Grade 8 Mathematics

Shape and Space

Shape and Space (Measurement and 3-D Objects and 2-D Shapes)—8.SS.2, 8.SS.3, 8.SS.4, 8.SS.5

Note: Specific learning outcome **8.SS.1** is addressed in the discussion of Number and Shape and Space (Measurement).

Enduring Understandings:

Many geometric properties and attributes of shapes are related to measurement.

The area of some shapes can be used to develop the formula for the area, surface area, and volume of other shapes.

While geometric figures are constructed and transformed, their proportional attributes are maintained.

All measurements are comparisons.

Length, area, volume, capacity, and mass are all measurable properties of objects.

The unit of measure must be of the same nature as the property being measured.

General Learning Outcomes:

Use direct or indirect measurement to solve problems.

Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Specific Learning Outcome(s):	Achievement Indicators:
8.SS.2 Draw and construct nets for 3-D objects. [C, CN, PS, V]	➛ Match a net to the 3-D object it represents. ➛ Construct a 3-D object from a net. ➛ Draw nets for a right circular cylinder, right rectangular prism, and right triangular prism, and verify [that the nets are correct] by constructing the 3-D objects from the nets. ➛ Predict 3-D objects that can be created from a net and verify the prediction.

continued

Specific Learning Outcome(s):	**Achievement Indicators:**
8.SS.3 Determine the surface area of ■ right rectangular prisms ■ right triangular prisms ■ right cylinders to solve problems. [C, CN, PS, R, V]	➛ Explain, using examples, the relationship between the area of 2-D shapes and the surface area of a 3-D object. ➛ Identify all the faces of a prism, including right rectangular and right triangular prisms. ➛ Describe and apply strategies for determining the surface area of a right rectangular or right triangular prism. ➛ Describe and apply strategies for determining the surface area of a right cylinder. ➛ Solve a problem involving surface area.
8.SS.4 Develop and apply formulas for determining the volume of right prisms and right cylinders. [C, CN, PS, R, V]	➛ Determine the volume of a right prism, given the area of the base. ➛ Generalize and apply a rule for determining the volume of right cylinders. ➛ Explain the relationship between the area of the base of a right 3-D object and the formula for the volume of the object. ➛ Demonstrate that the orientation of a 3-D object does not affect its volume. ➛ Apply a formula to solve a problem involving the volume of a right cylinder or a right prism.
8.SS.5 Draw and interpret top, front, and side views of 3-D objects composed of right rectangular prisms. [C, CN, R, T, V]	➛ Draw and label the top, front, and side views of a 3-D object on isometric dot paper. ➛ Compare different views of a 3-D object to the object. ➛ Predict the top, front, and side views that will result from a described rotation (limited to multiples of 90°) and verify predictions. ➛ Draw and label the top, front, and side views that result from a rotation (limited to multiples of 90°). ➛ Build a 3-D block object, given the top, front, and side views, with or without the use of technology. ➛ Sketch and label the top, front, and side views of a 3-D object in the environment, with or without the use of technology.

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of measuring length (cm, m) by
  - selecting and justifying referents for the units cm and m
  - modelling and describing the relationship between the units cm and m
  - estimating length using referents
  - measuring and recording length, width, and height
- Describing 3-D objects according to the shape of the faces, and the number of edges and vertices
- Demonstrating an understanding of area of regular and irregular 2-D shapes by
  - recognizing that area is measured in square units
  - selecting and justifying referents for the units cm^2 or m^2
  - estimating area by using referents cm^2 or m^2
  - determining and recording area (cm^2 or m^2)
  - constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area
- Solving problems involving 2-D shapes and 3-D objects
- Describing and constructing rectangular and triangular prisms
- Demonstrating an understanding of volume by
  - selecting and justifying referents for cm^3 or m^3 units
  - estimating volume by using referents for cm^3 or m^3
  - measuring and recording volume (cm^3 or m^3)
  - constructing rectangular prisms for a given volume
- Describing and providing examples of edges and faces of 3-D objects, and sides of 2-D shapes, that are
  - parallel
  - intersecting
  - perpendicular
  - vertical
  - horizontal
- Developing and applying a formula for determining the
  - perimeter of polygons
  - area of rectangles
  - volume of right rectangular prisms

- Developing and applying a formula for determining the area of
  - triangles
  - parallelograms
  - circles

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of perfect squares and square roots, concretely, pictorially, and symbolically (limited to whole numbers)

Background Information

Measurement

The key to understanding measurement is developing an understanding of the formulas for calculating surface area and volume and then being able to use the formulas to solve problems.

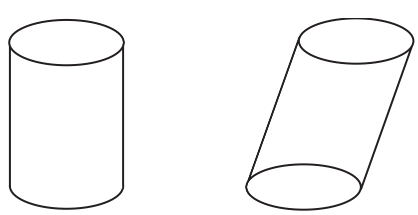
Determining the surface area and volume of right cylinders, right rectangular prisms, and right triangular prisms is an extension of already known formulas (area of a rectangle, area of a triangle, area of a circle, circumference of a circle, and volume of a rectangle) and the nets of these 3-D objects.

Definitions

cylinder

A geometric figure with two parallel and congruent, flat (plane) surfaces connected by one curved surface (curved face).

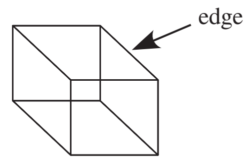
Examples:



edge

A line segment where two faces of a 3-D figure intersect.

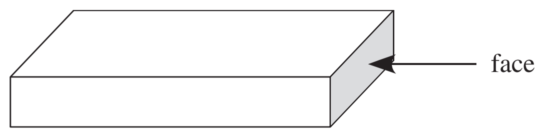
Example:



face

A flat surface of a solid.

Example:

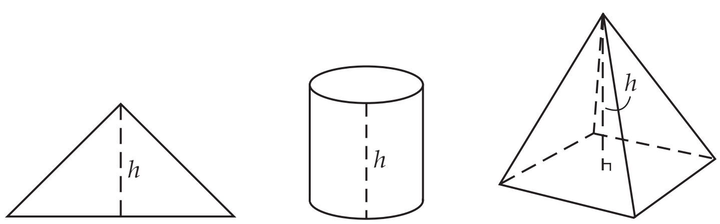


height

Can be used in the following ways:

- The measurement from base to top.
- The perpendicular distance from a vertex to the line containing the opposite side of a plane figure; the length of a perpendicular from the vertex to the plane containing the base of a pyramid or cone; the length of a perpendicular between the planes containing the bases of a prism or cylinder.

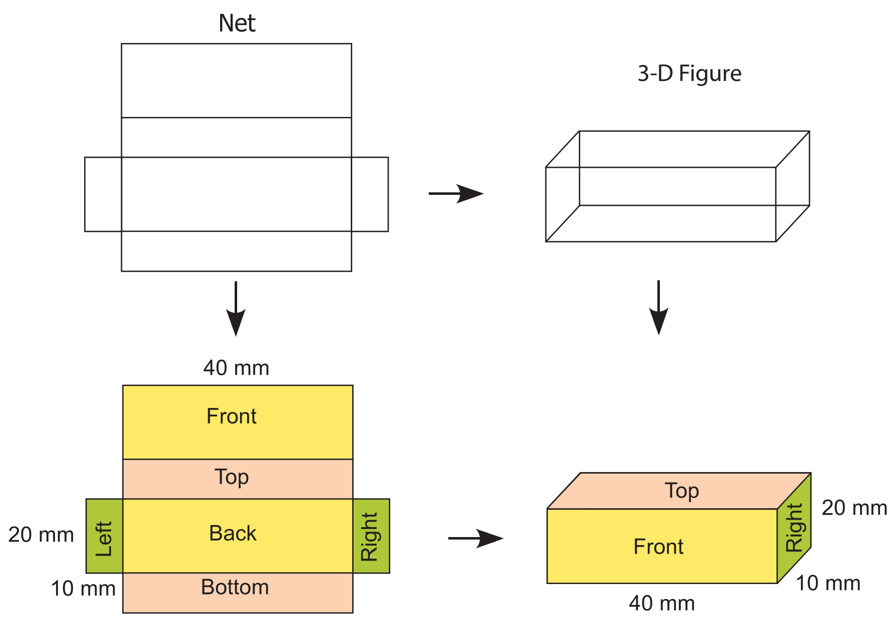
Examples:



net

The 2-D set of polygons of which a 3-D object is composed.

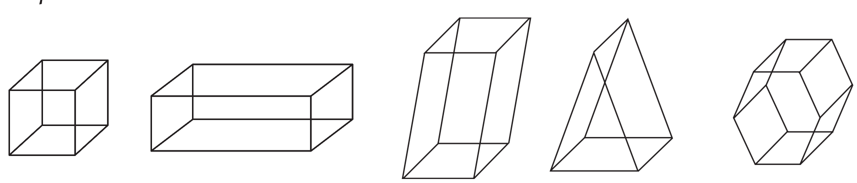
Example:



prism

A 3-D figure (solid) that has two congruent and parallel faces that are polygons (the bases); the remaining faces are parallelograms. The name of the prism is determined by the shape of the base.

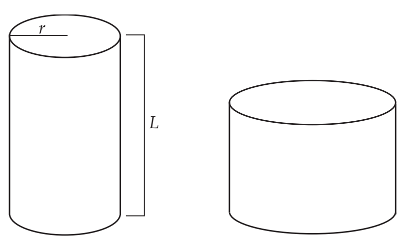
Examples:



right cylinder

A geometric figure with two parallel and congruent, flat (plane) surfaces connected at a right angle by one curved surface (curved face). A right cylinder has a 90° angle where the base and height meet.

Examples:



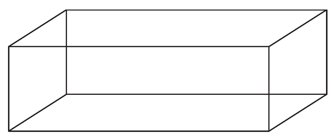
right prism

A prism that has a 90° angle where the base and height meet.

right rectangular prism

A prism whose six faces are rectangles; a prism with a rectangular base.

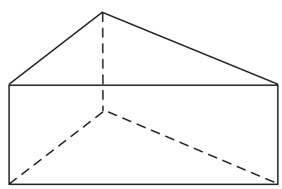
Example:



right triangular prism

A prism with a triangular base whose faces meet the base at right angles.

Example:



surface area

The sum of the areas of the faces or curved surface of a 3-D object.

three-dimensional (3-D) object

An object that has length, width, and height (e.g., prism, pyramid, cylinder, cone).

two-dimensional (2-D) shape

A figure that has two measures, such as length, width, or height (e.g., circle, square, triangle).

vertex

Can be used in the following ways:

- The common endpoint of two sides of a polygon.
- The common endpoint of two rays that form an angle.
- The common point where three or more edges of a 3-D solid meet.

view

A 2-D representation of a 3-D object.

volume

- In general, volume refers to an amount of space occupied by an object (e.g., solids, liquids, gas).
- In science, volume is expressed in cubic units (e.g., cubic centimetres (cm^3) and cubic metres (m^3)).
- In mathematics, volume means the same thing as *capacity*. Both volume and capacity are represented by the number of cubes (and parts of cubes) of a given size it takes to fill an object.

Determining Surface Area

Surface Area of a Right Prism

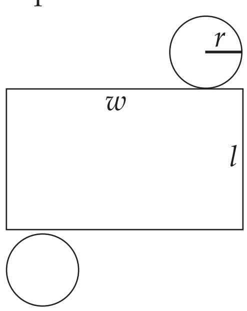
To determine the surface area of the right prism, determine the areas of each face and then add the areas together.

- **Surface Area of a Right Cylinder**

  To determine the surface area of a right cylinder, the shapes that make up the cylinder must be known. If you look at the net of a right cylinder, you will find that the shapes of the right cylinder are two circles (if there is a top and a bottom) and a rectangle.

  Example:

  A possible net of a right cylinder looks like this:



Area of a rectangle

$l \bullet w$

Area of a cirlce

πr^2

To determine the surface area of the right cylinder, determine the areas of the rectangle and the two circles, and then add the areas together.

Surface Area of Right Cylinder

= Area of rectangle + area of circles

$= (l \bullet w) + 2(\pi r^2)$

Surface area is measured in square units, written as cm^2, m^2, and so on.

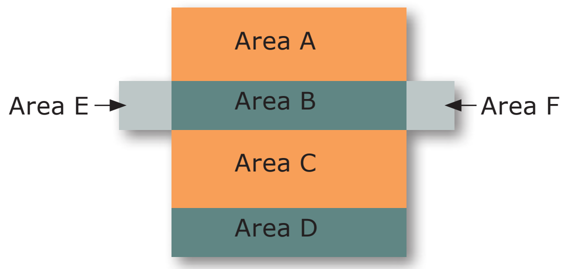
Note: The length of the rectangle is the same as the circumference of the circle.

■ Surface Area of a Right Rectangular Prism

To determine the surface area of a right rectangular prism, the shapes that make up the rectangular prism must be known. If you look at the net of a right rectangular prism, you will find that the shapes of the right rectangular prism are six rectangles, with opposite sides of the boxes the same.

Example:

A possible net of a right rectangular prism looks like this:



Area of a rectangle

$l \bullet w$

To determine the surface area of the right rectangular prism (Area A = Area C, Area B = Area D, Area E = Area F), you need to determine the areas of all six rectangles. Since opposite sides are equal, you only have to calculate the area of three rectangles, double each area, and add them.

Surface Area of Right Rectangular Prism

= 2(Area A) + 2(Area B) + 2(Area E)

$= 2(l_A + w_A) + 2(l_B + w_B) + 2(l_E + w_E)$

Note: l_A is the length of rectangle A, while l_B is the length of rectangle B. The length of rectangle A may or may not be the same as the length of rectangle B. Students need to be careful to use the correct dimensions to find each area. They are not expected to use the notation l_A.

Surface area is measured in square units, written as cm^2, m^2, and so on.

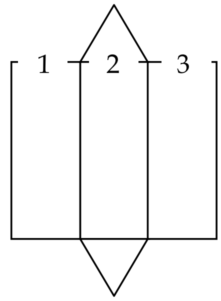
■ Surface Area of Right Triangular Prism

To determine the surface area of a right triangular prism, the shapes that make up the triangular prism must be known. If you look at the net of a right triangular prism, you will find that the shapes of the right triangular prism are three rectangles and two triangles, with the opposite triangles being the same size.

Example:

A possible net of a right triangular prism looks like this:

The rectangles may or may not be the same size, depending on the type of triangle the base is made from.

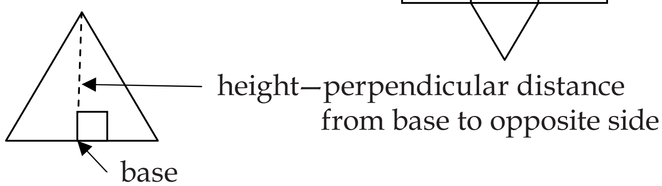


Area of a rectangle

$l \bullet w$

Area of a triangle

$\dfrac{b \bullet h}{2}$



To determine the surface area of the right triangular prism, you need to determine the area of the two triangles and the area of the three rectangles. You may be able to combine some areas if they contain similar measurements. A general formula for determining the surface area of a right triangular prism is as follows:

Surface Area of a Right Triangular Prism

= (area of rectangle 1) + (area of rectangle 2) + (area of rectangle 3) + 2(area of triangle)

$= (l_1 \bullet w_1) + (l_2 \bullet w_2) + (l_3 \bullet w_3) + 2\left(\dfrac{b \bullet h}{2}\right)$

Note: $l_1 \Rightarrow$ length of rectangle 1; $w_1 \Rightarrow$ width of rectangle 1.

Determining Volume

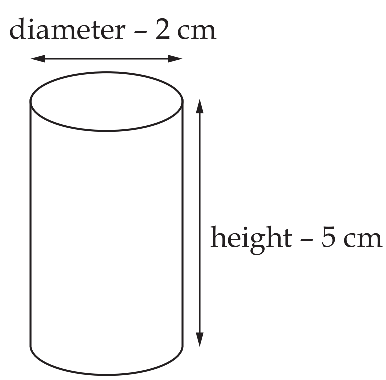
Volume of a Right Prism

To determine the volume of a right prism, determine the area of the base and multiply it by the height.

- **Volume of a Right Cylinder**

  The volume of a right cylinder is determined by multiplying the area of the base by the height of the cylinder.

Example:



Area of base $= \pi r^2$

$= \pi\ (1 \text{ cm})^2$

$= 3.14 \text{ cm}^2$

Volume $=$ Area of base $\bullet$ height

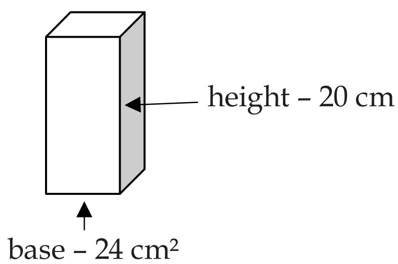
$= 3.14 \text{ cm}^2 \bullet 5 \text{ cm}$

$= 15.7 \text{ cm}^3$

- **Volume of a Right Rectangular Prism**

  Volume of a right rectangular prism is determined by multiplying the area of the base of the rectangular prism by the height of the rectangular prism.

  Example:

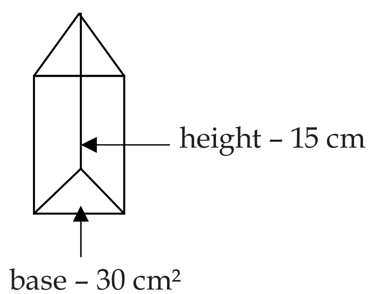


Volume of a rectangular prism
= area of base times height
= $24 \text{ cm}^2 \bullet 20 \text{ cm}$
= 480 cm^3

- **Volume of a Right Triangular Prism**

  The volume of a right triangular prism is determined by multiplying the area of the base of the triangular prism by the height of the triangular prism.

  Example:



Volume of a triangular prism
= area of base times height
= $30 \text{ cm}^2 \bullet 15 \text{ cm}$
= 450 cm^3

Mathematical Language

2-D shapes	net
3-D objects	orientation of a shape
area	radius
base of a prism	right cylinder
diameter	right rectangular prism
edge	right triangular prism
face	vertex
formula	view
height of a prism	volume

Assessing Prior Knowledge

Materials: BLM 8.SS.2.1: Measurement Pre-Assessment

Organization: Individual

Procedure:

1. Tell students that they will be extending their understanding of measurement over the next few lessons; however, you first need to find out what they already know about measurement.
2. Hand out copies of BLM 8.SS.2.1: Measurement Pre-Assessment and have students complete it individually, showing their work.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Determine the area of a rectangle.
- ❐ Determine the circumference of a circle.
- ❐ Determine the volume of a right rectangular prism.
- ❐ Determine the area of a circle.
- ❐ Determine the area of a triangle.

Suggestions for Instruction

- **Draw and label the top, front, and side views of a 3-D object on isometric dot paper.**
- **Sketch and label the top, front, and side views of a 3-D object in the environment, with or without the use of technology.**

Materials: Cereal or macaroni boxes (ideally, multiples of the same type and size of boxes), cubes/blocks, BLM 5–8.21: Isometric Dot Paper

Organization: Whole class/pairs/individual

Procedure:

1. Tell students that they will be learning about nets for 3-D objects.
2. Discuss the meaning of the following terms with the class:
   - 2-dimensional
   - 3-dimensional
   - net
   - net of a 3-D object
   - view (with respect to 3-D objects)
3. Place boxes around the room, showing different faces of the boxes. Have students draw the boxes from where they are sitting, so they will be drawing different views.
4. Have students, working in pairs, each choose one of their drawings. Without showing their drawings to each other, students take turns explaining how to draw their respective views while the partners try to replicate the drawings based on the explanations. Ask those students who were successful in having their partners draw an exact replica of their drawings, what key words they used to help their partners. (Observe whether students are able to use the terms *face*, *edge*, and *vertex* in their descriptions.)
5. Show students a right rectangular prism. To describe 3-D objects, one needs to count the number of faces, edges, and vertices on the objects.
   - A *face* is a flat or curved surface.
   - An *edge* is a line segment where two faces meet.
   - A *vertex* is a point where three or more edges meet.
6. Provide each student with a copy of BLM 5–8.21: Isometric Dot Paper. Ask students to redraw their boxes on the dot paper and identify the faces, edges, and vertices of their boxes.
7. Ask students the following questions:
   - What is the front of your box? What is the side? What is the top? Does it matter?
   - Were you able to see all the views when you labelled your box? Do you need to see all the views?
   - Why do you need to have only one side view if the top and front views are given?
8. Provide each student with 5 to 10 cubes/blocks. Have each student create an object and then draw and label the front, side, and top views of the object on the isometric dot paper provided (see BLM 5–8.21: Isometric Dot Paper).

Note: Students may need some time to explore how to use isometric dot paper. The dots are at angles, so students will need some time to learn how to connect the dots to draw and label their objects.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Draw and label the top, front, and side views of a 3-D object on isometric dot paper.
- ❒ Visualize 3-D objects in order to represent them in a 2-D picture.

Suggestions for Instruction

- **Compare different views of a 3-D object to the object.**
- **Predict the top, front, and side views that will result from a described rotation (limited to multiples of 90°) and verify predictions.**
- **Draw and label the top, front, and side views that result from a rotation (limited to multiples of 90°).**
- **Sketch and label the top, front, and side views of a 3-D object in the environment, with or without the use of technology.**

Materials: Various 3-D objects (e.g., books, rectangular erasers, boxes, CD cases) that students have gathered, BLM 5–8.21: Isometric Dot Paper, cubes, overhead or LCD projector

Organization: Individual/whole class

Procedure:

1. Ask each student to select one object from the assortment that has been gathered. Then have students do the following:
   - Draw and label the top, front, and side views of the object on the isometric dot paper provided.
   - Rotate the object 90° clockwise and draw and label the top, front, and side views of the object on the isometric dot paper.
   - Compare the different views of the 3-D object.
2. Show students a picture of any type of box using an overhead or LCD projector. Have them draw and label the top, front, and side views that result from a 270° clockwise turn. (Students may note that a 270° clockwise turn is the same as a 90° counter-clockwise turn.)
3. Show students a 2-D view of one of the objects and have them determine which object it could be.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Draw and label top, front, and side views of a 3-D object that has been rotated.
- ❐ Predict the top, front, and side views of an object to be rotated.
- ❐ Visualize the new image.
- ❐ Analyze the original view to determine the new view.
- ❐ Draw and label the views as they rotate.

Suggestions for Instruction

- **Draw and label the top, front, and side views of a 3-D object on isometric dot paper.**
- **Build a 3-D block object given the top, front, and side views, with or without the use of technology.**

Materials: Interlocking blocks/cubes, BLM 5–8.21: Isometric Dot Paper

Organization: Pairs

Procedure:

1. Pair up students in the class.
2. Have each student take approximately 20 blocks/cubes.
3. Ask each student to do the following:
   - Use six cubes to make a 3-D object and keep it hidden from your partner.
   - Use isometric dot paper to draw and label the top, front, and side views of your 3-D shape.
   - Exchange your isometric dot paper views with those of your partner and have your partner build the 3-D object you created.
   - Compare the original 3-D object with the partner's representation of it.
4. For the next rounds, increase the number of blocks to eight, and then to ten.

Observation Checklist

☑ Observe students’ responses to determine whether they can do the following:

- ❒ Draw and label the top, front, and side views of a 3-D object.
- ❒ Build a 3-D block from the top, side, and front views.

Suggestions for Instruction

- **Construct a 3-D object from a net.**
- **Draw nets for a right circular cylinder, right rectangular prism, and right triangular prism, and verify [that the nets are correct] by constructing the 3-D objects from the nets.**
- **Predict 3-D objects that can be created from a net, and verify the prediction.**
- **Match a net to the 3-D object it represents.**

Materials: BLM 8.SS.2.2: Nets of 3-D Objects, white paper

Organization: Individual

Procedure:

1. Hand out copies of BLM 8.SS.2.2: Nets of 3-D Objects. Have students, working individually, predict what a net will make, and then have them construct the object to verify their prediction.
2. Repeat the first step with a number of nets so that students are able to see how the nets make the 3-D objects.
3. Place the following three names in a container: right circular cylinder, right rectangular prism, and right triangular prism. Ask students to pick one of the names out of the container and draw the net for the selected 3-D object, using the white paper provided.
4. Have students construct a 3-D object from their net to determine whether their net is correct.
5. Repeat the process to observe whether students can construct nets for all three 3-D objects.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Predict a 3-D object based on the net.
- ❒ Construct a net for a right circular cylinder.
- ❒ Construct a net for a right rectangular prism.
- ❒ Construct a net for a right triangular prism.

Suggestions for Instruction

- **Construct a 3-D object from a net.**
- **Match a net to the 3-D object it represents.**

Materials: BLM 8.SS.2.3: 3-D Objects, white paper ($8\frac{1}{2} \times 11$), BLM 8.SS.2.4: Matching

Organization: Whole class/small group/individual

Procedure:

1. Ask students to suggest examples of 3-D objects. Record their suggestions on the whiteboard.
2. Hand out copies of BLM 8.SS.2.3: 3-D Objects. Ask students whether they notice anything about the objects represented that would help them determine the names of the objects.
3. Have students, working in small groups, draw nets of the 3-D objects shown on BLM 8.SS.2.3: 3-D Objects, using the white paper provided.
4. Ask students to construct the 3-D objects from the nets they drew to see whether they work.
5. Have students share their experience of creating the nets and constructing the 3-D object.
6. Hand out copies of BLM 8.SS.2.4: Matching. Have students complete the sheet individually.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Construct a net from a 3-D object.
- ❒ Match a net to the 3-D object it represents.

Suggestions for Instruction

- **Match a net to the 3-D object it represents.**

Materials: Various 3-D objects (cube, rectangular prism, triangular prism, trapezoid prism, cylinder, square pyramid, triangular pyramid), white paper ($8\frac{1}{2} \times 11$)

Organization: Individual/whole class

Procedure:

1. Have each student select from a container one of the following objects: cube, rectangular prism, triangular prism, trapezoid prism, cylinder, square pyramid, or triangular pyramid.
2. Have students, working individually, draw a net for their selected objects.
3. Using students' nets, construct a scavenger hunt around the class. Students need to locate and identify each net that was created.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify 3-D objects from their nets.

Suggestions for Instruction

- **Explain, using examples, the relationship between the area of 2-D shapes and the surface area of a 3-D object.**

Materials: Math journals

Organization: Whole class/individual

Procedure:

1. Tell students that they have constructed 3-D objects from nets, and now they are going to see how the area of 2-D shapes is connected to the surface area of 3-D objects.

2. Ask students: What is *area*? Have a conversation with students if they say area is length times width. Length times width is the way to calculate area (for a limited few 2-D shapes), but *area* is actually the number of square units that cover the surface that lies within a 2-D shape.
3. Come up with a class definition of *surface area*.
4. Have students respond to the following question in their math journals:

   Explain, using words and diagrams, the relationship between area and surface area. Give examples.

Observation Checklist

☑ Observe students' math journal responses to determine whether they can do the following:

- ❒ Relate the area of 2-D shapes with the surface area of 3-D objects.

Suggestions for Instruction

- **Explain, using examples, the relationship between the area of 2-D shapes and the surface area of a 3-D object.**
- **Identify all the faces of a prism, including right rectangular and right triangular prisms.**
- **Describe and apply strategies for determining the surface area of a right rectangular or right triangular prism.**

Materials: BLM 8.SS.3.1: Nets, square tiles, BLM 5–8.9: Centimetre Grid Paper (copied onto transparency), rulers

Organization: Pairs/whole class/individual

Procedure:

1. Pair up students, and provide each pair with two copies of a net of a right rectangular prism.
2. Have pairs use one net to construct the 3-D object.
3. Ask students to come up with a procedure for determining the surface area of the object. Let them know that square tiles, centimetre grid paper, and rulers are available if they need them.
4. As a class, discuss the various procedures that students used.
5. Repeat the process with right triangular prisms.
6. Have individual students find an example of a right triangular prism or a right rectangular prism in the classroom and find its surface area.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Communicate mathematically.
- ❒ Apply reasoning skills to develop a procedure for determining the surface area of a 3-D shape.
- ❒ Calculate the surface area of prisms.

Suggestions for Instruction

- **Describe and apply strategies for determining the surface area of a right cylinder.**

Materials: White paper, a variety of cylinders, rulers, string, a camera, word processing software

Organization: Whole class/pairs

Procedure:

1. Briefly review how to determine the area of a circle.
2. Pair up students and tell them that they will be working together to determine the surface area of a cylinder. Have each pair select one cylinder to work with.
3. Students can use paper, rulers, string, or anything else in the class they would like to use to help them determine the surface area of their selected cylinder.
4. Have students take pictures to document their steps, import these pictures into a word processor, and describe the process they followed.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Communicate mathematically.
- ❒ Describe a strategy for determining the surface area of a cylinder.

- **Describe and apply strategies for determining the surface area of a right cylinder.**

Materials: White paper

Organization: Individual

Procedure:

1. Provide each student with a piece of white paper.
2. Hold a sheet of paper in the landscape orientation, roll it to form a cylinder shape, and tape the paper together.
3. Ask students the following questions:
   - What was the original shape before the cylinder was formed? (rectangle)
   - How do you calculate the area of a rectangle? (length times width)
   - What is the length of the rectangle? (the height of the cylinder)
   - What is the width of the rectangle? (the circumference of the circle)
   - How do you calculate the circumference of a circle? ($\pi \bullet d$)
   - How do you calculate the area of the rectangular portion of the cylinder? ($\pi \bullet d \bullet h$)
   - What are the shapes at both ends of the cylinder? (circles)
   - How do you calculate the area of a circle? ($\pi \bullet r^2$)
   - How many circles are on a cylinder? (two)
   - How do you calculate the total surface area of a cylinder? [$2 (\pi \bullet r^2) + (h (\pi \bullet d))$]

Note: Students use the strategy for determining the surface area of a cylinder, but they do not need to memorize the formula.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Describe a method for determining the surface area of a cylinder.
- ❒ Apply a strategy for determining the surface area of a cylinder.
- ❒ Calculate the surface area of the cylinder.

Suggestions for Instruction

- **Solve a problem involving surface area.**

Materials: BLM 8.SS.3.2: Surface Area Problems, chart paper, math journals

Organization: Small group/whole class/individual

Procedure:

1. Have students form small groups, and provide them with copies of BLM 8.SS.3.2: Surface Area Problems.
2. Assign each group one of the surface area problems. Ask the groups to solve their assigned problem and record their answer on the chart paper provided. They must explain what method they chose for solving the problem, why they chose that method, and why they think their answer is reasonable.
3. Have each group present their problem and solution to the class. Provide opportunities for the other groups to ask questions and add to the responses.
4. Ask students to create and solve a new surface area problem in their math journals.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Solve surface area problems individually.
- ❒ Apply strategies for determining the surface area of 3-D objects.
- ❒ Communicate mathematically.

Suggestions for Instruction

- **Explain the relationship between the area of the base of a right 3-D object and the formula for the volume of the object.**
- **Determine the volume of a right prism, given the area of the base.**

Materials: A variety of boxes (e.g., cereal, facial tissue), centimetre cubes, rulers

Organization: Pairs/whole class

Procedure:

1. Pair up students, and have each pair select a box.
2. Let students know that their goal is to figure out a strategy to determine the volume of the box they have selected.
3. Have students share their strategies with the whole class. Ask guiding questions, such as the following:
   - How many centimetre cubes fill the bottom of your box? What is the area of the bottom of your box?
   - How many centimetre cubes stack up the corner of your box? What is the height of your box?
   - Can you use that information to determine the volume of your box? Explain.
   - Can you just use measurements to determine the volume of the box? Explain.
   - Why is it important to know the area of the base of the box in order to determine its volume?

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Develop a strategy to determine the volume of a box.

Suggestions for Instruction

- **Explain the relationship between the area of the base of a right 3-D object and the formula for the volume of the object.**
- **Determine the volume of a right prism, given the area of the base.**

Materials: Rulers

Organization: Whole class/individual

Procedure:

1. Tell students that they will now be exploring how to calculate the volume of right rectangular prisms and right triangular prisms.
2. Ask students the following questions:
   - What is volume? (*Volume* is the amount of space an object occupies. It is measured in cubic units.)

- Have you ever had to determine the volume of an object before? (Some students may say they have determined the volume of a cube by multiplying length • width • height.)
- Can you draw a cube and label the length, width, and height of the cube? Have students do this individually.
- If we multiply length by width, what do we determine? (the area of the base)
- Where have we calculated areas before? (when finding surface area)
- So, if we think of the prisms we have been working with, how can we determine the volume of the prisms? (area of the base times height of the prism)

3. Draw different rectangular and triangular prisms on the whiteboard. Write the area of the base and the height of each prism. Have students calculate the volume.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Connect their prior knowledge of area and surface area in order to determine the volume of right rectangular and right triangular prisms.
- ❒ Use the given information to calculate the volume of the prisms.

Suggestions for Instruction

- **Generalize and apply a rule for determining the volume of right cylinders.**

Materials: Paper

Organization: Whole class/pairs/individual

Procedure:

1. Draw at least four different cylinders on the whiteboard and label the area of the base, the height, and the volume.
2. Have students make a table and label the cylinder number, the radius, the area of the base, the height, and the volume.
3. Have students work with partners to see whether they can determine the relationship between the numbers provided. (The relationship is the area of the base times the height equals the volume.)

4. Provide students with four more cylinders with the area of the base and the height provided. Have them, independently, determine the volume of the right cylinder.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Generalize a rule for determining the volume of a cylinder.
- ❒ Connect the area of a circle with the volume of a cylinder.

Suggestions for Instruction

- **Demonstrate that the orientation of a 3-D object does not affect its volume.**

Materials: Variety of 3-D objects (2 identical objects of each shape) taped to tabletops in differing orientations, math journals

Organization: Pairs/whole class/individual

Procedure:

1. Have students, working in pairs, determine the volume of a variety of specified 3-D objects and make a note of anything interesting they discover as they determine the volumes.
2. Ask students to record their measurements, calculations, and observations in an organized fashion.
3. Discuss students' interesting discoveries as a class.
4. Have students explain, in their math journals, why orientation of a 3-D object does not affect its volume.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine the volume of 3-D objects.
- ❒ Explain why orientation does not affect volume.

- **Apply a formula to solve a problem involving the volume of a right cylinder or a right prism.**

Materials: BLM 8.SS.4.1: Volume Problems, chart paper, math journals

Organization: Small group/whole class/individual

Procedure:

1. Tell students that they will be solving problems that involve right cylinders and right prisms.
2. Divide students into small groups, and hand out chart paper and copies of BLM 8.SS.4.1: Volume Problems, which presents a variety of volume problems.
3. Ask groups to record their answers to the volume problems on the chart paper. They must explain what method they chose for solving the problem, why they chose that method, and why they think their answer is reasonable.
4. Have groups take turns presenting their problems to the class. Provide opportunities for the other groups to ask questions and add to the responses.
5. Ask students to create and solve a new volume problem in their math journals.

Observation Checklist

☑ Use students' math journal responses to determine whether they can do the following:
- ❒ Communicate mathematically.
- ❒ Apply effective strategies to solve volume problems.

Connecting Surface Area and Volume in Real Life

Introduction:

Krispee Oats Cereal Company wants to save as much money as possible. In order to do this, the company wants to have a high volume of cereal in the box, but a low surface area to avoid wasting money on the cardboard packaging. Students will design a cereal box that can hold 8750 cm^3 of cereal.

Purpose:

Students will demonstrate a comprehensive understanding of surface area and volume of right rectangular prisms, right triangular prisms, and right cylinders.

Curricular Links: Art, ELA

Materials/Resources: Cardboard, card stock, or other sturdy paper, markers, BLM 5–8.21: Isometric Dot Paper

Optional Materials: Construction paper or other coloured paper, paint

Organization: Individual

Scenario:

- You work for Krispee Oats Cereal Company. Your job is to create a cereal box that will hold 8750 cm^3 of cereal but will have a low surface area, as the company is trying to keep costs down and does not want to spend a lot of money on the cardboard packaging.
- You must meet the following expectations:
  - Demonstrate, using isometric dot paper, at least three different designs of the cereal box showing all measurements, ensuring that the cereal box will hold 8750 cm^3 of cereal.
  - Determine the cost of each of your cereal boxes if the cardboard costs $0.50 per square centimetre.
  - Create a net, including measurements, of your chosen design.
  - Choose the design that best meets the criteria for your company.
  - Construct the cereal box.
  - Decorate the cereal box to make it attractive to the consumer.

Assessment:

The following rubric can be used to assess achievement of the mathematics learning outcomes.

Criteria	Meeting Expectations	Developing to Meet Expectations	Beginning to Meet Expectations	Incomplete
The student				
■ is able to determine the dimensions of prisms that will hold a particular volume	❐ uses understanding of volume to determine three different possible dimensions for a particular volume	❐ uses understanding of volume to determine two different possible dimensions for a particular volume	❐ uses understanding of volume to determine one possible dimension for a particular volume	❐ does not determine a possible dimension for a particular volume
■ demonstrates an understanding of calculating the surface area of a prism	❐ includes clear step-by-step procedures for calculating the surface area of a prism	❐ includes most steps for calculating the surface area of a prism	❐ includes few steps for calculating the surface area of a prism	❐ includes no steps for calculating the surface area of a prism
■ demonstrates how to solve problems involving surface area	❐ demonstrates a comprehensive understanding of problem solving involving surface area by including clear step-by-step procedures for calculating the cost of materials	❐ demonstrates an understanding of problem solving involving surface area by including the cost of materials using surface area	❐ demonstrates minimal understanding of problem solving involving surface area when attempting to include the cost of materials using surface area	❐ does not attempt to show an understanding of solving problems involving surface area
■ demonstrates an understanding of a net	❐ accurately draws and labels a net of a 3-D object that represents the final product	❐ draws and labels a net of a 3-D object that may or may not represent the final product	❐ draws and labels a net of a 3-D object but it does not represent the final product	❐ does not draw or label a net of a 3-D object

Extension:

What would be the volume and surface area of the cardboard box that could hold 20 boxes of your final product?

Shape and Space (Transformations)—8.SS.6

Enduring Understandings:

Many geometric properties and attributes of shapes are related to measurement.

Tessellations are created using transformations.

General Learning Outcome:

Describe and analyze position and motion of objects and shapes.

Specific Learning Outcome(s):	Achievement Indicators:
8.SS.6 Demonstrate an understanding of tessellations by ▪ explaining the properties of shapes that make tessellating possible ▪ creating tessellations ▪ identifying tessellations in the environment [C, CN, PS, T, V]	➛ Identify in a set of regular polygons those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices. ➛ Identify in a set of irregular polygons those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices. ➛ Identify a translation, reflection, or rotation in a tessellation. ➛ Identify a combination of transformations in a tessellation. ➛ Create a tessellation using one or more 2-D shapes, and describe the tessellation in terms of transformations and conservation of area. ➛ Create a new tessellating shape (polygon or non-polygon) by transforming a portion of a tessellating polygon, and describe the resulting tessellation in terms of transformations and conservation of area. ➛ Identify and describe tessellations in the environment.

Prior Knowledge

Students may have had experience with the following:

- Identifying a single transformation (translation, rotation, or reflection) of 2-D shapes
- Demonstrating an understanding of angles by
  - identifying examples of angles in the environment
  - classifying angles according to their measure
  - estimating the measure of angles using 45°, 90°, and 180° as reference angles
  - determining angle measures in degrees
  - drawing and labelling angles when the measure is specified
- Describing and comparing the sides and angles of regular and irregular polygons
- Performing a combination of transformations (translations, rotations, or reflections) on a single 2-D shape, and drawing and describing the image
- Performing a combination of successive transformations of 2-D shapes to create a design, and identifying and describing the transformations
- Performing and describing transformations of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral vertices)

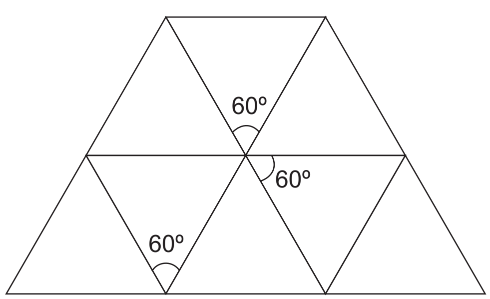
Background Information

Tessellations

Three regular polygons—equilateral triangles, squares, and hexagons—will tessellate the plane because their angles are a factor of 360º. Irregular polygons whose angles add to a factor of 360º will also tessellate the plane. Polygons that tessellate the plane can be used to make tessellations.

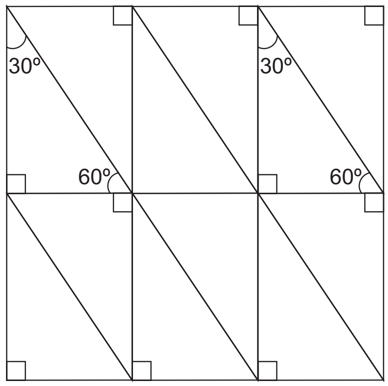
Examples:

Regular Polygon That Tessellates the Plane



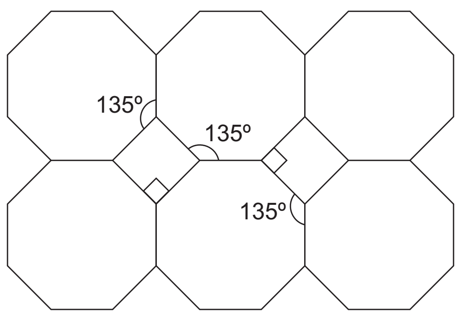
The angles of an equilateral triangle are each 60º. Therefore, no matter how you arrange the triangle, the shape will tessellate.

Irregular Polygon That Tessellates the Plane



The angles of every triangle add up to 180° (which is a factor of 360°). Therefore, all triangles should tessellate, although a little more manoeuvring may be needed if they are not equilateral triangles. The same is true of all quadrilaterals.

Combination of Polygons That Tessellate the Plane



Sometimes it may take a combination of regular and/or irregular polygons to create a tessellation. In this example, a regular octagon (with interior angle measure 135°) and a regular quadrilateral (with interior angle measure 90°) will tessellate, since the sum of the angles that meet at a point is (135° + 135° + 90° = 360°) a factor of 360°.

Polygons are transformed via translations (slides), reflections (flips), and rotations (turns) in order to tessellate the plane.

M. C. Escher is an artist famous for his work in tessellations. Escher-style tessellations begin with a regular polygon that tessellates. A portion of one side is removed and taped onto the opposite side to create a new shape that has the same area as the original shape. Inside the shape, creative images are filled in to make a work of art.

For more information, visit the following websites:

- The Official M. C. Escher Website. <www.mcescher.com/>.
- Tessellations.org. "Do-It-Yourself." <www.tessellations.org/>.

Note: These websites were current in June 2015. If they are not available, use a search engine with the key words "Escher" or "tessellations" to find other possible resources.

Mathematical Language

irregular polygon
quadrilaterals
reflection
regular polygon
rotation
tessellation
transformation
translation

Learning Experiences

Assessing Prior Knowledge

Materials: BLM 8.SS.6.1: Coordinate Image

Organization: Individual

Procedure:

1. Tell students that the purpose of this learning activity is to have them demonstrate their understanding of transformations.
2. Have students transform image ABCD on BLM 8.SS.6.1: Coordinate Image:
   - Translate ABCD [7, –2].
   - Reflect ABCD over the *x* axis.
   - Rotate ABCD 90° clockwise, with the origin as the point of rotation.
   - Locate ABCD in its new location using coordinates.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Translate images.
- ❒ Reflect images.
- ❒ Rotate images.

Suggestions for Instruction

- **Identify in a set of regular polygons those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices.**
- **Identify in a set of irregular polygons those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices.**

Materials: Regular polygons (square, equilateral triangle, regular hexagon, regular pentagon, regular octagon) and irregular polygons (rhombus, rectangle, parallelogram, isosceles triangle, trapezoid, irregular pentagon), BLM 8.SS.6.2: Tessellating the Plane, math journals

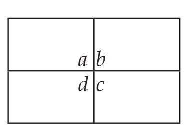
Organization: Small group/pairs/whole class/individual

Procedure:

1. Divide the class into small groups, and provide each group with a set of shapes. Using the Sort and Predict strategy, have students sort the shapes into the following two categories: regular polygons and irregular polygons.
2. Review the results and, as a class, generate the definitions of *regular polygons* and *irregular polygons*.
3. Using the regular hexagon, demonstrate how to tessellate the plane.
4. Use a regular pentagon to demonstrate that it does not tessellate the plane.
5. Using the Think-Pair-Share strategy, have students come up with the definition of *tessellating the plane*.
6. As a class, come up with a common definition of *tessellating the plane*.
7. Let the groups try to tessellate the rest of the shapes they have. Once they have done this, have students make a chart for the shapes, identifying each shape, noting the sum of the interior angles of the common vertices, and indicating whether or not the shapes tessellate the plane. (See BLM 8.SS.6.2: Tessellating the Plane.)

   Example:

   The following is a shape that tessellates. Four of these shapes create a tessellation. Angles a, b, c, and d form the interior angles of the common vertices. The sum of angles a, b, c, and d must equal 360° in order to tessellate.



8. Demonstrate to students how to measure the interior angles of the common vertices. Then have them measure the interior angles of the common vertices of those that tessellate the plane, as well as those that do not tessellate the plane.
9. Ask students:
   Why do some shapes tessellate the plane and others do not?
10. After discussing why some shapes tessellate the plane and others do not, have students answer the following questions in their math journals:
    - What are three regular polygons that tessellate the plane?
    - Why do they tessellate the plane?
    - What are three irregular polygons that tessellate the plane?
    - Why do they tessellate the plane?
    - Why don't regular pentagons and regular octagons tessellate the plane?

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify regular polygons that tessellate the plane.
- ❒ Identify irregular polygons that tessellate the plane.
- ❒ Explain why some polygons tessellate the plane and others do not.

Note: In their responses, students should indicate that only polygons whose interior angles measure 360° or whose interior angles add to a factor of 360° will tessellate the plane.

Suggestions for Instruction

- **Identify in a set of regular polygons those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices.**
- **Identify in a set of irregular polygons those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices.**

Materials: BLM 5–8.21: Isometric Dot Paper, BLM 5–8.22: Dot Paper

Organization: Individual

Procedure:

1. Tell students that they will be creating tessellations using dot paper and that they will need to label their shapes and angles.
2. Have students create and label the following:
   - a tessellation using one regular polygon
   - a tessellation using one irregular polygon
   - a tessellation using two or more regular polygons
   - a tessellation using two or more irregular polygons

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify a regular polygon that tessellates.
- ❒ Identify a combination of regular polygons that tessellate.
- ❒ Identify an irregular polygon that tessellates.
- ❒ Identify a combination of irregular polygons that tessellate.

Suggestions for Instruction

- **Identify a translation, reflection, or rotation in a tessellation.**
- **Identify a combination of transformations in a tessellation.**

Materials: BLM 8.SS.6.3: Tessellation Slideshow, BLM 8.SS.6.4: Tessellation Recording Sheet, 1 computer per small group, BLM 8.SS.6.5: Tessellation Transformation, math journals

Organization: Small group/whole class/individual

Procedure:

1. Tell students that they will view the 10 tessellation slides included in BLM 8.SS.6.3: Tessellation Slideshow. Hand out copies of BLM 8.SS.6.4: Tessellation Recording Sheet.
2. Divide the class into small groups. Ask each group to view the slideshow and decide which transformation was used for each of the 10 images, recording their decisions on the recording sheet provided.
3. As a class, review the slideshow and come to a consensus on which transformation was used for each tessellation.
4. Have students, individually, complete BLM 8.SS.6.5: Tessellation Transformation.

Observation Checklist

☑ Observe students’ responses to determine whether they can do the following:

- ❒ Identify a translation used in a tessellation.
- ❒ Identify a reflection used in a tessellation.
- ❒ Identify a rotation used in a tessellation.
- ❒ Identify a combination of transformations used in a tessellation.

Suggestions for Instruction

- **Create a tessellation using one or more 2-D shapes, and describe the tessellation in terms of transformations and conservation of area.**
- **Create a new tessellating shape (polygon or non-polygon) by transforming a portion of a tessellating polygon, and describe the resulting tessellation in terms of transformations and conservation of area.**

Materials: BLM 5–8.9: Centimetre Grid Paper, white paper, scissors, math journals

Organization: Individual

Procedure:

1. Ask students to list the various shapes that tessellate.
2. Hand out copies of BLM 5–8.9: Centimetre Grid Paper, and have students, individually, draw a shape of their choice that they know will tessellate.
3. Have students determine the area of the shape, and then cut out the shape.
4. Instruct students to draw the shape on the white paper provided. Have students choose a transformation that works with their shape and transform the shape to tessellate the plane.
5. Ask students to determine the area of the newly tessellated shapes.
6. Ask students to answer the following questions:
   - What do you notice when you compare the area of the original shape and the area of the new shape?
   - What statement can be made about the area of the tessellating shapes?
7. Ask students to explain, in their math journals, why the area of a tessellating shape stays the same.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Describe a tessellation using transformations.
- ❒ Reason why area is conserved in a tessellation.

- **Identify and describe tessellations in the environment.**

Materials: Paper and clipboard

Organization: Pairs or small groups

Procedure:

1. Tell students that they will be working in a small group to create a tessellation scavenger hunt and then exchange clues with another group. Give them the following parameters:
   - You must have between eight and ten points of interest.
   - A point of interest is a tessellation to which you want to draw attention.
   - All your points of interest must be located within the school grounds.
   - You need to provide a neat copy of your clues so that the other group can find your points of interest without difficulty.
   - You need to provide an answer key with a sketch of the tessellation and a clear description of its location.
2. Have each group exchange clues with another group and try to find the other group's tessellation points of interest.
3. Have students meet up with the group they exchanged with to make sure both groups have found all the tessellation points of interest.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify and describe tessellations in the environment.

Putting the Pieces Together

Escher Tessellations

Introduction:

This task allows students to research M. C. Escher and then create a piece of artwork that represents his use of tessellations.

Purpose:

Students will use the following skills: transformations to create tessellations, principles and elements of art, the inquiry process.

Curricular Links: Mathematics, Art, ELA, LwICT

Materials/Resources: Internet access, various art media (e.g., pencil crayons, paint, charcoal—depending on the comfort level of the teacher), manila tag paper to use for the template, white paper for the end project

Organization: Individual/small group

Scenario:

You are going to research M. C. Escher and identify how he used geometric shapes to create amazing works of art. Then, using his techniques, you will create your own Escher-like tessellation for display in the classroom or in the school hallway.

You will prepare a report on your research findings about Escher. You will explain how you used geometric shapes to create your tessellation and what transformations you used. You can choose the format of your report from the following options: a written report, a brochure, or a presentation (e.g., using *PowerPoint*, *Photo Story*, or *Movie Maker*).

Assessment:

The following rubric can be used to assess achievement of the mathematics learning outcomes.

Note: Other rubrics may be added to assess Art, ELA, and LwICT learning outcomes.

Criteria	Meeting Expectations	Developing to Meet Expectations	Beginning to Meet Expectations	Incomplete
The student				
■ demonstrates an understanding of tessellation by creating tessellations	❒ provides a creative tessellation that clearly models Escher's work	❒ provides a tessellation that models Escher's work	❒ provides a tessellation that somewhat models Escher's work	❒ does not provide a tessellation
■ demonstrates an understanding of tessellation by explaining the properties of shapes that make tessellating possible	❒ provides a clear explanation of how geometric shapes were used to create the tessellation, including reference to interior angles	❒ provides a general explanation of how geometric shapes were used to create the tessellation	❒ provides a vague or minimal explanation of how geometric shapes were used to create the tessellation	❒ provides no explanation of how geometric shapes were used to create the tessellation
	❒ provides a clear explanation of what transformations were used to create the tessellation	❒ provides a general explanation of what transformations were used to create the tessellation	❒ provides a vague or minimal explanation of what transformations were used to create the tessellation	❒ provides no explanation of what transformations were used to create the tessellation