Grade 7 Mathematics

Statistics and Probability

Statistics and Probability (Data Analysis) (7.SP.1, 7.SP.2)

Enduring Understanding(s):

Data can be described by a single value used to describe the set.

General Learning Outcome(s):

Collect, display, and analyze data to solve problems.

Specific Learning Outcome(s):	Achievement Indicators:
7.SP.1 Demonstrate an understanding of central tendency and range by ▪ determining the measures of central tendency (mean, median, mode) and range ▪ determining the most appropriate measures of central tendency to report findings [C, PS, R, T]	➛ Determine the mean, median, and mode for a set of data, and explain why these values may be the same or different. ➛ Determine the range of a set of data. ➛ Provide a context in which the mean, median, or mode is the most appropriate measure of central tendency to use when reporting findings. ➛ Solve a problem involving the measures of central tendency.
7.SP.2 Determine the effect on the mean, median, and mode when an outlier is included in a data set. [C, CN, PS, R]	➛ Analyze a set of data to identify any outliers. ➛ Explain the effect of outliers on the measures of central tendency for a data set. ➛ Identify outliers in a set of data and justify whether or not they are to be included in the reporting of the measures of central tendency. ➛ Provide examples of situations in which outliers would or would not be used in determining the measures of central tendency.

Prior Knowledge

Students may have had experience with the following:

- Differentiating between first-hand and second-hand data.
- Comparing the likelihood of two possible outcomes occurring, using words such as
  - less likely
  - equally likely
  - more likely
- Creating, labelling, and interpreting line graphs to draw conclusions.
- Selecting, justifying, and using appropriate methods of collecting data, including
  - questionnaires
  - experiments
  - databases
  - electronic media
- Graphing collected data and analyzing the graph to solve problems.

For more information on prior knowledge, refer to the following resource:

Manitoba Education and Advanced Learning. *Glance Across the Grades: Kindergarten to Grade 9 Mathematics*. Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/cur/math/glance_k-9/index.html.

Related Knowledge

Students should be introduced to the following:

- Comparing and ordering fractions, decimals (to thousandths), and integers by using
  - benchmarks
  - place value
  - equivalent fractions and/or decimals
- Constructing, labelling, and interpreting circle graphs to solve problems.
- Expressing probabilities as ratios, fractions, and percents.
- Identifying the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.
- Conducting a probability experiment to compare the theoretical probability (determined using a tree diagram, table, or another graphic organizer) and experimental probability of two independent events.

BACKGROUND INFORMATION

Students live in an information age that abounds with data. Various media continually offer information on fashion, entertainment, sports, finances, safety, health, and world events. Students encounter data regularly at school, in their marks, in science experiments, in social studies information, and so on. To be helpful, data needs to be categorized and understood.

Statistics help reduce large quantities of data to single values. The single value makes it much simpler to conceptualize and communicate about the information contained in the data. Statistics, however, are sometimes manipulated or presented in a manner that uses facts to mislead people and sway their opinions. By studying statistics, students develop their ability to understand and evaluate information presented in advertising, politics, and news reports, and to communicate their experience with data.

Measures of Central Tendency

In previous grades, students collected data first hand and from electronic sources, and learned when to use each source. In Grade 7, students are introduced to three statistical measures of central tendency: mean, median, and mode. Each is a numeric value attempting to represent an entire set of data. Each measure is an average with its own focus, strengths, and weaknesses. The more symmetrical the set of data is, the closer the measures of central tendency will be to one another. The more skewed the set of data is, the greater the difference between the values will be. The different measures are best used in different situations, although sometimes all three measures provide meaningful representations of the data.

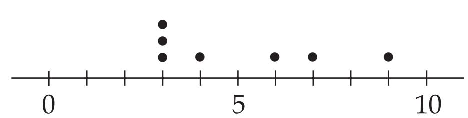
The measures of central tendency and range are discussed below:

- **Mean:** The arithmetic *mean* is commonly referred to as average, and is commonly used to assign student grades. The mean is the measure of central tendency most affected by outliers; therefore, it is best used when the range of values in the set is narrow. To find the mean, combine all the values in the set and then evenly redistribute them. The algorithm for calculating the mean is to sum all values in the set, and divide the combined value by the number of values in the set. The mean can also be found by finding the central balance point on a number line.

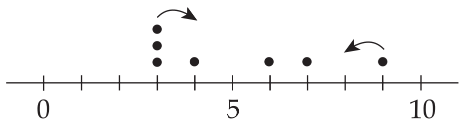
  Example:

  Given the numbers 3, 4, 6, 3, 3, 9, 7,

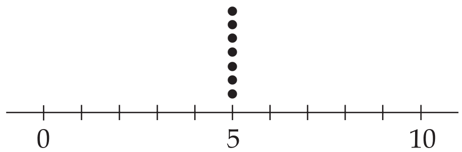
  a) plot the numbers on a number line



b) move the numbers toward the centre, allowing one move to the left for every one move to the right



c) continue this process until the numbers line up on one point



- **Median:** The *median* is the middle value in an ordered set of data. The median is easy to understand and easy to determine. To find the median, place all the values in the set, including repeated numbers, in numerical order and select the value in the middle. If there is no single middle value, add the two middle numbers together and divide by two. Because the median is the middle value, half the values in a data set will be greater than the median and half the values will be less than the median. The median represents the 50th percentile. (**Note:** Formal study of percentiles occurs in Grade 12 Essential Mathematics.) The median is less affected by outliers; therefore, it is more stable. It is the most appropriate measure of central tendency to represent a set of data containing extreme values.
- **Mode:** The *mode* is the most commonly occurring item in a set. A set of data may not have a mode, or it may have one mode, be bimodal, or have multiple modes. The mode may or may not indicate the centre of the data it represents. Generally, *outliers* (extreme values at either the high or the low end of the range) do not affect modes. Modes are very unstable, however, and a small change in the data can drastically change the mode. Because the mode identifies the most typical item in a set, it is useful for predicting the case in a particular situation. For example, if the mode for shirts sold is size 10, the buyers for a store can use the mode to help them decide which sizes to stock in the store's inventory.
- **Range:** The *range* describes a set of data by identifying the difference between the greatest value and the least value in a data set.

Understanding how and when to use the different statistical values gives students the ability to understand and communicate about data more clearly, and to use data wisely to make informed decisions.

When planning for student learning experiences, choose learning activities that emphasize concepts and understanding. Have students gather data for the purpose of answering questions. Allowing students to ask their own questions and collect their own data provides contexts and purposes for analyzing data and for exploring the different statistics. Students may, for example, wish to compare their classmates' habits or physical skills, integrate science or social studies content, or answer questions about world conditions or trends.

Mathematical Language

average

data

mean

measure of central tendency

median

mode

outlier

range

statistics

Venn diagram

Learning Experiences

Assessing Prior Knowledge

Materials:

- grid paper
- rulers

Organization: Individual or pairs

Procedure:

1. Review the concepts of formulating questions, collecting first- or second-hand data, and preparing bar graphs.
2. Have students work individually, or in pairs, to do the following:
   a) Formulate a survey question about peers that can be answered with numeric values.

      Sample Questions:

      - How many siblings are in your family? How many pets (or cell phones, televisions) does your family have?

- How many times a week do you eat a particular food, watch a movie, or participate in physical activity?
- How many hours do you sleep per night?
- How tall are you?
- How many pairs of mittens (or shoes, pants) do you own?
- How many countries have you visited?
- Compare the heights (or heart rates, lengths of names) of boys and girls in the class.

b) Gather the information.

c) Display the data in a bar graph.

d) Formulate a question about the population of the survey that could be answered using the information from the graph. Include an answer key to the question.

3. Have students present and display their work. These data sets can be used for subsequent learning experiences.

Variations:

- Have students choose a question and create bar graphs from data you provide or from the school census data available online.
- Rather than having students conduct surveys, have them research sources to collect data to answer specific questions (e.g., temperatures or rainfall amounts over a certain period of time, sizes of farms in a region, the price of a commodity).

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❐ Pose a survey question that can be answered with numeric values.
- ❐ Conduct a survey or search resources to obtain data.
- ❐ Display data in a bar graph.

Suggestions for Instruction

- **Determine the mean, median, and mode for a set of data, and explain why these values may be the same or different.**
- **Determine the range of a set of data.**

Materials:

- bar graphs from the surveys conducted in the previous learning experience (Assessing Prior Knowledge)
- presentation board
- math journals

Organization: Individual or pairs, whole class

Procedure:

1. Remind students that their surveys and graphs reveal interesting information about their peers.
2. Ask students to use the information in their graphs to create a general statement about a "typical" or average student in the class, or grade, or whatever group they surveyed.

   Examples:

   - How many siblings does a typical Grade 7 student have?
   - How many times does the typical Grade 7 student eat French fries in a week?
3. Have students work individually or with their partners to determine the best answer to their question, explain how they arrived at the answer, and explain why the answer represents a typical student.
4. Reassemble as a class and have students share their questions, answers, explanations, and justifications.
5. During the class discussion, encourage students to comment on and ask questions about their classmates' responses, and to present alternative answers.
6. Introduce vocabulary related to statistics as topics present themselves during the sharing.
7. Record the vocabulary on a presentation board. Include the three measures of central tendency (*mean*, *median*, and *mode*), the *range*, and *outliers* (if they are present).

   a) If students choose the most frequent response to represent the average student, introduce *mode*.

   b) If they find the arithmetic mean, or redistribute the items, introduce *mean* and discuss the methods they used to determine the value.

   c) If they use the middle value, introduce *median* and discuss the methods of finding the median.

d) If they discuss the range of values, introduce the *range* as the difference between these values.

e) If they mention anomalies such as a very high or very low value, introduce *outliers*.

8. Inform students that in using one value to represent a range of data, they have been exploring statistical measures of central tendency. Measures of central tendency will be studied in greater detail in the following learning experiences.

9. Have students record the new vocabulary terms and what they have learned about them in their math journals.

Variations:

- Have students combine the information from several surveys to create a profile of the typical Grade 7 student.
- Rather than using student survey data, provide students with several questions and sets of data. For example, provide data for several styles of T-shirts, each selling for a different price. Ask what would be a fair price for the T-shirts if they all sold for the same price. Or supply data for the amount of money individual students raised for a class trip. Ask how much money a typical student raised.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Demonstrate an understanding of mean, median, and mode for a set of data, and explain why these values may be the same or different.
- ❒ Demonstrate an understanding of the range of a set of data.

Suggestions for Instruction

- **Determine the mean, median, and mode for a set of data, and explain why these values may be the same or different.**
- **Determine the range of a set of data.**

Materials:

- BLM 7.SP.1.1: Finding the Centre of a Graph and Comparing the Values
- bar graphs from the previous learning experience or supplied data
- modelling clay, coloured cubes or blocks, and/or counters
- grid paper (1 cm)
- transparent rulers or grid strips
- presentation board
- math journals
- BLM 7.SP.1.2: Exploring Measures of Central Tendency (optional)

Organization: Individual or pairs, whole class

Procedure:

1. Distribute copies of BLM 7.SP.1.1: Finding the Centre of a Graph and Comparing the Values, and have students work individually or with a partner to complete it. Then reassemble as a class and discuss what students discovered.
2. Alternately, guide the class through the steps and have them record their learning in their math journals.
   a) Have students, working individually or in pairs, build a concrete model of their own graph, or a classmate's graph. Alternatively, supply students with data and have them prepare a graph and a model for the data. Students may use cubes or blocks, counters, and/or 1 cm grid paper and modelling clay.
   b) When students have completed their graphs, ask them to do the following:
      - Identify the range for the data represented in their graph, and record it (subtract the least value from the greatest value). Rearrange the graph to emphasize the range.
      - Identify any outliers or extreme values in the graph.
      - Find the mode, or most frequent value, represented in their graph, and record it. Explain how the graph could be rearranged to emphasize the mode.
      - Find the median or middle value in their graph, and record it. Explain how the graph could be rearranged to emphasize the median.
   c) Ask students to explore, on their own or with their partner, how to level the data and find its centre, or balance point. Emphasize that this is not the middle value or median. Students will be rearranging their graphs to emphasize the centre of the data, or the mean. They record the mean.

3. Have students reassemble as a class and share what they did to level the data, and discuss any questions or comments that arise. Strategies for levelling the data could include the following:
   a) Compress the modelling clay graph, while holding the sides and surface firm.
   b) Rearrange the blocks by taking blocks from the longer bars and placing them on the smaller bars until they are similar in height. If whole blocks cannot be shared evenly, it may be necessary to share fractions of a block.
   c) Place a ruler perpendicular to the bars of the graph, and adjust the position of the ruler until there are an equal number of blocks above the line and below the line. It may be necessary to position the ruler within a block if the mean is not a whole number.
4. Have students compare their three values, mode, median, and mean, and determine whether they each represent the data well, or whether one value represents the data better than the others, and why that may be.
5. Share students' reflections and discuss how each value is a measure of central tendency or a way to represent the average value. When the data set has a small range, the average values are similar, and each represents the graph. When there are outliers in the data, or the range is wide, the averages may be quite different from each other, and no average by itself represents the data well. Different measures are better for different situations, and sometimes more than one measure is needed to represent the data.

Variations:

- Supply students with graphs of data that controls the value of the averages (i.e., a whole-number mean if students are not prepared to work with decimals or fractions), or control the presence of modes or outliers.
- Have students rearrange the values of the bars to create multiple bar graphs that have the same mean. Ask why the different graphs have the same mean.
- Have students explore what effect rearranging the values of the bars of the graphs have on each of the average values. See BLM 7.SP.1.2: Exploring Measures of Central Tendency.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:
- ❒ Determine the mean, median, and mode for a set of data, and explain why these values may be the same or different.
- ❒ Determine the range of a set of data.
- ❒ Use reasoning and visualization to determine measures of central tendency.

Suggestions for Instruction

- **Determine the mean, median, and mode for a set of data, and explain why these values may be the same or different.**
- **Determine the range of a set of data.**

Materials:

- demonstration board
- magnets or self-stick notes
- number lines
- Unifix® or linking cubes
- BLM 7.SP.1.2: Exploring Measures of Central Tendency (optional)

Organization: Pairs, whole class

Procedure:

1. Review the three types of averages (mode, median, and mean), and how students found these values using graphs.
2. Present a set of data such as the following: 3, 4, 6, 3, 3, 9, 7. Ask students to use a Think-Pair-Share strategy (think about the question individually, discuss ideas with a partner, and then share responses with the class) to do the following:
   a) Identify the range in the data.
   b) Identify and explain how to determine the mean without making a graph.
   c) Identify and explain how to determine the median without making a graph.
   d) Identify and explain how to determine the mode without making a graph.
3. Introduce students to using a number line to find the centre of the data.
   a) Draw a number line on the demonstration board from 0 to 10.
   b) Place a square to represent each value on the corresponding point of the number line. Magnets or self-stick notes work well on a chalkboard or whiteboard. (In the data set specified above, there are three number 3s, so place three squares on 3.)
   c) The goal is to find the centre of all these values. Systematically move the self-stick notes from each end toward the centre until all the notes are stacked up on one point (e.g., a move of two jumps from the right toward the centre must be countered by a move of two jumps from the left toward the centre).
   d) For the above data set, the blocks will all line up on the mean 5.

Variations:

- Have students practise determining the mean and explore the effect of different values on averages. Alter the values in the above data set, but maintain a set of seven digits with a sum of 35. Record the measures of central tendency for each set, using BLM 7.SP.1.2: Exploring Measures of Central Tendency. Compare the values for different sets of data.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Determine the mean, median, and mode for a set of data, and explain why these values may be the same or different.
- ❒ Determine the range of a set of data.

Suggestions for Instruction

- **Determine the mean, median, and mode for a set of data, and explain why these values may be the same or different.**
- **Determine the range of a set of data.**
- **Provide a context in which the mean, median, or mode is the most appropriate measure of central tendency to use when reporting findings.**
- **Solve a problem involving the measures of central tendency.**
- **Analyze a set of data to identify any outliers.**
- **Explain the effect of outliers on the measures of central tendency for a data set.**
- **Identify outliers in a set of data and justify whether or not they are to be included in the reporting of the measures of central tendency.**
- **Provide examples of situations in which outliers would or would not be used in determining the measures of central tendency.**

Materials:

- BLM 7.SP.1.3A: Simone's Spelling Scores (Questions)
- BLM 7.SP.1.3B: Simone's Spelling Performance Record

Organization: Pairs or small groups, whole class (for Think-Pair-Share)

Procedure:

1. Distribute copies of BLM 7.SP.1.3A: Simone's Spelling Performance (Questions) and BLM 7.SP.1.3B: Simone's Spelling Performance Record.
2. Present a set of data such as Simone's spelling quiz results, scored out of 10. Her scores for the first seven quizzes were: 8, 8, 7, 9, 6, 10, and 8.
3. Ask students what score best represents Simone's spelling performance, and why they believe it to be so. In this set of data, the mean, median, and mode are all 8. The range is 4.
4. Simone writes three more quizzes, with scores of 3, 7, and 8. Have students identify and support her performance level now (mean 7.4, median 8, mode 8, range 7).
5. On the last three quizzes, Simone receives scores of 9, 10, and 0. Have students identify which one number will represent Simone's spelling performance. Ask students to support their choice using measures of central tendency and range (mean 7.2, median 8, mode 8, range 10).
6. Discuss students' choices and reasons. Include a discussion of
   a) the effect of outliers on the mean, median, and mode
   b) the influence of the range on the different measures
   c) possible reasons for the outliers (e.g., didn't study, called to the office, cheated, lost quiz)
   d) whether or not the outliers should be included in the data

Variations:

- Use students' own assignment or test scores.
- Use a different data set that does not represent school scores (e.g., prices for jeans, party sizes at a pizza restaurant, sizes of shoes or clothes).
- Have students research to obtain data to answer a question they pose.
- Have students generate random data to explore the effects of large outliers, or range size, on measures of central tendency.
- Ask students to analyze their findings and make a general statement regarding circumstances for which they recommend each measure of central tendency.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Determine the mean, median, and mode for a set of data, and explain why these values may be the same or different.
- ❒ Determine the range of a set of data.
- ❒ Provide a context in which the mean, median, or mode is the most appropriate measure of central tendency to use when reporting findings.
- ❒ Solve a problem involving the measures of central tendency.
- ❒ Analyze a set of data to identify any outliers.
- ❒ Explain the effect of outliers on the measures of central tendency for a data set.
- ❒ Identify outliers in a set of data, and justify whether or not they are to be included in the reporting of the measures of central tendency.
- ❒ Provide examples of situations in which outliers would or would not be used in determining the measures of central tendency.

Suggestions for Instruction

- **Determine the mean, median, and mode for a set of data, and explain why these values may be the same or different.**
- **Determine the range of a set of data.**
- **Provide a context in which the mean, median, or mode is the most appropriate measure of central tendency to use when reporting findings.**
- **Solve a problem involving the measures of central tendency.**

Materials:

- BLM 7.SP.1.4: Using Central Tendency to Choose a Quarterback
- spinners (optional)

Organization: Individual, small groups, whole class

Procedure:

1. Remind students that the mean, median, and mode are all measures of central tendency that represent an entire set of data. Students will now use these measures to choose a quarterback for a football game.
2. Distribute copies of BLM 7.SP.1.4: Using Central Tendency to Choose a Quarterback, and ask students to complete the page individually.
3. Then have students meet in small groups to discuss their thinking. Each group will choose one quarterback, and a spokesperson will present and justify the group's choice to the class.
4. As groups present and defend their choices, encourage students to respond to presentations with comments and questions.
5. Discuss what to do with errors, and when each measure (mean, median, and mode) is best used.

Variations:

- Have students generate additional data by using a spinner with sections for 0, 5, 10, 15, 20, and 25 yards, and then ask them to recalculate the measures and re-evaluate their decisions.
- Have students generate data for additional quarterbacks. Question whether their decisions are based on the same measures each time.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Determine the mean, median, and mode for a set of data, and explain why these values may be the same or different.
- ❒ Determine the range of a set of data.
- ❒ Provide a context in which the mean, median, or mode is the most appropriate measure of central tendency to use when reporting findings.
- ❒ Solve a problem involving the measures of central tendency.

Suggestions for Instruction

- **Provide a context in which the mean, median, or mode is the most appropriate measure of central tendency to use when reporting findings.**

Materials:

- BLM 7.SP.1.2: Exploring Measures of Central Tendency
- previously completed record sheets of data sets and measures of central tendency, including the information from the graphs produced in the Assessing Prior Knowledge learning experience
- spinners or pairs of number cubes (regular or multi-sided)

Organization: Pairs or small groups

Procedure:

1. Explain to students that they will be investigating sets of data to determine generalizations about which circumstances best match each measure of central tendency.
2. Have students work with a partner or in a small group.
3. Ask students to evaluate their previous records. If they require more data, or larger data sets, they can do the following:
   a) Randomly generate new data sets by spinning spinners or by rolling the number cubes and multiplying the displayed numbers.
   b) Research the legitimate answers to actual questions (e.g., salaries earned in specific companies, numbers of different sandwiches sold at a fast-food restaurant, flavours of ice cream sold, quantities of different drinks sold in the school cafeteria, teams winning championships).
4. Have students generate a list of guidelines for the types of data or circumstances they recommend for each measure of central tendency.
5. Discuss the guidelines as a class (refer to Background Information).

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Provide a context in which the mean, median, or mode is the most appropriate measure of central tendency to use when reporting findings.

Suggestions for Instruction

- **Solve a problem involving the measures of central tendency.**
- **Identify outliers in a set of data and justify whether or not they are to be included in the reporting of the measures of central tendency.**
- **Provide examples of situations in which outliers would or would not be used in determining the measures of central tendency.**

Materials:

- data sets from previous learning experiences
- paper

Organization: Individual, pairs or small groups

Procedure:

1. Explain that in this learning activity students will demonstrate their understanding and use of measures of central tendency.
2. Have students, individually, create a realistic question and an accompanying data set on one side of a sheet of paper, and a detailed solution to the question on the reverse side of the paper.
   - Questions may include outliers that would or would not be used in determining the central tendency.
   - Solutions require the range, outliers, and the mean, median, and mode to be identified. Ask students to identify the best measure to reflect the centre of that data and justify the choice.
3. Students then share their questions with a partner or a small group and demonstrate their ability to use and choose measures of central tendency to solve problems.
4. When students have solved a problem, they check their solution and discuss any discrepancies with the creator of the problem.

Variations:

- Prepare additional problems and data sets for students who require them.
- Provide students with several problems and data sets, and ask them to provide solutions for each.
- Have a statistics challenge in which individual students compete to solve the problems in front of a classroom audience, or in which teams of students compete against one another to find the best measure of central tendency in each case.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Solve a problem involving the measures of central tendency.
- ❒ Identify outliers in a set of data, and justify whether or not they are to be included in the reporting of the measures of central tendency.
- ❒ Provide examples of situations in which outliers would or would not be used in determining the measures of central tendency.

Statistics and Probability (Data Analysis) (7.SP.3)

Enduring Understanding(s):

Circle graphs show a comparison of each part to a whole using ratios.

Percents, fractions, decimals, and ratios are different representations of the same quantity.

General Learning Outcome(s):

Collect, display, and analyze data to solve problems.

Specific Learning Outcome(s):	Achievement Indicators:
7.SP.3 Construct, label, and interpret circle graphs to solve problems. [C, CN, PS, R, T, V]	➛ Identify common attributes of circle graphs, such as ▪ title, label, or legend ▪ the sum of the central angles is 360° ▪ the data is reported as a percent of the total and the sum of the percents is equal to 100% ➛ Create and label a circle graph, with or without technology, to display a set of data. ➛ Find and compare circle graphs in a variety of print and electronic media, such as newspapers, magazines, and the Internet. ➛ Translate percentages displayed in a circle graph into quantities to solve a problem. ➛ Interpret a circle graph to answer questions.

Prior Knowledge

Students may have had experience with the following:

- Differentiating between first-hand and second-hand data.
- Demonstrating an understanding of percent (limited to whole numbers) concretely, pictorially, and symbolically.
- Demonstrating an understanding of angles by
  - identifying examples of angles in the environment
  - classifying angles according to their measure
  - estimating the measure of angles using 45°, 90°, and 180° as reference angles
  - determining angle measures in degrees

- drawing and labelling angles when the measure is specified

- Selecting, justifying, and using appropriate methods of collecting data, including
  - questionnaires
  - experiments
  - databases
  - electronic media
- Graphing collected data and analyzing the graph to solve problems.
- Demonstrating an understanding of probability by
  - identifying all possible outcomes of a probability experiment
  - differentiating between experimental and theoretical probability
  - determining the theoretical probability of outcomes in a probability experiment
  - determining the experimental probability of outcomes in a probability experiment
  - comparing experimental results with the theoretical probability for an experiment

For more information on prior knowledge, refer to the following resource:

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Related Knowledge

Students should be introduced to the following:

- Solving problems involving percents from 1% to 100%.
- Demonstrating an understanding of circles by
  - describing the relationships among radius, diameter, and circumference of circles
  - relating circumference to pi
  - determining the sum of the central angles
  - constructing circles with a given radius or diameter
  - solving problems involving the radii, diameters, and circumferences of circles
- Expressing probabilities as ratios, fractions, and percents.
- Conducting a probability experiment to compare the theoretical probability (determined using a tree diagram, table, or another graphic organizer) and the experimental probability of two independent events.

BACKGROUND INFORMATION

The purpose of graphs is to display data. Students come to Grade 7 with experience in using line graphs to display continuous data, and bar graphs, double bar graphs, and pictographs to display discrete data. In Grade 7, students are introduced to circle graphs. Circle graphs are also referred to as pie charts.

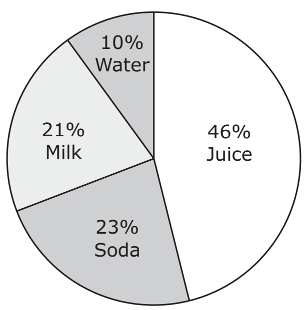
Circle Graphs (Pie Charts)

Various media use circle graphs to display comparative data. The *circle graph* displays the distribution of data, not the actual data values. The set of data is grouped into categories, and each category is expressed as a percent of the whole set of data.

Each sector of the graph represents a part-to-whole ratio. Circle graphs emphasize the relation between a category and the whole set of data, as well as the relation between different categories within the data set. Comparisons within circle graphs are most clear when the number of categories is small and when there is a definite variation in the size of the categories.

Example:

Lunch Beverages Chosen by 325 Students at Sveldon Middle School Cafeteria



Data			
Beverage Choice	**Number of Students**	**Percent**	**Angle Size**
Juice	150	46%	166°
Soda	75	23%	83°
Milk	68	21%	75°
Water	32	10%	36°
Totals	325	100%	360°

This circle graph shows that nearly half of the students eating in the school cafeteria choose juice as a lunch beverage, and that nearly equal numbers of students choose milk or soda.

Circle graphs may also be used to compare data sets of different size, as circle graphs compare ratios rather than definite quantities. The ratios regarding students' choices of beverage in the example above can be compared to choices made by students in other schools or in other regions. The comparisons may be used to answer questions or to solve problems (e.g., which school to target for a nutrition education program).

Circle graphs are also used effectively to display probability.

Experience with circles and central angles (learning outcome 7.SS.1), an understanding of decimals, percents, and fractions, and the ability to perform calculations with these values (learning outcomes 7.N.2, 3, 4, 5, and 7) make it easier for students to create and interpret circle graphs. An understanding of rounding is useful when constructing circle graphs (e.g., if the majority of percents or angle sizes have been rounded up or down, adjustments may be required to ensure the sum of percents totals 100%, and central angles represented in the graph total 360°).

Ways to Create Circle Graphs

There are many ways to create circle graphs. Several of these are described below. Each circle graph must have a descriptive title and must be labelled with the category names and corresponding percents, or be accompanied by a legend. The percents represented by the sectors must total 100%, and the sum of the central angles must equal 360°.

- **Make concrete representations.**
  - Divide students into categories, such as those who have pets, and those who do not have pets. Ask students in each group to stand side by side, equidistant from each other, and then have the two groups form a circle. Estimate the middle of the circle, and draw a line (perhaps using a skipping rope) from the centre of the circle to each point where the two groups meet.
  - Students could also use tokens to represent the numbers in the two groups. The tokens could be evenly spaced around a circle whose circumference has been divided into percents, and a line could be drawn from the centre to the points on the circumference midway between adjacent groups.
- **Join bars from a bar graph.**
  - Create a bar to represent the quantity in each group.
  - Colour the bars.
  - Then cut out each bar, and join the bars end to end with tape to create one long strip. Bring the ends of the strip together to create a circle.
  - Draw a line from the centre of the circle to each point where a new category begins.
- **Use fraction circles.**
  - Choose a fraction circle that matches the number of pieces of data. For example, if there are 10 marbles in a set, choose a circle divided into tenths. Each tenth represents one marble. If six of the marbles are blue, colour $\frac{6}{10}$ of the circle blue; if three of the marbles are yellow, colour $\frac{3}{10}$ of the circle yellow; and if the remaining marble is red, colour $\frac{1}{10}$ of the circle red.
  - Draw lines from the centre of the circle to the point on the circumference where the categories meet.
- **Calculate percents and use percent circles.**

- Express the number in each category as a fraction of the whole set.
- Convert each fraction to a decimal number and then to a percent.
- Use a circle divided into 100ths, or into 20ths, to represent intervals of 5% (see BLM 5–8.26: Percent Circle).
- Create sectors to represent the percent of each category.

- **Calculate percents and create central angles.**
  - Create a chart such as the one below.

Category	Quantity	Fraction of the Whole	Percent of the Whole	Percent Times 360° in the Circle	Size of the Central Angle

  - Draw a circle and one radius.
  - Use the radius to measure one of the central angles.
  - Use the subsequent radii to create successive central angles.

Mathematical Language

angle
circle graph
key
legend
percent
pie chart
sectors
sum
sum of the central angles

Assessing Prior Knowledge

Materials:

- grid paper
- markers
- rulers
- access to data sources (optional)

Organization: Whole class, individual or pairs

Procedure:

1. Use a class discussion to review the characteristics of graphs, including the visual display of data, descriptive titles, labelling of axes, scale, and plots.
2. Ask students to work individually or in pairs to collect data on some topic, and then display the data as a graph. Students may obtain data through surveys or observations, or they may research a topic (e.g., colour of clothes worn on a given day, movie, music, reading, or food preferences, language(s) spoken, number of siblings, populations, life spans).
3. Ask students to write questions that can be answered using the information in their graphs, and then have them write answers to these questions.
4. Post students' graphs, along with the accompanying questions and answers, around the room.
5. Have students participate in a Gallery Walk to view the displayed graphs. As a class, discuss the purpose and effectiveness of using graphs to display information about a topic.

Variations:

- Supply students with data or prepared graphs, rather than having them collect their own data.
- Supply prepared graphs and related questions for students to answer. Then discuss the characteristics and purposes of graphs.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Select, justify, and use appropriate methods of collecting data, including questionnaires, experiments, databases, and electronic media.
- ❒ Graph collected data and analyze the graph to solve problems.

Assessing Prior Knowledge

Materials:

- BLM 7.SP.3.1: Calculating the Percent of the Total
- index cards (optional)
- calculators (optional)

Organization: Individual or pairs, whole class

Procedure:

1. Review strategies for converting fractions to percents.
2. Distribute copies of BLM 7.SP.3.1: Calculating the Percent of the Total, and have students work individually or in pairs to find the percents presented in the scenarios.
3. When students have had sufficient time to respond to the questions, have them compare percents with a partner and resolve any discrepancies in their answers.
4. Reassemble as a class and discuss strategies students used to express portions of a whole as percents.

Variations:

- Have students create their own scenarios and questions regarding percents. Ask them to record the scenarios and questions on one side of an index card, and the solutions on the opposite side of the card. The cards may be used for drill games, learning activities, or Exit Slips.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially, and symbolically.

Assessing Prior Knowledge

Materials:

- BLM 7.SP.3.2: Percent of a Circle
- protractors
- calculators (optional)
- paper, compasses, and multi-sided number cubes or spinners (optional)

Organization: Individual, whole class

Procedure:

1. Review how to use a protractor to measure and draw angles.
2. Distribute copies of BLM 7.SP.3.2: Percent of a Circle, and have students, working individually, identify various percents of shaded circles, shade designated percents of circles, and draw angles to represent a percent of a circle.
3. Review and correct students' responses as a class, and discuss any questions that arise.

Variations:

- Provide students with additional practice in identifying various percents of shaded circles, shading various percents of circles, and drawing angles that correspond to a percent of a circle.
- Have students use an online computer game to identify the percent of a circle that has been shaded.

  Sample Website:

  Games are available on the following website:

  Scweb4free.com. *Circle Graphs Game*. 2009.
  www.scweb4free.com/circle.html.

  In this game, students view segmented circles and select a multiple-choice response to identify the percent of students who prefer hamburgers.
- Have students play a game in pairs, using multi-sided number cubes, paper, a compass, and protractors. Each student uses the compass or template to draw a circle, mark its centre, and draw a radius from the centre to the outside of the circle. The partners take turns rolling the number cubes. The product of the two numbers rolled equals the percent of the circle to shade. The percent $\times$ 360º indicates the size of angle to draw. Students shade each sector they draw. The first student to shade the entire circle wins.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially, and symbolically.
- ❒ Demonstrate an understanding of angles by drawing and labelling angles when the measure is specified.

Suggestions for Instruction

- **Identify common attributes of circle graphs, such as**
  - **title, label, or legend**
  - **the sum of the central angles is 360°**
  - **the data is reported as a percent of the total and the sum of the percents is equal to 100%**
- **Create and label a circle graph, with or without technology, to display a set of data.**
- **Interpret a circle graph to answer questions.**

Materials:

- a large open area with a marked centre (e.g., the centre of a basketball court, the pitcher's mound of a ball diamond)
- long cords or skipping ropes (about four)
- tape or pegs to hold down one end of the cords
- grid paper
- markers
- scissors
- rulers
- demonstration board

Organization: Whole class, individual

Procedure:

Part A

1. As a class, review the concept that graphs are visual ways to display data. Inform students that for this learning activity they will use different methods to create a graph called a circle graph. The circle graph enables them to divide a group into different categories, and allows them to compare the size of each category to each other and to the whole group.
2. Secure one end of the cords to the centre of a circle in an open area.
3. Ask students to line up in two categories, such as those who have pets, and those who do not. Record the categories and numbers in each category.
4. Have the lines follow their leader to form a circle around the centre point.
5. Have one student from where the two lines meet go to the centre of the circle and bring the end of one of the cords back to the circumference of the circle. Note how the circle has been divided into two sections, those who have pets, and those who do not.
6. Talk about which sector is smaller, and which is larger. Discuss whether most of the students have pets or whether most do not. Estimate the percent of the circle represented by each category. Discuss a descriptive title for the circle graph.
7. Repeat the procedure with other categories (e.g., favourite colours, number of siblings, ethnic backgrounds). The four cords accommodate four categories.
8. Stop the exercise after sufficient examples have been explored, and review what students learned about circle graphs.
9. Post the categories and numbers in each category for each of the graphs formed in Part A.

Part B

10. Demonstrate creating a circle graph for one set of data by colouring grids to represent each category, cutting the coloured grids into strips, taping the strips end to end, and then joining the ends to form a circle. Trace the circle, estimate the centre of the circle, and mark a point of the circumference where different colours meet. Use a ruler to connect the centre of the circle and the points on the circumference. Estimate the percent of the circle represented by each sector, record the percent, and label the sector. Title the graph. Use the data to write comparative statements about the categories and the whole set of data represented by the graph.
11. Have students, working individually, select one data set and then create their own circle graph and comparative statements for that data set. Post students' graphs.

Variation:

- As an alternative to using the open area, solicit questions from the class, record the data on the demonstration board, and have pairs of students act out scenarios using counters and circles (as described in the Background Information for learning outcome 7.SP.3).

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Identify common attributes of circle graphs, such as
  - title, label, or legend
  - the sum of the central angles is 360°
  - the data is reported as a percent of the total and the sum of the percents is equal to 100%
- ❒ Create and label a circle graph, with or without technology, to display a set of data.
- ❒ Interpret a circle graph to answer questions.

Suggestions for Instruction

- **Identify common attributes of circle graphs, such as**
  - **title, label, or legend**
  - **the sum of the central angles is 360°**
  - **the data is reported as a percent of the total and the sum of the percents is equal to 100%**
- **Create and label a circle graph, with or without technology, to display a set of data.**
- **Interpret a circle graph to answer questions.**

Materials:

- BLM 7.SP.3.3: Data Chart for Creating Circle Graphs
- BLM 5–8.26: Percent Circle
- fraction circles, available on the following website:
  Manitoba Education. "Middle Years Activities and Games." *Mathematics*.
  www.edu.gov.mb.ca/k12/cur/math/my_games/index.html.
- coloured counters (e.g., marbles, cubes, toy cars, or toy animals, in bags)
- markers or pencil crayons
- compasses

Organization: Whole class, individual or pairs

Procedure:

This learning experience will likely take more than one class and is divided into three parts. Use the same materials to create three graphs in Parts A to C.

Part A

1. As a class, review the characteristics and purposes of circle graphs.
2. Distribute fraction circles that have been divided into 10ths.
3. Have students, working individually or in pairs, randomly choose 10 items (or the number of divisions on the fraction circles) from a bag. Then ask students to do the following:
   a) Sort the items according to colour.
   b) Colour adjacent segments on a fraction circle to match the number of items of each colour.
   c) Draw bold lines to divide the colours and create sectors of each colour.
4. Ask students to label each sector with the applicable colour and the corresponding percent, or create a legend for the categories (e.g., if 6 of the 10 cubes selected were blue, then $\frac{6}{10}$ or 60% of the cubes were blue).
5. Have students write a title for their graph, as well as comparative statements relating to the graph.
6. Have students total the percents represented in each section of their graph, and record the totals. Ask students to make a general statement regarding the sum of the percents in each graph. Discuss why the sum is 100%. In the discussion, include the concept that each category is a part of the whole set and 100% represents the whole set.

Part B

7. Distribute copies of
   - BLM 7.SP.3.3: Data Chart for Creating Circle Graphs
   - fraction circles that have been divided into 20ths or 100ths (see BLM 5–8.26: Percent Circle)
8. Have students randomly select 5 to 30 coloured counters and sort them into colour groups. Students then complete the following process, using BLM 7.SP.3.3: Data Chart for Creating Circle Graphs:
   a) Record the colours in the Category column of the chart.
   b) Record the number of counters of each colour in the Quantity column.
   c) Calculate the total quantity.
   d) Write the quantity of each colour as a fraction of the total counters selected.
   e) Then convert that fraction to a percent.

   f) Add the percents. If it was necessary to round some of the percents, it is possible that they will not total 100%. If they do not total 100%, determine which percents were rounded up and which were rounded down and make adjustments to the most appropriate values. (The final two columns of the chart will be completed in Part C.)

9. Ask students to use the percents to create a circle graph using the percent circles. Each 100th mark represents 1% of the circle, or each 20th represents 5% of the circle.
10. Have students label the graph, including
    a) a title for the graph
    b) a label for each category (if there is insufficient room, a legend may be used instead of labels)
    c) the percent of the whole for each category
11. Have students write comparative statements related to the categories of the graph.

Part C

12. As a class, review how to use a protractor and how to draw angles of specific measures.
13. Demonstrate to students that
    - each sector of the circle represents an angle measure
    - there are 360º in a circle
14. Show students how to find the angle measure by finding a percent of 360º.
15. Have students calculate the angle measures and record them on BLM 7.SP.3.3: Data Chart for Creating Circle Graphs. Some of the angle measures may need to be rounded. When the angle measures are totalled, they may not equal 360º. If this is the case, review the rounding, and adjust the values up or down as necessary.
16. Ask students to draw a circle graph using the measures of the central angles for each category.
    a) Use a compass to draw a circle.
    b) Mark the middle of the circle.
    c) Draw one radius for the circle.
    d) Use the radius as a starting point to measure one angle.
    e) Use subsequent radii to create successive central angles.
17. Have students label the graph, including
    a) a title for the graph
    b) a label for each category (if there is insufficient room, a legend may be used instead of labels)
    c) the percent of the whole for each category
18. Have students write comparative statements related to the categories of the graph.
19. As a class, discuss the applications for using each method of creating a circle graph.

Variations:

- Supply data for each graph, rather than having students generate their own data.
- Have students use previously collected data, or collect their own data, to create circle graphs, using BLM 7.SP.3.3: Data Chart for Creating Circle Graphs (optional).
- Use technology to create circle graphs. For example, use graphing programs such as Graphical Analysis, spreadsheet programs such as Excel or Numbers, or online graphing programs.

  Sample Websites:

  Graphing programs are available on websites such as the following:

  Math Playground. "Circle Graphs." *Math Manipulatives.* 2010. www.mathplayground.com/piechart.html.

  U.S. Department of Education. Institute of Education Sciences, National Center for Education Statistics. "Create a Graph." *Kids' Zone, Learning with NCES.* http://nces.ed.gov/nceskids/createagraph/.

  Utah State University. "Data Analysis and Probability." *National Library of Virtual Manipulatives.* 1999–2010. http://nlvm.usu.edu/en/nav/topic_t_5.html.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Identify common attributes of circle graphs, such as
  - title, label, or legend
  - the sum of the central angles is 360°
  - the data is reported as a percent of the total and the sum of the percents is equal to 100%
- ❒ Create and label a circle graph, with or without technology, to display a set of data.
- ❒ Interpret a circle graph to answer questions.

Suggestions for Instruction

- **Identify common attributes of circle graphs, such as**
  - **title, label, or legend**
  - **the sum of the central angles is 360°**
  - **the data is reported as a percent of the total and the sum of the percents is equal to 100%**
- **Find and compare circle graphs in a variety of print and electronic media, such as newspapers, magazines, and the Internet.**
- **Translate percentages displayed in a circle graph into quantities to solve a problem.**
- **Interpret a circle graph to answer questions.**

Materials:

- BLM 7.SP.3.4: Comparing Examples of Circle Graphs
- BLM 7.SP.3.5: Translating Percentages in a Circle Graph into Quantities
- media sources (e.g., magazines, newspapers, advertisements, the Internet) for examples of circle graphs
- scissors
- glue or tape

Organization: Small groups

Procedure:

Part A

1. Divide the class into small groups. Have the students in each group search through various media sources to find five examples of circle graphs. Have them print or cut out the graphs, including any titles, legends, or captions that accompany the graphs.
2. Ask each group to analyze their selected graphs to determine whether or not each graph includes the common attributes of circle graphs.
3. Group members can take turns recording information about each graph on the chart provided on BLM 7.SP.3.4: Comparing Examples of Circle Graphs.
4. Have the groups analyze their graphs and pose two or three questions that can be answered using information from their graphs. They record their questions on the first page of BLM 7.SP.3.5: Translating Percentages in a Circle Graph into Quantities. Students then prepare answers to their questions and record them on the second page of the BLM.

Part B

5. Present to the class a graph prepared by one of the groups. Demonstrate how the percentages in the graph could be used to solve a problem.

   Example:

   > A graph about teenage music preferences shows the percentage of students who prefer different musical artists. A music store catering to teenagers could use this information to choose which products to stock in its store. If 62% of the students prefer artist B, and the store is spending $9500 on inventory this month, how much money should the store spend purchasing artist B's music?

6. Have students work together to prepare problems that could be solved using their graphs. Ask them to record the problems on the first page of BLM 7.SP.3.5: Translating Percentages in a Circle Graph into Quantities. Have students prepare solutions to their problems and record them on the second page of the BLM.
7. Post the completed pages for a Gallery Walk. Have students solve their classmates' problems and verify their solutions.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❐ Identify common attributes of circle graphs, such as
  - title, label, or legend
  - the sum of the central angles is 360°
  - the data is reported as a percent of the total and the sum of the percents is equal to 100%
- ❐ Find and compare circle graphs in a variety of print and electronic media, such as newspapers, magazines, and the Internet.
- ❐ Translate percentages displayed in a circle graph into quantities to solve a problem.
- ❐ Interpret a circle graph to answer questions.

Suggestions for Instruction

- **Create and label a circle graph, with or without technology, to display a set of data.**
- **Translate percentages displayed in a circle graph into quantities to solve a problem.**

Materials:

- BLM 5–8.26: Percent Circle
- fraction circles, available on the following website:
  Manitoba Education. "Middle Years Activities and Games." *Mathematics*.
  www.edu.gov.mb.ca/k12/cur/math/my_games/index.html.
- a variety of packages (of various sizes) containing coloured items (e.g., coloured beads, candy)
- compasses
- protractors
- BLM 7.SP.3.3: Data Chart for Creating Circle Graphs (optional)

Organization: Small groups

Procedure:

1. Have students, working in small groups, sort the contents of the supplied packages into various colours and use the information to create a circle graph. Students may use BLM 7.SP.3.3: Data Chart for Creating Circle Graphs to help organize their answers. They may create their graph with a compass and a protractor, or using fraction circles or percent circles provided.
2. Have students propose a question to be solved using the information from their graph. For example, if we buy 2000 candies, how many can we expect to be red?
3. Ask each group to exchange their graph with that of another group, and solve the new problem.
4. After solving the problem, students return it to its originators and discuss any discrepancies.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Create and label a circle graph, with or without technology, to display a set of data.
- ❒ Translate percentages displayed in a circle graph into quantities to solve a problem.

Putting the Pieces Together

Geometric Flags

Introduction:

Students use information from a group survey to create a flag representative of the group. Then they calculate the area of the spaces in the flag.

Purpose:

In this investigation, students will demonstrate the ability to do the following (connections to learning outcomes are identified in parentheses):

- Solve problems involving percents. (7.N.3)
- Construct circle graphs to solve problems. (7.SP.3)
- Perform geometric constructions, including parallel and perpendicular line segments and perpendicular and angle bisectors. (7.SS.3)
- Perform and describe transformations. (7.SS.5)
- Apply a formula for determining the area of triangles, parallelograms, and circles. (7.SS.2)

Students will also demonstrate the following mathematical processes:

- Communication
- Mental Mathematics and Estimation
- Problem Solving
- Reasoning
- Technology

Materials/Resources:

- ruler
- compass
- protractor
- right triangle
- coordinate grid
- paper (for the flags)
- art supplies
- calculator (optional)
- Mira (optional)
- tracing paper (optional)

Organization: Small groups, individual

Procedure:

Student Directions

1. Survey a group of people regarding their preferred colour. You may survey groups within your class, in different classes, in your family, and so on. If each person in the group chooses a different colour, you may wish to have the survey participants select from a list of three to five colours. You may have them rank the colours as first, second, and third choice.
2. Calculate the percent of the group that prefers each colour.
3. Construct a circle graph to represent the preferences.
4. Design a flag for the group using the group's preferred colours.
   a) Calculate the area of the flag to be covered with each preferred colour.
   b) Create your design using circles, triangles, parallelograms, and transformations. Include parallel and perpendicular line segments and perpendicular and angle bisectors.
   c) Adjust the size of each shape to match the area to be covered with each preferred colour.
5. Create a summary chart that shows the areas of the different shapes in each preferred colour, including
   a) the total area representing each colour
   b) the percent of the total area covered by each colour
   c) a circle graph that represents the percent of the area covered by each colour
6. Create a final copy of your flag.
7. Prepare a presentation about your flag. In the presentation, prove that your flag represents the group findings because the area covered by each colour in the flag is the same as the percent of the group that preferred each colour. Highlight some features of your flag and how you solved a problem in creating the flag.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Calculate percents.
- ❒ Construct circle graphs.
- ❒ Estimate and calculate areas of triangles, parallelograms, and circles.
- ❒ Solve a problem involving the percent of an area.
- ❒ Perform and describe a transformation in the flag.
- ❒ Construct and identify parallel and perpendicular line segments in the flag.
- ❒ Construct and identify perpendicular and angle bisectors in the flag.
- ❒ Communicate mathematical ideas effectively.

Statistics and Probability (Chance and Uncertainty) (7.SP.4, 7.SP.5, 7.SP.6)

Enduring Understanding(s):

Percents, fractions, decimals, and ratios are different representations of the same quantity.

The principles of the probability of a single event apply to the probability of independent events.

General Learning Outcome(s):

Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Learning Outcome(s):	Achievement Indicators:
7.SP.4 Express probabilities as ratios, fractions, and percents. [C, CN, R, T, V]	➛ Determine the probability of an outcome occurring for a probability experiment, and express it as a ratio, fraction, or percent. ➛ Provide an example of an event with a probability of 0 or 0% (impossible) and an event with a probability of 1 or 100% (certain).
7.SP.5 Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events. [C, ME, PS]	➛ Provide an example of two independent events, such as ▪ spinning a four-section spinner and an eight-sided die ▪ tossing a coin and rolling a twelve-sided die ▪ tossing two coins ▪ rolling two dice and explain why they are independent. ➛ Identify the sample space (all possible outcomes) for an experiment involving two independent events using a tree diagram, a table, or another graphic organizer.

(continued)

Specific Learning Outcome(s):	Achievement Indicators:
7.SP.6 Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, a table, or another graphic organizer) and experimental probability of two independent events. [C, PS, R, T]	➢ Determine the theoretical probability of an outcome for an experiment involving two independent events. ➢ Conduct a probability experiment for an outcome involving two independent events, with or without technology, to compare the experimental probability to the theoretical probability. ➢ Solve a probability problem involving two independent events.

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of fractions less than or equal to one by using concrete and pictorial representations to
  - name and record fractions for the parts of a whole or a set
  - compare and order fractions
  - model and explain that for different wholes, two identical fractions may not represent the same quantity
  - provide examples of where fractions are used
- Describing and representing decimals (tenths and hundredths) concretely, pictorially, and symbolically.
- Demonstrating an understanding of fractions by using concrete and pictorial representations to
  - create sets of equivalent fractions
  - compare fractions with like and unlike denominators
- Describing and representing decimals (tenths, hundredths, thousandths) concretely, pictorially, and symbolically.
- Relating decimals to fractions (tenths, hundredths, thousandths).
- Comparing and ordering decimals (tenths, hundredths, thousandths) by using
  - benchmarks
  - place value
  - equivalent decimals

- Describing the likelihood of a single outcome occurring, using words such as
  - impossible
  - possible
  - certain
- Comparing the likelihood of two possible outcomes occurring, using words such as
  - less likely
  - equally likely
  - more likely
- Demonstrating an understanding of ratio, concretely, pictorially, and symbolically.
- Demonstrating an understanding of percent (limited to whole numbers) concretely, pictorially, and symbolically.
- Demonstrating an understanding of multiplication and division of decimals involving
  - 1-digit whole-number multipliers
  - 1-digit natural number divisors
  - multipliers and divisors that are multiples of 10
- Demonstrating an understanding of probability by
  - identifying all possible outcomes of a probability experiment
  - differentiating between experimental and theoretical probability
  - determining the theoretical probability of outcomes in a probability experiment
  - determining the experimental probability of outcomes in a probability experiment
  - comparing experimental results with the theoretical probability for an experiment

For more information on prior knowledge, refer to the following resource:

Manitoba Education and Advanced Learning. *Glance Across the Grades: Kindergarten to Grade 9 Mathematics*. Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/cur/math/glance_k-9/index.html.

Related Knowledge

Students should be introduced to the following:

- Solving problems involving percents from 1% to 100%.
- Demonstrating an understanding of the relationship between repeating decimals and fractions, and terminating decimals and fractions.

- Comparing and ordering fractions, decimals (to thousandths), and integers by using
  - benchmarks
  - place value
  - equivalent fractions and/or decimals
- Demonstrating an understanding of central tendency and range by
  - determining the measures of central tendency (mean, median, mode) and range
  - determining the most appropriate measures of central tendency to report findings
- Determining the effect on the mean, median, and mode when an outlier is included in a data set.
- Constructing, labelling, and interpreting circle graphs to solve problems.

Note:
When working with the probability of two independent events, keep in mind that Grade 7 students have not yet had exposure to multiplication of fractions, but may have had exposure to multiplication of decimals.

Background Information

In our society, probability is used in making weather forecasts to express the likelihood of precipitation, in making medical inferences such as the likelihood of contracting an infection or a disease, in making correlations between lifestyle habits and health, in predicting election results, in determining the chances of winning a lottery or a draw, in understanding whether or not a game is fair, and so on.

The study of probability begins in Grade 5, with students describing the likelihood of a single event occurring and comparing the likelihood of two possible outcomes using the language of probability. Unless an event is the only possible outcome, or unless it is impossible for an outcome to occur, one can never be certain of an outcome, or of the number of times an outcome will occur in a given number of trials. Thus, in Grade 6, students determine and compare theoretical probability and experimental results.

Experimental Probability

Experimental probability describes what actually did happen in a real situation. Sometimes experimental probability is called *relative frequency*.

$$\text{Experimental Probability of an Event} = \frac{\text{Number of Observed Favourable Outcomes*}}{\text{Total Number of Trials}}$$

*A favourable outcome is the outcome that the experimenter is looking for.

If, for example, an experimenter was examining the probability of rolling a 4 when rolling a regular number cube, and rolled the number cube 50 times, and 10 of those times he or she rolled 4s, the experimental probability would be $\frac{10}{50}$ or $\frac{1}{5}$ or 0.2 or 20%.

Probability is a generalized statement used to predict future events. A generalization cannot be trusted for making predictions if it is based on a small number of trials; however, students can increase the number of trials in their experiments by combining trials conducted by different students, or by using computer applets for generating large numbers of number cube rolls, coin tosses, or spins.

Sample Websites:

Some applets are available on the following websites:

Shodor. "Experimental Probability." *Interactive*. 1994–2011. www.shodor.org/interactivate/activities/ExpProbability/.

Select the spinner, or the type of die, and the number of trials desired. The simulator tallies and calculates results.

Utah State University. "Data Analysis and Probability." *National Library of Virtual Manipulatives.* 1999–2010. http://nlvm.usu.edu/en/nav/topic_t_5.html.

In the Grades 6 to 8 section, select Coin Tossing or Spinners.

A larger number of trials will permit students to make a generalization in which they can have confidence. The larger the sample size is, the more similar the experimental results and the theoretical probability will be.

Theoretical Probability

Theoretical probability helps students to predict what is likely to happen in a given circumstance, but does not foretell what will happen for sure. To calculate the theoretical probability of an event, the events must occur randomly, not influenced by any outside force, and the events must be equally likely, or have the same chance of occurring.

$$\text{Theoretical Probability of an Event} = \frac{\text{Number of Outcomes Favourable* to the Event}}{\text{Total Number of Possible Equally Likely Outcomes}}$$

*A favourable outcome is the desired outcome.

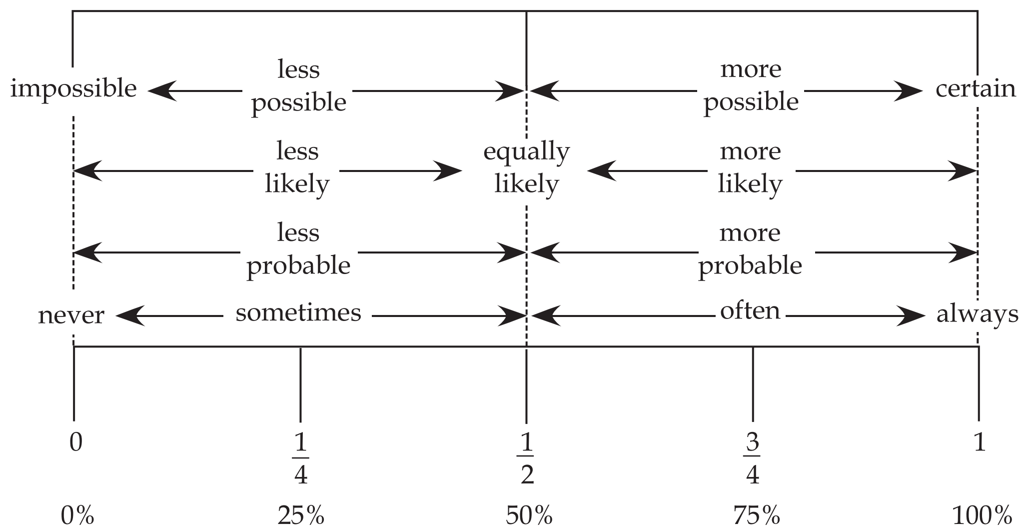
If, for example, an experimenter was examining the probability of rolling a 4 when rolling a regular number cube, he or she would know the number of favourable outcomes is 1, and the number of equally likely outcomes is 6, so the probability is $\frac{1}{6}$ or $0.1\overline{6}$ or $16.\overline{6}\%$.

Theoretical probability may be used to make predictions about future events, when events are equally likely. If the events are not equally likely, as when tossing an object and predicting whether it will land right-side up, upside down, or on its side, *experimental probability* may be used to make future predictions.

In Grade 7, students express probability as ratios, fractions, or percents. Probability ranges between impossible 0% and certain 100% (between $\frac{0}{1}$ and $\frac{1}{1}$ or between 0 and 1). A probability line with benchmarks may be used to illustrate the equivalent expressions.

Example:

Sample Probability Line



$$P = \frac{\text{Number of Favourable Outcomes}}{\text{Number of Possible Outcomes}}$$

Probability Investigations and Problems

Grade 7 students extend investigations of theoretical and experimental probability to include two independent events. They also solve probability problems involving two independent events.

To investigate probability with experiments, students will need access to devices that generate random results. These may include spinners, items to toss (e.g., coins, bicoloured tiles, number cubes, tacks), and items to draw from (e.g., cards from a deck, coloured blocks, tiles, marbles from a bag). Computer-simulated applets may also be used.

Sometimes probability problems involve situations that may be too dangerous, too expensive, or too difficult to experiment with. In these circumstances, a simulation can be chosen to mimic the experiment. For example, in a situation that requires investigating the possibility of whether a birth is male or female, the two outcomes can be represented with the two sides of a coin.

Organizing Outcomes of Probability Investigations

Students may begin studying the probability of two independent events with a concrete investigation, such as predicting the outcome of tossing two coins. The obvious outcomes are two heads, two tails, and one of each. Some students may conclude that there is a likelihood of $\frac{1}{3}$ for each combination. An investigation will generate different results, and the difference may interest students in organizing possible outcomes in a systematic way.

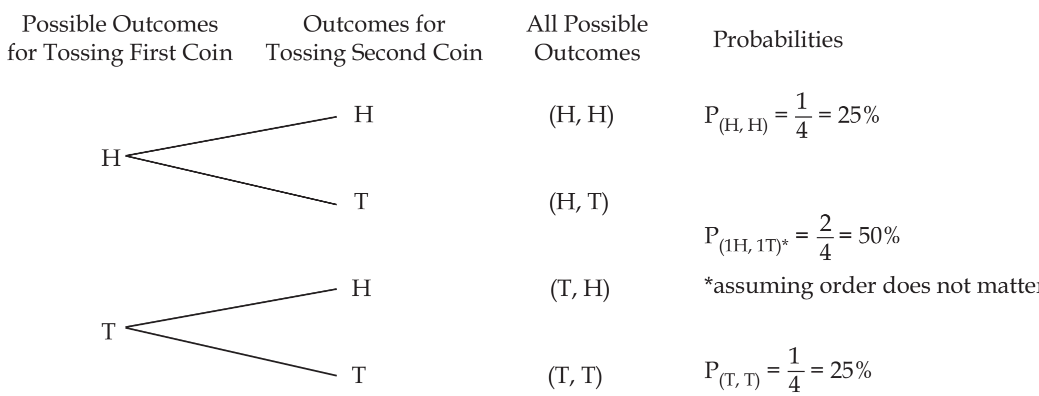
Outcomes of probability investigations can be organized with the use of a tree diagram and a frequency table or chart:

- **Tree Diagram:** A *tree diagram* lists the *possible outcomes* for each event in two columns and connects them with lines to form branches. The first column (or row) lists all possible outcomes for the first event. The second column (or row) lists all outcomes for the second event beside each of the outcomes in the first event. A tree diagram is used to describe theoretical probability.

  Example:

  Below is an example of a horizontal tree diagram of the possible outcomes for the independent events when tossing two coins.

Tree Diagram for Tossing Coins



$$\text{The Probability of Any Event} = \frac{\text{Number of Favourable Outcomes}}{\text{Number of Possible Outcomes}}$$

$$\text{e.g., } P_{(T, T)} = \frac{1}{4}$$

Note that the probabilities for the three outcomes listed above are not $\frac{1}{3}$ each, but rather $\frac{1}{4}$, $\frac{1}{4}$, and $\frac{1}{2}$ respectively.

- **Frequency table or chart:** When used to predict possible outcomes of an event, a *frequency table* or *chart* organizes all possible outcomes for two independent events. Each cell in the table indicates one possible outcome.

  Example:

First Coin Toss

Second Coin Toss	H	T
H	H, H	T, H
T	H, T	T, T

$$\text{The Probability of Any Event} = \frac{\text{Number of Favourable Outcomes}}{\text{Number of Possible Outcomes}}$$

$$\text{e.g., } P_{(1H,\ 1T)^*} = \frac{2}{4} = 50\% \qquad \text{* assuming order does not matter}$$

Organizing outcomes helps to reveal hidden outcomes, as in the examples above, where students may erroneously believe there are only three possible outcomes: 2 heads, 2 tails, and one of each.

To demonstrate a thorough understanding of probability, students should recognize the following:

- Outcomes must be equally likely. For example, a spinner that is $\frac{1}{2}$ red, $\frac{1}{4}$ yellow, and $\frac{1}{4}$ blue has $\frac{2}{4}$ possibilities for red.
- The probability of an outcome is equivalent when tossing the same number cube six times or when tossing six number cubes one time each. Or, if there are six red marbles and six white marbles in a bag, the chance of pulling either colour is the same as if there is one red marble and one white marble in the bag.
- For independent events, the probability of an outcome does not change based on the previous outcome. When an event such as rolling a sum of 7 on two number cubes has repeated itself several times, there is no increased likelihood that the next roll will or will not be a sum of 7. The likelihood of rolling or not rolling a sum of 7 remains the same, regardless of the outcome in the previous roll.
- Knowing the vocabulary terms is important. For example, *likely* means more than 50% of the time, not almost all the time.

MATHEMATICAL LANGUAGE

certain event
dependent event
event
experimental probability
favourable outcome
frequency table or chart
impossible event
independent event
likely
outcome
possible outcome
probability
random
relative frequency
sample size
sample space
theoretical probability
tree diagram

Assessing Prior Knowledge

Materials:

- a set of two multi-sided number cubes or spinners per group
- calculators (optional)
- BLM 7.SP.4.1: Recording Sheet for Fraction–Decimal–Percent Equivalents (optional)
- paper (optional)

Organization: Whole class, small groups (of three or five)

Procedure:

1. As a class, review writing equivalent fractions, decimals, and percents.
2. Organize the class into groups, and have each group designate a Person A, B, and C.
3. Demonstrate a few rounds of the game using three volunteers. The aim is to move as quickly and accurately as possible to create 10 equivalent fractions, decimals, and percents, using the following routine:
   a) **Person A:** Roll two number cubes.
   b) **Person B:** Use the numbers on the number cube to create a proper fraction.
   c) **Person C:** Express the fraction as an equivalent decimal number.
   d) **Person A:** Express the number as an equivalent percent.
   e) **Person B:** Roll two number cubes. The routine continues.
   f) Use BLM 7.SP.4.1: Recording Sheet for Fraction–Decimal–Percent Equivalents to record results. Person A begins the sheet after rolling the number cube. Person A records what Person B says, and then passes the sheet to Person B. Person B records what person C says, and passes the sheet to person C. The routine continues, with the sheet following behind the person who is answering.
4. Groups carry on with the game, racing to see who can be the first group to create 10 equivalent fractions, decimals, and percents correctly.
5. Play as many rounds as seems interesting and useful.

Variation:

- Play the game as a baseball game. The pitcher "pitches" two numbers. The batter gets to first base by naming the proper fraction, to second base by naming the decimal, and to third base by naming the percent. By finishing within a time limit, the batter gets to home base and earns a run. Failure to respond in a given time results in an out. The game can be played as a whole class or in pairs, using a paper baseball diamond and markers.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Name part-to-whole ratios as fractions and their decimal and percent equivalents.

Suggestions for Instruction

- **Determine the probability of an outcome occurring for a probability experiment, and express it as a ratio, fraction, or percent.**
- **Provide an example of an event with a probability of 0 or 0% (impossible) and an event with a probability of 1 or 100% (certain).**

Materials:

- BLM 7.SP.4.2: What Is the Probability?
- a bag of letter tiles (B, R, I, D, G, E) (optional)

Organization: Individual, whole class

Procedure:

1. Distribute copies of BLM 7.SP.4.2: What Is the Probability?
2. Have students answer the questions on the sheet on their own. The questions require students to
   a) identify outcomes and probabilities
   b) compare experimental and theoretical probability
   c) identify outcomes as impossible, certain, or more likely
   d) create and answer their own question related to probability
3. Reassemble as a class and have students share their responses to the questions, correcting any errors.

Variations:

- As a class, complete BLM 7.SP.4.2: What Is the Probability? and discuss any questions students may have.
- Have students work in small groups. Provide each group with the bag of letter tiles (B, R, I, D, G, E), and have them investigate the probability of drawing each letter in 36 trials. Combine the results of the small groups, recalculate the experimental probability, and compare the probability for the combined results to the results of the individual groups.
- Ask students to discuss why the experimental results and the theoretical probability are different from each other.
- Have students, working in pairs, choose their own letter tiles (or other devices) and prepare a similar question sheet and answer key. Groups exchange sheets, complete and correct the questions, and resolve any discrepancies between the responses.
- Have students design spinners, or situations, that would result in given probabilities.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Describe single events as impossible, possible, certain, less likely, more likely, or equally likely.
- ❒ Identify possible outcomes in a probability scenario.
- ❒ Differentiate between experimental and theoretical probability.
- ❒ Determine theoretical and experimental probability.
- ❒ Compare experimental results with theoretical probability.

Suggestions for Instruction

- **Determine the probability of an outcome occurring for a probability experiment, and express it as a ratio, fraction, or percent.**
- **Provide an example of an event with a probability of 0 or 0% (impossible) and an event with a probability of 1 or 100% (certain).**

Materials:

One set per pair of students:

- six-sided pencils
- notebooks
- markers
- BLM 7.SP.4.3: Experimental Probability Tally Sheet and Probability of Outcomes (optional)

Organization: Pairs, whole class

Procedure:

1. Have students, working in pairs, mark each side of a six-sided pencil with one possible outcome (e.g., numbers, student names, letters to form a word).
2. Students conduct their probability experiment, proceeding as follows:
   a) One student rolls the pencil by rubbing it between his or her palms, and then lays a hand on a notebook (to muffle the sound) and allows the pencil to roll down the hand onto the notebook.
   b) The student announces the outcome that landed facing up.
   c) The other student records the result on a tally sheet.
   d) Students continue rolling and recording until they have completed a set number of rolls, or until a set time has passed.
3. Students total their tally marks, and record the ratios of tallies for each outcome to the total number of tallies (outcome : total).
4. Students then record the probability for each outcome, expressing it as a fraction, a decimal, and a percent.
5. When groups have completed their experiment, reassemble as a class and ask students what they have discovered or learned from the experience.
   a) Compare the probabilities obtained by different groups. If students have used the same outcomes, consolidate the group data and calculate the probability for the combined results.
   b) Students may note the sum of the values in the decimal column totals about 1 (depending on rounding), and the sum of the percents is close to 100% (depending on rounding).

c) Ask students for an example of an outcome that is impossible for this experiment, and for an event that is certain.

Variations:

- Have all students mark their six-sided pencils with the same outcomes, and combine the class results to represent a larger trial.
- Events that are not equally likely, such as a paper cup landing upright, upside down, or on its side, may be used for this experiment.
- Compare students' results and discuss differences observed.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Determine the probability of an outcome occurring for a probability experiment, and express it as a ratio, fraction, or percent.
- ❒ Provide an example of an event with a probability of 0 or 0% (impossible) and an event with a probability of 1 or 100% (certain).

Suggestions for Instruction

- **Provide an example of an event with a probability of 0 or 0% (impossible) and an event with a probability of 1 or 100% (certain).**

Materials:

- rulers (30 cm)
- two colours of pens or highlighters
- demonstration board

Organization: Whole class, pairs

Procedure:

1. Remind students that probability is used to predict the likelihood that an event will happen. Ask whether it is possible to predict any event with certainty.

   Answer:

   The only events that can be predicted with certainty are those with probabilities of 0% or 100%.

2. Ask students for examples of events with a probability of 0% (impossible) or 100% (certain).

   Example:

   If a draw box contains only odd numbers, the probability of drawing an even number is 0% and the probability of drawing an odd number is 100%.

3. Have students create a probability line similar to the Sample Probability Line illustrated in the Background Information.

   Have students include some or all of the following vocabulary in their probability line:

   - impossible, less possible, more possible, certain
   - less likely, equally likely, more likely
   - less probable, more probable
   - never, sometimes, often, always

4. Have pairs of students challenge one another by describing situations that could match the probability of a point or region on the probability line.

   Example:

   - Student A describes a situation, and asks for an event that would match a probability descriptor: Given a bag containing the letters of the alphabet, identify an event that would be less likely than another event.
   - Student B replies: Given a bag containing the letters of the alphabet, it is less likely you'll draw a vowel than a consonant.
   - Student B then challenges Student A, and may ask for an impossible event in the same situation.
   - Student A could reply: Given the letters of the alphabet, it is impossible to draw a number.

Variations:

- Provide a template of a probability line with benchmarks, directional arrows, and blanks for students to fill in.
- As an extension, have students identify situations with events that have a 50% probability (or other value) of occurring.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Provide an example of an event with a probability of 0 or 0% (impossible) and an event with a probability of 1 or 100% (certain).

Suggestions for Instruction

- **Provide an example of two independent events, such as**
  - **spinning a four-section spinner and an eight-sided die**
  - **tossing a coin and rolling a twelve-sided die**
  - **tossing two coins**
  - **rolling two dice**

  and explain why they are independent.

Materials:

- BLM 7.SP.5.1: Which Conditions Affect Probability?
- coins
- colour counters
- bags
- ticket stubs with numbers (optional)
- BLM 7.SP.5.2: Examples of Two Independent Events (optional)

Organization: Small groups (of three or four students), whole class

Procedure:

1. Divide students into small groups.
2. Distribute copies of BLM 7.SP.5.1: Which Conditions Affect Probability?
3. Have each group determine the theoretical probability of each event occurring, decide whether specified conditions affect the theoretical probability of the event, and explain why or why not.
4. Reassemble as a class and discuss students' responses. Probability is based on the number of favourable outcomes and the number of possible outcomes. Only conditions that alter the number of outcomes (e.g., adding an extra item, not replacing an item that has been removed) will affect the theoretical probability of an event.

5. Inform students of the following:
   a) When one event does not affect the probability of an outcome of another event, the events are said to be *independent events*. Ask students to identify the independent events in the examples on BLM 7.SP.5.1.
   b) When one event does affect the probability of an outcome of another event, the events are said to be *dependent events*. Ask students to identify the dependent events in the examples on BLM 7.SP.5.1. Have students identify other events that would not represent independent events.
6. Ask students whether rolling a number cube before pulling a coloured block from a bag will have any effect on the colour of the block drawn.
7. Have students return to their small groups and list pairs of independent events, explaining why the two events are independent. Ask them to list some pairs of events that are not representative of independent events.

   Examples:

   Independent events could include
   - choosing a card from a deck (or a marble, other counter, number, or letter tile from a bag), returning it to the deck (or bag), and drawing a second item
   - tossing two coins, number cubes, or letter cubes
   - randomly choosing two items (e.g., names, numbers, menu items, clothing selections, colours, movies, transportation methods, vacation spots, games) from a selection (The first choice must not be removed from the set, for the events to remain independent.)
   - any combination of the above items

Variations:

- Provide concrete opportunities for students to experiment with determining whether or not probability is affected by certain conditions. For example, collect tickets or names on slips and conduct a draw to determine the probability of a particular winner. Conduct the experiment, first replacing a name each time it is drawn, and then not replacing a name after it has been drawn.
- Provide a chart (such as BLM 7.SP.5.2: Examples of Two Independent Events) for students to list sets of two independent events and to explain why the events are independent.
- Provide students with a list of pairs of events and ask them to identify whether or not the pairs represent independent events, and to explain why the events are or are not independent.

Observation Checklist

☑ Listen to and observe students’ responses to determine whether students can do the following:

- ❒ Provide an example of two independent events, such as
  - spinning a four-section spinner and an eight-sided die
  - tossing a coin and rolling a twelve-sided die
  - tossing two coins
  - rolling two dice

  and explain why they are independent.

Suggestions for Instruction

- **Identify the sample space (all possible outcomes) for an experiment involving two independent events using a tree diagram, a table, or another graphic organizer.**

Materials:

- coins (the same or different denominations)
- demonstration board
- math journals
- BLM 7.SP.4.3: Experimental Probability Tally Sheet and Probability of Outcomes (optional)

Organization: Pairs or small groups, whole class, individual

Procedure:

1. Make preparations for having students investigate the probability of outcomes when tossing two coins.
   a) Divide students into pairs or small groups.
   b) Inform students they will be experimenting to determine the possible outcomes, and the probability of each outcome, when tossing two coins. Have students create a recording sheet for their experiment (or distribute copies of BLM 7.SP.4.3: Experimental Probability Tally Sheet and Probability of Outcomes).
   c) Tell students they will identify all possible outcomes and predict the probability of each outcome.

d) Discuss whether the procedure for tossing the coins (tossing both coins at once, or tossing them one after the other) will affect the results of the experiment.

Neither will affect the probability; however, good experimental technique ensures that the same procedure is followed as closely as possible throughout an experiment.

2. Have students conduct the investigation in pairs or in small groups.
   a) Distribute two coins to each group.
   b) Have students conduct their experiment for a given number of coin tosses, or for a set period of time.
   c) Have students total their results.
   d) Consolidate students' data on the demonstration board.
3. Compare students' results to their predictions, and discuss differences or similarities.

   If students predicted a probability of $\frac{1}{3}$ for each outcome, the results will likely not support that prediction. This raises an opportunity for discussing how to determine the sample space (number of possible outcomes) for two independent events.
4. Calculate probability as the number of favourable outcomes out of the number of possible outcomes (the sample space).

$$P = \frac{\text{Number of Favourable Outcomes}}{\text{Number of Possible Outcomes}}$$

   a) Agree that the favourable outcomes are two heads, two tails, or one of each.
   b) Ask students to identify all possible outcomes.
5. Ask students to demonstrate, or demonstrate for them, systematic ways to organize the sample space for two independent events in a way that ensures all possible outcomes are identified. If students do not see the benefits of organizational charts, have them identify the sample space for events with multiple outcomes, such as rolling two number cubes.
6. Organizers include the following (see Background Information for examples):
   a) tree diagrams
   b) frequency charts or tables
   c) organized lists
7. Have students make a math journal entry illustrating methods to identify the sample space (all possible outcomes) for two independent events.

Variations:

- Use computer applets to simulate coin tosses.
- Replace the introductory investigation with a guided example of tossing two coins as two independent events. Demonstrate calculating the sample space with a tree

diagram, and then with a frequency chart or table, as illustrated in the Background Information.

- Demonstrate using organizers for identifying the sample space for an experiment involving two independent events, and then have students identify such an experiment and use two methods to identify the corresponding sample space. Students may use events from the list they created in the previous learning experience.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Identify the sample space (all possible outcomes) for an experiment involving two independent events using a tree diagram, a table, or another graphic organizer.

Suggestions for Instruction

- **Determine the theoretical probability of an outcome for an experiment involving two independent events.**
- **Conduct a probability experiment for an outcome involving two independent events, with or without technology, to compare the experimental probability to the theoretical probability.**

Materials:

- manipulatives (e.g., coins, number cubes, colour counters, marbles, letter tiles, bags)
- software programs or online applets (such as those listed in the Background Information)
- BLM 7.SP.6.1: Frequency Chart for Organizing Outcomes for Two Independent Events (optional)

Organization: Individual or pairs

Procedure:

1. Inform students they will design and conduct a probability experiment involving two independent events, as outlined in the following steps.
   a) Choose two independent events. Previous lists provide examples of events to choose from.
   b) Determine the sample space (all possible outcomes) by drawing a tree diagram, a frequency chart, or another organizer. BLM 7.SP.6.1: Frequency Chart for Organizing Outcomes for Two Independent Events may be provided as a template.

c) Determine the theoretical probability of an outcome for the experiment.

d) Conduct the experiment using manipulatives or computer applets or software programs.

e) Determine the experimental probability for the outcome.

f) Compare the experimental probability to the theoretical probability. Include descriptive statements and numerical statements in the comparisons. Propose an explanation for variations in the results.

2. Have students present their investigations to each other, or post the investigations for students to view.

Variations:

- Assign students two independent events to use for the experiment.
- Have all students work on the same investigation, and compile their results for comparing the theoretical and experimental probability.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Determine the theoretical probability of an outcome for an experiment involving two independent events.
- ❒ Conduct a probability experiment for an outcome involving two independent events, with or without technology, to compare the experimental probability to the theoretical probability.

Suggestions for Instruction

- **Solve a probability problem involving two independent events.**

Materials:

- demonstration board
- display board

Organization: Whole class, pairs

Procedure:

1. Identify games of chance that involve two independent events, such as the following:
   - Rock, Paper, Scissors (played with two people)
   - games that involve rolling number cubes (e.g., Lucky Seven)
   - games that involve drawing marbles from a bag (e.g., Player A draws a marble from a bag containing a given assortment of marbles, and returns the marble to the bag. Player B then draws a marble. If the colours match, Player B scores a point; if they don't match, Player A scores the point.)
2. Ask whether a game is fair, and how one judges whether or not it is fair.
3. As a class, investigate a game of chance, such as Rock, Paper, Scissors, to determine whether or not the game is fair.
4. Have individuals or pairs of students determine whether or not other games of chance are fair. Have them provide evidence for their conclusions.
5. Provide opportunities for students to play the games.
6. Use students' results to create a display entitled Would You Play This Game?

Variations:

- Have pairs of students play a game based on the probability of two independent events occurring. After a certain time has passed, ask students who thinks the game is fair. Challenge students to support their opinion using what they know about probability.
- Have students design a fair game based on the probability of two independent events occurring. Have them provide evidence that the game is fair. Host a Game Fair in which students introduce and play their games with others.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Solve a probability problem involving two independent events.

- **Solve a probability problem involving two independent events.**

Materials:

- BLM.7.SP.6.2: Probability Problems Involving Two Independent Events
- file cards (optional)

Organization: Individual, pairs

Procedure:

1. Present students with probability problems involving two independent events, such as those on BLM.7.SP.6.2: Probability Problems Involving Two Independent Events.
2. Ask students to complete the problems independently and then compare their responses with those of a classmate, resolving any discrepancies.

Variations:

- Ask students to design their own probability problems and solutions. Have students consolidate problems as an assignment sheet and an answer key, and share question sheets with others. Students compare their responses to the answer key, and discuss any discrepancies with the authors.
- Place student-created probability problems and solutions on large file cards. Students can pick a card or two, and complete the problems as part of a learning activity centre or as an Exit Slip.
- Play online probability games requiring players to solve probability problems involving both simple and independent events.

  Sample Website:

  For a sample probability game, refer to the following website:

  Math-Play.com. *Probability Game.* 2007.
  www.math-play.com/Probability-Game.html.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Solve a probability problem involving two independent events.

Defining the Average Potato

Introduction:

Students collect data, determine measures of central tendency, consider the range and the effect of outliers, and analyze circle graphs in order to define the average potato.

Purpose:

In this investigation, students will demonstrate the ability to do the following (connections to learning outcomes are identified in parentheses):

- Determine measures of central tendency (mean, median, and mode) and range. (7.SP.1)
- Determine the most appropriate measures of central tendency to report findings. (7.SP.1)
- Determine the effect on the mean, median, and mode when an outlier is included in the data. (7.SP.2)
- Construct circle graphs to solve problems. (7.SP.3)
- Express probabilities as ratios, fractions, and percents. (7.SP.4)

Students will also demonstrate the following mathematical processes:

- Communication
- Connections
- Mental Mathematics and Estimation
- Problem Solving
- Reasoning
- Technology

Materials/Resources:

- Each student will require
  - a potato
  - a data sheet
  - a way to label each potato (e.g., adhesive tape)
- Each group will require
  - a tape measure or string
  - a ruler (cm)
  - a mass scale

- a setup for measuring volume (e.g., water, a basin, a container with a wide mouth filled to the brim that sits inside another container with a pour spout, a measuring cup to measure the overflow, towels for cleaning up spills)
- a calculator
- a compass
- a protractor

Organization: Small groups (of four or five)

Procedure:

1. Ensure each member of your group has a potato, an assigned letter of the alphabet, and a copy of the Data Sheet for Determining the Average Potato.
2. Label your potato with the letter of the alphabet you were assigned. Give your potato a name that begins with that letter, and record the name on your data sheet. Count the number of eyes on your potato, and record the number on your data sheet.
3. Measure and record the length around (cm), the breadth (cm), the mass (gr), and the volume (mL) of your potato.
4. When the measuring is complete, have each person in your group clearly read and repeat his or her data aloud. Listen carefully to your classmates' data and record it accurately.
5. After all the data has been collected, work as a group to define the average potato.
   a) Have each group member take responsibility to calculate the measures of central tendency for one of the potato measures.
   b) Identify the range and any outliers, and discuss the effect of the outliers on the measures of central tendency.
   c) Prepare circle graphs for each potato measure to help define the average potato.
   d) As a group, agree on a definition for an average potato.
   e) Determine the number of potatoes that would fit your definition of an average potato. What is the experimental probability that a potato in the group would be an average potato, according to your definition? Explain.
   f) Prepare to present your group's definition and a defence of that definition to the class.
6. Present definitions to the class, and take part in a discussion. Does the class agree on a definition for an average potato? Explain.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Determine measures of central tendency.
- ❒ Determine the affect of outliers included in the data.
- ❒ Determine the most appropriate measures of central tendency to report findings.
- ❒ Construct circle graphs to solve problems.

Extension (Optional):

Students design an average-potato lottery and compare the experimental results to the theoretical probability of winning.

Purpose:

In this extension, students will demonstrate the ability to do the following (connections to learning outcomes are identified in parentheses):

- Express probabilities as ratios, fractions, and percents. (7.SP.4)
- Define the sample space for the probability experiment. (7.SP.5)
- Compare the theoretical and experimental probability of two independent events. (7.SP.6)

Procedure:

- Use the collected data and the agreed-upon definition to hold a potato lottery. Define the criteria for the lottery. The winner could receive the potatoes.
- Randomly select five or six potatoes from your group. Calculate the theoretical probability of selecting an average potato from the group twice in a row, given the first potato is returned to the bunch after it has been selected. Determine the experimental probability of the same event by holding a class lottery. The potatoes could serve as the lottery prize.

Data Sheet for Determining the Average Potato					
Name	**Length around (cm)**	**Breadth (cm)**	**Mass (gr)**	**Volume (mL)**	**Number of Eyes**
A					
B					
C					
D					
E					
F					
G					
H					
I					
J					
K					
L					
M					
N					
O					
P					
Q					
R					
S					
T					
U					
V					
W					
X					
Y					
Z					
Total					
Mean					
Median					
Mode					

NOTES