NUMERACY COUNTS

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Thinking Mathematically

Reasoning is a mathematical process that helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and to justify their mathematical thinking. Applying their reasoning skills to higher-order questions challenges students to think and develop a sense of wonder about mathematics.

In her research paper "Student Interaction in the Math Classroom: Stealing Ideas or Building Understanding," Dr. Catherine D. Bruce stressed the importance of the quality of math tasks—particularly how open-ended tasks, which have multiple solutions and/or which permit multiple solution strategies, offer students increased opportunities to explain and justify their reasoning. If a task involves a simple operation and single solution, there will be little or no opportunity to engage students.
http://en.copian.ca/library/research/what_works/student_interaction_math/student_interaction_math.pdf

The following charts compare closed and open-ended tasks.

Closed Tasks

$6 \times 4 =$ ______

???
I can`t remember the answer.

Characteristics:

These tasks

- have just one correct answer
- require arithmetical thinking
- develop procedural fluency

Open-Ended Tasks

Sam multiplied two numbers together and got an answer of 24. What might the two numbers be? Explain your thinking.

I could do 1×24, but I am going to use ½ and 48.

Characteristics:

These tasks

- require multiple solutions or multiple strategies to arrive at the answer
- ask students to explain, justify, and make predictions
- have multiple entry points so that all students can access the problem
- require mathematical thinking
- develop conceptual understanding

Students need to experience a balance between both question types, but many of the resources available to teachers (both print and online) tend to address closed tasks rather than open-ended/rich tasks. One way to address this issue is to transform closed tasks into ones that are more open-ended.

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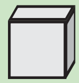
K–4 Examples

Closed Tasks	Open-Ended Tasks
432 + 167 = ________	I added two 3-digit numbers. The sum was close to 610. What might the two numbers have been? Show three possible solutions.
What is this object? 	I took a 3-D object out of a box and traced around one of its faces. The shape I drew was a square. Which 3-D object might I have taken out of the box?
Complete the following: 23 × 2 = ________ 51 × 3 = ________	Use the digits 1, 2, and 3. Fill in the empty boxes. $\square\square \times \square = ?$ How many different questions can you make? Which one gives the largest answer? Which one gives the smallest answer?

5–8 Examples

Closed Tasks	Open-Ended Tasks
Two numbers have a difference of 2.38. The smaller number is 3.12. What is the larger number?	Two numbers have a difference of 2.38. What could the numbers be if ■ the two numbers add up to 6? ■ one of the numbers is three times larger than the other?
List the factors of 80.	Common factors of two numbers are 2 and 6. What might the numbers be?
Solve: $n + 6 = 13 - 5$ $10 = 2c$	Create a word problem for the equation $n + 2 = 10$.

Questions for Reflection

- **How are open-ended tasks currently being used?**
- **What challenges do you see in trying to move to using more open-ended tasks?**
- **What resources/supports might be needed?**

Resources

The following are some useful resources for teaching mathematical reasoning.

- *Good Questions for Math Teaching: Why Ask Them and What to Ask (Grades K–6)* (2002) by Peter Sullivan and Pat Lilburn.
- *Good Questions for Math Teaching: Why Ask Them and What to Ask (Grades 5–8)* (2005) by Lainie Schuster and Nancy Canavan Anderson.
- *Good Questions: Great Ways to Differentiate Mathematics Instruction, 2nd Edition* (2012) by Marian Small.
- *Parallel and Open Task Problem-Solving Math Bank* by the Ontario Teachers' Federation www.otffeo.on.ca/en/resources/lesson-plans/parallel-open-task-problem-solving-math-bank/
- *Open-Ended Questions for Mathematics* by the Appalachian Rural Systemic Initiative (2000) https://www.uky.edu/OtherOrgs/ARSI/www.uky.edu/pub/arsi/openresponsequestions/mathorq.pdf