Grade 12
Applied Mathematics
Achievement Test

Marking Guide

June 2013
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Websites are subject to change without notice.

Disponible en français.

Available in alternate formats upon request.
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General Marking Instructions

Please do not make any marks in the test booklets. Any marks in a test booklet will have to be erased by departmental staff before the sample marking if the booklet is selected.

Please ensure that

- the student booklet number and the number on the Scoring Sheet are identical
- only a pencil is used to complete the Scoring Sheet
- each student’s final result is recorded, by booklet number, on the corresponding Scoring Sheet
- the Scoring Sheet is complete and a copy has been made for school records

Once marking is completed, please forward the Scoring Sheets to Manitoba Education in the envelope provided (for more information, see the administration manual).

Marking the Questions

Explanations for common errors for multiple-choice questions have been provided if applicable.

To receive full marks for a question, a student’s response must be complete and correct. Partial marks may be awarded for an “appropriate strategy” with execution errors. An appropriate strategy is defined as one that is consistent with the learning outcomes and mathematical processes associated with the question and, if properly executed, would lead to the correct answer.

Some questions require a form of explanation or justification from students. Depending on the student’s learning style, the explanation or justification can be given through a labelled diagram, in words, by showing mathematical operations for answer verification, or by referring to a software or calculator program. For this reason, appropriate flexibility is required when marking student responses.

Student Errors

As a guiding principle, students should only be penalized once for each error committed in the context of a test question. For example, students may choose an inappropriate strategy for a question, but carry it through correctly and arrive at an incorrect answer. In such cases, students should be penalized for having selected an inappropriate strategy for the task at hand, but should be given credit for having arrived at an answer consistent with their choice of strategy.

Errors which are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.
Communication Errors

The marks allocated to questions are primarily based on the concepts associated with the learning outcomes in the curriculum. For each question, shade in the circle on the Scoring Sheet that represents the mark awarded based on the concepts. A total of these marks will provide the preliminary mark.

Errors that are not related to the concepts are called “Communication Errors” and these will be indicated on the Scoring Sheet in a separate section (see example below). There will be a 0.5 mark deduction for each type of communication error committed, regardless of the number of errors committed for a certain type (i.e., committing a second error for any type will not further affect a student’s mark).

The total mark deduction for communication errors for any student response is not to exceed the marks given for that response. When multiple communication errors are made in a given response, any deductions are to be indicated in the order in which the errors occur in the response, without exceeding the given marks.

There is a maximum deduction of 3 marks (approximately 5% of the total test mark) for communication errors.

The student’s final mark is determined by subtracting the communication errors from the preliminary mark.

Example:

A student has a preliminary mark of 46. The student committed two E1 errors (0.5 mark deduction) and three E4 errors (0.5 mark deduction).

\[
\begin{align*}
\text{Preliminary Mark} & - \left( 0.5 \times \# \text{ of error types for a maximum deduction of 3 marks} \right) = \text{Final Mark} \\
46 & - (0.5 \times 2) = 45
\end{align*}
\]
Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix C provides examples of such irregularities as well as procedures to follow to report irregularities.

If a Scoring Sheet is marked with “0” and/or “NR” only (e.g., student was present but did not attempt any questions) please document this on the Irregular Test Booklet Report.

Assistance

If, during marking, any issue arises that cannot be resolved locally, please call Manitoba Education at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the marking keys.

Jennifer Maw  
Assistant Coordinator, Assessment Unit  
Grade 12 Applied Mathematics  
Telephone: 204-945-5886  
Toll-Free: 1-800-282-8069, ext. 5886  
Email: jennifer.maw@gov.mb.ca
Marking Keys

Please note that this Marking Guide contains screen captures taken from a TI–83 Plus graphing calculator.
Circle the graph below which best represents a cubic function.

A)  

B)  

C)  

D)  

Common Errors
A: exponential function  
B: logarithmic function  
C: quadratic function
Given the following function which represents the change in a town’s population with respect to time:

$$y = 1000(1.05)^x$$

Referring to the town, explain the meaning of:

a) “1000”

(1 mark)

“1000” is the initial population.

b) “1.05”

(1 mark)

“1.05” is the growth rate of 5%.

<table>
<thead>
<tr>
<th>Marking Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>① 1 mark for correct explanation in (a)</td>
</tr>
<tr>
<td>② 1 mark for correct explanation in (b)</td>
</tr>
</tbody>
</table>
Exemplar 1

Question No. 2

Given the following function which represents the change in a town’s population with respect to time:

\[ y = 1000(1.05)^x \]

Referring to the town, explain the meaning of:

a) “1000”

(1 mark)

“1000” is the population

b) “1.05”

(1 mark)

“1.05” is the time

0 marks:
→ no criteria met
Exemplar 2

Question No. 2

Given the following function which represents the change in a town’s population with respect to time:

\[ y = 1000 \times (1.05)^x \]

Referring to the town, explain the meaning of:

a) “1000”

(1 mark)

1000 = The point where the fx intercepts the y axis.

b) “1.05”

(1 mark)

1.05 = Whether the fx is increasing or decreasing and in this graph it is increasing.

0 marks:
→ no criteria met
A football player wants to kick a football so it will go over a crossbar that is 35 yards away and 3.33 yards high. (Diagram is not drawn to scale.)

The horizontal distance \( (d, \text{in yards}) \) and the height \( (h, \text{in yards}) \) that the football travels are represented by the following equation:

\[
h = -0.04d^2 + 1.51d
\]

How far above or below the crossbar will the football travel? Show your work.

\[
2\text{nd TRACE 1: value } x = 35, y = 3.85
\]

\[
3.85 - 3.33 = 0.52 \text{ yards}
\]

The football will travel 0.52 yards above the crossbar.

Marking Key

<table>
<thead>
<tr>
<th></th>
<th>1 mark for appropriate work</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1 mark for correct answer</td>
</tr>
</tbody>
</table>
A football player wants to kick a football so it will go over a crossbar that is 35 yards away and 3.33 yards high. (Diagram is not drawn to scale.)

The horizontal distance \(d\), in yards) and the height \(h\), in yards) that the football travels are represented by the following equation:

\[
h = -0.04d^2 + 1.51d
\]

How far above or below the crossbar will the football travel? Show your work.

\[
y = -0.04d^2 + 1.51d
\]

on calc

graph

2nd trace

maximum

14.25 m - 3.33 m

= 10.92 m

The football will reach a maximum of 10.92 m.

1 mark:

- 1 mark for correct answer

- 0.5 mark deduction (if applicable) for not stating or incorrectly stating the final answer
Exemplar 2

Question No. 3  Total: 2 marks

A football player wants to kick a football so it will go over a crossbar that is 35 yards away and 3.33 yards high. (Diagram is not drawn to scale.)

The horizontal distance \( (d, \text{ in yards}) \) and the height \( (h, \text{ in yards}) \) that the football travels are represented by the following equation:

\[
h = -0.04d^2 + 1.51d
\]

How far above or below the crossbar will the football travel? Show your work.

\[
\begin{align*}
h &= -0.04d^2 + 1.51d \\
(\text{height}) \\
&= -0.04(35)^2 + 1.51(35) \\
&= -49.00 + 52.85 \\
&= 3.85 \\
3.85 &= h
\end{align*}
\]

The football will travel 3.85 yards above the crossbar.

1 mark:

1 1 mark for appropriate work
The mass of a steel ball varies with respect to its diameter.

<table>
<thead>
<tr>
<th>diameter (mm)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass (g)</td>
<td>0</td>
<td>2</td>
<td>10</td>
<td>32</td>
<td>80</td>
</tr>
</tbody>
</table>

Determine the cubic equation that best represents the data. Sketch a clearly labelled graph of the equation.

Cubic equation: \( y = 0.01x^3 - 0.10x^2 + 0.61x - 0.06 \)
Question No. 4 continued

Marker Note(s):
→ No mark deduction for rounding errors, unless this results in a non-cubic equation.
→ Regression equations may vary depending on the software used.

<table>
<thead>
<tr>
<th>Marking Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>
The mass of a steel ball varies with respect to its diameter.

<table>
<thead>
<tr>
<th>diameter (mm)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
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<tr>
<td>mass (g)</td>
<td>0</td>
<td>2</td>
<td>10</td>
<td>32</td>
<td>80</td>
</tr>
</tbody>
</table>

Determine the cubic equation that best represents the data. Sketch a clearly labelled graph of the equation.

cubic equation: ____________________________
Exemplar 1 (continued)

2 marks:

1 → 1 mark for correct cubic equation
2 → 1 mark for correct graph with appropriate shape

3 → 0.5 mark deduction (if applicable) for not including one of the following in the equation: “y =”, “sin”, “ln”, or “x”, or for writing parameters separately from the equation
Exemplar 2

Question No. 4

The mass of a steel ball varies with respect to its diameter.

<table>
<thead>
<tr>
<th>diameter (mm)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass (g)</td>
<td>0</td>
<td>2</td>
<td>10</td>
<td>32</td>
<td>80</td>
</tr>
</tbody>
</table>

Determine the cubic equation that best represents the data. Sketch a clearly labelled graph of the equation.

cubic equation: \[ ax^3 + bx^2 + cx + d = 0.013x^3 + 0.097x^2 + 0.61x - 0.057 \]
Exemplar 2 (continued)

2 marks:

1. → 1 mark for correct cubic equation
2. → 1 mark for correct graph with appropriate shape

3. → 0.5 mark deduction (if applicable) for not including one of the following in the equation: “y =”, “sin”, “ln”, or “x”, or for writing parameters separately from the equation
A mass is suspended by a spring and is in a resting position 0.50 metres above a table.

The mass is pulled down 0.40 metres and is then released. The following information is obtained:

- It takes 1.20 seconds for the mass to return to its lowest position.
- The mass reaches a maximum height of 0.90 metres.

a) Determine the sinusoidal equation that best represents the distance of the mass with respect to the table as a function of time since it was released. Show your work.

Using SinReg: \( y = 0.40 \sin(5.24x - 1.57) + 0.50 \)

b) When will the mass be 0.75 metres above the table for the first time?

Marker Note(s):
→ Regression equations may vary depending on the software used.
A mass is suspended by a spring and is in a resting position 0.50 metres above a table.

The mass is pulled down 0.40 metres and is then released. The following information is obtained:

- It takes 1.20 seconds for the mass to return to its lowest position.
- The mass reaches a maximum height of 0.90 metres.

a) Determine the sinusoidal equation that best represents the distance of the mass with respect to the table as a function of time since it was released. Show your work.

\[ y = 104.642 \sin(0.2626x - 0.0828) + 100.63 \]

1 mark: 1 mark for appropriate work in (a)

b) When will the mass be 0.75 metres above the table for the first time?

\[ y_1 = 104.642 \sin(0.2626x - 0.0828) + 100.63 \]

\[ y_2 = 0.75 \]

Intersection \[ x = 1.69, y = 0.75 \]

It will be above the table at 0.75 metres for the first time after 1.
A mass is suspended by a spring and is in a resting position 0.50 metres above a table.

The mass is pulled down 0.40 metres and is then released. The following information is obtained:

- It takes 1.20 seconds for the mass to return to its lowest position.
- The mass reaches a maximum height of 0.90 metres.

a) Determine the sinusoidal equation that best represents the distance of the mass with respect to the table as a function of time since it was released. Show your work.

\[ a \sin(40x + c) + 0.50 \]

\[ (0.40) \sin( \frac{2\pi}{1.20} x ) + (0.50) \]

\[ 0.40 \sin(5.24 x) + (0.50) \]

\[ \frac{2\pi}{1.20} = 5.23598 \]

b) When will the mass be 0.75 metres above the table for the first time?

\[ h(0.75) = 0.40 \sin(5.24 \times 0.75) + 0.50 \]

\[ = 0.527 \]

\[ = 0.53 \text{ seconds after the mass is released.} \]
THIS PAGE WAS INTENTIONALLY LEFT BLANK.
It becomes easier and easier to see the headlights of an oncoming car the closer that it gets. The distance \( d \), in metres, between the car and an observer can be described as a function of the intensity \( I \), in lumens, of the headlight brightness:

\[
d = 350 - 72 \ln(I)
\]

a) Sketch a clearly labelled graph of the equation.

(2 marks)
Question No. 6 continued

b) Determine the distance to an oncoming car if the intensity of its headlights is 75 lumens.

(1 mark)

\[ d = 350 - 72 \ln(75) \]
\[ = 39.14 \text{ m} \]

OR

2nd TRACE 1: value \( x = 75, y = 39.14 \)

The car is 39.14 metres away.

The intensity of the headlights will be greatest when the distance is 0 metres.

The maximum intensity of the headlights is 129.17 lumens.

c) What is the maximum intensity of the headlights? Justify your answer.

(2 marks)

The intensity of the headlights will be greatest when the distance is 0 metres.

The maximum intensity of the headlights is 129.17 lumens.

OR

\[ Y_2 = 0 \]

2nd TRACE 5: intersect \( x = 129.17, y = 0 \)

The maximum intensity of the headlights is 129.17 lumens.

Marker Note(s):

→ For the appropriate shape mark to be awarded, the graph must curve, must not cross the y-axis and must show an appropriate x-intercept.
It becomes easier and easier to see the headlights of an oncoming car the closer that it gets. The distance \(d\), in metres, between the car and an observer can be described as a function of the intensity \(I\), in lumens, of the headlight brightness:

\[
d = 350 - 72 \ln(I)
\]

a) Sketch a clearly labelled graph of the equation.
Exemplar 1 (continued)

b) Determine the distance to an oncoming car if the intensity of its headlights is 75 lumens.

(1 mark)

\[ d = -72 \ln(75) + 350 \]

\[ = 39 \text{ m} \]

4 marks:

\(1\) → 1 mark for correct graph with appropriate shape in (a)

\(2\) → 1 mark for including: labels for the axes, units for the axes, and scales for the axes in (a)

\(3\) → 1 mark for correct answer in (b)

\(4\) → 1 mark for correct maximum intensity in (c)

\(5\) → 0.5 mark deduction (if applicable) for rounding too soon or rounding incorrectly

c) What is the maximum intensity of the headlights? Justify your answer.

(2 marks)

It should be a hundred lumens as the max intensity because you can only get a hundred percent.

In this case the max intensity is 129 lumens. This is because it must be measured out of a different percent or they can have that max because it's in lumens.
Exemplar 2

Question No. 6  
Total: 5 marks

It becomes easier and easier to see the headlights of an oncoming car the closer that it gets. The distance \((d, \text{ in metres})\) between the car and an observer can be described as a function of the intensity \((I, \text{ in lumens})\) of the headlight brightness:

\[ d = 350 - 72 \ln(I) \]

a) Sketch a clearly labelled graph of the equation.

(2 marks)
Exemplar 2 (continued)

b) Determine the distance to an oncoming car if the intensity of its headlights is 75 lumens.

(1 mark)

\[ y = 75 \quad x = 45.58 \, \text{meters} \]

\[ 45.58 \, \text{meters} \]

c) What is the maximum intensity of the headlights? Justify your answer.

(2 marks)

\[ \text{infinite intensity because there is no y intercept} \]

4 marks:

1 → 1 mark for correct graph with appropriate shape in (a)
2 → 1 mark for correct answer in (b)
3 → 1 mark for appropriate justification in (c)
4 → 1 mark for correct maximum intensity in (c)

Note: No mark awarded for labels, units, and scale in (a) since axes are reversed. Marks 3, 4, and 5 were awarded based on the student’s answer in (a).
PROBABILITY

Learning Outcome: 12A.P.4  Question Type: Multiple Choice

Question No. 7 and Answer  Total: 1 mark

Licence plates in Ontario contain 4 upper case letters followed by 3 digits, with repetition allowed. Circle the maximum possible number of licence plates that begin with the letters: MMBA, MANI, or BNTP.

A) 2160
B) 2880
✓ C) 3000
D) 4000

Common Errors
A: 3×10×9×8
B: 4×10×9×8
D: 4×10×10×10

Learning Outcome: 12A.P.1  Question Type: Multiple Choice

Question No. 8  Total: 1 mark

A bag contains 6 white marbles, 8 blue marbles, 2 yellow marbles, and 4 green marbles. What are the odds in favour of selecting a white marble?

✓ A) 6:14
B) 6:20
C) 14:6
D) 20:6

Common Errors
B: probability
C: odds against selection
D: probability reversed
A cookie jar contains 10 chocolate chip cookies, 12 double chocolate cookies, and 15 oatmeal cookies. Allison says that the odds against selecting a cookie with chocolate are 15 to 37. Ryan says that the odds against are 15 to 22. Who is correct? Explain your answer.

Ryan is correct. Odds are expressed as \( \text{part} : \text{part} \)
while probabilities are expressed as \( \text{part} : \text{whole} \).

OR

Ryan is correct.

\[
\begin{array}{ccc}
\text{Oatmeal} & \text{Chocolate} & \text{Double Chocolate} \\
15 & 12 & 10 \\
\end{array}
\]

\[= 15 : 22\]

Marking Key

1 mark for correct explanation
Question No. 9  

A cookie jar contains 10 chocolate chip cookies, 12 double chocolate cookies, and 15 oatmeal cookies. Allison says that the odds against selecting a cookie with chocolate are 15 to 37. Ryan says that the odds against are 15 to 22. Who is correct? Explain your answer.

Ryan is correct as his answer is in the proper format.

Allison just kept hers as a fraction and didn’t change to odds.

0 marks:  
→ no criteria met
A cookie jar contains 10 chocolate chip cookies, 12 double chocolate cookies, and 15 oatmeal cookies. Allison says that the odds against selecting a cookie with chocolate are 15 to 37. Ryan says that the odds against are 15 to 22. Who is correct? Explain your answer.

Ryan is right because the sum of the two numbers needs to add up to 37.

Ryan is right.
Describe a situation containing mutually exclusive events. Explain why the events are mutually exclusive.

- Selecting a 5 and an even number from a set of cards numbered from 1 to 10. These are mutually exclusive events since 5 is not an even number.

*Other answers are possible.*
Exemplar 1

<table>
<thead>
<tr>
<th>Question No. 10</th>
<th>Total: 2 marks</th>
</tr>
</thead>
</table>

Describe a situation containing mutually exclusive events. Explain why the events are mutually exclusive.

In school, taking either math or history which are in the same time block. This is mutually exclusive because you can only take one. They are in the same time block so you cannot take both, making it mutually exclusive.

2 marks:
1 → 1 mark for appropriate example
2 → 1 mark for appropriate explanation
Describe a situation containing mutually exclusive events. Explain why the events are mutually exclusive.

rolling a dice and flipping a coin. They are mutually exclusive because doing one doesn’t affect the outcome of the other.

1 mark:
① → 1 mark for appropriate example
An organization consisting of 15 women and 19 men must create a 10-person committee.

a) How many committees can be created that will include 4 women and 6 men? Show your work.

(2 marks)

\[
\begin{align*}
\text{women: } & 15C_4 = 1365 \\
\text{men: } & 19C_6 = 27132 \\
\end{align*}
\]

4 women and 6 men: \(1365 \times 27132 = 37035180\) committees

b) If a 10-person committee is randomly selected, what is the probability that the committee will include 4 women and 6 men? Show your work.

(2 marks)

\[
\begin{align*}
\frac{37035180}{34C_{10}} &= \frac{37035180}{131128140} \\
&= 0.28 = 28.24\% \\
\end{align*}
\]

Marking Key

1. 1 mark for appropriate work in (a)
2. 1 mark for correct answer in (a)
3. 1 mark for appropriate work in (b)
4. 1 mark for correct answer in (b)
Exemplar 1

An organization consisting of 15 women and 19 men must create a 10-person committee.

a) How many committees can be created that will include 4 women and 6 men? Show your work.

\[
15C_4 + 19C_6
\]
\[
1365 + 27132
\]
\[
= 28497 \text{ ways}
\]

(2 marks)

b) If a 10-person committee is randomly selected, what is the probability that the committee will include 4 women and 6 men? Show your work.

\[
\frac{15C_4 + 19C_6}{34C_{10}} \frac{28497}{131128140}
\]

28497: 131128140

3 marks:

\[\rightarrow\] 1 mark for appropriate work in (a)
\[\rightarrow\] 1 mark for appropriate work in (b)
\[\rightarrow\] 1 mark for correct answer in (b)
An organization consisting of 15 women and 19 men must create a 10-person committee.

a) How many committees can be created that will include 4 women and 6 men? Show your work.

\[
\begin{align*}
15 \binom{4}{4} & \times 19 \binom{6}{6} = 1 \\
11 \binom{4}{4} & \times 13 \binom{6}{6} = 1 \\
7 \binom{4}{4} & \times \binom{6}{6} = \frac{1}{3} \\
3 \binom{4}{4} & + \binom{6}{6} \\
\end{align*}
\]

You can have 3 committees of 4 women and 6 men.

b) If a 10-person committee is randomly selected, what is the probability that the committee will include 4 women and 6 men? Show your work.

\[
\frac{\left(15 \binom{4}{4} \times 19 \binom{6}{6}\right)}{34 \binom{10}{10}} = 0.28 \times 100 = 28\% \text{ of picking 4 women and 6 men.}
\]
You have been asked to create a four-character password for your computer using:

- the 26 upper case letters of the alphabet (A, B, C, …)
- the 26 lower case letters of the alphabet (a, b, c, …)
- the digits from 0 to 9
- the symbols: ~ ! @ # $ % ^ & *

a) How many different four-character passwords are possible if any of the letters, digits, or symbols can be used for each character if repetition is allowed?

\[(1 \text{ mark})\]
\[26 + 26 + 10 + 9 = 71 \text{ characters} \]
\[71 \times 71 \times 71 \times 71 = 71^4 = 25\,411\,681 \text{ passwords}\]

b) How many different four-character passwords are possible if repetition is not allowed?

\[(1 \text{ mark})\]
\[71P_4 = 71 \times 70 \times 69 \times 68 = 23\,319\,240 \text{ passwords}\]

c) How many four-character passwords begin with a letter and end with a digit if repetition is allowed?

\[(1 \text{ mark})\]
\[52 \times 71 \times 71 \times 10 = 2\,621\,320 \text{ passwords}\]
Question No. 12 continued

(d) How many different four-character passwords containing at least one symbol are possible if repetition is allowed? Show your work.

(2 marks)

Total = all passwords – passwords without symbols

= $71^4 - 62^4$

= $25 411 681 - 14 776 336$

= $10 635 345$

OR

1 symbol: $(9 \times 62 \times 62 \times 62) \times 4$

= 8 579 808

2 symbols: $(9 \times 9 \times 62 \times 62) \times \frac{4!}{(2!2!)}$

= 1 868 184

3 symbols: $(9 \times 9 \times 9 \times 62) \times 4$

= 180 792

4 symbols: $(9 \times 9 \times 9 \times 9)$

= 6561

8 579 808 + 1 868 184 + 180 792 + 6561 = 10 635 345 passwords

Marking Key

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 mark for correct answer in (a)</td>
</tr>
<tr>
<td>2</td>
<td>1 mark for correct answer in (b)</td>
</tr>
<tr>
<td>3</td>
<td>1 mark for correct answer in (c)</td>
</tr>
<tr>
<td>4</td>
<td>1 mark for appropriate work in (d)</td>
</tr>
<tr>
<td>5</td>
<td>1 mark for correct answer in (d)</td>
</tr>
</tbody>
</table>
You have been asked to create a four-character password for your computer using:

- the 26 upper case letters of the alphabet (A, B, C, …) 26
- the 26 lower case letters of the alphabet (a, b, c, …) 26
- the digits from 0 to 9 10
- the symbols: ~!@#$%^&* 9

a) How many different four-character passwords are possible if any of the letters, digits, or symbols can be used for each character if repetition is allowed?

(1 mark)

\[ 71 \cdot 71 \cdot 71 \cdot 71 = 25,411,681 \text{ passwords possible} \]

b) How many different four-character passwords are possible if repetition is not allowed?

(1 mark)

\[ 71 \cdot 70 \cdot 69 \cdot 68 = 23,319,240 \text{ passwords possible} \]

c) How many four-character passwords begin with a letter and end with a digit if repetition is allowed?

(1 mark)

\[ 52 \cdot 71 \cdot 71 \cdot 10 = 2,621,320 \text{ passwords} \]
Exemplar 1 (continued)

d) How many different four-character passwords containing at least one symbol are possible if repetition is allowed? Show your work.

(2 marks)

\[
\begin{align*}
\text{Case } \#1, \text{ 1 symbol} & \quad \frac{9}{5} \cdot \frac{62}{5} \cdot \frac{62}{5} \cdot \frac{62}{5} = 2144952 \\
\text{Case } \#2, \text{ 2 symbols} & \quad \frac{9}{5} \cdot \frac{9}{5} \cdot \frac{62}{5} \cdot \frac{62}{5} = 311364 \\
\text{Case } \#3, \text{ 3 symbols} & \quad \frac{9}{5} \cdot \frac{9}{5} \cdot \frac{9}{5} \cdot \frac{62}{5} = 45198 \\
\text{Case } \#4, \text{ 4 symbols} & \quad \frac{9}{5} \cdot \frac{9}{5} \cdot \frac{9}{5} \cdot \frac{9}{5} = 6561 \\
\end{align*}
\]

\[2508075 \text{ passwords possible.}\]

4 marks:
- 1 mark for correct answer in (a)
- 1 mark for correct answer in (b)
- 1 mark for correct answer in (c)
- 1 mark for appropriate work in (d)
You have been asked to create a four-character password for your computer using:

- the 26 upper case letters of the alphabet (A, B, C, …)
- the 26 lower case letters of the alphabet (a, b, c, …)
- the digits from 0 to 9
- the symbols: ~ ! @ # $ % ^ & *

a) How many different four-character passwords are possible if any of the letters, digits, or symbols can be used for each character if repetition is allowed?

\[
\text{Number of passwords} = 26 \times 26 \times 26 \times 26 = 456976
\]

b) How many different four-character passwords are possible if repetition is not allowed?

\[
\begin{align*}
\text{Number of passwords} & = 26 \times 25 \times 24 \times 23 + 26 \times 25 \times 24 \times 23 + 10 \times 9 \times 8 \times 7 + 9 \times 8 \times 7 \times 6 \\
& = 358800 + 358800 + 5040 + 3024 \\
& = 725,664
\end{align*}
\]

c) How many four-character passwords begin with a letter and end with a digit if repetition is allowed?

\[
\begin{align*}
\text{Number of passwords} & = 52 \times 52 \times 52 \\
& = 14068 \text{ letters} \\
& = 14068 \text{ possible passwords}
\end{align*}
\]

\[
\begin{align*}
14068 + 1000 + 81 & = 15149
\end{align*}
\]
Exemplar 2 (continued)

d) How many different four-character passwords containing at least one symbol are possible if repetition is allowed? Show your work.

(2 marks)

\[
\begin{align*}
26 \times 26 \times 26 &= 7034 \\
26 \times 26 \times 26 &= 7034 \\
10 \times 10 \times 10 &= 1000 \\
9 \times 9 \times 9 \times 9 &= 6561
\end{align*}
\]

\[= 21629\]
Brigitte invests $5000.00 at an interest rate of 6% for 5 years. Circle the compounding period below that would maximize the rate of return on the investment.

✓ A) daily

B) monthly

C) quarterly

D) semi-annually

Circle the asset below which is most likely to depreciate in value.

A) rare coin collection

B) classic car

C) house

✓ D) computer
Mr. Chang is 64 and plans to retire next year. His portfolio includes the following investments:

- $50 000.00 in mutual funds
- $100 000.00 in stocks
- $20 000.00 in guaranteed investment certificates (GICs)

a) Does this portfolio include an appropriate level of risk for Mr. Chang at this stage in his life? Explain your answer.

(1 mark)

No. Given that Mr. Chang is so close to retirement, this portfolio involves too much risk because of the high proportion of stock.

*Other answers are possible.*
Question No. 15 continued

b) Mr. Chang’s investments had the following returns last year: mutual funds increased by 12.00%, stocks decreased by 4.00%, and GICs had an annual interest rate of 3.00%. Calculate the average rate of return for the year for this portfolio. Show your work.

(2 marks)

<table>
<thead>
<tr>
<th>Type of investment</th>
<th>Principal ($)</th>
<th>Return ($)</th>
<th>End of the year ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mutual funds</td>
<td>50 000.00</td>
<td>6000.00</td>
<td>56 000.00</td>
</tr>
<tr>
<td>stocks</td>
<td>100 000.00</td>
<td>-4000.00</td>
<td>96 000.00</td>
</tr>
<tr>
<td>GICs</td>
<td>20 000.00</td>
<td>600.00</td>
<td>20 600.00</td>
</tr>
<tr>
<td>Total:</td>
<td>= $170 000.00</td>
<td>= $2600.00</td>
<td>= $172 600.00</td>
</tr>
</tbody>
</table>

Average rate of return = \( \frac{($172 600.00 - $170 000.00)}{$170 000.00} \times 100 \)

= 1.53%

OR

Average rate of return = \( \frac{-$2600.00}{$170 000.00} \times 100 \)

= 1.53%

Marking Key

1. 1 mark for appropriate explanation in (a)
2. 1 mark for correct total return amount or correct total end of the year amount in (b)
3. 1 mark for correct answer in (b) consistent with work
Mr. Chang is 64 and plans to retire next year. His portfolio includes the following investments:

- $50 000.00 in mutual funds
- $100 000.00 in stocks
- $20 000.00 in guaranteed investment certificates (GICs)

a) Does this portfolio include an appropriate level of risk for Mr. Chang at this stage in his life? Explain your answer.

(1 mark)

No, assuming Mr. Chang lives in Canada a wise option would be to invest money in a savings account. If he had entered money into an account that he couldn't spend, the money would accumulate and increase in value, he would have more money than he initially entered. Mutual funds you are able to spend money you put in, "Daily Access". Stocks are a risky investment because you risk the chance of losing money so I don't think this was wise especially since he was retiring. Even though the Canadian Pension Plan directly takes money off your cheques (if you choose), assuming Mr. Chang doesn't have CPP listed in his portfolio I think that would have been an appropriate investment of money as he grew older.
b) Mr. Chang’s investments had the following returns last year: mutual funds increased by 12.00%, stocks decreased by 4.00%, and GICs had an annual interest rate of 3.00%. Calculate the average rate of return for the year for this portfolio. Show your work.

(2 marks)

<table>
<thead>
<tr>
<th>Type of investment</th>
<th>Principal ($)</th>
<th>Return ($)</th>
<th>End of the year ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mutual funds</td>
<td>50 000.00</td>
<td>6 000</td>
<td>$ 56 000</td>
</tr>
<tr>
<td>stocks</td>
<td>100 000.00</td>
<td>-4 000</td>
<td>$ 96 000</td>
</tr>
<tr>
<td>GICs</td>
<td>20 000.00</td>
<td>600</td>
<td>$ 20 600</td>
</tr>
<tr>
<td><strong>Total:</strong></td>
<td></td>
<td><strong>2 600</strong></td>
<td><strong>$172 600</strong></td>
</tr>
</tbody>
</table>

\[ \text{MF} = 50 \, 000 \times 1.12 = 56 \, 000 \]
\[ 56 \, 000 - 50 \, 000 = 6 \, 000 \]
\[ = 6 \, 000 \]

\[ S = 100 \, 000 \times 0.04 = 4 \, 000 \]
\[ 100 \, 000 - 4 \, 000 = 96 \, 000 \]
\[ * \text{lost money} * \]

\[ \text{GIC's} = 20 \, 000 \times 1.03 = 20600 \]
\[ 20 \, 600 - 20 \, 000 \]
\[ = 600 \]

**2 marks:**

1. 1 mark for appropriate explanation in (a)
2. 1 mark for correct total return amount or correct total end of the year amount in (b)
Mr. Chang is 64 and plans to retire next year. His portfolio includes the following investments:

- $50,000.00 in mutual funds
- $100,000.00 in stocks
- $20,000.00 in guaranteed investment certificates (GICs)

a) Does this portfolio include an appropriate level of risk for Mr. Chang at this stage in his life? Explain your answer.

(1 mark)

No, he should have more like maybe a mortgage, a car. He should own more things of value. He is 64, he should have more than 170,000.
Exemplar 2 (continued)

b) Mr. Chang’s investments had the following returns last year: mutual funds increased by 12.00%, stocks decreased by 4.00%, and GICs had an annual interest rate of 3.00%. Calculate the average rate of return for the year for this portfolio. Show your work.

(2 marks)

<table>
<thead>
<tr>
<th>Type of investment</th>
<th>Principal ($)</th>
<th>Return ($)</th>
<th>End of the year ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mutual funds</td>
<td>50 000.00</td>
<td>+ 12%</td>
<td>56 000</td>
</tr>
<tr>
<td>stocks</td>
<td>100 000.00</td>
<td>- 4%</td>
<td>96 000</td>
</tr>
<tr>
<td>GICs</td>
<td>20 000.00</td>
<td>+ 3%</td>
<td>20 600</td>
</tr>
<tr>
<td><strong>Total:</strong></td>
<td></td>
<td></td>
<td><strong>172 600</strong></td>
</tr>
</tbody>
</table>

Mutual funds

\[
\text{End of year} = \text{Principal} \times (1 + \text{Return}) = 50 000 \times 1.12 = 56 000
\]

Average rate of return:

\[
\text{Average rate of return} = \left( \frac{\text{Current value} - \text{Previous}}{\text{Previous}} \right) \times 100
\]

\[
= \left( \frac{56 000 - 50 000}{50 000} \right) \times 100 = 12\%
\]

Stocks

\[
\text{End of year} = \text{Principal} \times (1 - \text{Return}) = 100 000 \times 0.96 = 96 000
\]

\[
\left( \frac{96 000 - 100 000}{100 000} \right) \times 100 = -4\%
\]

GICs

\[
\text{End of year} = \text{Principal} \times (1 + \text{Interest rate}) = 20 000 \times 1.03 = 20 600
\]

\[
\left( \frac{20 600 - 20 000}{20 000} \right) \times 100 = 3\%
\]

1 mark:

\[\rightarrow\text{1 mark for correct total return amount or correct total end of the year amount in (b)}\]
Amar dreams of retiring at the age of 55. He had planned on starting to save for his retirement at the age of 50, but his financial advisor does not agree. He recommends that Amar starts to save sooner.

a) If Amar invests $1000.00 on his 25th birthday and contributes $200.00 every month to an account that earns 8.00% compounded monthly, what will be the value of the investment on his 55th birthday? Show your work.

\[
\text{(2 marks)}
\]

The value of the investment will be $309,007.62.

b) If Amar invests $1000.00 on his 50th birthday, how much will he have to contribute every month to match the final value of the investment in (a)? Assume that the interest rate and the compounding periods are the same.

\[
\text{(1 mark)}
\]

Amar will have to contribute $4185.23 every month.
c) Calculate the difference between Amar’s total contribution in (a) and in (b)? Show your work.

(2 marks)

Total contribution in (a): \((360)(200.00) + 1000.00 = 73000.00\)

Total contribution in (b): \((60)(4185.23) + 1000.00 = 252113.80\)

\(252113.80 - 73000.00 = 179113.80\)

**Marker Note(s):**

→ A maximum of 1 error is allowed in the input values of a financial template in (a) and (b) (award the mark for appropriate work, but not the mark for correct answer).

**Marking Key**

<table>
<thead>
<tr>
<th></th>
<th>1 mark for appropriate work in (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1 mark for correct answer in (a)</td>
</tr>
<tr>
<td>3</td>
<td>1 mark for correct answer in (b) consistent with work in (a)</td>
</tr>
<tr>
<td>4</td>
<td>1 mark for appropriate work in (c)</td>
</tr>
<tr>
<td>5</td>
<td>1 mark for correct answer in (c) consistent with work in (a) and (b)</td>
</tr>
</tbody>
</table>
Amar dreams of retiring at the age of 55. He had planned on starting to save for his retirement at the age of 50, but his financial advisor does not agree. He recommends that Amar starts to save sooner.

a) If Amar invests $1000.00 on his 25th birthday and contributes $200.00 every month to an account that earns 8.00% compounded monthly, what will be the value of the investment on his 55th birthday? Show your work.

\[
\begin{align*}
N &= 360 \\
I &= 8 \\
PV &= 1000 \\
Pt &= -200 \\
fv &= \ ? = $287136.16 \\
p/y &= 12 \\
c/y &= 12
\end{align*}
\]

b) If Amar invests $1000.00 on his 50th birthday, how much will he have to contribute every month to match the final value of the investment in (a)? Assume that the interest rate and the compounding periods are the same.

\[
\begin{align*}
N &= 60 \\
I &= 8 \\
PV &= 1000 \\
Pt &= \ ? = -$3928.12 \\
fv &= 287136.16 \\
p/y &= 12 \\
c/y &= 12
\end{align*}
\]
Exemplar 1 (continued)

c) Calculate the difference between Amar’s total contribution in (a) and in (b)? Show your work.

(2 marks)

\[
\begin{align*}
\text{a) } \quad & 360 \times 200 = \$ 72 000 \\
\text{b) } \quad & 3.92 \times 8.12 \times 60 = \$ 2 356 872 \\
\text{Option b earns less in interest and has to pay the difference of } \$ -16 368 72 \\
\text{in monthly payments.}
\end{align*}
\]

<table>
<thead>
<tr>
<th>4 marks:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 → 1 mark for appropriate work in (a)</td>
</tr>
<tr>
<td>3 → 1 mark for correct answer in (b) consistent with work in (a)</td>
</tr>
<tr>
<td>4 → 1 mark for appropriate work in (c)</td>
</tr>
<tr>
<td>5 → 1 mark for correct answer in (c) consistent with work in (a) and (b)</td>
</tr>
<tr>
<td>6 → 0.5 mark deduction (if applicable) for not stating or incorrectly stating the final answer</td>
</tr>
</tbody>
</table>
Question No. 16  
Total: 5 marks

Amar dreams of retiring at the age of 55. He had planned on starting to save for his retirement at the age of 50, but his financial advisor does not agree. He recommends that Amar starts to save sooner.

a) If Amar invests $1000.00 on his 25th birthday and contributes $200.00 every month to an account that earns 8.00% compounded monthly, what will be the value of the investment on his 55th birthday? Show your work.

\[
\begin{align*}
N &= 760 \\
I\% &= 8 \\
pv &= -1000 \\
pmt &= -200 \\
p/\% &= 12 \\
c/\% &= 12 \\
Fv &= 309007.62
\end{align*}
\]

(2 marks)

b) If Amar invests $1000.00 on his 50th birthday, how much will he have to contribute every month to match the final value of the investment in (a)? Assume that the interest rate and the compounding periods are the same.

\[
\begin{align*}
N &= 300 \\
I\% &= 8 \\
pv &= -1000 \\
fv &= 309007.62 \\
p/\% &= 12 \\
c/\% &= 12 \\
pmt &= 317.20
\end{align*}
\]

(1 mark)
Exemplar 2 (continued)

c) Calculate the difference between Amar’s total contribution in (a) and in (b)? Show your work.

(2 marks)

\[ \begin{align*}
\text{a) } &= 73000 \\
\text{b) } &= 96160
\end{align*} \]

\[ \begin{align*}
\text{\text{96160}} \\
\begin{array}{c}
\text{73000} \\
\hline
\text{23160}
\end{array}
\end{align*} \]

4 marks:

- 1 mark for appropriate work in (a)
- 1 mark for correct answer in (a)
- 1 mark for appropriate work in (c)
- 1 mark for correct answer in (c) consistent with work in (a) and (b)

- 0.5 mark deduction (if applicable) for not stating or incorrectly stating the final answer
The Reimers have purchased a house valued at $250 000.00 and have made a down payment of $25 000.00.

a) Calculate their monthly mortgage payment if they obtain a mortgage amortized over 15 years at an interest rate of 5.50% compounded semi-annually. Show your work.

(2 marks)

Their monthly mortgage payment is $1831.05.
b) How much equity will the Reimers have in their house after 5 years if the value of the house appreciates at a rate of 2.00% per year? Show your work.

(3 marks)

\[
\text{equity} = \text{appreciated value of the house} - \text{balance owing}
\]

\[
\text{equity} = \frac{250\,000.00 \times 1.02^5}{169197.5283} - 169197.53
\]

\[
= \frac{276\,020.20}{169197.5283} - 169197.53
\]

\[
= 106\,822.67
\]

OR

\[
\text{appreciation} = \left(\frac{250\,000.00 \times 1.02^5}{169197.5283}\right) - 250\,000.00
\]

\[
= 26\,020.20
\]

\[
\text{equity} = \text{down payment} + \sum \text{Pn} (1, 60) + \text{appreciation}
\]

\[
= 25\,000.00 + 55\,802.47 + 26\,020.20
\]

\[
= 106\,822.67
\]

Marker Note(s):

→ A maximum of 1 error is allowed in the input values of a financial template in (a) and (b) (award the mark for appropriate work, but not the mark for correct answer).

<table>
<thead>
<tr>
<th>Marking Key</th>
</tr>
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<tbody>
<tr>
<td>① 1 mark for appropriate work in (a)</td>
</tr>
<tr>
<td>② 1 mark for correct answer in (a)</td>
</tr>
<tr>
<td>③ 1 mark for correct calculation of appreciation in (b)</td>
</tr>
<tr>
<td>④ 1 mark for correct calculation of balance owing or sum of the principal paid in (b) consistent with answer in (a)</td>
</tr>
<tr>
<td>⑤ 1 mark for correct equity in (b) consistent with work</td>
</tr>
</tbody>
</table>
The Reimers have purchased a house valued at $250 000.00 and have made a down payment of $25 000.00.

a) Calculate their monthly mortgage payment if they obtain a mortgage amortized over 15 years at an interest rate of 5.50% compounded semi-annually. Show your work.

\[ \begin{align*}
N &= 180 \\
I\% &= 5.5 \\
PV &= -250 000 \\
\text{APR} &= 2034.50 \\
FV &= 0 \\
p\% &= 12 \\
c\% &= 2
\end{align*} \]
Exemplar 1 (continued)

b) How much equity will the Reimers have in their house after 5 years if the value of the house appreciates at a rate of 2.00% per year? Show your work.

(3 marks)

\[
25000 \times 1.02 = 25000 \times 1.02 = 260100 \times 1.02 = 2650100 \times 1.02
\]

\[
= 270608.04 \times 1.02 = 276020.20
\]

\[
190 \times 2034.50 = 366390
\]

\[
276020.20 - 366390
\]

\[
-90369.8
\]
The Reimers have purchased a house valued at $250 000.00 and have made a down payment of $25 000.00.

\[ 250,000 - 25,000 = 225,000 \]

a) Calculate their monthly mortgage payment if they obtain a mortgage amortized over 15 years at an interest rate of 5.50% compounded semi-annually. Show your work.

\[ n = 180 \]
\[ i = 5.5\% \]
\[ PV = 225,000 \]
\[ PMT = ? \]
\[ FV = 0 \]
\[ P/Y = 12 \]
\[ C/Y = 2 \]

Reimers have to pay \$1831.05 per month.
Exemplar 2 (continued)

b) How much equity will the Reimers have in their house after 5 years if the value of the house appreciates at a rate of 2.00% per year? Show your work.

(3 marks)

\[
\begin{align*}
FV &= 250,000 (1.02)^3 = 265,302 \\
\begin{array}{l}
\text{PV} = 225,000 \\
\text{IMT} = -1831.05 \\
\text{FV} = ? \quad \text{Alpha solve} \\
\text{IP} = 12 \\
\text{CI} = 2
\end{array}
\end{align*}
\]

\[
265,302 - 169,197.52 = 96,104.48
\]

4 marks:

1 → 1 mark for appropriate work in (a)
2 → 1 mark for correct answer in (a)
3 → 1 mark for correct calculation of balance owing or sum of the principal paid in (b) consistent with answer in (a)
4 → 1 mark for correct equity in (b) consistent with work
5 → 0.5 mark deduction (if applicable) for rounding too soon or rounding incorrectly
A cake mix will produce 230 cubic inches of batter. You are using cylinder-shaped baking cups that have a diameter of 3 inches and a depth of 2 inches for the batter. How many cupcakes will you be able to make? Show your work.

radius = 1.5 in.

\[ V = \pi r^2 h \]

\[ = \pi (1.5 \text{ in.})^2 (2 \text{ in.}) \]

\[ = 14.14 \text{ in}^3 \text{ for each cupcake} \]

\[ \frac{230 \text{ in}^3}{14.14 \text{ in}^3/\text{cupcake}} = 16.27 \text{ cupcakes} \]

I will be able to make 16 cupcakes.

Accept the following answers: 16, 16.27, or 17.
Exemplar 1

Question No. 18  

A cake mix will produce 230 cubic inches of batter. You are using cylinder-shaped baking cups that have a diameter of 3 inches and a depth of 2 inches for the batter. How many cupcakes will you be able to make? Show your work.

\[ r = \frac{3}{2} = 1.5 \]

\[ V_{\text{cupcake}} = \pi r h + 2 \pi r^2 \]

\[ = (2 \pi (1.5)(2)) + (2 \pi (1.5)^2) \]

\[ = 32.99 \text{ in}^3 \]

\[ \frac{230}{32.99} = 6.97 \]

You would be able to make approximately 7 cupcakes.

1 mark:

\[ \Rightarrow \rightarrow 1 \text{ mark for correct answer} \]
A cake mix will produce 230 cubic inches of batter. You are using cylinder-shaped baking cups that have a diameter of 3 inches and a depth of 2 inches for the batter. How many cupcakes will you be able to make? Show your work.

\[
\pi r^2 h = \pi (1.5^2)(2) = 14.14 \text{ in}^3 \text{ to } 14^3 \text{ to make full cupcakes.}
\]

\[
\frac{230}{14^3} = 16.43 \rightarrow 16 \text{ full cupcakes}
\]

You will be able to make 16 cupcakes.

2 marks:

1. 1 mark for appropriate work
2. 1 mark for correct answer
3. 0.5 mark deduction (if applicable) for rounding too soon or rounding incorrectly
A goat is tied to the corner of a barn with a 50-foot rope. The barn measures 60 feet by 40 feet. Calculate the total area outside of the barn that is available to the goat. Show your work.

Total area = $\frac{3\pi (50 \text{ ft.})^2}{4} + \frac{\pi (10 \text{ ft.})^2}{4}$

= $5890.49 + 78.54$

= $5969.03 \text{ ft}^2$

Marker Note(s):
→ Deduct a maximum of 1 mark if student calculates only one of the two areas correctly.

<table>
<thead>
<tr>
<th>Marking Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>
A goat is tied to the corner of a barn with a 50-foot rope. The barn measures 60 feet by 40 feet. Calculate the total area outside of the barn that is available to the goat. Show your work.

\[ \pi r^2 = \text{circle} \]
\[ \pi 50^2 = \frac{7853.981634}{4} \]
\[ \frac{1}{4} \text{ of circle } = 1963.495404 \times 3 \]
\[ 3/4 \text{ of circle } = 5890.49 \text{ square ft.} \]

1 mark: ☑ 1 mark for appropriate work
Exemplar 2

Question No. 19

A goat is tied to the corner of a barn with a 50-foot rope. The barn measures 60 feet by 40 feet. Calculate the total area outside of the barn that is available to the goat. Show your work.

\[
\frac{1}{2} \times 40 \times 50 = \text{Area 1} \\
\frac{1}{2} \times 10 \times 10 = \text{Area 2} \\
\pi \times 50^2 = \text{Area C} \\
\text{Total area available} = 7853.98 - 1000 - 50 = 6803.98 \text{ ft}^2
\]

0 marks: 
→ no criteria met
The Manitoba Beach Volleyball Association has asked you to design a souvenir beach ball according to the following information:

- The beach ball must have a volume between 1 and 3 cubic feet.
- The plastic material costs $0.15 per ft².
- Labour and other materials cost $1.25 per beach ball.
- The Association wants to make a profit of 80% of the cost of making each beach ball.

Based on your design, what is the minimum selling price for each souvenir beach ball?

Show your work.

\[
V = 1 \text{ ft}^3 = \frac{4}{3} \pi r^3
\]

\[
r = 0.62 \text{ ft.}
\]

Surface area = \(4\pi r^2\)

\[
= 4\pi (0.62 \text{ ft.})^2
\]

\[
= 4.84 \text{ ft}^2
\]

Cost of production = $4.84(0.15) + $1.25 = $1.98

Profit = $1.98 \times 0.80 = $1.58

Selling price per beach ball = $1.98 + $1.58 = $3.56

OR

\[
V = 2 \text{ ft}^3 = \frac{4}{3} \pi r^3
\]

\[
r = 0.78 \text{ ft.}
\]

Surface area = \(4\pi r^2\)

\[
= 4\pi (0.78 \text{ ft.})^2
\]

\[
= 7.65 \text{ ft}^2
\]

Cost of production = $7.65(0.15) + $1.25 = $2.40

Profit = $2.40 \times 0.80 = $1.92

Selling price per beach ball = $2.40 + $1.92 = $4.32

OR

\[
V = 3 \text{ ft}^3 = \frac{4}{3} \pi r^3
\]

\[
r = 0.89 \text{ ft.}
\]

Surface area = \(4\pi r^2\)

\[
= 4\pi (0.89 \text{ ft.})^2
\]

\[
= 10.06 \text{ ft}^2
\]

Cost of production = $10.06(0.15) + $1.25 = $2.76

Profit = $2.76 \times 0.80 = $2.21

Selling price per beach ball = $2.76 + $2.21 = $4.97

Other answers are possible.

Marker Note(s):

→ The answer may vary as a result of rounding.

**Marking Key**

<table>
<thead>
<tr>
<th></th>
<th>1 mark for correct radius</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1 mark for correct surface area consistent with radius</td>
</tr>
<tr>
<td>3</td>
<td>1 mark for correct cost of production consistent with surface area</td>
</tr>
<tr>
<td>4</td>
<td>1 mark for correct selling price consistent with surface area</td>
</tr>
</tbody>
</table>
Exemplar 1

Question No. 20  

The Manitoba Beach Volleyball Association has asked you to design a souvenir beach ball according to the following information:

- The beach ball must have a volume between \( \frac{1}{3} \) and 3 cubic feet.
- The plastic material costs $0.15 per ft\(^2\).
- Labour and other materials cost $1.25 per beach ball.
- The Association wants to make a profit of 80% of the cost of making each beach ball.

Based on your design, what is the minimum selling price for each souvenir beach ball? Show your work.

\[
\text{Volume} = \frac{4}{3} \pi r^3 \\
\text{Volume} = 1.77 \text{ ft}^3 \\
\text{Surface area} = 7.07 \text{ ft}^2 \\
4 \cdot \pi \cdot 0.75^2 = 7.07 \text{ ft}^2 \\
0.15 \cdot 7.07 = \$1.06 + 1.25 = 2.31 \text{ per ball (cost to make)} \\
2.31 \cdot 1.8 = \$4.16 \\
\text{Each ball will be sold for } \$4.16 \text{ each}
\]

4 marks:

1. → 1 mark for correct radius
2. → 1 mark for correct surface area consistent with radius
3. → 1 mark for correct cost of production consistent with surface area
4. → 1 mark for correct selling price consistent with surface area
5. → 0.5 mark deduction (if applicable) for not stating or incorrectly stating the final answer
Exemplar 2

Question No. 20

Total: 4 marks

The Manitoba Beach Volleyball Association has asked you to design a souvenir beach ball according to the following information:

- The beach ball must have a volume between 1 and 3 cubic feet.
- The plastic material costs $0.15 per ft².
- Labour and other materials cost $1.25 per beach ball.
- The Association wants to make a profit of 80% of the cost of making each beach ball.

Based on your design, what is the minimum selling price for each souvenir beach ball? Show your work.

$$r = 0.75$$

$$\text{Volume} = \frac{4 \pi r^3}{3} = 2.36 \text{ ft}^3$$

$$\text{S.A} = 4 \pi r^2 = 7.06 \ldots \text{ ft}^2$$

Cost for plastic = $0.15 \times 7.06 = $1.06

Total cost for manufacturing ball = $1.06 + $1.25 = $2.31

$$2.31 = 20\% \ of \ the \ total \ cost$$

$$2.31 \times 5 = $11.55$$

$11.55 \times 0.8 = $9.24

$2.31 + $9.24 = 11.55

The organization will have to charge

$11.55 for each ball.

3 marks:

1. → 1 mark for correct radius
2. → 1 mark for correct surface area consistent with radius
3. → 1 mark for correct cost of production consistent with surface area
LOGICAL REASONING

Learning Outcome: 12A.L.3  Question Type: Multiple Choice

Question No. 21 and Answer  Total: 1 mark

Given the statement: “If the temperature outside is below –40°C, then schools will be closed.” Circle the contrapositive below.

A) “If schools are closed, then the temperature outside is below –40°C.”

✓ B) “If schools are not closed, then the temperature outside is not below –40°C.”

C) “If the temperature outside is not below –40°C, then schools will not be closed.”

D) “Schools will be closed if and only if the temperature outside is below –40°C.”

Common Errors
A: converse
C: inverse
D: biconditional
A sample of 100 families was surveyed regarding the electronic devices they have in their homes. The Venn diagram below shows the number of families that have a computer, a DVD player, or a Blu-Ray player.

a) How many families have all three electronic devices in their homes?

(1 mark)

5

b) How many families do not have any of these electronic devices in their homes?

(1 mark)

100 – (14 + 65 + 5 + 3 + 5 + 4 + 2) = 2
Marc wrote the statement: “An isosceles triangle is equilateral.”

a) Rewrite the statement in “if-then” form.

(1 mark)

“If a triangle is isosceles, then it is equilateral.”

b) Provide a counter-example to show that the “if-then” statement in (a) is false.

(1 mark)

See the diagram below for a counter-example.

Other answers are possible.

<table>
<thead>
<tr>
<th>Marking Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
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</tbody>
</table>
Exemplar 1

Question No. 23  
Total: 2 marks

Marc wrote the statement: “An isosceles triangle is equilateral.”

a) Rewrite the statement in “if-then” form.

(I mark)

If it is an isosceles triangle, then it is equilateral

b) Provide a counter-example to show that the “if-then” statement in (a) is false.

(I mark)

If it is equilateral it doesn’t always have to be an isosceles triangle

1 mark:

1 → 1 mark for correct “if-then” statement in (a)
Marc wrote the statement: “An isosceles triangle is equilateral.”

a) Rewrite the statement in “if-then” form.

(1 mark)

If a triangle is equilateral, then it must be an isosceles triangle.

b) Provide a counter-example to show that the “if-then” statement in (a) is false.

(1 mark)

If a triangle is not equilateral, then it is not an isosceles triangle.
The following students attend the same school and participate in the extracurricular activities as indicated below.

The basketball team consists of:

\[ B = \{ \text{Jacquie, Lisa, Mangu, Maya, Nora, Sabrina} \} \]

The student tutoring group consists of:

\[ T = \{ \text{Jacquie, Mangu, Paul, Sabrina, Sam, Simon} \} \]

The volleyball team consists of:

\[ V = \{ \text{Nick, Paul, Pieter, Quinton, Sam, Simon} \} \]

a) Identify the two sets from above that are disjoint.

\((1 \text{ mark})\)

The basketball and volleyball teams are disjoint.

b) Determine \( B \cap T \).

\((1 \text{ mark})\)

\[ \{ \text{Jacquie, Mangu, Sabrina} \} \]
The following students attend the same school and participate in the extracurricular activities as indicated below.

The basketball team consists of:

\[ B = \{ \text{Jacquie, Lisa, Mangu, Maya, Nora, Sabrina} \} \]

The student tutoring group consists of:

\[ T = \{ \text{Jacquie, Mangu, Paul, Sabrina, Sam, Simon} \} \]

The volleyball team consists of:

\[ V = \{ \text{Nick, Paul, Pieter, Quinton, Sam, Simon} \} \]

a) Identify the two sets from above that are disjoint.

(1 mark)

The basketball team and basketball team are disjointed.

b) Determine \( B \cap T \).

(1 mark)

Jacquie, Mangu, and Sabrina are in both basketball and tutoring.

1 mark:

- \( \Theta \) → 1 mark for correct answer in (b)
- \( \ominus \) → 0.5 mark deduction (if applicable) for not stating or incorrectly stating the final answer
The following students attend the same school and participate in the extracurricular activities as indicated below.

The basketball team consists of:

\[ B = \{ \text{Jacquie, Lisa, Mangu, Maya, Nora, Sabrina} \} \]

The student tutoring group consists of:

\[ T = \{ \text{Jacquie, Mangu, Paul, Sabrina, Sam, Simon} \} \]

The volleyball team consists of:

\[ V = \{ \text{Nick, Paul, Pieter, Quinton, Sam, Simon} \} \]

a) Identify the two sets from above that are disjoint.

(1 mark)

\[ \cap (A \cap C) = \emptyset \]

Basketball and volleyball are disjoint.

b) Determine \( B \cap T \).

(1 mark)

\[ B \cap T \]

3 are in both.

1 mark:

① → 1 mark for correct answer in (a)
Appendices
Appendix A:
Table of Questions by Unit and Learning Outcome

<table>
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Legend for Units:
A: Relations and Functions
B: Probability
C: Financial Mathematics
D: Design and Measurement
E: Logical Reasoning

Legend for Question Types:
MC: Multiple Choice
SA: Short Answer
LA: Long Answer
# Appendix B:

Table of Questions by Type and Learning Outcome

<table>
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Total = 37

**Legend for Question Types:**
- MC: Multiple Choice
- SA: Short Answer
- LA: Long Answer

**Legend for Units:**
- A: Relations and Functions
- B: Probability
- C: Financial Mathematics
- D: Design and Measurement
- E: Logical Reasoning
Appendix C:
Irregularities in Provincial Tests

A Guide for Local Marking

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the Department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all “NR”) or only incorrect responses (“0”)

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the Department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the Department along with the test materials.
Irregular Test Booklet Report

Test: ____________________________________________________________

Date marked: ____________________________________________________

Booklet No.: ____________________________________________________

Problem(s) noted: ________________________________________________

_________________________________________________________________

_________________________________________________________________

_________________________________________________________________

_________________________________________________________________

_________________________________________________________________

Question(s) affected: _____________________________________________

_________________________________________________________________

_________________________________________________________________

_________________________________________________________________

_________________________________________________________________

Action taken or rationale for assigning marks: ______________________

_________________________________________________________________

_________________________________________________________________

_________________________________________________________________

_________________________________________________________________

_________________________________________________________________