons of water per day? Show your work.

\(2\) marks

3 marks:

1 \(\rightarrow\) 1 mark for appropriate work in (a)
2 \(\rightarrow\) 1 mark for correct answer in (a)
3 \(\rightarrow\) 1 mark for appropriate work in (b)
4 \(\rightarrow\) 0.5 mark deduction (if applicable) for rounding too soon or rounding incorrectly
Exemplar 1

<table>
<thead>
<tr>
<th>Question 8</th>
<th>Total: 1 mark</th>
</tr>
</thead>
</table>

Brien states that taking a driver’s education course and passing the road test on the first attempt are dependent events. Explain why Brien is correct.

They would be dependent because the course teaches proper driving.

0 marks:  
→ no criteria met
Exemplar 2

Question 8  

Brien states that taking a driver’s education course and passing the road test on the first attempt are dependent events. Explain why Brien is correct.

As the driver’s ed. course allows students to drive sooner and helps prepare for their driving test, it is likely that a higher percentage of students that take the course pass on their first try than the percent of students that pass on the first try without the course. In this case, they would be dependant.

1 mark:

1 mark for appropriate explanation
Determine the number of paths you can use to go from point A to point B if you can only move south or east. Show your work.

355 ways to go from Point A → Point B

2 marks:
1 → 1 mark for appropriate work
2 → 1 mark for correct answer
Determine the number of paths you can use to go from point A to point B if you can only move south or east. Show your work.

\[ 35 + 20 = 55 \]

you can go from A to B 55 ways!

0 marks:
\[ \rightarrow \text{no criteria met} \]
Exemplar 1

Question 10

John has 24 coins in his piggy bank and 6 of them are quarters. He reaches into his piggy bank and pulls out a coin at random.

a) Determine the probability that the coin will be a quarter.

\[
\frac{6}{24} = 0.25 \%
\]

(1 mark)

b) Determine the odds against the coin being a quarter.

(1 mark)

2 marks:

1 → 1 mark for correct answer in (a)

2 → 1 mark for correct answer in (b)

4 → 0.5 mark deduction (if applicable) for not stating or incorrectly stating the final answer
Exemplar 2

Question 10  

John has 24 coins in his piggy bank and 6 of them are quarters. He reaches into his piggy bank and pulls out a coin at random.

a) Determine the probability that the coin will be a quarter.

(1 mark)

\[
\frac{6}{24} = \frac{1}{4}
\]

b) Determine the odds against the coin being a quarter.

(1 mark)

\[
\frac{24 - 6}{6} = \frac{18}{6} = \frac{3}{4}
\]

1 mark:
1 → 1 mark for correct answer in (a)
A group of 6 friends is going to a concert. How many different ways can they sit in a row if Jasmin and Leena must sit beside each other? Show your work.

\[ 5P_5 \times 2P_2 = 120 \times 2 = 240 \]

The six friends can sit 240 different ways.
A group of 6 friends is going to a concert. How many different ways can they sit in a row if Jasmin and Leena must sit beside each other? Show your work.

\[
\frac{1 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 1 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = 24 \\
\frac{4 \cdot 3 \cdot 1 \cdot 1 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 \cdot 1} = 24 \\
\frac{4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 \cdot 1} = 24
\]

\[120\] ways they can sit in a row
Exemplar 1

<table>
<thead>
<tr>
<th>Question 12</th>
<th>Total: 2 marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Among a group of students, 65% will attend a hockey game, 55% will go out for supper, and 30% will attend a hockey game and go out for supper. Determine the percentage of students who will neither attend a hockey game nor go out for supper. Show your work.</td>
<td></td>
</tr>
</tbody>
</table>

Among a group of students, 65% will attend a hockey game, 55% will go out for supper, and 30% will attend a hockey game and go out for supper.

Determine the percentage of students who will neither attend a hockey game nor go out for supper. Show your work.

\[
\begin{align*}
&35 + 30 + 25 = 90 \\
&90 + 10 = 100 \\
&\text{final answer not stated}
\end{align*}
\]

2 marks:

1 → 1 mark for appropriate work
2 → 1 mark for correct answer
4 → 0.5 mark deduction (if applicable) for not stating or incorrectly stating the final answer
Among a group of students, 65% will attend a hockey game, 55% will go out for supper, and 30% will attend a hockey game and go out for supper.

Determine the percentage of students who will neither attend a hockey game nor go out for supper. Show your work.

\[
\begin{align*}
65\% & \text{ hockey} \\
-30\% & \text{ both} \\
\hline
35\% \\
55\% & \text{ supper} \\
-30\% & \text{ both} \\
\hline
25\% \\
35\% + 25\% & = 60\% \\
100\% & \\
-60\% & \\
= 40\% \\
\end{align*}
\]

1 mark:
\(\Rightarrow\) 1 mark for appropriate work
A fisherman knows that the probability of catching a fish depends on the weather. If it is raining, the probability of catching a fish is 30%. If it is not raining, the probability of catching a fish is 10%. During an average fishing season, it rains 20% of the time.

Determine the probability that the fisherman will catch a fish on any given day. Show your work.
A fisherman knows that the probability of catching a fish depends on the weather. If it is raining, the probability of catching a fish is 30%. If it is not raining, the probability of catching a fish is 10%. During an average fishing season, it rains 20% of the time.

Determine the probability that the fisherman will catch a fish on any given day. Show your work.
A school’s drama club includes 14 members: 8 boys and 6 girls. Four members are selected to attend a workshop.

a) How many possible groups of 4 members can be selected if there are no restrictions?

\[
\binom{14}{4} = \frac{14 \cdot 13 \cdot 12 \cdot 11}{4 \cdot 3 \cdot 2 \cdot 1} = 1,001 \text{ possibilities if no restrictions.}
\]

(1 mark)

b) How many possible groups of 4 members can be selected if at least one boy must be in the group? Show your work.

(2 marks)

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Case 1)} & \text{Case 2)} & \text{Case 3)} & \text{Case 4)} \\
\hline
1 \text{ boy} & 2 \text{ boys} & 3 \text{ boys} & 4 \text{ boys} \\
\hline
8 \cdot 7 \cdot 6 \cdot 5 & 8 \cdot 7 \cdot 6 \cdot 5 & 8 \cdot 7 \cdot 6 \cdot 5 & \mathbf{8 \cdot 7 \cdot 6 \cdot 5} \\
\hline
= 960 & = 1680 & = 2016 & \mathbf{= 1680} \\
\hline
\end{array}
\]

\[
960 + 1680 + 1680 + 2016 = 6336 \quad \Rightarrow \text{there are 6336 possible groups.}
\]

2 marks:
- 1 mark for appropriate work in (b)
- 1 mark for correct answer in (b)
A school’s drama club includes 14 members: 8 boys and 6 girls. Four members are selected to attend a workshop.

a) How many possible groups of 4 members can be selected if there are no restrictions?

\[ 14 \binom{4}{2} = 1001 \]

1001 possible groups

(1 mark)

b) How many possible groups of 4 members can be selected if at least one boy must be in the group? Show your work.

\( \begin{align*}
1^3 & \quad 8 \cdot 6 \cdot 5 \cdot 4 = 960 \\
2^3 & \quad 8 \cdot 7 \cdot 6 \cdot 5 = 1680 \\
3^3 & \quad 8 \cdot 7 \cdot 6 \cdot 5 = 2016 \\
4^3 & \quad 8 \cdot 7 \cdot 6 \cdot 5 = 1680
\end{align*} \)

\[ 960 + 1680 + 2016 + 1680 = 6336 \]

6336 possible groups

2 marks:
1 → 1 mark for correct answer in (a)
3 → 1 mark for correct answer in (b)
THIS PAGE WAS INTENTIONALLY LEFT BLANK.
Marko wants a new sports car. He wonders whether buying or leasing the car would be the better option.

**Option 1: Buying**
- purchase price of $30 000.00 (taxes included)
- payments every two weeks
- down payment of $5000.00
- interest rate of 3.00%, compounded every two weeks

**Option 2: Leasing**
- monthly payments of $300.00 (taxes included) for 5 years
- residual value of $15 000.00 (taxes included)

a) If Marko chooses Option 1 and wants to pay off the car over a five-year period, how much would his payment be every two weeks? Show your work.

\[ \text{Buying} \]
\[ N = 130 \]
\[ I\% = 3 \]
\[ PV = $28900 \]
\[ PMT = 0 \Rightarrow \$239.53 \]
\[ FU = 0 \]
\[ P/Y = 26 \]
\[ C/Y = 26 \]
\[ \text{END} \]

\[ 30000(1.13) = 33900 \]
\[ -5000 \]
\[ \$28900 \]

Marko’s payment will be $239.53.
b) Calculate the total cost of Option 2 if Marko purchases the car for its residual value at the end of the lease.

(1 mark)

\[
\begin{align*}
N &= 60 \\
I &= \\
PV &= \\
PMT &= -$339 \\
FV &= 0 \\
p/4 &= 12 \\
c/4 &= 12 \\
\text{END}
\end{align*}
\]

\[
\begin{align*}
300(1.13) &= $339 \\
15000(1.13) &= $16950 \\
339(60) &= 20340 \\
+16950 &= 37290 \\
\end{align*}
\]

c) Which option should Marko choose? Explain your reasoning.

(1 mark)

I think leasing would be the better option. The total cost of the vehicle ends up being the same. In this case there’s no interest on the lease. If you have $17000 to buy the car after the lease is up you’ll be left with the vehicle that gave you easier monthly payments instead of every two weeks.
Marko wants a new sports car. He wonders whether buying or leasing the car would be the better option.

**Option 1: Buying**
- purchase price of $30 000.00 (taxes included)
- payments every two weeks
- down payment of $5000.00
- interest rate of 3.00%, compounded every two weeks

**Option 2: Leasing**
- monthly payments of $300.00 (taxes included) for 5 years
- residual value of $15 000.00 (taxes included)

a) If Marko chooses Option 1 and wants to pay off the car over a five-year period, how much would his payment be every two weeks? Show your work.

(2 marks)

\[
N = 26 \times 5 = 130 \\
\text{I\\%} = 3 \\
\text{PV} = -35 000 \\
? \text{PMT} = \\
\text{FV} = 0 \\
\text{P/Y} = 26 \\
\text{C/Y} = 26 \\
\text{END}
\]

Marko’s payments would be $290.08
b) Calculate the total cost of Option 2 if Marko purchases the car for its residual value at the end of the lease.

\[ \begin{align*}
300 \times 60 &= 18000 \\
5 \times 12 &= 60 \\
15000 + 18000 &= 33000
\end{align*} \]

(1 mark)

2 marks:
1 → 1 mark for appropriate work in (a)
2 → 1 mark for correct total cost of Option 2 in (b)
3 → 0.5 mark deduction (if applicable) for not including the units in the final answer

(1 mark)

Leasing is the better option because it is $2000 cheaper.

c) Which option should Marko choose? Explain your reasoning.

(1 mark)
Mr. Van Wyck’s assets are worth $650 000.00. The mortgage on his house is $250 000.00 and he owes $130 000.00 in total on his credit lines and credit cards.

a) Calculate Mr. Van Wyck’s net worth.

\[ \text{Net} = \text{assets} - \text{liabilities} \]
\[ \text{Net} = 650 000 - 380 000 = 270 000 \]

b) Calculate Mr. Van Wyck’s debt to equity ratio. Based on your answer, do you think the bank will lend him money? Explain.

\[ \text{DtoE} = \left( \frac{\text{liabilities} - \text{mortgage}}{\text{Networth}} \right) \times 100 \]
\[ \frac{380 000 - 250 000}{270 000} \times 100 \]
\[ = 48.14 \]
Mr. Van Wyck’s assets are worth $650 000.00. The mortgage on his house is $250 000.00 and he owes $130 000.00 in total on his credit lines and credit cards.

a) Calculate Mr. Van Wyck’s net worth.

(1 mark)

\[
\text{net worth} = \text{total assets} - \text{total liabilities}
\]

\[
\text{net worth} = (650 000) - (250 000 + 130 000)
\]

\[
\text{net worth} = 650 000 - 380 000
\]

\[
\text{net worth} = 270 000
\]

b) Calculate Mr. Van Wyck’s debt to equity ratio. Based on your answer, do you think the bank will lend him money? Explain.

(2 marks)

\[
d to e = \left( \frac{\text{total liabilities} - \text{mortgage}}{\text{net worth}} \right) \times 100
\]

\[
d to e = \left( \frac{380 000 - 250 000}{270 000} \right) \times 100
\]

\[
d to e = \frac{130 000}{270 000} \times 100
\]

\[
d to e = 48.15\%
\]

Yes. Because I believe that this is a good rate.

2 marks:
① → 1 mark for correct net worth in (a)
② → 1 mark for correct debt to equity ratio in (b)
Exemplar 1

**Question 19**

Francis makes a one-time investment of $12 000.00 in a registered retirement savings plan at 5.00%, compounded semi-annually. He plans to withdraw the money when he retires in 30 years.

a) Determine the value of the investment when Francis retires. Show your work.

(2 marks)

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[ A = 12000.00\left(1 + \frac{0.05}{2}\right)^{30(2)} \]

\[ = 52797.48 \]

b) Calculate his rate of return over the 30 years. Show your work.

(2 marks)

\[ \frac{52797.48 - 12000}{12000} = 3.4 \]

\[ 3.4 \times 100 = 340.1\% \]

**4 marks:**

1 → 1 mark for appropriate work in (a)
2 → 1 mark for correct answer in (a)
3 → 1 mark for appropriate work in (b)
4 → 1 mark for correct answer in (b)
5 → 0.5 mark deduction (if applicable) for rounding too soon or rounding incorrectly
Exemplar 2

Francis makes a one-time investment of $12 000.00 in a registered retirement savings plan at 5.00%, compounded semi-annually. He plans to withdraw the money when he retires in 30 years.

a) Determine the value of the investment when Francis retires. Show your work.

$2 marks$

\[
\begin{align*}
N &= 30 \\
\text{\textit{I\%}} &= 5 \\
P \text{\textit{V}} &= 12000.00 \\
P \text{\textit{MT}} &= 0 \\
F \text{\textit{V}} &= \frac{52797.48}{1} \\
P \text{\textit{ly}} &= 1 \\
P \text{\textit{ly}} &= 2
\end{align*}
\]

The value of the investment when Francis retires is $52797.48.

b) Calculate his rate of return over the 30 years. Show your work.

$2 marks$

\[
\text{Rate of Return} = \frac{52797.48 - 12000.00}{12000.00} \times 100
\]

\[
= \frac{40797.48}{12000} \times 100
\]

\[
= 3.40
\]

$4 marks:

1. → 1 mark for appropriate work in (a)
2. → 1 mark for correct answer in (a)
3. → 1 mark for appropriate work in (b)
4. → 1 mark for correct answer in (b)

5. → 0.5 mark deduction (if applicable) for not stating or incorrectly stating the final answer
Exemplar 1

Question 20

Therese and Alphonse purchased a house valued at $354,000.00. They made a $60,000.00 down payment and obtained a mortgage amortized over 25 years at an interest rate of 4.75%, compounded semi-annually.

a) Determine Therese and Alphonse’s monthly mortgage payment. Show your work.  

(2 marks)

\[ \begin{align*} 
N &= 300 \\
I &= 4.75 \\
PV &= 297,000 \\
* PMT &= 1685.34 \\
FV &= 0 \\
P|Y &= 12 \\
C|Y &= 2 \\
\end{align*} \]

Therese and Alphonse’s monthly mortgage payment is $1685.34.

b) What will be the balance owing on the mortgage after 5 years?  

(1 mark)

\[ \text{bal}(60) = 261,822.89 \]

After 5 years, they will still owe $261,822.89.
c) After the initial 5-year period, Therese and Alphonse renegotiate their mortgage. The bank offers them an interest rate of 2.25%, compounded semi-annually. If their monthly payment remains the same, how much sooner will they be able to pay off their mortgage? Show your work.

(2 marks)

If they switch to this new mortgage plan, their mortgage will be paid off in 15.3 years. This creates a total of 20.3 years. That's 4.7 years sooner than the original 25 years.

4 marks:

1 → 1 mark for appropriate work in (a)
3 → 1 mark for correct balance owing in (b)
4 → 1 mark for appropriate work in (c)
5 → 1 mark for correct answer in (c)

→ 0.5 mark deduction (if applicable) for rounding too soon or rounding incorrectly
Exemplar 2

Question 20  

Therese and Alphonse purchased a house valued at $354 000.00. They made a $60 000.00 down payment and obtained a mortgage amortized over 25 years at an interest rate of 4.75%, compounded semi-annually.

a) Determine Therese and Alphonse’s monthly mortgage payment. Show your work.

(2 marks)

\[
\begin{align*}
N &= 50 \\
I &= 4.75 \\
PV &= -294,000 \\
PMT &= ? \\
FV &= 0 \\
p &= 2 \\
C/Y &= 2
\end{align*}
\]

b) What will be the balance owing on the mortgage after 5 years?

(1 mark)

\[ 9,178.21 \]
c) After the initial 5-year period, Therese and Alphonse renegotiate their mortgage. The bank offers them an interest rate of 2.25%, compounded semi-annually. If their monthly payment remains the same, how much sooner will they be able to pay off their mortgage? Show your work.

(2 marks)

\[
\begin{align*}
N &= 7(30) \\
I &= 2.25 \\
PV &= -259,178.21 \\
PMT &= 1018.52 \\
FV &= 0 \\
\rho/y &= 2 \\
c/y &= 2
\end{align*}
\]

After the five year period, it will only take 15 more years to pay off their mortgage.

3 marks:
1. 1 mark for appropriate work in (a)
2. 1 mark for correct balance owing in (b)
3. 1 mark for appropriate work in (c)
4. 0.5 mark deduction (if applicable) for rounding too soon or rounding incorrectly
Philippa wants to cover her dining room floor with linoleum. The floor measures 14 ft. \times 12\text{ ft.}

The linoleum costs $13.99 per square yard and must be purchased in whole units.

What will be the total cost for the flooring, including taxes? Show your work.
(Note: GST = 5\%, PST = 8\%)

\[
SA = 14 \times 12 = 168\text{ ft}^2
\]

\[
168 \times 13.99 = 2350.32
\]

\[
\text{GST} = \frac{2350.32 \times 0.05}{100} = 117.52
\]

\[
\text{PST} = \frac{2350.32 \times 0.08}{100} = 188.03
\]

\[
\text{Total Cost} = 2350.32 + 117.52 + 188.03 = 2655.87
\]

**1 mark:**

\(\text{Total Cost} \rightarrow 1\text{ mark for correct total cost}\)
Philippa wants to cover her dining room floor with linoleum. The floor measures 14 ft. × 12 ft. The linoleum costs $13.99 per square yard and must be purchased in whole units.

What will be the total cost for the flooring, including taxes? Show your work.
(Note: GST = 5%, PST = 8%)

You will need \( \frac{14 \times 12}{3} = 56 \) square yards.

\[20 \times 13.99 = 279.8\]

\[279.8 \times 0.13 = 36.374\]

\[279.8 + 36.374 = 316.174 \text{ round to}\]

\[\$316.17\]

You will have to pay \( \$316.17 \) for the flooring.

2 marks:

1. 1 mark for appropriate work
2. 1 mark for correct total cost
Mackenzie Construction was awarded the contract to build gravel shoulders along the highway between Wabowden and Thompson. (Diagram is not drawn to scale.)

The gravel shoulders will be
- along a 22 mile segment of the highway
- on both sides of the highway
- 10 feet wide
- 20 inches deep

**Note:** 1 mile = 5280 feet

How many truckloads of gravel will be needed for the project if a truck holds 20 cubic yards of gravel? Show your work.

\[
\text{Volume} = l \times w \times h
\]

**Conversions:**

\[
22 \text{ mi.} \times \frac{1 \text{ mile}}{5280 \text{ ft.}} = 116160 \text{ ft}
\]

\[
20 \text{ in.} \times \frac{1 \text{ ft}}{0.083} = 240.963 \text{ ft}
\]

Volume of gravel:

\[
116160 \text{ ft.} \times 240.963 \text{ ft.} \times 10 \text{ ft.}
\]

\[
= 279902620.8 \text{ ft}^3
\]

\[
279902620.8 \text{ ft}^3 \div \frac{1 \text{ yd}^3}{27 \text{ ft}^3} = 10366763.73 \text{ yd}^3
\]

\[
\text{Amount &: truck loads space}
\]

\[
\frac{10366763.73 \text{ yd}^3}{20 \text{ yd}^3} = 518338 \text{ loads of trucks.}
\]

**2 marks:**

- 1 mark for correct conversion from cubic feet to cubic yards
- 1 mark for correct number of truckloads

- 0.5 mark deduction (if applicable) for not using whole units appropriately
Mackenzie Construction was awarded the contract to build gravel shoulders along the highway between Wabowden and Thompson. (Diagram is not drawn to scale.)

The gravel shoulders will be:
- along a 22 mile segment of the highway
- on both sides of the highway
- 10 feet wide
- 20 inches deep

**Note:** 1 mile = 5,280 feet

How many truckloads of gravel will be needed for the project if a truck holds 20 cubic yards of gravel? Show your work.

\[
\text{Volume of 1 gravel shoulder:} \quad 10 \times 1 \frac{2}{3} \times 5280 = 88000 \text{ ft}^3
\]
\[\times 2 \text{ for both shoulders} \]
\[= 176000 \text{ ft}^3\]
\[176000 \text{ ft}^3 \times \frac{1}{9} \text{ yd}^3 \]
\[= 19555.56 \text{ yd}^3\]
\[19555.56 \div 20 \]
\[= 977.78\]

**2 marks:**
- 1 mark for appropriate work calculating volume of gravel
- 1 mark for correct number of truckloads

\[\therefore 977.78 \text{ truckloads of gravel is needed}\]
Given the following situation:

- the universal set \( U = \{ \text{positive integers less than 10} \} \)
- \( A = \{ 2, 3, 4, 5, 6 \} \)
- \( B = \{ \text{even numbers of } U \} \)

Determine \( A \cap B \).
Given the statement: “If I live in Winnipeg, then I live in Manitoba.”

a) Write the inverse of the given statement.

(1 mark)

If I don’t live in Winnipeg, then I’m not living in Manitoba.

b) Is the given statement biconditional? Explain.

(1 mark)

If I live in Manitoba, then I live in Winnipeg.

No because I can live in Steinbach.

c) Write the contrapositive of the given statement.

(1 mark)

If I don’t live in Manitoba, then I don’t live in Winnipeg.

3 marks:

1 → 1 mark for writing the inverse in (a)
2 → 1 mark for correct answer in (b)
3 → 1 mark for writing the contrapositive in (c)
Appendices
# Appendix A:
## Table of Questions by Unit and Learning Outcome

<table>
<thead>
<tr>
<th>Unit</th>
<th>Question</th>
<th>Type</th>
<th>Learning Outcome</th>
<th>Mark</th>
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**Legend for Units:**
A: Relations and Functions  
B: Probability  
C: Financial Mathematics  
D: Design and Measurement  
E: Logical Reasoning

**Legend for Question Types:**
MC: Multiple Choice  
SA: Short Answer  
LA: Long Answer
## Appendix B:
### Table of Questions by Type and Learning Outcome

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### Legend for Question Types:
- **MC**: Multiple Choice
- **SA**: Short Answer
- **LA**: Long Answer

### Legend for Units:
- **A**: Relations and Functions
- **B**: Probability
- **C**: Financial Mathematics
- **D**: Design and Measurement
- **E**: Logical Reasoning
Appendix C:
Irregularities in Provincial Tests

A Guide for Local Marking

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an Irregular Test Booklet Report should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all “NR”) or only incorrect responses (“0”)

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an Irregular Test Booklet Report.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an Irregular Test Booklet Report documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the department along with the test materials.
Irregular Test Booklet Report

Test: ____________________________

Date marked: ______________________

Booklet No.: _______________________

Problem(s) noted: ______________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

Question(s) affected: ______________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

Action taken or rationale for assigning marks: ______________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________