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After the administration of this test, print copies of this resource will be available for purchase from the Manitoba Learning Resource Centre. Order online at www.manitobalrc.ca.

This resource will also be available on the Manitoba Education and Training website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time. Available in alternate formats upon request.
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General Marking Instructions

Please ensure that

- the student booklet number matches the number on the Scoring Sheet
- only a pencil is used to complete the Scoring Sheet
- the final test mark is recorded on the Scoring Sheet
- the Scoring Sheet is complete and a copy has been made for school records

Do not make any marks in the student booklets. Booklets may be selected by Manitoba Education and Training for sample marking.

Once marking is completed, please forward the Scoring Sheets to Manitoba Education and Training using the envelope provided (for more information, see the administration manual).

Marking

Explanations for student errors for selected-response questions have been provided, if applicable.

To receive full marks for a question, a student’s response must be complete and correct. Partial marks may be awarded for an “appropriate strategy” with execution errors. An appropriate strategy is defined as one that is consistent with the learning outcomes and mathematical processes associated with the question and, if properly executed, would lead to the correct answer.

Some questions require a form of explanation or justification from students. Explanation or justification can be given through a labelled diagram, in words, by showing mathematical operations for answer verification, or by providing output from a technological tool. For this reason, appropriate flexibility is required when marking student responses.

Students are expected to round all final answers to two decimal places unless otherwise indicated in the question, or if the answer terminates to a whole number or one decimal place. More than two decimal places are acceptable if rounded correctly, except for monetary values or when the context of the question implies whole units be used (e.g., people, cans of paint).

Errors

Marks are deducted if conceptual or communication errors are committed. A 0.5 mark deduction will also apply each time a student makes one of the following errors:

- an arithmetic error
- a procedural error (not a conceptual error)
- a lack of clarity in the explanation, the description, or the justification

Conceptual Errors

As a guiding principle, students should only be penalized once for each error committed in the context of a test question. For example, students may choose an inappropriate strategy for a question, but carry it through correctly and arrive at an incorrect answer. In such cases, students should be penalized for having selected an inappropriate strategy for the task at hand, but should be given credit for having arrived at an answer consistent with their choice of strategy.
Communication Errors

Communication errors are errors that are not related to the concepts and are tracked on the *Scoring Sheet* in a separate section. There is a 0.5 mark deduction for each type of communication error committed, with a maximum deduction of 3 marks from the total test mark. Each communication error can only be deducted once per test and committing a second error of the same type does not further affect a student’s mark.

### Final Answer
- does not include a percent sign
- does not identify the answer (e.g., TVM solver, Venn diagram)
- does not use the given contextual variables
- incorrectly states the final answer

### Notation
- does not include braces when using set notation
- does not include a box when using a Venn diagram
- does not include one of the following in the equation: “y =”, “sin”, “ln”, or “x”, or writes parameters separately from the equation
- does not change “y ~” to “y =” when writing an equation

### Transcription/Transposition
- makes a transcription error (inaccurate transferring of information)
- makes a transposition error (changing order of digits)
- inaccurately plots one point on a scatter plot

### Whole Units
- does not use whole units for materials purchased in design and measurement questions
- does not use whole units in contextual questions involving discrete data (e.g., people)

### Units
- does not include the dollar sign for monetary values
- uses incorrect units of measure
- does not include the units in the final answer
- confuses square and cubic units (e.g., cm² instead of cm³, or vice versa)
- does not include units with labels on a graph

### Rounding
- rounds incorrectly
- rounds too soon
- does not express the answer to the appropriate number of decimal places, including monetary values to two decimal places

When a given response includes multiple types of communication errors, deductions are indicated in the order in which the errors occur in the response. No communication errors are recorded for work that has not been awarded marks. The total deduction may not exceed the marks awarded.
**Scoring**

The marks allocated to questions are based on the concepts associated with the learning outcomes in the curriculum. For each question, shade in the circle on the *Scoring Sheet* that represents the mark awarded based on the concepts. A total of these marks will provide the preliminary mark.

The student’s final mark is determined by subtracting the communication errors from the preliminary mark.

**Example:**

A student has a preliminary mark of 46. The student committed one E1 error (0.5 mark deduction) and three E6 errors (0.5 mark deduction).

<table>
<thead>
<tr>
<th>Final Answer</th>
<th>Notation</th>
<th>Transcription/Transposition</th>
<th>Whole Units</th>
<th>Units</th>
<th>Rounding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Errors</td>
<td>Preliminary Mark – (Number of error types × 0.5) = Final Mark</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>–</td>
<td>(2 × 0.5)</td>
<td>=</td>
<td>45</td>
<td></td>
</tr>
</tbody>
</table>

**Irregularities in Provincial Tests**

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If a *Scoring Sheet* is marked with “0” only (e.g., student was present but did not attempt any questions) please document this on the *Irregular Test Booklet Report*.

**Assistance**

If any issue arises that cannot be resolved locally during marking, please call Manitoba Education and Training at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the marking keys.

Yongfei Wu  
Assessment Consultant  
Grade 12 Applied Mathematics  
Telephone: 204-945-4035  
Toll-Free: 1-800-282-8069, ext.  
4035 Email: yongfei.wu@gov.mb.ca
Marking Keys

Please note that this *Marking Guide* contains screen captures taken from a TI–84 Plus graphing calculator.
RELATIONS AND FUNCTIONS

Question 1  
Learning Outcome: 12.A.R.3  Question Type: Selected Response  
Total: 1 mark

Select the best answer.

Which of the following types of functions could have a range of \( \{ y \mid -2 \leq y \leq 12, \ y \in \mathbb{R} \} \)?

A) exponential

B) **sinusoidal**

C) cubic

D) logarithmic
Select the best answer.

Which of the following functions has a domain of \((0, \infty)\)?

A) \(y = 3^x\)

B) \(y = -3x^2 + 18x - 18\)

C) \(y = 3x^3 + 2x^2 + x\)

D) \(y = 3\ln x\)
The power generated by a wind turbine can be modelled by the equation:

\[ P = 2.06w^3 + 0.56w^2 - 3.38w + 5 \]

where \( P \) represents the power in kilowatts (kW) and \( w \) represents the wind speed in metres per second (m/s).

The wind turbine starts to generate electricity when the power is 5000 kW or greater.

What wind speed is required for the turbine to start generating electricity?

\[
\begin{align*}
Y_2 &= 5000 \\
\text{CALC} 5: \text{intersect} &\quad (13.384 9..., 5000) \\
x &= 13.38 \\
\end{align*}
\]

A wind speed of 13.38 m/s is required.

**Marker Note(s):**

→ Award mark 1 for an answer of \( x = 13.39 \text{ m/s} \) when Desmos is used.

**Marking Key**

1 mark for correct answer
One cup of coffee contains 94 mg of caffeine. Once consumed, the amount of caffeine in the human body is reduced by 16% every hour.

a) Determine the exponential equation that models the remaining amount of caffeine in the body as a function of time in hours. Show your work.

\[ y = 94(0.84)^x \]

b) Using your equation in (a), determine the remaining amount of caffeine in the body 19 hours after consuming one cup of coffee.

\[ y = 94(0.84)^{19} \]

The remaining amount of caffeine in the body is 3.42 mg.

c) If one cup of espresso coffee, containing 125 mg of caffeine, was consumed instead of the cup of coffee, describe how your equation in (a) would change.

The initial value would change from 94 to 125.

\[ y = 125(0.84)^x \]
A bridge is designed to move up and down with the wind. The graph below represents the vertical position of a point at the top of the bridge on a windy day.

**Vertical Position as a Function of Time**

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>0</th>
<th>7.5</th>
<th>15</th>
<th>22.5</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Position (cm)</td>
<td>40</td>
<td>0</td>
<td>-40</td>
<td>0</td>
<td>40</td>
</tr>
</tbody>
</table>

\[ y = 40 \sin (0.21x + 1.57) \]

*Other equations are possible.*

b) Determine how many times the point at the top of the bridge is at its maximum height during the first 60 seconds.

(1 mark)

The point at the top of the bridge is at its maximum height 3 times during the first 60 seconds.

**Marker Note(s):**

→ Award 1 mark in (a) for correct table with incorrect regression.

**Marking Key**

1. 0.5 mark for correct “a” value in equation in (a)
2. 0.5 mark for correct “b” value in equation in (a)
3. 0.5 mark for correct “c” value in equation in (a)
4. 0.5 mark for correct “d” value in equation in (a)
5. 1 mark for consistent answer in (b)
During a science class experiment, Karlie stands on a ladder and throws a ball up in the air. Her partner records the time from when the ball leaves her hand until it hits the ground. She determines that the equation representing the height of the ball as a function of time is:

\[ h(t) = -4.9t^2 + 10t + 4 \]

where \( h \) represents the height in metres and \( t \) represents the time in seconds.

a) Create a clearly labelled graph of the equation given the context of this question.

(3 marks)
b) Determine the amount of time the ball is 4 metres or higher from the ground during this experiment.

\[(1\text{ mark})\]

\[Y_2 = 4\]

\[\text{CALC} 5: \text{intersect } (0, 4) \text{ and } (2.040\ldots, 4)\]

\[x_1 = 0, x_2 = 2.04\]

Time at 4 m or higher = 2.04 – 0 = 2.04 s

The ball is 4 metres or higher for 2.04 seconds.

Marking Key

<table>
<thead>
<tr>
<th>Mark</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 mark for communicating the context of the graph with appropriate title and/or labels in (a)</td>
</tr>
<tr>
<td>2</td>
<td>1 mark for using an appropriate domain and range (i.e., window settings/grid range) for the context of the question in (a)</td>
</tr>
<tr>
<td>3</td>
<td>1 mark for an appropriate shape that illustrates key characteristics of the function (e.g., maximum, minimum, asymptotes, intercepts) in (a)</td>
</tr>
<tr>
<td>4</td>
<td>1 mark for correct time at 4 metres or higher in (b)</td>
</tr>
</tbody>
</table>
Select the best answer.

A building has 8 doors that can be used to either enter or exit.

How many ways can Cindy enter and exit the building if she must enter through one door and exit through another?

A) 15
B) 16
C) 56
D) 64

Student Error
A: 8+7
B: 8+8
D: 8×8
Question 8

Learning Outcome: 12.A.P.5

There are 2 principals and 5 teachers lining up in a row for a photograph that will appear in the local paper.

Determine the number of ways they can line up in a row if a principal must stand on each end. Show your work.

\[
\begin{align*}
2 \times 5 \times 4 \times 3 \times 2 \times 1 \times 1 &= 240 \\
\uparrow & \quad \uparrow \\
\text{principal} & \quad \text{principal}
\end{align*}
\]

There are 240 ways.

OR

\[
2! \times 5! = 240 \text{ ways}
\]

There are 240 ways.

OR

\[
2P_2 \times 5P_5 = 240 \text{ ways}
\]

There are 240 ways.

Marking Key

- 0.5 mark for 2! or 2P_2 (principals)
- 0.5 mark for 5! or 5P_5 (teachers)
- 1 mark for consistent product
The probability that Brian participates in a study group before his test is 0.70. If he participates in a study group, the probability that he will get an A on his test is 0.80. If he does not participate in a study group, the probability that he will get an A on his test is 0.40.

a) Use a graphic organizer to show all possible outcomes for this situation.

(b) Determine the probability that Brian will get an A on his test. Show your work.

\[ P(\text{gets an A}) = P(\text{study group, gets an A}) + P(\text{no study group, gets an A}) \]
\[ = (0.70)(0.80) + (0.30)(0.40) \]
\[ = 0.56 + 0.12 \]
\[ = 0.68 \]

The probability is \( \frac{17}{25} \), 0.68, or 68%.

**Marking Key**

<table>
<thead>
<tr>
<th>Marking Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mark for appropriate graphic organizer in (a)</td>
</tr>
<tr>
<td>0.5 mark for ( P(\text{study group, gets an A}) ) in (b)</td>
</tr>
<tr>
<td>0.5 mark for ( P(\text{no study group, gets an A}) ) in (b)</td>
</tr>
<tr>
<td>1 mark for consistent sum in (b)</td>
</tr>
</tbody>
</table>
Each housing area in Manitoba is assigned a postal code.

- Each postal code is made up of 3 letters and 3 digits that alternate.
- The postal code must begin with the letter R.
- The letters D, F, I, O, Q, U are not used.
- Repetition is allowed.

Determine the number of postal codes that can be created in Manitoba. Show your work.

\[
\begin{align*}
1 \times 10 \times 20 & \times 10 \times 20 \times 10 = 400\,000 \\
R & \text{Digit} & \text{Letter} & \text{Digit} & \text{Letter} & \text{Digit}
\end{align*}
\]

There are 400 000 postal codes that can be created in Manitoba.

**Marker Note(s):**
→ Deduct a maximum of 0.5 mark if repetition is not demonstrated.

**Marking Key**

<table>
<thead>
<tr>
<th>1</th>
<th>0.5 mark for considering R</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.5 mark for considering letter factors</td>
</tr>
<tr>
<td>3</td>
<td>0.5 mark for considering digit factors</td>
</tr>
<tr>
<td>4</td>
<td>0.5 mark for consistent product</td>
</tr>
</tbody>
</table>
From his home, Rasik has to take either a bus or a train downtown and then take a different bus or train to work. He has the following transportation options:

- 5 different bus routes or 3 different train routes from home to downtown
- 4 different bus routes or 2 different train routes from downtown to work

a) Determine the number of routes Rasik can take from home to work.

(1 mark)

$$8 \times 6 = 48$$

There are 48 routes.

b) Determine the number of routes Rasik can take from home to work by only taking the bus.

(1 mark)

$$\text{bus only routes} = 5 \times 4$$

$$= 20$$

c) If Rasik randomly chooses his routes, what are the odds that he takes only a bus from home to work?

(1 mark)

$$\text{total – bus only routes} = 48 - 20$$

$$= 28$$

The odds are 20:28 or 5:7.

Marking Key

| 1 | 1 mark for correct answer in (a) |
| 2 | 1 mark for correct number of bus only routes in (b) |
| 3 | 1 mark for consistent odds in (c) |
Question 12  

Learning Outcome: 12.A.P.6  

At an amusement park, there are 11 adults and 17 teenagers wanting to go on a boat ride. The boat has a 9-passenger capacity.

If there are at most 2 adult passengers, determine the number of ways 9 passengers can be chosen to go on the boat ride. Show your work.

Case 1: 2 adults, 7 teenagers: \( \binom{11}{2} \times \binom{17}{7} = 1069640 \)

Case 2: 1 adult, 8 teenagers: \( \binom{11}{1} \times \binom{17}{8} = 267410 \)

Case 3: 0 adults, 9 teenagers: \( \binom{11}{0} \times \binom{17}{9} = 24310 \)

\[ 1069640 + 267410 + 24310 = 1361360 \]

There are 1,361,360 ways.

Marker Note(s):
→ Award a maximum of 0.5 mark for correctly describing cases with no calculations.
→ Award a maximum of 1 mark if permutations are used instead of combinations.
→ Award a maximum of 1 mark if sums are used instead of products when calculating cases.

Marking Key

1. 0.5 mark for correct product in Case 1 or \( \binom{11}{2} \times \binom{17}{7} \)
2. 0.5 mark for correct product in Case 2 or \( \binom{11}{1} \times \binom{17}{8} \)
3. 0.5 mark for correct product in Case 3 or \( \binom{11}{0} \times \binom{17}{9} \)
4. 0.5 mark for consistent sum
Of the 41 students in Grade 12,

- 21 students are in drama club
- 17 students are in chess club
- 14 students are in environmental club
- 8 students are in chess club and drama club
- 10 students are in drama club and environmental club
- 3 students are in chess club and environmental club only
- 2 students are in all three clubs

a) Draw a Venn diagram to represent this situation.

(3 marks)

b) What is the probability that a randomly selected student is not in any of these clubs?

(1 mark)

\[
\frac{10}{41} = 0.2439
\]

The probability is \( \frac{10}{41} \), 0.24, or 24.39%.

Marking Key

1. 1 mark for correctly placing the number of students in two or more clubs in (a)
2. 1 mark for consistent number of students in exactly one club in (a)
3. 1 mark for consistent number of students not in any clubs in (a)
4. 1 mark for consistent answer in (b)
FINANCIAL MATHEMATICS

Question 14

Learning Outcome: 12.A.FM.1

Question Type: Selected Response

Select the best answer.

Eric wants to borrow money from his bank.

Which of the following compounding periods results in the least amount of interest paid on the loan?

A) monthly

B) semi-annually

C) quarterly

D) daily
Jeannette’s new job involves a lot of driving. She wants to lease a new vehicle. Her friend Simon is trying to convince her not to lease.

State two reasons that Simon could use to convince her not to lease.

**Reasons:**
- She could be penalized for exceeding the limit of kilometres driven.
- The vehicle cannot be customized.
- There is no potential for equity.
- Her job might change and her driving requirements may also change.

*Other reasons are possible.*

**Marking Key**

<table>
<thead>
<tr>
<th></th>
<th>Marking Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 mark for the first appropriate reason</td>
</tr>
<tr>
<td>2</td>
<td>1 mark for the second appropriate reason</td>
</tr>
</tbody>
</table>
Question 16  

Total: 5 marks

Learning Outcomes: 12.A.FM.1, 12.A.FM.3  

Question Type: Constructed Response

Himesh wants to retire in 35 years. After meeting with his financial advisor, he determines that his portfolio will consist of the following two investments:

**Investment 1**: $15 000.00 in a mutual fund that earns an interest rate of 6.50%, compounded monthly.

**Investment 2**: regular biweekly deposits of $180.00 in a tax-free savings account (TFSA), with an interest rate of 3.75%, compounded biweekly. (Assume the initial value of the TFSA is 0.)

a) What will be the value of each investment when Himesh retires? Show your work.

(4 marks)

<table>
<thead>
<tr>
<th>Investment 1</th>
<th>Investment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>N=35</td>
<td>N=910</td>
</tr>
<tr>
<td>I%=6.5</td>
<td>I%=3.75</td>
</tr>
<tr>
<td>PV=15000</td>
<td>PV=0</td>
</tr>
<tr>
<td>PMT=0</td>
<td>PMT=180</td>
</tr>
<tr>
<td>FV=145025.69</td>
<td>FV=338449.99</td>
</tr>
<tr>
<td>P/Y=12</td>
<td>P/Y=26</td>
</tr>
<tr>
<td>C/Y=12</td>
<td>C/Y=26</td>
</tr>
<tr>
<td>FMT:END</td>
<td>FMT:END</td>
</tr>
</tbody>
</table>

The value of Investment 1 will be $145 025.69.
The value of Investment 2 will be $338 449.99.

b) Himesh’s goal is to have $500 000.00 in his portfolio by the time he retires. Determine if he will meet his goal. Justify your answer by using your investment values in (a).

(1 mark)

$145 025.69 + $338 449.99 = $483 475.68

No, he will not meet his goal.

OR

No, he will not meet his goal because the sum of his portfolio is less than $500 000.00.

<table>
<thead>
<tr>
<th>Marking Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
</tbody>
</table>
Kaia purchased a house for $400 000.00. At the time of purchase, she made a down payment of $100 000.00. The remaining balance of the mortgage was financed at an interest rate of 4.30%, compounded semi-annually, over 25 years.

a) How much is Kaia’s monthly mortgage payment? Show your work.

\[
N=300 \\
I%=4.5 \\
P=300000 \\
PMT=-1627.22697 \\
FV=0 \\
P/Y=12 \\
C/Y=2 \\
PMT:BEGIN
\]

Kaia’s monthly mortgage payment is $1627.23.

b) Kaia’s house appreciates in value at an average rate of 2.00% per year. What will the value of her house be after 15 years?

\[
\$400\ 000.00 \times 1.02^{15} = \$538\ 347.34
\]

The value of her house will be $538 347.34.

c) How much equity will Kaia have in the house after 15 years? Show your work.

\[
\text{Equity} = \$538\ 347.34 - \$158\ 761.79 \\
= \$379\ 585.55
\]

Kaia will have $379 585.55 in equity after 15 years.

OR

\[
\sum \text{Prin}(1.180)^{\sum \text{Prin}(1.180)} \\
\text{Equity} = \text{appreciation} + \text{principal paid} + \text{down payment} \\
= \$138\ 347.34 + \$141\ 238.21 + \$100\ 000.00 \\
= \$379\ 585.55
\]

Kaia will have $379 585.55 in equity after 15 years.

**Marking Key**

<table>
<thead>
<tr>
<th>Marking Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1 mark for appropriate work in (a)</td>
</tr>
<tr>
<td>2 1 mark for consistent answer in (a)</td>
</tr>
<tr>
<td>3 1 mark for correct answer in (b)</td>
</tr>
<tr>
<td>4 0.5 mark for consistent calculation of balance owing or of principal paid in (c)</td>
</tr>
<tr>
<td>5 0.5 mark for consistent balance owing or principal paid in (c)</td>
</tr>
<tr>
<td>6 0.5 mark for consistent calculation of equity in (c)</td>
</tr>
<tr>
<td>7 0.5 mark for consistent equity in (c)</td>
</tr>
</tbody>
</table>
Dimitri owns a house with a mortgage of $175 000.00. He has $165 000.00 in other loans and his net worth is $300 000.00.

a) Determine his debt-to-equity ratio.

\[
\text{Debt-to-equity ratio (\%)} = \left( \frac{\text{Total liabilities} - \text{Mortgage}}{\text{Net worth}} \right) \times 100
\]

\[
= \left( \frac{340 000.00 - 175 000.00}{300 000.00} \right) \times 100
\]

\[
= 55\%
\]

His debt-to-equity ratio is 55%.

b) Determine his total assets.

\[
\text{Total assets} = \text{Net worth} + \text{Total liabilities}
\]

\[
= 300 000.00 + 340 000.00
\]

\[
= 640 000.00
\]

His total assets are $640 000.00.

c) Dimitri wants to borrow $10 000.00 to take his family on a vacation. Using his debt-to-equity ratio in (a), explain if the bank will lend him the money.

\[
\text{No, the bank will not lend him the money since his debt-to-equity ratio is above 50\%.}
\]
**DESIGN AND MEASUREMENT**

<table>
<thead>
<tr>
<th>Question 19</th>
<th>Total: 1 mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learning Outcome: 12.A.D.1</td>
<td>Question Type: Selected Response</td>
</tr>
</tbody>
</table>

Select the best answer.

Maala needs to shovel a 1 m × 2.8 m walkway after a 30 cm snowfall. The maximum amount of snow each full shovel can remove is 0.04 m³ per scoop.

Assuming each scoop contains the maximum amount of snow, how many scoops will it take for her to remove all the snow from the walkway?

A) 21  
B) 70  
C) 336  
D) 2100

**Student Error**  
B: 1 × 2.8 ÷ 0.04  
C: 280 × 30 × 0.04  
D: 1 × 2.8 × 30 ÷ 0.04
Hugo creates a scale model of a planet for art class. He wants to paint it.

- The radius of the model is 20 cm.
- He has one bottle of paint that will cover 12 000 cm².
- It is assumed that each coat applied requires the same amount of paint.

How many full coats of paint can be applied using the one bottle? Show your work.

Surface area \( = 4\pi r^2 \)
\[ = 4\pi (20)^2 \]
\[ = 5026.55 \text{ cm}^2 \]

\[
\frac{12000 \text{ cm}^2}{5026.55 \text{ cm}^2} = 2.39
\]

There are 2 full coats that can be applied.

Marking Key

<table>
<thead>
<tr>
<th></th>
<th>Marking Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 mark for correct surface area</td>
</tr>
<tr>
<td>2</td>
<td>1 mark for consistent number of full coats</td>
</tr>
</tbody>
</table>
You are entered in a sandcastle building contest. Pails in the following shapes are available for use:

Shapes are not drawn to scale.

Your sandcastle design must:

- be built on a 2 ft. × 1 ft. area
- use each shape at least once
- include a minimum of 5 shapes
- be no more than 3 shapes high

a) Draw or describe a sandcastle that meets the above criteria.

(2 marks)

Sample Design 1:

On the bottom level of my sandcastle, I placed two cubes beside each other. On top of each cube, I placed a cylinder. I then placed one cone on top of the cylinders, in the middle.

OR

Sample Design 2:

On the bottom level of my sandcastle, I placed two cubes beside each other. On top of each cube, I placed a cylinder. I then placed one cone on top of each cylinder.

Other answers are possible.
b) Calculate the volume of sand required, in cubic feet, to build your sandcastle. Show your work.

\((3 \text{ marks})\)

cube \(= (12)^3 \)  
\(= 1728 \text{ in}^3 \)  
\(= 1 \text{ ft}^3 \)

cylinder \(= \pi (3)^2 (9) \)  
\(= 254.469 \text{ in}^3 \)  
\(= 0.147 \text{ ft}^3 \)

cone \(= \frac{\pi (3)^2 (11)}{3} \)  
\(= 103.673 \text{ in}^3 \)  
\(= 0.060 \text{ ft}^3 \)

Sample Design 1: \[2 \text{ cubes} + 2 \text{ cylinders} + 1 \text{ cone}\]

\[V = 2(1728) + 2(254.469) + 103.673\]

\[= 3456 + 508.938 + 103.673\]

\[= 4068.611 \text{ in}^3\]

\[\frac{4068.611}{12^3} = 2.35 \text{ ft}^3\]

Sample Design 2: \[2 \text{ cubes} + 2 \text{ cylinders} + 2 \text{ cones}\]

\[V = 2(1728) + 2(254.469) + 2(103.673)\]

\[= 3456 + 508.938 + 207.345\]

\[= 4172.283 \text{ in}^3\]

\[\frac{4172.283}{12^3} = 2.41 \text{ ft}^3\]

Other answers are possible.

c) If the organizers of the contest purchased sand for $1.11/\text{ft}^3$, what is the cost of the sand needed to build your design? Assume whole units do not need to be used since the sand has already been purchased.

\((1 \text{ mark})\)

Sample Design 1: \[2.35 \text{ ft}^3 \times \$1.11/\text{ft}^3 = \$2.61\]

OR

Sample Design 2: \[2.41 \text{ ft}^3 \times \$1.11/\text{ft}^3 = \$2.68\]

Other answers are possible.

<table>
<thead>
<tr>
<th>Marking Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
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<td>5</td>
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<td>7</td>
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<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>
An online app awards 0 points, 3 points, or 7 points for performing tasks. Once your score adds up to 25 points or more, you pass the level.

State the scores that are not possible at any time during any level. Show your work using the list of numbers below.

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 \\
6 & 7 & 8 & 9 & 10 \\
11 & 12 & 13 & 14 & 15 \\
16 & 17 & 18 & 19 & 20 \\
21 & 22 & 23 & 24 & 25 \\
\end{array}
\]

Scores that are not possible: 1, 2, 4, 5, 8, 11

Marking Key

1. 0.5 mark for correctly eliminating all multiples of 3
2. 0.5 mark for correctly eliminating all multiples of 7
3. 0.5 mark for correctly eliminating at least one other score
4. 0.5 mark for correctly eliminating all combinations of 3 and 7
Question 23

Learning Outcome: 12.A.I.2

The universal set of living things includes:

- the set of all birds \((B)\)
- the set of all plants \((P)\)
- the set of all owls \((O)\)
- the set of all dogs \((D)\)
- the set of all mammals \((M)\)
- the set of all roses \((R)\)

a) Give one example of two disjoint subsets.

(1 mark)

\[B \text{ and } P\]

OR

\[P \text{ and } O\]

OR

\[M \text{ and } R\]

Other examples are possible.

b) Using set notation, give one example of a set that is a subset of another.

(1 mark)

\[O \subset B\]

OR

\[D \subset M\]

OR

\[R \subset P\]

Marking Key

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 mark for correct answer in (a)</td>
</tr>
<tr>
<td>2</td>
<td>1 mark for correct answer in (b)</td>
</tr>
</tbody>
</table>
A movie theatre employee tracked the number of people who purchased popcorn ($P$) and chocolate ($C$) in one month. The results are given below.

![Venn Diagram]

a) What percentage of people purchased popcorn?

1 mark

\[ 57\% + 16\% = 73\% \]

The percentage of people that purchased popcorn is 73%.

b) Describe what $(P \cup C)'$ represents in this situation.

1 mark

This represents the percentage of people that did not purchase popcorn or chocolate at this theatre.

<table>
<thead>
<tr>
<th>Marking Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>
Given the following conditional statement:

“If you live in Manitoba, then you live in Canada.”

a) Identify what type of statement is logically equivalent to the above conditional statement.

(1 mark)

Select the best answer.

A) converse

B) inverse

C) contrapositive

b) Write the statement you identified in (a).

(1 mark)

“If you do not live in Canada, then you do not live in Manitoba.”

Marking Key

<table>
<thead>
<tr>
<th>Marking Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>
Exemplars

Exemplars may contain screen captures taken from software or Internet pages.
Exemplar 1

Question 3

The power generated by a wind turbine can be modelled by the equation:

\[ P = 2.06w^3 + 0.56w^2 - 3.38w + 5 \]

where \( P \) represents the power in kilowatts (kW) and \( w \) represents the wind speed in metres per second (m/s).

The wind turbine starts to generate electricity when the power is 5000 kW or greater.

What wind speed is required for the turbine to start generating electricity?
The power generated by a wind turbine can be modelled by the equation:

\[ P = 2.06w^3 + 0.56w^2 - 3.38w + 5 \]

where \( P \) represents the power in kilowatts (kW) and \( w \) represents the wind speed in metres per second (m/s).

The wind turbine starts to generate electricity when the power is 5000 kW or greater.

What wind speed is required for the turbine to start generating electricity?

When \( P = 5000 \)

\[ w = 13.38 \]

The wind speed required is 13.38 m/s.

1 mark:
1 → 1 mark for correct answer
5 → does not include the units in the final answer
One cup of coffee contains 94 mg of caffeine. Once consumed, the amount of caffeine in the human body is reduced by 16% every hour.

a) Determine the exponential equation that models the remaining amount of caffeine in the body as a function of time in hours. Show your work.

\[
y = ab^x\]

[Desmos graph]

\[
a = 111.915 \quad b = 0.83996
\]

b) Using your equation in (a), determine the remaining amount of caffeine in the body 19 hours after consuming one cup of coffee.

\[
y = (111.915)(0.83996)^{19}
\]

After 19 hours the body would have \[4.072\ mg\]

c) If one cup of espresso coffee, containing 125 mg of caffeine, was consumed instead of the cup of coffee, describe how your equation in (a) would change.

The equation would change the percentage of caffeine every hour and make it a larger amount remaining after every hour.

2 marks:

\[\rightarrow 1\ mark\ for\ correct\ rate\ of\ change\ in\ equation\ in\ (a)\]
\[\rightarrow 1\ mark\ for\ consistent\ answer\ in\ (b)\]
\[\rightarrow 1\ mark\ for\ rounds\ too\ soon\]
Exemplar 2

Question 4

One cup of coffee contains 94 mg of caffeine. Once consumed, the amount of caffeine in the human body is reduced by 16% every hour.

a) Determine the exponential equation that models the remaining amount of caffeine in the body as a function of time in hours. Show your work.

(b marks)

<table>
<thead>
<tr>
<th>hrs</th>
<th>caffeine</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>94</td>
</tr>
<tr>
<td>1</td>
<td>15.04</td>
</tr>
<tr>
<td>2</td>
<td>2.41</td>
</tr>
<tr>
<td>3</td>
<td>0.39</td>
</tr>
<tr>
<td>4</td>
<td>0.06</td>
</tr>
</tbody>
</table>

\[ y = 94 \times (0.84)^x \]

b) Using your equation in (a), determine the remaining amount of caffeine in the body 19 hours after consuming one cup of coffee.

(1 mark)

\[ x = 19, \quad y = 6.6 \times 10^{-14} \]

(c) If one cup of espresso coffee, containing 125 mg of caffeine, was consumed instead of the cup of coffee, describe how your equation in (a) would change.

(1 mark)

\[ y = 315.74 \times 0.08^x \]

the values are the same, just a faster depreciation in a cup of espresso

2 marks:

1 → 1 mark for correct initial value in equation in (a)
2 → 1 mark for consistent answer in (b)
3 → does not include the units in the final answer
4 → rounds too soon in (a)
5 → does not express the answer to the appropriate number of decimal places in (b)
Exemplar 3

Question 4

One cup of coffee contains 94 mg of caffeine. Once consumed, the amount of caffeine in the human body is reduced by 16% every hour.

a) Determine the exponential equation that models the remaining amount of caffeine in the body as a function of time in hours. Show your work.

(2 marks)

\[
y(t) = 94 \cdot 0.84^t
\]

b) Using your equation in (a), determine the remaining amount of caffeine in the body 19 hours after consuming one cup of coffee.

(1 mark)

\[
y(19) \approx 3.42 
\]

3.42 mg

(c) If one cup of espresso coffee, containing 125 mg of caffeine, was consumed instead of the cup of coffee, describe how your equation in (a) would change.

(1 mark)

The "a" value would increase.

4 marks:

1 → 1 mark for correct initial value in equation in (a)
2 → 1 mark for correct rate of change in equation in (a)
3 → 1 mark for consistent answer in (b)
4 → 1 mark for appropriate description in (c)

→ does not include one of the following in the equation: "y =", "sin", "ln", or "x", or writes parameters separately from the equation
→ rounds too soon
A bridge is designed to move up and down with the wind. The graph below represents the vertical position of a point at the top of the bridge on a windy day.

Vertical Position as a Function of Time

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>0</th>
<th>7.5</th>
<th>15</th>
<th>22.5</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Position (cm)</td>
<td>40</td>
<td>0</td>
<td>-40</td>
<td>0</td>
<td>40</td>
</tr>
</tbody>
</table>

\[ y = 40 \sin(0.21(x+1.57)) \]

b) Determine how many times the point at the top of the bridge is at its maximum height during the first 60 seconds.

\[ \text{Period} = \frac{30}{60} \times 2 \text{ times} \]

1.5 marks:
1. 0.5 mark for correct “a” value in equation in (a)
2. 0.5 mark for correct “b” value in equation in (a)
3. 0.5 mark for correct “d” value in equation in (a)
Exemplar 2

Question 5  Total: 3 marks

A bridge is designed to move up and down with the wind. The graph below represents the vertical position of a point at the top of the bridge on a windy day.

(2 marks)

b) Determine how many times the point at the top of the bridge is at its maximum height during the first 60 seconds.

(1 mark)
During a science class experiment, Karlie stands on a ladder and throws a ball up in the air. Her partner records the time from when the ball leaves her hand until it hits the ground. She determines that the equation representing the height of the ball as a function of time is:

\[ h(t) = -4.9t^2 + 10t + 4 \]

where \( h \) represents the height in metres and \( t \) represents the time in seconds.

a) Create a clearly labelled graph of the equation given the context of this question.
Exemplar 1 (continued)

b) Determine the amount of time the ball is 4 metres or higher from the ground during this experiment.

(1 mark)
Exemplar 2

Question 6  
Total: 4 marks

During a science class experiment, Karlie stands on a ladder and throws a ball up in the air. Her partner records the time from when the ball leaves her hand until it hits the ground. She determines that the equation representing the height of the ball as a function of time is:

\[ h(t) = -4.9t^2 + 10t + 4 \]

where \( h \) represents the height in metres and \( t \) represents the time in seconds.

a) Create a clearly labelled graph of the equation given the context of this question.

(3 marks)
Exemplar 2 (continued)

b) Determine the amount of time the ball is 4 metres or higher from the ground during this experiment.

(1 mark)

\[ y \geq 4 \]

The ball is 4 meters or higher from the ground for 2.041 seconds.

3 marks:

1 → 1 mark for communicating the context of the graph with appropriate title and/or labels in (a)

2 → 1 mark for an appropriate shape that illustrates key characteristics of the function (e.g., maximum, minimum, asymptotes, intercepts) in (a)

3 → 1 mark for correct time at 4 metres or higher in (b)
There are 2 principals and 5 teachers lining up in a row for a photograph that will appear in the local paper.

Determine the number of ways they can line up in a row if a principal must stand on each end. Show your work.

\[ 2P_2 + 5P_5 = 122 \text{ ways} \]

1 mark:
① → 0.5 mark for 2! or \( 2P_2 \) (principals)
② → 0.5 mark for 5! or \( 5P_5 \) (teachers)
There are 2 principals and 5 teachers lining up in a row for a photograph that will appear in the local paper.

Determine the number of ways they can line up in a row if a principal must stand on each end. Show your work.

\[ \underline{2 \ 5 \ 5 \ 5 \ 5 \ 1} = 6,250 \text{ ways} \]

1.5 marks:
1. → 0.5 mark for 2! or $2! \ P_2$ (principals)
2. → 1 mark for consistent product
The probability that Brian participates in a study group before his test is 0.70. If he participates in a study group, the probability that he will get an A on his test is 0.80. If he does not participate in a study group, the probability that he will get an A on his test is 0.40.

a) Use a graphic organizer to show all possible outcomes for this situation.

(1 mark)

b) Determine the probability that Brian will get an A on his test. Show your work.

(2 marks)

\[ \begin{align*}
0.70 \times 0.80 &= 0.56 \\
0.30 \times 0.40 &= 0.12 \\
\text{The probability that Brian will get a } A \text{ on his test is } 68\%.
\end{align*} \]

3 marks:
1 → 1 mark for appropriate graphic organizer in (a)
2 → 0.5 mark for \(P(\text{study group, gets an A})\) in (b)
3 → 0.5 mark for \(P(\text{no study group, gets an A})\) in (b)
4 → 1 mark for consistent sum in (b)
Exemplar 2

Question 9

The probability that Brian participates in a study group before his test is 0.70. If he participates in a study group, the probability that he will get an A on his test is 0.80. If he does not participate in a study group, the probability that he will get an A on his test is 0.40.

a) Use a graphic organizer to show all possible outcomes for this situation.

(1 mark)

\[
\begin{align*}
P &= \text{participate} \\
\bar{P} &= \text{does not participate} \\
A &= \text{gets an A} \\
\bar{A} &= \text{does not get an A}
\end{align*}
\]

\[
\begin{array}{c}
0.7 & \quad P \\
0.3 & \quad \bar{P}
\end{array}
\]

\[
\begin{array}{c}
0.80 & \quad A \\
0.20 & \quad \bar{A}
\end{array}
\]

\[
\begin{array}{c}
0.40 & \quad A \\
0.60 & \quad \bar{A}
\end{array}
\]

b) Determine the probability that Brian will get an A on his test. Show your work.

(2 marks)

\[
\begin{align*}
\text{P}(\text{study group, gets an A}) &= (0.7)(0.8) + (0.3)(0.6) \\
&= 0.56 + 0.18 \\
&= 0.74
\end{align*}
\]

2.5 marks:

1. 1 mark for appropriate graphic organizer in (a)
2. 0.5 mark for \(P(\text{study group, gets an A})\) in (b)
3. 1 mark for consistent sum in (b)
Exemplar 1

Question 10

Each housing area in Manitoba is assigned a postal code.

- Each postal code is made up of 3 letters and 3 digits that alternate.
- The postal code must begin with the letter R.
- The letters D, F, I, O, Q, U are not used.
- Repetition is allowed.

Determine the number of postal codes that can be created in Manitoba. Show your work.

\[
\begin{array}{cccccccc}
R & \# & 9 & \# & 20 & \# & 9 & \# \\
& \times & 1 & \times & 9 & \times & 20 & \times 9 \\
\end{array}
\]

\[= \boxed{291600\text{ postal codes}}\]

1.5 marks:

1. → 0.5 mark for considering R
2. → 0.5 mark for considering letter factors
3. → 0.5 mark for consistent product
Exemplar 2

Question 10  

Each housing area in Manitoba is assigned a postal code.

- Each postal code is made up of 3 letters and 3 digits that alternate.
- The postal code must begin with the letter R.
- The letters D, F, I, O, Q, U are not used.
- Repetition is allowed.

Determine the number of postal codes that can be created in Manitoba. Show your work.

\[
\text{Case 1:} \quad R - 10 - 19 - 9 - 18 - 8 = 2\,461,240 \text{ postal codes}
\]

1.5 marks:

1. → 0.5 mark for considering R
2. → 0.5 mark for considering digit factors
3. → 0.5 mark for consistent product
Exemplar 1

Question 11

From his home, Rasik has to take either a bus or a train downtown and then take a different bus or train to work. He has the following transportation options:

- 5 different bus routes or 3 different train routes from home to downtown
- 4 different bus routes or 2 different train routes from downtown to work

(a) Determine the number of routes Rasik can take from home to work.

(1 mark)

\[ \text{Bus} = \frac{5}{H} \times \frac{4}{W} = 20 \]

He can take 26 different routes

\[ \text{Train} = \frac{3}{H} \times \frac{2}{W} = 6 \]

(b) Determine the number of routes Rasik can take from home to work by only taking the bus.

(1 mark)

20

(c) If Rasik randomly chooses his routes, what are the odds that he takes only a bus from home to work?

(1 mark)

\[ \frac{20}{26} \]

2 \, : \, 6

2 marks:

1 mark for correct number of bus only routes in (b)
1 mark for consistent odds in (c)
Exemplar 2

Question 11  

From his home, Rasik has to take either a bus or a train downtown and then take a different bus or train to work. He has the following transportation options:

- 5 different bus routes or 3 different train routes from home to downtown
- 4 different bus routes or 2 different train routes from downtown to work

(a) Determine the number of routes Rasik can take from home to work.

(1 mark)

\[
\binom{5+3}{1} \cdot \binom{4+2}{1} = 48
\]

or

\[
\binom{8}{1} \cdot \binom{6}{1} = 48
\]

48 routes

(b) Determine the number of routes Rasik can take from home to work by only taking the bus.

(1 mark)

\[
\binom{5}{1} \cdot \binom{4}{1} = 20
\]

c) If Rasik randomly chooses his routes, what are the odds that he takes only a bus from home to work?

(1 mark)

\[
\frac{\binom{5}{1} \cdot \binom{4}{1}}{\binom{8}{1} \cdot \binom{6}{1}} = \frac{20}{48} = \frac{5}{12}
\]

\[
\frac{5}{12} \text{ or } 0.42 \text{ or } 41.67\%
\]

2 marks:

1 → 1 mark for correct answer in (a)
2 → 1 mark for correct number of bus only routes in (b)
Exemplar 1

Question 12

At an amusement park, there are 11 adults and 17 teenagers wanting to go on a boat ride. The boat has a 9-passenger capacity.

If there are at most 2 adult passengers, determine the number of ways 9 passengers can be chosen to go on the boat ride. Show your work.

\[
\begin{align*}
\text{no adult} & \quad 17 \quad 16 \quad 15 \quad 14 \quad 13 \quad 12 \quad 11 \quad 10 \quad 9 \quad 8 \quad 8 \quad 2 \quad 1 \quad 6 \quad 4 \quad 2 \quad 8 \quad 0 \quad 0 \\
\text{1 adult} & \quad 9 \quad 17 \quad 16 \quad 15 \quad 14 \quad 13 \quad 12 \quad 11 \quad 10 \quad 8 \quad 8 \quad 2 \quad 1 \quad 6 \quad 2 \quad 8 \quad 0 \quad 0 \\
\text{2 adults} & \quad 9 \quad 8 \quad 17 \quad 16 \quad 15 \quad 14 \quad 13 \quad 12 \quad 11 \quad 7 \quad 0 \quad 5 \quad 7 \quad 2 \quad 9 \quad 0 \quad 2 \quad 4 \quad 0 \\
\end{align*}
\]

\[
2.47 \times 10^{10}
\]

0.5 mark:
0.5 mark for consistent sum
Exemplar 2

Question 12

At an amusement park, there are 11 adults and 17 teenagers wanting to go on a boat ride. The boat has a 9-passenger capacity.

If there are at most 2 adult passengers, determine the number of ways 9 passengers can be chosen to go on the boat ride. Show your work.

\[ \text{Math} \rightarrow \text{prob} \rightarrow \binom{n}{r} \]

\[ \begin{align*}
\text{w/ 2 adults: } \binom{11}{2} &= 55 \\
\text{w/o adults: } \binom{17}{9} &= 19,448 \\
\text{w/ 1 adult: } (\binom{11}{1} + \binom{17}{8}) &= 24,310 \\
\end{align*} \]

\[ \binom{11}{2} + \binom{17}{9} + (\binom{11}{1} + \binom{17}{8}) = 19,503 + 24,310 + 68,134 = 68,134 \]

\[ \therefore \text{There are 68,134 ways 9 people can be chosen with at most 2 adults} \]

1 mark:

③ → 0.5 mark for correct product in Case 3 or \( \binom{11}{0} \times \binom{17}{9} \)

④ → 0.5 mark for consistent sum
Exemplar 1

Question 13

Of the 41 students in Grade 12,

- 21 students are in drama club
- 17 students are in chess club
- 14 students are in environmental club
- 8 students are in chess club and drama club
- 10 students are in drama club and environmental club
- 3 students are in chess club and environmental club only
- 2 students are in all three clubs

a) Draw a Venn diagram to represent this situation.

(3 marks)

![Venn Diagram]

b) What is the probability that a randomly selected student is not in any of these clubs?

(1 mark)

\[ \frac{41 - (5 + 6 - 8 - 2 - 1 - 8 - 3)}{41} = \frac{8}{41} = 0.1951 \]

19.51% chance

3 marks:

① → 1 mark for consistent number of students in exactly one club in (a)
② → 1 mark for consistent number of students not in any clubs in (a)
③ → 1 mark for consistent answer in (b)
④ → does not include a box when using a Venn diagram
Exemplar 2

Question 13

Of the 41 students in Grade 12,

• 21 students are in drama club
• 17 students are in chess club
• 14 students are in environmental club
• 8 students are in chess club and drama club
• 10 students are in drama club and environmental club
• 3 students are in chess club and environmental club only
• 2 students are in all three clubs

a) Draw a Venn diagram to represent this situation.

(b) What is the probability that a randomly selected student is not in any of these clubs?

\[ \frac{10}{41} = 0.2424 \approx 24\% \]
Exemplar 1

Question 15

Total: 2 marks

Jeannette’s new job involves a lot of driving. She wants to lease a new vehicle. Her friend Simon is trying to convince her not to lease.

State two reasons that Simon could use to convince her not to lease.

- She’s not investing for something that’s hers.
- She won’t get to keep the vehicle and then she would have to lease a new car, and there is no ending to paying a higher amount.
Exemplar 2

Question 15

Jeannette’s new job involves a lot of driving. She wants to lease a new vehicle. Her friend Simon is trying to convince her not to lease.

State two reasons that Simon could use to convince her not to lease.

- Buying would be cheaper in the longrun (if planning to sell later)
- She may not have the job very long

1.5 marks:
1 → 1 mark for the first appropriate reason
2 → 1 mark for the second appropriate reason
5 → 0.5 mark deduction for lack of clarity
Himesh wants to retire in 35 years. After meeting with his financial advisor, he determines that his portfolio will consist of the following two investments:

**Investment 1:** $15 000.00 in a mutual fund that earns an interest rate of 6.50%, compounded monthly.

**Investment 2:** regular biweekly deposits of $180.00 in a tax-free savings account (TFSA), with an interest rate of 3.75%, compounded biweekly. (Assume the initial value of the TFSA is 0.)

**a)** What will be the value of each investment when Himesh retires? Show your work.

\[
\begin{align*}
1) &\quad N = 420 \\
&\quad I\% = 6.5 \\
&\quad PV = -15,000 \\
&\quad PMT = 0 \\
&\quad FV = ? \\
&\quad P/Y = 12 \\
&\quad C/Y = 12 \\
&\quad FV = \$145,025.69
\end{align*}
\]

\[
\begin{align*}
2) &\quad N = 910 \\
&\quad I\% = 3.75 \\
&\quad PV = 0 \\
&\quad PMT = -180 \\
&\quad FV = 7 \\
&\quad P/Y = 26 \\
&\quad C/Y = 26 \\
&\quad FV = \$378,449.99
\end{align*}
\]

**b)** Himesh’s goal is to have $500 000.00 in his portfolio by the time he retires. Determine if he will meet his goal. Justify your answer by using your investment values in (a).

No, he will be \$16,474.32 short of \$500,000.00

**4.5 marks:**

1 → 1 mark for appropriate work for Investment 1 in (a)
2 → 1 mark for consistent value of Investment 1 in (a)
3 → 1 mark for appropriate work for Investment 2 in (a)
4 → 1 mark for consistent value of Investment 2 in (a)
5 → 1 mark for appropriate justification in (b)
A† → 0.5 mark deduction for arithmetic error
Exemplar 2

Question 16

Total: 5 marks

Himesh wants to retire in 35 years. After meeting with his financial advisor, he determines that his portfolio will consist of the following two investments:

**Investment 1:** $15,000.00 in a mutual fund that earns an interest rate of 6.50%, compounded monthly.

**Investment 2:** regular biweekly deposits of $180.00 in a tax-free savings account (TFSA), with an interest rate of 3.75%, compounded biweekly. (Assume the initial value of the TFSA is 0.)

a) What will be the value of each investment when Himesh retires? Show your work.

(4 marks)

\[
A = P \left(1 + \frac{r}{n}\right)^{nt}
\]

\[
A = 15000 \left(1 + \frac{0.065}{12}\right)^{12 \times 35}
\]

\[
= 1,117,775.53
\]

\[
A = 180 \left(1 + \frac{0.0375}{26}\right)^{26 \times 35}
\]

\[
= 16,407.08
\]

b) Himesh’s goal is to have $500,000.00 in his portfolio by the time he retires. Determine if he will meet his goal. Justify your answer by using your investment values in (a).

(1 mark)

\[
1,117,775.53 + 16,407.08 = 1,134,182.61 > 500,000
\]

Yes, he will reach his goal.

2 marks:

- \(\rightarrow\) 1 mark for consistent value of Investment 1 in (a)
- \(\rightarrow\) 1 mark for appropriate justification in (b)
- \(\rightarrow\) does not include the dollar sign for monetary values
Exemplar 1

Question 17

Kaia purchased a house for $400 000.00. At the time of purchase, she made a down payment of $100 000.00. The remaining balance of the mortgage was financed at an interest rate of 4.30%, compounded semi-annually, over 25 years.

a) How much is Kaia’s monthly mortgage payment? Show your work.

(2 marks)

\[ N = 300 \]
\[ I = 4.30\% \]
\[ PV = 300 000 \]
\[ \text{R}_{\text{mt}} = -1627.23 \]
\[ \text{FV} = 0 \]
\[ \text{R}_{\text{y}} = 12 \]
\[ \text{c}_{\text{y}} = 2 \]

b) Kaia’s house appreciates in value at an average rate of 2.00% per year. What will the value of her house be after 15 years?

(1 mark)

\[ 400 000 (1.02)^{15} = 538 347.34 \]

\[ \text{Market value} \]

\[ \text{Amount owing} = 158 761.79 \]

\[ 538 347.34 - 158 761.91 = 379 585.43 \]

5 marks:

1 → 1 mark for appropriate work in (a)
2 → 1 mark for consistent answer in (a)
3 → 1 mark for correct answer in (b)
4 → 0.5 mark for consistent calculation of balance owing or principal paid in (c)
5 → 0.5 mark for consistent balance owing or principal paid in (c)
6 → 0.5 mark for consistent calculation of equity in (c)
7 → 0.5 mark for consistent equity in (c)
8 → makes a transcription error (inaccurate transferring of information)
9 → does not include the dollar sign for monetary values
Question 17  Total: 5 marks

Kaia purchased a house for $400 000.00. At the time of purchase, she made a down payment of $100 000.00. The remaining balance of the mortgage was financed at an interest rate of 4.30%, compounded semi-annually, over 25 years.

a) How much is Kaia’s monthly mortgage payment? Show your work.

(2 marks)

\[
\begin{align*}
N &= (25 \times 12) \\
I &= 4.3 \\
P &= \frac{400 000 - 100 000}{1} = 300 000 \\
PMT &= \frac{300 000 \times 0.02681}{0.12} = -1627.23 \\
FV &= 0 \\
p/y &= 12 \\
c/y &= 2
\end{align*}
\]

b) Kaia’s house appreciates in value at an average rate of 2.00% per year. What will the value of her house be after 15 years?

(1 mark)

\[
\begin{align*}
(400 000)(1.02)^{15} &= 538 347.34
\end{align*}
\]

c) How much equity will Kaia have in the house after 15 years? Show your work.

(2 marks)

\[
\begin{align*}
DP + \text{Principal} \times (1, 180) + \text{amount of appreciation} \\
100,000 + (-1412 \times 8.21) + 138348.34 \\
= 97110.13
\end{align*}
\]

4.5 marks:

1 \rightarrow 1 mark for appropriate work in (a)
2 \rightarrow 1 mark for consistent answer in (a)
3 \rightarrow 1 mark for correct answer in (b)
4 \rightarrow 0.5 mark for consistent calculation of balance owing or of principal paid in (c)
5 \rightarrow 0.5 mark for consistent balance owing or principal paid in (c)
6 \rightarrow 0.5 mark for consistent calculation of equity in (c)
7 \rightarrow makes a transcription error (inaccurate transferring of information)
Dimitri owns a house with a mortgage of $175 000.00. He has $165 000.00 in other loans and his net worth is $300 000.00.

a) Determine his debt-to-equity ratio.

\[ \text{debt to equity ratio} = \frac{340 000 - 175 000.00}{300 000.00} \times 100 \]

\[ = 55\% \]

b) Determine his total assets.

His total assets are $300,000.00.

c) Dimitri wants to borrow $10 000.00 to take his family on a vacation. Using his debt-to-equity ratio in (a), explain if the bank will lend him the money.

No the bank will not lend him money because his debt to equity ratio is greater than 32\%.

1 mark:  
①→ 1 mark for correct answer in (a)
Exemplar 2

Question 18  
Total: 3 marks

Dimitri owns a house with a mortgage of $175 000.00. He has $165 000.00 in other loans and his net worth is $300 000.00.

a) Determine his debt-to-equity ratio.

(1 mark)

\[
\frac{175,000 + 165,000}{300,000} = \frac{340,000}{300,000} = 1.1333 = 113.33\%
\]

b) Determine his total assets.

(1 mark)

\[
\text{Assets} = 300,000 + 340,000 = 640,000
\]

No, because his debt is too high with his mortgage and the bank might think that Dimitri can’t handle it.

2 marks:

1 → 1 mark for correct answer in (a)
2 → 1 mark for consistent answer in (b)
3 → does not include the dollar sign for monetary values
Hugo creates a scale model of a planet for art class. He wants to paint it.

- The radius of the model is 20 cm.
- He has one bottle of paint that will cover 12 000 cm².
- It is assumed that each coat applied requires the same amount of paint.

How many full coats of paint can be applied using the one bottle? Show your work.

\[
\pi r^2 = \pi (20)^2 = 1256.64 \text{ cm}^2
\]

\[
12000 \div 1256.64 = 9.55 \quad \text{so 10 bottle}
\]

**1 mark:**

- 1 mark for consistent number of full coats
- Uses incorrect units of measure
- Rounds incorrectly
Hugo creates a scale model of a planet for art class. He wants to paint it.

- The radius of the model is 20 cm.
- He has one bottle of paint that will cover 12 000 cm².
- It is assumed that each coat applied requires the same amount of paint.

How many full coats of paint can be applied using the one bottle? Show your work.

\[
\text{Sphere S.A.} = 4\pi r^2
\]
\[
= 4\pi (20)^2
\]
\[
= 5026.55
\]

\[
\text{Coats} = \frac{12000 - 5026.55}{5026.55} = 0.97345 \text{ full coats of paint using one bottle}
\]

**1 mark:**

1 mark for correct surface area
Exemplar 1

Question 21

You are entered in a sandcastle building contest. Pails in the following shapes are available for use:

Shapes are not drawn to scale.

Your sandcastle design must:

- be built on a 2 ft. × 1 ft. area
- use each shape at least once
- include a minimum of 5 shapes
- be no more than 3 shapes high

a) Draw or describe a sandcastle that meets the above criteria.

(2 marks)
Exemplar 1 (continued)

b) Calculate the volume of sand required, in cubic feet, to build your sandcastle. Show your work.

(3 marks)

\[ \text{cube} \quad V = l^3 = 1^3 \times 3 = 3 \text{ ft}^3 \]

\[ \text{cone} \quad \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (0.25)^2 (0.916 \text{ ft}) \]

\[ \frac{1}{3} \pi (0.57525) = 0.60 \text{ ft}^3 \]

\[ \text{cylinder} \quad \pi r^2 h = \pi (0.25)^2 (0.75) \]

\[ \pi (0.046875) = 0.15 \text{ ft}^3 \]

3 ft\(^3\) + 0.60 ft\(^3\) + 0.15 ft\(^3\) = 3.75 ft\(^3\)

\[ 3 ft^3 + 0.60 \text{ ft}^3 + 0.15 \text{ ft}^3 = 3.75 \text{ ft}^3 \]

c) If the organizers of the contest purchased sand for $1.11/\text{ft}^3$, what is the cost of the sand needed to build your design? Assume whole units do not need to be used since the sand has already been purchased.

(1 mark)

\[ \$1.11 \times 3 = \$3.33 \]

\[ \$1.11 \times 0.75 = \$0.83 \]

3.33 + 0.83 = \$4.16

5 marks:

\[ \square \rightarrow 0.5 \text{ mark for appropriate use of each shape at least once in (a)} \]

\[ \square \rightarrow 0.5 \text{ mark for including a minimum of 5 shapes in (a)} \]

\[ \square \rightarrow 0.5 \text{ mark for no more than 3 shapes high in (a)} \]

\[ \square \rightarrow 0.5 \text{ mark for correct volume of cube(s) in (b)} \]

\[ \square \rightarrow 0.5 \text{ mark for correct volume of cone(s) in (b)} \]

\[ \square \rightarrow 0.5 \text{ mark for correct volume of cylinder(s) in (b)} \]

\[ \square \rightarrow 1 \text{ mark for correct conversion to cubic feet in (b)} \]

\[ \square \rightarrow 1 \text{ mark for consistent cost in (c)} \]

\[ \square \rightarrow \text{ makes a transcription error (inaccurate transferring of information)} \]

\[ \square \rightarrow \text{ rounds incorrectly} \]
Exemplar 2

Question 21

You are entered in a sandcastle building contest. Pails in the following shapes are available for use:

![Shapes diagram]

Your sandcastle design must:

- be built on a 2 ft. × 1 ft. area
- use each shape at least once
- include a minimum of 5 shapes
- be no more than 3 shapes high

a) Draw or describe a sandcastle that meets the above criteria.

(2 marks)

Base, 2 ft by 1 ft
2 cubes along base.
2 cylinders in the middle of each cube
1 cone on top of the cylinders in between them.
Exemplar 2 (continued)

b) Calculate the volume of sand required, in cubic feet, to build your sandcastle. Show your work.

(3 marks)

\[
\begin{align*}
1 \times 1 \times 1 \times 2 &= 2 \text{ ft}^3 \\
(3)^2 \times 2 &= 56.55 \text{ ft}^3 \\
\frac{(3)^2}{3} \times 2 &= 207.35 \text{ ft}^3 \\
\text{I would need } 265.895 \text{ ft}^3 \text{ of sand.}
\end{align*}
\]

c) If the organizers of the contest purchased sand for $1.11/\text{ft}^3$, what is the cost of the sand needed to build your design? Assume whole units do not need to be used since the sand has already been purchased.

(1 mark)

\[265.895 \times 1.11 = 295.14\]

4 marks:

1 → 0.5 mark for appropriate area in (a)
2 → 0.5 mark for appropriate use of each shape at least once in (a)
3 → 0.5 mark for including a minimum of 5 shapes in (a)
4 → 0.5 mark for no more than 3 shapes high in (a)
5 → 0.5 mark for correct volume of cube(s) in (b)
6 → 0.5 mark for correct volume of cone(s) in (b)
7 → 0.5 mark for correct volume of cylinder(s) in (b)
8 → 1 mark for consistent cost in (c)
9 → 0.5 mark deduction for procedural error
10 → uses incorrect units of measure
An online app awards 0 points, 3 points, or 7 points for performing tasks. Once your score adds up to 25 points or more, you pass the level.

State the scores that are not possible at any time during any level. Show your work using the list of numbers below.

Scores that are not possible: 1, 2, 4, 5, 8, 11, 18

1.5 marks:
- 0.5 mark for correctly eliminating all multiples of 7
- 0.5 mark for correctly eliminating at least one other score
- 0.5 mark for correctly eliminating all combinations of 3 and 7
An online app awards 0 points, 3 points, or 7 points for performing tasks. Once your score adds up to 25 points or more, you pass the level.

State the scores that are not possible at any time during any level. Show your work using the list of numbers below.

Scores that are not possible: $1, 2, 4, 5, 8, 11, 25$

1.5 marks:
1. $0.5$ mark for correctly eliminating all multiples of 3
2. $0.5$ mark for correctly eliminating all multiples of 7
3. $0.5$ mark for correctly eliminating at least one other score
The universal set of living things includes:

- the set of all birds ($B$)
- the set of all plants ($P$)
- the set of all owls ($O$)
- the set of all dogs ($D$)
- the set of all mammals ($M$)
- the set of all roses ($R$)

a) Give one example of two disjoint subsets.

(1 mark)

b) Using set notation, give one example of a set that is a subset of another.

(1 mark)

<table>
<thead>
<tr>
<th>2 marks:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 → 1 mark for correct answer in (a)</td>
</tr>
<tr>
<td>2 → 1 mark for correct answer in (b)</td>
</tr>
<tr>
<td>3 → incorrectly states the final answer</td>
</tr>
</tbody>
</table>
Exemplar 2

Question 23

The universal set of living things includes:

- the set of all birds ($B$)
- the set of all plants ($P$)
- the set of all owls ($O$)
- the set of all dogs ($D$)
- the set of all mammals ($M$)
- the set of all roses ($R$)

a) Give one example of two disjoint subsets.

(1 mark)

\[ O \subseteq B \quad D \subseteq M \quad : \text{subsets} \]

\[ \emptyset \cup D \quad : \text{empty set} \]

b) Using set notation, give one example of a set that is a subset of another.

(1 mark)

\[ P \subseteq P \]

\[ P : \{ \text{Red roses, white roses} \} \]

\[ P : \{ \text{Red roses, white roses, tulips, grass, trees} \} \]

1 mark:

⇒ 1 mark for correct answer in (b)
A movie theatre employee tracked the number of people who purchased popcorn ($P$) and chocolate ($C$) in one month. The results are given below.

a) What percentage of people purchased popcorn?

(1 mark)

73%.

b) Describe what $(P \cup C)'$ represents in this situation.

(1 mark)

$18\%$

1.5 marks:

1 → 1 mark for correct answer in (a)
2 → 1 mark for correct description in (b)
15 → 0.5 mark deduction for lack of clarity
Exemplar 2

Question 24

Total: 2 marks

A movie theatre employee tracked the number of people who purchased popcorn ($P$) and chocolate ($C$) in one month. The results are given below.

```
<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>57%</td>
<td>16%</td>
<td>9%</td>
</tr>
<tr>
<td></td>
<td>18%</td>
<td></td>
</tr>
</tbody>
</table>
```

a) What percentage of people purchased popcorn?

(1 mark)

$$57 \% + 16 \% = 73\% \text{ of people}$$

b) Describe what $(P \cup C)'$ represents in this situation.

(1 mark)

$(P \cup C)'$ represents all the people who did not purchase both popcorn and chocolate, 84%
Appendices
## Appendix A:
### Table of Questions by Unit and Learning Outcome

<table>
<thead>
<tr>
<th>RELATION AND FUNCTIONS</th>
<th>Question</th>
<th>Learning Outcome</th>
<th>Mark</th>
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<tbody>
<tr>
<td>1</td>
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<td>12.A.R.3</td>
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<th>LOGICAL REASONING</th>
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Appendix B:  
Irregularities in Provincial Tests  
A Guide for Local Marking

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student or only incorrect responses (“0”)

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the department along with the test materials.
Irregular Test Booklet Report

Test: ____________________________________________

Date marked: ____________________________________________

Booklet No.: ____________________________________________

Problem(s) noted: ____________________________________________

Question(s) affected: ____________________________________________

Action taken or rationale for assigning marks: ____________________________________________