GRADE 5 MATHEMATICS

Patterns and Relations

Grade 5: Patterns and Relations (Patterns) (5.PR.1)

Enduring Understandings:

Number patterns and relationships can be represented using variables.

General Outcome:

Use patterns to describe the world and solve problems.

Specific Learning Outcome(s):	Achievement Indicators:
5.PR.1 Determine the pattern rule to make predictions about subsequent elements. [C, CN, PS, R, V]	➛ Extend a pattern with or without concrete materials, and explain how each element differs from the preceding one. ➛ Describe, orally or in writing, a pattern using mathematical language, such as one more, one less, five more. ➛ Write a mathematical expression to represent a pattern, such as $r + 1$, $r - 1$, $r + 5$. ➛ Describe the relationship in a table or chart using a mathematical expression. ➛ Determine and explain why a number is or is not the next element in a pattern. ➛ Predict subsequent elements in a pattern. ➛ Solve a problem by using a pattern rule to determine subsequent elements. ➛ Represent a pattern visually to verify predictions.

Prior Knowledge

Students may have had experience with the following:

- Describing, extending, comparing, and creating increasing patterns using manipulatives, diagrams, and numbers
- Describing, extending, comparing, and creating decreasing patterns using manipulatives, diagrams, and numbers
- Identifying and describing patterns found in tables including a multiplication chart
- Adding and subtracting whole numbers less than 10 000

- Multiplying whole numbers less than 100 by whole numbers less than 10
- Dividing whole numbers less than 100 by whole numbers less than 10

Background Information

Although there are different types of patterns, the learning experiences that follow focus on **increasing/decreasing patterns**. The elements that make up these patterns are called terms. Each term builds on the previous term. Consequently, these patterns are often referred to as **growing patterns**. For example, 2, 4, 6, 8, 10. . . and 1, 2, 4, 8, 16. . . are two common increasing patterns.

Using a table to model an increasing/decreasing pattern can help students organize their thinking. It can also help them generalize the patterns symbolically. There are two types of generalizations (rules) that can be made: recursive and explicit. A **recursive generalization** tells how to find the value of a term given the value of the preceding term. An **explicit generalization** expresses the relationship between the value of the term and the term number. For example, consider this pattern:

♥ ♥♥♥ ♥♥♥♥♥ ♥♥♥♥♥♥♥ ♥♥♥♥♥♥♥♥♥

The pattern can be organized into a table like this.

Term	1	2	3	4	5
Term Value	1	3	5	7	9

The recursive generalization that describes this pattern is $n + 2$, since the value of each term is two more than the preceding term. If the pattern were continued, the value of the sixth term would be 11 since 9 + 2 = 11. However, to find the value of the 100th term, you would need to find the value of each of the 99 preceding terms.

It is easier to predict the value of subsequent terms with an explicit generalization. Notice that in the above pattern, if you double a term number and subtract 1, you get the value of the term. For example:

> The value of the third term is $2 \times 3 - 1 = 5$ and the value of the fifth term is $2 \times 5 - 1 = 9$. Thus, the explicit generalization that describes the pattern is $2n - 1$. If the pattern were continued, the value of the 100th term would be 199, since $2 \times 100 - 1 = 199$.

When helping students recognize patterns, it is important to remember that they may not see the pattern in the same way as you. Therefore, it is important that you ask students to explain their thinking. Having students describe their reasoning can also help them realize that often there is more than one way to look at a pattern.

Mathematical Language

Decreasing pattern

Increasing pattern

Pattern

Term

Term number

Term value

Learning Experiences

Assessing Prior Knowledge

Materials: K-W-L charts (BLM 5–8.18) and large chart paper sheets

Organization: Pairs/Whole class

Procedure:

a) Tell students that in the next few lessons they will be learning about patterns, but before they begin you need to find out what they already know about them.

b) Pose the question: "What is a pattern?" Have students think about the question and then share their thoughts with their partners. Ask students to work with their partners to complete the first two columns of their K-W-L chart.

c) Post the large sheets of chart paper on the board. Make a class K-W-L chart by asking each pair of students to share their ideas with the rest of the class and to write their responses on the chart paper. Encourage students to explain what they know about patterns by asking them questions such as

- "What is a pattern?"
- "How do you know the example you gave is a pattern?"
- "What comes next in your pattern? How do you know?"
- "How can you describe your pattern?"
- "Is there another way you could describe your pattern?"

d) Ask students to record what they learned from the discussion in the third column of their chart.

e) Have students maintain and revisit their chart throughout the unit on patterns.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ define what a pattern is
- ❑ identify examples and non-examples of patterns
- ❑ identify different types of patterns (e.g., Do they know what a repeating pattern is? Do they know what an increasing pattern is?)
- ❑ extend a pattern.
- ❑ describe a pattern
- ❑ use the vocabulary associated with patterns correctly (e.g., Do they use words such as "core," "repeating," "increasing/decreasing," and "term" correctly?)

Use their responses to clear up any misconceptions they might have about patterns and to enhance their ability to create, extend, analyze, and describe patterns.

- **Extend a pattern with or without concrete materials, and explain how each element differs from the preceding one.**
- **Describe, orally or in writing, a pattern using mathematical language.**
- **Write a mathematical expression to represent a pattern.**
- **Describe the relationship in a table or chart using a mathematical expression.**
- **Predict subsequent elements in a pattern.**
- **Represent a pattern visually to verify predictions.**

Materials: Coloured tiles, overhead of the problem, and copies of the worksheet (BLM 5.PR.1.1).

Organization: Whole class/Pairs

Procedure:

a) Present Problem A to students.

Encourage students to discuss the pattern by asking them questions, such as the following:

- "What does Angela's pattern look like?"
- "How many tiles will Angela need to make the fifth term of her pattern?"
- "Draw the next two terms in Angela's pattern. Was your prediction right?"
- "How many tiles do you think Angela will need to make the tenth term? Why?"

b) Begin to fill in a table. Explain that a table can help them think about the pattern. Fill in the values for the first three terms. Relate each number you write to the pattern.

Term	1	2	3							
Term Value	4	5	6							

c) Tell students that they should work with their partner to find out how many tiles Angela needs to make the tenth term, and record their findings on the following worksheet.

d) Have students share their answers with the other members of the class and explain their reasoning. Draw a completed table showing the number of tiles needed for the first 10 terms, and ask students how they could find the number of tiles needed for any term in the pattern.

Encourage students to look at the relationship between a term and the number of tiles in the term. (Students should note that the number of tiles is always three more than the term number, so the rule for finding the number of tiles in any term of the pattern is $n + 3$). Show students how to record the rule using a mathematical expression.

Term	1	2	3	4	5	6	7	8	9	10
Term Value	4	5	6	7	8	9	10	11	12	13

Observation Checklist

Observe students' responses to determine whether they can do the following:

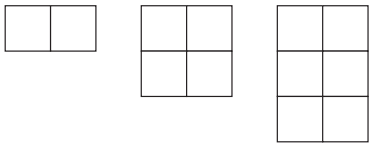
- ❑ extend a pattern with or without concrete materials, and explain how each element differs from the preceding one
- ❑ describe, orally and in writing, a pattern using mathematical language, such as "one more," etc.
- ❑ write a mathematical expression to represent a pattern, such as $n + 3$
- ❑ describe the relationship in a table or chart using a mathematical expression
- ❑ predict subsequent elements in a pattern
- ❑ solve a problem by using a pattern rule to predict subsequent elements
- ❑ represent a pattern visually to verify predictions

Materials: Coloured tiles and pattern blocks

Organization:

Procedure:

a) Ask students to use the coloured tiles to construct and then draw the next two terms in this pattern.



b) Encourage students to discuss the patterns by asking them questions, such as the following:

- "What patterns do you notice?"
- "How does the second term differ from the first term? The third term?"
- "Are there other patterns that describe how the pattern grows? What are they?"

c) Ask students to make a table of values for the pattern, and then predict the number of tiles needed to make the tenth term.

Term	1	2	3	4	5	6	7	8	9	10
No. of Tiles	2	4								20

d) Have students explain the pattern that they used to predict the tenth term. Ask them to write a rule for finding the value of any term in the pattern ($2n$), and then use their rule to predict the number of tiles in the 25th term.

e) Have students draw or use pattern blocks to create a pattern that uses the same numbers as the tile pattern they just discussed. Have them share their patterns with the other members of the class.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ extend a pattern with or without the use of concrete materials, and explain how each element differs from the preceding one
- ❑ describe, orally or in writing, a pattern using mathematical language, such as "one more" or "one less"
- ❑ write a mathematical expression to represent a pattern, such as n +2
- ❑ describe the relationship in a table or chart using a mathematical expression
- ❑ predict subsequent elements in a pattern
- ❑ recognize that different patterns can have the same rule
- ❑ create a rule that fits a given pattern

Materials: Paper, pencil, and an overhead of the problem (BLM 5.PR.1.2)

Organization: Whole class

Procedure:

a) Present the following problem:

- A fence is constructed using posts and boards. Between adjacent posts is one board, as shown on the transparency. How many boards will you need if you build a fence with 90 posts?

b) Have students discuss the problem and suggest strategies for solving it. Try their strategies. If students do not think of creating a pattern, ask them to draw a picture to show what the fence would look like if it were made with three posts. Four posts? Then ask, "How many boards are needed if the fence is made with three posts? Four posts? How many boards do you think are needed if the fence is made with five posts? Draw a picture to check your prediction. Were you right? What could you do to find out the number of boards needed for a fence with 90 posts?"

c) Ask students to work with their partner to solve the problem.

d) Have students share their answers and explain their reasoning. Write their strategies on the board and encourage students to describe a rule that they could use to find the number of boards needed to make any number of posts (n – 1, where n represents the number of posts).

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ solve a problem by using a pattern rule to determine subsequent elements
- ❑ describe orally a pattern using mathematical language
- ❑ represent a pattern visually to verify predictions
- ❑ construct a table of values
- ❑ describe the relationship in a table or chart using mathematical expressions
- ❑ explain why a number is or is not the next element in a pattern

- **Extend a pattern with or without concrete materials, and explain how each element differs from the preceding one.**
- **Describe, orally or in writing, a pattern using mathematical language.**
- **Write a mathematical expression to represent a pattern.**
- **Predict subsequent elements in a pattern.**

Materials: Pattern blocks and overhead or copies of the problem (BLM 5.PR.1.3)

Organization: Whole class

Procedure:

a) Give students the following activity:

- Miguel started to build this pattern with the triangle pattern blocks.

1. Construct and draw the next two terms in Miguel's pattern.
2. Complete the table of values:

Term	1				
No. of Triangles	3				

3. Predict the tenth term of the pattern.
4. Describe using mathematical symbols any pattern (rule) you used to determine the tenth term.

b) Have students share their solutions with the other members of the class and explain their reasoning.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ extend a pattern using concrete materials, and explain how each element differs from the preceding one
- ❑ describe a pattern using mathematical language, such as "one more"
- ❑ write a mathematical expression to represent a pattern, such as $3r$
- ❑ predict subsequent elements in a pattern

Materials: Paper and pencils

Organization: Whole class

Procedure:

a) Present the following problem.

- Mr. Olsen's class was studying addition. Corry, who likes to add, decided to create a pattern involving the sums of two numbers. These are the first three terms in his pattern:

0 + 1 = 1	2 + 0 = 2	3 + 0 = 3
1 + 0 = 1	0 + 2 = 2	0 + 0 = 3
	1 + 1 = 2	1 + 2 = 3
		2 + 1 = 3

If Corry continues his pattern, how many different ways will he write 75 as the sum of two whole numbers?

b) Have students discuss the problem and suggest strategies they could use to solve the problem. Have students try their strategies. If no one suggests trying to find a rule to describe the pattern, ask students to write the next two term terms in Corry's pattern and then create a table of values.

No.	1	2	3						
No. of Ways	2	3	4						

c) Ask, "How many different ways can Corry write 10 as the sum of two whole numbers? 15? 75? What rule can you use to describe the number of ways any whole number can be written as the sum of two whole numbers?" ($n + 1$)

d) Extend the activity by asking: "How many different ways can you write the number 75 as the sum of two whole numbers if combinations that use the same addends but in a different order only count as one way? For example, 0 + 1 and 1 + 0 would only count as one way."

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ extend a pattern and explain how each element differs from the preceding one
- ❑ describe orally a pattern using mathematical language, such as "one more"
- ❑ describe the relationship in a table or chart using mathematical language
- ❑ solve a problem by using a pattern rule to determine subsequent elements
- ❑ predict subsequent elements in a pattern
- ❑ determine and explain why a number is or is not the next number in a pattern

■ **Represent a pattern visually to verify predictions.**

Materials: Overhead of the pattern (BLM 5.PR.1.4), paper and crayons or markers

Organization: Individual/Whole class

Procedure:

a) Ask students to complete the following activity:

1. Here are two rules: $n + 5$ and $4n$.

   Ask students, "Which rule fits this pattern? How do you know the rule fits the pattern?"

2. Design a pattern that fits the other rule.
3. Ask them to explain how they know their pattern fits the rule.

b) Have students share their answers with the other members of the class. Encourage them to explain their reasoning.

c) Have students revisit their K-W-L chart. Have them share their ideas and add them to the class K-W-L chart.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ identify that the rule represents a given pattern
- ❑ create a pattern that fits a given rule
- ❑ recognize that different patterns can have the same rule

- **Extend a pattern with or without concrete materials, and explain how each element differs from the preceding one.**
- **Describe, orally or in writing, a pattern using mathematical language.**
- **Write a mathematical expression to represent a pattern.**

Materials: Copies of the activity sheet (BLM 5.PR.1.5)

Organization: Individual

Procedure:

a) Ask students to complete the activity sheet.

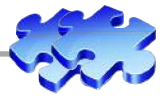
b) Have students share their answers with the other members of the class. Encourage students to explain their reasoning.

c) Have students revisit their K-W-L chart. Have students share their ideas and add them to the class K-W-L chart.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ extend a pattern without concrete materials
- ❑ explain how each element differs from the preceding one
- ❑ describe orally a pattern using mathematical language, such as "one more than"
- ❑ write a mathematical expression to represent a pattern, such as $n + 2$
- ❑ determine and explain why a number is or is not the next element in a pattern



How Tall?

Purpose:

The purpose of this investigation is to have students apply their knowledge of number concepts, patterns, and measurement. In particular, the investigation was designed to reinforce students' ability to

- apply estimation strategies
- recall multiplication and division facts
- demonstrate an understanding of fractions
- represent decimals to thousandths
- relate fractions to decimals
- measure the length of objects
- determine the rule for a pattern to make predictions

The specific concepts and skills that students demonstrate will depend on the strategy that they use to solve the problem.

In addition, the investigation is designed to enhance students' ability to

- solve problems
- reason mathematically
- communicate mathematically
- make connections

Materials: Metre sticks and several copies of a large hand (the size of bristol board)

Organization: Small groups

Procedure:

a) Hang copies of the large hand around the room before students come into the room.

b) Have students look around the room and ask them what they notice. Tell students that they just had a visitor in the room and he left his handprints all over the room. Ask, "Who could have left these prints? How big do you think this person is?"

c) Tell students that their job is to figure out how tall the visitor was. Have the groups discuss the problem and devise a strategy for determining the height of the visitor.

d) Have the groups carry out their strategy for solving the problem.

e) Have the groups share their solutions with the other members of the class and explain their reasoning.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ use mathematical language correctly
- ❑ carry out mathematical procedures correctly, such as measuring and computing
- ❑ use mathematics confidently to solve problems
- ❑ use a variety of strategies to ensure the correctness of their solution
- ❑ communicate and reason mathematically
- ❑ contribute to the mathematical discussion

Extension:

Examine proportions in the book *Jim and the Beanstalk* by Raymond Briggs.

- The giant claims that he is going to eat three fried boys on a piece of toast. How big would the toast have to be to fit three fifth-graders?
- How big would the giant be based on the size of his/her toast?

NOTES

Grade 5: Patterns and Relations (Variables and Equations) (5.PR.2)

Enduring Understandings:

Number patterns and relationships can be represented by variables.

General Outcome:

Represent algebraic expressions in multiple ways.

Specific Learning Outcome(s):	Achievement Indicators:
5.PR.2 Solve problems involving single-variable (expressed as symbols or letters), one-step equations with whole-number coefficients, and whole-number solutions. [C, CN, PS, R]	➛ Express a problem in context as an equation where the unknown is represented by a letter variable. ➛ Solve a single-variable equation with the unknown in any of the terms (e.g., $n + 2 = 5$, $4 + a = 7$, $6 = r - 2$, $10 = 2c$). ➛ Create a problem in context for an equation.

Prior Knowledge

Students may have had experience with the following:

- Expressing a problem as an equation in which a symbol is used to represent an unknown number
- Solving one-step equations involving a symbol to represent an unknown number

Background Information

The learning experiences in this section focus, in part, on translating word problems into equations and then solving them. This is not new to students. In the Early Years, students were introduced to whole-number operations through routine problems. In the beginning, students solved these problems using concrete and pictorial representations. Later, they translated these problems into equations, often using an empty square to represent the unknown value.

Consequently, the learning experiences provide students with additional experience with solving routine problems and, at the same time, beginning the transition to using letters to represent unknown quantities. They also serve as an informal introduction to the terms "equation," "mathematical expression," and "variable." These terms are defined as follows:

An **equation** is a mathematical sentence stating that one or more quantities are equal. Equations that contain variables, such as $3 + x = 21$ and $2y + 3 = 15$, are sometimes referred to as open sentences, while equations that have no variables, such as $3 + 5 = 8$ and $24 \div 3 = 8$, are referred to as closed sentences.

A **mathematical expression** comprises numbers, variables, and operation signs, but does not contain a relational symbol such as $=$, $\neq$, $<$, $>$, $\leq$, and $\geq$. For example, $6x + 3$ and $\frac{x}{4} - 8$ are mathematical expressions.

A **variable** is a symbol for a number or group of numbers in a mathematical expression or equation.

Mathematical Language

Equation

Solution

Unknown

Learning Experiences

Assessing Prior Knowledge

Materials: None

Organization: Pairs/Whole class

Procedure:

a) Tell students they will be solving some problems involving equations, but before you can give them these problems you need to know what they already know about equations.

b) Pose this question: "What is an equation?" Have students think about the question for a few moments, then share their answer with their partner.

c) Have students share their answers with the other members of the class. Encourage discussion by asking students questions, such as the following:

- "What is an example of an equation?"
- "What does the equation tell you?"
- "Is 6×7 an equation? Why or why not?"
- "Is $5 = 14 - 9$ an equation? Why or why not?"
- "Is $n \div 4 = 16$ an equation?"
- "When do you use equations?"

d) Write students' responses on the board or overhead. Place their responses under the headings "Things we know about equations" and "Things we need to think about." Use the list to help you plan subsequent lessons.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ describe what an equation is
- ❑ recognize examples and non-examples of equations
- ❑ provide examples of equations that contain variables (unknown quantities)
- ❑ provide examples of equations that do not contain variables
- ❑ recognize that an expression can be on either side of the equal sign (e.g., $5 = 14 - 9$ is the same as $14 - 9 = 5$)

- **Express a problem in context as an equation where the unknown is represented by a letter variable.**

Materials: Overhead of the problems (BLM 5.PR.2.1), copies of the activity (5.PR.2.2)

Organization: Pairs/individual

Procedure:

a) Ask students to work with their partner to solve the following problem:
   - Make sure students understand what the phrase "best fits" means (i.e., the equation should reflect the structure of the problem. For example, if the action in the problem indicates items are being combined, the equation should express the addition of quantities in the order they are given). Also, stress that they do not have to solve the equation.

b) Give students Problem A. Have students share their answer and explain their reasoning.

c) Give students Problem B. Have students share their answer and explain their reasoning.

d) Have students complete the activity sheet.

Observation Checklist

Examine students' responses to determine whether they can do the following:

- ❑ identify the equation that reflects the structure of the problem
- ❑ express a problem in context as an equation where the unknown is represented by a letter variable

Materials: Math journal

Organization: Individual/Whole class

Procedure:

a) Present this problem:

- Nancy and Jessica were asked to write an equation for this story. I want to buy 35 pencils. Pencils come in packages of 7. How many packages do I need to buy? Nancy wrote $7 \times n = 35$ and Jessica wrote $35 \div 7 = n$. Who is right? Why?

b) Ask students to think about the problem and then record their answer in their math journals.

c) Have students discuss their answer with the other members of the class.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ identify an equation that fits a given story problem
- ❑ recognize that both equations fit the problem

- **Express a problem in context as an equation where the unknown is represented by a letter variable.**
- **Solve a single-variable equation with the unknown in any of the terms.**

Materials: Copies of the word problems (BLM 5.PR.2.3)

Organization: Whole class

Procedure:

a) Ask students to write an equation to fit a problem and then solve it. Make sure students know that they should use a letter to represent the unknown value.

b) Have students share their answer and explain their reasoning. Encourage discussion of the problem by asking them questions, such as the following:

- "What equation did you use to solve the problem?"
- "Did anyone use a different equation? What is it?"
- "Do the equations have the same solution?"
- "How do you know your answer is correct?"
- "What strategy did you use to solve the problem?"
- "Did anyone use a different strategy? What is it?"

c) Repeat the activity with similar problems.

d) Ask students to write their own problems. Have them give their problems to the other members of the class and ask them to solve them. Make a class booklet of the problems that they write, and share the book with another class.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ express a problem in context as an equation where the unknown is represented by a letter variable
- ❑ solve a single variable equation where the unknown is represented by a letter variable
- ❑ create a problem in context for an equation
- ❑ use appropriate strategies to solve an equation
- ❑ justify the solution to an equation

- **Express a problem in context as an equation where the unknown is represented by a letter variable.**
- **Solve a single-variable equation with the unknown in any of the terms.**
- **Create a problem in context for an equation.**

Materials: Paper and pencils

Organization: Small groups

Procedure:

a) Ask the groups to select one equation from each of the following lists:

$x + 9 = 25$	$25 - y = 14$	$26 = 2c$
$7 + r = 35$	$y - 8 = 16$	$5x = 60$
$z = 25 + 83$	$x = 62 - 23$	$4k = 32$

b) Ask the groups to write one word problem for each equation that they chose.

c) Have each group exchange its problems with another group. Have the groups solve each other's problems.

d) Have several groups share their problems and solutions with the other members of the class. Encourage students to discuss their solutions by asking them questions, such as the following:

- "What equation did you use to solve the problem?"
- "Why did you choose that equation?"
- "Is there another equation that could be used to solve the problem?"
- "How do you know your solution is correct?"
- "What strategy did you use to solve the equation?"
- "Is there another strategy that could be used to solve the problem? What is it?"

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ create a problem in context for an equation
- ❑ solve a single variable equation with the unknown in any of the terms
- ❑ express a problem in context as an equation where the unknown is represented by a letter variable
- ❑ use an appropriate strategy to solve an equation
- ❑ justify their solution to an equation

- **Solve a single-variable equation with the unknown in any of the terms.**

Materials: Copies of the activity (BLM 5.PR.2.4)

Organization: Individual/Whole class

Procedure:

a) Have students complete the activity. Do an example with the class before letting students work on their own.

b) Have students share their answers with the other members of the class. Encourage students to justify their solutions and explain the strategy that they used to solve the equations.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ solve a single variable equation with the unknown represented by a letter variable
- ❑ identify the variable in an equation
- ❑ use an appropriate strategy to solve the equation
- ❑ justify their solution to an equation