Grade 5 Mathematics

Number

Grade 5: Number (5.N.1)

Enduring Understandings:

The position of a digit in a number determines its value.

Each place value position is 10 times greater than the place value position to its right.

General Outcome:

Develop number sense.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
5.N.1 Represent and describe whole numbers to 1 000 000. [C, CN, T, V]	➛ Write a numeral using proper spacing without commas (e.g., 934 567 and not 934,567). ➛ Describe the pattern of adjacent place positions moving from right to left. ➛ Describe the meaning of each digit in a numeral. ➛ Provide examples of large numbers used in print or electronic media. ➛ Express a given numeral in expanded notation (e.g., 45 321 = [4 x 10 000] + [5 x 1000] + [3 x 100] + [2 x 10] + [1 x 1] or 40 000 + 5000 + 300 + 20 + 1). ➛ Write the numeral represented in expanded notation.

PRIOR KNOWLEDGE

Students may have had experience with the following:

- Representing and describing whole numbers to 10 000 pictorially and symbolically
- Comparing and ordering whole numbers to 10 000
- Demonstrating an understanding of addition of numbers with answers to 10 000
- Demonstrating an understanding of subtraction of 3- and 4-digit numbers

Background Information

For students to work effectively with large numbers, they need to have a good understanding of the structure of our numeration system. The Hindu-Arabic, or base-10, numeration system that we use today originated in India around 500 CE, and was carried to other parts of the world by Arab people. The system gradually replaced the use of Roman numerals and the abacus in trade and commerce in Europe and, by the 16th century, was predominant. The features of the system that led to its acceptance and the computational procedures we use today include the following:

1. It consists of 10 **digits** (symbols), 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, that are used in combination to represent all possible numbers.
2. It has a base number. In this system, 10 ones are replaced by one group of 10, 10 tens are replaced by one hundred, 10 hundreds are replaced by one thousand, and so on. The number of objects grouped together is called the **base** of the system. Thus, the Hindu-Arabic system is a base-10 system.
3. It has **place value**. Each place in a numeral has its own value. For any place in the system, the next position to the left is 10 times greater and the position to the right is one-tenth as large.
4. It has a symbol for zero. The symbol has two functions. It is a placeholder in numerals like 5027, where it indicates there are no hundreds (or 50 hundreds). It is also the number that indicates the size of the set that has no objects in it.
5. It is additive and multiplicative. The value of a numeral is found by multiplying each place value by its corresponding digit and then adding all the resulting products. Expressing a numeral as the sum of its digits times their respective place values is called **expanded notation**. For example, the expanded notation for 8273 is (8 x 1000) + (2 x 100) + (7 x 10) + (3 x 1) or 8000 + 200 + 70 + 3.

Consequently, the focus of the learning experiences that follow is on helping students conceptualize the magnitude of large numbers and understanding the characteristics of our numeration system that allow us to read, write, and interpret the numerals for these numbers.

Mathematical Language

Base

Digit

Expanded notation

Hundred thousand

One million

Place value

Ten thousand

Learning Experiences

Assessing Prior Knowledge

Materials: BLM 5.N.1.1: Place Value

Organization: Individual

Procedure:

a) Tell students that in the next few lessons they will be learning about numbers greater than 10 000, but before they begin you need to find out what they already know about large numbers.

b) Ask students to complete the activity found on BLM 5.N.1.1.

Observation Checklist

Use students’ responses to the questions to determine whether they can do the following:

❑ compare and order whole numbers in the thousands

❑ write numbers in words

❑ identify the place value position of the digits in a numeral

❑ identify the value of each digit in a numeral

■ **Provide examples of large numbers used in print or electronic media.**

Materials: *A Million Dots* by Andrew Clements, calculators, stopwatch or timer with a second hand.

Organization: Whole class/Small groups

Procedure:

a) Ask students, “How many dots do you think you can draw in one minute? If we counted all the dots everyone in the class makes in one minute, how many dots do you think we would have altogether? Do you think we would have a million dots?”

b) Explain that a million is a big number and they are going to find out what a million dots looks like.

c) Read *A Million Dots*.

d) After reading the book, ask students whether they want to change their estimates of the number of dots that they can draw in one minute Have students draw dots for one minute. When they finish, have them suggest ways to count the dots. Encourage them to think about making groups of tens to facilitate the counting process.

e) Have students use the total number of dots that they make in one minute to determine how long it would take
   - one person to make a million dots
   - the class to make a million dots

f) Have each group decide what else they could do to show how big a million is. Help them devise and carry out a plan for showing the magnitude of the number. For example, students could determine
   - the length of 1 million loonies laid end to end
   - the number of pages a telephone book would need to have to list 1 million people
   - the number of boxes of toothpicks they would need to make a million

g) Have each group share their plans and what they found out about 1 million with the other members of the class.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ make reasonable estimates
- ❏ solve computational problems with and without using technology
- ❏ develop and carry out a plan for solving a problem
- ❏ indicate that they have a sense of the magnitude of 1 million

- **Describe the pattern of adjacent place positions moving from right to left.**

Materials: Blank Hundred Square (BLM 5–8.6), scissors, tape, or stapler.

Organization: Small groups

Procedure:

a) Write the numbers 600 and 60 on the board or on an overhead. Ask students, "Would you rather have 600 pennies or 60 pennies?" Have students explain their reasoning.

b) Explain that the place of a digit within a number is important because it tells us the value of the digit, and today they will be learning more about place value.

c) Point to each digit in the numeral 600 and ask students, “What is the place value position of this digit?” Write students’ responses on the board or overhead, and show students that the ones can be represented with a square, the tens with a strip of 10 squares, and hundreds with a grid of 100 squares.

d) Ask students, “What place value position comes next? How can we use the hundred squares to show 1000?” Let students explore different ways to arrange the hundred squares to make 1000. Each student should then make a 1000-strip by taping or stapling ten of the hundred squares together.

e) Ask students questions about the relationship between the different place value positions. For example:
   - “How many hundreds in one thousand?”
   - “How many times larger is one thousand than one hundred?”
   - “How many tens are in one hundred?”
   - “How many times larger is one hundred than ten?”
   - “How many tens are in one thousand?”
   - “How many times larger is one thousand than ten?”

f) Ask students, “What place value position comes next? How much larger than the thousands position should the new place value position be? Why do you think this? How can we use the 1000-strips to show the next place value position?” Have groups of 10 students staple or tape their 1000 strips together. When students finish making their 10 000 square strips, ask them questions about the relationship between the different place value positions similar to the ones in part (e).

g) Have students in each group work together to answer these questions: “What place value position comes next? What is the relationship of this position to the other place value positions? What would a model of this place value position look like?” Have each group share its answers with the other members of the class. Encourage students to explain their reasoning.

h) Repeat part (g) to introduce students to the millions position.

Observation Checklist

Observe students’ responses to determine whether they can do the following:

- ❑ recognize that each place is 10 times greater than the place to its right
- ❑ describe the various relationships among the place value positions (e.g., 10 000 is 100 times greater than 100 and 1000 times greater than 10)
- ❑ describe place value positions to the millions

- **Write a numeral using proper spacing without commas.**

Materials: Paper and pencils, overhead copy of place value chart—whole numbers (BLM 5–8.7)

Organization: Whole class/Pairs

Procedure:

a) Write a number on the board or overhead (e.g., 62 893), and ask students, “How do you read the number? How do place value patterns help us read numbers?”

b) Show students a place value chart. Explain that when we read and write large numbers, we group the digits into threes. Each group of three forms a family. Each family has a different last name and is separated from the other families by a space. The family on the far right is the ones. The family to its immediate left is the thousands. The next family on the left is the millions. In each family, there is a place for ones, tens, and hundreds.

c) Tell students that, for the remaining time, they will be focusing on numbers in the thousands family. Record a number in the place value chart (e.g., 425 679), and explain how to read the number and what each digit in the number means. Do three or four more examples.

d) Have students work with their partner. Students need to sit so one person in a pair can see the board and the other one cannot. Write a number on the board (e.g., 286 164). Students facing the board read the number to their partner. Their partner writes the number down. Students then compare the number they wrote down with the number on the board. Repeat the activity several times, giving each student an opportunity to be both the “reader” and the “writer.”

Observation Checklist

Observe students’ responses to determine whether they can do the following:

- ❏ read a number correctly
- ❏ write a numeral correctly with proper spacing

- **Describe the meaning of each digit in a numeral.**

Materials: Number cards with the numbers 0 through 9 (BLM 5–8.5) with one number per card, and large strips of paper showing the place value headings (BLM 5.N.1.2), one for each group

Organization: Small groups (group size depends on the size of the numbers)

Procedure:

a) Put the place value column headings on the walls so they are just above the students' heads.

b) Say a number (e.g., 90 372). Students in each group must find the appropriate number cards, then arrange themselves into a line underneath the column headings showing the number you said. Encourage students to tell what each digit in the number means. Have students repeat the activity several more times.

c) Expand the place value column headings to include hundred thousands and have students form 6-digit numbers.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ identify the place value position of each digit in a number
- ❑ describe the meaning of each digit in a number

- **Describe the meaning of each digit in a numeral.**

Materials: None

Organization: Whole class

Procedure:

a) Tell students they will be doing some number calisthenics. Explain that they will be acting out a number you write on the board or overhead projector. They must act out the number by doing in sequence:

   - as many hops on their left foot as specified by the value of the digit in the hundred-thousands
   - as many jumping jacks as the value of the digit in the ten-thousands position
   - as many clap-your-hands as in the value in the thousands position
   - as many touch-your-toes as the value in the hundreds position
   - as many hops on their right foot as the value of the digits in the tens place
   - as many finger snaps as the value of the digits in the ones position

   For example, for the number 243 167, students would do

   - 2 hops on their left foot
   - 4 jumping jacks
   - 3 clap-your-hands
   - 1 touch-your-toes
   - 6 hops on their right foot
   - 7 finger snaps

b) Have students act out 527 483; 298 645; and 738 295. When students are familiar with the movements for each place value position, have them act out 3-digit, 4-digit, 5-digit, and 6-digit numbers.

c) Vary the activity by having students choose the numbers that they act out.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ identify the correct place value position of each digit in the number
- ❏ identify the value of each digit in a number

- **Write a numeral using proper spacing without commas.**

Materials: Paper and pencils

Organization: Whole class

Procedure:

a) Write four or five numbers on the board that use different arrangements of the same digits. For example:

   10 053   10 503   10 530   13 005   13 530

b) Read one of the numbers (e.g., ten thousand five hundred thirty). Ask the students to tell you which one you chose. Have students explain how they knew which number you read. Encourage students to describe what the "zeros" in each numeral mean.

c) Continue reading the numbers and having students identifying them. When they finish identifying all the numbers, have them order the numbers from smallest to largest and then write the numbers in expanded notation.

d) Repeat the activity using different sets of numbers.

e) Vary the activity by using six-digit numbers instead of five-digit numbers.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ describe the meaning of each digit in a numeral
- ❑ describe the pattern of place value positions moving from right to left
- ❑ identify the place value position of each digit in a numeral
- ❑ express a numeral in expanded notation

- **Write a numeral using proper spacing without commas.**
- **Express a given numeral in expanded notation.**

Materials: Dice

Organization: Small groups

Procedure:

a) Distribute six dice to each group. Tell students that they will be tossing the dice five times. The first time they roll the dice they should create a six-digit number with the numbers that they roll. They should record the number and then write it in expanded notation. Next, they should remove one die and roll the remaining five dice to create a five-digit number. Again, they should write the number in both standard notation and expanded notation. Students should continue removing a die, creating a number with the numbers that are rolled, and recording the numbers in standard notation and expanded notation until they have one die left.

b) Have each group share its results with the other members of the class.

c) Vary the activity by having students write the number in standard notation and in words.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ write a numeral using proper spacing without commas
- ❑ express a given numeral in expanded notation
- ❑ write a numeral in expanded notation

- **Write a numeral using proper spacing without commas.**
- **Express a given numeral in expanded notation.**
- **Write the numeral represented in expanded notation.**

Materials: Calculators, overhead of the excerpt from *Which Do You Prefer – Chunky or Smooth?* (BLM 5.N.1.3)

Organization: Individual

Procedure:

a) Have students read the excerpt shown below. Ask them to rewrite the number words using numerals and rewrite the numerals using number words. In addition, have students write each of the numbers in expanded notation.

   In her book called *Which Do You Prefer – Chunky or Smooth?*, Heather Brazier tells us the following:

   > On an average day in Canada …we consume eighty thousand, eight hundred forty-nine kilograms of peanut better. Of the total, 20 212 kg are chunky… (46)

b) Have students figure out how much smooth peanut better must be eaten by Canadians on an average day. Have them write their answer as a numeral and in words.

c) Throughout the year, have students bring in examples of large numbers that they find in newspapers or magazines. Keep a class chart that shows the number in numerals, expanded form, and words.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ write numbers in word form
- ❏ write the numeral for a number written in words
- ❏ write a numeral in expanded notation
- ❏ provide examples of large numbers used in print or electronic media

- **Describe the meaning of each digit in a numeral.**

Materials: Calculators

Organization: Whole class

Procedures:

a) Ask students to show 83 247 on their calculators. Tell them that their goal is to change the 2 to 0 (zero it) by subtracting one number. When students finish, ask them the following:
   - "What number do you have on your calculator now?"
   - "What number did you subtract to wipe out the 2?"
   - "Why did you subtract that number?"

b) Continue asking students to show five-digit and six-digit numbers on their calculators. After you name a number for them to show on their calculators, ask them to zero a digit in one of the place value positions. Encourage students to describe what they did and why they did it to zero a digit.

c) Ask students to add a number to wipe out a digit (e.g., adding 4 can wipe out the 6 in 506).

d) Vary the activity by telling students that they can use either addition or subtraction to wipe out a digit.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ identify the place value position of each digit in a numeral
- ❏ identify the value of each digit in a numeral
- ❏ use technology to compute sums and differences

Materials: Number cards (BLM 5–8.5) (one set per student)

Organization: Groups of three or four

Procedure:

a) Tell students that they will be playing a place value game. Explain how to play the game.

   1. Hand each student a complete set of cards. Once students are in the group, all cards should be combined together.
   2. Shuffle the cards and lay them face down in the playing area.
   3. Players take turns drawing five cards from the deck.
   4. Players arrange the cards in their hands so that they have the largest possible number.
   5. One player says, "Let's see the numbers" and everyone lays their cards face up in front of them. A card cannot be moved after it has been placed face-up on the playing surface.
   6. Players take turns reading the number that they created. The player who has the largest number and reads the number correctly wins a point.
   7. The winner is the person with the most points after five rounds of the game.

b) Demonstrate how to play the game and answer any questions that students may have. Have students play the game.

c) Vary the game by having students

   - draw six cards instead of five
   - create the smallest possible number with their cards

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ read 5-digit and 6-digit numbers correctly
- ❑ use place value concepts to determine which of two or more numbers is the largest
- ❑ use place value concepts to determine which of two or more numbers is the smallest

- **Write a numeral using proper spacing without commas.**
- **Describe the meaning of each digit in a numeral.**

Materials: Calculators, paper and pencils

Organization: Pairs

Procedure:

a) Tell students that they will be playing a game called "Give and Take." Explain how to play the game.

   1. Players write down a six-digit number containing no zeros and no identical digits. Players keep their numbers hidden from each other throughout the game.
   2. Players take turns being the giver and the taker. Each player tries to increase his or her number by taking digits from the other player.
   3. A turn begins when the asker says: "Give me your x's, where 'x' can be any digit from 1 through 9." (e.g., "Give me your 7s.").
   4. If that digit is in the giver's number, the giver announces its place value (e.g., "You get 700." If the digit is not in the giver's number, the giver announces this by saying, "You get zero.").

      Note that the value of a digit that is asked for depends on its position in the giver's number. If 7 is asked for and the number is 325 714, then the giver says, "You get 700." If the giver's number is 372 514, then the giver says " you get 70 000."
   5. As soon as the giver responds with the number, the asker adds that amount to his or her number (e.g., + 700) and the giver subtracts that amount from his or her number (e.g., –700).
   6. Players' numbers change with each new addition or subtraction. Players always use the most recent form of their numbers when adding, subtracting, or announcing the place value of a digit. Players keep track of their changing number by adding and subtracting from their original number and its successors. For example:

      325 714 + 20 000 = 345 714

      345 714 – 5 000 = 340 714
   7. If the same digit appears two or more times in a giver's number during play, the giver can say either of its values (e.g., for 845, 218, the giver can say 8 and not mention the 800 000).
   8. The game ends after each player has had five turns as asker. Players check each other's addition and subtractions. The player with the largest number is the winner.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ identify the correct place value position of each digit
- ❑ identify the value of each digit in a number
- ❑ use calculators correctly to determine sums and differences
- ❑ write a numeral with the proper spacing with no commas

NOTES

Grade 5: Number (5.N.2)

Enduring Understandings:

Computational estimations produce approximate answers.

General Outcome:

Develop number sense.

Specific Learning Outcome(s):	**Achievement Indicators:**
5.N.2 Apply estimation strategies, including ■ front-end rounding ■ compensation ■ compatible numbers in problem-solving contexts. [C, CN, ME, PS, R, V]	➢ Provide a context for when estimation is used to ■ make predictions ■ check reasonableness of an answer ■ determine approximate answers ➢ Describe contexts in which overestimating is important. ➢ Determine the approximate solution to a problem not requiring an exact answer. ➢ Estimate a sum or product using compatible numbers. ➢ Estimate the solution to a problem using compensation, and explain the reason for compensation. ➢ Select and use an estimation strategy to solve a problem. ➢ Apply front-end rounding to estimate ■ sums (e.g., 253 + 615 is more than 200 + 600 = 800) ■ differences (e.g., 974 – 250 is close to 900 – 200 = 700) ■ products (e.g., the product of 23 x 24 is greater than 20 x 20 or 400 and less than 25 x 25 or 625) ■ quotients (e.g., the quotient of 831 ÷ 4 is greater than 800 ÷ 4 or 200)

Prior Knowledge

Students may have had experience with the following:

- Adding whole numbers with sums less than 10 000
- Subtracting whole numbers with differences less than 10 000
- Using different strategies to estimate sums and differences
- Multiplying a 1-digit whole number times a 2-digit or 3-digit whole number
- Using a personal strategy to estimate a product
- Dividing a 2-digit whole number dividend by a 1-digit whole number divisor
- Using a personal strategy to estimate a quotient

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of multiplication (1- and 2-digit multipliers and up to 4-digit multiplicands)
- Demonstrating an understanding of division (1- and 2-digit divisors and up to 4-digit dividends)

Background Information

Computational estimation is the process of determining approximate answers to computational problems. Students who are skillful estimators have a good grasp of basic facts, place value, and the operations of addition, subtraction, multiplication, and division. They are also adept at mental mathematics and flexible in their use of estimation strategies, such as the ones described below.

Front-End Estimation:

Front-end rounding involves identifying the most significant (left-most) digits in a question, performing the appropriate operation, and determining the place value of the digits. For example:

- 654 + 714 + 435 is more than 1700 since 6 + 7 + 4 = 17 (and annex the zeros)
  (or since 600 + 700 + 400 = 1700)
- 532 − 285 is approximately 300 since 5 − 2 = 3 (and annex the zeros)
  (or since 500 − 200 = 300)
- 4 x 728 is more than 2800 since 4 x 7 = 28 (and annex the zeros)
  (or since 4 x 700 = 2800)
- 926 ÷ 3 is more than 300 since 9 ÷ 3 = 3 (and annex the zeros)
  (or since 900 ÷ 3 = 300)

Note: It is important for teachers to emphasize estimation skills. Discourage students from calculating first, then estimating (e.g., "I know 2.5 + 4.7 is 7.2, so I will estimate it is close to 7.").

Although front-end rounding can be used with any operation, it is most powerful when adding and multiplying. With these two operations, the computation is always underestimated.

Compatible Numbers:

This strategy involves searching for pairs of numbers that are easy to compute. When using this strategy, students look at all the numbers in a problem, and change or round the numbers so they can be paired usefully with another number. It is particularly effective for division. For example, in the question 2270 ÷ 6, rounding the dividend to 2300 (the closest hundred) or 2000 (the closest 1000) does not facilitate the estimation process. However, rounding it to 2400 (a compatible number because it is divisible by 6) makes estimating the quotient easier.

This strategy is also useful for addition. For example, when adding several numbers, students look for numbers that can be paired or grouped together to make multiples of 10.

about 100

25 + 45 + 63 + 81 or about 100 27 45 63 + 81 about 100

about 100

Therefore, the sum of 27 + 45 + 63 + 81 is about 200

Compensation:

Compensation involves refining, or adjusting, an original estimate that was obtained with another strategy. For example, the front-end estimation of 220 for the sum 86 + 23 + 72 + 55 can be adjusted to 240, since 6 + 3 and 2 + 5 (the numbers in the ones position) are both close to 10. Similarly, the front-end estimation of 2400 for 43 x 62 can be adjusted to 2600 since (3 x 60) + (2 x 40) would be greater than 200. Also, for 44 x 54, for example, you can round one number up and one number down and compute 50 x 50 for an estimate of 2500, rather than front-end rounding for an estimate of 2000.

In many instances, different strategies can be applied to the same problem. The choice of strategies depends on the students, the numbers, and the operations involved. Teachers need to help students become aware of the various strategies and help them develop confidence in their ability to estimate. To do this, they need to

- engage students in discussions about the strategies they used to estimate the solution to a computational problem (Sharing strategies can lead to the development and use of new strategies.)
- accept a range of estimates in order to help students understand that there is no one "right" estimate
- encourage students to identify real-world situations that involve estimations

- incorporate estimation throughout their instructional programs (Like problem solving, estimation should not be taught in isolated units.)

Mathematical Language

Annex

Approximate

Compatible numbers

Compensation

Estimate

Estimation

Front-end rounding

Learning Experiences

Assessing Prior Knowledge

Materials: None

Organization: Individual

Procedure:

a) Tell students you need to know what they know about computational estimation so you can help them become better estimators. To find out what they know, give them some problems to estimate. Tell students that you will show them several problems, one at a time, and they will have an appropriate amount of time (decide this based on individual students—approximately 30 seconds) to estimate the solution to each problem. They must record their estimate before their time is up.

b) Give students the following problems:

1. $84 + 27 + 35 + 62$
2. $892 + 154$
3. $4821 + 3179$
4. $628 - 147$
5. $5372 - 3124$
6. 8×12
7. 4×356
8. $86 \div 4$

c) Have students share their estimates and the strategies they used to determine them.

Observation Checklist

- ❑ Check students' responses to the problems to determine whether they can estimate the solutions to addition, subtraction, multiplication, and division problems. Use the class discussion to find out what strategies students use to make their estimates.

■ Provide a context for when estimation is used.

Materials: Markers and newspapers/magazines

Organization: Whole class

Procedure:

a) Tell students that they are going to investigate the use of estimated and exact numbers.

b) Give students copies of different newspapers. Ask them to circle the numbers used in the headlines and articles. Next, have students review the context for the use of each circled number to determine whether the numbers in the headlines or articles refer to exact or estimated (approximate) values. For example, have students decide whether these statements taken from a newspaper refer to exact or estimated values:
   - One million people evacuated from New Orleans
   - The condo resold for $134 000
   - Last year, 13 142 tons of scrap metal were recycled

c) Engage students in a discussion about the numbers they found in the newspapers. Encourage them to explain why they think a given number is exact or estimated. Have students discuss why estimated numbers are often used in newspaper articles (e.g., estimated numbers are easier to interpret and use).

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ identify real-world examples of estimated numbers
- ❑ distinguish between exact and estimate (approximate) numbers
- ❑ give a reasonable explanation why a number is either exact or estimated

Materials: Situation Cards (BLM 5.N.2.1) and index cards

Organization: Small groups

Procedure:

a) Give each group a card with one of the situations on it.

b) Ask students to decide whether the situation on their card requires an estimated answer or an exact answer, and to list the reasons for their response.

c) Have each group read its situation to the other members of the class, and explain why they think the situation requires an estimated or exact answer.

d) Have each group create a situation card. Each group should record, on a separate piece of paper, the reasons why they think the situation they created requires an estimated or exact answer. Have the groups exchange cards and decide whether the new situation they were given requires an estimated or exact answer and why they think so. Each group should compare its response with the response of the group who created the situation.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ provide a context for when estimation is used to approximate an answer
- ❏ provide a context for when estimation is used to predict an answer
- ❏ distinguish between situations that require an exact answer and those that require an estimated answer
- ❏ give reasonable explanations of why a situation requires either an exact or estimated answer

- **Provide a context for when estimation is used.**
- **Determine the approximate solution to a problem not requiring an exact answer.**

Materials: None

Organization: Pairs/Whole class/Small groups

Procedure:

a) Present students with the following problem:

> The 28 students in Mr. Nelson's fifth-grade class are planning a Halloween party. The students decided to make a fruit punch for everyone to drink at the party. They know that a can of juice makes eight cups of punch. How many cans of juice should they buy?

b) Give students time to solve the problem, and then have them share their solutions with their partner.

c) Have students discuss their solutions and share their reasoning with the other members of the class. Help them recognize that there are times we need an estimate because there is not enough information to determine an exact answer (e.g., we do not know how thirsty students will be).

d) Ask each group to identify other situations that require an estimate because there is not enough information to compute an exact answer.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ identify situations that require an estimate because there is not enough information to compute an exact answer
- ❑ provide a context when estimating is used to make predictions
- ❑ provide reasonable estimates
- ❑ give reasonable explanations for their estimates

■ **Describe contexts in which overestimating is important.**

Materials: Copies of estimation situations (BLM 5.N.2.2)

Organization: Pairs/Large group

Procedure:

a) Ask students to read each of the situations, and decide whether an overestimate or underestimate is needed.

b) Have students discuss their answers with their partners. Then have students share their answers and the reasons for them with the other members of the class.

c) Have students describe other estimation situations and decide whether an underestimate or an overestimate is needed.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ identify and describe contexts in which overestimating is important
- ❏ identify and describe contexts in which underestimating is important

- **Determine the approximate solution to a problem not requiring an exact answer.**

Materials: Number tiles or number cards (BLM 5–8.5), calculators

Organization: Individual/Pairs/Whole class

Procedure:

a) Ask students to complete the following activity:

   Explain that they must use the number tiles 4–9 to create 3-digit by 1-digit multiplication problems. The products of the problems must be as close to the target as possible. They get three tries for each target number. They should record each problem they create and its solution. Tell them they can use their calculators to find the solution to the problems they create.

   Target

   _____ _____ _____ x _____ = 5000

   _____ _____ _____ x _____ = 8000

   _____ _____ _____ x _____ = 7000

   _____ _____ _____ x _____ = 4000

b) When students finish, have them compare their estimates with a partner and explain the strategies they used to create the problems.

c) Have students share the strategies they used to create the problems with the other members of the class.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ select and use an estimation strategy to estimate the product of two numbers
- ❑ make reasonable adjustments to their estimates of the product of two numbers
- ❑ explain the strategy they used to estimate the product of two numbers

Materials: Dice, calculators, paper and pencil

Organization: Pairs

Procedure:

a) Tell students that they will be playing an estimation game with their partner. Explain how the game is played.

1. One player tosses three dice and creates a 3-digit number with the numbers that are rolled. This number becomes the dividend.
2. The other player tosses one die, and the number that is rolled becomes the divisor. Both players should record the problem.
3. Players record the problem, then quickly and silently write an estimate of the quotient of the two numbers. Players should not take more than 10 or 15 seconds to write their estimates. A 15-second timer could be used to time student estimates, or a third student could ensure a fair time period has elapsed.
4. Players use a calculator to find the quotient of the two numbers. Each player's score is the difference between his or her estimate and the exact answer.
5. The person with the lowest score after five rounds of the game is the winner.

b) Demonstrate how to play the game and answer any questions students may have. Have students play the game.

c) Have students share some of the problems they created and the strategies that they used to estimate the quotient.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ select and use an estimation strategy for division
- ❏ use technology to determine the solution to a division problem
- ❏ describe the strategy they used to determine an estimate

- **Apply front-end rounding to estimate sums, differences, products, and quotients.**
- **Estimate the solution to a problem using compensation, and explain the reason for compensation.**

Materials: Paper and pencil

Organization: Whole class

Procedure:

a) Present students with the following problem:

> Mrs. Martin's class was estimating the sums of addition problems. Matty said that the sum of 66 + 23 + 74 is about 150. How did Matty get her estimate?

b) Have students share their ideas about how the sum was estimated. Then ask, "Is the sum of 66 + 23 + 74 over or under 150? How do you know? How could you get a closer estimate of the sum?"

c) Explain that when we use the front-end rounding strategy to estimate the sum of two or more numbers, we can always adjust our estimate by looking at the other digits in the problem. For example, if we look at the digits in the ones position in the problem 66 + 23 + 74, 6 + 3 is close to 10. So the sum of the ones is greater than 10. Therefore, we can adjust our estimate to 160.

d) Do two or three more examples, and then ask students to use the front-end rounding strategy to estimate the sum of the following problems, and then adjust their estimates to get a closer approximation of the solution.

- 62 + 49 + 88 + 21
- 14 + 23 + 85 + 91
- 19 + 30 + 83 + 54 + 57

d) Have students share their estimates. Encourage them to describe how they adjusted their estimates to get a closer approximation.

e) Have students use front-end rounding to estimate the sums of problems with 3- and 4-digit numbers, and adjust their estimates to get a closer approximation.

f) Use a similar approach to help students learn how to adjust problems involving subtraction, multiplication, and division.

Observation Checklist

Observe students' responses to determine whether they can do the following:

❏ use front-end rounding to estimate the sum (difference, product, quotient) of two or more numbers

❏ estimate the solution to a problem using compensation, and explain the reasons for compensating

NOTES

Grade 5: Number (5.N.3)

Enduring Understandings:

Proficiency with the basic facts facilitates estimation and computation with larger and smaller numbers.

Multiplication and division are inverse operations.

General Outcome:

Develop number sense.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
5.N.3 Apply mental math strategies to determine multiplication facts and related division facts to 81 (9 x 9). [C, CN, ME, R, V] Recall of multiplication facts to 81 and related division facts is expected by the end of Grade 5.	➛ Describe the mental mathematics strategy used to determine a basic fact, such as ■ skip-count up by one or two groups from a known fact (e.g., if 5 x 7 = 35, then 6 x 7 is equal to 35 + 7 and 7 x 7 is equal to 35 + 7 + 7) ■ skip-count down by one or two groups from a known fact (e.g., if 8 x 8 = 64, then 7 x 8 is equal to 64 – 8 and 6 x 8 is equal to 64 – 8 – 8) ■ halving/doubling (e.g., for 8 x 3 think 4 x 6 = 24) ■ use patterns when multiplying by 9 (e.g., for 9 x 6, think 10 x 6 = 60, then 60 – 6 = 54; for 7 x 9, think 7 x 10 = 70, and 70 – 7 = 63) ■ repeated doubling (e.g., if 2 x 6 is equal to 12, then 4 x 6 is equal to 24, and 8 x 6 is equal to 48) ■ repeated halving (e.g., for 60 ÷ 4, think 60 ÷ 2 = 30 and 30 ÷ 2 = 15) ■ relating multiplication to division facts (e.g., for 7 x 8, think 56 ÷7 =) ■ use multiplication facts that are squares (1 x 1, 2 x 2, up to 9 x 9) ➛ Refine personal strategies to increase efficiency (e.g., for 7 x 6, use known square 6 x 6 + 6 instead of repeated addition 6 + 6 + 6 + 6 + 6 + 6 + 6).

Prior Knowledge

Students may have had experience with the following:

- Explaining the properties of 0 and 1 for multiplication and the property of 1 for division
- Describing and applying mental mathematics strategies, such as
  - skip-counting from a known fact
  - using halving, doubling
  - using doubling and adding one more group
  - using patterns in the 9s facts
  - using repeated doubling

  to develop an understanding of basic multiplication facts to 9 x 9 and related division facts

Recall of the multiplication and related division facts up to 5 x 5 is expected by the end of Grade 4.

- Using arrays to represent multiplication facts

Background Information

Calculations people do on a daily basis involve knowing basic math facts. For this reason, basic facts continue to be an integral part of the mathematics curriculum. In the Early Years, students are expected to recall some facts and use strategies to determine others in order to help them learn more sophisticated mathematics. If students entering the Middle Years do not have the strategies to determine or recall the basic facts, then teachers need to teach the strategies and help students work towards these skills.

Thinking strategies provide students with different approaches for arriving at an answer. Students also strengthen their number sense and learn to adapt these strategies when working with larger numbers. Once students have had time to practice these strategies in game or activity settings, then different methods can be implemented to help students develop and maintain the ability to recall and be able to determine the facts that are appropriate for the grade level.

"Learning math facts is a developmental process where the focus of instruction is on thinking and building number relationships. Facts become automatic for students through repeated exposure and practice. When a student recalls facts, the answer should be produced without resorting to inefficient means, such as counting. When facts are automatic, students are no longer using strategies to retrieve them from memory." (Manitoba Education. *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*, p. 12)

As part of a balanced mathematics program, it is useful to be able to add, subtract, multiply, and divide quickly. It is also important to know facts without having to rely on inefficient methods of working them out. “While computational recall is important, it is only a part of a comprehensive mathematical background that includes more complex computation, an understanding of mathematical concepts, and the ability to think and reason to solve problems.” (Seeley 1).

In *About Teaching Mathematics*, Marilyn Burns writes the following:

> What about using timed tests to help children learn their basic facts? This makes no instructional sense. Children who perform well under time pressure display their skills. Children who have difficulty with skills, or who work more slowly, run the risk of reinforcing wrong learning under pressure. In addition, children can become fearful and negative toward their math learning. Also, timed tests do not measure children’s understanding. . . . It doesn’t ensure that students will be able to use the facts in problem-solving situations. Furthermore, it conveys to children that memorizing is the way to mathematical power, rather than learning to think and reason to figure out answers (2000, p. 157).

According to John A. Van de Walle (2006), speed (using timed tests) “is effective only for students who are goal oriented and who can perform in pressure situations. The pressure of speed can be debilitating and provides no positive benefits. The value of timed tests as a learning tool can be summed up as follows:

Timed Tests

- Cannot promote reasoned approaches to fact mastery
- Will produce few long-lasting results
- Reward few
- Punish many
- Should generally be avoided

“If there is any defensible purpose for a timed test of basic facts it may be for diagnosis—to determine which combinations are mastered and which remain to be learned. Even for diagnostic purposes there is little reason for a timed test more than once every couple of months” (pp. 95–96).

Research shows that timed tests can be more harmful than beneficial and are associated with lowering the level of fact mastery (Isaacs & Carroll, 1999). Prematurely demanding speed of fact retrieval can cause anxiety and can undermine understanding (Isaacs & Carroll). Timed tests cause many students to become quicker at immature approaches (Isaacs & Carroll; Ezbicki, 2008). For this reason, it is recommended that students master strategies for efficient fact retrieval prior to practicing those facts for fluency (accuracy and speed) (Woodward, 2006). Timed tests discourage the use of thinking strategies, as students are less likely to explore the more sophisticated strategies necessary to make progress if they are being timed (Isaacs & Carroll).

Daily timed tests, and even weekly or monthly timed tests, are unnecessary (Isaacs & Carroll, 1999). However, timed tests can be used every few months or so to assess fact proficiency (Isaacs & Carroll).

Principles and Standards for School Mathematics defines computational fluency as ". . . having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate *flexibility* in the computational methods they choose, *understand* and can explain these methods, and produce accurate answers *efficiently*. The computational methods that a student uses should be based on mathematical ideas that the student understands well, including the structure of the base-ten number system, properties of multiplication and division, and number relationships" (p. 152).

Many of the following activities should be used multiple times throughout the school year to encourage repeated exposure and practice for the basic facts.

Mathematical Language

Divide

Dividend

Division

Divisor

Factor

Multiplication

Multiply

Product

Quotient

Assessing Prior Knowledge

Materials: Math journals, Mental Math Strategies checklist (BLM 5–8.8)

Organization: Individual/Whole class

Procedure:

a) Ask students to solve each of the following problems in two different ways:
   - Rosa is planning to arrange 48 books on six shelves. If she puts an equal number of books on each shelf, how many books will she put on each shelf?
   - Mark has a six-page photo album. How many pictures does Mark have if each page holds eight pictures?

b) Have students share their solutions with the other members of the class. Encourage students to explain their reasoning by asking questions, such as:
   - "Which strategy did you use to solve the problem?"
   - "What is another strategy you could use to solve the problem?"
   - "Will the strategy work for other problems involving division (multiplication)? Show me."
   - "Which strategy do you prefer to use? Why?"

Observation Checklist

Use students' responses and BLM 5–8.8 to determine which strategies students know. Also, examine their responses to determine whether they can do the following:

- ❑ identify problem situations that call for the operation of multiplication
- ❑ identify problem situations that call for the operation of division
- ❑ describe and apply a thinking strategy to determine the product or quotient of two whole numbers
- ❑ describe and apply more than one thinking strategy to determine the product or quotient of two whole numbers

- **Describe the mental mathematics strategy used to describe a basic fact.**
- **Refine personal strategies to increase efficiency.**

Materials: Activity sheets that show the same array repeated about eight times. For example:

1. xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx	2. xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx	3. xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx	4. xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx
5. xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx	6. xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx	7. xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx	8. xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx

Organization: Individual/Whole class

Procedure:

a) Show students an 8 x 7 array. Ask them to split the array to show a strategy for finding the product of 8 x 7 and have them describe the corresponding number sentences. For example:

    xxxxxxx
    xxxxxxx
    xxxxxxx
    xxxxxx
    xxxxxxx
    xxxxxxx
    xxxxxxx

    xxxxxxx

   7 x 7 = 49
   1 x 7 = 7

   Therefore, 8 x 7 = 49 + 7 = 56

Ask students to split the remaining arrays in different ways to show the various strategies that can be used to find the product of 8 x 7. Have them record the corresponding number sentences for each strategy that they find.

b) Have students share the strategies they found with the other members of the class. Ask, "What strategy is the easiest? Why do you think this?"

c) Repeat the activity for other multiplication facts and have students compare the strategies that they find for each fact.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ describe the mental math strategy used to determine a multiplication fact such as skip counting by 1 or 2 groups from a known fact
- ❑ identify and apply different mental math strategies to determine a multiplication fact such as doubling, repeated doubling (e.g., 8 x 7 = [2 x 7] + [2 x 7] + [2 x 7] + [2 x 7] = 56), or doubling plus one or two groups (e.g., 8 x 7 = [3 x 7] + [3 x 7] + [2 x 7] = 56)

Materials: Paper and pencils

Organization: Whole class/Pairs

Procedure:

a) Have students study the following array and then ask them how knowing the facts of 5 can help them with other facts.

x x x x x x
x x x x x x
x x x x x x
x x x x x x
x x x x x x
x x x x x x
x x x x x x

5 x 6 = 30
2 x 6 = 12

Therefore, 7 x 6 = 42.

b) Have students describe how using a "think 5 facts" strategy can help them determine these fact problems. If necessary, have students draw the corresponding arrays.

9 x 6 =

8 x 3 =

6 x 4 =

7 x 7 =

c) Have students make a list of other facts that they could determine easily using a "think 5 facts" strategy. Have them share their lists with their partner and describe how they would use the strategy to solve each fact that they listed.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ describe how the "facts of 5" strategy can be used to determine a basic multiplication fact
- ❑ identify multiplication facts that can be determined using a "facts of 5" strategy

Materials: Paper and pencil

Organization: Pairs

Procedure:

a) Give students a fact problem such as 6 x 8. Have the first student in each pair do one part of the problem (e.g., 4 eights is 32). The second student must finish the problem—in this case, 2 eights is 16. The first student then adds the two parts together to determine the product. Have the students record the strategy that they used. Encourage students to split the problem into parts that are easy to find.

b) Repeat the activity several times, but have students switch roles.

c) Have students share the strategies they used with the other members of the class and discuss which strategies are the easiest and most efficient to use.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ identify and apply different mental math strategies to determine a multiplication fact
- ❏ describe the mental math strategies used to determine a multiplication fact

Materials: Paper and pencil

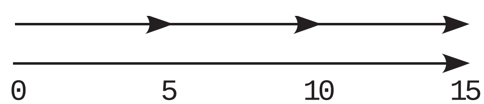
Organization: Whole class

Procedure:

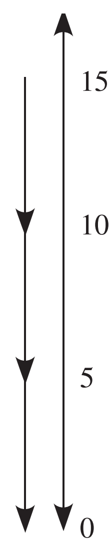
a) Ask students to write four number sentences using only the numbers 3, 5, and 15. When they finish, ask them to show the facts ($3 \times 5 = 15$, $5 \times 3 = 15$, $15 \div 3 = 5$, and $15 \div 5 = 3$) on number lines.

For example:

$3 \times 5 = 15$



$15 \div 5 = 3$



c) Ask students what they notice about the facts and how you showed them on the number lines. (Students should notice the inverse relationship between multiplication and division, although they may use the terms "opposite" or "backwards.")

d) Have students write four math sentences for each of the following triplets of numbers:

6 7 42

8 3 24

4 5 20

9 8 72

e) Ask students how a division fact can be determined by thinking a multiplication fact. Then have them use the relationship between multiplication and division to describe the thinking strategy for solving the following division problems. For example, for 36 ÷ 9 = , think 'some number' x 9 = 36 since 4 x 9 = 36, 36 ÷ 9 must equal 4.

- 48 ÷ 6 =   16 ÷ 4 =
- 63 ÷ 7 =   27 ÷ 3 =
- 18 ÷ 3 =   40 ÷ 8 =
- 32 ÷ 4 =   49 ÷ 7 =
- 56 ÷ 8 =   72 ÷ 9 =

Observation Checklist

Observe students' responses to determine whether they can do the following:

❑ identify the multiplication and division facts that can be associated with a given triplet of numbers

❑ identify the inverse of a multiplication fact

❑ identify the inverse of a division fact

❑ describe how a "think multiplication" strategy can be used to recall a division fact

❑ recognize the inverse relationship between multiplication and division

Materials: A deck of cards with the face cards removed.

Organization: Small groups

Caution: In some communities, playing cards are seen as a form of gambling and discouraged. Please be aware of local sensitivities before introducing this activity.

Procedure:

a) Tell students that they are going to play a variation of the game " I spy" with the members of their group. Explain how the game is played.

   1. Lay the cards on the playing surface face up in five rows of 8.
   2. Players take turns challenging each other to find two cards that have a specific product. The two cards must be next to each other horizontally, vertically, or diagonally. For example, if a 5 and a 3 are next to each other, a player could say, "I spy two cards whose product is 15."
   3. The other players look for the cards. The player who finds the right combination takes the two cards. If the combination cannot be found then the player who posed the "I spy" question takes the two cards.
   4. If a player makes an error or there is no such combination of cards, nobody collects any cards and the next player takes his or her turn.
   5. As cards are removed, the remaining cards should be rearranged to fill in the spaces.
   6. The game is over when all the cards have been picked up. The winner is the player with the most cards.

b) Demonstrate how the game is played and answer any questions students might have. Have students play the game.

Variation: Instead of using playing cards, copy four sets of number cards for each student (BLM 5–8.5). Omit the "0" card.

Observation Checklist

Observe students' responses to determine which

- ❑ facts that they can recall easily
- ❑ facts that they have difficulty recalling

Materials: Tic-tac-toe grids (BLM 5.N.3.1)

Organization: Pairs

Procedure:

a) Tell students that they will be playing "Multiplication Tic-tac-toe." Explain how the game is played.

1. Players decide who will be "X" and who will be "O."
2. Each player lists the numbers from 1 to 9 in a column beside the multiplication grid.
3. The first player crosses out any number in his or her column of numbers.
4. Beginning with the second player, the game proceeds in this manner. During a turn, a player crosses off any number in his or her column of 9 numbers that has not been crossed off. The player then multiplies that number by the last number crossed off by his or her opponent. If the product is on the tic-tac-toe board and not yet crossed off the player places an x or O over the product. For example:
   - player 1 crosses off the number 9 in his or her column of numbers
   - player 2 crosses off the number 7 in his or her column of numbers (Since 7 x 9 = 63, player 2 places an X [or O] over the 63 on the grid.)
   - player 1 crosses off 5 in his or her column of numbers (Since 5 x 7 = 35, player 1 places an O [or X] over the 35 on the grid.)
5. The game ends when any of the following occur:
   - a player gets three marks in a row (as in tic-tac-toe)
   - all of the numbers on the grid are marked off with either an X or an O
   - all nine numbers in a player's column of numbers are marked off

b) Demonstrate the game and answer any questions that students might have. Have the students play the game.

c) Have students play the game using these grids

d) Have students make their own multiplication tic-tac-toe grids and use them to play the game with their partner.

e) Have students discuss the strategies that they used to win the game.

Observation Checklist

Observe students' responses to determine which

- ❑ facts that they can recall easily
- ❑ facts that they have difficulty recalling
- ❑ strategies they are using to win the game

Materials: Copies of the division puzzle (BLM 5.N.3.2)

Organization: Individual/Pairs

Procedure:

a) Tell students that their task is to find the ten division facts that are hidden horizontally and vertically in the puzzle. Explain that two adjacent squares can be used to form a 2-digit number and show them that the numbers 8, 1, 9, and 9 in the first row form the fact 81 ÷ 9 = 9

b) Have students find the remaining facts. They should circle each fact that they find and then compare their answers with their partner.

c) Vary the activity by creating multiplication puzzles or by creating combined multiplication and division puzzles.

d) Have students create their own fact puzzles and exchange them with the other members of the class.

Observation Checklist

Check students' responses for the following facts:

- ❑ 81 ÷ 9 = 9 49 ÷ 7 = 7 56 ÷ 7 = 8 24 ÷ 3 = 8
- ❑ 63 ÷ 7 = 9 40 ÷ 5 = 8 9 ÷ 3 = 3 42 ÷ 6 = 7
- ❑ 16 ÷ 4 = 4 25 ÷ 5 = 5

Materials: A set of 27 cards for each group, two cards for each of the numbers 1 through 9 (BLM 5–8.5), and nine cards with the word "everyone" on it and a number from 1 through 9 under it (BLM 5.N.3.3); one-minute timers, paper and pencils

Organization: Small groups

Procedure:

a) Tell students that they will be playing a game involving the basic facts for division. Explain how the game is played.

   1. Shuffle the cards and place them face down on the playing area.
   2. The player whose birthday comes first starts the game.
   3. The first player turns over the top card. The player has two minutes to write as many division facts as he or she can that have the number on the card as a quotient. The second player acts as the timer and says "Stop!" when two minutes are up.
   4. The first player receives one point for each correct division fact that has the number on the card as the quotient.
   5. The second player turns over the next card and writes as many division facts as he or she can that have the number on the card as a quotient. The first player acts as the timer, and says "Stop!" when two minutes are up. The second player receives one point for each correct fact.
   6. If a player turns over an "everyone" card, all players write down as many division facts as they can that have the number on the card as a quotient. One player volunteers to be the timekeeper, and says "Stop!" when two minutes are up. Each player receives one point for each fact that has the number on the card as a quotient.
   7. The first player to get 50 points is the winner.

b) Demonstrate how the game is played and answer any questions students might have. Have students play the game.

Observation Checklist

Observe students' responses to determine which facts they

- ❑ recall easily
- ❑ have difficulty recalling

Materials: Number cards (BLM 5–8.5), observation form (BLM 5–8.1)

Organization: Groups of 3

Procedure:

a) Tell students that they will be playing a game involving the basic facts for multiplication and division. Explain how the game is played.

   1. Shuffle the cards and place them face down in a pile in the centre of the playing area.
   2. Two students sit facing each other while the third student sits so they can see the other two. The two players who are facing each other are the guessers. The third student is the caller.
   3. Each guesser chooses one card from the deck without looking at the card and holds it up to his or her forehead.
   4. The caller states the product of the numbers on the cards.
   5. The guesser who is the first to figure out what number is on his or her card wins both cards.
   6. The player who has the most cards after 10 rounds is the winner.
   7. The winner becomes the caller for the next game.

b) Demonstrate how to play the game and answer any questions students might have. Have students play the game.

Observation Checklist

Observe students' responses to determine which facts they

- ❏ recall easily
- ❏ have difficulty recalling
- ❏ use a strategy for
- ❏ have difficulty using a strategy for

Use the observation form (BLM 5–8.1) to assess how well students work together.

Materials: Single digit multiplication chart activity (BLM 5.N.3.4)

Organization: Whole class and individual

Procedure:

Demonstrate the patterns that exist in the multiplication chart. Have students colour in the facts according to each pattern.

a) The doubles:

x	0	1	2	3	4	5	6	7	8	9
0										
1			2							
2		2	4	6	7	10	12	14	16	18
3			6							
4			8							
5			10							
6			12							
7			14							
8			16							
9			18							

b) The five facts (clock facts): 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60

c) The "nifty nines": Each decade is one less than the number of nines and each answer's digits add up to nine.

9 x 9 = **8**1

8 x 9 = **7**2

7 x 9 = **6**3

6 x 9 = **5**4

5 x 9 = **4**5

4 x 9 = **3**6

3 x 9 = **2**7

d) The zeros and the facts multiplied by one.

e) Facts with 3: Doubles and one more group.

f) Facts with 4: Double Double

g) The last nine facts: These are often the most difficult for students: 6 x 6, 6 x 8, 7 x 6, 8 x 6, 7 x 8, 8 x 7, 7 x 7, and 8 x 8. Have students suggest strategies for learning these facts.

By the end, students should have a fully coloured multiplication chart by fact pattern.

More practice with multiplication facts and charts can be found at the following websites:

- http://illuminations.nctm.org/ActivityDetail.aspx?ID=155
- http://eworkshop.on.ca/edu/resources/guides/NSN_vol_3_Multiplication.pdf
- http://dvl.ednet.ns.ca/mental-math
- http://www.aplusmath.com/cgi-bin/Homework_Helper/mtable
- http://www.mathsisfun.com/tables.html
- http://nzmaths.co.nz/taxonomy/term/195
- https://www.det.nsw.edu.au/eppcontent/glossary/app/.../factsheet/4024.pdf

Observation Checklist

Observe students' responses to determine which facts they

- ❑ recall easily
- ❑ have difficulty recalling
- ❑ use patterns for recall
- ❑ have other strategies to recall

Use the observation form (BLM 5–8.1) to assess students.

Materials: Blank multiplication chart—numbers from 0 to 9 across the top and 0 to 9 along the side, three pencil crayons—green, yellow, and red

Organization: Individual

Procedure:

On a blank multiplication chart, have students shade in the answers to the multiplication facts that they know for sure with the green pencil crayon. If they think they know, have them shade the square in yellow. For those facts they do not know, have them shade the square in red. Walk around while students are doing this and point to a green square to ensure students know these facts periodically. For the red facts, have students write these down for study, practise, and review. Put into a math portfolio to revisit throughout the year.

Usually students know the zeros, the ones, the fives, the nines, doubles, etc., but have trouble with the 6 x 7, 8 x 7, etc. Students should practise the ones they don't know.

Observation Checklist

Observe students' responses to determine which facts they

- ❑ recall easily
- ❑ have difficulty recalling

Use the observation form (BLM 5–8.1) to assess students.

Materials: Race Around the Clock activity (BLM 5.N.3.5) or hand-draw the clock shape on individual white boards, 9-sided number cube

Organization: Individual

Procedure:

Have students write the numbers from 1 to 9 in the boxes on the inside edge of the clock in random order and placement. **Note:** Some numbers will be repeated. Roll the number cube and students write the number rolled in the centre of the clock. Say "go," and students write the multiplication fact for each number around the clock face in the outside circle as quickly as they can.

Note: This is not a timed testing activity. Each student can assess her or his own progress and practise the facts they don't know.

Observation Checklist

Observe students' responses to determine which facts they

- ❑ recall easily
- ❑ have difficulty recalling

Use the observation form (BLM 5–8.1) to assess students.

Materials: Individual white boards or sheets of paper/student notebook or journal

Organization: Individual

Procedure:

Students work on multiplication fact practice exercises. Say a math fact (4 x 7) and students write their answer on the left side of the T-chart. After 5 seconds, say the answer (28) and the students write that answer on the righthand side opposite their answer. If the student is correct, the answers will match; if not, they will know they must practise that fact. Practice 10 to 15 facts each day. The important part of this exercise is for students to hear the correct answer to the multiplication fact and then to reinforce or correct that fact.

Student Answer	Teacher Answer

Variation: Instead of using playing cards, copy four sets of number cards for each student (BLM 5–8.5). Omit the “0” card.

Note: This activity can also be used for addition, subtraction, or division facts. The 10 to 15 facts of the day may be all one operation or may be a combination of operations.

Observation Checklist

Observe students’ responses to determine which facts they

- ❑ recall easily
- ❑ have difficulty recalling

Use the observation form (BLM 5–8.1) to assess students.

Materials: Playing cards with face cards removed

Caution: In some communities, playing cards are seen as a form of gambling and should be discouraged. Please be aware of local sensitivities before introducing this activity.

Organization: Pairs

Procedure:

To practise multiplication facts, have students divide a deck of cards with the face cards removed into two piles. Each student takes a pile and they will play against each other. Each student turns his or her first card over and the first person to say the multiplication fact wins the hand. Play continues. Matching players of like abilities makes a more enjoyable game.

Variation: Instead of using playing cards, copy four sets of number cards for each student (BLM 5–8.5). Omit the "0" card.

Observation Checklist

Observe students' responses to determine which facts they

- ❏ recall easily
- ❏ have difficulty recalling

Use the observation form (BLM 5–8.1) to assess students.

Materials: Playing cards with face cards removed and 6-sided number cube

Caution: In some communities, playing cards are seen as a form of gambling and should be discouraged. Please be aware of local sensitivities before introducing this activity.

Organization: Pairs

Procedure:

Roll the number cube to arrive at target number (e.g., 4). Have students turn over five playing cards or do this using an overhead projector, document camera, or smart board (e.g., 4 5 1 8 7). The challenge is for students to arrive at the target number using these 5 numbers and all four operations. A possible solution is $\underline{8} \times \underline{5} = 40$, $40 \div \underline{4} = 10$, $10 - \underline{7} = 3$, $3 + \underline{1} = 4$.

Observation Checklist

Observe students' responses to determine which facts they

- ❏ recall easily
- ❏ have difficulty recalling

Use the observation form (BLM 5–8.1) to assess students.

Grade 5: Number (5.N.4)

Enduring Understandings:

There are many strategies that can be used to compute the answers to computational problems.

Strategies for computing the answers to computational problems involve taking apart and combining numbers in a variety of ways.

General Outcome:

Develop number sense.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
5.N.4 Apply mental mathematics strategies for multiplication, such as ■ annexing then adding zeros ■ halving and doubling ■ using the distributive property [C, ME, R]	➢ Determine the products when one factor is a multiple of 10, 100, or 1000 by annexing zero or adding zeros (e.g., for 3 x 200, think 3 x 2 and then add two zeros). ➢ Apply halving and doubling when determining a product (e.g., 32 x 5 is the same as 16 x 10). ➢ Apply the distributive property to determine a product involving multiplying factors that are close to multiples of 10 [e.g., 98 x 7 = (100 x 7) – (2 x 7)].

PRIOR KNOWLEDGE

Students may have had experience with the following:

- Modelling a multiplication problem using the distributive property
- Multiplying a 1-digit number times a 2-digit whole number or 3-digit whole number
- Using arrays to represent multiplication problems
- Adding and subtracting whole numbers less than 10 000
- Connecting concrete representations of multiplication problems with symbolic representations

Related Knowledge

Students should be introduced to the following:

- Determining multiplication facts to 81 and the related division facts

Background Information

The term **mental math** is most commonly used to describe computation that is done without paper and pencil or any calculation device such as a computer or calculator. A focus on mental math can help students become more adept at reasoning with numbers and enable them to gain new insights into operations and number relationships. It can also help them become adept at estimating, a skill that has become more important because of its practicality and the widespread use of computers and calculators.

Mental math usually involves the use of nonstandard algorithms such as repeated doubling or doubling and halving. Perhaps the most commonly used mental math strategy is the dropping and reattaching of common zeros. For example, to find the product of 3 x 70, think 3 x 7 = 21 and then "tack on" the zero that was dropped to get 210. The terminology " add zero" should be avoided, since it is misleading. 21 + 0 is 21 not 210.

Teachers can help students become adept at mental computation by making mental math an integral part of their instructional programs. In particular, they need to

- help students develop mental math strategies that make sense to them
- provide frequent practice sessions that are about 10 minutes in duration
- help students develop confidence by gradually increasing the complexity of the mental computations
- encourage students to use mental math whenever possible
- encourage students to develop their own mental math strategies
- make sure that students know the difference between mental math and estimation
- model the use of mental math and estimation

Mathematical Language

Doubling

Factor

Halving

Product

Multiple

Multiplication

Assessing Prior Knowledge

Note: This activity is also used to assess students' readiness for outcome 5.N.5.

Materials: Paper and pencils

Organization: Individual/Whole class

Procedure:

a) Ask students to solve the following problem in two different ways.
   - There are eight rows of chairs in the school auditorium. If each row has 45 chairs, how many chairs are there altogether?

b) Have students share their solutions and strategies with the other members of the class. Record the strategies students use on the board or overhead, and encourage discussion by asking questions, such as:
   - "Is there another strategy you could use to solve the problem? What is it?"
   - "Which strategy is easier to use? Why do you think it is easier?"
   - "Will the strategy work for other problems involving multiplication? Show me."
   - "Which strategy do you prefer to use? Why?"

Observation Checklist

Use students' responses to determine which strategies they know. Also, examine their responses to determine whether they can do the following:

- ❏ identify problem situations that require the operation of multiplication
- ❏ determine the correct product of a 1-digit whole number times a 2-digit whole number
- ❏ use more than one strategy to solve a multiplication problem involving a 1-digit whole number times a 2-digit whole number.
- ❏ determine the product of a 1-digit number times a 2-digit number using the distributive property.

- **Determine the products when one factor is a multiple of 10, 100, or 1000 by annexing zero or adding zeros.**

Materials: Base-10 blocks, strings of 100 beads, or other place-value materials, centimetre grid paper (BLM 5–8.9), copies of multiplication problems (BLM 5.N.4.1)

Organization: Pairs/Whole class

Procedure:

a) Have students explore multiplying by powers of ten by asking them to solve the problems. Let students know that they can use materials or draw diagrams to help them solve the problems.

b) Have students explain the strategies that they used to solve the problems. Encourage students to discuss the similarities among the problems and any patterns that they see by asking them questions such as:
   - "How are the problems alike?"
   - "How are the solutions alike?"
   - "Why do you think this is so?"

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ recognize that the problems describe multiplication situations
- ❏ use models (materials or diagrams) appropriately to solve the problems
- ❏ represent the problems symbolically (e.g., 4 x 20 = 80)
- ❏ recognize that the problems involve calculating the products of multiples of 10
- ❏ recognize that the solutions involve a power or multiple of 10

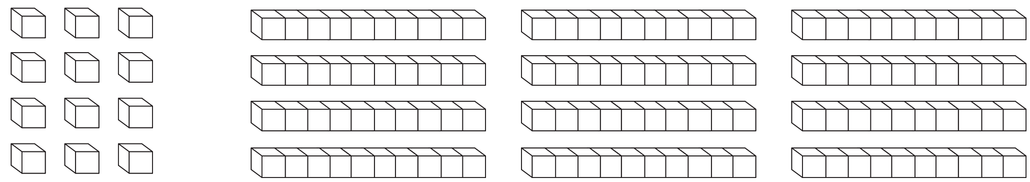
Materials: Base-10 blocks and calculators

Organization: Whole class

Procedure:

a) Show students these arrays.

4 x 3 ones   4 x 3 tens



4 x 3 = 12   4 x 3 tens = 12 tens = 120

Have students compare the two arrays and find the products. Encourage discussion by asking:

- "How are the two arrays alike?" (Both have 4 rows of three.)
- "How are the two arrays different?" (The first has ones in each row, the second has tens.)
- "How are the products different?" (In the first we have ones; in the second we have tens.)

b) Repeat part (a) several times. For example, have students compare these arrays and record the corresponding multiplication sentences.

- 8 x 2 ones and 8 x 2 tens ➝ 8 x 2 = 16 and 8 x 2 tens = 16 tens = 160
- 3 x 8 ones and 3 x 8 tens ➝ 3 x 8 = 24 and 3 x 8 tens = 24 tens = 240
- 7 x 5 ones and 7 x 5 tens ➝ 7 x 5 = 35 and 7 x 5 tens = 35 tens = 350
- 2 x 9 ones and 2 x 9 tens ➝ 2 x 9 = 18 and 2 x 9 tens = 18 tens = 180
- 4 x 6 ones and 4 x 6 tens ➝ 4 x 6 = 24 and 4 x 6 tens = 24 tens = 240
- 9 x 3 ones and 9 x 3 tens ➝ 3 x 9 = 27 and 9 x 3 tens = 27 tens = 270

Help students generalize their findings by asking them questions such as:

- "What patterns do you see?"
- "What conclusions can you draw?"
- "What do you know about the product of a number times a multiple of 10?"
- "What rule can you use when multiplying a number of ones by a multiple of 10?"

c) Provide students with a variety of mental math exercises. For example, ask students to solve these problems mentally:

- 8 x 60
- 3 x 70
- 20 x 4
- 60 x 5
- 9 x 40
- 30 x 5
- 10 x 7

d) Extend students' knowledge of multiplying by a multiple of ten by having them use their calculators to solve problems like the following:

- 15 x 10   125 x 10
- 15 x 20   125 x 20
- 15 x 30   125 x 30
- 15 x 40   125 x 40
- 15 x 50   125 x 50
- 15 x 60   125 x 60
- 15 x 70   125 x 70
- 15 x 80   125 x 80
- 15 x 90   125 x 90

Have students record any patterns that they see and determine a rule for multiplying a number by a multiple of 10.

e) Use a similar procedure for multiplying ones times hundreds and ones times thousands.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ recognize that ones times tens is tens
- ❑ recognize that when multiplying by ten, the product has a zero in the ones position
- ❑ find the product of ten times a number
- ❑ recognize the rule that to find the product of a number times a multiple of ten, multiply the number times the digit in the tens position, then "tack on" a zero to show tens

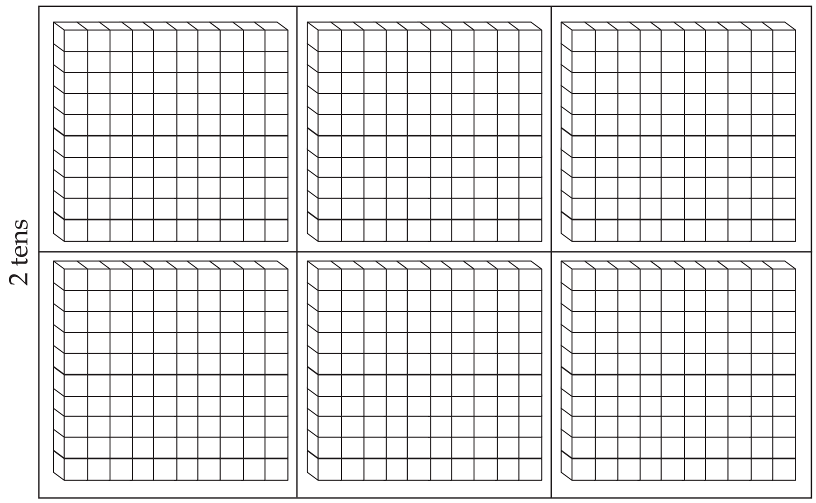
Use similar criteria for multiplying by 100 and 1000.

Materials: Base-10 blocks, base-10 grid paper (BLM 5–8.10), or graph paper

Organization: Whole class

Procedure:

a) Ask students to use the base-10 blocks or graph paper to make a 2-tens by 3-tens array.



When students finish, ask the following questions:

- "How many rows are there?" (2 tens)
- "How many columns are there?" (3 tens)
- "How many hundreds are there?" (6 hundreds)
- "What multiplication sentence does the array illustrate?" (2 tens x 3 tens = 6 hundreds; 20 x 30 = 600)

b) Repeat part (a) several times. For example, have students make these arrays and record the corresponding multiplication sentences.

- 1 ten x 6 tens ⟶ 1 tens x 6 tens = 6 hundreds ⟶ 10 x 60 = 600
- 5 tens x 3 tens ⟶ 5 tens x 3 tens = 15 hundreds ⟶ 50 x 30 = 1500
- 8 tens x 4 tens ⟶ 8 tens x 4 tens = 32 hundreds ⟶ 80 x 40 = 3200
- 3 tens x 1 ten ⟶ 3 tens x 1 ten = 3 hundreds ⟶ 10 x 30 = 300
- 9 tens x 5 tens ⟶ 9 tens x 5 tens = 45 hundreds ⟶ 90 x 50 = 4500
- 6 tens x 6 tens ⟶ 6 tens x 6 tens = 36 hundreds ⟶ 60 x 60 = 3600

Help students generalize their findings by asking them questions such as:

- "What patterns do you see?"
- "What conclusions can you make?"
- "What do you know about the product of a multiple of ten times a multiple of ten?"
- "What can you use to find the product of a multiple of ten times a multiple of ten?"

c) Provide students with a variety of mental math activities. For example, ask students to solve these problems mentally:

- 20 x 40
- 30 x 60
- 70 x 20
- 90 x 80
- 10 x 60
- 40 x 50
- 20 x 50
- 70 x 60

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ recognize that tens x tens is hundreds
- ❏ recognize that when multiplying a multiple of ten times a multiple of ten, there are zeros in the ones and the tens position of the product
- ❏ find the product of a multiple of ten times a multiple of ten
- ❏ recognize the rule that to find the product of a multiple of ten times a multiple of ten, find the product of the digits in the tens position, then "tack on" two zeros to show hundreds

Materials: Calculators and copies of the game sheet cut into strips (BLM 5.N.4.2)

Organization: Pairs

Procedure:

a) Tell students that they will be playing a game with their partner. Explain how to play the game.

1. Players take turns selecting tasks from the pile.
2. The other player uses his or her calculator to carry out the task. If the player can perform the task, he or she scores a point. For example, suppose a player is given the task to change 2 into 120 using multiplication, in one input. The player enters 2 into his or her calculator, and then inputs "x 60 =" to get 120.
3. The player with the most points after 10 rounds of the game is the winner.

b) Demonstrate how to play the game and answer any questions students might have. Have students play the game.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ recognize multiples of 10
- ❏ recognize multiples of 100
- ❏ determine the missing factor in multiplication problems involving multiples of 10 and 100

- **Apply halving and doubling when determining a product.**

Materials: Counters

Organization: Whole class

Procedure:

a) Have students make a rectangular array with eight rows of three counters. Ask them how many counters they have altogether. “What strategy did you use to find the total?” “What is another way you could find the total?” Focus on partitioning strategies, such as double four threes or five threes and three threes. Record the appropriate number sentence(s) on the board. Record the number sentence 8 x 3 = 24 on the board or overhead.

x x x
x x x
x x x
x x x
x x x
x x x
x x x
x x x

b) Ask students to work out how they can change their array to show 4 x 6. If necessary, show students how the 8 x 3 array can be partitioned to illustrate 4 x 6. Ask, “How many counters do you have altogether?” Record the number sentence 4 x 6 = 24 on the board and note that 8 x 3 and 4 x 6 have the same product. Record 8 x 3 = 4 x 6. Ask students why they think this is so. Help students make the connection between their actions on the materials and the generalization that one set of factors can be changed into the other by doubling and halving.

x x x	x x x x x x
x x x	x x x x x x
x x x	x x x x x x
x x x	x x x x x x
x x x	
x x x	
x x x	
x x x	

c) Have students use their counters to test if the following statements are true:

$4 \times 5 = 2 \times 10$

$2 \times 8 = 4 \times 4$

$4 \times 3 = 2 \times 6$

$4 \times 10 = 8 \times 5$

d) Ask students to match each of the following multiplication problems with an equivalent problem. Have students use their counters to check their answers.

1.	3 x 16	6 x 4
2.	20 x 3	3 x 28
3.	7 x 4	6 x 8
4.	9 x 4	14 x 2
5.	6 x 14	10 x 6
6.	12 x 2	18 x 2

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ double a number
- ❑ find half of a number
- ❑ recognize that the product is the same if one factor is doubled and the other factor is halved
- ❑ determine an equivalent multiplication problem by doubling and halving its factors

Materials: Paper and pencil

Organization: Whole class/Small groups

Procedure:

a) Tell students that they will be doing a mental math activity. Explain that you will be giving them some multiplication problems. You will only say a problem once and they will only have enough time to record the answer.

b) Give students the following problems:

1. What is 6 doubled?
2. What is half of 14?
3. What is 10 doubled?
4. What is 8 doubled?
5. What is half of 24?
6. What is half of 18?
7. What is 3 doubled plus half of 10?
8. What is half of 4 plus 11 doubled?
9. What is 12 doubled plus 5 doubled?
10. What is half of 30 plus half of 20?

c) Have students share their answers with the other members of the class.

d) Have each group discuss the following question: "What mental math strategies can you use to find the product of 15 x 18?" Have the groups share their strategies. Encourage students to discuss the advantages and disadvantages of using each of the suggested strategies.

e) If no one suggests doubling and halving, show students the strategy. Explain that one way to determine the product of 15 x 18 is to double 15 and multiply it by half of 18. Since 15 doubled is 30 and half of 18 is 9, the product of 15 times 18 is the same as 30 times 9, which is equal to 270. Do two or three more examples such as

1. 32 x 21
2. 16 x 42
3. 8 x 84
4. 4 x 168
5. 2 x 336
6. 1 x 672

f) Ask students to use the doubling and halving strategy to determine the following products:

1. 12 x 45
2. 16 x 15
3. 14 x 25
4. 8 x 35
5. 18 x 25

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ double a number
- ❑ find half a number
- ❑ determine the product of two numbers by applying a doubling and halving strategy

■ **Apply the distributive property to determine a product involving multiplying factors that are close to multiples of 10.**

Materials: Base-10 blocks or counters

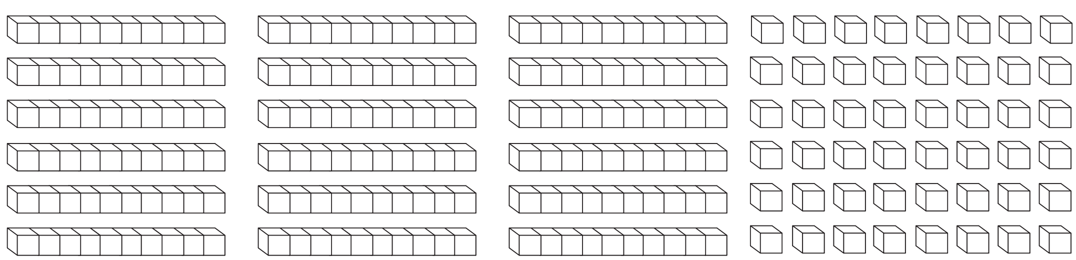
Organization: Whole class

Procedure:

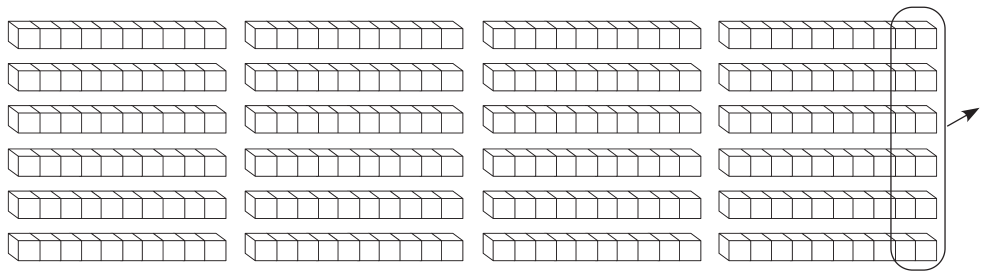
a) Present students with the following problem:

- ■ Ellen keeps the stamps she collects in a book. There are 38 stamps on each page of her book. If there are six pages in her book, how many stamps does she have altogether?

b) Ask students to identify a strategy that they would use to solve the problem. As students explain their strategy, model it with the materials. If no one suggests using the distributive property of multiplication over subtraction, explain the strategy while modelling it with materials.



There are 6 pages with 38 stamps on each page, so we need to make a 6 x 38 array. If we add 2 to each 38, we have six 40s or 240. Now take the 12 that we added away, which leaves 228.



c) Do two or three more examples and then ask students to solve the following problems using the strategy. Let students use materials to help them solve the problems.

1. 3 x 29 = ______
2. 7 x 18 = ______
3. 4 x 49 = _______
4. 2 x 58 = _______
5. 8 x 28 = _______

d) Have students share their answers and explain how they used the strategy to solve each of the problems.

e) Work with students to represent their solutions concretely (i.e., with base-10 blocks), pictorially (as above), and symbolically (i.e., 6 x 38 = 6 x 40 — 6 x 2 or "6 groups of 38 is the same as 6 groups of 40 subtract 6 groups of 2).

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ apply the distributive property to determine a product when one of the factors is close to a multiple of ten
- ❏ calculate the correct product of a 1-digit number times a multiple of ten
- ❏ calculate the correct difference between two whole numbers
- ❏ explain how to use the strategy to find the product when one of the factors is close to a multiple of ten

Materials: Copies of the Products activity (BLM 5.N.4.3) and base-10 materials

Organization: Individual/Partner

Procedure:

a) Have students complete the activity.

b) Have students check their answers with their partner. If discrepancies arise, have students use materials to determine the correct answer.

c) Have students discuss the strategies that they used to find the products in part B. Encourage students to describe the strategies that they used and explain why they choose them. Encourage students to use concrete, pictorial, and symbolic representations.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ apply the distributive property to determine a product when one of the factors is close to a multiple of 10
- ❑ calculate the correct product of a 1-digit number times a multiple of 10
- ❑ calculate the correct difference between two whole numbers
- ❑ recognize when to use the distributive strategy and the halving and doubling strategy

NOTES

Grade 5: Number (5.N.5)

Enduring Understandings:

There are a variety of strategies that can be used to compute the answers to computational problems.

Strategies for computing the answers to computational problems involve taking apart and combining numbers in a variety of ways.

General Outcome:

Develop number sense.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
5.N.5 Demonstrate an understanding of multiplication (1- and 2-digit multipliers and up to 4-digit multiplicands), concretely, pictorially, and symbolically, by ■ using personal strategies ■ using the standard algorithm ■ estimating products to solve problems. [C, CN, ME, PS, V]	➢ Illustrate partial products in expanded notation for both factors [e.g., for 36 x 42, determine the partial products for (30 + 6) x (40 + 2)]. ➢ Represent both 2-digit factors in expanded notation to illustrate the distributive property [e.g., to determine the partial product of 36 x 42, (30 + 6) x (40 + 2) = 30 x 40 + 30 x 2 + 6 x 40 + 6 x 2 = 1200 + 60 + 240 + 12 = 1512]. ➢ Model the steps for multiplying 2-digit factors using an array and base-10 blocks, and record the process symbolically. ➢ Describe a solution procedure for determining the product of two 2-digit factors using a pictorial representation, such as an area model. ➢ Model and explain the relationship that exists between an algorithm, place value, and number properties. ➢ Determine products using the standard algorithm of vertical multiplication. (Numbers arranged vertically and multiplied using single digits, which are added to form a final product.) ➢ Solve a multiplication problem in context using personal strategies, and record the process. ➢ Refine personal strategies such as mental math strategies to increase efficiency when appropriate [e.g., 16 x 25 think 4 x (4 x 25) = 400].

Prior Knowledge

Students may have had experience with the following:

- Using personal strategies to solve multiplication problems involving a 1-digit number times a 2-digit or 3-digit number
- Using arrays to represent multiplication problems
- Connecting concrete representations of multiplication problems with symbolic representations
- Modelling a multiplication product using the distributive property
- Estimating the product of a 1-digit number and a 2-digit or 3-digit number
- Adding and subtracting whole numbers less than 10 000
- Demonstrating an understanding of area of regular 2-D shapes

Related Knowledge

Students should be introduced to the following:

- Determining multiplication facts to 81
- Determining the product of two numbers when one of the factors is a multiple of 10, 100, or 1000
- Determining the product of a multiple of 10 times a multiple of 10

Background Information

An **algorithm** is a system of finite procedures for solving a particular class of problems. The best known algorithms are the traditional paper-and-pencil procedures for adding, subtracting, multiplying, and dividing. Along with these standard algorithms, mathematics instruction should include an emphasis on understanding through mental mathematics, estimation, the use of technology, the development of invented procedures, and the use of alternative algorithms, such as area model multiplication, and adding up to solve subtraction problems.

By encouraging students to develop their own computation strategies and allowing them to use alternative algorithms, the emphasis in mathematics instruction is shifted to reasoning, problem solving, and conceptual understanding. Providing students with opportunities to invent their own strategies and use alternative algorithms enhances their number and operation sense. Students become more flexible in their thinking, more aware of the different ways to solve a problem, and more adept at selecting the most appropriate procedure for solving a problem. Discussion of the algorithms or strategies and their relationship to place value and number properties can also help students develop better reasoning and communication skills.

The standard algorithm for multiplication is where numbers are arranged vertically and multiplied using single digits, which are added to form a final product. Students are expected to use the standard algorithm as one of the tools for computation.

When teaching the traditional algorithm for multiplication, it is important to follow the concrete, pictorial, and symbolic sequence of teaching. The important idea is to allow students to construct meaning, not memorize procedures without understanding. Students' misconceptions (or their fuzzy understandings) can be reinforced by a poorly understood algorithm. Students should be able to explain the relationship that exists among an algorithm, place value, and number properties.

Teachers can facilitate students' understanding and use of a variety of computational strategies by

- providing a supporting and accepting environment
- allowing time for exploration and experimentation
- embedding computational tasks in real-life situations
- allowing students to discuss, analyze, and compare their solution strategies
- encouraging discussion that focuses on place value and number properties when defending the choice of a particular algorithm or strategy
- understanding that a child needs to be efficient at computation and that this looks different for each student

Note: For further information regarding using models and methods of division to build understanding, see Appendix 2: Models for Multiplying 2-Digit Numbers by 2-Digit Numbers. Also see resources such as the following:

- "Big Ideas for Teaching Mathematics Grades 4–8" by Marian Small (found in Chapter 2 of *Varied Approaches for Multiplication and Division*)
- "Making Math Meaningful to Canadian Students K–8" by Marian Small (found in Chapter 10 of *Computational Strategies: Operations with Whole Numbers*)
- *Teaching Student Centered Mathematics* by John A. Van de Walle

Mathematical Language

Array

Expanded notation

Factor

Multiplication

Multiply

Partial product

Product

Assessing Prior Knowledge

Note: This activity is also used to assess students' readiness for outcome 5.N.4.

Materials: Paper and pencils

Organization: Individual/Whole class

Procedure:

a) Ask students to solve the following problem in two different ways:
   - There are eight rows of chairs in the schools' auditorium. If each row has 45 chairs, how many chairs are there altogether?

b) Have students share their solutions and strategies with the other members of the class. Record their strategies on the board or overhead, and encourage discussion by asking questions, such as the following.
   - "Is there another strategy you could use to solve the problem? What is it?"
   - "Which strategy is easier to use? Why do you think it is easier?"
   - "Will the strategy work for other problems involving multiplication? Show me."
   - "Which strategy do you prefer to use? Why?"

Observation Checklist

Use students' responses to determine which strategies they know. Also, examine their responses to determine whether they can do the following:

- ❏ identify problem situations that require the operation of multiplication
- ❏ determine the correct product of a 1-digit number times a 2-digit number
- ❏ use more than one strategy to solve a multiplication problem involving a 1-digit number times a 2-digit number
- ❏ determine the product of a 1-digit number times a 2-digit number using the distributive property

- **Model the steps for multiplying 2-digit factors using an array and base-10 blocks, and record the process symbolically.**
- **Describe the solution procedure for determining the product of two 2-digit factors using a pictorial representation, such as an area model.**

Materials: Dot paper

Organization: Whole class

Procedure:

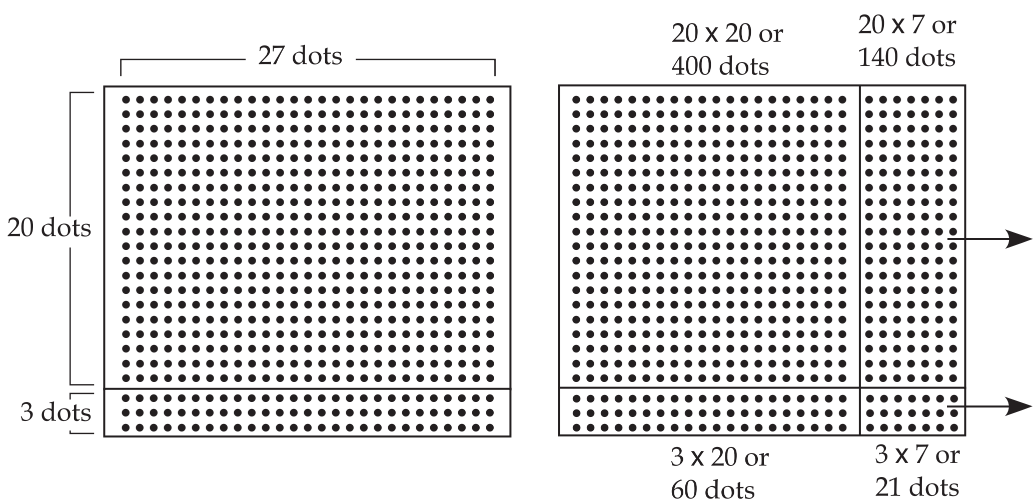
Note: Before beginning this task, you may need to determine the students' understanding of area as the amount of space taken up by a 2-D shape.

a) Present students with the following situation and ask them what they need to do to solve the problem:
   - There are 23 rows of cars in the shopping mall's parking lot. Each row has 27 cars in it. How many cars are parked in the lot?

b) Have students model the scenarios using base-10 blocks. Each unit cube represents one parking stall.

c) Have students draw a border around an array (23 x 27) that represents the cars in the parking lot. Ask them to partition the array in a way that will make it easier to find the total number of dots.

d) Have students share how they partitioned the array. Introduce the following ways of partitioning the array if students do not suggest them, and discuss how they are related. Students should not yet be required to record the process symbolically.

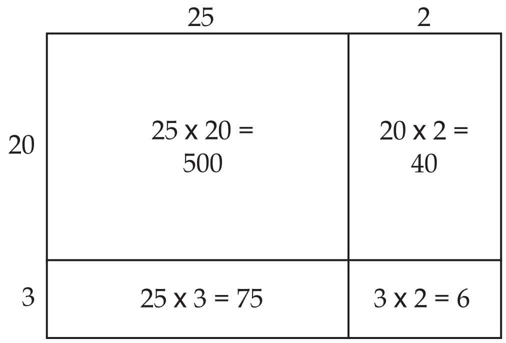


Since $20 \times 27 = 20 \times (20 + 7) = (20 \times 20) + (20 \times 7)$

and $3 \times 27 = 3 \times (20 + 7) = (3 \times 20) + (3 \times 7)$

then $(20 + 3) \times 27 = (20 \times 20) + (20 \times 7) + (20 \times 3) + (3 \times 7)$

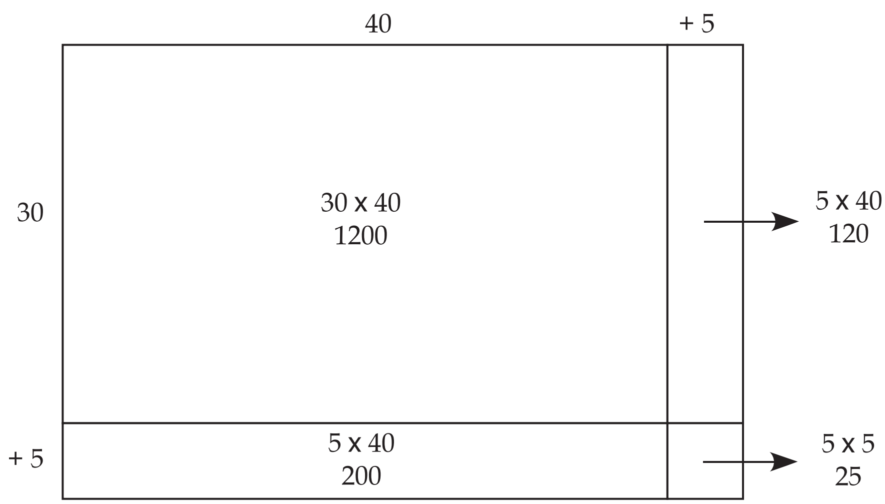
Or



Total Area = 500 + 40 + 75 + 6
Total Area = 621

Emphasize that the whole numbers can be "broken apart" into more convenient pieces in order to make the computation easier. Make sure that students understand that this will not affect their answer.

e) Do another example of using place value to find the product of two numbers—that is, show students how the product of 35 x 45 can be found by partitioning an array into four parts that can be associated with the expanded forms of the factors and finding the sum of the parts.



Once students become comfortable, they can draw the representation without needing the appropriate number of dots.

Use the sum of the individual (partial) products to find the total value of the original product:

35 x 45

= (30 + 5) x (40 + 5)

= (30 x 40) + (30 x 5) + (5 x 40) + (5 x 5)

= 1200 + 150 + 200 + 25

= 1575

f) Ask students to solve each of the following problems by partitioning an array and finding the sum of the parts, as illustrated in part (e). Have them represent these sentences concretely and pictorially in as many ways as possible.

- 42 x 15
- 22 x 37
- 21 x 24
- 18 x 23

Observation Checklist

Examine students' responses to determine whether they can do the following:

- ❏ illustrate partial products in expanded notation for both factors
- ❏ represent both 2-digit factors in expanded notation to illustrate the distributive property
- ❏ describe a solution procedure for determining the product of two 2-digit numbers using pictorial representations
- ❏ calculate correct sums and products
- ❏ model a 2 x 2 digit multiplication concretely and pictorially

Examine students' responses to determine whether any errors are due to

- ❏ carelessness
- ❏ not knowing a basic multiplication or addition fact
- ❏ a procedural error (e.g., renaming and regrouping incorrectly when finding the sum of two numbers or not finding all the partial products in a multiplication problem)

- **Illustrate partial products in expanded notation for both factors.**
- **Represent both 2-digit factors in expanded notation to illustrate the distributive property.**
- **Model the steps for multiplying 2-digit factors using an array and base-10 blocks, and record the process symbolically.**
- **Describe a solution procedure for determining the product of two 2-digit factors using a pictorial representation, such as an area model.**

Materials: Base-10 blocks

Organization: Whole class/Pairs

Procedure:

a) Ask students to solve the following problem:

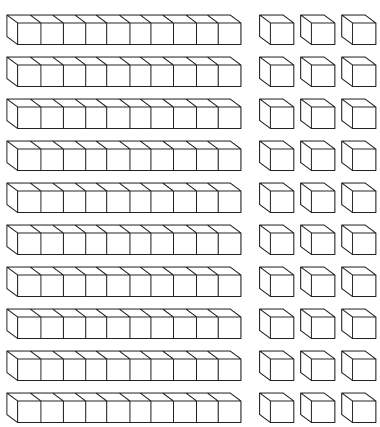
- The students enrolled in the community centre's swimming program are lined up in rows to get their picture taken. There are 12 rows with 13 students in each row. How many students are enrolled in the swimming program?

b) Have students share their solutions to the problem. Encourage them to discuss the strategies they used by asking the following questions:

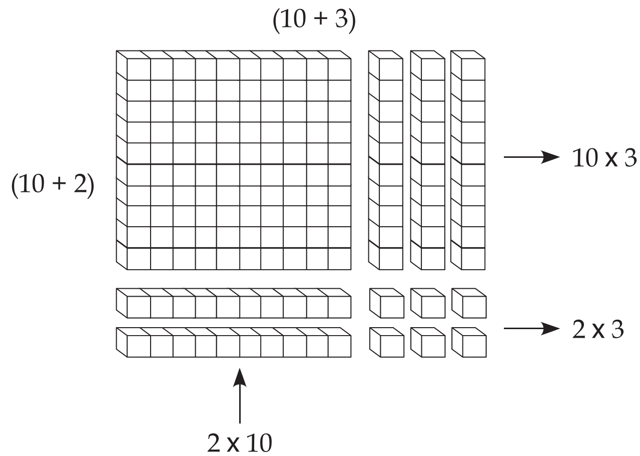
- "How did you find your answer?"
- "Will your strategy work for other problems?"
- "Is there another strategy you could use?"
- "Which strategy is more efficient?"

c) If students do not suggest using the distributive property, illustrate the strategy by making an array with the base-10 blocks. The array should consist of 12 rows of 13 blocks (or 13 rows of 12 blocks).

12 x 13 = 12 (10 + 3)

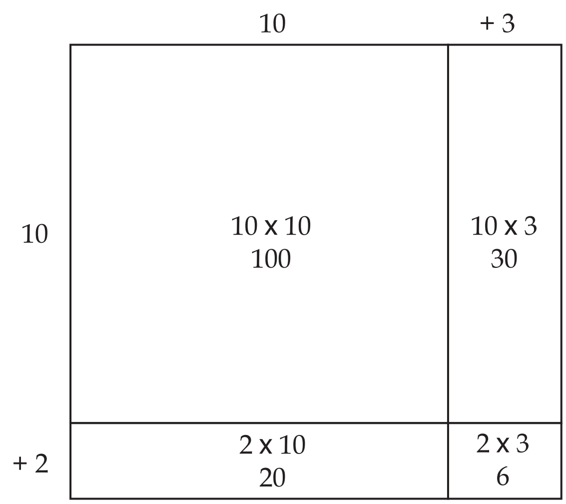


Since 10 longs equal 1 flat and 10 units equal 1 long, the array can be simplified by exchanging these blocks.



d) Model several more 2 x 2 digit multiplication scenarios using base-10 blocks.

e) Emphasize that the array has been partitioned into four parts. Each part represents a partial product. These four parts can be shown in a diagram called an area model:



The product of the original question is simply the sum of each for the four sections.

f) The partial products are the result of expressing each factor in expanded notation, and multiplying each addend in the first factor by each addend in the second factor. The product is the sum of the partial products and can be expressed symbolically:

$$12 \times 13 = (10 + 2) \times (10 + 3) = (10 \times 10) + (10 \times 3) + (2 \times 10) + (2 \times 3) = 156.$$

This can also be represented symbolically as:

$$\begin{array}{rl} 12 & \\ \times\ 13 & \\ \hline 6 & (3 \times 2) \\ 30 & (3 \times 10) \\ 20 & (10 \times 2) \\ 100 & (10 \times 10) \\ \hline 156 & \end{array}$$

g) Do one or two more examples, then have students use the base-10 blocks to solve these problems. Encourage students to record the procedure they used for each problem, concretely, pictorially, and symbolically.

- 15 x 25
- 18 x 27
- 21 x 26
- 22 x 28
- 32 x 31

h) Students should choose one of the questions and explain how each representation relates to place value and number properties.

Note: As students get more practice with multiplication, another symbolic representation can be presented:

$$\begin{array}{rl} 12 & \\ \times\ 13 & \\ \hline 36 & (3 \times 12) \\ 120 & (10 \times 12) \\ \hline 156 & \end{array}$$

Observation Checklist

Check students' responses to determine whether they can do the following:

- ❑ illustrate partial products in expanded notation for both factors
- ❑ represent both 2-digit factors in expanded notation to illustrate the distributive property
- ❑ model the procedure for finding the product of two 2-digit factors using an array and base-10 blocks
- ❑ record the procedure symbolically
- ❑ solve a multiplication problem in context using personal strategies, and record the procedure concretely, pictorially, and symbolically

Examine students' responses to determine if a mistake is due to

- ❑ carelessness
- ❑ not knowing a basic multiplication or addition fact
- ❑ a procedural error (e.g., renaming and regrouping incorrectly when finding the sum of two numbers or not finding all the partial products in a multiplication problem)

- **Illustrate partial products in expanded notation for both factors.**
- **Represent both 2-digit factors in expanded notation to illustrate the distributive property.**

Materials: Number cards (BLM 5–8.5), paper and pencils.

Organization: Groups of 2–4/Whole class

Procedure:

a) Tell students that they will be playing a game involving multiplication of 2-digit numbers. Explain how the game is played.

   1. Shuffle the cards and place them face down in a pile in the centre of the playing area.
   2. Each player draws four cards and arranges them to form two 2-digit whole numbers that will give him or her the largest possible product.
   3. Players place their numbers face up in front of them. Each player writes his or her numbers as the sum of tens and ones, then multiplies each member of the first number times each member of the second number.

      For example, if 1, 3, 5, and 7 are drawn, a player can form the numbers 71 and 53, then find the product of the two numbers by multiplying (70 + 1)(50 + 3)—that is, (70 +1)(50 + 3) = (70 x 50) + (70 x 3) + (1 x 50) + (1 x 3) = 3500 + 210 + 50 + 3 = 3763.

4. Players check each other's calculation. The player with the largest product gets one point.
5. The winner is the player with the most points after five rounds.

b) Demonstrate how the game is played and answer any questions students might have. Have students play the game.

c) Have students discuss the strategies that they used to win the game. Begin the discussion by asking questions, such as "What four cards did you draw? How did you decide which numbers to form with the cards you drew? What other numbers could you have made? How did you decide which numbers would give you the largest product?"

d) Vary the activity by having students

- use the four cards to form a 1-digit by 3-digit multiplication to give the largest possible product
- use four cards and ask students to form either a 2-digit by 2-digit or a 1-digit by 3-digit multiplication
- use five cards to form a 2-digit by 3-digit or 1-digit by 4-digit multiplication
- use six cards to form a 3-digit by 3-digit or 2-digit by 4-digit multiplication
- have students arrange the cards to form the smallest product

Observation Checklist

Observe students' responses to determine whether they can do the following:

❏ illustrate partial products in expanded notation for both factors

❏ present both 2-digit factors in expanded notation to illustrate the distributive property

❏ determine the correct product of two 2-digit whole numbers

❏ give reasonable estimates of the product of two 2-digit numbers

❏ recognize that the larger the factors, the greater the product

- **Multiply 2-digit numbers.**

Materials: Copies of the basic facts multiplication table (BLM 5–8.11)

Organization: Whole class

Procedure:

a) Show students a copy of the multiplication table and ask them to describe any patterns they see.

b) Show students the square labelled “A”

2	3
4	6

and have them find the product of the opposite pairs of vertices (2 x 6 and 3 x 4). Ask them what they notice about the products.

c) Now show students the square labelled “B”

18	21	24	27
24	28	32	36
30	35	40	45
36	42	48	54

and have them find the product of the opposite pairs of vertices (18 x 54 and 36 x 27). Again, ask students what they notice about the products. Ask students if they think this would be true for other squares.

d) Have students work with their partners to find the product of the opposite pairs of vertices of 10 different squares on the multiplication table. Make sure students vary the size of the squares.

e) Have students discuss their findings with the other members of the class.

f) Extend the activity by asking students to determine whether the pattern they found for squares also holds for rectangles.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ recall the basic facts for multiplication
- ❑ determine the correct product of a 1-digit number times a 2-digit number
- ❑ determine the correct product of a 2-digit number times a 2-digit number
- ❑ use a strategy for calculating the product of two numbers that is mathematically correct and efficient and can be generalized (applied to other multiplication problems)
- ❑ recognize that the products of the opposite pairs of vertices of squares (and other rectangles) on the multiplication table are equal

Materials: Number cards (BLM 5–8.5), dice, paper and pencils

Organization: Pairs

Procedure:

a) Tell students that they will be playing the game "target." Explain how the game is played.

1. Shuffle the cards and place them face down in a pile in the centre of the playing area (the cards will need to be reshuffled after each round of play).
2. Players take turns rolling a die and drawing four cards.
3. On a turn, a player rolls the die and uses the chart shown below to determine the target range of the product.

Number Rolled	Target Range
1	500 or less
2	501 — 1000
3	1001 — 3000
4	3001 — 5000
5	5001 — 7000
6	more than 7000

4. The player then draws four cards and uses them to form two numbers whose product the player thinks falls within the target range. The player does not have to use all four cards. A number cannot begin with a zero.

5. The player multiplies the two numbers. The other player checks his or her partner's calculations. If the product is in the target range, he or she gets one point. If the product is outside the range, no points are awarded.
6. The first player to get five points is the winner.

b) Demonstrate how the game is played and answer any questions students might have. Have students play the game.

c) Vary the activity by having students draw five or six cards.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ give reasonable estimates of the product of a 2-digit number times a 1-digit or 2-digit number
- ❑ determine the product of 2-digit numbers using the distributive property
- ❑ use concrete or pictorial representations to determine the product of two numbers
- ❑ use an efficient strategy for finding the product of two numbers

- **Illustrate partial products in expanded notation for both factors.**
- **Represent both 2-digit factors in expanded notation to illustrate the distributive property.**
- **Model and explain the relationship that exists between an algorithm, place value, and number properties.**
- **Determine products using the standard algorithm of vertical multiplication.**

Materials: Copies of Multiplication Methods activity (BLM 5.N.5.1)

Organization: Whole class/Small Groups (3 or 4)

Procedure:

a) Explain to students that they will be practising three different strategies or algorithms for finding the products of two numbers. Although the strategies could be called different names, for this activity, we will call them

   i) Area Model

   ii) Vertical Method

   iii) Compressed Vertical Method

b) As a whole class, go through the following example. Each of these three methods should have been discussed with students prior to the activity and the first example is to activate their knowledge.

Multiply 26 x 57. Use each of the three methods and be prepared to explain the method using appropriate mathematical language.

Method 1: Area Model

Work:

	20	+	6
50	1000		300
+			
7	140		42

Explanation:

Write each factor in expanded notation.

Find each of the four partial products:

50 x 20 = 1000
50 x 6 = 300
7 x 20 = 140
7 x 6 = 42

Add the four partial products.

1000 + 300 + 140 + 42 = 1482

Or

$$\begin{array}{r} 1000 \\ 300 \\ 140 \\ +\quad 42 \\ \hline 1482 \end{array}$$

Method 2: Vertical Method

Work:

$$\begin{array}{r} 26 \\ \times\quad 57 \\ \hline 42 \\ 140 \\ 300 \\ 1000 \\ \hline 1482 \end{array}$$

Explanation:

Find the partial products:

7 x 6 = 42
7 x 20 = 140
50 x 6 = 300
50 x 20 = 1000

Add the partial products.

Note: When finding the partial products, remember that the 2 in 26 is not a 2 but 20. Similarly, the 5 in 56 is not a 5 but 50.

Method 3: Compressed Vertical Method

Work:	Explanation:
$\begin{array}{r} {}^{3}4\ \ 26 \\ \times\ \ 57 \\ \hline 182 \\ 1300 \\ \hline 1482 \end{array}$	Find the partial products: 7 x 26 = 182* 50 x 26 = 1300** Add the partial products.

Notes:

* To find the partial product 7 x 26, multiply 7 x 6 to get 42. Place the 2 below the line in the ones place and put the 4 beside the 2 to show that you need to add 4 tens to the product of 7 and 20 or 4 to the product of 7 and 2 tens. Since 7 x 2 = 14, and 14 + 4 = 18, you have 18 tens so you can write 18 to the left of the 2 to show 18 tens. You really did 7 x 26 = (7 x 20) + (7 x 6) = 140 + 42 = 182.

** To find the partial product 50 x 26, you can think of the partial product 5 x 26 if you place a zero in the ones place under the 2. Now proceed as in Note 1: multiply 5 x 6 to get 30. Put the 0 in the tens place under the 8 and put the 3 beside the 2 to show that you need to add 3 to the product of 5 and 2. Since 5 x 2 + 3 = 13, put 13 to the left of the digits you just put on the second row. Note that you were really multiplying 50 x 20 and adding 300 to get 1000 + 300 (or 50 x 26 = (50 x 20) + 50 x 6) = 1000 + 300 = 1300).

c) Arrange the students in groups of 3 (or 4). Have the students number off from 1 to 3 (or 4). The students will each solve multiplication problems using a specified method. They will then discuss their solutions and talk about the efficiency of each method.

d) There will be 3 rounds for this activity. For each round, assign a particular method to each group member

■ Round 1:	Area Model:	Person 1 (and Person 4)
	Vertical Method:	Person 2
	Compressed Vertical Method:	Person 3
■ Round 2:	Area Model:	Person 2
	Vertical Method:	Person 3 (and Person 4)
	Compressed Vertical Method:	Person 1
■ Round 3:	Area Model:	Person 3
	Vertical Method:	Person 1
	Compressed Vertical Method:	Person 2 (and Person 4)

e) For each round, write the multiplication question and the answer on the board. Each person in the group is to do her or his own work. After everyone in the group is finished, have the students discuss the following questions:

   Did we all get the right answer? If no, have students check the incorrect answers to try to correct the mistake.

   - Which method seemed to be the most efficient? Explain.

f) After the discussion, the group should work together to explain each step in each method.

g) Questions such as the following could be used (be sure to vary the size of the factors each time so students get practice with different types of questions):

   - 9 x 483 = 4347
   - 52 x 149 = 7748
   - 6743 x 28 = 188 804

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ determine the product of two numbers using the distributive property
- ❏ explain the steps in each of the three procedures
- ❏ record each procedure symbolically
- ❏ use place value knowledge when performing multi-digit multiplication

Examine students' responses to determine if a mistake is due to:

- ❏ carelessness
- ❏ not knowing a basic multiplication or addition fact
- ❏ procedural error (e.g., renaming and regrouping incorrectly when finding the sum of two numbers or not finding all the partial products in a multiplication problem)

Grade 5: Number (5.N.6)

Enduring Understandings:

There are many strategies that can be used to determine the answers to computational problems.

Strategies for computing the answers to computational problems involve taking apart and combining numbers in a variety of ways.

General Outcome:

Develop number sense.

Specific Learning Outcome(s):	**Achievement Indicators:**
5.N.6 Demonstrate an understanding of division (1- and 2-digit divisors and up to 4-digit dividends), concretely, pictorially, and symbolically, and interpret remainders by ■ using personal strategies ■ using the standard algorithm ■ estimating quotients to solve problems. [C, CN, ME, PS]	➛ Model the division process as equal sharing using base-10 blocks, and record it symbolically. ➛ Explain that the interpretation of a remainder depends on the context: ■ ignore the remainder (e.g., making teams of 4 from 22 people) ■ round up the quotient (e.g., the number of five passenger cars required to transport 13 people) ■ express remainders as fractions (e.g., five apples shared by two people) ■ express remainders as decimals (e.g., measurement or money) ➛ Model and explain the relationship that exists between algorithm, place value, and number properties. ➛ Determine quotients using the standard algorithm of long division. (The multiples of the divisor are subtracted from the dividend). ➛ Solve a division problem in context using personal strategies, and record the process. ➛ Refine personal strategies, such as mental math strategies to increase efficiency when appropriate (e.g., $860 \div 2$ think $86 \div 2 = 43$ then $860 \div 2$ is 430).

Prior Knowledge

Students may have had experience with the following:

- Solving division problems involving 2-digit whole number dividends by 1-digit whole number divisors
- Using personal strategies to solve division problems involving 1-digit whole number divisors and 2-digit whole number dividends
- Estimating the quotients of division problems involving 1-digit whole number divisors and 2-digit whole number dividends
- Relating division to multiplication
- Adding and subtracting whole numbers to 10 000

Related Knowledge

Students should be introduced to the following:

- Determining basic multiplication and division facts to 81
- Using strategies such as front-end rounding, compensation, and compatible numbers to estimate the answers to computational problems
- Dividing multiples of 10 and 100 by whole numbers less than 10

Background Information

An algorithm is a system of finite procedures for solving a particular class of problems. The best known algorithms are the traditional paper-and-pencil procedures for adding, subtracting, multiplying, and dividing. Along with these standard algorithms, mathematics instruction should include an emphasis on understanding through mental mathematics, estimation, the use of technology, the development of invented procedures, and the use of alternative algorithms, such as equal sharing, equal grouping, and strategic dividing.

By encouraging students to develop their own computation strategies and allowing them to use alternative algorithms, the emphasis in mathematics instruction has shifted to reasoning, problem solving, and conceptual understanding. Providing students with opportunities to invent their own strategies and use alternative algorithms enhances their number and operation sense. Students become more flexible in their thinking, more aware of the different ways to solve a problem, and more adept at selecting the most appropriate procedure for solving a problem. Discussion of the algorithms or strategies and their relationship to place value and number properties can also help students develop better reasoning and communication skills.

The standard algorithm for division is subtracting multiples of the divisor from the dividend. Students are expected to use the standard algorithm as one of the tools for computation.

When teaching the traditional algorithm for division, it is important to follow the concrete, pictorial, and symbolic sequence of teaching. The important idea is to allow students to construct meaning, not memorize procedures without understanding. Students' misconceptions (or their fuzzy understandings) can be reinforced by a poorly understood algorithm. Therefore, be sure that a student has a thorough understanding of the operations and of taking apart and combining numbers (i.e., using place value) before beginning work on developing the traditional algorithm. Students should be able to explain the relationship that exists among an algorithm, place value, and number properties.

Teachers can facilitate students' understanding and use of a variety of computational strategies by

- providing a supporting and accepting environment
- allowing time for exploration and experimentation
- embedding computational tasks in real-life situations
- allowing students to discuss, analyze, and compare their solution strategies
- encouraging discussion that focuses on place value and number properties when defending the choice of a particular algorithm or strategy
- understanding that a child needs to be efficient at computation and that this looks different for each student

Note: For further information regarding using models and methods of division to build understanding, see Appendix 1: Models for Dividing 3-Digit Numbers by 1-Digit Numbers.

Mathematical Language

Division

Divisible

Divisor

Dividend

Quotient

Remainder

Assessing Prior Knowledge

Materials: Math journals

Organization: Individual/Whole class

Procedure:

a) Present students with the following situation:
   - Manuel's calculator is broken. He needs to find the quotient of 57 ÷ 6, but he has forgotten how to divide. Help him out by explaining how he can find the quotient.

b) Have students share their answers. Record their strategies on the board or overhead and encourage discussion of them by asking questions, such as the following:
   - "What strategy could Manuel use to find the quotient of 57 ÷ 6?"
   - "Will your strategy work for other division problems? Show me."
   - "What is another strategy he could use?"
   - "How are the strategies alike? How do they differ?"
   - "Which strategy do you prefer? Why?"

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ correctly use the terms *dividend*, *divisor*, *quotient*, *remainder*, and *divide*
- ❏ recognize that division involves partitioning into equal parts
- ❏ recognize that division involves forming equal-sized groups
- ❏ describe a strategy that is mathematically correct
- ❏ use an efficient strategy that can be used for all division problems
- ❏ recognize that the division is uneven (i.e., there is a remainder of 3)

- **Model the division process as equal sharing using base-10 blocks, and record it symbolically.**

Materials: Base-10 materials such as base-10 blocks or digi-blocks

Organization: Whole class

Procedure:

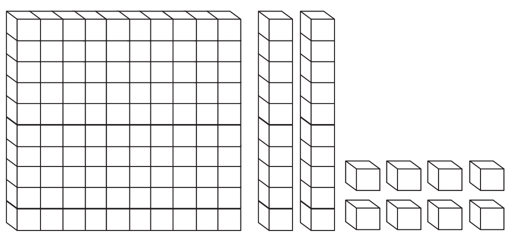
a) Ask students to solve the following problem:

   - Marcy works in the library. She needs to put the same number of books on four shelves. There are 128 books. How many books should she put on each shelf?

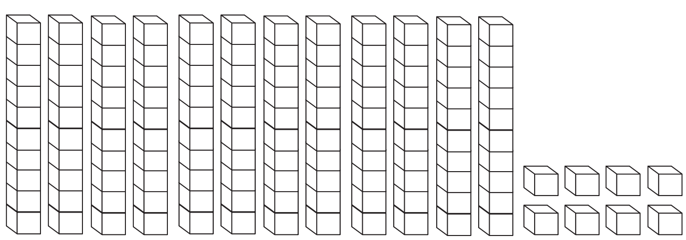
b) Have the students share their solutions with the other members of the class. Encourage discussion by asking questions, such as the following:

   - "What strategy did you use to determine the solution to the problem?"
   - "Is there another strategy you could use to solve the problem?"
   - "How are the strategies alike?"
   - "Which strategy is more efficient? Why do you think this?"
   - "How do you know your solution is correct?"

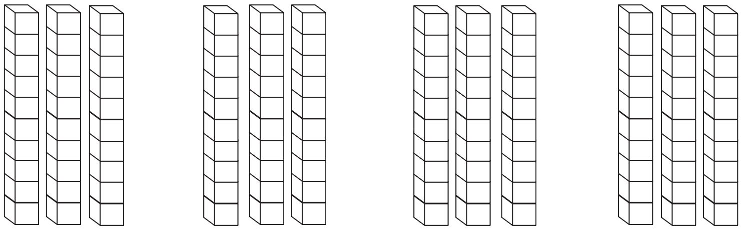
c) Model the division process as equal sharing if students do not suggest this strategy. For example, show students 128 with the blocks.



Explain one strategy by telling students that 128 needs to be partitioned into four equal parts. Since there are not enough flats (100s) to put one in each part, the flat can be exchanged for 10 longs so 128 becomes 12 tens and 8 ones.

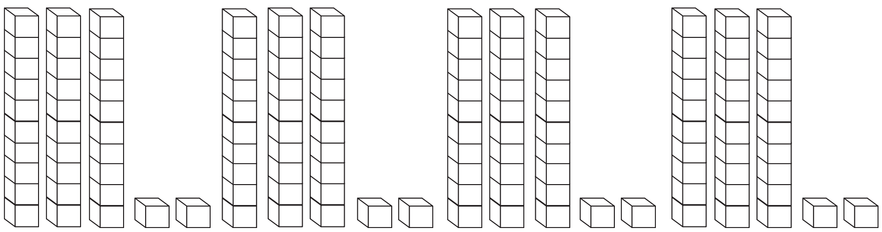


Next, partition the tens into four equivalent parts.



Then partition the ones into four equivalent parts.

Now, put the pieces together to make four parts with 32 in each part.



Record the process as 128 ÷ 4 = 12 tens and 8 ones ÷ 4 = 3 tens and 2 ones, or 32, or as

$$\begin{array}{r} \text{3 tens, 2 ones} \\ 4\overline{)\text{12 tens, 8 ones}} \end{array} \text{ or } \begin{array}{r} 30+2 \\ 4\overline{)120+8} \end{array}$$

d) Do two or three more examples, and then have students use the strategy to solve the following problems. Have students record their answers symbolically.

- 156 ÷ 3 =
- 216 ÷ 4 =
- 317 ÷ 2 =
- 438 ÷ 5 =
- 624 ÷ 4 =

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ model the division process as equal sharing using base-10 blocks
- ❑ record the division process symbolically

- **Model the division process as equal sharing using base-10 blocks, and record it symbolically.**
- **Explain that the interpretation of a remainder depends on the context.**
- **Solve a division problem in context using personal strategies, and record the process.**

Materials: Base-10 materials such as base-10 blocks or digi-blocks.

Organization: Whole class/Small groups

Procedure:

a) Have students solve the following problem:

   - Max and Joshua are responsible for setting up tables for the school's banquet. Eight people can sit at each table. How many tables do Max and Joshua need to set up if there are 150 people going to the banquet?

b) Have students share their answers and discuss how the remainder should be interpreted. Encourage students to explain why the quotient needs to be increased by 1.

c) Explain that remainders are a common occurrence in division, and how they are interpreted depends on the problem statement.

d) Have each group of students solve the following problems and decide how the remainder should be interpreted.

   - Some students will be wearing sashes for the spring school dance festival. Four metres of ribbon are needed for each sash. Mr. Sanchez has 135 metres of ribbon. How many sashes can he make?
   - Sally has 26 cookies to share evenly with three friends. How many cookies does each get?
   - Marshall saved $157.00. He spent half of the money he saved to buy three video games. How much did he spend on the video games?
   - Seventy-four people are going on a camping trip. Six people can go in each car. How many cars are needed?

e) Have the groups share their answers and discuss how they interpreted the remainder. Encourage students to explain their reasons for how they handled the remainder. Summarize the discussion by stressing that there are four ways the remainder can be handled. Depending on the problem statement, the remainder can be expressed as a fraction or a decimal; it can be dropped or it can require the quotient to be increased by 1.

f) Assign each group one of the ways that a remainder can be interpreted, and ask them to write a problem that involves the interpretation that you gave them. Have the groups give their problem to the other members of the class to solve and decide how to interpret the remainder.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ model the division process as equal sharing using base-10 materials
- ❏ solve a division problem using personal strategies, and record the process
- ❏ interpret the remainder with respect to the context
- ❏ write a division problem that involves dropping the remainder, increasing the quotient by 1, expressing the remainder as a fraction, or expressing the remainder as a decimal

- **Model the division process as equal sharing using base-10 blocks, and record it symbolically.**
- **Explain that the interpretation of a remainder depends on the context.**

Materials: Dice, base-10 blocks or digi-blocks

Organization: Pairs

Procedure:

a) Tell students that they will be doing an activity involving division. Explain that they will be rolling a die four times and using the four numbers that they roll to create as many division problems as they can that have no remainder. Each problem that they create must involve a 3-digit whole number dividend and a 1-digit whole number divisor. Let students know that they can use the materials to help them determine whether there is a remainder, and that they should record both the numbers that they rolled and the problems that they created that have no remainders.

   If they are convinced that there are no problems that can be created that have a remainder of zero, they should record the numbers that they rolled, and explain in writing why they think this is so.

b) Do an example of the activity with the students. For instance, suppose 2, 5, 6, and 4 are rolled. One problem that has no remainder that can be made with the numbers is $564 \div 2$; another is $256 \div 4$.

c) Have students repeat the activity, but this time have them create problems with the largest possible remainder.

Observation Checklist

Analyze students' responses to determine whether they can do the following:

- ❏ devise a strategy determining all the possible problems that can be considered
- ❏ solve division problems in context using personal strategies, and record the process
- ❏ use a strategy that is efficient, mathematically correct, and can be generalized (work for all problems)
- ❏ recognize division problems that have no remainders
- ❏ recognize division problems that have remainders
- ❏ identify the largest possible remainder for a given divisor

■ Solve a division problem in context using personal strategies, and record the process.

Materials: Division problem cards (BLM 5.N.6.1), timer or clock with second hand, calculator for each group.

Organization: Groups of 3/Whole class

Procedure:

a) Tell students that they will be playing a game that involves estimating quotients. Explain how the game is played.

   1. Shuffle the cards and place them face down in the middle of the playing area.
   2. One student in each group acts as the timer. The other two students play against each other.
   3. The timer turns over a card and says, "Go!" The other two students now have 10 seconds to write down their estimate of the quotient.
   4. When the 10 seconds are up, the timer says, "Stop!" and the players must put their pencils down. The timer then uses the calculator to find the quotient.
   5. The player whose estimate is closest to the actual answer wins the card. If there is a tie, no player receives a card.
   6. The game is over when there are no cards left. The player with the most cards wins the game.

b) Demonstrate how the game is played and answer any questions that students might have. Have students play the game.

c) Have students discuss the strategies they used to estimate the quotients. Begin the discussion by asking questions, such as the following:
   - "How did you estimate the quotient of 748 ÷ 2?"
   - "How can you get a closer estimate?"
   - "Is there another strategy you could use to estimate the quotient?"

d) Have the winner of the game become the timer and play the game again.

e) Have students make up their own division problem cards and use them to play the game.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ give reasonable estimates of the quotients
- ❑ use technology to solve division problems
- ❑ use strategies such as "front-end rounding" and "compensation" to determine reasonable estimates of the quotient
- ❑ determine basic facts for division

- **Model the division process as equal sharing using base-10 blocks, and record the process symbolically.**
- **Explain that the interpretation of a remainder depends on the context.**
- **Solve a division problem in context using personal strategies, and record the process.**

Materials: Base-10 blocks or digi blocks for students who need them; paper and pencils

Organization: Whole class/Small groups

Procedure:

a) Ask students to solve the following problem: 24 ÷ 3 = . Explain that there are other problems related to 24 ÷ 3 that also have a quotient of 8, but these problems have a remainder. For example: 25 ÷ 3 is 8 with a remainder of 1. Ask students to find another problem related to 24 ÷3 that has a quotient of 8 and a remainder of 2 (26 ÷ 3). Have students find all the division problems with a remainder related to 36 ÷ 4 = (37 ÷ 4 = , 38 ÷ 4 = , and 39 ÷ 4 =).

b) Assign each group one of the division problems shown below, and ask them to find all the related division problems as well as the remainder for each problem.

- 150 ÷ 5 = (151 ÷ 5 = , 152 ÷ 5 = , 153 ÷ 5 = , 154 ÷ 5 =)
- 432 ÷ 6 =
- 217 ÷ 7 =
- 344 ÷ 8 =
- 918 ÷ 9 =

c) Have each group share its findings with the other members of the class and record their answers on the board. Encourage discussion by asking questions, such as the following:

- "What patterns do you see?"
- "If you divide a 634 by 12, how many related division problems could you write? Why?"
- "What do you know about the remainders when you divide by 12?"
- "When you divide a number, what is the smallest remainder you can have?"
- "What is the largest remainder you can have?"

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ model the division process as equal sharing using base-10 blocks
- ❑ solve division problems in context using a personal strategy and record the process symbolically
- ❑ recognize that the larger the divisor, the more related division problems
- ❑ recognize that the smallest possible remainder is 0 and the largest possible remainder is always 1 less than the divisor
- ❑ recognize that the number of possible remainders is the same as the divisor (e.g., when dividing by 3, there are 3 possible remainders: 0, 1, and 2)

- **Explain that the interpretation of a remainder depends on the context.**
- **Solve a division problem in context using personal strategies, and record the process.**

Materials: Two dice and a playing board for each group (BLM 5.N.6.2). One die should have the numbers 1, 3, 5, 7, 7, and 9 written on it and the other die should have the numbers 1, 2, 4, 6, 6, and 8 written on it.

Organization: Groups with 3 or 4 students

Procedure:

a) Tell students that they will be playing a game involving remainders in division. Explain how the game is played.

   1. Players take turns, rotating clockwise.
   2. During a turn, a player crosses off any unused number on the playing board. He or she then chooses one of the dice and rolls it. Next, the player divides the number rolled into the number that was crossed off and finds the remainder.
   3. The remainder for the division problem is the player's score for that round.

      For example, a player crosses off 113 and chooses to roll the die with the numbers 1, 3, 5, 7, 7, and 9 on it. If the number rolled is a 5, the player divides 5 into 113 and gets a quotient of 22 with a remainder of 3. The player's score for that round is 3.
   4. If a player notices a mistake that another player makes, he or she gets that player's score for the turn.
   5. The game ends when all the numbers have been crossed off. The player with the largest cumulative score is the winner.

b) Demonstrate how the game is played and answer any questions students might have. Have students play the game.

c) Have students play the game again, but this time the player with the smallest cumulative score is the winner.

d) Have students create their own division game board and use it to play the game with their partner.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ solve division problems in context using a personal strategy, and record the process symbolically
- ❑ use a strategy that is efficient, is mathematically correct, and can be generalized
- ❑ develop a strategy for playing the game
- ❑ recognize problems that have been solved incorrectly

For further information, see

- Ontario Education. *Number Sense and Numeration, Grades 4 to 6*. Volume 4. "Learning about Division in the Junior Grades."
- *Big Ideas from Dr. Small* by Marian Small. "Grades 4 to 8: Whole Number Operations. Varied Approaches for Multiplication and Division." p. 34.

Notes

Grade 5: Number (5.N.7)

Enduring Understandings:

Equivalent fractions are fractions that represent the same value.

Different strategies can be used to compare fractions with unlike denominators.

General Outcome:

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
5.N.7 Demonstrate an understanding of fractions by using concrete and pictorial representations to ■ create sets of equivalent fractions ■ compare fractions with like and unlike denominators [C, CN, PS, R, V]	➛ Create a set of equivalent fractions and explain why there are many equivalent fractions for any fraction using concrete materials. ➛ Model and explain that equivalent fractions represent the same quantity. ➛ Determine if two fractions are equivalent using concrete materials or pictorial representations. ➛ Formulate and verify a rule for developing a set of equivalent fractions. ➛ Identify equivalent fractions for a fraction. ➛ Compare two fractions with unlike denominators by creating equivalent fractions. ➛ Position a set of fractions with like and unlike denominators on a number line (vertical or horizontal), and explain strategies used to determine the order.

PRIOR KNOWLEDGE

Students may have had experience with the following:

- Demonstrating an understanding of fractions less than or equal to one using concrete and pictorial representations
- Naming and recording fractions for the parts of a whole or a set
- Comparing and ordering fractions with like numerators or like denominators
- Providing examples of where fractions are used
- Relating fractions to decimals

BACKGROUND INFORMATION

Equivalent fractions are fractions that represent the same value. For example, $\frac{2}{3}$, $\frac{4}{6}$, $\frac{6}{9}$, and $\frac{8}{12}$ are different names for the same number. Students need an understanding of equivalence in order to compare, order, add, and subtract fractions.

Students often find fractions confusing and difficult to comprehend. Difficulties with learning fractions can arise from instruction that emphasizes procedural knowledge rather than conceptual knowledge. They also arise because the whole number concepts that students learned do not always apply to fractions. For example, when the numerators are the same and the denominators are different, the larger of two fractions is determined by comparing denominators using order ideas that are the inverse of those for whole numbers. For example, 4 is less than 5, but $\frac{1}{5}$ is less than $\frac{1}{4}$. However, when the denominators are the same, the larger of two fractions is determined by comparing the numerators using whole number concepts. For example, 3 is less than 5 so $\frac{3}{8}$ is less than $\frac{5}{8}$.

To help students overcome their difficulties with learning fractions, instruction should focus on developing concepts rather than on abstract rules. Learning experiences that emphasize exploration and the manipulation of a variety of concrete materials and pictorial representations are key to helping students develop meaning for fraction concepts. Rules for manipulating fractions should only be introduced after students develop an understanding of the concepts. If the rules are introduced too soon, students end up memorizing them. Rules that are memorized without understanding are often forgotten or applied inappropriately.

Mathematical Language

Denominator

Equivalent

Fraction

Greater than

Less than

Numerator

Learning Experiences

Assessing Prior Knowledge

Materials: Copies of concept description sheet (BLM 5–8.2)

Organization: Individual

Procedure:

a) Tell students that in the next few lessons they will be learning about fractions, but before they begin you need to find out what they already know about fractions.

b) Ask students to complete the concept development sheet. Let them know that it is all right if they cannot think of anything to put in a section. They will have another opportunity to complete the sheet when they learn more about fractions.

c) When students finish, begin a discussion of fractions by asking, "What is a fraction? What is an example of a fraction?" As the discussion progresses, clear up any misconceptions students may have and make sure that they see a variety of examples and non-examples.

d) Have students complete the concept development sheet again.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ recognize that a whole can be a region or a collection of objects
- ❑ recognize that the parts of a whole must be equivalent
- ❑ give appropriate examples and non-examples of fractions
- ❑ write a symbol for a fraction
- ❑ know the terms "denominator" and "numerator"

- **Determine if two fractions are equivalent using concrete materials or pictorial representations.**
- **Create a set of equivalent fractions and explain why there are many equivalent fractions for any fraction using concrete materials.**
- **Identify equivalent fractions for a fraction.**

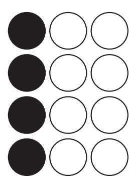
Materials: Two-coloured chips

Organizations: 3 groups of students

Procedure:

a) Give each student 12 chips and divide the students into three groups: A, B, and C. Give each group different instructions. For example, tell Team A to show $\frac{1}{3}$, Team B to show $\frac{2}{6}$, and Team C to show $\frac{4}{12}$.

b) When students finish, have them compare their results. Students should note that everyone has turned over four chips and the instructions yield the same results. Explain that $\frac{1}{3}$, $\frac{2}{6}$, and $\frac{4}{12}$ are equivalent fractions because they are different names for the same amount.



$$\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$$

c) Give different instructions that yield equivalent results. For example, ask group A to show $\frac{2}{3}$, group B to show $\frac{4}{6}$, and group C to show $\frac{8}{12}$. Ask questions such as, "Why is $\frac{2}{3}$ equivalent to $\frac{4}{6}$?" "Is $\frac{2}{3}$ the same as $\frac{4}{6}$ even when you start with 18 chips?" "Why?" Have students verify their answer by manipulating the chips.

d) Continue the activity, but change the number of chips and the instructions that result in equivalent fractions.

e) Vary the activity by having one team give the instruction (e.g., show $\frac{1}{2}$ and the other two teams give instructions that they think will yield the same results when carried out). Have students carry out the instructions to see if they create equivalent fractions.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ determine if two fractions are equivalent using concrete materials
- ❑ create a set of equivalent fractions and explain why they are equivalent
- ❑ identify equivalent fractions for a fraction

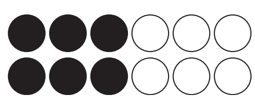
Materials: Egg cartons and counters

Organization: Pairs

Procedure:

a) Have students use their egg cartons and counters to show you $\frac{1}{12}$ of a dozen. Then ask students to show you $\frac{2}{12}$ of a dozen. Now have students show you $\frac{1}{6}$ of a dozen. When students show two counters in their egg cartons to illustrate $\frac{1}{6}$ of a dozen, say, "But you just said that is $\frac{2}{12}$ of a dozen!" Ask, "Which is it: $\frac{1}{6}$ or $\frac{2}{12}$? Why are they equivalent?"

b) Ask students to use their egg cartons to find as many equivalent fractions as they can (students should find fractions equivalent to $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{3}{4}$, and $\frac{5}{6}$). Have students record their findings pictorially and symbolically.



$$\frac{1}{2} = \frac{3}{6} = \frac{6}{12}$$

c) Ask students to find two fractions that are not equivalent and explain why they are not equivalent. Again, have students record their findings pictorially and symbolically.

d) Have students share their findings with the other members of the class. Encourage students to explain why two fractions are equivalent or not equivalent.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ model and explain that equivalent fractions represent the same quantity
- ❑ distinguish between equivalent and non-equivalent fractions
- ❑ identify equivalent fractions for a fraction
- ❑ determine if two fractions are equivalent using concrete materials or pictorial representations

- ■ **Model and explain that equivalent fractions represent the same quantity.**
- ■ **Determine if two fractions are equivalent using concrete materials or pictorial representations.**
- ■ **Identify equivalent fractions for a fraction.**

Materials: Fraction blocks or fraction bars

Organization: Small groups

Procedure:

Note: Colours may vary based on materials used. Be sure to determine values before you begin.

a) Ask students to show you the yellow block. Tell them that this block represents one-half of the whole pink fraction block. Have them name each of the other blocks as a fraction of the pink block.

b) Ask students to use their blocks to find out

- ■ how many reds fit on the yellow block, and to write a number sentence that shows the relationship between the red and yellow blocks $\left(\frac{2}{4}=\frac{1}{2}\right)$
- ■ how many blues fit on the yellow block, and to write a number sentence that shows the relationship
- ■ how many greens fit on the yellow block, and to write a number sentence that shows the relationship
- ■ what one-eighth would look like, and to find out how many eights would fit on the yellow block (Have them write a number sentence that shows this relationship.)

c) Ask students to list all the fractions they found that are equivalent to one-half $\left(\text{Answer: } \frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{6}{12}=\frac{4}{8}\right)$, and to describe any patterns or relationships that they see.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ name the fractional part of each block in relation to the larger pink block
- ❑ create a set of equivalent fractions and explain why there are many equivalent fractions using concrete materials
- ❑ model and explain that equivalent fractions represent the same quantity
- ❑ identify patterns or relationships (e.g., the denominator is twice the numerator)

- **Determine if two fractions are equivalent using concrete materials or pictorial representations.**
- **Identify equivalent fractions for a fraction.**

Materials: Two-coloured chips, fraction blocks or fraction bars

Organization: Pairs

Procedure:

a) Tell students that you will be giving them some fractions and it is their job to determine if they are equivalent. Explain that they can use chips or fraction blocks to help them decide.

b) Have students determine whether the following fractions are equivalent:

- $\frac{3}{5}$ and $\frac{9}{15}$
- $\frac{2}{8}$ and $\frac{4}{12}$
- $\frac{3}{6}$ and $\frac{9}{18}$
- $\frac{7}{10}$ and $\frac{14}{20}$
- $\frac{5}{9}$ and $\frac{18}{27}$

c) Have students share their answers with the other members of the class. Encourage discussion by asking questions, such as the following:
   - "How do you know that $\frac{3}{5}$ is equivalent to $\frac{9}{15}$?"
   - "Why is $\frac{2}{8}$ not equivalent to $\frac{4}{12}$?"
   - "What fraction is equivalent to $\frac{4}{12}$? How do you know?"

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ identify equivalent fractions
- ❑ distinguish between equivalent and non-equivalent fractions
- ❑ model and explain that two equivalent fractions represent the same amount

- **Determine if two fractions are equivalent using concrete materials or pictorial representations.**

Materials: A set of 40 equivalent fraction cards for each group of students (BLM 5.N.7.1).

Organization: Groups of 2 to 4 students

Procedure:

a) Tell students that they will be playing a game involving equivalent fractions. Explain how the game is played.
   1. Deal five cards to each player. The remaining cards are placed in a pile face down in the middle of the playing area.
   2. Players take turns asking another player for a card. For example, "Please give me the card that shows $\frac{1}{2}=\frac{2}{4}$." If the player has the card, he or she must give it to the asker. When a player receives a requested card, the player lays the two matched cards aside.
   3. If the player does not have the requested card, the asker draws a card from the pile in the middle of the playing area. If it matches a card in his or her hand, the cards are set aside. If it does not match any of the cards in his or her hand, the player keeps the card.
   4. The game is over when one player runs out of cards. The player with the most pairs is the winner.

b) Demonstrate how to play the game and answer any questions students may have. Have students play the game.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ identify pictorial representations of equivalent fractions
- ❑ match pictorial representations of equivalent fractions with symbolic statements
- ❑ determine if two fractions are equivalent using concrete materials or pictorial representations

Materials: Math journals

Organization: Individual

Procedure:

a) Ask students to answer the following question in their math journals:

- Are these fractions equivalent? Explain how you reached your answer.

$$\frac{3}{4} \qquad \frac{6}{12}$$

b) Have students share their answers with the other members of the class. Encourage them to explain their reasoning.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ determine if two fractions are equivalent
- ❑ explain how they know $\frac{3}{4} \neq \frac{6}{12}$ or demonstrate using pictorial representations that $\frac{3}{4} \neq \frac{6}{12}$ (If pictorial representations are used, make sure their diagrams match their explanations.)

- **Identify equivalent fractions for a fraction.**
- **Create a set of equivalent fractions and explain why there are many equivalent fractions for any fraction using concrete materials.**
- **Determine if two fractions are equivalent using concrete materials or pictorial representations.**
- **Formulate and verify a rule for developing a set of equivalent fractions.**

Materials: Two-coloured chips and a set of fraction cards for each group of students (BLM 5.N.7.2)

Organization: Small groups

Procedure:

a) Ask students to sort the cards into piles according to how much of each diagram is shaded. Have students turn over each pile and record the fractions on the back of the cards. For example, $\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}$, and $\frac{18}{24}$. Have them describe any patterns they see, and then ask them to name other fractions that belong to each set and explain how they know they are equivalent.

b) Have students complete each of the following patterns. Tell them that they can draw pictures or use materials to help them complete each pattern.

$$\frac{1}{8}, \frac{2}{16}, \frac{3}{24}, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_$$

$$\_\_\_\_\_, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \_\_\_\_\_, \_\_\_\_\_, \frac{7}{28}$$

$$\_\_\_\_\_, \_\_\_\_\_, \frac{3}{15}, \frac{4}{20}, \frac{5}{25}, \frac{6}{30}, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_$$

c) Ask students to share their answers and explain their reasoning.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ identify equivalent fractions for a fraction
- ❑ create a set of equivalent fractions and explain why there are many equivalent fractions
- ❑ determine if two fractions are equivalent using pictorial representations
- ❑ formulate a rule for determining equivalent fractions

- **Compare two fractions with unlike denominators by creating equivalent fractions.**

Materials: Egg cartons, counters, fraction bars (BLM 5–8.12), Cuisenaire rods, or fraction blocks, clock face (BLM 5–8.13)

Organization: Individual or pairs

Procedure:

a) Ask students to use their egg cartons and counters to answer questions, such as identifying which is larger among the following:
   - $\frac{1}{4}$ dozen or $\frac{1}{3}$ dozen
   - $\frac{7}{12}$ dozen or $\frac{1}{2}$ dozen
   - $\frac{3}{4}$ dozen or $\frac{4}{6}$ dozen
   - $\frac{1}{2}$ dozen or $\frac{5}{12}$ dozen
   - $\frac{2}{3}$ dozen or $\frac{5}{6}$ dozen

b) Have students share their answers and explain the strategies they used to determine which fraction is the larger.

c) If an analog clock is not in the classroom, draw a picture of a clock face on the board and then ask questions such as identifying which is greater among the following:
   - $\frac{1}{2}$ hour or $\frac{5}{6}$ hour
   - $\frac{2}{4}$ hour or $\frac{1}{3}$ hour
   - $\frac{5}{12}$ hour or $\frac{2}{3}$ hour
   - $\frac{3}{4}$ hour or $\frac{1}{2}$ hour
   - $\frac{10}{12}$ hour or $\frac{2}{4}$ hour
   - $\frac{2}{3}$ hour or $\frac{7}{12}$ hour

d) Repeat the activity, but have students use a different material. For example, have students use fraction bars to answer questions such as identifying which is larger among the following:

- $\frac{5}{8}$ or $\frac{2}{3}$
- $\frac{1}{6}$ or $\frac{1}{4}$
- $\frac{3}{10}$ or $\frac{5}{6}$
- $\frac{3}{4}$ or $\frac{7}{8}$
- $\frac{3}{4}$ or $\frac{11}{12}$
- $\frac{1}{3}$ or $\frac{2}{4}$
- $\frac{5}{12}$ or $\frac{3}{8}$
- $\frac{1}{2}$ or $\frac{2}{6}$

e) Have students share their answers and explain the strategies they used to determine which fraction is the larger.

Observation Checklist

Observe students to determine whether they can do the following:

- ❏ represent a given fraction with concrete materials
- ❏ compare two fractions with unlike denominators using concrete materials

Materials: Graph paper

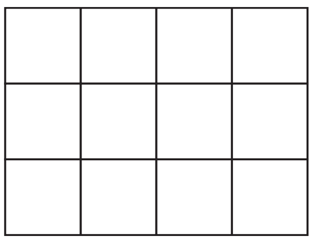
Organization: Small groups/Whole class

Procedure:

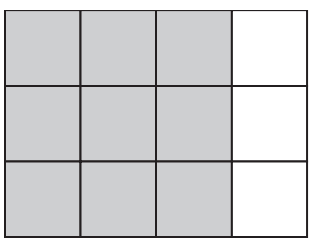
a) Present students with the following problem:

- Sammy had a piece of graph paper on his desk. His teacher asked, "Which is larger, $\frac{2}{3}$ or $\frac{3}{4}$?" After a few minutes, Sammy said, "I think I see a new way to find out." How do you think Sammy did it?

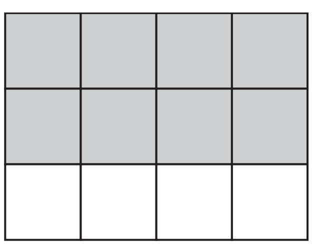
b) Have each group explain how they think Sammy used the graph paper to help him compare the two fractions. If none of the groups use the graph paper to find common denominators, explain that Sammy could have used the graph paper to mark off a rectangle that was four squares one way and three squares the other way.



Each column is $\frac{1}{4}$ of the rectangle, so $\frac{3}{4}$ would look like this:



Each row is $\frac{1}{3}$, so $\frac{2}{3}$ would look like this:



$\frac{3}{4} = \frac{9}{12}$ and $\frac{2}{3} = \frac{8}{12}$, $\frac{9}{12}$ is greater than $\frac{8}{12}$, so $\frac{3}{4}$ is greater than $\frac{2}{3}$.

c) Do two or three more examples, then ask students to use the method to decide which fraction is larger:

- $\frac{3}{4}$ and $\frac{4}{6}$
- $\frac{3}{5}$ and $\frac{2}{3}$
- $\frac{4}{9}$ and $\frac{5}{6}$
- $\frac{2}{7}$ and $\frac{4}{5}$
- $\frac{3}{4}$ and $\frac{2}{5}$

d) Have students share their answers. Encourage them to describe and show how they used the graph paper to help them find fractions with common denominators.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ represent fractions pictorially
- ❑ compare two fractions with unlike denominators by creating equivalent fractions
- ❑ describe and show how they found equivalent fractions

Materials: Spinner with the numbers 1-12 on them (BLM 5–8.14), copies of the recording sheet (BLM 5.N.7.3), fraction strips, pattern blocks or other fraction manipulatives, an overhead transparency of the recording sheet

Organization: Pairs/Whole class

Procedure:

a) Tell students that they will be playing a game with their partner that involves comparing fractions. Explain how to play the game.

1. Players take turns spinning the spinner four times.
2. After each spin, the player writes the number in one of the boxes or on one of the lines beside the bottom box. Once a number has been written, it cannot be changed. After four spins, there will be a fraction (formed by the numbers in the boxes) and two "rejected" numbers that can be put in the trash can.

3. After both players have created their fractions, they can use the materials or draw pictures to model their fractions and decide which is the larger. The players then write a sentence to show the comparison $\left(\text{e.g., } \frac{3}{6} > \frac{1}{4} \text{ or } \frac{3}{4} = \frac{9}{12}\right)$.
4. The player who made the larger fraction scores one point. If the fractions are equivalent, both players score a point.
5. The winner is the player with the most points after four rounds.

c) Demonstrate how to play the game and answer any questions students might have. Have students play the game.

d) Vary the game by having one player spin the spinner four times and both players use the same numbers to complete the game board.

e) Have students discuss their experience playing the game. Encourage them to describe any strategy they used that worked well for them.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ model fractions appropriately using concrete and pictorial representations
- ❑ compare fractions with unlike denominators by
  - ■ creating equivalent fractions
  - ■ using concrete materials or pictorial representations
  - ■ using a personal strategy
- ❑ recognize equivalent fractions
- ❑ show symbolically which of two fractions is larger $\left(\text{e.g., } \frac{2}{3} > \frac{1}{2}\right)$
- ❑ recognize that if the denominators are the same, the fraction with the larger numerator is the larger
- ❑ recognize that if the numerators are the same, the fraction with the smaller denominator is the larger

Materials: Math journals and pencils

Organization: Individual/Whole class

Procedure:

a) Write these questions on the board or overhead.
   - "Which is larger: $\frac{6}{8}$ or $\frac{4}{5}$? How do you know?"

b) Ask students to think about the questions and then record their answers in their math journals.

c) When students finish, have them share their answers. Encourage them to discuss the strategies they used by asking them questions, such as:
   - "How do you know that $\frac{4}{5}$ is larger than $\frac{6}{8}$?"
   - "Is there another strategy you could use to show that $\frac{4}{5}$ is larger than $\frac{6}{8}$?"
   - "Which strategy do you prefer? Why?"

Observation Checklist

Observe students' responses to determine the strategy they used to find the larger fraction. For example, some students may find the larger fraction by

- ❑ creating equivalent fractions with like denominators $\left(\text{e.g., } \frac{32}{40} \text{ and } \frac{30}{40}\right)$
- ❑ creating equivalent fractions with like numerators $\left(\text{e.g., } \frac{12}{15} \text{ and } \frac{12}{16}\right)$
- ❑ comparing the fractions to a benchmark (e.g., deciding whether $\frac{6}{8}$ or $\frac{4}{5}$ is closer to 1)

Also, observe students' responses to determine whether they can do the following:

- ❑ communicate their ideas effectively
- ❑ use appropriate diagrams or pictures (when used in their explanation)
- ❑ recognize that the more pieces into which a whole is divided, the smaller the pieces.
- ❑ recognize equivalent fractions
- ❑ use appropriate procedures to find equivalent fractions

- **Position a set of fractions with like and unlike denominators on a number line (vertical or horizontal), and explain strategies used to determine the order.**

Materials: Fraction cards (BLM 5.N.7.2), vertical and horizontal number lines marked with 0, $\frac{1}{2}$, and 1.

Organization: Whole class/Pairs

Procedure:

a) Show students the cards with fractions on them, and explain that their task is to place the fractions in order from smallest to largest. Select one card and prop it on the chalkboard tray. Show the other cards one at a time and have a student place them on the tray. Have students explain their reasoning.

b) Draw a number line on the board and mark the points 0, $\frac{1}{2}$, and 1. Have students place each fraction on the number line and explain their reasoning.

c) Have students work with their partner to position each of the following sets of fractions on the number line:

- $\frac{1}{12}, \frac{1}{6}, \frac{1}{3}, \frac{2}{3}, \frac{3}{4}, \frac{11}{12}$
- $\frac{1}{16}, \frac{2}{8}, \frac{3}{8}, \frac{9}{16}, \frac{7}{8}$
- $\frac{1}{9}, \frac{1}{4}, \frac{1}{3}, \frac{5}{9}, \frac{2}{3}, \frac{6}{8}$

d) Have students create their own set of fractions to order and place on the number line. Have them justify, in writing, their placement of the fractions.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ use an appropriate (mathematically correct) strategy for comparing fractions with unlike denominators
- ❏ position a set of fractions with like and unlike denominators on a number line
- ❏ explain the strategies that they used to position the fractions

NOTES

Grade 5: Number (5.N.8, 5.N.9)

Enduring Understandings:

Decimals are symbols for common fractions whose denominators are powers of ten.

Decimals are an extension of the base-10 numeration system.

Fractions and decimals can be used interchangeably.

General Outcome:

Develop number sense.

Specific Learning Outcome(s):	**Achievement Indicators:**
5.N.8 Describe and represent decimals (tenths, hundredths, thousandths), concretely, pictorially, and symbolically. [C, CN, R, V]	➣ Write the decimal for a concrete or pictorial representation of part of a set, part of a region, or part of a unit of measure. ➣ Represent a decimal using concrete materials or a pictorial representation. ➣ Represent an equivalent tenth, hundredth, or thousandth for a decimal, using a grid. ➣ Express a tenth as an equivalent hundredth and thousandth. ➣ Express a hundredth as an equivalent thousandth ➣ Describe the value of each digit in a decimal.
5.N.9 Relate decimals to fractions (tenths, hundredths, thousandths). [CN, R, V]	➣ Write a decimal in fractional form. ➣ Write a fraction with a denominator of 10, 100, or 1000 as a decimal. ➣ Express a pictorial or concrete representation as a fraction or decimal (e.g., 250 shaded squares on a thousandth grid can be expressed as 0.250 or $\frac{250}{1000}$).

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of fractions less than or equal to one using concrete and pictorial representations
- Naming and recording fractions for the parts of a whole or a set
- Comparing and ordering fractions with like numerators or like denominators
- Providing examples of where fractions are used
- Describing and representing decimals (tenths and hundredths) concretely, pictorially, and symbolically
- Relating fractions to decimals (to hundredths)

Related Knowledge

Students should be introduced to the following:

- Using concrete or pictorial representations to create equivalent fractions
- Measuring the length of an object in millimetres, centimetres, or metres
- Stating the relationship between millimetres and centimetres, centimetres and metres, millimetres and metres

Background Information

Knowledge of decimals is necessary to deal effectively with everyday situations involving money, measurement, probability, and statistics. However, many students lack the understanding of decimals needed to deal with these situations in meaningful ways. Many of their misconceptions about decimals stem from their efforts to apply whole number concepts to decimals. For example, some students believe that 0.143 is greater than 0.43 because 143 is larger than 43, while others believe that 0.150 is ten times larger than 0.15 since 150 is ten times larger than 15.

Instruction that focuses on the meaning of decimals can help students overcome or avoid these misconceptions. In particular, learning experiences need to emphasize two interpretations of decimals: first, decimals are just another symbol for common fractions whose denominators are powers of ten; and second, decimals are an extension of the base-10 numeration system. Allowing students to manipulate concrete and pictorial representations of decimals and helping them make connections between their actions on these representations and the symbols for decimals can facilitate their understanding of these two interpretations.

Mathematical Language

Decimal

Decimal point

Denominator

Equivalent

Fraction

Hundredths

Numerator

Tenths

Thousandths

Learning Experiences

Assessing Prior Knowledge

Materials: Paper and pencil

Organization: Individual/Whole class

Procedure:

a) Tell students that in the next few lessons they will be learning about decimals, but before they begin you need to find out what they already know about decimals. Have students write a letter telling you what they know about decimals.

b) When students finish, have them share what they know about decimals with the other members of the class. Use the discussion to clear up any misconceptions they might have about decimals.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ relate fractions to decimals (to hundredths)
- ❑ describe and represent decimals pictorially and symbolically
- ❑ identify examples and non-examples of decimals
- ❑ identify examples of where decimals are used
- ❑ recognize that decimals are an extension of the place value system

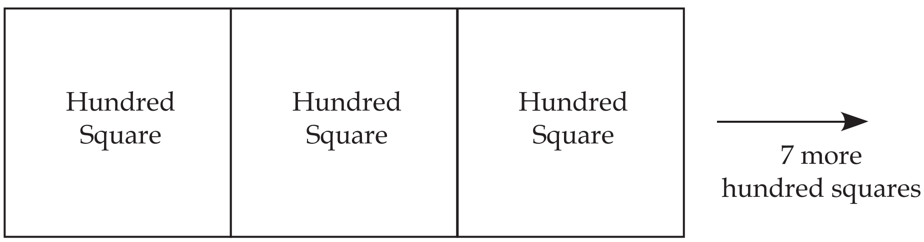
- **Write the decimal for a concrete or pictorial representation of a part of a set, part of a region, or part of a unit of measure.**
- **Represent a decimal using concrete materials or a pictorial representation.**
- **Represent an equivalent tenth, hundredth, or thousandth for a decimal, using a grid.**

Materials: Hundred squares (BLM 5–8.6) and tape

Organization: Whole class

Procedure:

a) Give each student 10 hundred squares. Tell them that the squares are pieces of a whole and that they will be using them to learn about decimals. Ask students to arrange the pieces in a strip to show a whole. Have them tape the pieces together, as shown below.



b) Name the piece "one whole strip." Ask students, "How many hundred squares make one whole strip? If there are 10 hundred squares in a whole, what is a hundred square called (1 tenth)? How many rows of ten small squares are there? How do you know? If there are 100 columns of ten small squares, what is each column of ten small squares called (1 hundredth)? How many small squares are in the whole? How do you know? If there are 1000 small squares in the whole, what is each small square called (1 thousandth)?"

c) Ask students to count parts of the strips by tenths. Have them point to each tenth as they count. Next, have them count by hundredths. Tell students that since it would take too long to count by thousandths, you want them to count by 10 thousandths. Ask, "What part of the whole is 10 thousandths (1 hundredth)?" Have students point to each 10 thousandth as they count 10 thousandths, 20 thousandths, …, 100 thousandths, …, 1000 thousandths.

d) Write the decimals 0.1 and 0.01 on the board or overhead. Ask students to use their strips to show what each decimal means. Then ask, "How do you think we should write the decimal for one-thousandth? Why?" Discuss that it makes sense to use the next place to the right for thousandths.

e) Write the following decimals on the board. Have students read each decimal and illustrate it with their strip.

■ 0. 007	■ 0. 015
■ 0. 065	■ 0. 105
■ 0. 504	■ 0. 253
■ 0. 183	■ 0. 724
■ 0. 965	■ 0. 008

Note: Students will need this "thousands strip" for a later activity.

Observation Checklist

Observe students' responses to determine the strategy they used to find the larger fraction. For example, some students may find the larger fraction by

- ❏ identifying tenths, hundredths, and thousandths
- ❏ counting by tenths, hundredths, and 10 thousandths
- ❏ reading decimals correctly
- ❏ associating symbols for decimals with a concrete representation
- ❏ representing decimals with concrete materials

- **Represent a decimal using concrete materials or a pictorial representation.**

Materials: Metre sticks (one for each pair of students) and an overhead transparency of a metre stick

Organization: Pairs

Procedure:

a) Ask students to find 0.1 of their metre stick. Record the decimal on the board or overhead, and then ask students to explain their choice.

b) Repeat the activity—that is, have students use their metre sticks to show the following:

■ 0.01	■ 0.008
■ 0.001	■ 0.680
■ 0.040	■ 0.308
■ 0.125	■ 0.882
■ 0.268	■ 0.913

Write the decimals on the board or overhead and have students explain their choices.

c) Vary the activity by asking students questions such as:
   - "Is 0.001 closer to 0 or to 1?"
   - "Is 0.275 closer to 0.200 or 0.300?"
   - "What decimals come between 0.775 and 0.780?"
   - "What decimal comes immediately before 0.431?"
   - "What decimal comes immediately after 0.599?"

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ represent decimals with concrete materials
- ❑ associate symbols for decimals with concrete materials
- ❑ identify the decimal that comes immediately before or after a given decimal
- ❑ identify decimals that come between two given decimals

- **Represent a decimal using concrete materials or a pictorial representation.**
- **Express a tenth as an equivalent hundredth and thousandth.**
- **Express a hundredth as an equivalent thousandth.**
- **Describe the value of each digit in a decimal.**

Materials: The thousandths strips that students made and used in a prior activity for these outcomes

Organization: Whole class/Pairs

Procedure:

a) Have students decide what part of their strip represents 0.600. Next, ask students to use their strip to help them name a decimal equivalent to 0.600 (0.6 and 0.60). If students experience difficulty completing this task, have them count by tenths (and then hundredths) until 0.600 is reached.

b) Repeat the activity, but this time ask students to decide what part of their strip represents 0.350. Then have students use their strip to find a decimal equivalent to 0.350 (0.3 + 0.05 and 0.35). Continue to give students other thousandths and ask them to find decimal equivalents.

c) Have students use their strips to find decimals equivalent to 0.45 (0.450, 0.40 + 0.05, 0.4 + 0.05) and 0.3 (0.30 and 0.300).

d) Have students work with their partner to find decimals that are equivalent to

- 0.125 (0.1 + 0.20 + 0.005, 0.12 + 0.005, 0.1 + 0.025)
- 0.328
- 0.8
- 0.630
- 0.72
- 0.034
- 0.295
- 0.044
- 0.638

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ represent a decimal using concrete materials
- ❏ describe the value of each digit in a decimal
- ❏ express tenths as equivalent hundredths and thousandths
- ❏ describe thousandths as equivalent tenths and hundredths
- ❏ describe hundredths as equivalent tenths and thousandths

Materials: Thousandth grid (BLM 5–8.15) (Students should use one grid for each decimal number.), crayons or markers

Organization: Whole class/Small groups

Procedure:

a) Ask students to use the grid paper to represent these decimals.

0.2	0.4	0.7	0.9
0.20	0.40	0.70	0.90
0.200	0.400	0.700	0.900

b) When students finish, ask questions about the relationships among the decimals, such as:
   - "What do you notice about two-tenths, twenty-hundredths, and two-hundred-thousandths?"
   - "How are these decimals alike?"
   - "How do they differ?"

c) Have students use the grid paper to find decimals that are equivalent to 0.10, 0.300, 0.5, 0.60, and 0.8. Encourage students to discuss their findings by asking questions similar to the following:
   - "What decimals are equivalent to ten hundredths? How do you know?"
   - "How are the decimals alike?"
   - "How do they differ?"
   - "What rule do your observations suggest?"

d) Have groups consider these questions: "What is the difference between the values of the 5 in (1) and the values in (2)? What does this tell you? How do you know?"
   1. 5, 50, 500
   2. 0.5, 0.50, 0.500

e) Have the groups share their answers with the other members of the class.

Observation Checklist

Observe students' responses to determine whether they can do the following:

❑ demonstrate the meaning of a decimal by representing it pictorially

❑ represent an equivalent tenth, hundredth, or thousandth using a grid

❑ recognize and explain the similarities and differences among equivalent decimals

❑ represent a tenth as an equivalent hundredth or thousandth

❑ recognize that annexing a zero to the right of a whole number changes the place and value of each digit

❑ recognize that annexing a zero to the right of a decimal changes the name but not the place or value of each digit

❑ describe the value of each digit in a whole number or decimal

Materials: Base-10 blocks or digi-blocks and a place value mat (BLM 5–8.16)

Organization: Whole class

Procedure:

a) Show students the large block and tell them that, for this activity, the block will represent one, a flat will represent one-tenth, the long will represent one-hundredth, and the small cube will represent one-thousandth. To help students get used to the new number names for the pieces, ask them to use their blocks to show you:

■ two-tenths	five-hundredths	six-thousandths
■ nine-tenths	two-hundredths	three-thousandths
■ five-tenths	four-hundredths	nine-thousandths

b) Continue asking students to show you different numbers with their base-10 blocks until they can do it quickly and easily.

c) Represent the following decimals with the blocks and ask students to write the corresponding symbols for the decimals.

■ 2.156	■ 1.903
■ 0.189	■ 5.341
■ 1.057	■ 3.204
■ 3.420	■ 2.651

Encourage students to discuss their answers by asking them questions such as:

- "What decimal did you write?"
- "What is the value of the 1 in 2.156?"
- "What is the place value of the 8 in 0.189?"
- "What is the value of the 2 in 3.204? In 3.420?"
- "What decimal is equivalent to 3.420?"

d) Vary the activity by naming a decimal (e.g., 1.254) and having students represent it with their blocks. After they represent the number, ask them to record the corresponding symbol (decimal). Again, encourage students to discuss their answers by asking questions similar to the ones in part (c).

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ write the decimal that corresponds to its concrete representation
- ❑ represent a decimal using concrete or pictorial representations
- ❑ state the value of each digit in a decimal
- ❑ name the place value position of each digit in a decimal
- ❑ recognize equivalent decimals

- **Write a decimal in fractional form.**
- **Write a fraction with a denominator of 10, 100, or 1000 as a decimal.**
- **Express a pictorial or concrete representation as a fraction or decimal.**

Materials: Thousandths grid (BLM 5–8.15) and coloured pencils

Organization: Pairs

Procedure:

a) Give students several copies of the thousandths grid. Have students observe that each grid is a 20-by-50 rectangle containing 1000 small squares.

b) Ask students to shade in 350 squares, and then write under the grid the decimal and fraction names for the shaded-in squares.

c) Next, have students shade in 250 squares. Ask students to write the basic fraction and decimal name for the shaded squares $\left(\frac{250}{1000} \text{ and } 0.250\right)$ Encourage students to use their grids to help them find equivalent numbers. Ask:
   - "What decimal is equivalent to $\frac{250}{1000}$? How do you know?"
   - "What fraction with a smaller numerator and denominator is equivalent to $\frac{250}{1000}$? $\left(\frac{25}{100} \text{ or } \frac{1}{4}\right)$ How do you know?"
   - "What other fraction is equivalent to $\frac{250}{1000}$? How do you know?"

d) Repeat the activity several times. For example, have students shade

■ 125 squares	■ 600 squares
■ 184 squares	■ 750 squares
■ 375 squares	■ 100 squares
■ 500 squares	■ 267 squares

   Encourage students to write as many decimal and fraction names as they can for each shaded grid.

e) Have students share their answers and explain their reasoning.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ express a pictorial representation as a fraction
- ❑ express a pictorial representation as a decimal
- ❑ identify equivalent fractions for a pictorial representation
- ❑ identify equivalent decimal for a pictorial representation
- ❑ explain their reasoning

- **Write a decimal in fractional form.**
- **Write a fraction with a denominator of 10, 100, or 1000 as a decimal.**

Materials: Fraction and decimal equivalent cards (BLM 5.N.8&9.1)

Organization: Pairs

Procedure:

a) Tell students that they will be playing a game that involves matching decimals with their equivalent fractions.

b) Tell students that they will need to shuffle their cards and spread them face up on their playing area. When you say go, they should begin matching the fraction/decimal cards without saying anything to their partner. The first pair to match the decimal/fraction cards correctly wins.

c) Demonstrate how to play the game and answer any questions students might have. Have them play the game.

d) Repeat the activity, but this time put a time limit on it. For example, give students a minute to complete the activity. The pair that matches the most number of cards correctly wins.

e) Have students make their own equivalent fraction/decimal cards and use them to play the game.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ associate a fraction with a denominator of 10 with an equivalent decimal
- ❏ associate a fraction with a denominator of 100 with an equivalent decimal
- ❏ associate a fraction with a denominator of 1000 with an equivalent decimal
- ❏ identify fractions and decimals that are not equivalent

Materials: Fraction and decimal equivalent cards (BLM 5.N.8&9.1)

Organization: Small groups

Procedure:

a) Tell students that they will be playing concentration with the fraction/decimal cards.

b) To play the game, have students spread the cards face down on the playing area. Have students take turns turning over two cards. If the cards match, the player keeps the cards and takes another turn. If the cards do not match, the player turns them back over and the next player takes a turn. Play continues until all the cards have been matched. The player with the most cards is the winner.

c) Demonstrate how to play the game and answer any questions students might have. Have students play the game.

d) Have students play the game again with the fractions/decimal equivalent cards that they made (see previous activity).

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ associate a fraction with a denominator of 10 with an equivalent decimal
- ❏ associate a fraction with a denominator of 100 with an equivalent decimal
- ❏ associate a fraction with a denominator of 1000 with an equivalent decimal
- ❏ identify fractions and decimals that are not equivalent

Materials: A coin and paper and pencils

Organization: Whole class

Procedure:

a) Tell students that their job is to record the numbers that you will be reading to them. They can record a number as a decimal or as a fraction (they must choose one representation, but can change which they choose for each new number). They will score a point if they record the number correctly. After they record the number, you will toss a coin. If the coin lands heads up, everyone who wrote the number correctly as a decimal scores another point. If the coin lands tails up, everyone who wrote the number correctly as a fraction scores another point. Students keep their own score.

b) Read these numbers to students:

- three thousandths
- one-hundred-fifteen thousandths
- sixty-two thousandths
- eighty thousandths
- forty hundredths
- six tenths
- one-hundred-five thousandths
- nine tenths
- five-hundred-three thousandths
- three hundredths

c) Select 10 more numbers to read to the class. Repeat the activity, but this time let the student who scored the most points read the numbers.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ associate an oral representation of a number with the correct decimal symbol for the number
- ❑ associate an oral representation of a number with the correct fraction symbol for the number
- ❑ read a decimal numeral correctly
- ❑ read a fraction correctly

NOTES

Grade 5: Number (5.N.10)

Enduring Understandings:

Decimals are an extension of the base-10 numeration system.

General Outcome:

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
5.N.10 Compare and order decimals (tenths, hundredths, thousandths) by using ■ benchmarks ■ place value ■ equivalent decimals [CN, R, V]	➛ Order a set of decimals by placing them on a number line (vertical or horizontal) that contains the benchmarks 0.0, 0.5, 1.0. ➛ Order a set of decimals including only tenths using place value. ➛ Order a set of decimals including only hundredths using place value. ➛ Order a set of decimals including only thousandths using place value. ➛ Explain what is the same and what is different about 0.2, 0.20, and 0.200. ➛ Order a set of decimals including tenths, hundredths, and thousandths using equivalent decimals.

Prior Knowledge

Students may have had experience with the following:

- Describing and representing decimals (tenths and hundredths)

Related Knowledge

Students should be introduced to the following:

- Describing and representing decimals to thousandths concretely, pictorially, and symbolically

Mathematical Language

Decimal

Equivalent

Benchmark

Tenths

Hundredths

Thousandths

Learning Experiences

- **Order a set of decimals including only tenths using place value.**
- **Order a set of decimals including only hundredths using place value.**

Materials: Math journals, base-10 blocks, place value mats (BLM 5–8.16)

Organization: Pairs

Procedure:

a) Show students a flat and tell them that, for this activity, a flat will represent one, a long will represent one-tenth, and the small cube will represent one-hundredth. To help students get used to the new number names for the pieces, ask them to use their blocks to show you the following:
   - nine-tenths
   - two-tenths
   - three-tenths
   - one-hundredth
   - six-hundredths
   - fourteen-hundredths

b) Continue asking students to show you different numbers with their base-10 blocks until they can do it quickly and easily.

c) Have students use their blocks to represent each pair of numbers and decide which number is larger. Ask students to record their answers.
   - 0.5 and 0.8
   - 0.2 and 0.7
   - 1.3 and 1.2
   - 7.8 and 7.9
   - 0.34 and 0.54
   - 0.21 and 0.12
   - 0.53 and 0.08
   - 0.07 and 0.42

d) Vary the activity by asking students to

- make a number larger than a given number (e.g., ask students to make a number greater than 2.6)
- make a number smaller than a given number (e.g., ask students to make a number less than 0.36)
- make a number between two given numbers (e.g., ask students to make a number between 0.1 and 0.4)
- look at each number in a pair of numbers, decide which number is greater, and then check their answers with the blocks (e.g., ask students to circle the larger number [0.51 or 0.43], and then use their blocks to check their answers)

e) Ask students to answer the following questions in their math journals:

- "Which number is smaller: 2.8 or 2.3? How do you know?"
- "Which number is larger: 0.53 or 0.06? How do you know?"

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ order a set of decimals including only tenths using place value
- ❏ rename hundredths as tenths and hundredths (e.g., Do students recognize that fifty-three–hundredths is the same as five-tenths and three-hundredths)
- ❏ order a set of decimals including only hundredths using place value
- ❏ identify numbers greater than (or less than) a given number
- ❏ identify numbers that come between two given numbers

- **Order a set of decimals including only tenths using place value.**
- **Order a set of decimals including only hundredths using place value.**
- **Order a set of decimals including only thousandths using place value.**

Materials: One die per group, recording sheet (BLM 5.N.10.1)

Organization: Small groups

Procedure:

a) Tell students that they will be playing a game called "Less Than." Explain how the game is played.
   1. Designate one player to roll the die. When the die is rolled, players write the resulting number in one of their boxes before the die is rolled again. Once a number is written on a line, it cannot be changed.
   2. A round of the game consists of four rolls of the die. If the two numbers that are generated from the four rolls of the die are in the correct order, the player scores one point. If the numbers are not in the correct order, the player scores a zero.
   3. Play four rounds. The winner is the player with the most points.

b) Demonstrate how to play the game and answer any questions students may have. Have students play the game.

c) Vary the game by having students
   - create and order three decimal numbers (___ · ___ < ___ · ___ < ___ · ___)
   - create and order decimals in the hundredths or thousandths (e.g., 0 · ____ ____ < 0 · ____ ___)

Observation Checklist

Depending on the version of the game that is played, check students' responses to determine whether they can order a set of decimals

❑ including only tenths using place value

❑ including only hundredths using place value

❑ including only thousandths using place value

Materials: Paper and pencils

Organization: Individual

Procedure:

a) Have students solve problems like the following:

- The split times for Dimitri and Euclid were 10.321 and 10.209 seconds respectively. Who had the fastest time?
- The masses of five eggs were as follows: 0.042 kg, 0.053 kg, 0.051 kg, 0.049 kg, and 0.056 kg. Place the masses in order from smallest to largest.
- To make a miniature toy car, you need tires with a width between 0.465 cm and 0.472 cm. Will a tire with a width of 0.469 cm work? Explain your answer.
- At a swimming competition, June scored 9.80, Nora scored 9.75, Debbie scored 9.79, and Alexia scored 9.81. What must Tina score to win the competition? Explain your answer.

b) Have students share their solutions to the problems and explain their reasoning.

Observation Checklist

Check students' responses to determine whether they can do the following:

- ❏ solve problems involving the comparing and ordering of decimals to thousandths
- ❏ compare and order hundredths
- ❏ compare and order thousandths

Materials: Paper and pencil

Organization: Individual

Procedure:

a) Ask students to name a decimal that is
   - greater than 5.9 and less than 6
   - greater than 9 and less than 9.1
   - greater than 0.63 and less than 0.64
   - greater than 8.9 and less than 9.15
   - greater than 7.8 and less than 7.62

b) Have students use the digits 0 through 9 to complete the answer to each statement. A digit cannot be used more than once.
   - greater than 0.52 but less than 0.53 0.5__ __
   - greater than 0.614 but less than 0.62 0.6 __ __
   - greater than 83.07 but less than 83.079 83.0 __ __
   - greater than 367.821 but less than 367.831 376.8 __ __

Observation Checklist

Monitor students' responses to determine whether they can do the following:

- ❑ identify a number between two given numbers
- ❑ compare and order decimals including only tenths
- ❑ compare and order decimals including only hundredths
- ❑ compare and order decimals including only thousandths

- **Order a set of decimals by placing them on a number line (vertical or horizontal) that contains the benchmarks, 0.0, 0.5, 1.0.**

Materials: Decimal cards (BLM 5.N.10.2), as well as grid paper, metre sticks, or base-10 blocks available for students who would like to use them

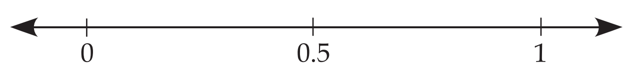
Organization: Pairs or small groups/Whole class

Procedure:

a) Give each pair of students a set of cards and ask them to sort the cards into three groups: those that are close to zero, those that are close to five-tenths, and those that are close to 1.

b) When students finish sorting their cards, have them share their answers with the other members of the class. Encourage students to explain their reasoning by asking them questions such as:
   - "How do you know when a decimal is close to five-tenths?"
   - "How do you know when a number is close to zero?"
   - "How do you know when a number is close to one?"
   - "Are the number of decimal places important in determining the size of a decimal? Why?"
   - "Which decimals are hardest to order? Why?"

c) Draw either a horizontal or vertical number line on the board and label the points 0, 0.5, and 1.



Ask students to estimate where one-tenth would be on the number line. Have a student tape a card showing one-tenth on the number line. Continue having students estimate and show where the numbers on their decimal cards would be on the number line.

d) Ask each pair of students to make their own decimal cards that show hundredths and thousandths (e.g., 0.20 and 0.132). Have them exchange their cards with another pair and then sort the cards into three groups: decimals that are close to one, decimals that are close to five-tenths, and decimals close to zero.

e) Have students share the numbers and their explanations of how they grouped them with the other members of the class. Encourage them to explain their reasoning by asking them questions similar to those in part (b).

f) Select several of the number cards that the students created and have students estimate and show where the numbers are on the number line.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ identify decimals that are close to the benchmarks of 0, 0.5, and 1
- ❏ explain how they know when a decimal is close to 0, 0.5, and 1
- ❏ order a set of decimals consisting of tenths and hundredths using place value
- ❏ order a set of decimals consisting of hundredths and thousandths using place value
- ❏ order a set of decimals by placing them on a number line that contains the benchmarks 0, 0.5, and 1
- ❏ order a set of tenths, hundredths, and thousandths by using equivalent decimals

■ Order a set of decimals including tenths, hundredths, and thousandths using equivalent decimals.

Materials: Blank dice (Write the decimals 0.4, 0.3, 0.9, 0.74, 0.04, and 0.60 on each die.)

Organization: Pairs

Procedure:

a) Tell students that they are going to play a place value game caller "Larger." Explain how the game is played.

1. Both players roll a die at the same time. After a roll, each player records the resulting decimal numbers in a table like the one shown below.

My Number																
My Partner's Number																

2. Players circle the larger decimal number. The player with the larger number scores one point. If a tie occurs, circle both decimal numbers and give each player a point.
3. The winner of the game is the person with the most points after 20 rounds of the game.

b) Demonstrate how the game is played and answer any questions students may have. Have students play the game.

c) Vary the game by writing different decimals on the dice.

Observation Checklist

Observe students' responses to the game to determine whether they can do the following:

- ❑ order sets of decimals using place value
- ❑ order tenths and hundredths using equivalent decimals

■ **Order a set of decimals including only thousandths using place value.**

Materials: Players' averages and batting criteria (see below), nine pieces of paper for each student or pair of students

Organization: Individual or pairs/Large group

Procedure:

a) Present students with the following scenario:

- You have just been hired to coach the school's softball team. You have to make up the batting order for the game today but there is no one around who can help you. You know from your coaching experience that a player's batting average, on-base average, and slugging average are good indications of where a player should bat in the lineup. These are the averages of your players:

Player	Batting Average	On-Base Average	Slugging Average
Arnez	.310	.335	.414
Britton	.415	.477	.318
Brock	.297	.331	.355
Charles	.338	.355	.614
Jackson	.429	.461	.586
Lamar	.323	.360	.576
Rolen	.273	.341	.358
Santos	.248	.305	.315
White	.423	.416	.399

b) Tell students that they must make up cards that list each player's statistics, and then arrange them in a batting order that is most closely aligned to the following criteria. When they finish, they should make a lineup card to share with the rest of the class.

Player	Batting Average	On-Base Average	Slugging Average
1	High	Highest	Low–Medium
2	High	Medium–High	Medium
3	Highest	High	High
4	Medium	Medium	Highest
5	Medium	Medium	High
6	Medium	Medium	Medium
7	Low–Medium	Low–Medium	Low–Medium
8	Low–Medium	Low–Medium	Low–Medium
9	Lowest	Lowest	Lowest

c) Before students begin working on the activity, make sure they understand that the higher the batting average, the more often the player gets a hit; the higher the on-base average, the more often the player gets on base; and the higher the slugging average, the more often a player gets an extra base hit (doubles, triples, and homers).

d) Have students share their lineups with the other members of the class. Encourage them to explain why their lineups satisfy the given criteria.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ order a set of decimals in the thousandths using place value
- ❑ create a lineup card that fits the criteria
- ❑ explain the reasons for the placement of each batter in the lineup

Grade 5: Number (5.N.11)

Enduring Understandings:

Adding and subtracting decimals is similar to adding and subtracting whole numbers.

General Outcome:

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
5.N.11 Demonstrate an understanding of addition and subtraction of decimals (to thousandths), concretely, pictorially, and symbolically, by ■ using personal strategies ■ using the standard algorithms ■ using estimation ■ solving problems [C, CN, ME, PS, R, V]	➣ Estimate a sum or difference using front-end estimation (e.g., for 6.3 + 0.25 + 306.158, think 6 + 306, so the sum is greater than 312) and place the decimal in the appropriate place. ➣ Correct errors of decimal point placements in sums and differences without using paper and pencil. ➣ Explain why keeping track of place value positions is important when adding and subtracting decimals. ➣ Predict sums and differences of decimals using estimation strategies. ➣ Solve a problem that involves addition and subtraction of decimals, to thousandths. ➣ Model and explain the relationship that exists between an algorithm, place value, and number properties. ➣ Determine the sum and difference using the standard algorithms of vertical addition and subtraction. (Numbers are arranged vertically with corresponding place value digits aligned.) ➣ Refine personal strategies, such as mental math, to increase efficiency when appropriate (e.g., 3.36 + 9.65 think, 0.35 + 0.65 = 1.00, therefore, 0.36 + 0.65 = 1.01 and 3 + 9 = 12 for a total of 13.01).

Prior Knowledge

Students may have had experience with the following:

- Using compatible numbers when adding and subtracting decimals (to hundredths)
- Estimating the sums and differences of problems involving addition and subtraction of decimals (to hundredths)
- Using mental math strategies to solve problems involving addition and subtraction of decimals (to hundredths)

Related Knowledge

Students should be introduced to the following:

- Describing and representing decimals to thousandths concretely, pictorially, and symbolically
- Comparing and ordering decimals to thousandths
- Modelling and explaining the relationship between mm and cm units and mm and m units

Mathematical Language

Addition

Difference

Estimate

Subtraction

Sum

Assessing Prior Knowledge

Materials: Paper and pencil

Organization: Individual

Procedure:

a) Ask students to solve the following problems:

   1. Roberta was getting ready for the first day of school. She bought a set of five pens for \$2.57, a binder for \$4.35, and a box of three-hole paper for \$5.15. What was the total cost of her school supplies?
   2. Joe is saving money to buy a new video game. He has \$19.50, and tomorrow Mr. Mitchell is giving him \$4.75 for mowing his lawn. If the video game cost \$39.95, how much more money does Joe need to save?

b) Have students share their solutions to the questions. Encourage them to explain the strategies they used to solve the problems.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ identify the operation(s) needed to solve a computational problem
- ❏ calculate sums and differences of decimals (limited to hundredths)
- ❏ solve one- and two-step problems
- ❏ use appropriate strategies to solve addition and subtraction problems involving decimals

- **Solve a problem that involves addition and subtraction of decimals, to thousandths.**

Materials: Base-10 blocks, two different-coloured dies for each group, and a place value mat (BLM 5–8.16)

Organization: Small groups

Procedure:

a) Show students the large block and explain that they will be playing a game that involves letting the block represent one. Ask, "If the block represents one, what does the flat represent? Why? What does the long represent? Why? What does the small cube represent? Why?"

b) To help students become familiar with the new names for the base-10 block, ask them to use their blocks to show
   - 5 tenths
   - 8 tenths
   - 6 hundredths
   - 3 hundredths
   - 12 hundredths
   - 2 thousandths
   - 7 thousandths
   - 15 thousandths

   Continue to ask students to represent different decimals with the blocks until they can do it quickly and easily.

c) Tell students that the game they will be playing is called "Race to One." Explain how the game is played.
   1. Let one coloured die represent hundredths and the other coloured die represent thousandths.
   2. Players take turns rolling the die and using their blocks to represent the number on their place value mats.
   3. When players get 10 cubes, they trade them for a long. When they have 10 longs, they trade them for a flat, and when they have 10 flats, they trade them for a block.
   4. The first player to get a block wins the game.

d) Demonstrate how the game is played and answer any questions students might have. Have students play the game.

e) After students have played the game several times, have them use a recording sheet like the one shown below.

Turn	Number Rolled	Total
1	0.006	0.006
2	0.012	0.018

f) Repeat the activity for subtraction—that is, have students play "Race to Zero." For this version of the game, students start with the large block and, on each turn, remove the number of blocks that represents the number rolled on the die. The first player to remove all of his or her blocks is the winner. After students have played the game several times, have them use a recording sheet like the one shown above.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ use concrete materials to solve problems involving addition and subtraction of decimals to thousandths
- ❏ make groups of ten to trade, when necessary
- ❏ exchange materials for smaller or larger units when necessary

Materials: Decimal grid paper that shows thousandths (BLM 5–8.15), metre sticks, and base-10 blocks

Organization: Individual/Large group

Procedure:

a) Tell students that they will be solving some problems involving decimals. Explain that they can use a strategy of their own choosing and that grid paper, metre sticks, and base-10 blocks are available if they want to use materials to help them solve the problems.

b) Present students with these problems:

- Marco bought two bananas. The mass of the first banana was 0.057 kg, and the mass of the second was 0.45 kg. What was the combined mass of the bananas?
- At the swim meet, Marsha scored 9.234 points for her high dive. Maxine scored 6.192 points for her high dive. How much higher was Marsha’s score than Maxine’s?
- Nadia has a piece of string that is 1.12 m long. If she uses 0.509 m to tie a package, how much string does she have now?
- Dimitri ran the first leg of a race in 10.123 seconds, the second leg of the race in 9.72 seconds, and the last leg of the race in 11.658 seconds. How long did it take him to run the entire race?

c) When students finish solving a problem, have them share their solutions with the other members of the class. Encourage students to explain the strategies they use to solve the problems.

d) Continue to give students problems like those above.

Observation Checklist

Observe students’ responses to determine whether they can do the following:

- ❏ identify the operation needed to solve the problem
- ❏ solve problems involving addition and subtraction of decimals to thousandths
- ❏ use a variety of strategies to solve a problem
- ❏ calculate correctly

- **Place the decimal point in a sum or difference using front-end estimation.**
- **Explain why keeping track of place value positions is important when adding and subtracting decimals.**
- **Predict sums and differences of decimals using estimation strategies.**

Materials: Paper and pencils

Organization: Pairs/Whole class

Procedure:

a) Tell students that you will be giving them some problems. Explain that you do not want them to find the answers to the problems. Instead, you want them to list everything that they know about them. For example, there are two things we know about this problem:

   0.3 + 0.7 + 0.5 = _____

   The answer is in tenths and the answer is greater than one.

b) Ask students to make a list of everything they know about the following problems:

   1. 8.245 + 0.28 + 1.35
   2. 15.921 – 9.468
   3. 11.03 + 4.8 + 12.143
   4. 10.186 – 4.795
   5. 19.823 + 0.45 + 6.782
   6. 4 – 0.357

c) Have students share their answers with the other members of the class. Encourage students to explain their reasoning by asking them questions such as:

   - "How do you know the answer is about 9?"
   - "How do you know the answer is in thousandths?"
   - "Why is it important to keep track of place value positions when adding? Subtracting?"
   - "How do you know that the answer is less than 3?"

d) Instead of having students list everything they know about a problem, have them list what the answer cannot be. For example, the answer to the problem 5.123 + 4.382 cannot be less than 9. The total number in the thousandths place cannot be greater than 10.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ predict sums and differences of decimals using estimation strategies (e.g., front-end estimation or front-end estimation with compensation)
- ❑ explain why it's important to keep track of place value positions when adding and subtracting decimals
- ❑ predict the place of the decimal point in sums and differences using front-end estimation

- **Explain why keeping track of place value positions is important when adding and subtracting decimals.**
- **Solve a problem that involves the addition and subtraction of decimals, limited to thousandths.**

Materials: Number cards (BLM 5–8.5) (one set for each student), number frames (BLM 5.N.11.1)

Organization: Pairs

Procedure:

a) Ask students to arrange cards 1–6 on their number mats to create a problem with the largest possible sum. Have them record their solution. Have students share their solutions with the other members of the class and explain their reasoning.

b) Repeat the activity. Have students use cards 1–6 to create problems with the
   - smallest possible sum
   - largest possible difference
   - smallest possible difference

c) Have students use the digit cards 0, 1, 4, 6, 7, and 9 to create problems with the
   - largest possible sum
   - smallest possible sum
   - largest possible difference
   - smallest possible difference

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ solve problems involving addition and subtraction of decimals (limited to thousandths)
- ❑ compare and order decimals to thousandths
- ❑ identify the place value position of each digit in a decimal (limited to thousandths)
- ❑ identify the value of each digit in a decimal (limited to thousandths)
- ❑ calculate sums and differences involving decimals using appropriate strategies

- ■ **Predict sums and differences of decimals using estimation strategies.**
- ■ **Solve a problem that involves addition and subtraction of decimals, to thousandths.**

Materials: Copies of the activity (BLM 5.N.11.2)

Organization: Individual

Procedure:

a) Have students complete the activity.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ solve problems involving addition and subtraction of decimals (limited to thousandths)
- ❑ predict sums and differences involving decimals using estimation strategies
- ❑ calculate sums and differences involving decimals using appropriate computational strategies

Materials: Paper and pencils, number fan (BLM 5–8.17)

Organization: Pairs/Whole class

Procedure:

a) Ask students to make a decimal greater than 0.8 using their number fans. Make a list of their suggestions to show them a variety of answers.

b) Tell students that they will be going on a hunt for decimals. Explain that you will be giving them different clues and their job is to work with their partner to find an answer that fits the clue and to show their response on a number fan.

   1. A decimal between 3.25 and 3.26
   2. Two decimals whose sum is 9.346
   3. Two decimals with a difference of 0.821
   4. Three decimals whose sum is 4.734

c) Have students share their answers. Make a list of their answers to each statement so students can see a variety of solutions. Encourage students to share the strategies that they used to determine their answers.

d) Have each pair of students make up clues and exchange them with another pair of students.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ solve problems involving addition and subtraction of decimals (limited to thousandths)
- ❏ order decimals including tenths, hundredths, and thousandths
- ❏ recognize the relationship between operations
- ❏ calculate sums and differences involving decimals using appropriate computational strategies
- ❏ explain the strategies they used to determine the answers to the statements

Planning a Party

Purpose:

The intent of this investigation is to have students apply their knowledge of number and measurement concepts to a real-world situation. In particular, the investigation is designed to reinforce students' ability to

- apply estimation strategies
- compute whole numbers
- demonstrate an understanding of fractions
- represent decimals to thousandths
- compare and order decimals
- add and subtract decimals
- demonstrate an understanding of units of measure and their relationship to each other

In addition, the investigation enhances students' ability to

- solve problems
- reason mathematically
- make connections to other subjects (science and ELA)
- make connections to the real world
- communicate mathematically

Materials/Resources: Price list and purchase order (see below)

Organization: Small groups

Procedure:

a) Present the following situation:
   - You have been asked to plan a party for the students in your class. You need to order food, beverages, and entertainment for the party. You must plan two hours of entertainment. You have $25.00 plus $2.00 per person to spend on the party.

b) Tell students that they should use the following price list to decide which items they would order for the party. Explain that they can specify the flavours of the drink and food that they want (e.g., apple juice and chocolate cake). They can also name a specific movie or game (sports or video) that they would like to have at the party. Anything else they need for the party, such as plates and utensils, will be provided.

Price List:

Beverages:

No name pop	$0.98 for 2 L
Brand name pop	$1.98 for 2 L
No name juice	$1.12 for 1 L
Brand name juice	$2.04 for 1 L
Milk	$5.00 for 4 L

Food:

Apples	4 for $1.00
Watermelon	1/2 for $3.50
No name chips	$4.99 for a 500 g bag
Brand name chips	$1.25 for 100 g bag
Hot dogs	$2.79 per dozen
Hot dog buns	$2.50 for 10
Microwave popcorn	$2.99 for 3 bags
20 cm by 30 cm cake	$12.00
40 cm by 60 cm cake	$20.00
Cupcakes/muffins	$0.50 each
Large carrots	$1.44 for 12 carrots
Baby carrots	$2.99 for 30 carrots
Party sub	$15.00 for 10 people
Individual subs	$1.75 per person

Entertainment:

Movie rental	$5.49
Game system rental	$21.75
Video game rental	$4.26
Karaoke machine	$28.15
Community centre rental	$11.00 per hour
Sporting equipment rental	$1.50 per person
Sports centre rentals	$2.25 per person

c) Explain that when they have decided on what items they want for the party, they should complete the following purchase order:

Purchase Order

Item	Reasons	Amount You Will Buy?	How Much Each Person Will Get	Cost
			Total Cost	

d) Have students complete the investigation and then share their plans for the party. Encourage students to explain their reasoning.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ give reasonable estimates of the amount of food and drink needed for the party
- ❏ calculate correctly the cost of an item
- ❏ calculate correctly the amount of an item needed
- ❏ provide appropriate reasons for choosing an item
- ❏ calculate correctly the total cost of the party
- ❏ calculate the cost of the party to be less than or equal to the amount of money available
- ❏ plan entertainment that is within the two-hour limit.
- ❏ plan entertainment that is fun and inclusive for all
- ❏ select food and drinks that meet everyone's needs
- ❏ select food and drinks that reflect the school's policy on nutrition