Grade 8 Mathematics

Patterns and Relations

Patterns and Relations (Patterns)—8.PR.1

Enduring Understandings:

Words, tables, graphs, expressions, and equations are different representations of the same pattern.

Variables are used to describe mathematical relationships.

General Learning Outcome:

Use patterns to describe the world and solve problems.

Specific Learning Outcome(s):	Achievement Indicators:
8.PR.1 Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V]	➛ Determine the missing value in an ordered pair for an equation of a linear relation. ➛ Create a table of values for the equation of a linear relation. ➛ Construct a graph from the equation of a linear relation (limited to discrete data). ➛ Describe the relationship between the variables of a graph.

Prior Knowledge

Students may have had experience with the following:

- Representing and describing patterns and relationships using charts and tables to solve problems
- Identifying and explaining mathematical relationships using charts and diagrams to solve problems
- Determining the pattern rule to make predictions about subsequent elements
- Representing and describing patterns and relationships using graphs and tables
- Demonstrating an understanding of oral and written patterns and their corresponding relations
- Constructing a table of values from a relation, graphing the table of values, and analyzing the graph to draw conclusions and solve problems
- Identifying and plotting points in the four quadrants of a Cartesian plane using ordered pairs

Increasing/Decreasing Linear Patterns

Although there are different types of patterns, the learning experiences that follow focus on *increasing/decreasing* ***linear*** *patterns*. The elements that make up these patterns are called *terms*. Each term builds on the previous term. Consequently, these patterns are often referred to as *growing patterns*. For example, 2, 4, 6, 8, 10 . . . and 0, 3, 6, 9, 12 . . . are two common increasing linear patterns.

Using a table to model an increasing/decreasing pattern can help students organize their thinking. It can also help them generalize the patterns symbolically. There are two types of generalizations that can be made: recursive and explicit.

- A *recursive generalization* tells how to find the value of a term given the value of the preceding term.
- An *explicit generalization* expresses the relationship between the value of the term and the term number.

Example:

Consider this pattern: ♥ ♥♥♥ ♥♥♥♥♥ ♥♥♥♥♥♥♥ ♥♥♥♥♥♥♥♥♥

The pattern can be organized into a table like this:

Term	1	2	3	4	5
Term Value	1	3	5	7	9

The recursive generalization that describes this pattern is $n + 2$ (where n is the previous term value), since the value of each term is two more than the preceding term. If the pattern were continued, the value of the sixth term would be 11, since 9 + 2 = 11. However, to find the value of the 100th term, you would need to find the value of each of the 99 preceding terms.

It is easier to predict the value of subsequent terms with an explicit generalization. Notice that in the above pattern, if you double a term number and subtract 1 ($2n – 1$), you get the value of the term (where n is the term number).

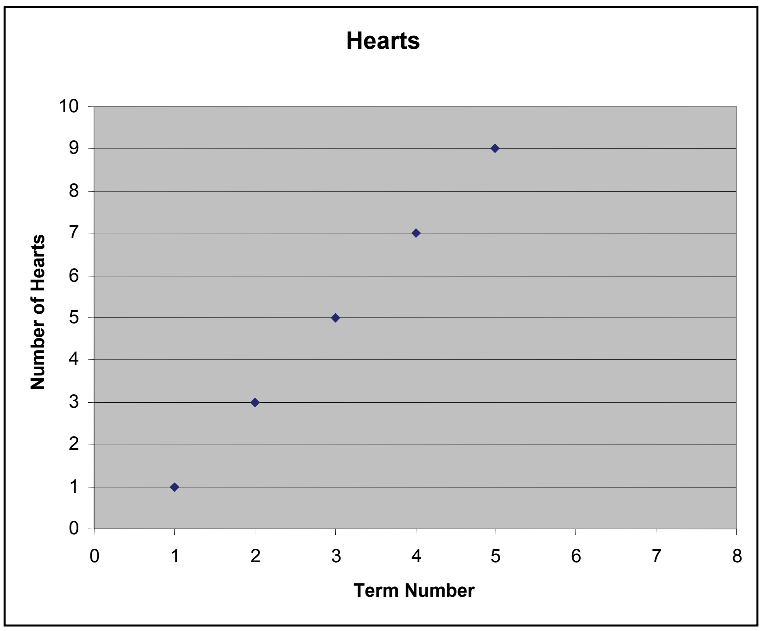
The value of the third term is 2 • 3 – 1 = 5, and the value of the fifth term is 2 • 5 – 1 = 9. Thus, the explicit generalization that describes the pattern is 2*n* – 1. If the pattern were continued, the value of the 100th term would be 199, since 2 • 100 – 1 = 199.

When helping students to recognize patterns, it is important to remember that they may not see the patterns in the same way you do. Therefore, ask students to explain their thinking. Having students describe their reasoning can also help them realize that often there is more than one way to look at a pattern.

Graphing linear relationships can often help students recognize both the recursive and explicit generalizations.

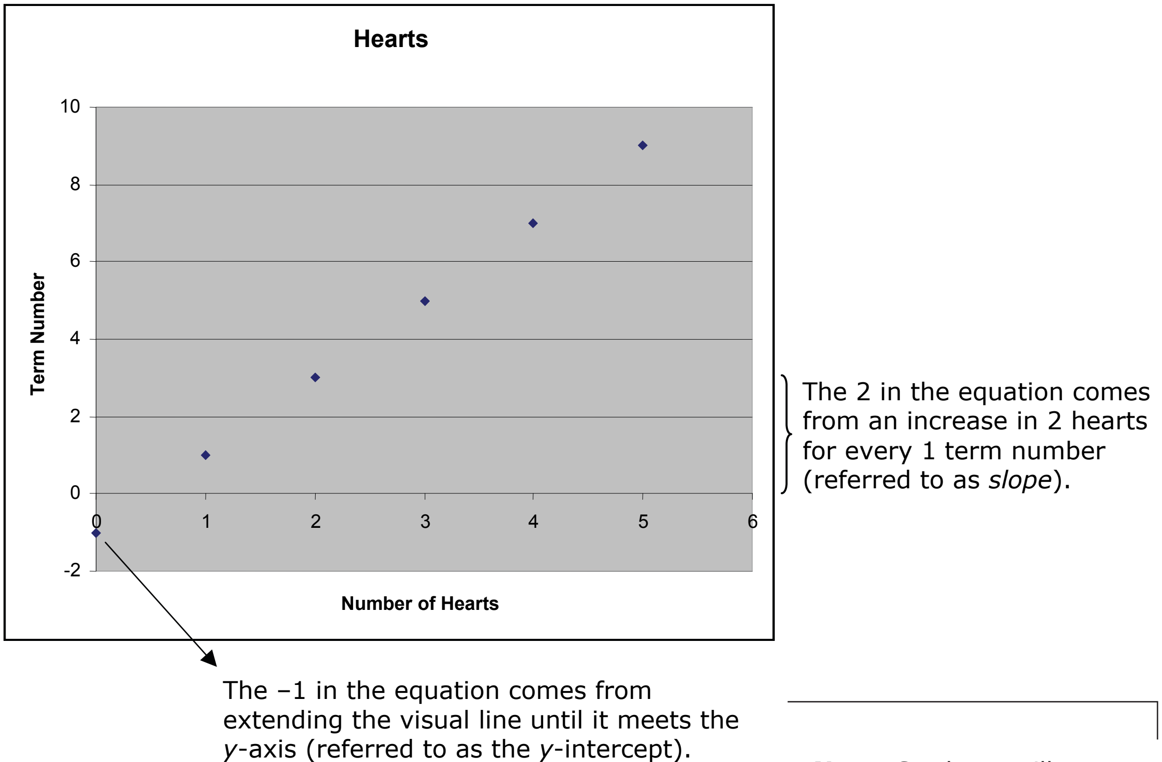
Example:

The graph for the above pattern would look like this:



The number of hearts increases by 2 each time, showing the recursive generalization.

The explicit generalization of $2n - 1$ can be seen in the graph:



The data in the above graphs are displayed as *discrete data*. This occurs when it would not make sense to plot points in between graphed values (e.g., the term number 1.5 does not exist). If you were to connect the points, it would be referred to as *continuous data* (e.g., the graph of the distance walked in an amount of time).

Note: Students will not formally encounter the slope and the *y*-intercept until Senior Years mathematics. It is important, however, for students to analyze linear relationships expressed graphically. If students make these generalizations, provide them with the correct terminology.

Mathematical Language

equation

expression

formula

linear relation

pattern

relation

table of values

variable

x-value

y-value

Learning Experiences

Assessing Prior Knowledge

Materials: BLM 8.PR.1.1: Patterns Pre-Assessment

Organization: Individual

Procedure:

1. Tell students that they will be extending their understanding of patterns over the next few lessons; however, you first need to determine what they already know about patterns.
2. Hand out copies of BLM 8.PR.1.1: Patterns Pre-Assessment.
3. Have students complete all sections of the BLM to the best of their ability.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Extend a pattern pictorially.
- ❒ Describe a pattern in own words.
- ❒ Construct a table of values from a relation.
- ❒ Make predictions from a pattern.
- ❒ Graph relations.
- ❒ Write relations algebraically.

Suggestions for Instruction

- **Determine the missing value in an ordered pair for an equation of a linear relation.**

Materials: BLM 8.PR.1.2: Determine the Missing Values

Organization: Whole class/individual

Procedure:

1. Put the following chart on the whiteboard.

x	1		3	4		
y	0	2	4			

2. Ask students whether they can fill in the missing x and y values. Ask them how they determined the missing values.
3. Ask students to record the pattern as an equation. Record the following ordered pairs, one at a time, on the board: (8, y), (x, 20), (0, y), (x, –4). Ask students to determine the missing values for the pattern.
4. Discuss student responses. Record any suggestions students make and evaluate the various methods used to determine the missing values for the ordered pairs.
5. Have students individually complete BLM 8.PR.1.2: Determine the Missing Values.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine the missing values in a table.
- ❒ Determine the missing value in an ordered pair for a linear equation.

Suggestions for Instruction

- **Determine the missing value in an ordered pair for an equation of a linear relation.**

Materials: BLM 8.PR.1.3: Break the Code, projector

Organization: Whole class/pairs

Procedure:

1. Display BLM 8.PR.1.3: Break the Code to the class (using a projector).
2. As a class, work through the ordered pairs to break the code.
3. Have students work in pairs, creating codes for one another and then exchanging them with their partners.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine the missing value in an ordered pair for a linear equation.

Suggestions for Instruction

- **Create a table of values for the equation of a linear relation.**
- **Construct a graph from the equation of a linear relation (limited to discrete data).**

Materials: BLM 8.PR.1.4: Linear Relations

Organization: Whole class/individual

Procedure:

1. As a class, work through the following problems. Discuss how you select the numbers in the table of values. Model how you set up the tables and how you graph the data. Also, discuss the linear relations that are represented in the graphs.

   Example 1:

   - The circumference of a circle is approximately three times its diameter. The precise relationship can be expressed as $C = \pi d$, where C is the circumference, π is a constant of 3.14159 . . . , and d is the diameter.
   - Make a table of values to show the relationship.
   - Construct a graph from the ordered pairs you recorded from the equation.

   Example 2:

   - The following is a linear relation: $y = 3x + 1$.
   - Make a table of values to show this relationship.
   - Construct a graph from the ordered pairs.

2. Discuss the graphs with students to ensure they understand correct graphing technique, including selecting a title, labelling axes, recording dependent variable on the *y* axis, recording independent variable on the *x* axis, spacing numbers appropriately, and so on.
3. Hand out copies of BLM 8.PR.1.4: Linear Relations. Have students complete the learning activity individually.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Independently create a table of values for the equation of a linear relation.
- ❒ Construct a graph from the equation of a linear relation.

Suggestions for Instruction

- **Describe the relationship between the variables of a graph.**

Materials: BLM 8.PR.1.5: Graphs

Organization: Individual/pairs/whole class

Procedure:

1. Hand out copies of BLM 8.PR.1.5: Graphs.
2. Ask students to create a T-table or a T-chart to represent the *x* and *y* values found on the graphs.
3. Ask students to describe the relationship between the *x* and *y* values found on the graphs and in the T-chart using a formula.
4. Have students join a partner and share the relationships they each identified. Extend the discussion by having students identify other variables that could result in a graph that looks like the ones on BLM PR.1.5.
5. Discuss the identified relationships as a class.
6. Have students find a graph (e.g., in newspapers, magazines) and see whether they can identify a linear relationship in the graph.
7. Have students individually draw a graph that reflects a linear relationship, and then exchange it with their respective partners, who must identify the linear relationship between the variables on the graph.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Communicate mathematically.
- ❒ Visualize a mathematical relationship that will create a linear relation.
- ❒ Describe the relationship between variables of a graph.

Notes

Patterns and Relations (Variables and Equations)—8.PR.2

Enduring Understandings:

The principles of operations used with whole numbers also apply to operations with decimals, fractions, and integers.

Number sense and mental mathematics strategies are used to estimate answers and lead to flexible thinking.

Preservation of equality is used to solve equations.

General Learning Outcome:

Represent algebraic expressions in multiple ways.

Specific Learning Outcome(s):	Achievement Indicators:
8.PR.2 Model and solve problems using linear equations of the forms ■ $ax = b$ ■ $\frac{x}{a} = b, a \neq 0$ ■ $ax + b = c$ ■ $\frac{x}{a} + b = c, a \neq 0$ ■ $a(x + b) = c$ concretely, pictorially, and symbolically, where *a*, *b*, and *c* are integers. [C, CN, PS, V]	➛ Model a problem with a linear equation, and solve the equation using concrete models. ➛ Verify the solution to a linear equation using a variety of methods, including concrete materials, diagrams, and substitution. ➛ Draw a visual representation of the steps used to solve a linear equation and record each step symbolically. ➛ Solve a linear equation symbolically. ➛ Identify and correct errors in an incorrect solution of a linear equation. ➛ Solve a linear equation by applying the distributive property (e.g., $2(x + 3) = 5$; $2x + 6 = 5$; . . .). ➛ Solve a problem using a linear equation, and record the process.

Prior Knowledge

Students may have had experience with the following:

- Applying mental mathematics strategies for multiplication, such as
  - annexing, then adding zeros
  - halving and doubling
  - using the distributive property
- Solving problems involving single-variable (expressed as symbols or letters), one-step equations with whole-number coefficients and whole-number solutions
- Demonstrating and explaining the meaning of preservation of equality, concretely, pictorially, and symbolically
- Demonstrating an understanding of preservation of equality by
  - modelling preservation of equality, concretely, pictorially, and symbolically
  - applying preservation of equality to solve equations
- Explaining the difference between an expression and an equation
- Evaluating an expression given the values of the variable(s)
- Modelling and solving problems that can be represented by one-step linear equations of the form $x + a = b$, concretely, pictorially, and symbolically, where a and b are integers
- Modelling and solving problems that can be represented by linear equations of the form:
  - $ax + b = c$
  - $ax = b$
  - $\frac{x}{a} = b, a \neq 0$

  concretely, pictorially, and symbolically, where a, b, and c are whole numbers

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically
- Demonstrating an understanding of multiplication and division of integers, concretely, pictorially, and symbolically

BACKGROUND INFORMATION

Linear Equations

Linear equations enable students to solve problems and make predictions based on a linear relationship. Understanding that linear equations form a balance will assist with understanding the relationships found in other mathematical concepts (e.g., Pythagorean theorem, area of a triangle, area of a circle).

The learning experiences related to specific learning outcome 8.PR.2 focus, in part, on translating word problems into equations and then solving them. This is not new to students. In the Early Years, students were introduced to whole-number operations through routine problems. At first, students solved these problems using concrete and pictorial representations. Later, they translated these problems into equations, often using an empty square to represent the unknown value.

Consequently, the learning experiences suggested for this learning outcome provide students with additional experience with solving routine problems. At the same time, students begin the transition to using letters to represent unknown quantities.

The suggested learning experiences also serve as an informal introduction to the terms *equation*, *mathematical expression*, and *variable*, which are defined as follows:

- An *equation* is a mathematical sentence stating that one or more quantities are equal. Equations that contain variables, such as $3 + x = 21$ and $2y + 3 = 15$, are sometimes referred to as *open sentences*, while equations that have no variables, such as $3 + 5 = 8$ and $24 \div 3 = 8$, are referred to as *closed sentences*.
- A *mathematical expression* comprises numbers, variables, and operation signs, but does not contain a relational symbol such as =, ≠, <, >, ≤, or ≥. For example, $6x + 3$ and $\frac{x}{4} - 8$ are mathematical expressions.
- A *variable* is a symbol for a number or a group of numbers in a mathematical expression or an equation.

In Grade 8, students are expected to solve two-step linear equations where coefficients and constants are integers, as well as apply the distributive property to equations in order to solve them.

Two-step linear equations have two operations within the expression and, therefore, the most efficient method to solve the problem involves two steps.

Example:

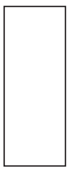
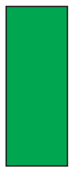
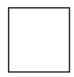
$2x + 6 = 5$

In the expression on the left, the two operations are multiplication ($2 \cdot x$) and addition (+ 6). To solve the equation, the opposite operations are used to balance the equation.

Solving Linear Equations Concretely, Pictorially, and Symbolically

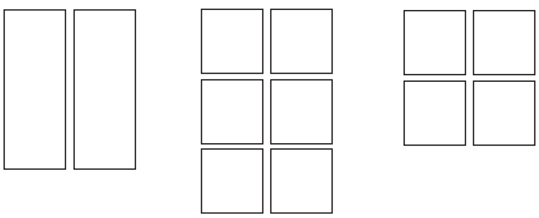
It is important for students to explore the solving of equations concretely, pictorially, and symbolically. An algebra balance (or pan balance) and algebra tiles are both appropriate concrete materials to use in demonstrating the preservation of equality.

Note: As algebra tiles have different colours, it is important to define which tile represents positive and which represents negative each time you use a new manipulative, and stay consistent with its meaning throughout its use.

 Represents $+x$  Represents $-x$  Represents $+1$  Represents -1

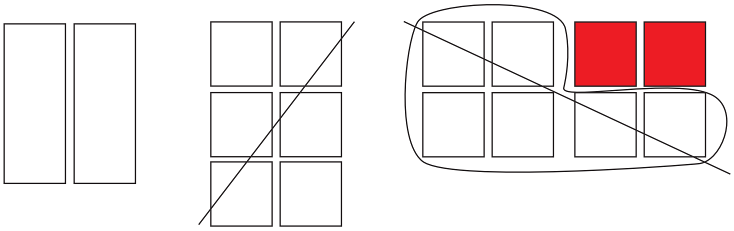
Solving Equations Concretely/Pictorially

$2x \quad + \quad 6 \quad = \quad 4$

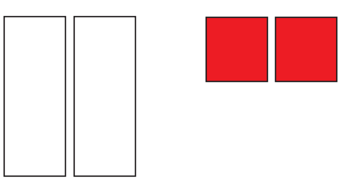


Remove (+6) from each side. There are not six (+1s) to remove from the right-hand side, so two zero pairs need to be added.

$2x \quad + \quad 6 - 6 \quad = \quad 4 + (2 - 2) - 6$

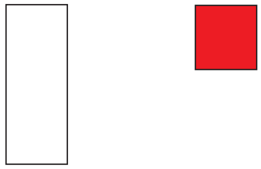


$2x \quad = \quad -2$



Simplify.

$x = -1$ Determine the value of each x.



Solving Equations Symbolically

$2x + 6 = 4$

$2x + 6 - 6 = 4 - 6$ Step 1: Subtract 6 from both expressions.

$2x = -2$

$\frac{2x}{2} = \frac{-2}{2}$ Step 2: Divide both expressions by 2.

$x = -1$

It is important to work with students to develop a procedure to solve equations so that they understand the mathematics involved. The important principles include the following:

- **Making zeros:** The reason you subtract 6 (in step 1 above) is to try to isolate the variable. 6 – 6 creates an addition of zero, leaving one term on the left-hand side. This is formally referred to as the *zero property of addition* or the *identity property of addition*.
- **Making ones:** The reason you divide by 2 (in step 2 above) is to try to isolate the variable, x. Dividing by 2 creates a multiplication by one, leaving just the variable on the left-hand side. This is formally referred to as the *inverse relationship* or the *identity property of multiplication*.
- **Preservation of equality:** To maintain balance and equality, you must do the same thing to both sides.

Distributive property is a property of real numbers that states that the product of the sum or the difference of two numbers is the same as the sum or difference of their products. Students should be familiar with the concept of distributive property as a multiplication strategy from Grade 5 [$a\ (b + c) = ab + ac$].

Examples:

Multiplication over Addition

2(15 + 4)
= 2 • 15 + 2 • 4
= 30 + 8

$2(2y + 3)$
$= 2 \cdot 2y + 2 \cdot 3$
$= 4y + 6$

Multiplication over Subtraction

$4(12 - 8)$
$= 4 \bullet 12 - 4 \bullet 8$
$= 48 - 32$

$3(5t - 3)$
$= 3 \bullet 5t - 3 \bullet 3$
$= 15t - 9$

Using the Distributive Property to Solve Equations

There are two different methods to solve the equation below. Neither is more correct than the other. Students should be encouraged to discover both, and use either freely on a case-by-case basis.

$2(x + 3) = 5$

$2 \bullet x + 2 \bullet 3 = 5$ Apply the distributive property.

$2x + 6 = 5$

$2x + 6 - 6 = 5 - 6$ Subtract 6 from both expressions.

$2x = -1$

$\frac{2x}{2} = \frac{-1}{2}$ Divide both expressions by 2.

$x = -\frac{1}{2}$

OR

$2(x + 3) = 5$

$\frac{2(x+3)}{2} = \frac{5}{2}$ Divide both expressions by 2.

$x + 3 = 2\frac{1}{2}$

$x + 3 - 3 = 2\frac{1}{2} - 3$ Subtract 3 from both sides.

$x = -\frac{1}{2}$

Note: Preservation of equality is the most important concept to consider when solving linear equations. It can be compared to a rule in a game. **Making zeros** and **making ones** are strategies for solving linear equations. When first solving linear equations, allow students to try operations, and then ask whether the operations or strategies were useful.

For example: solve $3b - 5 = 7$

A student may want to subtract 3 from each side.

Done correctly, this gives 3b – 8 = 4.

While this strategy is not wrong, it is not useful because there are still three terms in the equation and the student is no closer to isolating the variable, which is the "goal" of the "game."

Allowing students to use inefficient strategies at first and discussing how efficient they are will help students understand more efficient strategies, while not introducing too many rules to memorize.

Mathematical Language

balance
constant
equation
equivalent
evaluate
expression
formula
one-step linear equation
opposite operation
substitution
two-step linear equation
variable

Learning Experiences

Assessing Prior Knowledge

Materials: BLM 8.PR.2.1: Algebra Pre-Assessment, algebra tiles or balance scale, chips

Organization: Individual

Procedure:

1. Tell students that they will be extending their understanding of solving equations; however, you first need to determine what they already know about solving equations.
2. Hand out copies of BLM 8.PR.2.1: Algebra Pre-Assessment, and let students know what manipulatives are available to them. Have students solve the questions individually, showing their work.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Model and solve problems that can be represented by one-step linear equations of the form $x + a = b$ concretely, where a and b are whole numbers.
- ❒ Model and solve problems that can be represented by one-step linear equations of the form $x - a = b$ pictorially, where a and b are whole numbers.
- ❒ Model and solve problems that can be represented by one-step linear equations of the form $ax = b$ pictorially, where a and b are whole numbers.
- ❒ Model and solve problems that can be represented by one-step linear equations of the form $\frac{x}{a} = b$ symbolically, where a and b are whole numbers and $a \neq 0$.
- ❒ Communicate mathematically.
- ❒ Apply mental mathematics and reasoning strategies in order to compute mathematically.

Suggestions for Instruction

- **Model a problem with a linear equation, and solve the equation using concrete models.**

Materials: Chart paper,
Kroll, Virginia. *Equal Shmequal: A Math Adventure*. Illus. Philomena O'Neill. Watertown, MA: Charlesbridge Publishing, Inc., 2005. Print.

Organization: Pairs/whole class

Procedure:

1. Tell students that you will read them a story called *Equal Shmequal*, in which the animals use a see-saw to make equal teams. Students will need to explain how the see-saw could be used to represent linear equations.
2. Read the story.
3. Have students pair up and use the see-saw to represent an equation of their own on chart paper.

4. Have students present their work to the class, and allow time for discussion. Where necessary, facilitate discussion using guiding questions such as the following:
   - When is a see-saw balanced?
   - When are numbers balanced?
   - If you make one side of the see-saw heavier, what happens?
   - If you make the value of one side of the equation higher, what happens?
   - How can you re-establish balance on a see-saw?
   - How can you re-establish balance in an equation?
5. Post students' work on the classroom wall so that students can refer to them.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Communicate mathematically.
- ❒ Visualize a balance with symbolic algebraic representations.

Suggestions for Instruction

- **Model a problem with a linear equation, and solve the equation using concrete models.**

Materials: BLM 5–8.20: Algebra Tiles, paper ($8\frac{1}{2} \times 11$)

Organization: Whole class/individual

Procedure:

1. Remind students that in earlier grades they learned how to solve equations using positive whole numbers. Now they are going to learn how to solve equations using positive and negative integers.
2. Distribute $+x$, $-x$, +1, and –1 algebra tiles to all students. Review the representations of the algebra tiles. It does not matter which representation you choose to use, providing you are consistent.
3. Have students make an equal sign (=) on paper.
4. Write the following on the whiteboard: $4x = -16$.
5. Have students model the equation at their desks.
6. Ask students to determine the value of x by manipulating the algebra tiles. Discuss the various strategies students use to determine the value of x.

7. Always verify that the solution is correct. Start with $4x = -16$. Substitute –4 for x and determine that $4 \bullet -4 = -16$. Note that –16 will appear on both sides of the equal sign, so the solution is correct.
8. Repeat the process with simple equations until students feel comfortable using algebra tiles.
9. Repeat the process with equations containing negative x values, such as $-x + 3 = 2$, $\frac{-x}{2} = -2$.
   Note: These equations are more difficult because students will end up with a negative x equals a negative number. However, we are looking for the positive value of x. Students will need to apply their understanding of integers to this concept to determine the variable.
10. Write the following equations on the whiteboard and have students solve the equations concretely:
    $-2x = -8$, $x - 4 = -8$, $\frac{2x}{4} = 4$, $5x + 2 = 12$

Observation Checklist

☑ As students are working on the equations, circulate to observe whether they are able to do the following:

- ❒ Model and solve linear equations using models.

Suggestions for Instruction

- **Draw a visual representation of the steps used to solve a linear equation and record each step symbolically.**
- **Verify the solution to a linear equation using a variety of methods, including concrete materials, diagrams, and substitution.**

Materials: BLM 5–8.20: Algebra Tiles, white paper ($8\frac{1}{2} \times 11$)

Organization: Individual

Procedure:

1. Tell students that they will be building on what they learned in the previous learning experience. Ask them to model the following equation using algebra tiles: $2x + 4 = 10$
2. Provide each student with a white piece of paper and ask students to fold the paper in half.

3. Have them draw the equation on the top left-hand side of the paper, using the appropriate colours.
4. Ask them to continue with their diagram to solve the problem, working down the left side of the page.
5. When they have solved the problem pictorially, ask them to write the symbolic equation on the top right-hand side of the paper.
6. Have them work down the page, solving the equation so that each step of the symbolic solution matches the pictorial step across from it.
7. Always have students check their work when they have a solution to see whether the result balances the equation. Students will substitute the result for the variable and work through the equation to see whether it balances.

Observation Checklist

☑ As students are working on the equations, circulate to observe whether they are able to do the following:
- ❒ Model and solve linear equations pictorially.
- ❒ Model and solve linear equations symbolically.
- ❒ Verify the solutions of linear equations.

Suggestions for Instruction

- **Draw a visual representation of the steps used to solve a linear equation and record each step symbolically.**
- **Verify the solution to a linear equation using a variety of methods, including concrete materials, diagrams, and substitution.**

Materials: BLM 5–8.20: Algebra Tiles, brown lunch bags (2 per group), self-adhesive notes with a large equal sign (=) written on them

Organization: Pairs

Procedure:

1. Have students form groups of two. Give each pair an equal sign and two bags.
   - Bag A contains +1, –1, +*x*, and –*x* algebra tiles.
   - Bag B contains +1 and –1 algebra tiles.
2. Have partner A take a handful of tiles from bag A and place the tiles on one side of the equal sign.

3. Have partner B take a handful of tiles from bag B and place the tiles on the other side of the equal sign.
4. Have students work together to simplify and solve the equations concretely and record their process either pictorially or symbolically.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Model and solve linear equations concretely.
- ❒ Model and solve linear equations pictorially.
- ❒ Model and solve linear equations symbolically.

Suggestions for Instruction

- **Solve a linear equation symbolically.**
- **Solve a linear equation by applying the distributive property (e.g., $2(x + 3) = 5$; $2x + 6 = 5$; . . .).**

Materials: BLM 8.PR.2.2: Solving Equations Symbolically

Organization: Individual/pairs/whole class

Procedure:

1. As a review of the distributive property, write the following on the whiteboard: $7 \bullet 16$. Ask students whether they can think of a simple way to determine this product. Record and discuss all ideas.
2. Record the following on the whiteboard: $2(a + 4)$. Is there another way to write this expression? Record and discuss students' responses.
3. Ask students whether they would be able to solve the equation if you made the expression $2(a + 4)$ into an equation, $2(a + 4) = 12$. Record and discuss students' responses.
4. Provide each student with a copy of BLM 8.PR.2.2: Solving Equations Symbolically.
5. Have students use the Think-Pair-Share strategy, following these steps:
   - Complete the BLM individually.
   - Share responses with a partner.
   - Make notes of any questions that arise.
   - Find a new partner to share responses with.
6. Discuss students' results as a class and allow opportunity for questions as needed.

7. Ask students to solve, and verify their solutions to, the following questions in their math journals:

   $\frac{-x}{5} = -30$, $-4k = 24$, $3(y + 6) = 27$, $-4(r - 6) = 12$

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Solve linear equations symbolically.
- ❒ Solve linear equations by applying the distributive property.
- ❒ Verify the solutions of linear equations.

Suggestions for Instruction

- **Solve a linear equation symbolically.**
- **Solve a linear equation by applying the distributive property (e.g., $2(x + 3) = 5$; $2x + 6 = 5$; . . .).**

Materials: BLM 8.PR.2.3: Algebra Match-up, scrap paper

Organization: Pairs

Procedure:

1. Have students choose a partner. Provide each pair of students with a set of algebra match-up cards (copied from BLM 8.PR.2.3: Algebra Match-up).
2. Tell students that they will be playing *Algebra Match-up*. (**Note:** The game is played like *Go Fish*.)
3. Explain the rules for *Algebra Match-up*.
   - Deal 5 cards to each player.
   - Player A asks player B for the pair to one of his or her cards. For example, Player A asks for a card by stating, "Twice a number is equal to 6."
   - If player B has the solution, he or she will respond, "Yes, you may have $y = 3$," and will give player A that card. If player B does not have the solution, he or she will say, "No, take a card."
   - Play continues until all cards have been matched up.
   - The player with the most pairs at the end of the round wins.
   - Players are allowed to use scrap paper to try to determine matching pairs.
4. Have students play the game.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Solve linear equations symbolically.
- ❒ Apply the distributive property in order to solve equations.
- ❒ Use mental mathematics and reasoning strategies to determine the solution to equations.

Suggestions for Instruction

- **Identify and correct errors in an incorrect solution of a linear equation.**

Materials: BLM 8.PR.2.4: Analyzing Equations, chart paper, BLM 8.PR.2.5: Analyzing Equations Assessment

Organization: Pairs/whole class/individual

Procedure:

1. Hand out copies of BLM 8.PR.2.4: Analyzing Equations.
2. Have students, working in pairs, analyze the solutions to the equations presented on the BLM. Ask them to determine whether or not the equations were solved correctly. For any equations that were not solved correctly, students must record the error and provide a corrected solution on chart paper. Students need to be prepared to share their responses with the class.
3. As a class, discuss the solutions presented by students. Post the correct solutions on the classroom wall.
4. Have students create their own linear equation and provide a solution, either with or without an error. Have each student exchange his or her example with a partner, who must review the solution to determine whether or not there is an error. Partners share their findings with each other.
5. Provide students with BLM 8.PR.2.5: Analyzing Equations Assessment. Ask them to analyze the solutions, determine whether they are correct or incorrect, and explain their reasoning.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify and correct errors in an incorrect solution of a linear equation.
- ❒ Solve equations symbolically.
- ❒ Communicate solution errors using mathematical language.

Suggestions for Instruction

- **Solve a problem using a linear equation, and record the process.**

Materials: BLM 8.PR.2.6: Solving Problems Using a Linear Equation, chart paper

Organization: Small group/whole class

Procedure:

1. Tell students that they will be applying what they know about solving equations to problem-solving scenarios.
2. Divide the class into small groups, and provide students with copies of BLM 8.PR.2.6: Solving Problems Using a Linear Equation.
3. Have students work together to determine methods of solving the problems presented on the BLM. Have the groups present their solution methods to the class.
4. As a class, discuss the various solution methods presented. If a group does not come up with an algebraic method of solving the problems, offer that as a possible solution.
5. Tell students that it is important for them to practise representing word problems algebraically. Offer several more problems for students to represent algebraically (but not to solve them).
6. Have students, working in small groups, develop scenarios in which one would use linear equations to solve the problems, and have them record their problems on chart paper.
7. Have groups exchange problems and use algebraic reasoning to solve the problems.
8. Present, discuss, and post the problems and solutions generated by students.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Solve problems using a linear equation.
- ❒ Demonstrate an understanding of preservation of equality by modelling preservation of equality.

Putting the Pieces Together

Business and Marketing Analyst

Introduction:

This task allows students to put themselves in the role of a business and marketing analyst whose job is to determine how much of a product must be sold to break even, how much profit will be made if the product is sold out, and how much to sell the product for, making comparisons and setting a price for the product.

Purpose:

Students will be able to solve one- and two-step linear equations and use numbers to compare profit and loss.

Curricular Links: ELA

Materials/Resources: Calculator, white paper for business report, poster paper (if required for presentation)

Organization: Individual/small group

Scenario:

- You are a business and marketing analyst for Jamie Lee Foods.
- You know that it costs the company $250 to bake 100 dozen cookies.
- It will cost the company $250 to package the cookies individually.
- It will cost the company $175 to package the cookies in groups of 6.
- You need to determine the following:
  - If the cookies are sold individually for 50 cents each, how many cookies would be needed to cover all expenses (break even)? How much profit would be made if all the cookies were sold? Is selling the cookies for 50 cents each reasonable, or would it be better to sell them for 75 cents each?
  - If the cookies are sold as a 6-pack, what would be a reasonable sale price? Explain.

- You must prepare a written report that will be shared with the president and vice-president of Jamie Lee Foods. You must show all calculations and provide clear and concise explanations for determining what you think would be the best option for Jamie Lee Foods.

Assessment:

The following rubric can be used to assess achievement of the mathematics learning outcomes.

Criteria	Meeting Expectations	Developing to Meet Expectations	Beginning to Meet Expectations	Incomplete
The student				
■ sets up linear equations	❒ sets up all linear equations correctly using numbers and variables to calculate solutions	❒ sets up some linear equations correctly using numbers and variables to calculate solutions	❒ sets up few linear equations using numbers and variables to calculate solutions	❒ does not set up linear equations to show calculations
■ solves linear equations	❒ makes correct calculations that clearly communicate the mathematical processes required to determine the various options	❒ makes some errors in calculations and/or is somewhat disorganized when communicating the mathematical processes required to determine the various options	❒ makes many errors in calculations and/or is very disorganized when communicating the mathematical processes required to determine the various options	❒ does not provide calculations
■ analyzes costs to determine the best option for the company	❒ provides clear and organized communication of cost breakdowns and decisions for the best option	❒ provides somewhat unclear and disorganized communication of cost breakdowns and decisions for the best option	❒ provides very unclear and disorganized communication of cost breakdowns and decisions for the best option	❒ does not include a cost analysis

Extension:

Students could come up with an actual business plan on a smaller scale and then make and sell the product and donate profits to charity.

Notes