Foundational Experiences

Foundational Experiences

For everything we learn, we rely on prior experiences to help us understand, connect, and remember concepts and skills. When foundational experiences precede formal instruction for a mathematics topic, students can access those experiences in their memories to make sense of the mathematical ideas underlying the formal terminology, symbols, and arithmetic.

During instruction for foundational experiences, students have opportunities to reason, problem solve, estimate, visualize, communicate, use technology, and connect ideas—that is, opportunities to experience mathematics through the seven mathematical processes of the curriculum. Foundational experiences should be accessible and appealing, and they should sustain engagement over time.

Students may not always be interested in mathematical concepts and skills, but they are more likely to be interested when they are actively engaged with them. If students find the experience is worth talking about (e.g., with peers, their teacher, and others), then they are demonstrating their interest while building their foundational understanding of the content.

Patterned Mental Math

The intention of the "patterned mental math" style of questions is to have students notice patterns when doing a series of mental math problems that the teacher has carefully selected and sequenced. The patterns may be practice for some students and foundational experiences for others, depending on the focus of the patterned mental math and the prior experience of the students. This style of mental math lends itself to explore a variety of arithmetic and algebra topic areas. An experienced teacher will recognize topics for which this format would be helpful to students.

The process begins with students receiving a blank template. See page 18 for samples. The teacher orally provides mental math problems for students to write down and solve using the spaces on the template. The first three groups of questions are prepared ahead of time and given orally by the teacher to the students. The fourth group on the template is reserved for students to follow the structure they notice in the first three groups of questions and to make up their own questions and solutions for parts (a) to (d). The fourth group will allow students to follow and test the pattern.

Radical Square Hunt

This experience is intended for students to use areas of squares to explore irrational numbers as support for IAP outcome A2. This will be a foundational experience for students learning about irrational (radical) numbers and their representations. For those familiar with square roots, the experience will be an opportunity to solidify and connect their knowledge of area of squares and representations of radical numbers.

Powering Up and Down

This section is intended to describe students' visual and tactile foundational experiences that allow them to explore and experience powers and exponential growth as support for IAP outcome A3. Some students may have worked through some of these foundational experiences in Grade 9 in preparation for their work on powers with integral bases (outcomes 9N1 and 9N2). These experiences will be foundational for the development of their understanding of powers and exponential growth (or decay).

Patterned Mental Math

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Understanding common factors and trinomial factors*

1. The following series of questions is support for IAP outcome A5. The questions are intended to help students gain a deeper understanding of factoring a difference of squares while purposefully doing mental math. A student would have created the completed template (shown below) after being given oral instructions from the teacher, such as the following:

   "In section 1a, write the expression '5 subtract 1' and write the result."

   "In section 1b, write the expression and the result of 5 plus 1."

   "In section 1c, write the product of the answers from a and b."

   "In section 1d, write the expression and the result of '5 squared subtract 1 squared'."

   "In section 2a, write the expression and the result of '10 subtract 1'."

   "In section 2b, write the expression and the result of '10 plus 1'."

   "In section 2c, write the product of the answers from a and b."

   "In section 2d, write the expression and the result of '10 squared subtract 1 squared'."

   "In section 3a, write the expression and the result of '–5 subtract 1'."

   "In section 3b, write the expression and the result of '–5 plus 1'."

   "In section 3c, write the product of the answers from a and b."

   "In section 3d, write the expression and the result of '–5 squared subtract 1 squared'."

   "In section 4, using the structure of questions 1, 2, and 3 above, make up your own."

   "What do you notice? What do you wonder? Write three things in the space at the bottom. Compare what you wrote with what your neighbour wrote."

Patterned Mental Math

Date Monday

Name Pat

1a $5 - 1 = 4$	2a $10 - 1 = 9$
1b $5 + 1 = 6$	2b $10 + 1 = 11$
1c product $4 \times 6 = 24$	2c $9 \times 11 = 99$
1d $5^2 - 1^2 = 24$	2d $10^2 - 1^2 = 99$

3a $-5 - 1 = -6$	4a $2 - 1 = 1$
3b $-5 + 1 = -4$	4b $2 + 1 = 3$
3c $(-6)(-4) = 24$	4c $1 \times 3 = 3$
3d $(-5)^2 - (1)^2 = 24$	4d $2^2 - 1^2 = 3$

The product of the difference and sum is the same value as the difference of squares.
The pattern works when adding and subtracting 1 even when starting with a negative number.
Could I add or subtract something other than 1 and still follow the pattern?

This may be a foundational experience for students who have not yet come to understand formal algebra as generalized arithmetic. This experience may help them make a general rule. For students with algebra experience, the teacher may ask the class (or some students), “Can you show that the pattern works for all cases, generally?”

As a next step, students could be asked to make a square that is 5×5 (using tiles, graph paper, etc.) and subtract a square of size 1×1. They should demonstrate (by rearranging pieces) and explain why, when the square is subtracted, the area is the same as a 4×6 rectangle.

Patterned Mental Math

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Multiplication of polynomial expressions concretely, pictorially, symbolically*

2. The following series of questions is support for IAP outcome A4 to have students do purposeful mental math to connect their understanding of the distributive property to multiplication of polynomial expressions. A teacher could begin with the following oral instructions to explore another pattern while doing mental math.

   "In section 1a, write the expression '7 plus 1' and write the result."

   "In section 1b, write the expression and the result of '7 times the answer from a'."

   "In section 1c, write the product of '7 times 7'."

   "In section 1d, write the sum of 7 and the answer from c."

   "In section 2a, write the expression and the result of '6 plus 1'."

   "In section 2b, write the expression and the result of '6 times the answer from a'."

   "In section 2c, write the product of '6 times 6'."

   "In section 2d, write the sum of 6 and the answer from c."

   The instructions and pattern would proceed, as with the first example, through the group 3 questions given by the teacher and the group 4 questions where students use the pattern structure to make up their own questions. This is followed by three statements of "notice and wonder." This example could help some students explore and articulate the distributive property.

Patterned Mental Math Date ____________

Name _______________

1a
1b
1c
1d

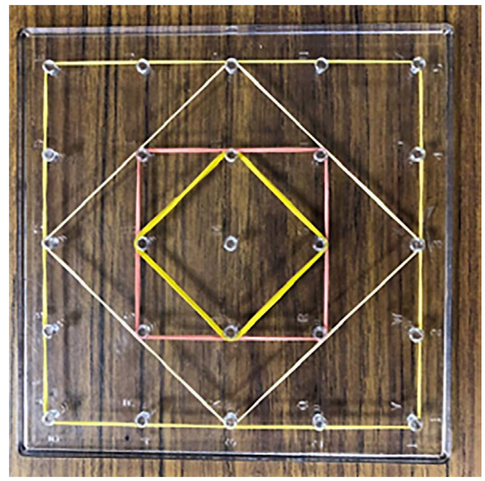
2a
2b
2c
2d

3a
3b
3c
3d

4a
4b
4c
4d

Radical Square Hunt

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Understanding irrational numbers: representing, identifying, simplifying, ordering*



You can have students use geoboards and elastics to create various squares where the smallest square on the geoboard (1 × 1) represents an area of 1 unit. Alternatively, you can use grid paper where the smallest square (1 cm × 1 cm or possibly ¼ inch × ¼ inch) represents an area of 1 unit. Initially, work within a 4 × 4 graph paper grid or geoboard (see page 21).

Ask students, "What whole number areas can you represent by creating squares on the geoboard (or graph paper with vertices at grid intersection points)?"

As squares of different sizes are created, encourage students to count the areas on the geoboard by physically pointing at each 1 unit^2 area. Students should write the multiplication statement for length times width equals area in ways that are meaningful to them. For example, students could write the following:

For the large yellow square above $4 \times 4 = 16$; $\sqrt{16} \times \sqrt{16} = 16$

For the tan-coloured square above $\sqrt{8} \times \sqrt{8} = 8$; $2\sqrt{2} \times 2\sqrt{2} = 8$

Initially, with a geoboard, students may only create squares with areas of 1, 4, 9, and 16. If so, ask the students what other whole number areas can be represented with squares on the geoboard. Or ask, "Can a square with an area of 8 be created?" Some students will probably start to create squares using oblique lines as sides (rather than vertical and horizontal). If necessary, ask students to draw the diagonal of the square with area of 4 to get them thinking about lengths other than horizontal and vertical. The teacher will need to pay attention to what students are saying and what they are trying in order to determine the next good question to ask to probe their thinking. Note: On a 4 × 4 grid, the squares that can be created have whole number areas of 1, 2, 4, 5, 8, 9, 10, and 16—it is not necessary that all students find all of these areas.

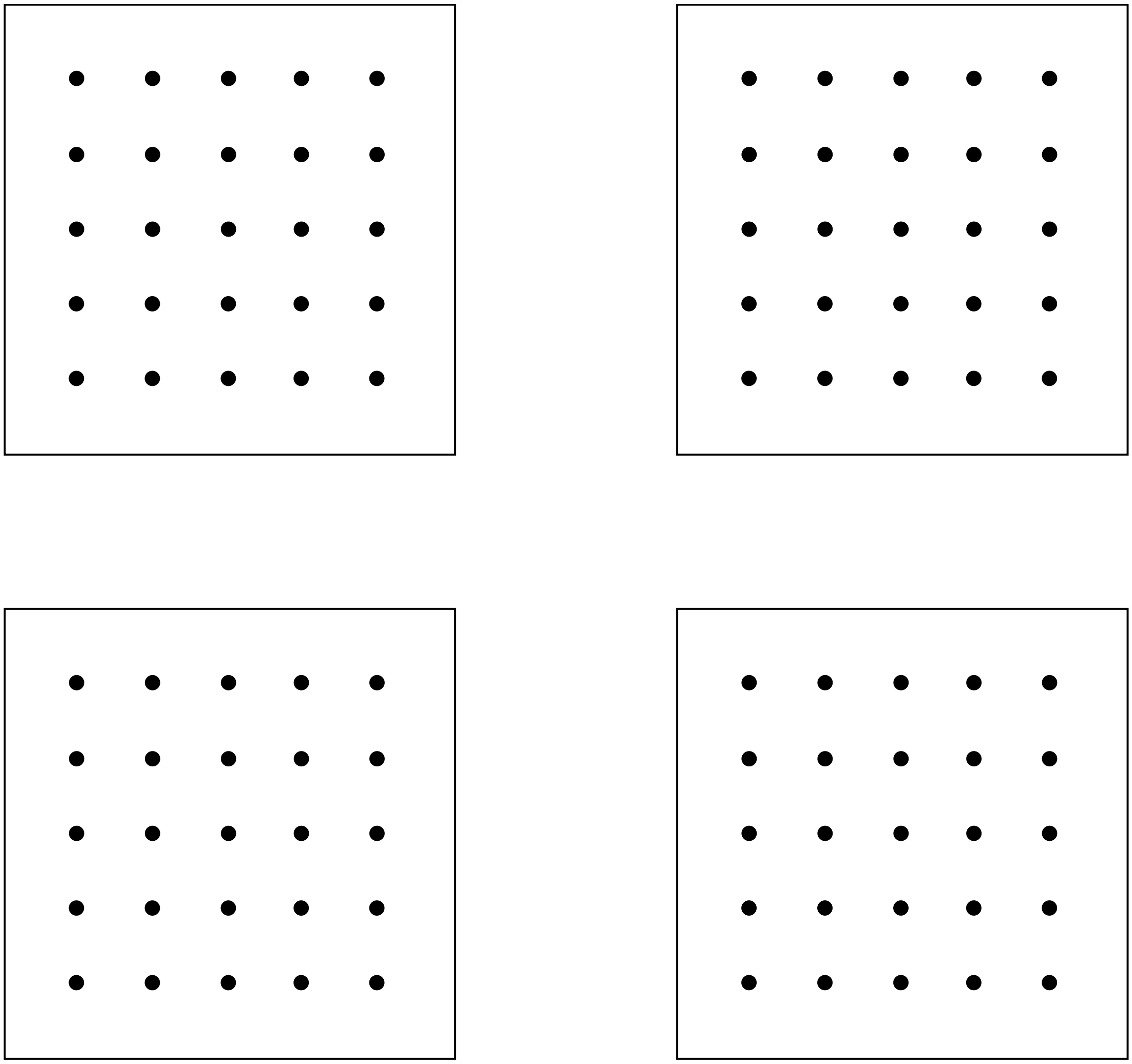
Teachers can help students to make use of their foundational experiences within the above activity. The following is an example for outcome 10I.A.2: "Demonstrate an understanding of irrational numbers by: representing, identifying, and simplifying irrational numbers; ordering irrational numbers." By referring to the students' foundational experiences with the radical square hunt, teachers can help students to link their understandings of the Pythagorean theorem with the mathematics of the new outcome.

As a next step, students could use the Pythagorean theorem to explore the relationship between the areas and the lengths of the sides of the squares they have created. They could be asked, “Compare the lengths of sides of the squares with areas of 2 and 8.” To extend the examples, students could use graph paper to work with a larger 5×5 grid. Now, they could be asked, “Create a square with an area of 18.” [Note: Squares with areas of 13, 17, and 25 are also possible.] “What do you notice and wonder when you compare the lengths of the sides of the squares with areas 2, 8, and 18?” “Imagine an even larger grid. Are there other square areas that could be related?”

The next step would have students notice, think about, and express some of the square side-length relationships (that is, a square with an area of 2 has a side of $\sqrt{2}$; a square with an area of 8 has a side of $\sqrt{8}$, which is the same length as $2\sqrt{2}$ or three times the length of the square with side $\sqrt{2}$). Similarly, a square with an area of 18 has a side of $\sqrt{18}$, which is the same length as $3\sqrt{2}$ (that is, three times the length of the square with side $\sqrt{2}$). They may notice the pattern $1\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}$ and wonder what is the next area of square in this pattern. They may wonder what other square areas can be related this way, or they may notice $\sqrt{5}$ and $\sqrt{20}$ have this same relationship. With encouragement, students can inquire about and express such relationships collaboratively.

This record sheet is for students to keep track of their work with elastics on a geoboard.

Radical Square Hunt



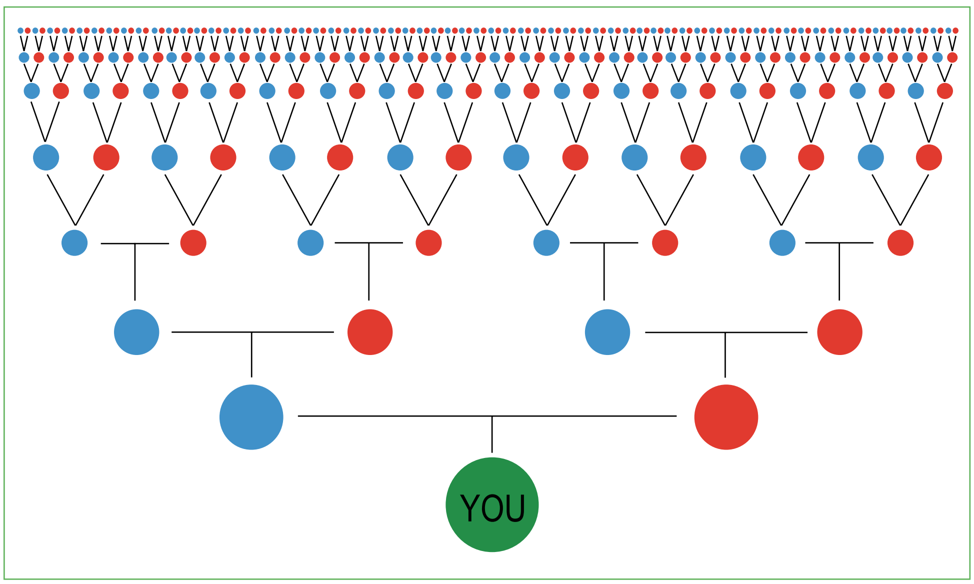
Powering Up and Down

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Understanding powers with integral and rational exponents*

Students can engage in these foundational experiences with the concept of powers without relying on formal symbolic representations of powers, these activities can precede formal instruction with the topic.

The following list of experiences adds to the depth of student understanding of powers and exponential growth.

1. **Family Tree:** How many direct ancestors did you have when Treaty 1 was signed in 1871? How many ancestors did you have when Columbus landed on the East Coast of North America? How many direct ancestors do you have that are alive today? How can you represent the patterns within your ancestry with objects? …with drawings? …with numbers? …with words?



2. **Shampoo Sales:** In the 1980s, advertisements for a shampoo company used the math of doubling to show how quickly a good idea can be spread. After trying the shampoo, the model said, "I told two friends and they told two friends, and so on, and so on, and so on." With each level of telling two friends, the number of images of the model doubled. What number of images do you expect to be displayed in the advertisement at each successive stage? When does the display of images get out of hand? How can you better represent the array of images so that you can show many iterations? What does this pattern suggest about the power of online communication?

   Here is one of the many variations on the advertisement: www.youtube.com/watch?v=brC_jK6stBs (CTB).

3. **Ebola Virus:** In the chart below, which sections of the pattern of contagion approach doubling? What questions might you be able to explore with doubling if the monthly doubling of new cases did model the spread of Ebola? What information about this disease and its treatment might help with your thinking?

Ebola in Guinea, Liberia, and Sierra Leone		
Date (Month)	**New Cases per Month**	**Deaths per Month (40%)**
2014-03-31	120	48
2014-04-30	114	45
2014-05-31	75	30
2014-06-30	290	116
2014-07-31	723	289
2014-08-31	1730	692
2014-09-30	3501	1400
2014-10-31	6987	2794
		(CDC)

4. **Number Line:** Create a number line on a length of paper, counting by centimetres. Use one-centimetre grid paper to cut out lengths or rectangles, doubling each time. Attach the cut-outs to the number line. Describe the pattern. Describe the length of the number line you'll need to keep going.

5. **Hundreds Board:** Start with a photocopy of a hundreds board with numbers, and have a few copies of a blank hundreds board (without numbers) on hand. Glue squares of coloured paper onto the hundreds board to show the positions of the doubling numbers {1, 2, 4, 8, …}. Aim to show the first ten or so. Describe some of your thinking as you experience the power of doubling in this activity.

6. **Climbing the Doubling Ladder:** Have students create a doubling ladder as a table of values (see below). Just as in counting, you start at zero; when you start doubling, you start at one. Students will notice that doubling starting at zero is not very interesting. With the exception of students who have an interest and are ready for further exploration, it is intended that students go up the doubling ladder, starting at the "seed" value on the ground at step 0. Building a ladder with five to eight steps is usually sufficient for students to be able to construct mental images of the ladder steps that go beyond what they have recorded on paper.

Steps	Doubling Value
...	...
3	8 = 1 x 2 x 2 x 2
2	4 = 1 x 2 x 2
1	2 = 1 x 2
seed	1
...	...

Have students individually create a table as above and fill in the steps and the doubling values above the seed.

In groups of 2 or 3, instruct them to move up and down the steps of the doubling ladder and keep track of the doubling value for each of the following tasks.

A. Start at the ground and count up 4 steps.
   - What is the doubling value?
   - Count up 8 steps. What is the doubling value?
   - How many steps do you need to go up before reaching a 3-digit number? What is its value?
   - How many steps do you need to go up before reaching a 4-digit number? What is its value?

B. Start at the ground and count up 3 steps.
   - Start at step 1 and count up 3 steps.
   - Start at step 3 and count up 3 steps.
   - Describe and explain any patterns that you notice.

C. Start at step 4 and count down 3 steps (record the starting value and the resulting doubling value).
   - Start at step 4 and go down 3 steps.
   - Start at step 10 and go down 3 steps.
   - Start at step 5 and go down 3 steps.
   - Describe and explain any patterns that you notice.

D. Start at step 3, then up, up, down, down, down.
  - Start anywhere, then up, up, down, down, down.
  - Describe and explain any patterns that you notice.

After each section of tasks, students should be given an opportunity to share with the group the patterns they notice. Teachers might want to have some groups talk to others, as needed, to be sure that all students have an opportunity to see the patterns developing. After the experience, it is worthwhile to debrief as a whole class about what they noticed in general and what they wondered about. Some extensions and possible wonderings after this experience include the following:*

"What doubling values appear if the steps of the ladder go below ground?"

"What numbers appear if it is a tripling ladder instead of a doubling ladder?"

7. **Staying on the Doubling Ladder:** This experience is intended to help students develop a strong sense of the numbers on the doubling ladder and how powers interact with the four basic operations. Have students individually create a table, as shown below, and fill in the steps and the doubling values above the seed. Alternatively, students may use a doubling ladder that they have previously created.

Steps	Doubling Value
...	...
3	8 = 1 x 2 x 2 x 2
2	4 = 1 x 2 x 2
1	2 = 1 x 2
seed	1
...	...

Students could be asked to work in groups of 2 or 3 as they work on the following tasks.

* The following articles describe Manitoba teachers and students engaging in these activities:

Slivinski, Peter, Steven Erickson, and Ralph T. Mason. "Mathematics 9: Designing Foundational Experiences for the Hardest Topics." *MERN Journal*, vol. 11, 2015, pp. 50–57.

Mason, Ralph T., and Steven Erickson. "Foundational Experiences for Secondary Mathematics: A New Approach to Curriculum?" *MERN Journal*, vol. 11, 2015, pp. 17–20. Available online at http://mbtrc.org/data/documents/Journal-V11.pdf.

Pick two numbers on the ladder. Determine if you can perform the operation (each one separately) and obtain a result that is also a number on the doubling ladder. Is it always true? Sometimes? If so, under what circumstances?

- Add them.
- Subtract them.
- Multiply them.
- Divide them.

Debrief with students about the patterns they see. They may ask if they can pick the same number twice (as in 4 + 4). You may wish to encourage such exploration.

8. **Undoubling on the Doubling Ladder:** By cutting rectangles from grid paper, students can easily make rectangles to represent the values on the doubling ladder, up to 64. Lead them in undoubling, starting with 64 and either folding or cutting to undouble. When they get down to a rectangle of area 1, undoubling will make values below the seed. As they proceed, have students record these "undoubling numbers" on their ladder. To help students visualize the results of undoubling, invite them to cut out a "magnified seed," a ten-by-ten square to represent 1. With the magnified seed, students can make fraction and decimal representations for several layers of the doubling ladder below the seed level of 1.

   Extension: Repeat activity with a new seed. Are any observations and descriptions of patterns still true? Why or why not?