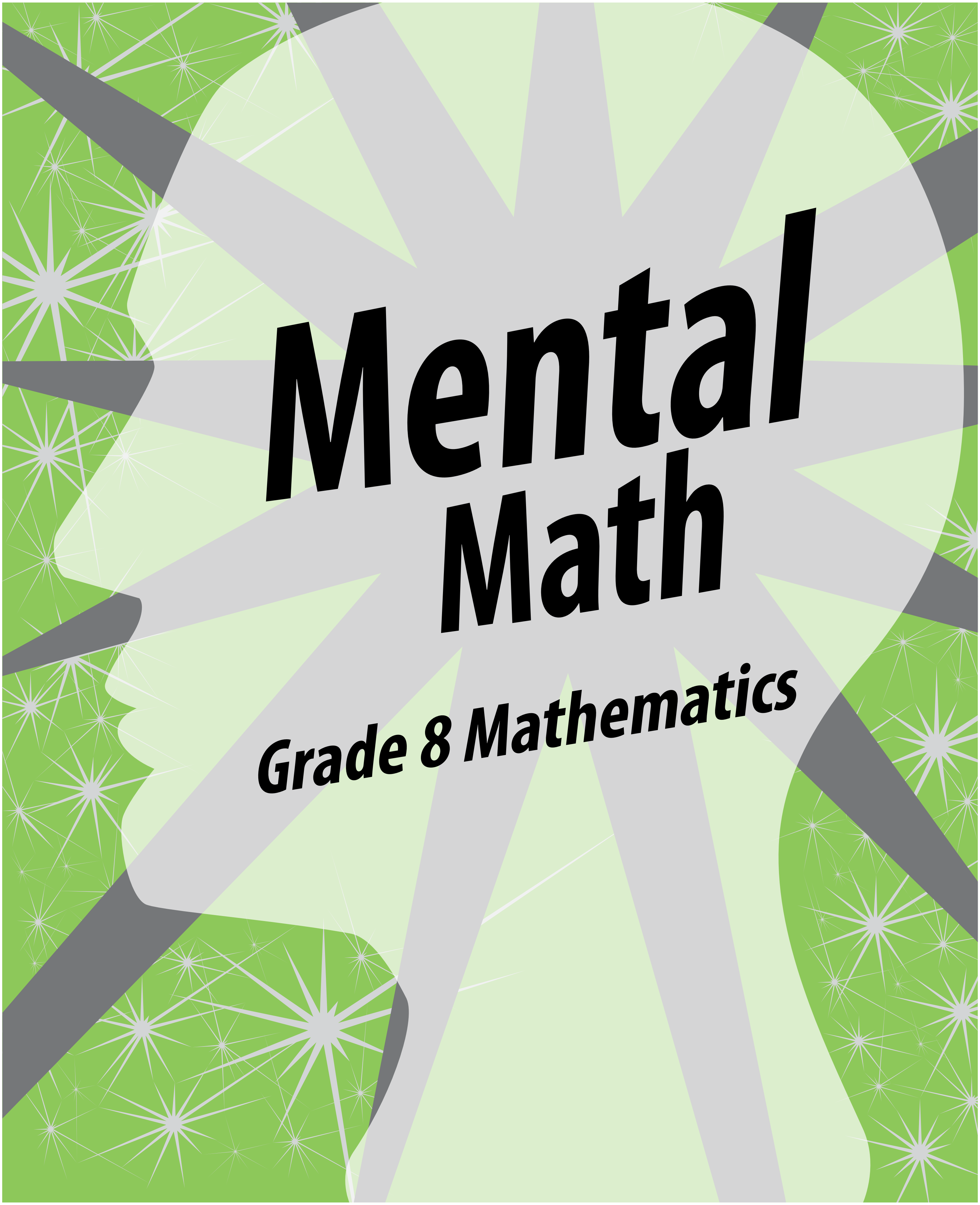

# Mental Math

## Grade 8 Mathematics

Manitoba

# Mental Math: Grade 8 Mathematics

Support Document

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Schools are encouraged to share this resource with parents, guardians, and communities, as appropriate.

Any websites referenced in this resource are subject to change without notice. Educators are advised to preview and evaluate websites and online resources before recommending them for student use.

Print copies of this resource can be purchased from the Manitoba Learning Resource Centre (stock number 80733). Order online at www.mtbb.mb.ca.

This resource is also available on the Manitoba Education and Training website at www.edu.gov.mb.ca/k12/cur/math/supports.html.

While the department is committed to making its publications as accessible as possible, some of the mathematical formulas, equations, and graphics in this document are not fully accessible at this time.

Available in alternate formats upon request.

# Contents

# ACKNOWLEDGEMENTS

Manitoba Education and Training wishes to thank the members of the *Mental Math: Grade 8 Mathematics* Document Development Team for their contribution to this document. Their dedication and hard work have made this document possible.

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# Introduction

# INTRODUCTION

*Mental Math: Grade 8 Mathematics* is a complement to the Grade 8 Mathematics curriculum. This document is intended for use in helping students to develop strategies for performing mental calculations and estimations more flexibly, efficiently, and accurately.

## Why Mental Mathematics and Estimation?

Mental math and estimation are essential components of the Manitoba Kindergarten to Grade 8 Mathematics curriculum and are one of the seven processes to be thoroughly integrated into the teaching and learning of Grade 8 Mathematics learning outcomes. Manitoba's teachers are asked to report student success in mental math and estimation on provincial student report cards, as well as on provincial assessments. This guide provides possible methods for both teaching and assessing these skills.

The Grade 8 Mathematics curriculum states:

> "Mental mathematics and estimation is a combination of cognitive strategies that enhances flexible thinking and number sense. It is calculating mentally without the use of external memory aids. It improves computational fluency by developing efficiency, accuracy, and flexibility." (Manitoba Education and Advanced Learning, 2013, 12)

Mental calculation requires the use of knowledge of numbers and mathematical operations. Mental calculation is at the root of the estimation process, and it allows individuals to determine whether results obtained with a calculator or a pen and paper are reasonable.

> "Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Estimation is also used to make mathematical judgements and to develop useful, efficient strategies for dealing with situations in daily life. When estimating, students need to know which strategy to use and how to use it." (Manitoba Education and Advanced Learning, 2013, 12)

Students who have experience with mental math and estimation develop the ability to work flexibly with numbers. Practising mental calculation and estimation is important in the development of number sense and it is foundational in gaining a clear understanding of place value, mathematical operations, and basic numeracy.

Mental calculation and estimation are very practical skills that can be used to do quick calculations at times when a pencil or a calculator is unavailable. Using mental calculation strategies can also eliminate some steps in written calculations and help simplify processes as students progress through the grades. Teachers should provide opportunities for their students to use mental math and estimation on a regular basis and encourage their students to find examples of the usefulness of mental calculation in their lives, such as when shopping, travelling, playing sports, or doing other everyday activities.

## Strategies

Through a thorough implementation of the Manitoba Kindergarten to Grade 8 curriculum, Grade 8 students will have learned many of the mental math and estimation strategies outlined in this document in previous grades. Students may not have used these methods for several years, however, or they may not have seen how the strategies they learned connect to their current studies.

Students may also not realize that the strategies that are most effective for mental calculation are often not the same strategies that are most effective for written calculation. These are two very different skills that require two very different processes. In order to encourage flexibility, efficiency, and accuracy, it is important that this distinction is made.

## Document Features

This document includes three main sections: this introduction, a section describing strategies, and a series of mental mathematics questions organized by *learning targets*.

*Learning targets* are groups of related outcomes derived from *Glance Across the Grades – Manitoba 2015*. In the same way, this document consists of thirteen (13) learning targets related to the Grade 8 Mathematics learning outcomes.

| Learning Target (as used in Glance Across the Grades) | Specific Learning Outcomes | Number of Pages |
|---|---|---|
| Operations with Whole Numbers (Multiplication/Division) | 8.N.1, 8.N.7 | 12 |
| Representation of Rational Numbers | 8.N.4 | 8 |
| Operations with Rational Numbers | 8.N.2, 8.N.3, 8.N.5, 8.N.6, 8.N.8 | 11 |
| Patterning and Algebraic Thinking | 8.PR.1 | 4 |
| Algebraic Representations with Equations | 8.PR.2 | 5 |
| Length | 8.SS.1 | 3 |
| Area | 8.SS.3 | 3 |
| Volume (Capacity) | 8.SS.4 | 3 |
| Identifying, Sorting, Comparing, and Constructing | 8.SS.2, 8.SS.5 | 3 |
| Position and Motion | 8.SS.6 | 2 |
| Collection, Organization, and Analysis of Data | 8.SP.1 | 2 |
| Probability | 8.SP.2 | 2 |

The learning target and strategies of focus are identified on each page. The questions on each page are divided into three different categories:

- Six **prior learning questions** from earlier grade levels (These are meant for reflection and practice, as well as for formative assessment.)
- Four **Grade 8 questions** related to the learning target of study
- Two **other questions** for teachers to insert their own questions

The answers to the questions are provided in the column on the extreme right-hand side of each page. The flexible and efficient use of strategies should always remain the focus of these mental math activities. It is critical that students reflect on their processes more than on the correctness of their answers in order for them to further develop their mental math abilities.

A blank template is also provided in a section titled Reproducible Sheets. Teachers may use it to prepare additional question sheets.

A file in Word format is available in the Mathematics Group on the Maple (Manitoba Professional Learning Environment) site at www.mapleforem.ca. It is provided to enable teachers to add or modify questions to suit the needs of their students.

## Methodology

Teachers should ensure that mental calculation exercises are short in duration because they require significant sustained concentration. Although these calculations should be done within a certain period of time, it is important that teachers avoid emphasizing speed. Although speed may be a factor in basic recall, it is not the primary goal of developing the mental computational methods targeted in this document. Ensuring that mental math and estimation occur regularly in a classroom will encourage students to see these skills as important in their daily lives and to build their abilities. Possible methods to do this in a class include the following:

- Incorporating Number Talks: A five- to ten-minute class conversation crafted around a mental math problem and the flexible methods students use to arrive at an answer (Parrish)
- Projecting portions of this program at the front of the class, visually or orally, and discussing the strategies used afterwards
- Making use of a template such as the one provided on page 13 to encourage communication among students about strategies
- Student journaling
- Teacher facilitation of discussion and small group work
- Presentation of portions of this program as a game with competing teams
- Student creation of mental math questions
- Questioning, reflecting, and discussing methods used
- Discussing errors in the process that led to an incorrect answer

The primary goal of this program is to improve student use of mental math strategies. Through communication about the strategies and the methods being used in class, students can self-assess their own strategy choice and become more comfortable with trying new processes. Students should be encouraged to continually look for, and practice, the processes they discover to be the most efficient and accurate.

## Assessment

Exercises from this document can be used as assessments *for*, *as*, and *of* learning.

- Assessment *for* learning *(Prior Learning Questions section)*: The first six questions on each page should serve as formative assessment. Results from this can be used to pinpoint student learning gaps from previous years and direct future instruction. These prior learning sections are exclusively for use as formative assessment. They cannot factor into summative assessments because their material deals exclusively with outcomes from previous grades.
- Assessment *as* learning: This takes place through quality student self-assessment and reflection. Discussions and reflections help to solidify student understanding. Without this effort, it is unlikely that students will grow in their mental math and estimation abilities. Quality feedback is critical to improved student success.
- Assessment *of* learning: This can take place through the grade-specific sections of the question pages, as well as through any related discussions and activities that students take part in related to these grade-specific sections. Teachers can use these results both formatively and summatively.

Mental math and estimation skills can be assessed in a variety of ways. Although written products give teachers some insight into student thinking, both observations and conversations can provide additional information on student ability in mental math and estimation, and should be used as an additional source of assessment where possible. Rubric use, with a focus on communication skills related to mental math and estimation, can be a method of formatively assessing student conversations and making classroom observations a part of summative data.

Student discussion is an effective way for students to present and self-assess their own thinking. Students are required to be clear and concise when explaining their reasoning to others, and in turn they are given the opportunity to learn new approaches from other students and the teacher. These exchanges about the strategies and results will also allow the teacher to identify the difficulties encountered by some. Subsequently, the teacher can help students discover new, relevant, useful, and important strategies.

# Report Cards

The following is the draft version of Manitoba Education and Training's *Manitoba Report Card Grade Scale Mathematics Achievement Profiles* for mental mathematics and estimation

This profile should be used throughout assessment processes, as well as when assigning a mark to the mental math section of report cards. Students are expected to connect and apply mental math and estimation strategies with skills and knowledge, as well as to communicate mental mathematics and estimation strategies concretely, orally, and written in the form of pictures/diagrams, words, symbols/numbers, graphs, and/or charts.

**Manitoba Report Card Grade Scale Mathematics Achievement Profiles—Grades 1 to 8 (DRAFT April 2017)**
**Subject Category: Mental Mathematics and Estimation**

| Category Indicator | Extent to which the student is meeting grade-level[1] learning outcomes across the provincial report card grading scale | | | | |
|---|---|---|---|---|---|
| | **Not demonstrated (ND)** | **Limited (1)** | **Basic (2)** | **Good (3)** | **Very good to excellent (4)** |
| Connects and applies mental math and estimation strategies with skills and knowledge | Does not yet demonstrate the required understanding and application of concepts and skills. | Requires considerable, ongoing teacher support and/or direction to<br>— connect knowledge of mental math and/or estimation to the understanding of number properties and relationships, and<br>— use mental math and/or estimation strategies and/or must work through the strategies on paper or with concrete objects.<br>May have computational errors. | Requires occasional teacher or peer support to connect knowledge of mental math and/or estimation to the understanding of number properties and relationships.<br>Chooses and uses modelled and familiar mental math and/or estimation strategies.<br>May have computational errors. | Accurately connects knowledge of mental math and/or estimation to the understanding of number properties and relationships.<br>Uses mental math and/or estimation strategies accurately. | Flexibly and efficiently connects knowledge of mental math and estimation to the understanding of number properties and relationships.<br>Efficiently and flexibly chooses and uses mental math and estimation strategies. |
| | | | | | |
| Communicates mental mathematics and estimation strategies<br>▪ Concretely<br>▪ Orally<br>▪ Written<br>o Pictorial/Diagrams<br>o Words<br>o Symbolic/Numbers<br>o Graphs/Charts | Has persistent learning difficulties which hinder or prevent acquisition of knowledge and skills. | Uses very little mathematical vocabulary.<br>Requires considerable, ongoing teacher support to describe mental math and estimation strategies used for computation. | Requires occasional teacher or peer support to use mathematical vocabulary and describe the mental math and estimation strategies used for computation. | May require occasional prompting for clarification.<br>Uses mathematical vocabulary to explain the mental math and estimation strategies used for computation. | Uses a broad range of mathematical vocabulary clearly, completely, and precisely to explain the mental math and estimation strategies used for computation. |

1 As developmentally appropriate for the time of year towards attaining end-of-grade academic outcomes or academic outcomes described in an individual education plan. References in the table to "assistance," etc, do not refer to adaptations defined as "a change in the teaching process, materials, assignments or pupil products to assist a pupil to achieve the expected learning outcomes."' (www.edu.gov.mb.ca/k12/specedu/programming/adaptation.html)

## Grade 8 Mental Math Outcomes

In the Manitoba Grade 8 Mathematics curriculum, the learning outcomes listed below are the only ones that include a specific [ME] designation. This designation means that when addressing the following outcomes, teachers **must** integrate mental math and estimation as a process.

- **8.N.2.** Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers). [C, CN, **ME**, R, T]
- **8.N.6.** Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically. [C, CN, **ME**, PS]
- **8.N.8.** Solve problems involving positive rational numbers. [C, CN, **ME**, PS, R, T, V]
- **8.PR.1.** Graph and analyze two-variable linear relations. [C, **ME**, PS, R, T, V]

Teachers **may** integrate and assess mental math and estimation into all learning outcomes, however, and this document is meant to provide support in doing so.

> "Mental mathematics is a combination of strategies that enhances flexible thinking and number sense. Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Estimation is often used to make mathematical judgments and to develop useful, efficient strategies for dealing with situations in daily life. Strategies in mental mathematics and estimation enable students to calculate mentally without the use of external aids. In the process, they improve their computational fluency—developing efficiency, accuracy, and flexibility."
>
> —Manitoba Education and Advanced Learning (2015)

# Notes

# Reproducible Sheets

# *Mental Math*

## Grade 8 Mathematics

| | |
|---|---|
| **Learning Target:** | |
| **Strategies of Focus:** | |

| **Prior Learning** | **Answers** |
|---|---|
| 1. | |
| 2. | |
| 3. | |
| 4. | |
| 5. | |
| 6. | |
| **Grade-Level Questions** | |
| 7. | |
| 8. | |
| 9. | |
| 10. | |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

| | |
|---|---|
| **Learning Target:** | |
| **Strategies of Focus:** | |

| **Prior Learning** | **Answers** |
|---|---|
| 1. | |
| 2. | |
| 3. | |
| 4. | |
| 5. | |
| 6. | |
| **Grade-Level Questions** | |
| 7. | |
| 8. | |
| 9. | |
| 10. | |
| **Other Questions** | |
| 11. | |
| 12. | |

# Mental Math Student Communication Template

1. Students reason through a mental math or estimation problem mentally, without showing their work, and then they put their response directly in the answer section of the template.
2. Students then explain the steps they performed mentally through writing or drawing.
3. Students can then share their approach with another student and document the method used by someone else.
4. Students can then reflect on the best strategy, in their opinion.

| Question: | | Answer: |
|---|---|---|
| Method used to calculate mentally: | A method used by someone else: | |
| Preferred method and reason: | | |

Example

| **Question:** The Oakville Rangers train for 2 hours and 30 minutes each week. How many minutes per week does the team train? | | **Answer:** 150 minutes |
|---|---|---|
| **Method used to calculate mentally:** $30 + 30 = 1$ hour<br>So<br>$30 + 30 + 30 + 30 = 2$ hours<br>So $30 + 30 + 30 + 30 + 30 = 2\frac{1}{2}$ hours<br>$= 150$ minutes | **A method used by someone else:** $2 \times 60 = 120$<br>$120 + 30 = 150$ minutes | |
| **Preferred method and reason:** I prefer the other student's method because it uses fewer steps. My answer was correct, but took longer to get. | | |

| Question: | | Answer: |
|---|---|---|
| Method used to calculate mentally: | A method used by someone else: | |
| Preferred method and reason: | | |

| Question: | | Answer: |
|---|---|---|
| Method used to calculate mentally: | A method used by someone else: | |
| Preferred method and reason: | | |

# Teaching Mental Math and Estimation Strategies

# Teaching Mental Math and Estimation Strategies

Dozens of mental math and estimation strategies exist, and many go by several different names. This document provides a limited number of strategies and in no way should be taken as an exhaustive list of all of the mental math and estimation strategies available. Strategies can be found online or through teacher guides, and can even be developed within your own classes.

## Curriculum and This Support Document

The Manitoba curriculum does not set specific grades as required teaching for many of these strategies. Some Manitoba divisions have found success in dividing up the most common strategies for specific grade-level focus. Although most of the strategies can be used at all grade levels, organizing a cross-grade method of focusing on common strategies may reduce the amount of focus that teachers feel they need to place on teaching each of the many strategies.

BLM 5-8.8, from the Grades 5–8 Blackline Masters package, is attached in order to help you and your school develop an approach to focusing on several of these strategies while aligning with specific grade-level curricular outcomes.

Strategy posters and teaching methods are also attached for teacher and student support.

This Grade 8 Mental Math document is designed as a support to the existing curriculum and is in no way a mandated program. Individual teachers should use it as they see fit.

## Topics

| | |
|---|---|
| S-1 | Standard algorithms |
| S-2 | Adding from left to right |
| S-3 | Adding parts |
| S-4 | Adding compatible numbers |
| S-5 | Compensation, adding and subtracting |
| S-6 | Subtracting, balancing both elements |
| S-7 | Subtracting from left to right |
| S-8 | Subtracting parts |
| S-9 | Subtracting: Think integers |
| S-10 | Subtracting: Think adding |
| S-11 | Division: Think multiplication |
| S-12 | Multiplying and dividing parts |
| S-13 | Annexing zeros |
| S-14 | Moving the decimal point |
| S-15 | Compensation, multiplying and dividing |
| S-16 | Dividing, balancing both elements |
| S-17 | Doubling and/or halving |
| S-18 | Commutative property |
| S-19 | Associative law |
| S-20 | Distributive law |
| S-21 | Spatial reasoning |
| S-22 | Hybrid methods |
| S-23 | Memorization |
| S-24 | Estimation: Compatible numbers |
| S-25 | Estimation: Common rounding |
| S-26 | Estimation: Front-end rounding |
| S-27 | Estimation: Money |

# BLM 5–8.8: Mental Math Strategies

The following list compiles mental math strategies as found in *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes*. Note: This resource is meant for teacher information, not as a list of strategies that students should memorize.

| Grade 1 | Grade 2 | Grade 3 | Grade 4 | Grade 5 | Grade 6 | Grade 7 |
|---|---|---|---|---|---|---|
| 1.N.10. | 2.N.8.<br>2.N.10. | 3.N.6.<br>3.N.7.<br>3.N.10.<br>3.N.11.<br>3.N.12. | 4.N.4.<br>4.N.5.<br>4.N.6.<br>4.N.11. | 5.N.2.<br>5.N.3.<br>5.N.4. | 6.N.8. | 7.N.2. |

| Grade | Concept | Strategy | Meaning | Example |
|---|---|---|---|---|
| 1 | Addition | Counting on | Students begin with a number and count on to get the sum. Students should begin to recognize that beginning with the larger of the two addends is generally most efficient. | *for* 3 + 5<br>*think* 5 + 1 + 1 + 1 *is* 8;<br>*think* 5, 6, 7, 8 |
| 1 | Subtraction | Counting back | Students begin with the minuend and count back to find the difference. | *for* 6 − 2<br>*think* 6 − 1 − 1 *is* 4;<br>*think* 6, 5, 4 |
| 1, 2 | Addition | Using one more | Starting from a known fact and adding one more. | *for* 8 + 5 *if you know* 8 + 4 *is* 12 *and one more is* 13 |
| 1, 2 | Addition | Using one less | Starting from a known fact and taking one away. | *for* 8 + 6 *if you know* 8 + 7 *is* 15 *and one less is* 14 |
| 1, 2, | Addition<br>Subtraction | Making 10 | Students use combinations that add up to ten and can extend this to multiples of ten in later grades. | 4 + ____ *is* 10<br>7 + ____ *is* 10;<br>*so* 23 + ____ *is* 30 |

*(continued)*

## BLM 5–8.8: Mental Math Strategies (Continued)

| Grade | Concept | Strategy | Meaning | Example |
|---|---|---|---|---|
| 1 | Addition<br>Subtraction | Starting from known doubles | Students need to work to know their doubles facts. | 2 + 2 *is* 4<br>*and* 4 − 2 *is* 2 |
| 1, 2, 3 | Subtraction | Using addition to subtract | This is a form of part-part-whole representation. Thinking of addition as:<br>part + part = whole<br>Thinking of subtraction as:<br>whole − part = part | *for* 12 − 5<br>*think* 5 + ____ = 12<br>*so* 12 − 5 *is* 7 |
| 2 | Addition<br>Subtraction | The zero property of addition | Knowing that adding 0 to an addend does not change its value, and taking 0 from a minuend does not change the value. | 0 + 5 = 5;<br>11 − 0 = 11 |
| 2, 3 | Addition<br>Subtraction | Using doubles | Students learn doubles, and use this to extend facts:<br>using doubles<br>doubles plus one (or two)<br>doubles minus one (or two) | *for* 5 + 7<br>*think* 6 + 6 *is* 12;<br>*for* 5 + 7<br>*think* 5 + 5 + 2 *is* 12<br>*for* 5 + 7<br>*think* 7 + 7 − 2 *is* 12 |
| 2, 3 | Addition<br>Subtraction | Building on known doubles | Students learn doubles, and use this to extend facts. | *for* 7 + 8<br>*think* 7 + 7 *is* 14<br>*so* 7 + 8 *is* 14 + 1 *is* 15 |
| 3 | Addition | Adding from left to right | Using place value understanding to add 2-digit numerals. | *for* 25 + 33<br>*think* 20 + 30 *and* 5 + 3<br>*is* 50 + 8 *or* 58 |

*(continued)*

## BLM 5–8.8: Mental Math Strategies (Continued)

| Grade | Concept | Strategy | Meaning | Example |
|---|---|---|---|---|
| 3 | Addition<br>Subtraction | Making 10 | Students use combinations that add up to ten to calculate other math facts and can extend this to multiples of ten in later grades. | *for* $8 + 5$<br>*think* $8 + 2 + 3$ *is*<br>$10 + 3$ *or* 13 |
| 3 | Addition<br>Subtraction | Compensation | Using other known math facts and compensating. For example, adding 2 to an addend and taking 2 away from the sum. | *for* $25 + 33$<br>*think* $25 + 35 - 2$ *is*<br>$60 - 2$ *or* 58 |
| 3 | Addition | Commutative property | Switching the order of the two numbers being added will not affect the sum. | $4 + 3$ *is the same as*<br>$3 + 4$ |
| 3, 4 (decimals) | Addition<br>Subtraction | Compatible numbers | Compatible numbers are friendly numbers (often associated with compatible numbers to 5 or 10). | *for* $4 + 3$ *students may think* $4 + 1$ *is* 5 *and* 2 *more makes* 7 |
| 3 | Multiplication<br>Division | Array | Using an ordered arrangement to show multiplication or division (similar to area). | *for* $3 \times 4$ *think*<br>• • • •<br>• • • •<br>• • • •<br>*for* $12 \div 3$ *think*<br>• • • •<br>• • • •<br>• • • • |
| 3 | Multiplication | Commutative property | Switching the order of the two numbers being multiplied will not affect the product. | $4 \times 5$ *is the same as*<br>$5 \times 4$ |
| 3 | Multiplication | Skip-counting | Using the concept of multiplication as a series of equal grouping to determine a product. | *for* $4 \times 2$<br>*think* 2, 4, 6, 8<br>*so* $4 \times 2$ *is* 8 |
| 4 | Multiplication | Zero property of multiplication | Multiplying a factor by zero will always result in zero. | $30 \times 0$ *is* 0<br>$0 \times 15$ *is* 0 |

*(continued)*

## BLM 5–8.8: Mental Math Strategies (Continued)

| Grade | Concept | Strategy | Meaning | Example |
|---|---|---|---|---|
| 4 | Multiplication<br>Division | Multiplicative identity | Multiplying (dividing) a factor (dividend) by one will not change its value. | 1 × 12 *is* 12<br>21 ÷ 1 *is* 21 |
| 4, 5 | Multiplication<br>Division | Skip-counting from a known fact | Similar to the counting on strategy for addition. Using a known fact and skip-counting forward or backward to determine the answer. | *for* 3 × 8<br>*think* 3 × 5 is 15 *and skip-count by threes* 15, 18, 21, 24 |
| 4, 5 | Multiplication<br>Division | Doubling or halving | Using known facts and doubling or halving them to determine the answer. | *for* 7 × 4, *think the double of* 7 × 2 *is* 28<br>*for* 48 ÷ 6, *think the double of* 24 ÷ 6 *is* 8 |
| 4 | Multiplication<br>Division | Using the pattern for 9s | Knowing the first digit of the answer is one less than the non-nine factor and the sum of the product's digits is nine. | *for* 7 × 9 *think one less than* 7 *is* 6 *and* 6 *plus* 3 *is nine, so* 7 × 9 *is* 63 |
| 4, 5 | Multiplication | Repeated doubling | Continually doubling to get to an answer. | *for* 3 × 8, *think* 3 × 2 *is* 6, 6 × 2 *is* 12, 12 × 2 *is* 24 |
| 4 | Division | Using multiplication to divide | This is a form of part-part-whole representation. Thinking of multiplication as:<br>part × part = whole<br>Thinking of division as:<br>whole ÷ part = part | *for* 35 ÷ 7<br>*think* 7 × ____ = 35<br>*so* 35 ÷ 7 *is* 5 |
| 4, 5 | Multiplication | Distributive property | In arithmetic or algebra, when you distribute a factor across the brackets:<br>$a \times (b + c) = a \times b + a \times c$<br>$(a + b) \times (c + d) = ac + ad + bc + bd$ | *for* 2 × 154<br>*think* 2 × 100 *plus* 2 × 50 *plus* 2 × 4 *is* 200 + 100 + 8 *or* 308 |

(continued)

# BLM 5–8.8: Mental Math Strategies (Continued)

| Grade | Concept | Strategy | Meaning | Example |
|---|---|---|---|---|
| 5 | Division | Repeated halving | Continually halving to get a number. | *for* 32 ÷ 4, *think* 32 ÷ 2 is 16 *and* 16 ÷ 2 *is* 8 *so* 32 ÷ 4 *is* 8 |
| 5 | Multiplication | Annexing zeros | When multiplying by a factor of 10 (or a power of ten), taking off the zeros to determine the product and adding them back on. | *for* 4 × 700, *think* 4 × 7 *is* 28 *and add two zeros to make* 2800 |
| 5 | Multiplication | Halving and doubling | Halving one factor and doubling the other. | *for* 24 × 4, *think* 48 × 2 *is* 96 |
| 6, 7 | Division | Dividing by multiples of ten | When dividing by 10, 100, etc., the dividend becomes smaller by 1, 2, etc. place-value positions. | *for* 76.3 ÷ 10 *think* 76.3 *should become smaller by one place-value position so* 76.3 ÷ 10 *is* 7.63 |

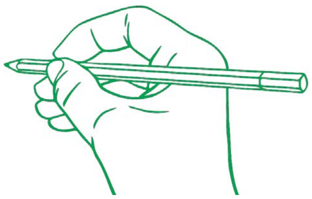
# *Mental Math*

## Grade 8 Mathematics

S–1

**Sample Strategy**

### Standard algorithms and mental math



Although students need to become proficient in performing standard algorithms on paper, their cumbersome nature make them difficult for use with mental math.

**EXAMPLE 1**

$$\begin{array}{r} {}^{1\,1}\phantom{3} \\ 173 \\ +\ \ 67 \\ \hline 240 \end{array}$$

7 + 3 = 10, remember 0, carry 1
1 + 7 + 6 = 14, remember 4, carry 1
1 + 1 = 2, remember 2

Reverse 0, 4, 2 to 2, 4, 0 and mentally reassign place value to **240**.

**EXAMPLE 2**

$$\begin{array}{r} \$91 \\ \times\ \ 13 \\ \hline 273 \\ 910 \\ \hline \$1183 \end{array}$$

1 × 3 = 3, remember 3
9 × 3 = 27, remember 27

Rearrange mentally into 273.

Add a place-holder zero.

1 × 1 = 1, remember 1
9 × 1 = 9, remember 9

Rearrange mentally into 910.

3 + 0 = 3, remember 3
7 + 1 = 8, remember 8
2 + 9 = 11, remember 11

Rearrange mentally and reassign place value and units to **1183**.

Mental mathematics strategies need to be analyzed for efficiency. In most cases, standard algorithms are very useful for paper-and-pencil arithmetic but are not efficient for doing mental calculations.

# Mental Math: Grade 8 Mathematics

## Teaching Strategies for Sample Strategy S–1

S–1

### Standard algorithms and mental math

The standard algorithms for addition, subtraction, multiplication, and division work efficiently for paper-and-pencil tasks and should be encouraged for use with these. For mental math, however, using the standard algorithms often requires more steps than other methods outlined in this document.

The goals of mental math and estimation, as outlined by the Manitoba curriculum, are to enhance flexible thinking and number sense, and to improve computational fluency by developing efficiency, accuracy, and flexibility. It is important that students have a variety of methods at their disposal in order to choose the method that will, for them, be the most efficient and accurate method. It is also important that they have a conceptual understanding of the method they use in order to gain flexibility. For these reasons, it is necessary that students have additional tools for solving math mentally.

Students, and even some adults, find standard algorithms to be the most effective mental math strategy because of their own familiarity with them. Although these methods may not have the flexibility or efficiency of other methods, and the large number of steps increases the likelihood of error, standard algorithms may be an effective, although inefficient, approach for some in the same way that skip-counting for multiplication or counting back for subtraction can be effective, but inefficient.

Students should be encouraged to find their own most efficient and least cumbersome method of performing mental math in a variety of situations. They can learn this through both direct and indirect teaching methods.

> "Although results differ depending on what and how manipulatives are used in learning situations, learning with manipulatives is correlated positively with later development of mental mathematics (Gravemeijer), achievement, and understanding (Sowell). Conceptual knowledge originates in the inventive activities of the learner through actions on objects rather than from sensory impression or social transmission derived from teachers and parents (Piaget)." (MacKenzie)

Many of the teaching strategies in this document suggest the use of manipulatives to demonstrate and involve students in the process of developing their own conceptual understanding and mental math skills. The use of manipulatives also serves as an effective method of communication for students. In addition, students will be more likely to retain information and ideas when involved actively in their own learning through the use of these tools in their math classes.

*I hear and I forget*
*I see and I remember*
*I do and I understand.*
*—Ancient Chinese Proverb*

# *Mental Math*

## Grade 8 Mathematics

S–2

**Sample Strategy**

**Place-value partitioning: Adding from left to right**

Starting from the right and working left can be an efficient method to solve addition questions mentally …

EXAMPLE 1

$$\begin{array}{r} 46 \\ +\ 38 \\ \hline \end{array}$$

6 + 8 = 14

40 + 30 = 70

70 + 14 = **84**

…but often, a more efficient method for mentally adding is to start from the left side of the equation.

EXAMPLE 2

$$\begin{array}{r} 46 \\ +\ 38 \\ \hline \end{array}$$

40 + 30 = 70

6 + 8 = 14

70 + 14 = **84**

EXAMPLE 3

$$\begin{array}{r} 25.6 \\ +\ 13.7 \\ \hline \end{array}$$

20 + 10 = 30

5 + 3 = 8

30 + 8 = 38

0.6 + 0.7 = 1.3

38 + 1.3 = **39.3**

The benefit of adding from left to right is that you do not need to store as many numbers in your head as with the standard algorithm. You never have to regroup (carry), and place value is always maintained.

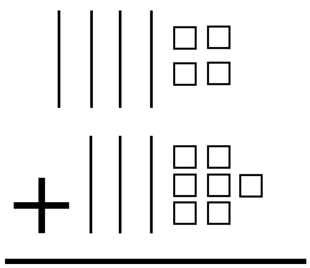
## Teaching Strategies for Sample Strategy S–2

S–2

### Place-value partitioning: Adding from left to right

**Use Base-10 blocks to show that adding from left to right can be an efficient method of performing mental math.**

**Example 1:** 44 + 37



Show 44 + 37 with Base-10 blocks.

- show adding from left to right by adding groups of tens first, then regrouping units into tens if possible, and finally counting the remaining units

  40 + 30 + 10 + 1 = 81
- compare this to trying to add the total, starting with the units first

  4 + 7 = 11

  11 + 70 = 81
- reflect: Is it easier to add tens first, units first, or does it really matter?
- reflect: Is it quicker to report using one of the methods?

Students may find that the first approach is more efficient. By starting with the largest place value and working progressively smaller, you can avoid mentally sorting out tens from ones. You can also report the answer as you work through the question.

**Example 2:** Working left to right, 333 + 156

Add the hundreds first and right away you can report, "*four hundred*...."

Add the tens and report, "eighty-...."

Add the units and report "nine."

Working memory does not need to be used to store the value of each digit. As you continue working through the question, you report the answer at the same time. When these steps involve regrouping (carrying), a simple step of substituting the previous number corrects the answer.

When working from right to left, each calculated digit needs to be stored in working memory. An additional final step is necessary to reverse the order of all of these digits. The conceptual importance of place value can be lost with this approach.

Working right to left, for example, with 333 + 156:

3 + 6 = 9 (Store this in your memory.)

3 + 5 = 8 (Store this in your memory. Notice how place value loses all significance.)

3 + 1 = 4 (Store this in your memory.)

Reverse all the digits: 9, 8, 4 becomes 4, 8, 9.

Now reassign a place value to all of the digits and report.

4, 8, 9 becomes *four hundred eighty-nine*.

**With paper and pencil, the standard algorithm is generally more efficient because any regrouping/carrying is addressed immediately, and erasing is not needed. With mental computation, however, students should realize, and be encouraged to implement, working from left to right as a more efficient process.**

# *Mental Math*

## Grade 8 Mathematics

S–3

**Sample Strategy**

**Break down numbers and add their parts**

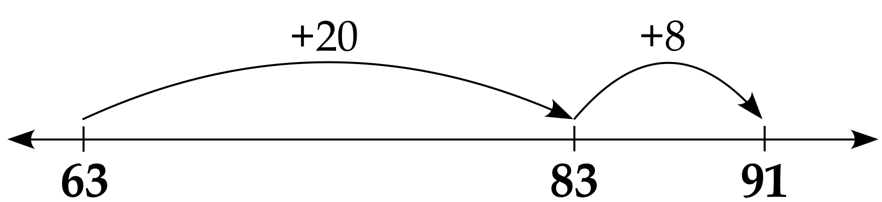
Here's another method of doing addition in your head:

**EXAMPLE 1**

$$\begin{array}{r} 63 \\ +\ 28 \\ \hline \end{array}$$

63 + 20 = 83

83 + 8 = **91**



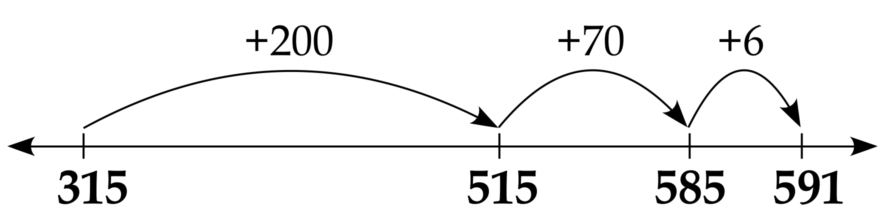

Break the numbers up and add the parts in the order that works best for you.

**EXAMPLE 2**

$$\begin{array}{r} 315 \\ +\ 276 \\ \hline \end{array}$$

315 + 200 + 70 + 6

= **591**




When you break down numbers and add their parts, you never have to regroup (carry) and place value is always maintained.

# Mental Math: Grade 8 Mathematics

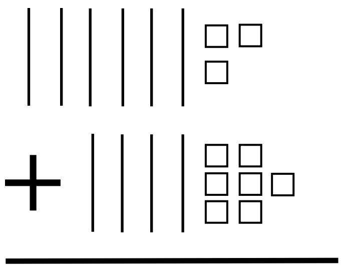
## Teaching Strategies for Sample Strategy S–3

S–3

### Break down numbers and add their parts

**Use Base-10 blocks to show that numbers can be broken apart and added in many different ways.**

**Example 1:** 63 + 47



Show 63 + 47 with Base-10 blocks.

Reflect: How many different ways can the total be found?

63 + 7 = 70

70 + 40 = 110

60 + 40 = 100

100 + 3 + 7 = 110

3 + 7 + 60 + 40 = 110

etc.

Reflect: Which ways are most efficient or easiest to use? Why?

**Example 2:** Using a number line to show 12.3 + 48.7

Have students use a metre stick or tape measure as a number line.

Show 12.3 cm + 48.7 cm

Reflect: How many different ways can you add these numbers to get the answer?

| | |
|---|---|
| 12.3 cm + 40 = 52.3 cm<br>52.3 + 8 = 60.3 cm<br>60.3 + 0.7 = 61 cm | 12 cm + 48 = 60 cm<br>60 cm + 0.3 + 0.7 = 61 cm |
| 12.3 cm + 0.7 = 13 cm<br>13 + 40 = 53 cm<br>53 + 8 = 61 cm | 0.7 cm + 0.3 = 1 cm<br>1 + 2 + 8 = 11 cm<br>11 + 10 + 40 = 61 cm |



Reflect: Which ways are most efficient or easiest to use? Why?

**Students should come to some conclusions, such as that starting with compatible (friendly) numbers is sometimes easiest, and that numbers can be broken apart and worked with in many ways. Doing mental computation from left to right is generally one of the most efficient methods of adding numbers mentally, but when friendly numbers are present, starting wherever they occur may be even more efficient.**

# *Mental Math*

## Grade 8 Mathematics

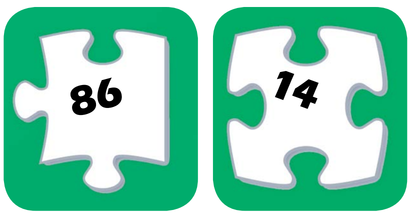
S–4

**Sample Strategy**

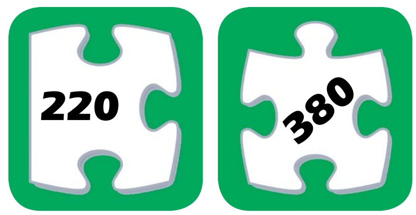
**Finding compatible numbers (also known as friendly numbers)**

Compatible numbers are pairs of numbers that are easy to add in your head.

The following are examples of compatible numbers:




The sum equals 100.




The sum equals 600.

**EXAMPLE 1** Find the pairs of compatible numbers that add up to 300.

| | | |
|---|---|---|
| 140 | 85 | 160 |
| 118 | 217 | 73 |
| 215 | 182 | 83 |

140 + 160

118 + 182

215 + 85

**EXAMPLE 2** Find the pairs of compatible numbers that add up to 800.

| | | |
|---|---|---|
| 250 | 175 | 567 |
| 333 | 440 | 467 |
| 625 | 550 | 360 |

250 + 550

333 + 467

625 + 175

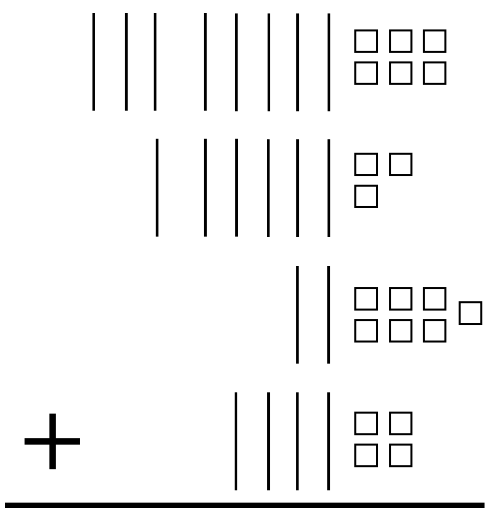
## Teaching Strategies for Sample Strategy S–4

S–4

### Finding compatible numbers (also known as friendly numbers)

**Use Base-10 blocks to show that grouping compatible numbers makes them easier to add.**

**Example 1:** 86 + 63 + 27 + 44



Model 86 + 63 + 27 + 44, using Base-10 blocks.

Reflect: What is the most efficient method of finding the total?

- Adding all of the units and all of the tens separately and then adding them both together (similar to the algorithm)
- Counting all
- Adding tens and counting up
- Regrouping in some other method

Encourage students to look for groups of friendly numbers.

80 + 20 = 100

60 + 40 = 100

6 + 4 = 10

3 + 7 = 10

100 + 100 + 10 + 10 = 220

**An algorithmic approach to an addition question with this many addends is very difficult to do mentally. When working with more than two addends, looking first for compatible numbers is often the most efficient strategy. Even if there are some addends that are not compatible, mentally computing visible, friendly numbers reduces the number of steps necessary to solve an addition question of this type.**

Try the following, searching for compatible numbers:

| | | | |
|---|---|---|---|
| 38 + 62 + 1 | 136 + 893 + 7 + 64 | 61 + 76 + 239 + 824 | 8.09 + 7.91 + 2 |
| $\frac{1}{4} + \frac{1}{2} + \frac{3}{4}$ | 43% + 2% + 37% | \$4.23 + \$6.55 + \$3.45 | 2.3 km + 700 m |

# *Mental Math*

## Grade 8 Mathematics

**S–5**

**Sample Strategy**

**Compensation for adding and subtracting, using compatible numbers**

Sometimes it is easier to do addition and subtraction in your head by creating your own compatible numbers and adjusting the total.

**EXAMPLE 1**

$$\begin{array}{r} 650 \\ +\ 375 \\ \hline \end{array}$$

650 + 350 = 1000

1000 + 25 = **1025**

**EXAMPLE 2**

$$\begin{array}{r} 1250 \\ -\ \ 753 \\ \hline \end{array}$$

1250 – 750 = 500

500 – 3 = **497**

**EXAMPLE 3**

$$\begin{array}{r} 53 \\ +\ 39 \\ \hline \end{array}$$

39 + 1 = 40

53 + 40 = 93

93 – 1 = **92**

**EXAMPLE 4**

$$6\frac{1}{5} - 2\frac{4}{5}$$

$$6\frac{1}{5} - \left(2\frac{4}{5} + \frac{1}{5}\right) = 6\frac{1}{5} - 3$$

$$= 3\frac{1}{5}$$

$$3\frac{1}{5} + \frac{1}{5} = \mathbf{3\frac{2}{5}}$$

# Mental Math: Grade 8 Mathematics

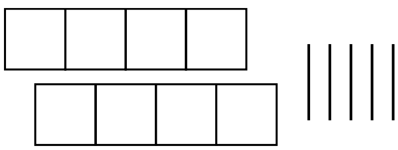
## Teaching Strategies for Sample Strategy S–5

S–5

### Compensation for adding and subtracting, using compatible numbers

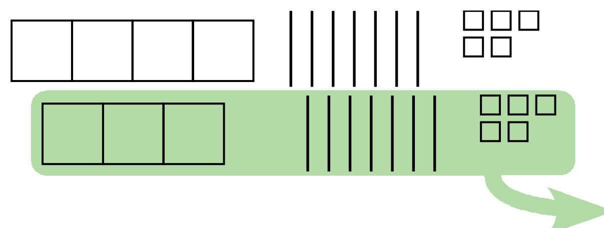
**Use Base-10 blocks to demonstrate the extra work involved in regrouping, which can be eliminated by using compatible (friendly) numbers.**

**Example 1:** Have students model 850 − 375 using Base-10 blocks

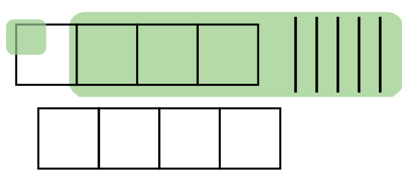


This will require several steps to complete. A ten will need to be broken down into ones, and a hundred will need to be broken into tens.

After regrouping the 850, pulling 375 units away will leave 4 hundreds, 7 tens, and 5 units = 475.



Ask students to look at the question of 850 − 375 as 850 − 350 (a compatible number) − 25.



850 − 375 = 850 − 350 − 25

850 − 350 = 500

500 − 25 = 475

By creating friendly numbers, the borrowing/regrouping process is eliminated, simplifying the process.

Doing this work mentally eliminates the need to regroup (or borrow) when doing subtraction equations. Using compatible numbers also maintains place value.

For example: 823 − 730 =

Using the method of creating compatible numbers:

823 − 730 = 823 − 723 − 7
823 − 723 = 100
100 − 7 = *ninety-three*

Using the algorithmic process mentally:

3 − 0 = 3
2 − 3 doesn't work, so regroup tens (borrow)
12 − 3 = 9
8 changed to a 7, so 7 − 7 = 0
Reverse the numbers of 3, 9, 0 to 093 and reassign place value = *ninety-three*

**It should become apparent to students that, although the algorithmic method works well on paper, mentally it is a much more cumbersome process than using compatible numbers.**

# *Mental Math*

## Grade 8 Mathematics

S–6

**Sample Strategy**

**Subtracting, balancing both elements**

When you add the same number to the two elements of a subtraction question, the difference between the two does not change.

EXAMPLE 1

$$\begin{array}{r} 4.32 \\ -\ 1.95 \\ \hline \end{array}$$

$4.32 + 0.05 = 4.37$

$1.95 + 0.05 = 2$

$4.37 - 2 = \mathbf{2.37}$

EXAMPLE 2

$$\begin{array}{r} 96.3 \\ -\ 30.1 \\ \hline \end{array}$$

$30.1 + (-0.1) = 30$

$96.3 + (-0.1) = 96.2$

$96.2 - 30 = \mathbf{66.2}$

EXAMPLE 3

$$6\frac{1}{3} - 1\frac{2}{3}$$

$1\frac{2}{3} + \frac{1}{3} = 2$

$6\frac{1}{3} + \frac{1}{3} = 6\frac{2}{3}$

$6\frac{2}{3} - 2 = \mathbf{4\frac{2}{3}}$

## Teaching Strategies for Sample Strategy S–6

S–6

### Subtracting, balancing both elements

**Use volume to demonstrate that adding or subtracting the same number from both elements in a subtraction question will always result in the same difference.**

Required materials: 2 measuring cups
a water source
measuring spoons

1. Fill one measuring cup up to the 500 mL mark and the other to the 750 mL mark. Ask students to find the difference between the two in mL.
2. Add 100 mL to both cups and ask students to find the difference between the two.
3. Add 125 mL more to both cups and ask students to again find the difference between the two.
4. Continue adding equivalent amounts of water to each measuring cup until students see clearly that the difference will remain constant as long as an equal amount is added to both measuring cups.
5. Remove 150 mL from each measuring cup and ask students to find the difference.
6. Remove 50 mL from each measuring cup and ask students to find the difference.
7. Continue removing equivalent amounts of water from each measuring cup until students realize that the difference will remain constant as long as an equal amount is subtracted from both measuring cups.

**Extension:** Try the same activity, using the imperial system and fractions of cups.

Have students develop a rule from this activity. It should be similar to the following:

**When you add the same number to the two elements of a subtraction question, the difference between the two does not change.**

$$(a - b) = (a + n) - (b + n)$$

**and**

$$(a - b) = (a - n) - (b - n)$$

# Mental Math

## Grade 8 Mathematics

S–7

**Sample Strategy**

**Subtract starting from the left: Place-value positioning**

Mentally subtracting when regrouping (borrowing) is involved takes a small additional step, but it is a very effective and useful strategy.

EXAMPLE 1

$$\begin{array}{r} 468 \\ -\ 323 \\ \hline \end{array}$$

Scan the question. No regrouping is needed.

400 – 300 = 100

60 – 20 = 40

8 – 3 = 5

100 + 40 + 5 = **145**

EXAMPLE 2

$$\begin{array}{r} 9514 \\ -\ 6233 \\ \hline \end{array}$$

Scan the question. Regrouping will be needed for the tens place. Adjust the hundreds place to reflect this.

9000 – 6000 = 3000

**4**00 – 200 = 200

110 – 30 = 80

4 – 3 = 1

3000 + 200 + 80 + 1 = **3281**

# Mental Math: Grade 8 Mathematics

## Teaching Strategies for Sample Strategy S–7

S–7

## Subtract starting from the left: Place-value positioning

**Explain various mental math strategies to demonstrate that starting subtraction from the left side is efficient.**

Have students mentally solve the following question using a Mental Math Student Communication Template and the method described in the Reproducible Sheets section. (Notice that no regrouping is involved in this specific question and that there are many other possible methods of mentally solving this equation, such as compatible numbers or compensation.)

| Question: 974 − 343 | Answer: 631 |
|---|---|
| Method used to calculate mentally (a right-to-left, standard algorithm approach):<br>1. 4 − 3 = 1 (mentally store)<br>2. 7 − 4 = 3 (mentally store)<br>3. 9 − 3 = 6 (mentally store)<br>4. Mentally retrieve and reverse the order from 1, 3, 6 to 6, 3, 1 and reassign place value<br>5. Say the answer as "**six hundred thirty-one**" | A method used by someone else (a left-to-right approach):<br>1. 900 − 300 = say, "***six hundred...***"<br>2. 70 − 40 = say, "***thirty...***"<br>3. 4 − 3 = say, "***one.***" |
| **Preferred method and reason:**<br>Second method takes fewer steps and less mental storing. There is less chance for error. | |

Getting the correct answer is not as important as the process used to solve this question. Many students may get the wrong answer by following the algorithmic approach because the many steps involving mental storing and retrieval can become difficult to manage. Guide students to see that, although the algorithmic approach is usually the most effective method when working with pencil and paper, it is not often an effective mental math strategy. Also note that the left-to-right approach maintains the importance of place value.

Try the following question, which involves regrouping:

| Question: 814 − 78 | Answer: 736 |
|---|---|
| Method used to calculate mentally (right to left):<br>1. 4 − 8 requires regrouping<br>2. 14 − 8 = 6 (store mentally)<br>3. Recall that 1 was changed to 0. Regroup.<br>4. 10 − 7 = 3 (store mentally)<br>5. Recall that 8 was changed to 7. 7 − 0 = 7<br>6. Mentally retrieve and reverse the order from 6, 3, 7 to 736<br>7. Say the answer as, "**seven hundred thirty-six**." | A method used by someone else (left to right):<br>1. 800 − 0 = 800 (store mentally)<br>2. 10 − 70 = problem! Regroup 800 to 700 and say, "***seven hundred...***"<br>3. 110 − 70 = 40 (store mentally)<br>4. 4 − 8 = problem! Regroup 40 to 30 and say," ***thirty...***"<br>5. 14 − 8 = say, "***six.***" |
| **Preferred method and reason:**<br>The second method is more efficient in this case. It also maintains place value throughout. This method would not be as efficient with paper and pencil, however, because there would be a lot of erasing. | |

# *Mental Math*

## Grade 8 Mathematics

S–8

**Sample Strategy**

**Subtracting one part at a time: Place-value positioning**

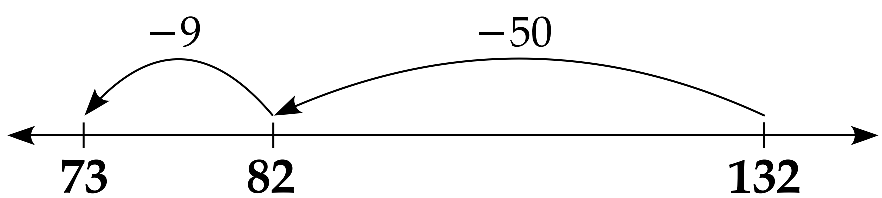
When solving a subtraction question that requires regrouping (borrowing), try subtracting one part at a time.

**EXAMPLE 1**

$$\begin{array}{r} 132 \\ -\ \ 59 \\ \hline \end{array}$$

$132 - 50 = 82$

$82 - 9 = \mathbf{73}$

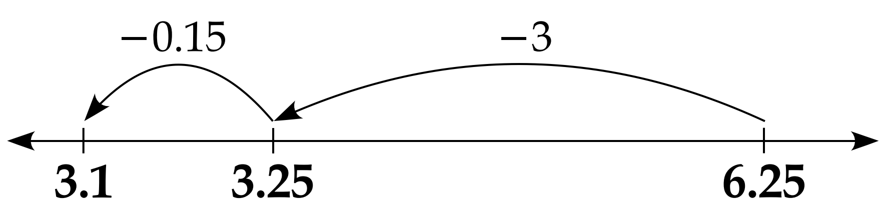



**EXAMPLE 2**

$$\begin{array}{r} 6.25 \\ -\ 3.15 \\ \hline \end{array}$$

$6.25 - 3 = 3.25$

$3.25 - 0.15 = \mathbf{3.1}$



## Subtracting one part at a time: Place-value positioning

**Use number lines to demonstrate the effectiveness of counting-back subtraction strategies when used with breaking up numbers into their smaller parts.**

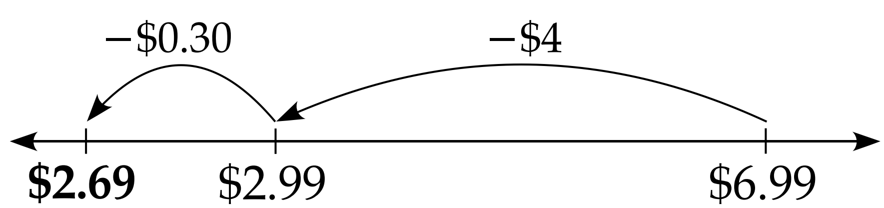
Have students find 29 cm − 15 cm using a ruler, and explain their process to a partner.

Share possible strategies with the class:

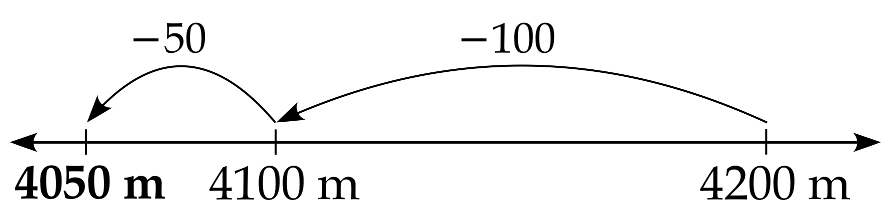
- Some may have used a strategy of counting back, starting with 29 and counting back by ones 15 times. This may have worked with this situation, but for larger numbers or decimals it would generally be inefficient.
- Some may have used a strategy of compensation, changing 29 to 30, subtracting 15, then subtracting 1 more to compensate. This is an efficient strategy for subtraction.
- Some may have seen the difference immediately as 14, or thought algebraically 15 + x = 29. This may have been possible for this situation, but not likely possible for decimals or larger numbers.
- Subtracting 1 part at a time is an effective strategy. Start at 29, subtract 10 to get to 19, and subtract 5 more to get to 14 cm. Breaking up the subtrahend will work in all situations.

Try drawing number lines and breaking up the subtrahend for the following questions. Note that number lines in this case do not have to be to scale.

$6.99 − $4.30



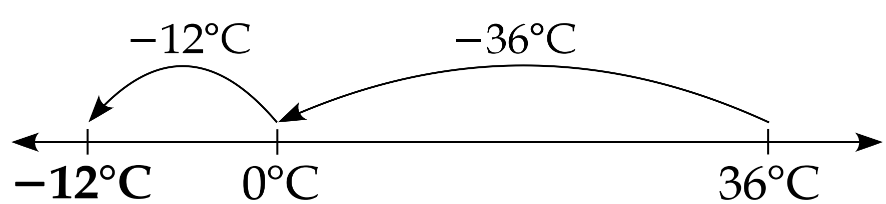

4.2 km − 150 m




In some situations, students may find that breaking up the subtrahend into other combinations of numbers may become even more efficient:

36°C − 48°C

36°C − 48°C = 36°C − (36°C + 12°C)
= 36°C − 36°C − 12°C



# *Mental Math*

## Grade 8 Mathematics

S–9

**Sample Strategy**

### Subtraction: Thinking integers

When working on a subtraction question that involves regrouping (borrowing), a possible method is to make use of the way integers subtract.

EXAMPLE 1

$$\begin{array}{r} 243 \\ -\ 67 \\ \hline \end{array}$$

$200 - 0 = 200$

$40 - 60 = -20$

$3 - 7 = -4$

$200 - 20 - 4 = \mathbf{176}$

EXAMPLE 2

$$\begin{array}{r} \$9.95 \\ -\ \$6.87 \\ \hline \end{array}$$

$900¢ - 600¢ = 300¢$

$90¢ - 80¢ = +10¢$

$5¢ - 7¢ = -2¢$

$300¢ + 10¢ - 2¢ = \mathbf{\$3.08}$

EXAMPLE 3

$$34\frac{1}{4} - 16\frac{3}{4}$$

$30 - 10 = 20$

$4 - 6 = -2$

$\frac{1}{4} - \frac{3}{4} = -\frac{2}{4} = -\frac{1}{2}$

$20 - 2 - \frac{1}{2} = \mathbf{17\frac{1}{2}}$

Working from left to right allows place value to be maintained.

# Mental Math: Grade 8 Mathematics

## Teaching Strategies for Sample Strategy S–9

S–9

### Subtraction: Thinking integers

Using a method such as this requires that students have a very strong ability to work with negative integers, as well as a strong conceptual understanding of integers and the subtraction process. Teaching and encouraging the use of this strategy should only be done after students have a thorough understanding of both.

Potential sources of confusion for students:

- Students who are not entirely proficient at using the standard algorithm may get parts of this process confused with the standard algorithm and end up with misconceptions of both processes.
- Students in Early Years have often developed a misconception that a number cannot be subtracted from a number that is smaller than it. This misconception may have been encouraged through the use of the standard algorithm, which requires regrouping when a number is subtracted from a smaller number. Students with this misconception do not have a strong understanding of integers, and this method would not be appropriate for them.

Have students consider and discuss this situation through group discussion.

| | | | | |
|---|---|---|---|---|
| 5 | 4 | .1 | 3 | |
| − | 9 | .7 | 4 | |
| 50 | −5 | −0.6 | −0.01 | **= 44.39** |

Why does this method work?

How does this method work?

Does this method always work? Explain and give examples. If you find examples where this method doesn't work, use calculators to double-check.

Is this method something you would use? Why or why not?

**The benefits of mentally using this thinking integer method are that work can be done from left to right and regrouping is entirely avoided. By avoiding regrouping, there are fewer numbers that students have to keep in their working memory and fewer opportunities for error. Place value is also maintained throughout this process.**

# *Mental Math*

## Grade 8 Mathematics

**S–10**

**Sample Strategy**

**Subtraction: Using addition to subtract/thinking addition**

One method of solving subtraction questions is to reframe them in your mind as addition questions.

**EXAMPLE 1**

$$\begin{array}{r} 764 \\ -\ 698 \\ \hline \end{array}$$

$698 + \underline{\qquad} = 764$

$698 + \underline{\quad 2 \quad} = 700$

$700 + \underline{\quad 64 \quad} = 764$

$2 + 64 = \mathbf{66}$

**EXAMPLE 2**

$$\begin{array}{r} \$29.95 \\ -\ \$15.34 \\ \hline \end{array}$$

$\$15.34 + \underline{\qquad\quad} = \$29.95$

$\$15.34 + \underline{\ \$14.00\ } = \$29.34$

$\$29.34 + \underline{\ \ \$0.61\ \ } = \$29.95$

$\$14.00 + \$0.61 = \mathbf{\$14.61}$

**EXAMPLE 3**

$$7\frac{5}{8} - 4\frac{7}{8}$$

$4\frac{7}{8} + \underline{\qquad} = 7\frac{5}{8}$

$4\frac{7}{8} + \underline{\quad \frac{1}{8} \quad} = 5$

$5 + \underline{\quad 2\frac{5}{8} \quad} = 7\frac{5}{8}$

$2\frac{5}{8} + \frac{1}{8} = 2\frac{6}{8} = \mathbf{2\frac{3}{4}}$

# Mental Math: Grade 8 Mathematics

## Teaching Strategies for Sample Strategy S–10

S–10

### Subtraction: Using addition to subtract/thinking addition

Thinking addition is likely one of the first ways most of your students learned to subtract single-digit numbers. Relate subtraction to the opposite operation of addition.

**Show the opposite relationship of addition and subtraction using manipulatives and part-part-whole relationships.**

**Example 1**

Show 255 − 55 = 200 with Base-10 blocks.

255 is the whole, 55 is one of its parts, and 200 is another one of its parts

Have students write out as many pairs of equations as they can to represent this:

$255 - 55 = 200 \Rightarrow 200 + 55 = 255$

$255 - 200 = 55 \Rightarrow 55 + 200 = 255$

$100 + 100 + 50 + 5 = 255 \Rightarrow 255 - 5 - 50 - 100 = 100$

$255 - 5 - 5(10) = 200 \quad \Rightarrow 200 + 5(10) + 5 = 255$

etc.

**Example 2**

Practise making change with money.

$20.00 paid for a $16.75 T-shirt

$20.00 − $16.75 = **$3.25**

Think addition:

$16.75 + $3 = $19.75

$19.75 + $0.25 = $20.00

$3 + $0.25 = **$3.25**

**Example 3**

Show $3\frac{1}{6} - 1\frac{5}{6}$ with pattern blocks.

$3\frac{1}{6} - 1\frac{5}{6} = 1\frac{2}{6} = 1\frac{1}{3}$

Think addition:

$1\frac{5}{6} + \underline{\quad\quad} = 3\frac{1}{6}$

$1\frac{5}{6} + \underline{\ \frac{1}{6}\ } = 2$

$2 + \underline{\ 1\frac{1}{6}\ } = 3\frac{1}{6}$

$1\frac{1}{6} + \frac{1}{6} = 1\frac{2}{6} = \mathbf{1\frac{1}{3}}$

**Reaffirm for students that subtraction, in every possible situation, can be viewed as the opposite process to addition.**

# *Mental Math*

## Grade 8 Mathematics

S–11

**Sample Strategy**

**Division: Using multiplication to divide/thinking multiplication**

One method of solving division questions is to reframe them in your mind as multiplication questions.

**EXAMPLE 1** $444 \div 11.1$

$11.1 \times \_\_\_\_ = 444$

$11.1 \times \underline{\ 4\ } = 44.4$

So, $11.1 \times \underline{\ 40\ } = 444$

$444 \div 11.1 = \mathbf{40}$

**EXAMPLE 2** $3 \div \frac{1}{2}$

$\frac{1}{2} \times \_\_\_\_ = 3$

$\frac{1}{2} \times \underline{\ 6\ } = 3$

$3 \div \frac{1}{2} = \mathbf{6}$

**EXAMPLE 3** $120 \div 0$

$0 \times \_\_\_\_ = 120$

No number will fit this!

# Mental Math: Grade 8 Mathematics

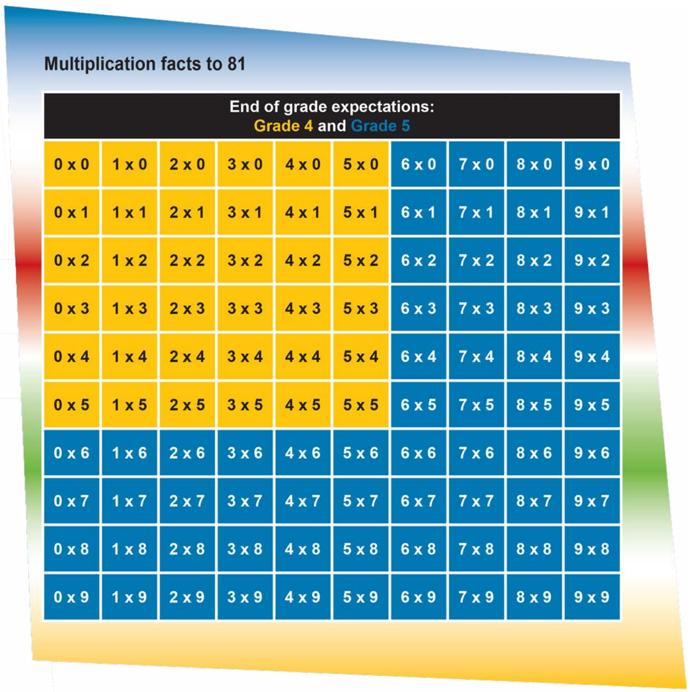
## Teaching Strategies for Sample Strategy S–11

S–11

### Division: Using multiplication to divide/thinking multiplication

- Recall of the multiplication and related division facts up to $5 \times 5$ is expected by the end of Grade 4.
- Recall of multiplication facts to 81 and related division facts is expected by the end of Grade 5.
- Students are no longer expected to develop recall to $12 \times 12$ at any grade level, but are expected to have multiple strategies to perform this type of double-digit multiplication mentally.

Students should be aware of division's inverse relationship to multiplication.



Multiplication facts to 81

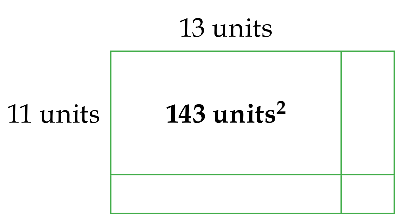
End of grade expectations:
Grade 4 and Grade 5

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 x 0 | 1 x 0 | 2 x 0 | 3 x 0 | 4 x 0 | 5 x 0 | 6 x 0 | 7 x 0 | 8 x 0 | 9 x 0 |
| 0 x 1 | 1 x 1 | 2 x 1 | 3 x 1 | 4 x 1 | 5 x 1 | 6 x 1 | 7 x 1 | 8 x 1 | 9 x 1 |
| 0 x 2 | 1 x 2 | 2 x 2 | 3 x 2 | 4 x 2 | 5 x 2 | 6 x 2 | 7 x 2 | 8 x 2 | 9 x 2 |
| 0 x 3 | 1 x 3 | 2 x 3 | 3 x 3 | 4 x 3 | 5 x 3 | 6 x 3 | 7 x 3 | 8 x 3 | 9 x 3 |
| 0 x 4 | 1 x 4 | 2 x 4 | 3 x 4 | 4 x 4 | 5 x 4 | 6 x 4 | 7 x 4 | 8 x 4 | 9 x 4 |
| 0 x 5 | 1 x 5 | 2 x 5 | 3 x 5 | 4 x 5 | 5 x 5 | 6 x 5 | 7 x 5 | 8 x 5 | 9 x 5 |
| 0 x 6 | 1 x 6 | 2 x 6 | 3 x 6 | 4 x 6 | 5 x 6 | 6 x 6 | 7 x 6 | 8 x 6 | 9 x 6 |
| 0 x 7 | 1 x 7 | 2 x 7 | 3 x 7 | 4 x 7 | 5 x 7 | 6 x 7 | 7 x 7 | 8 x 7 | 9 x 7 |
| 0 x 8 | 1 x 8 | 2 x 8 | 3 x 8 | 4 x 8 | 5 x 8 | 6 x 8 | 7 x 8 | 8 x 8 | 9 x 8 |
| 0 x 9 | 1 x 9 | 2 x 9 | 3 x 9 | 4 x 9 | 5 x 9 | 6 x 9 | 7 x 9 | 8 x 9 | 9 x 9 |

Having students practise solving equations involving multiplication and related division facts can be one way to build these basic skills.

| | | | |
|---|---|---|---|
| $30 \div 6 = y$ | $56 \div 7 = m$ | $42 \div 7 = f$ | $12 \div \frac{1}{4} = d$ |
| $6 \times y = 30$ | $7 \times m = 56$ | $7 \times f = 42$ | $0.25 \times d = 12$ |
| $y \times 6 = 30$ | $m \times 7 = 56$ | $f \times 7 = 42$ | $d \times 25\% = 12$ |
| $30 \div y = 6$ | $56 \div m = 7$ | $42 \div f = 7$ | $\frac{12}{d} = 0.25$ |

Algebra tiles can also be effectively used to show that there is an inverse relationship between the area of an array and the length of one of its sides.




$13 \times 11 = 143$
$11 \times 13 = 143$
$143 \div 11 = 13$
$143 \div 13 = 11$

# *Mental Math*

## Grade 8 Mathematics

S–12

**Sample Strategy**

### Place-value partitioning: Multiplication and division

It can often be easier to multiply or divide in your head when you break down a number and start from the left.

**EXAMPLE 1**

$$\begin{array}{r} 635 \\ \times \quad 4 \\ \hline \end{array}$$

$600 \times 4 = 2400$

$30 \times 4 = 120$

$5 \times 4 = 20$

$3400 + 120 + 12 = \mathbf{2540}$

**EXAMPLE 2**

$$1452 \div 4$$

$1000 \div 4 = 250$

$400 \div 4 = 100$

$52 = 40 + 12$

$40 \div 4 = 10$

$12 \div 4 = 3$

$250 + 100 + 10 + 3 = \mathbf{363}$

**EXAMPLE 3**

$$2 \times 4\frac{3}{8}$$

$2 \times 4 = 8$

$2 \times \frac{3}{8} = \frac{6}{8}$

$8 + \frac{6}{8} = 8\frac{6}{8} = \mathbf{8\frac{3}{4}}$

## Teaching Strategies for Sample Strategy S–12

S–12

### Place-value partitioning: Multiplication and division

**Explain various mental math strategies to demonstrate that place-value partitioning for multiplication and division is often most efficient.**

Have students mentally solve the following question using a Mental Math Student Communication Template and the method described in the Reproducible Sheets section.

| Question: 912 × 7 | Answer: 6384 |
|---|---|
| Method used to calculate mentally (a right-to-left, standard algorithmic approach):<br>1. 2 × 7 = 14 (mentally store the 4, regroup the 10 as 1)<br>2. 1 × 7 + 1 = 8 (mentally store)<br>3. 9 × 7 = 63 (mentally store)<br>4. Mentally retrieve and reverse the order from 4, 8, 63 to 63, 8, 4 and reassign place value<br>5. Say the answer as "**six thousand three hundred eighty-four**." | A method used by someone else (a left-to-right approach):<br>1. Scan, 2 × 7 is the only column that will require regrouping.<br>2. 900 × 7 = say, "***six thousand three hundred...***"<br>3. 10 × 7 = 70 (mentally store)<br>4. 2 × 7 = 14 (mentally store)<br>5. 70 + 14 = say, "***eighty-four.***" |
| **Preferred method and reason:**<br>Second method takes fewer steps and less mental storing. There is less chance for error. | |

Getting the correct answer is not as important as the process used to solve this question. Many students may get the wrong answer by following the algorithmic approach because the many steps involving mental storing and retrieval can become difficult to manage. Guide students to see that, although the algorithmic approach is usually the most effective method when working with pencil and paper, it is not often an effective mental math strategy.

Try the following question, which involves dividing:

| Question: 2052 ÷ 6 | Answer: 342 |
|---|---|
| Method used to calculate mentally (right to left):<br>1. 6 does not go into 2*<br>2. 20 ÷ 6 = 3 (remember this)<br>3. 3 × 6 = 18<br>4. 20 − 18 = 2<br>5. Drop down the 5 to make 25.<br>6. 25 ÷ 6 is 4 (remember this)<br>7. 4 × 6 = 24<br>8. 25 − 24 = 1, bring down the 2 to make 12.<br>9. 12 ÷ 6 = 2 (remember this)<br>10. Reassign place value, and say the answer as, "**three hundred forty-two**." | A method used by someone else (left to right):<br>1. 2052 = 1800 + 240 + 12 (store these mentally)<br>2. 1800 ÷ 6 = say, "***three hundred...***"<br>3. 240 ÷ 6 = say, "***forty-...***"<br>4. 12 ÷ 6 = say, "***two.***" |
| **Preferred method and reason:**<br>The second method is more efficient in this case. It also maintains place value throughout. | |

* **Vocabulary like this can be problematic. Students may come to believe that division can only take place with smaller numbers fitting into larger ones. $2 \div 6 = \frac{1}{3}$, but for the algorithm, we treat this as invalid.**

# *Mental Math*

## Grade 8 Mathematics

S–13

**Sample Strategy**

**Annexing zeros and working with powers of ten/cutting and pasting zeros**

In multiplication and division, annexing zeros allows for quick mental computation of whole numbers that are multiples of powers of ten.


**EXAMPLE 1**

$$\begin{array}{r} 6000 \\ \underline{\times \quad 30} \end{array}$$

$6\cancel{000} \times 3\cancel{0} = \mathbf{180\ 000}$

**EXAMPLE 2**

$$17\ 500 \div 25$$

$17\ 5\cancel{00} \div 25 = \mathbf{7\underline{00}}$

**EXAMPLE 3**

$$88\ 000 \div 400$$

Cancel out zeros where possible for division questions

$88\ 0\cancel{00} \div 4\cancel{00} = 880 \div 4 =$

$88\cancel{0} \div 4 = 22\underline{\mathbf{0}}$

# Mental Math: Grade 8 Mathematics

## Teaching Strategies for Sample Strategy S–13

S–13

### Multiplying and dividing by a power of 10

Have students mentally solve the following questions using a Mental Math Student Communication Template and the method described in the Reproducible Sheets section.

Annexing zeros algorithm for multiplication:

1. Cut all the trailing zeros for numbers being multiplied.
2. Multiply the remaining numbers.
3. Paste all the zeros back.

<table>
<tr><td>Question: $6000 \times 30$</td><td>Answer: 180 000</td></tr>
<tr><td>Method used to calculate mentally (using annexing zeros):<br>$6\cancel{000} \times 3\cancel{0} = 18\underline{\mathbf{0\,000}}$</td><td>A method used by someone else:<br>$6000 \times 30 = (6 \times 1000) \times (3 \times 10)$<br>$= 6 \times 3 \times 1000 \times 10$<br>$= 18 \times 10\,000$<br>$= 180\,000$</td></tr>
<tr><td colspan="2">Preferred method and reason:<br>Annexing zeros method takes only one step.</td></tr>
</table>

Annexing zeros algorithm for division:

1. Permanently cancel out zeros from both the dividend and divisor where possible.
2. Cut the remaining zeros from either the dividend or divisor.
3. Divide the remaining numbers.
4. Paste the zeros from the second step.

<table>
<tr><td>Question: $6300 \div 90$</td><td>Answer: 70</td></tr>
<tr><td>Method used to calculate mentally (using annexing zeros):<br>$630\cancel{0} \div 9\cancel{0} = 630 \div 9$<br>$63\cancel{0} \div 9 = 7\underline{\mathbf{0}}$</td><td>A method used by someone else:<br>$6300 \div 90 = (630 \times 10) \div (9 \times 10)$<br>$= (630 \div 9) \times (10 \div 10)$<br>$= 630 \div 9 \times 1$<br>$= 630 \div 9$<br>$= 70$</td></tr>
<tr><td colspan="2">Preferred method and reason:<br>Annexing zeros method takes fewer steps.</td></tr>
</table>

# *Mental Math*

## Grade 8 Mathematics

S–14

**Sample Strategy**

### Imagining a moving decimal point: Multiplication and division

Although the decimal point never really moves, imagining it as shifting can simplify the process of mentally multiplying and dividing by powers of ten.

| Thousands | Hundreds | Tens | Ones | Decimal | Tenths | Hundredths | Thousandths |
|---|---|---|---|---|---|---|---|
| | | | | • | | | |

| × 0.001 | × 0.01 | × 0.1 | × 1 | × 10 | × 100 |
|---|---|---|---|---|---|
| ÷ 1000 | ÷ 100 | ÷ 10 | ÷ 1 | ÷ 0.1 | ÷ 0.01 |

**EXAMPLE 1** $6000 \times 200$

6~~000~~ × 2~~00~~ = 12
shift decimal right one position for every zero in the question:

= **1 200 000**

**EXAMPLE 2** $90\,000 \times 0.06$

9~~0 000~~ × ~~0.0~~6 = 54
shift decimal right four times and left twice:

= **5400**

**EXAMPLE 3** $72 \div 0.09$

72 ÷ ~~0.0~~9 = 8
shift decimal right twice:

= **800**

## Imagining a moving decimal point: Multiplication and division

**Show by induction that the decimal point can be seen as shifting for multiplication and division questions.**

Have students answer the following series of questions and others similar to it, with or without calculators, noticing where the decimal points appear in the final answers. Have students develop rules:

| | | | |
|---|---|---|---|
| $40 \times 1000$ | $4 \times 1000$ | $0.4 \times 1000$ | $4 \div 0.001$ |
| $40 \times 100$ | $4 \times 100$ | $0.4 \times 100$ | $4 \div 0.01$ |
| $40 \times 10$ | $4 \times 10$ | $0.4 \times 10$ | $4 \div 0.1$ |
| $40 \times 1$ | $4 \times 1$ | $0.4 \times 1$ | $4 \div 1$ |
| $40 \times 0.1$ | $4 \times 0.1$ | $0.4 \times 0.1$ | $4 \div 10$ |
| $40 \times 0.01$ | $4 \times 0.01$ | $0.4 \times 0.01$ | $4 \div 100$ |
| $40 \times 0.001$ | $4 \times 0.001$ | $0.4 \times 0.001$ | $4 \div 1000$ |

Rules developed may include the following:

**For multiplying a number by a power of 10 greater than 1**

Ignore the trailing zeros and multiply. Mentally shift the decimal place in your answer **right** one place for every trailing zero in the question.

**For multiplying a number by a power of 10 less than 1**

Ignore leading zeros and decimals and multiply. Mentally shift the decimal place in your answer **left** one place for every decimal place in the question.

**For dividing a number by a power of 10 greater than 1**

Ignore the trailing zeros and divide. Mentally shift the decimal place in your answer **left** one place for every trailing zero in the question.

**For dividing a number by a power of 10 less than 1**

Ignore leading zeros and decimals and divide. Mentally shift the decimal place in your answer **right** one place for every decimal place in the question.

**Multiplying by 10 is the same as dividing by 0.1.**

**Multiplying by 100 is the same as dividing by 0.01.**

# *Mental Math*

## Grade 8 Mathematics

S–15

**Sample Strategy**

**Compensation: Multiplication and division**

Add or subtract a group to a portion of the question in order to make it easier to solve. Compensate your final answer by performing the opposite operation on it.

EXAMPLE 1 $\$0.73 \times 4$

$\$0.73 + \$0.02 = \$0.75$

$\$0.75 \times 4 = \$3.00$

$\$3.00 - (\$0.02 \times 4) = \mathbf{\$2.92}$

EXAMPLE 2 $7992 \div 8$

7992 + 1 group of 8 = 8000

$8000 \div 8 = 1000$

$1000 - 1 = \mathbf{999}$

EXAMPLE 3 $5\frac{7}{8} \times 4$

$6 \times 4 = 24$

$24 - \left(4 \times \frac{1}{8}\right) =$

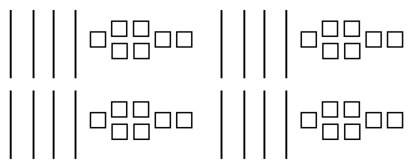
$24 - \frac{1}{2} = \mathbf{23\frac{1}{2}}$

## Compensation: Multiplication and division

**Show students that using compensation strategies for multiplication and division simplifies mental computations.**

### Compensation with Multiplication

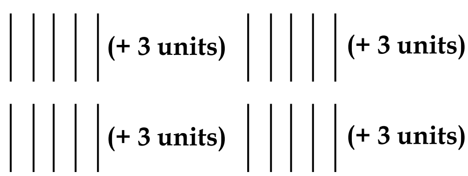
1. Demonstrate the question 47 × 4 with Base-10 blocks by showing 4 groups of 47.
2. Have students find the total using their own preferred method. They will get 188.



*four groups of forty-seven*
*= 188 total*

3. Show how rounding 47 to 50 makes this problem easier to solve. All that is required is that 3 be added to each of the four groups (3 × 4 = 12 in all).

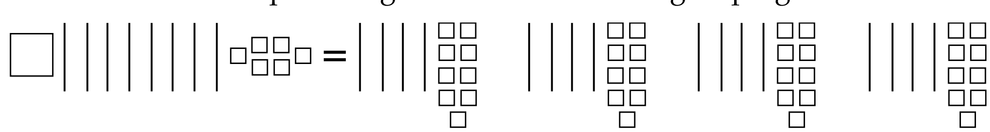
   Students should be able to see fairly quickly that four groups of 50 is 200, and that this is easier to model and solve.
4. In order to do this, four extra groups of 3 had to be added, so in order to compensate for this, four groups of 3 need to be removed from the final answer. 200 − 12 = 188




*= 200, with 12 extra units*
*200 − 12 = 188*

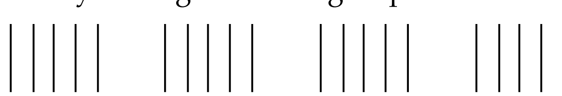
### Compensation with Division

1. Model 196 ÷ 4 with Base-10 blocks, and have students show and solve the equation. Showing this with Base-10 blocks will require a significant amount of regrouping.



*196 ÷ 4 = 49*

2. Have students try adding one more group of 4 to 196 to try the same question.



*200 ÷ 4 = 50*

3. Have students finish the compensation process by mentally removing one unit from each group, and they should see that 196 ÷ 4 = 49.

# *Mental Math*

## Grade 8 Mathematics

S–16

**Sample Strategy**

### Dividing: Balancing both elements

When you multiply the same number to the two elements of a division question, or divide both elements by the same number, the ratio between the two does not change and the answer always remains the same.

$$a \div b = a(c) \div b(c) \qquad a \div b = \frac{a}{c} \div \frac{b}{c}$$

**EXAMPLE 1** $7.5 \div 1.5$

$7.5(2) \div 1.5(2) = 15 \div 3$

$= \mathbf{5}$

**EXAMPLE 2** $240 \div 48$

$= (240 \div 6) \div (48 \div 6)$

$= 40 \div 8 = \mathbf{5}$

**EXAMPLE 3** $6\frac{2}{3} \div 1\frac{2}{3}$

$= \frac{20}{3} \div \frac{5}{3}$

$= \left(\frac{20}{3} \times 3\right) \div \left(\frac{5}{3} \times 3\right)$

$= 20 \div 5 = \mathbf{4}$

# Mental Math: Grade 8 Mathematics

## Teaching Strategies for Sample Strategy S–16

S–16

### Dividing: Balancing both elements

**Show, using counters, that the ratio between elements of a division question remains the same when both elements are divided by the same number (except for zero).**

- Have students show 36 ÷ 6 using counters. Students should have six distinct groups of 6.
- Using this arrangement, have students pile two counters from within the same group on top of each other. Their model will still represent 36 ÷ 6 = 6, but their piles of counters will be showing 18 ÷ 3 = 6 (eighteen piles, divided into groups of 3, equals six). They are modelling that

$$\begin{aligned} 36 \div 6 &= \frac{36}{2} \div \frac{6}{2} \\ &= 18 \div 3 \\ &= 6 \end{aligned}$$

- Have students regroup each of their piles so that each group now has two piles of 3, showing both 36 ÷ 6 = 6, and 12 ÷ 2 = 6 (twelve piles, divided into groups of 2, equals six). They will be modelling that

$$\begin{aligned} 36 \div 6 &= \frac{36}{3} \div \frac{6}{3} \\ &= 12 \div 2 \\ &= 6 \end{aligned}$$

- Have students try the same with 40 counters to show that

$$40 \div 4 = 20 \div 2 = 10 \div 1$$

**Use $a \div b = a(c) \div b(c)$ to show why, when dividing two fractions, we can multiply by the reciprocal to get the same result.**

$$\begin{aligned} &\frac{a}{b} \div \frac{c}{d} \\ &= \frac{a}{b}\left(\frac{d}{c}\right) \div \boxed{\frac{c}{d}\left(\frac{d}{c}\right)} \\ &= \frac{a}{b}\left(\frac{d}{c}\right) \div 1 \\ &= \frac{a}{b}\left(\frac{d}{c}\right) \end{aligned}$$

We multiply by the inverse because it will equal 1!

$$\boldsymbol{\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}}$$

$$\begin{aligned} &\frac{1}{3} \div \frac{4}{7} \\ &= \frac{1}{3}\left(\frac{7}{4}\right) \div \frac{4}{7}\left(\frac{7}{4}\right) \\ &= \frac{1}{3}\left(\frac{7}{4}\right) \div 1 \\ &= \frac{1}{3}\left(\frac{7}{4}\right) \end{aligned}$$

$$\boldsymbol{\frac{1}{3} \div \frac{4}{7} = \frac{1}{3} \times \frac{7}{4}}$$

# *Mental Math*

## Grade 8 Mathematics

S–17

**Sample Strategy**

## Doubling AND halving

In multiplication, doubling one factor and halving the other will give you the same result.

**EXAMPLE 1** $15 \times 12$

$15 \times 12 =$

$30 \times 6 = \mathbf{180}$

**EXAMPLE 2** $12\frac{1}{2} \times 4$

$12\frac{1}{2} \times 4 =$

$25 \times 2 = \mathbf{50}$

## Doubling OR halving

Using known facts, double them or halve them to determine the answer.

**EXAMPLE 3** $7 \times 16$

Think the double of $7 \times 8$

$7 \times 8 = 56$

$56 \times 2 = \mathbf{112}$

**EXAMPLE 4** $164 \div 8$

Think repeated halving

$164 \div 2 = 82$

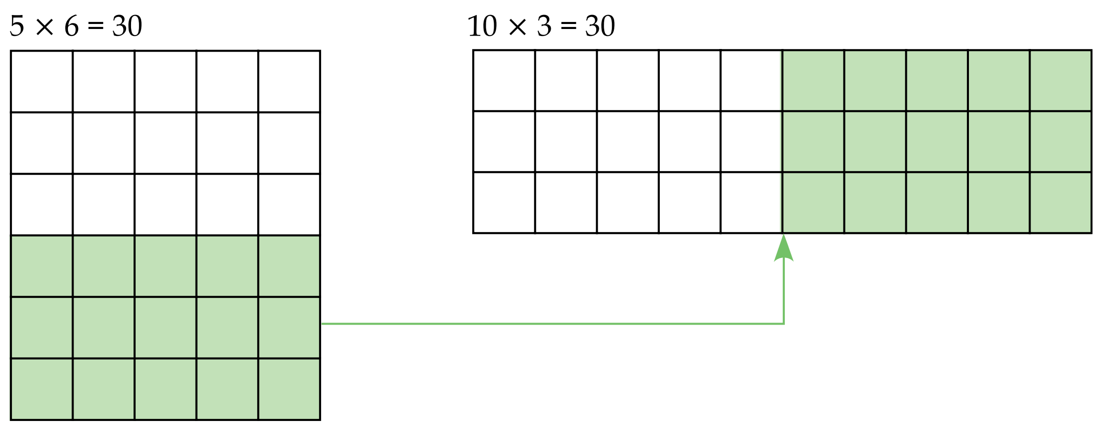
$82 \div 2 = 41$

$41 \div 2 = \mathbf{20\frac{1}{2}}$

## Doubling and/or halving

**Show using arrays (an area model) the effects of doubling and/or halving elements of a multiplication and division question.**

### Doubling AND Halving




- Have students, using grid paper, come up with other examples of where doubling and halving works well.
- Ask students to develop guidelines as to where this process might be effective in mental math situations. Students should eventually see that this method works best when working with at least one even number, or that when one element ends in a five, doubling it makes it a power of ten and it becomes easier to work with.
- A large selection of practice questions are available at: https://nzmaths.co.nz/sites/default/files/DoublingAndHalvingSheet.pdf.

### Doubling OR Halving

- This process can be used to solidify basic multiplication and division facts.
- Encouraging students to think of doubles and halves when working with 6s and 8s is an effective step towards developing automaticity.

  $6 \times 3$ is double $3 \times 3$

  $6 \times 4$ is double $3 \times 4$ OR double $6 \times 2$

  $6 \times 6$ is double $3 \times 6$ OR quadruple $3 \times 3$

  $6 \times 7$ is double $3 \times 7$

  $7 \times 8$ is double $7 \times 4$ OR quadruple $7 \times 2$

  $48 \div 12$ is half $48 \div 6$

  $24 \div 3$ is double $24 \div 6$

# *Mental Math*

## Grade 8 Mathematics

S–18

**Sample Strategy**

**Properties and laws: Commutative property**

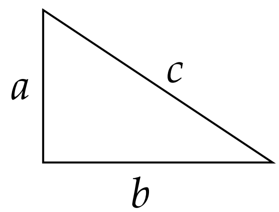
When adding or multiplying numbers together, switch them around in ways that make them easier to work with.

$a + b = b + a$

$a \times b = b \times a$

EXAMPLE 1

Find the hypotenuse.



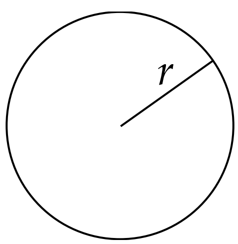

$a^2 + b^2 = c^2$

so

$b^2 + a^2 = c^2$

EXAMPLE 2

Find the diameter.




$d = 2r$

so

$r \times 2 = d$

EXAMPLE 3

Solve.

$$\frac{2}{3} \times \frac{6}{7}$$

$$\begin{aligned}\frac{2}{3} \times \frac{6}{7} &= \frac{6}{3} \times \frac{2}{7} \\ &= 2 \times \frac{2}{7} \\ &= \frac{4}{7}\end{aligned}$$

# *Mental Math*

## Grade 8 Mathematics

S–19

**Sample Strategy**

### Properties and laws: Associative property

When adding or multiplying many numbers together, order them in ways that make them easier to work with.

$(a + b) + c = a + (b + c)$

$(a \times b) \times c = a \times (b \times c)$

**EXAMPLE 1**

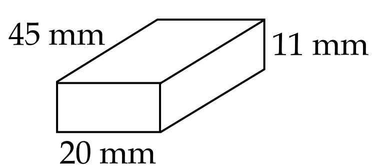
$$\begin{array}{r} 42 \\ 17 \\ 18 \\ 3 \\ 85 \\ +\ \ 4 \\ \hline \end{array}$$

$42 + 18 = 60$
$17 + 3 = 20$
$85 + 4 = 89$
$60 + 20 + 89 = \mathbf{169}$

**Or,** keep a running total:
$42 + 18 = 60$
$60 + 10$ (from the 17) $= 70$
$70 + 7$ (from the 17) $+ 3 = 80$
$80 + 5$ (from the 85) $+ 4 = 89$
$89 + 80 = \mathbf{169}$

**EXAMPLE 2**

Find the volume.




$45 \times 11 \times 20$
$= 45 \times 20 \times 11$
$= 900 \times 11$
$= \mathbf{9900\ mm^2}$

**EXAMPLE 3**

Find 36% of 25

$= 25 \times (36 \times 0.01)$
$= (25 \times 0.01) \times 36$
$= 25\%$ of 36
$= \mathbf{9}$

# *Mental Math*

## Grade 8 Mathematics

S–20

**Sample Strategy**

**Properties and laws: Distributive property**

$a \times (b + c) = a \times b + a \times c$

$a \times (b - c) = a \times b - a \times c$

**EXAMPLE 1**

$7 \times 92.1$

$7 \times 92.1$
$= (7 \times 90) + (7 \times 2) + (7 \times 0.1)$
$= 630 + 14 + 0.7$
$= \mathbf{644.7}$

**EXAMPLE 2**

Simplify:
$4(3 - 2x) + 1$

$= (4 \times 3) - (4 \times 2x) + 1$
$= 12 - 8x + 1$
$= \mathbf{13 - 8x}$

**EXAMPLE 3**

$9465 \div 11$

$9465 \div 11 = (8800 + 660 + 5) \div 11$
$8800 = 11 \times \mathbf{800}$
$660 = 11 \times \mathbf{60}$
$\frac{5}{11}$ **left over**

$= \mathbf{860\frac{5}{11}}$

# Mental Math: Grade 8 Mathematics

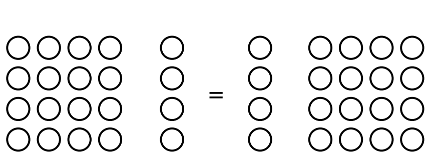
## Teaching Strategies for Sample Strategies S–18 to S–20

### Properties and laws: Commutative / Associative / Distributive

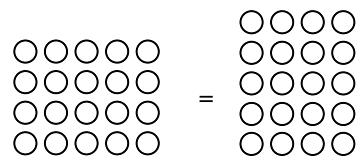
**Use counters to develop specific rules about commutative, associative, and distributive properties.**

#### Commutative Property

Show students a model of the commutative property with counters or blocks.



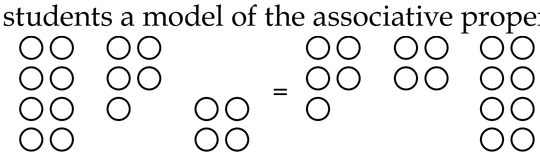
$16 + 4 = 4 + 16$



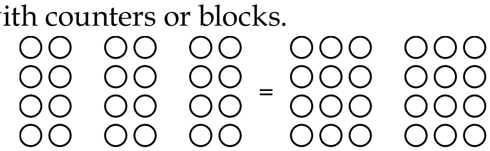
$4 \times 5 = 5 \times 4$

#### Associative Property

Show students a model of the associative property with counters or blocks.



$8 + 5 + 4 = 5 + 4 + 8$



$(2 \times 4) \times 3 = 2 \times (4 \times 3)$

- Have students try several examples of their own to determine if these properties will always hold true.
- Have students determine if these processes will work for subtraction and division, and explain why not.
- Have students develop rules with variables, such as:

  $a + b = b + a$

  $(a \times b) \times c = a \times (b \times c)$
- Have students use a number line to determine if the properties hold true when adding or multiplying whole numbers by negative integers.

#### Distributive Property

Have students analyze and explain why the following methods of multiplication work.

$$\begin{array}{r} 16 \\ \times\ 32 \\ \hline 12 \\ 20 \\ 180 \\ 300 \\ \hline \mathbf{512} \end{array}$$

$5 \times 4 = (4 \times 4) + (1 \times 4)$

$5 \times 4 = (3 \times 4) + (2 \times 4)$

$5 \times 4 = (2 \times 4) + (3 \times 4)$

$5 \times 4 = (1 \times 4) + (4 \times 4)$

$5 \times 4 = (2 \times 4) + (1 \times 4) + (2 \times 4)$

$5 \times 4 = (6 \times 4) - (1 \times 4)$

$5 \times 4 = (8 \times 4) - (3 \times 4)$

# *Mental Math*

## Grade 8 Mathematics

S–21

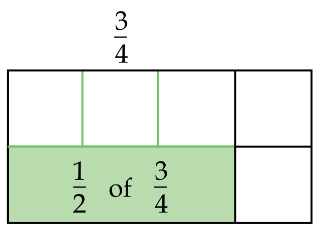
**Sample Strategy**

**Advanced: Spatial reasoning/visualization**

Forming a mental picture can be an effective way of reasoning through many math questions.

**EXAMPLE 1**

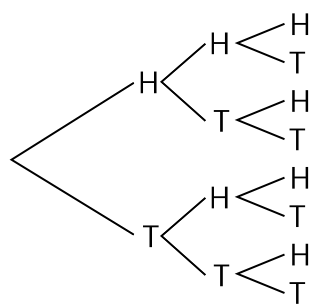
$$\frac{1}{2} \times \frac{3}{4}$$




$= \frac{3}{8}$

**EXAMPLE 2**

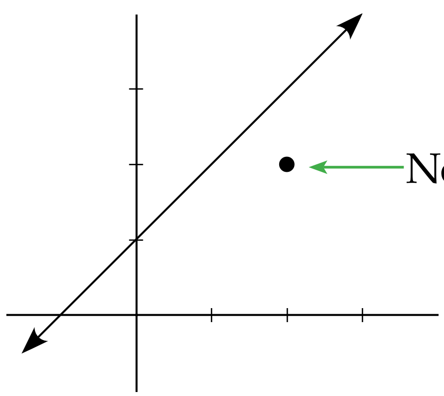
What is the probability of three tails in three coin tosses?




$= \frac{1}{8}$

**EXAMPLE 3**

Does the equation of the line $y = x + 1$ pass through (2, 2)?



No, it does not.

# *Mental Math*

## Grade 8 Mathematics

S–22

**Sample Strategy**

**Advanced: Hybrid approaches/A combination of strategies**

With increased flexibility comes the ability to integrate multiple mental math strategies in the solution of a single problem.

**EXAMPLE 1**

$52 \times 16$

$100 \times 16 = 1600$

$1600 \div 2 = 800$

Add 2 more groups of 16 ($16 \times 2 = 32$)

$800 + 32 = \mathbf{832}$

**EXAMPLE 2**

Calculate the cost of a $14.99 dinner after a 15% tip.

Round $14.99 to $15, because 15% of 1¢ will be insignificantly small.

$$\begin{aligned}\$15 \times 15\% &= (0.15 \times 10) + \frac{1}{2}(0.15 \times 10)\\ &= 1.50 + 0.75\\ &= 1.50 + 0.50 + 0.25\\ &= \$2.25\end{aligned}$$

$$\begin{aligned}\$2.25 + \$14.99 &= \$2.25 + \$15.00 - \$0.01\\ &= \mathbf{\$17.24}\end{aligned}$$

**EXAMPLE 3**

Siju runs 5 metres per second. How fast is that in km per hour?

5 metres $\times$ 60 seconds = 300 metres per minute

$(3\cancel{00} \times 6\cancel{0}$ minutes$) \div 1\cancel{000}$ metres $= 3 \times 6$

= **18 km/hr.**

# *Mental Math*

## Grade 8 Mathematics

S–23

**Sample Strategy**

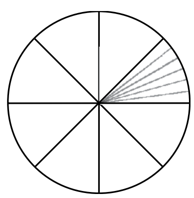
### Memorization/automaticity

Once a conceptual understanding of basic facts, procedures, or formulas is developed, automaticity through memorization can be effectively used in order to make complex problem solving more efficient by freeing up working memory.

- Not all facts, formulas, or procedures need to be memorized, but for some it is important (i.e., $\pi \approx 3.14$, $50\% = \frac{1}{2}$, $8 \times 7 = 56$).
- Without a conceptual understanding, it becomes difficult for students to make connections across Middle Years topics.

**EXAMPLE 1**

$$\frac{1}{8} \times \frac{1}{6}$$



$= \frac{1}{48}$

Find one-sixth of one-eighth:

$8 \times 6 = 48$

OR

$8 \times 6 = (6 \times 6) + 12 = 48$

OR

$8 + 8 + 8 + 8 + 8 + 8 = 48$

Memorized process:

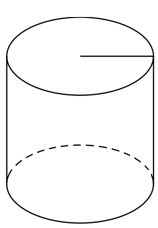
Multiply across

$1 \times 1 = 1$

$8 \times 6 = 48$ $\quad = \frac{1}{48}$

**EXAMPLE 2**

Explain two methods of finding the volume of the cylinder.



1. Memorized formula: $V = \pi r^2 h$
2. The volume of any right prism or cylinder has to be the area of the face times the height: $(3.14 \times r \times r) \times h = V$

# Mental Math: Grade 8 Mathematics

## Teaching Strategies for Sample Strategy S–23

S–23

### Memorization/automaticity

**Students prepare to commit many facts, procedures, and formulas to memory through regular and routine applications of efficient math strategies.**

**What about timed testing and flash cards?**

- "Drill should only be used when an efficient strategy is in place." (Van de Walle and Folk, 169)
- Games can also be used to reinforce strategies, ultimately leading to automaticity.
- Although students may be able to memorize exclusively through drill work, without a foundation in conceptual understanding they will potentially lack the ability to make connections to future related areas of math.
- Forced memorization activities can make math appear to be an abstract system of complicated rules and procedures that are all dependent on each other. Flexibility, reason, and proofs don't fit well with this interpretation of math or with timed testing.
- Timed tests have been linked to math anxiety.
- Practice makes permanence. Repeated mistakes on timed tests can reinforce incorrect answers. Correct answers can also reinforce inefficient strategies.

**What about memorizing procedures such as long division and double-digit multiplication?**

- Students should understand why these processes work in order to make effective use of them. The standard long division and double-digit multiplication processes work because of place-value partitioning. The processes can be demonstrated and recreated with manipulatives. By reinforcing how these processes work, students can understand why they work and can better commit the processes to memory.
- Standard processes of long division and double-digit multiplication always work, but these processes are not often the most efficient method to solve a mental math problem, and they have little flexibility to them. Using these processes may demonstrate an ability to follow a rote procedure, but this does not necessarily demonstrate an understanding of the underlying math.

**What about memorizing formulas?**

- Grade 8 curriculum learning outcomes dealing with formulas all state that the formulas are to be "developed and applied" by students. This emphasis on student development of formulas and rules requires that students use their knowledge and reasoning skills to create formulas and clearly understand why those formulas work. Students may develop formulas that resemble the standard ones used, and they may ultimately find memorizing them to be an effective strategy.

# *Mental Math*

## Grade 8 Mathematics

S–24

**Sample Strategy**

### Estimation strategies: Rounding using compatible numbers

Use nearly equal, simple-to-use values in order to perform mental math estimations efficiently and effectively.

Round to compatible numbers that are easy to compute mentally.

**EXAMPLE 1**

Approximate:

$$\begin{array}{r} 27 \\ 45 \\ 63 \\ +\ 81 \\ \hline \end{array}$$

*about 100*

$27 + 45 + 63 + 81 \approx \mathbf{200}$

*about 100*

**EXAMPLE 2**

Estimate: $19.1 \div 3$

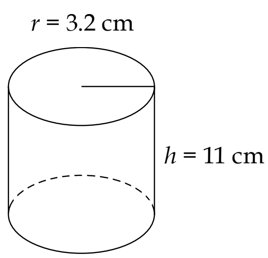
$19.1 \approx 18$

$18 \div 3 = \mathbf{6}$

*Using the standard method of rounding would result in 20 ÷ 3, which is no easier to solve than 19.1 ÷ 3.*

**EXAMPLE 3**

Will this can hold 250 mL of soup?




*In this case, round down all values in order to ensure that the final estimation is an* ***underestimate****.*

$V = \pi r^2 h$

$\approx 3 \times 9 \times 10 = \mathbf{270\ mL}$

Yes, this can will hold 250 mL of soup.

# Mental Math: Grade 8 Mathematics

## Background Information for Sample Strategies S–24 to S–27

## Estimation strategies: Rounding and estimating

### Methods of Rounding

There are many methods of rounding, and choosing the method to use depends entirely on the reason the rounding is being done. Students need to be familiar with different forms of rounding and should be able to reason through which method would be the best to use, depending on the context of the question.

### Four Situations Where Rounding is Important

1. When using a simpler, nearly equal value to make mental estimation more efficient and communication simpler.

   *Examples*

   $\pi \approx 3.14 \left(\pi \approx 3\text{, or } \pi \approx \frac{22}{7} \text{ may be even more useful in some instances}\right)$

   $\frac{2}{3} \approx 67\%$

   $227 \div 3 \approx 225 \div 3 = 75$

2. When communicating useful information.

   *Examples*

   After calculation, a price may appear to have three decimal places. The dollar system operates with only two decimal places. ($9.457 becomes $9.46.)

   100 students fit on each bus. How many buses are needed to transport 201 students? (201 ÷ 100 = 2.01 buses. Even though 2.01 buses is much closer to 2 than 3, 3 buses are required.)

3. When a calculation comes to a very precise result but displaying it in that way would present misinformation.

   *Examples*

   We round, and say that there are 7 billion people on Earth. (Reporting a precise number, such as 7 463 403 434, when that number is always changing, would present misinformation.)

   9 out of 10 dentists recommend a certain type of toothpaste. In a group of 15 dentists, how many recommend it? $\left(13\frac{1}{2}\text{? Rounding it makes sense. There is no such thing as } \frac{1}{2} \text{ of a dentist.}\right)$

4. When exact numbers are not needed to answer a question.

   *Example*

   Will $67 be enough for four $13 apps? ($13 ≈ $15; $15 × 4 = $60, so $67 is enough.)

# *Mental Math*

## Grade 8 Mathematics

**S–25**

**Sample Strategy**

### Estimation strategies: Common method—Half-round up

The most common method of rounding is to examine the value of the digit to the right of the one being rounded. If the digit is 1, 2, 3, or 4, we round down. If it is 5, 6, 7, 8, or 9, we round up.

Rounding results in a multiple of ten, which is often easy to work with.

**EXAMPLE 1**

Approximate: $94.2 \times 67.8$

$94.2 \times 67.8 \approx 90 \times 70$

$90 \times 70 =$ **6300**

**EXAMPLE 2**

Estimate: $2\,145\,648 - 389\,482$

$2\,145\,648 - 389\,482$

$\approx 2\,100\,000 - 400\,000 =$ **1 700 000**

**EXAMPLE 3**

Round to the nearest hundredth: 4012.235

$4012.23\underline{5}$

Because the underlined number is 5 or greater, we round the hundredths up to **4012.24.**

If 1, 2, 3, and 4 always round down and 5, 6, 7, 8, 9 always round up, could it ever become problematic that more numbers round up than down? If 5 is right in the middle, why does it round up all of the time? Also, what do you do with −7.5? Rounding up brings us closer to zero to −7! This rounding method is not perfect, but it can be very useful in certain situations and is widely used in industry and business.

# *Mental Math*

## Grade 8 Mathematics

S–26

**Sample Strategy**

**Estimation strategies: Front-end rounding and estimation**

There are several methods of front-end rounding and estimation.

- Keep the largest place value and truncate the rest.
- Round using the common half-round up method for the largest place value of each number.

**EXAMPLE 1**

Front-end estimation using the first method:

$36\ 548 \times 712$

$\approx 30\ 000 \times 700$
$= 30\ 000 \times 700$
$= \mathbf{21\ 000\ 000}$

This method will always provide an underestimate.

**EXAMPLE 2**

Front-end estimation using the common half-round up method:

$36\ 548 \times 712$

$\approx 40\ 000 \times 700$
$= 40\ 000 \times 700$
$= \mathbf{28\ 000\ 000}$

The second method of front-end estimation will always provide an equal or closer estimate to the first. In some cases this second method will provide an underestimate and, in others, an overestimate.

# *Mental Math*

## Grade 8 Mathematics

S–27

**Sample Strategy**

### Estimation strategies: Round to the nearest 5

With the elimination of the penny from circulation in 2013, Canada has implemented a rounding system to the nearest 5¢ for cash payments.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# Mental Math Questions by Learning Target

# *Mental Math*

## Grade 8 Mathematics

A–1

| | |
|---|---|
| **Learning Target** | Operations with Whole Numbers (Number Strand: 8.N.1, N.7) |
| **Strategies of Focus** | Annexing Zeros |

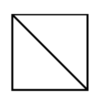
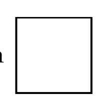
| **Prior Learning** | **Answers** |
|---|---|
| 1. Multiply: $90 \times 40$ | 3600 |
| 2. Divide: $14,000 ÷ 70 | $200 |
| 3. Divide: 25 000 kg ÷ 5 | 5000 kg |
| 4. Miss Kokum budgets $450 for her grandson's lunches for the year. How many $5 lunches can he buy? | 90 lunches |
| 5. How many cm are in 4 km? | 400 000 cm |
| 6. How many mL in a 4 L jug of milk? | 4000 mL |
| **Grade 8 Questions** | |
| For questions 7 to 10, solve and simplify. | |
| 7. $\sqrt{10\ 000} \div \sqrt{100}$ | 10 |
| 8. $\sqrt{10\ 000} \times \sqrt{900}$ | 3000 |
| 9. $\sqrt{200\ 000 \div 500}$ | 20 |
| 10. $6000 \times 1100 + \sqrt{1}$ | 6 600 001 |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

**A–2**

| | |
|---|---|
| **Learning Target** | Operations with Whole Numbers (Number Strand: 8.N.1, N.7) |
| **Strategies of Focus** | Visualization |

| **Prior Learning** | **Answers** |
|---|---|
| 1. What is the volume of a cube that has side lengths of 3 cm? | 27 $cm^3$ |
| 2. Which is heavier and by how much?<br>a 36 kg bag of rocks    a 100 kg bag of feathers | The feathers are 64 kg heavier. |
| 3. Which is closer to the perimeter of a classroom?<br>3800 cm    38 000 cm    80 cm    380 000 cm | 3800 cm |
| 4. One dog can run 32 km/h. Jerry wants his dogsled to go 96 km/h. How many dogs does he need? | Can't be done. 32 km/h is the maximum speed. |
| 5. A tree is 40 feet tall. There are 12 inches in each foot. How tall is the tree in inches? | 480 inches |
| 6. This 40-foot tree needs to be cut into 10-inch logs. How many logs will you get? | 48 logs |
| **Grade 8 Questions** | |
| 7. If one of the triangles shown here has an area of 18 $cm^2$, what must be the side length of the square?  | 6 cm |
| 8. A square has an area of 36 square units. What is the perimeter? | 24 units |
| 9. A cube has sides that are 4 cm long. What is the surface area? | 96 $cm^2$ |
| 10. What is the area of the square? 8 mm  | 64 $mm^2$ |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–3

| | |
|---|---|
| **Learning Target** | Operations with Whole Numbers (Number Strand: 8.N.1, N.7) |
| **Strategies of Focus** | Various |

| **Prior Learning** | **Answers** |
|---|---|
| 1. True or False: $14 = 14 \times 1 + 0$ | True |
| 2. True or False: $(9 \times 9) + (9 \times 5) = (9 \times 7) + (9 \times 7)$ | True |
| 3. Solve: 42 cm $\times$ 1000 | 42 000 cm<br>420 m |
| 4. Solve: 4.2 L $\times$ 0.01 | 0.042 L<br>42 mL |
| 5. Solve: 3.49 kg $\div$ 1000 | 0.00349 kg<br>3.49 g |
| 6. Solve: 3.49 kg $\div$ 0.001 kg | 3490 groups<br>**no units** |
| **Grade 8 Questions** | |
| For questions 7 to 10, simplify and solve. | |
| 7. $\sqrt{64} + \sqrt{144}$ | 20 |
| 8. $\sqrt{1}$ | 1 |
| 9. $\sqrt{10\ 000} \times \sqrt{16}$ | 400 |
| 10. $\sqrt{121} - \sqrt{144}$ | −1 |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–4

| | |
|---|---|
| **Learning Target** | Operations with Whole Numbers (Number Strand: 8.N.1, N.7) |
| **Strategies of Focus** | Various |

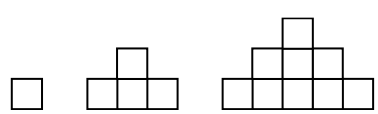
| Prior Learning | Answers |
|---|---|
| 1. Solve: $13 \times 25$ | 325 |
| 2. Solve: $15 \times 7$ | 105 |
| 3. Solve: $98 + 124$ | 222 |
| 4. List the factors of 9. | 1, 3, 9 |
| 5. Solve: $400 \div 20$ | 20 |
| 6. Solve: $501 \div 3$ | 167 |
| **Grade 8 Questions** | |
| 7. Solve and simplify: $\sqrt{16} + \sqrt{16}$ | 8 |
| 8. How many unit squares would you need to fill a larger square with a side length of 10 units? | 100 unit squares |
| 9. Simplify: $\sqrt{36}$ | 6 |
| 10. Which of the following are perfect squares? 7, 49, or 490 | 49 |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–5

| | |
|---|---|
| **Learning Target** | Operations with Whole Numbers (Number Strand: 8.N.1, N.7) |
| **Strategies of Focus** | Various |

| **Prior Learning** | **Answers** |
|---|---|
| For questions 1 to 5, solve using the operations indicated. | |
| 1. $1 \div 1 \times 1$ | 1 |
| 2. $1 - 1 \div 1$ | 0 |
| 3. $7 \times 8 + 1$ | 57 |
| 4. $8 + 20(4 \times 6)$ | 488 |
| 5. $(488 - 8) \div 24$ | 20 |
| 6. True or False: $12 \times 15 = 6 \times 30$ | True |
| **Grade 8 Questions** | |
| For questions 7 to 9, simplify and solve. | |
| 7. $\sqrt{10\ 000} \times (22 \times 2)$ | 4400 |
| 8. $\sqrt{64} \times (-0.01)$ | $-0.08$ |
| 9. $\sqrt{100} + \sqrt{100} \times \sqrt{100}$ | 110 |
| 10. How many squares are in figure 100?  | 10 000 squares |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

**A–6**

| | |
|---|---|
| **Learning Target** | Operations with Whole Numbers (Number Strand: 8.N.1, N.7) |
| **Strategies of Focus** | Various |

| **Prior Learning** | **Answers** |
|---|---|
| 1. What are the prime factors of 81? | 3 |
| 2. What are the prime factors of 36? | 2 and 3 |
| 3. What are the factors of 17? | 1 and 17 |
| 4. What are the first four multiples of twenty-two? | 22, 44, 66, 88 |
| 5. What are the first three common multiples of 6 and 4? | 12, 24, 36 |
| 6. What are the composite numbers between 10 and 20? | 12, 14, 15, 16, 18 |
| **Grade 8 Questions** | |
| 7. Simplify: $\sqrt{9}$ | 3 |
| 8. Simplify: $\sqrt{16}$ | 4 |
| 9. Simplify: $\sqrt{81}$ | 9 |
| 10. The length of a side of a square piece of paper is $\sqrt{64}$ cm. What is the area of the square? | 64 $cm^2$ |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–7

| | |
|---|---|
| **Learning Target** | Operations with Whole Numbers (Number Strand: 8.N.1, N.7) |
| **Strategies of Focus** | Annexing Zeros |

| **Prior Learning** | **Answers** |
|---|---|
| Solve questions 1 to 6 using the operation indicated. | |
| 1. $24 \times 3000$ | 72 000 |
| 2. $1200 \div 20$ | 60 |
| 3. $848\,000 \div 800$ | 1060 |
| 4. $110 \times 6000$ | 660 000 |
| 5. $1000 \div 0$ | Undefined |
| 6. $0 \div 1000$ | 0 |
| **Grade 8 Questions** | |
| 7. Solve and simplify: $\sqrt{49} \times (-10)$ | $-70$ |
| 8. Solve and simplify: $0 \times \sqrt{11}$ | 0 |
| 9. Which of the following are true statements? $8 = \sqrt{16}$ $\quad \sqrt{16} = 32$ $\quad \sqrt{16} = 2 + 2$ | $\sqrt{16} = 2 + 2$ |
| 10. Solve and simplify: $\sqrt{36} + \sqrt{81} + \sqrt{1}$ | 16 |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–8

| | |
|---|---|
| **Learning Target** | Operations with Whole Numbers (Number Strand: 8.N.1, N.7) |
| **Strategies of Focus** | Visualization |

| **Prior Learning** | **Answers** |
|---|---|
| 1. Use <, >, or =.<br>seventeen million seventeen thousand seventy-six ____ 17 170 076 | < |
| 2. Express in standard form:<br>fifty-four thousand, seven hundred two | 54 702 |
| 3. Express in standard form:<br>three hundred thirty million, forty-nine thousand, eleven | 330 049 011 |
| 4. What is the area of a square tabletop with a side length of 40 cm? | 1600 $cm^2$<br>0.16 $m^2$ |
| 5. A $20 debt is shared equally among five students. How much does each student owe? | $4 |
| 6. A shed covers an area of 24 square metres in a 10 m by 10 m green space. What is the size of the green space not covered? | 76 $m^2$ |
| **Grade 8 Questions** | |
| 7. How much longer is the side length of a square that has an area of 81 $cm^2$ than one that has an area of 36 $cm^2$? | 3 cm |
| 8. Solve and simplify: $\sqrt{25} - \sqrt{36}$ | −1 |
| 9. A square has an area of 100 $cm^2$. What is the perimeter? | 40 cm |
| 10. Starting at $1 \times 1$ on a multiplication chart, in what direction do the perfect squares all form a straight line? | Diagonally downward and to the right |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–9

| | |
|---|---|
| **Learning Target** | Operations with Whole Numbers (Number Strand: 8.N.1, N.7) |
| **Strategies of Focus** | Association, Distribution, and Commutative Properties |

| **Prior Learning** | **Answers** |
|---|---|
| 1. Solve: $6000 \times 900$ | 5 400 000 |
| 2. Solve: $35 + 46 + 35 + 24 + 4$ | 144 |
| 3. Solve: $24 + 32 + 68 + 26$ | 150 |
| 4. Solve: $81 + 73 + 19 + 68 + 27$ | 268 |
| 5. Express in standard form: $4\,000\,000 + 180\,000 + 756$ | 4 180 756 |
| 6. Express in standard form: $40\,000 + 2000 + 70 + 8$ | 42 078 |
| **Grade 8 Questions** | |
| For questions 7 to 10, solve and simplify. | |
| 7. $0 \times \sqrt{3600}$ | 0 |
| 8. $\sqrt{36} \times \sqrt{36}$ | 36 |
| 9. $\sqrt{36 \times 36}$ | 36 |
| 10. $-\sqrt{98 \div 2}$ | −7 |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

**A–10**

| | |
|---|---|
| **Learning Target** | Operations with Whole Numbers (Number Strand: 8.N.1, N.7) |
| **Strategies of Focus** | Thinking Multiplication |

| **Prior Learning** | **Answers** |
|---|---|
| 1. Is 831 mL ÷ 4 greater or less than 200 mL? | Greater |
| 2. Solve: $24 \times 2 + 1 \times 2$ | 50 |
| 3. Estimate the total cost using rounding to the nearest dollar: \$4.69 + \$3.10 + \$4.69 + \$3.10 | \$16 |
| 4. Solve: $4900 \div 0$ | Undefined |
| 5. Is 91 a prime number? | No ($7 \times 13$) |
| 6. Is 1009 divisible by 3? | No |
| **Grade 8 Questions** | |
| 7. Which of the following are perfect squares? 64, 640, 6400, 64 000, 640 000 | 64, 6400, 640 000 |
| 8. Which of the following are perfect squares? 0.1, 1, −10, 100, 1000 | 1, 100 |
| 9. Which of the following are perfect squares? −49, 7, 49, 70, 490, 700 | 49 |
| 10. Name the perfect squares between −10 and 10. | 1, 4, 9 |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–11

| | |
|---|---|
| **Learning Target** | Operations with Whole Numbers (Number Strand: 8.N.1, N.7) |
| **Strategies of Focus** | Various |

| **Prior Learning** | **Answers** |
|---|---|
| 1. Place the following integers from lowest to highest: 3, −2, 1, −5, −1 | −5, −2, −1, 1, 3 |
| 2. Use <, >, or =: −14 _______ −15 | −14 > −15 |
| 3. What are all of the factors of 27? | 1, 3, 9, 27 |
| 4. What are the prime factors of 60? | 2, 3, 5 |
| 5. What are the first four multiples of 7? | 7, 14, 21, 28 |
| 6. What are the common factors of 16 and 24? | 1, 2, 4, 8 |
| **Grade 8 Questions** | |
| 7. Use <, >, or =: $\sqrt{1600}$ ______ $7 \times 7$ | 40 < 49 |
| 8. Which numbers are NOT perfect squares? 16, 18, 9, 34, 144 | 18, 34 |
| 9. Solve and simplify: $-1 \times \sqrt{37 + 9 + 3}$ | −7 |
| 10. Solve and simplify: $\sqrt{64} \div (-2)$ | −4 |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

**A–12**

| | |
|---|---|
| **Learning Target** | Operations with Whole Numbers (Number Strand: 8.N.1, N.7) |
| **Strategies of Focus** | Place-Value Partitioning |

| Prior Learning | Answers |
|---|---|
| 1. What is Tim's salary if he earns \$901 per paycheque for 40 weeks? | \$36,040 |
| 2. Calculate: $2 + 29\,400 \div 7$ | 4202 |
| 3. A construction crew of 15 people earns \$45,165 for building a home. How much does each receive? | \$3011 |
| 4. The price of a box of doughnuts is \$5.79. Can this cost be divided evenly among three students? | Yes, if paid by debit, credit, or cheque. No, if by cash (\$1.93 each). |
| 5. How many cm are in $\frac{1}{2}$ a kilometre? | 50 000 cm |
| 6. Betty goes into debt \$4 each day. When she reaches −\$1000, the bank will cancel her credit card. How long does she have? | 250 days |
| **Grade 8 Questions** | |
| 7. You owe \$126 and pay it off in weekly payments of \$6. How long until you have paid off your debt? | 21 weeks |
| 8. Solve: $354 \times (-3)$ | −1062 |
| 9. Solve and simplify: $\sqrt{-256 \div (-4)}$ | 8 |
| 10. Temperatures over 4 days were −5°C, 0°C, 2°C, and −9°C. What is the average? | −3°C |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–13

| | |
|---|---|
| **Learning Target** | Representation of Rational Numbers (Number Strand: 8.N.4) |
| **Strategies of Focus** | Doubling and/or Halving |

| **Prior Learning** | **Answers** |
|---|---|
| For questions 1 to 6, solve as indicated. | |
| 1. $14 \times 35$ | $7 \times 70 = 490$ |
| 2. $144 \div 4$ | $144 \div 2 \div 2 = 36$ |
| 3. $\frac{1}{2} \times \frac{6}{8}$ | $1 \times \frac{3}{8} = \frac{3}{8}$ |
| 4. $15 \times 16$ | $30 \times 8 = 240$ |
| 5. $250 \times 36$ | $500 \times 18 = 1000 \times 9 = 9000$ |
| 6. $1.25 \times 24$ | $5 \times 6 = 30$ |
| **Grade 8 Questions** | |
| 7. Find unit rate: A snowmobile travels 400 km in 8 hours. | 50 km/h |
| 8. Express as a ratio in lowest form: 30 kids to 2 adults | 15 : 1 |
| 9. A pancake recipe uses 3 cups of flour and 4 eggs. How many eggs are needed when $4\frac{1}{2}$ cups of flour are used? | 6 eggs |
| 10. A chili recipe uses 6 jalapeños, 4 cans of tomato sauce, and 2 green peppers. How many jalapeños are needed in a batch using 6 cans of tomato sauce? | 9 jalapeños |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

**A–14**

| | |
|---|---|
| **Learning Target** | Representation of Rational Numbers (Number Strand: 8.N.4) |
| **Strategies of Focus** | Rounding Strategies |

| Prior Learning | Answers |
|---|---|
| 1. Estimate: $2\frac{1}{8} \times 3\frac{7}{9}$ | $\approx 8$ |
| 2. Estimate: $211.15 \div 3$ | $\approx 70$ |
| 3. Estimate: $4.1 \times 24.86$ | $\approx 100$ |
| 4. Which will be the greatest?<br>$4.3 \times 3$  $3 \times 4\frac{1}{3}$  $25.75 \div 2$  $1.5(7 \times 0.5)$ | $3 \times 4\frac{1}{3}$ |
| 5. Order the following from least to greatest: $200\%, \frac{9}{4}, 0.256, 0.2501, \frac{4}{9}$ | $0.2501, 0.256$<br>$\frac{4}{9}, 200\%, \frac{9}{4}$ |
| 6. Round to the nearest tenth:<br>fifty-four thousand, seven hundred two, and forty-five hundredths | 54 702.5 |
| **Grade 8 Questions** | |
| 7. What is the cost per sock if six pairs are on sale for \$17.99? | \$1.50 per sock |
| 8. Which is the better deal?<br>five hockey pucks for \$21.97 or eight for \$29.99 | 8 for \$29.99 |
| 9. Which is the better deal?<br>a dozen mini-doughnuts for \$3.07 or 25¢ each | 25¢ each |
| 10. Which is slower?<br>1.345 metres per second or 900 kilometres per hour | 1.345 m/s |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–15

| | |
|---|---|
| **Learning Target** | Representation of Rational Numbers (Number Strand: 8.N.4) |
| **Strategies of Focus** | Memorization |

| **Prior Learning** | **Answers** |
|---|---|
| 1. Express as a part-to-part ratio: 4 cans of water, 1 can of juice | $4:1$ |
| 2. Express as a part-to-whole ratio: 4 cans of water, 1 can of juice | $4:5$ |
| 3. Express $\frac{1}{4}$ as a percent and a decimal to two decimal points. | 25%, 0.25 |
| 4. Express $\frac{1}{10}$ as a percent and a decimal to two decimal points. | 10%, 0.10 |
| 5. Express $\frac{1}{20}$ as a percent and a decimal to two decimal points. | 5%, 0.05 |
| 6. Express $\frac{1}{3}$ as a percent and a decimal to two decimal points. | $\approx$ 33%, 0.33 |
| **Grade 8 Questions** | |
| 7. Is $\frac{2}{3}$ closer to 0.66 or 0.67? | 0.67 |
| 8. From the set of whole numbers from 1 to 9, what is the ratio of prime numbers to composite numbers in lowest terms? | $1:1$ |
| 9. Write each part-to-part ratio as a fraction of the whole in simplest terms: $4:6$, $2:30$, $1:9$ | $\frac{2}{5}, \frac{1}{16}, \frac{1}{10}$ |
| 10. Write each part-to-whole ratio as a fraction in simplest terms: $4:6$, $2:30$, $1:9$ | $\frac{2}{3}, \frac{1}{15}, \frac{1}{9}$ |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–16

| | |
|---|---|
| **Learning Target** | Representation of Rational Numbers (Number Strand: 8.N.4) |
| **Strategies of Focus** | Halving and/or Doubling |

| **Prior Learning** | **Answers** |
|---|---|
| 1. Ben drives down the Number 1 Highway at 100 km/h. How far will he travel in one hour? | 100 km |
| 2. How far will he travel in half an hour? | 50 km |
| 3. How far will he drive in fifteen minutes? | 25 km |
| 4. How far will he drive in ten and one-half hours? | 1050 km |
| 5. There are 24 teachers and 400 students in Dawson Trail School. What is the teacher-to-student ratio? | 3 : 50 |
| 6. The distance from Long Plain to Portage la Prairie is 25 km. How long will it take you to bike if you travel at 12.5 km/h? | 2 hours |
| **Grade 8 Questions** | |
| In your grocery cart, there are 2.8 kg of apples, 2.5 kg of oranges, 5.5 kg of potatoes, 3.2 kg of cereal, 0.4 kg of salami, and 4.2 kg of watermelon. Answer questions 7 to 10 using this information. | |
| 7. What is the ratio, in simplest form, of fruit to vegetables? | 19 : 11 |
| 8. What is the ratio of cereal to salami? | 8 : 1 |
| 9. What is the ratio of oranges to potatoes? | 5 : 11 |
| 10. What is the ratio of potatoes to watermelon to salami? | 55 : 42 : 4 |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

**A–17**

| | |
|---|---|
| **Learning Target** | Representation of Rational Numbers (Number Strand: 8.N.4) |
| **Strategies of Focus** | Finding Compatible Numbers |

| **Prior Learning** | **Answers** |
|---|---|
| 1. Solve: $\frac{1}{5}+\frac{3}{4}+\frac{4}{5}$ | $1\frac{3}{4}$ |
| 2. Solve: $\frac{1}{2}+\frac{3}{5}+1\frac{1}{2}+\frac{5}{25}$ | $2\frac{4}{5}$ |
| 3. Eve spends 28% of her day sleeping, 32% at school, 6% doing homework, and 4% doing chores. What percentage of time does she have left in a day? | 30% |
| 4. Solve: $\frac{1}{6}+7\frac{5}{6}-\frac{5}{7}$ | $7\frac{2}{7}$ |
| 5. Jaylow walks 1.64 km to school, 2.36 km at recesses, and 1.64 km home. How many km does she walk? | 5.64 km |
| 6. A brand of cereal is 17% sugar, 40% flour, and 3% tuna. What is the ratio, in lowest terms, of sugar and tuna to all other ingredients? | 1 : 4 |
| **Grade 8 Questions** | |
| 7. You earn \$10/hr. cleaning rides at the fair. From Monday to Saturday, you work 4.5 hr., 3.75 hr., 2.5 hr., 7 hr., 7.25 hr., and 8 hr. How much do you earn? | \$330 |
| 8. Your dad gives you \$16/week for helping with farm work. If the chores take you 15 minutes every weekday morning, 30 minutes after school every day, $3\frac{1}{2}$ hours on Saturday, and 45 minutes on Sunday, what is your hourly pay? | \$2/hour |
| 9. A tree grows 21.7 cm in spring, 16.3 cm in summer, 7.6 cm in fall, and 2.4 cm in winter. What is its average growth rate per month this year? | 4 cm per month |
| 10. A mall parking lot has 25 trucks, 59 vans, 25 bicycles, 92 cars, 8 motorcycles, and 41 SUVs. What is the ratio of motorized to non-motorized vehicles? | 9 : 1 |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–18

| | |
|---|---|
| **Learning Target** | Representation of Rational Numbers (Number Strand: 8.N.4) |
| **Strategies of Focus** | Benchmarks |

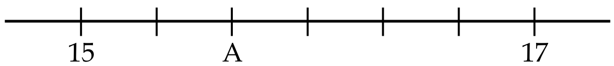
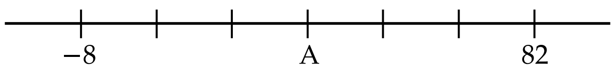
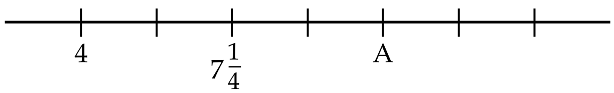
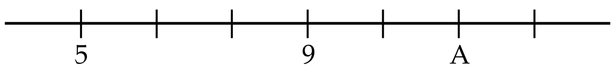
| **Prior Learning** | **Answers** |
|---|---|
| 1. Which is closer to $\frac{1}{2}$? $\frac{1}{3}$ 0.33 33% 67% | $\frac{1}{3}$ |
| 2. Which is closer to 45%? $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{3}$ $\frac{2}{5}$ | $\frac{2}{5}$ |
| 3. Who scored closer to the class average of 75% on the test? Zander: $\frac{40}{60}$ Malik: $\frac{44}{60}$ Jarak: $\frac{55}{60}$ | Malik |
| 4. Order the following from least to greatest: $\frac{3}{7}$ $\frac{1}{2}$ $\frac{9}{5}$ $\frac{5}{9}$ $\frac{1}{11}$ | $\frac{1}{11}$ $\frac{3}{7}$ $\frac{1}{2}$ $\frac{5}{9}$ $\frac{9}{5}$ |
| 5. Which fraction is closest to 4.2? $4\frac{5}{7}$ $\frac{9}{2}$ $\frac{4}{2}$ $4\frac{1}{10}$ | $4\frac{1}{10}$ |
| 6. Which of the following fractions is equivalent to a repeating decimal? $2\frac{7}{8}$ $\frac{4}{6}$ $\frac{3}{6}$ $\frac{9}{6}$ | $\frac{4}{6}$ |
| **Grade 8 Questions** | |
| 7. Which is the better deal? \$3 for four of your favourite songs, or \$2 for three of your favourite songs | \$2 for 3 |
| 8. Who is faster? James Waabooz ran 4 km in 9 minutes; Shirley Mikinaakose ran 5 km in 10 minutes | Shirley Mikinaakose |
| 9. Put the following in order from fastest to slowest: a snail at 2 km in 11 hours, a turtle at 3 km in 12 hours, a goldfish at 1 km in 2 hours, and a three-legged horse at 7 km in 16 hours | goldfish, horse, turtle, snail |
| 10. Mr. Cone cuts lawns for 8 hours and gets \$70 and Mrs. Cube works in a store and earns \$10/hr. Is Mr. Cone making more or less than Mrs. Cube per hour? | Less |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

**A–19**

| | |
|---|---|
| **Learning Target** | Representation of Rational Numbers (Number Strand: 8.N.4) |
| **Strategies of Focus** | Various |

| **Prior Learning** | **Answers** |
|---|---|
| For questions 1 to 4, find the value of A. | |
| 1.   | $15\frac{2}{3}$ |
| 2.   | 37 |
| 3.   | $10\frac{1}{2}$ |
| 4.   | $11\frac{2}{3}$ |
| 5. What fraction of a dollar is a quarter? | $\frac{1}{4}$ |
| 6. Six out of the 25 students in this math class will mistakenly wear their grandmothers’ clothes to school every Tuesday. What is the percentage of students who continually make this mistake? | 24% |
| **Grade 8 Questions** | |
| 7. Find unit rate: a snowmobile travels 400 km in 8 hours | 50 km/h |
| 8. Is the following a rate or a ratio? 30 kids to 2 adults | Ratio |
| 9. Is the following a rate or a ratio? 10 miles in 2 hours | Rate |
| 10. A map has a scale of 1 : 100 000. How far is it between Winnipeg and Thompson if the distance on the map is 7.5 cm? | 7.5 km |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–20

| | |
|---|---|
| **Learning Target** | Representation of Rational Numbers (Number Strand: 8.N.4) |
| **Strategies of Focus** | Working from left to right |

| Prior Learning | Answers |
|---|---|
| 1. Calculate: 0.124 + 0.264 | 0.388 |
| 2. Calculate: 12.4 + 13.5 + 68.5 | 94.4 |
| 3. Calculate: 146 + 78 + 101 | 325 |
| 4. Calvin gets paid $10/hr. to play online video games. He plays for 15.4 minutes, then for 14.2 minutes, and then for 10.4 minutes. How much should he get paid? | $6.67 |
| 5. After a party, 80% of a cheese pizza, 75% of a pepperoni pizza, and 45% of an anchovy pizza are left. How many full pizzas can be made from the leftovers? | 2 pizzas |
| 6. Tickets to the concert cost $10 plus 15% tax. Does Ivan have enough if he finds the following amounts under couch cushions: $3.25, $4.15, $1.60, and $2.35? | No (He needs 15 cents more.) |
| **Grade 8 Questions** | |
| 7. Teams in a 10-hour paintball tournament use 2436, 1253, and 1011 paintballs each. How many paintballs did they use per hour? | 470 paintballs/hr. |
| 8. The distances from East Kildonan to Point Douglas to the Maples and back are 5.21 km, 8.15 km, and 12.64 km. How long does it take to bike this trip at 13 km/h? | 2 hours |
| 9. Mallory spends the following amounts of time on her phone in one day: 18 minutes, $16\frac{1}{2}$ minutes, $18\frac{1}{2}$ minutes, $16\frac{1}{4}$ minutes, 27 minutes, and $23\frac{3}{4}$ minutes. What is the ratio of time Mallory spends on her phone to the time she spends away from her phone? | 1 : 11 |
| 10. Ty's penalty minutes in a hockey game were 4:00, 2:15, and 3:45. The game was 60 minutes long. What was the ratio of time spent in the penalty box to total game time? | 1 : 6 |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

**A–21**

| | |
|---|---|
| **Learning Target** | Operations with Rational Numbers (Number Strand: 8.N.2, N.3, N.5, N.6, N.8) |
| **Strategies of Focus** | Estimation |

| **Prior Learning** | **Answers** |
|---|---|
| 1. What two whole numbers will $7\frac{2}{5} + 9\frac{9}{11}$ be between? | 17 and 18 |
| 2. What two whole numbers will $16\frac{1}{7} + 21\frac{5}{11}$ be between? | 37 and 38 |
| 3. Will $5\frac{2}{5} + 13\frac{1}{3}$ be closer to 18 or 19? | 19 |
| 4. Will $\frac{42}{5} + \frac{7}{8}$ be closer to 9 or 10? | 9 |
| 5. Will $15\frac{2}{5} - 1\frac{1}{3}$ be closer to 13 or 14? | 14 |
| 6. Will $\frac{12}{5} - 1\frac{1}{3}$ be closer to 0 or 1? | 1 |
| **Grade 8 Questions** | |
| 7. Is $\sqrt{34}$ closer to 5 or to 6? | 6 |
| 8. Estimate: $\sqrt{17}$ | $\approx 4.1$ |
| 9. $\sqrt{n}$ is between 9 and 10. What whole number might $n$ equal? | $82 \le n \le 99$ |
| 10. $\sqrt{n}$ is between 6 and 6.5. What whole number might $n$ equal? | $37 \le n \le 42$ |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–22

| | |
|---|---|
| **Learning Target** | Operations with Rational Numbers (Number Strand: 8.N.2, N.3, N.5, N.6, N.8) |
| **Strategies of Focus** | Estimation |

| **Prior Learning** | **Answers** |
|---|---|
| 1. The drive from Gimli to Emerson is 200 km. How long will it take to drive this at 100 km/h? | 2 h |
| 2. The drive from Melita to Gretna is 300 km. About how long will it take to drive this at 80 km/h? | $3\frac{1}{2}$ to 4 h (3 h, 45 min.) |
| 3. The drive from Minot to Grand Forks is 210 miles. About how long will it take to get there driving 68 miles per hour? | ≈ 3 hr. |
| 4. How fast do you bike in km/h if it takes you 4 minutes to bike a kilometre? | 15 km/h |
| 5. Estimate the time each Grade 8 student spends in math class this month. | ≈ 16 to 22 hours |
| 6. Estimate the value of a stack of quarters 1 metre tall. A roll of quarters is 7 cm long and worth $10. | ≈ $135 to $145 |
| **Grade 8 Questions** | |
| 7. What two whole numbers is $\sqrt{30}$ between? | 5 and 6 |
| 8. Less than, greater than, or equal? 8 _____ $\sqrt{80}$ | $<$ |
| 9. Less than, greater than, or equal? 9 _____ $\sqrt{99}$ | $<$ |
| 10. Less than, greater than, or equal? 11 _____ $\sqrt{66}$ | $>$ |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–23

| | |
|---|---|
| **Learning Target** | Operations with Rational Numbers (Number Strand: 8.N.2, N.3, N.5, N.6, N.8) |
| **Strategies of Focus** | Estimation |

| **Prior Learning** | **Answers** |
|---|---|
| 1. The distance from The Pas to Winnipeg is 628 km. At an average speed of 80 km/h, about how long would this trip take? | Slightly less than 8 hours |
| 2. Four classes with 21 students each need to take a field trip to the box factory. Each bus holds 25 students. How many buses are needed? | 4 buses |
| 3. Sales taxes in Manitoba are 8% PST and 5% GST. What is the approximate total cost for a new phone priced at $99? | ≈ $113 |
| 4. You received a terrible haircut. You want to tip 5% or slightly less. The final bill is $20.99. What should you tip? | Below $1.05 |
| 5. Beth is a great waitress. She expects a 15% tip from a man who had a meal worth $12.35. He left her $1.30. Is this reasonable? | Not really, as this is under 11% |
| 6. Estimate the cost: 10% off a shirt worth $29.97 | ≈ $27 |
| **Grade 8 Questions** | |
| 7. Less than, greater than, or equal? $\sqrt{20}$ _____ 4.9 | < |
| 8. Less than, greater than, or equal? $\sqrt{96}$ _____ $9\frac{1}{10}$ | > |
| 9. Less than, greater than, or equal? $\sqrt{625}$ _____ 25 | = |
| 10. Less than, greater than, or equal? 3 _____ $\sqrt{6}$ | > |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

**A–24**

| | |
|---|---|
| **Learning Target** | Operations with Rational Numbers (Number Strand: 8.N.2, N.3, N.5, N.6, N.8) |
| **Strategies of Focus** | Rounding |

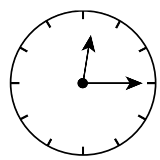
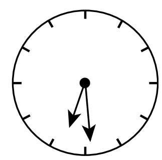
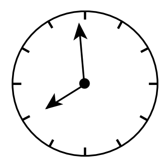
| **Prior Learning** | **Answers** |
|---|---|
| 1. Which of the following is closest to $17.58 \times 13.40$? <br> 228 3228.58 28.5814 428 | 228 |
| 2. Solve $1.5 \times 1.5$ to the nearest tenth. | 2.3 |
| 3. Estimate the value of a 9% tip on a \$21.99 haircut. | ≈ \$2 (\$1.98) |
| 4. A bag of raisins weighs 885 grams. Kyle needs 300 g to make cookies for a party starting in 12 minutes. If $\frac{1}{3}$ of a bag is all he has, should he try to get to the store before his party? | No, he has nearly enough. |
| 5. If Ana completes $\frac{3}{8}$ of her homework, and Sara completes $\frac{4}{9}$, who is closer to being done? | Sara |
| 6. A recipe calls for 500 mL of tomato paste. Gerri wants to triple the recipe. Cans only come in 355 mL. How many cans will she need? | 5 cans |
| **Grade 8 Questions** | |
| 7. Less than, greater than, or equal? $\sqrt{1}$ _____ 1 | = |
| 8. Less than, greater than, or equal? $\sqrt{2}$ _____ 2 | < |
| 9. Less than, greater than, or equal? $\sqrt{11}$ _____ 66 | < |
| 10. Less than, greater than, or equal? $\sqrt{7} \times \sqrt{7}$ _____ $\sqrt{49}$ | = |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

**A–25**

| | |
|---|---|
| **Learning Target** | Operations with Rational Numbers (Number Strand: 8.N.2, N.3, N.5, N.6, N.8) |
| **Strategies of Focus** | Memorization |

| **Prior Learning** | **Answers** |
|---|---|
| 1. How many months have 30 days each year? | 4 |
| 2. How many days are there in most years? | 365 |
| 3. What meal time does each most closely represent?   | Lunch<br>Supper |
| 4. What time does this represent?  | $\approx$ 7:58 |
| 5. Is 10:00 AM in the morning or at night? | Morning |
| 6. Calculate: $-9 \times 9$ | $-81$ |
| **Grade 8 Questions** | |
| 7. Simplify: $\sqrt{\sqrt{16}}$ | 2 |
| 8. Solve and simplify: $\sqrt{\sqrt{25} + \sqrt{16}}$ | 3 |
| 9. Solve and simplify: $\dfrac{\sqrt{100}}{\sqrt{4}}$ | 5 |
| 10. Solve and simplify: $\sqrt{\dfrac{100}{4}}$ | 5 |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–26

| | |
|---|---|
| **Learning Target** | Operations with Rational Numbers (Number Strand: 8.N.2, N.3, N.5, N.6, N.8) |
| **Strategies of Focus** | Various |

| **Prior Learning** | **Answers** |
|---|---|
| 1. Jim watches 12% of a movie, then 14%, then 16%, and then 28%. How much is left to watch? | 30% |
| 2. Audrey ate $\frac{1}{4}$ of a chocolate bar, then $\frac{3}{8}$, and then another $\frac{1}{4}$. How much is left? | $\frac{1}{8}$ |
| 3. Solve: $-33 + 19$ | $-14$ |
| 4. If it is 5°C in Boissevain and −17°C in Brochet, what is the temperature difference? | 22°C |
| 5. Solve: $34 \div 0.5$ | 68 |
| 6. If one gummy worm costs \$0.29, how much do 100 gummy worms cost? | \$29 |
| **Grade 8 Questions** | |
| For questions 7 to 10, solve and simplify. | |
| 7. $\sqrt{4} \times \sqrt{4}$ | 4 |
| 8. $\sqrt{81} \times \sqrt{81}$ | 81 |
| 9. $\sqrt{73.5} \times \sqrt{73.5}$ | 73.5 |
| 10. $\sqrt{a} \times \sqrt{a}$ | $a$ |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–27

| | |
|---|---|
| **Learning Target** | Operations with Rational Numbers (Number Strand: 8.N.2, N.3, N.5, N.6, N.8) |
| **Strategies of Focus** | Various |

| **Prior Learning** | **Answers** |
|---|---|
| 1. What whole number is $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$ closest to? | 1 |
| 2. One-fifth of the 55 Grade 12 students have cars. What percentage is this? | 20% |
| 3. If you run at 4 m/s for 400 seconds, how many kilometres will you run? | 1.6 km |
| 4. William needs 80% on his next math test. If it's out of 60 marks, how many marks does he need? | 48 marks |
| 5. Solve: $\frac{2}{5}+\frac{57}{10}$ | $6\frac{1}{10}$ |
| 6. Solve: $\frac{3}{5}-\frac{8}{25}$ | $\frac{7}{25}$ |
| **Grade 8 Questions** | |
| 7. Estimate the cost: 10% off a shirt worth $49.23 | ≈ $44 ($44.31) |
| 8. What is $\frac{1}{3}$ of a half of a pizza? | $\frac{1}{6}$ |
| 9. What percent is 16 cents of 2 dollars? | 8% |
| 10. What percent is 16 cents of 50 cents? | 32% |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–28

| | |
|---|---|
| **Learning Target** | Operations with Rational Numbers (Number Strand: 8.N.2, N.3, N.5, N.6, N.8) |
| **Strategies of Focus** | Various |

| **Prior Learning** | **Answers** |
|---|---|
| 1. Evaluate: $43 \div 0.5$ | 86 |
| 2. How many cm are equivalent to 63 mm? | 6.3 cm |
| 3. A pair of $200 hockey skates is on sale for 40% off. What is the cost? | $120 |
| 4. If you read 140 pages of a 338-page book, how many more do you have to read? | 198 pages |
| 5. If you buy an item costing $8.78 with a $10 bill, how much change will you get in Canada? | $1.20 |
| 6. If you buy an item costing $9.31 with a $10 bill, how much change will you get in Canada? | 70¢ |
| **Grade 8 Questions** | |
| 7. What is the area of a $8\frac{1}{2}$-inch by 11-inch piece of paper? | $93\frac{1}{2}$ inches$^2$ |
| 8. How many half-dozen packages of eggs does it take to get 360 eggs? | 60 packages |
| 9. Solve and simplify: $\sqrt{64}+(-10)$ | $-2$ |
| 10. Solve and simplify: $-\sqrt{144}\times -10$ | 120 |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–29

| | |
|---|---|
| **Learning Target** | Operations with Rational Numbers (Number Strand: 8.N.2, N.3, N.5, N.6, N.8) |
| **Strategies of Focus** | Various |

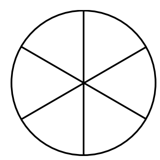
| **Prior Learning** | **Answers** |
|---|---|
| 1. If you buy an item worth \$55.55 and pay with 3 twenty-dollar bills, how much change will you get in Canada? | \$4.45 |
| 2. If you buy an item worth \$134.17 and pay with 3 fifty-dollar bills, how much change will you get in Canada? | \$15.85 |
| 3. If you buy an item worth \$134.17 and pay with 3 fifty-dollar bills, how much change will you get in North Dakota? | \$15.83 |
| 4. Beth is 42 years old. Her son, Spencer, is 14 years old. How old was Beth when she had Spencer? | 27 or 28 years old |
| 5. Which of the following numbers are divisible by 7? 98 119 784 77 | All of them |
| 6. Solve: $-42 + (-8)$ | $-50$ |
| **Grade 8 Questions** | |
| 7. Ken needs $\frac{2}{3}$ of a cup of milk to make a batch of pancakes. He only has $\frac{2}{5}$ of a cup. Does he have enough to make a half-batch? | Yes |
| 8. Solve: $\frac{3}{4} \div \frac{1}{2}$ | $1\frac{1}{2}$ |
| 9. Harli gets 5% of farm earnings for helping on her family farm. How much does she get when the farm earns \$400? | \$20 |
| 10. Solve: $\frac{5}{7} \times \frac{14}{10}$ | 1 |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–30

| | |
|---|---|
| **Learning Target** | Operations with Rational Numbers (Number Strand: 8.N.2, N.3, N.5, N.6, N.8) |
| **Strategies of Focus** | Various |

| **Prior Learning** | **Answers** |
|---|---|
| 1. What is 75% of 4? | 3 |
| 2. How much is 80% of 50? | 40 |
| 3. How many slices in $\frac{1}{4}$ of this pizza?  | $1\frac{1}{2}$ slices |
| 4. How many slices are there in $2\frac{1}{2}$ of these pizzas? | 15 slices |
| 5. How many slices in $\frac{7}{12}$ of these pizzas? | $3\frac{1}{2}$ slices |
| 6. How many slices in 75% of a pizza? | $4\frac{1}{2}$ slices |
| **Grade 8 Questions** | |
| 7. Solve: $\frac{3}{4} \times \frac{3}{4}$ | $\frac{9}{16}$ |
| 8. What is 25% of $2\frac{1}{2}$? | $\frac{5}{8}$ |
| 9. Solve: $2 \times \frac{1}{4}$ | $\frac{1}{2}$ |
| 10. Solve: $2 \times 3\frac{1}{4}$ | $6\frac{1}{2}$ |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–31

| | |
|---|---|
| **Learning Target** | Operations with Rational Numbers (Number Strand: 8.N.2, N.3, N.5, N.6, N.8) |
| **Strategies of Focus** | Various |

| **Prior Learning** | **Answers** |
|---|---|
| 1. What is the price of 3 apps at \$1.99 each? | \$5.97 |
| 2. Transcona's hockey team has 15 players. Five are taller than 2 m. What fraction does this represent? | $\frac{1}{3}$ |
| 3. Is $-48.7 \div -3$ positive or negative? | Positive |
| 4. Solve: $\frac{1}{5} + \frac{1}{4}$ | $\frac{9}{20}$ |
| 5. MegaFürniture Store sells 305 skrivbord kits from Monday to Friday. How many units is this per day, on average? | 61 kits |
| 6. Write $\frac{4}{5}$ as a decimal. | 0.8 |
| **Grade 8 Questions** | |
| 7. Solve: $\frac{1}{4} \times \frac{1}{4}$ | $\frac{1}{16}$ |
| 8. Solve: $\frac{3}{4} \times \frac{1}{4}$ | $\frac{3}{16}$ |
| 9. What whole numbers will $3\frac{2}{5} \times 2\frac{1}{2}$ be between? | 8 and 9 |
| 10. Solve and simplify: $\sqrt{\frac{5}{2} \div \frac{1}{10}}$ | 5 |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–32

| | |
|---|---|
| **Learning Target** | Patterning and Algebraic Thinking (Patterns and Relations: 8.PR.1) |
| **Strategies of Focus** | Various |

| **Prior Learning** | **Answers** |
|---|---|
| 1. What number, increased by five, gives two? | –3 |
| 2. What number, doubled, gives sixteen? | 8 |
| 3. The sum of what number and five gives negative ten? | –15 |
| 4. The sum of what number and negative five gives ten? | 15 |
| 5. The sum of what number and negative five gives negative ten? | –5 |
| 6. The product of what number and negative eight gives sixteen? | –2 |
| **Grade 8 Questions** | |
| For questions 7 to 10, solve for $y$, where $x = 5$. | |
| 7. $2x + 1 = y$ | $y = 11$ |
| 8. $y = 2x + 1$ | $y = 11$ |
| 9. $3(2x + 1) = y$ | $y = 33$ |
| 10. $3(2x + 1) = 3y$ | $y = 11$ |
| **Other Questions** | |
| 11. | |
| 12. | |

# Mental Math

## Grade 8 Mathematics

A–33

| | |
|---|---|
| **Learning Target** | Patterning and Algebraic Thinking (Patterns and Relations: 8.PR.1) |
| **Strategies of Focus** | Distribution and Commutative Property |

| **Prior Learning** | **Answers** |
|---|---|
| 1. What number, divided by three, gives fifteen? | 45 |
| 2. Does $2x + 7$ always equal $7 + 2x$? | Yes |
| 3. Does $2x - 7$ always equal $7 - 2x$? | No |
| 4. Does $(2x) \times 7$ always equal $7 \times 2x$? | Yes |
| 5. Does $2x \div 7$ always equal $7 \div 2x$? | No |
| 6. Does $\frac{2x}{7}$ always equal $2x \div 7$? | Yes |

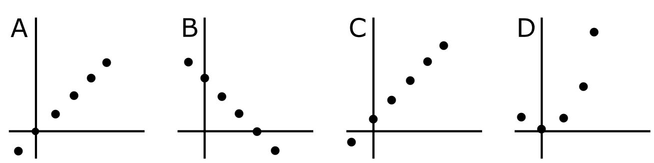
**Grade 8 Questions**

7. Complete the table of values for $y = 3x + 4$.

| $x$ | −10 | 1 | 10 | 50 |
|---|---|---|---|---|
| $y$ | | | | |

Answer: −26, 7, 34, 154

8. Which graph would best match $y = 3x + 4$?




Answer: C

9. Does $3x + 4 = 3(x + 4)$? How do the brackets make a difference?

Answer: $3(x + 4) = 3x + 12$
Distribution

10. Solve for $x$: $3(x - 1) = 9$

Answer: $x = 4$

**Other Questions**

11.

12.

# *Mental Math*

## Grade 8 Mathematics

A–34

| | |
|---|---|
| **Learning Target** | Patterning and Algebraic Thinking (Patterns and Relations: 8.PR.1) |
| **Strategies of Focus** | Distribution and Commutative Property |

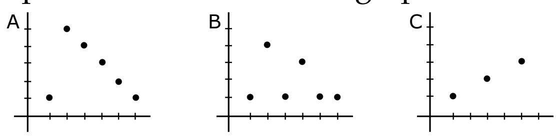
| Prior Learning | Answers |
|---|---|
| 1. Is $2x + 8 - 9 = (2x + 8) - 9$? Why? | Yes, the commutative property. |
| 2. Is $x - 9$ always equal to $x + (-9)$? | Yes |
| 3. Find the error for the equation $2(m + 7) = r$. <br> \| $m$ \| 0 \| 3 \| 6 \| 10 \| <br> \| $r$ \| 0 \| 20 \| 26 \| 34 \| | $m = 0$ <br> $r = 0$ |
| 4. Solve: $49 \div 7$ | 7 |
| 5. Solve: $7 \div 49$ | $\frac{1}{7}$ |
| 6. Solve: $0 \div 49$ | 0 |
| **Grade 8 Questions** | |
| 7. Complete the table of values for $2x = y$. <br> \| $x$ \| −3 \| 10 \| 50 \| 100 \| <br> \| $y$ \| \| \| \| \| | −6, 20, 100, 200 |
| 8. Will graphs of $y = 3x - 4$ and $y = -4 + 3x$ be parallel, perpendicular, intersecting, or will they be the same line? | Same line |
| 9. Will graphs of $y = 2(x - 4)$ and $y = 2x - 4$ be parallel, perpendicular, intersecting, or will they be the same line? | Parallel |
| 10. Will graphs of $y = 2x$ and $y = -2x$ be parallel, perpendicular, intersecting, or will they be the same line? | Perpendicular |
| **Other Questions** | |
| 11. | |
| 12. | |

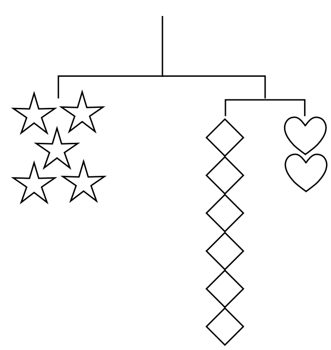
# *Mental Math*

## Grade 8 Mathematics

**A–35**

| | |
|---|---|
| **Learning Target** | Patterning and Algebraic Thinking (Patterns and Relations: 8.PR.1) |
| **Strategies of Focus** | Visualization |

| **Prior Learning** | **Answers** |
|---|---|
| For questions 1 to 6, use the graphs shown below.  | |
| 1. Which of the above graphs is a linear relation? | C |
| 2. Write the missing value for (___, 4) on graph A. | $y = 3$ |
| 3. Create a table of values for graph C. | $x$: 1, 3, 5; $y$: 1, 2, 3 |
| 4. What is the value for $y$ on graph B where $x = 4$? | $y = 3$ |
| 5. Find the missing value for (5, ___) on graph B. | $x = 1$ |
| 6. Which graph could represent the growth of a tomato plant over time? | C |
| **Grade 8 Questions** | |
| 7. If a star = 12 and a diamond = 5, what does a heart equal? | Heart = 15 |
| 8. If a star = 24, what does a diamond equal? | Diamond = 10 |
| 9. If a diamond = $\frac{1}{2}$, what does a heart equal? | Heart = $1\frac{1}{2}$ |
| 10. Which shape is the lightest? | Diamond |
| **Other Questions** | |
| 11. | |
| 12. | |



# *Mental Math*

## Grade 8 Mathematics

**A–36**

| | |
|---|---|
| **Learning Target** | Algebraic Representations with Equations (Patterns and Relations: 8.PR.2) |
| **Strategies of Focus** | Various |

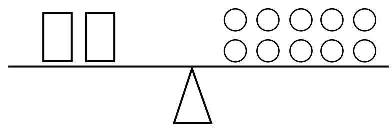
| **Prior Learning** | **Answers** |
|---|---|
| Use the diagram shown on the right to answer questions 1 to 6.  | |
| 1. How many circles are equal to 1 rectangle? | 5 circles |
| 2. How many rectangles are equal to 100 circles? | 20 rectangles |
| 3. How many circles are equal to 5 rectangles? | 25 circles |
| 4. How many rectangles will balance 25 circles? | 5 rectangles |
| 5. If you removed 3 circles from the right, how much of a rectangle would need to be removed for the scale to stay balanced? | $\frac{3}{5}$ of a rectangle |
| 6. If you add 9 circles to the left, how many rectangles do you need to add to the right to stay balanced? | $1\frac{4}{5}$ rectangles |
| **Grade 8 Questions** | |
| 7. Complete the table of values for $-2x = y$. | $-6$, $-20$, $-100$, $-200$ |
| 8. Solve for $x$: $\frac{x}{2} + 5 = 11$ | $x = 12$ |
| 9. If you add the same number to both sides of this equation, will $x$ change? $\frac{x}{2} + 5 + \square = 11 + \square$ | No |
| 10. If you subtract the same number from both sides of this equation, will $x$ change? Why or why not? $\frac{x}{5} + 2 - \square = 16 - \square$ | No, preservation of equality |
| **Other Questions** | |
| 11. | |
| 12. | |

Table for question 7:

| $x$ | 3 | 10 | 50 | 100 |
|---|---|---|---|---|
| $y$ | | | | |

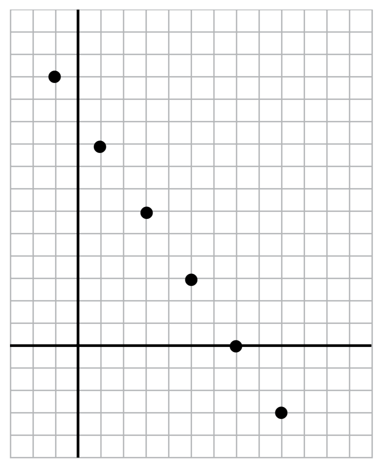
# Mental Math

## Grade 8 Mathematics

**A–37**

| | |
|---|---|
| **Learning Target** | Algebraic Representations with Equations (Patterns and Relations: 8.PR.2) |
| **Strategies of Focus** | Various |

<table>
<tr><th>Prior Learning</th><th>Answers</th></tr>
<tr><td>Use the graph shown below to answer questions 1 to 6.</td><td></td></tr>
<tr><td>1. Make a table of values for the graph.</td><td><table><tr><td>$x$</td><td>−1</td><td>1</td><td>3</td><td>5</td><td>7</td><td>9</td></tr><tr><td>$y$</td><td>12</td><td>9</td><td>6</td><td>3</td><td>0</td><td>−3</td></tr></table></td></tr>
<tr><td>2. What would be the value of $y$ for $x = 11$?</td><td>$y = -6$</td></tr>
<tr><td>3. What would be the value of $y$ for $x = 0$?</td><td>$y = 10\frac{1}{2}$</td></tr>
<tr><td>4. What is the missing value for the ordered pair (3,___)?</td><td>6</td></tr>
<tr><td>5. What is the missing value for the ordered pair (___, 3)?</td><td>5</td></tr>
<tr><td>6. Would a graph of $y = 6$ be parallel, perpendicular, or intersect this graph?</td><td>Intersect</td></tr>
<tr><th>Grade 8 Questions</th><th></th></tr>
<tr><td>7. Solve for $x$: $\frac{x}{30} = 3$</td><td>$x = 90$</td></tr>
<tr><td>8. Solve for $x$: $\frac{x}{2} + 3 = 11$</td><td>$x = 16$</td></tr>
<tr><td>9. Solve for $x$: $7(x + 3) = -49$</td><td>$x = -10$</td></tr>
<tr><td>10. Solve for $m$: $(13 - m) \div 2 = 6$</td><td>$m = 1$</td></tr>
<tr><th>Other Questions</th><th></th></tr>
<tr><td>11.</td><td></td></tr>
<tr><td>12.</td><td></td></tr>
</table>

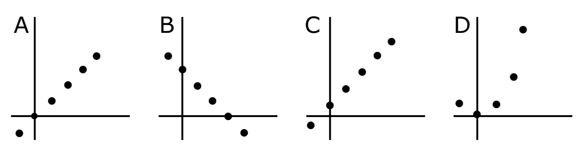


# *Mental Math*

## Grade 8 Mathematics

**A–38**

| | |
|---|---|
| **Learning Target** | Algebraic Representations with Equations (Patterns and Relations: 8.PR.2) |
| **Strategies of Focus** | Visualization and Estimation |

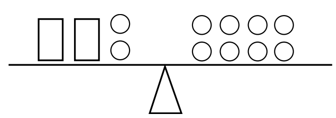
| **Prior Learning** | **Answers** |
|---|---|
| 1. Write the next three terms: −19, −13, −7, −1, ___, ___, ___ | 5, 11, 17 |
| 2. Find the missing number in the sequence: −10, ___, −16, −19 | −13 |
| 3. Find the missing number in the sequence: $\frac{3}{4}$, $1\frac{1}{4}$, ___, $2\frac{1}{4}$ | $1\frac{3}{4}$ |
| 4. Pile 1 has 4 blocks, pile 2 has 6 blocks, and pile 3 has 8 blocks. Will there be more than 100 blocks in pile 50? | Yes |
| 5. Wall 1 has 999 bottles, wall 2 has 851 bottles, and wall 3 has 703. Which wall will be the first to have fewer than 450 bottles? | Wall 5 |
| 6. Bowl 1 has 17 fish, bowl 2 has 25 fish, and bowl 3 has 33 fish. Will any bowl ever have an even number of fish? | No |
| **Grade 8 Questions** | |
| 7. Which graph shown on the right would best match $2x = y$?  | A |
| 8. Does the line $y = x$ pass through (1, 2)? | No |
| 9. Does the line $y = x$ pass through (4, 4)? | Yes |
| 10. Would $y = x + 10$ be closer to passing through (0, 9) or (12, 0)? | (0, 9) |
| **Other Questions** | |
| 11. | |
| 12. | |

# Mental Math

## Grade 8 Mathematics

**A–39**

| | |
|---|---|
| **Learning Target** | Algebraic Representations with Equations (Patterns and Relations: 8.PR.2) |
| **Strategies of Focus** | Visualization |

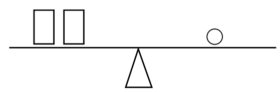
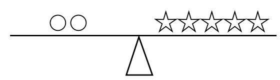
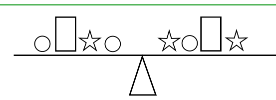
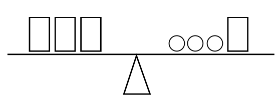
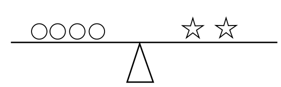
| **Prior Learning** | **Answers** |
|---|---|
| Write an equation for the word problems given in questions 1 to 5. | |
| 1. A number increased by 5 gives 2 | $x + 5 = 2$ |
| 2. Double a number makes 16 | $2x = 16$ |
| 3. A number divided by three equals fifteen | $\frac{x}{3} = 15$ |
| 4. Two-thirds of a number is 7 | $\frac{2}{3}x = 7$ |
| 5. Five is half of a number | $5 = \frac{1}{2}x$ |
| 6. How many circles are equal to one rectangle?  | 3 circles |
| **Grade 8 Questions** | |
| 7. What is the equation of the straight line passing through (0, 0) and (9, 9)? | $y = x$ |
| 8. What is the equation of the straight line passing through (0, 4) and (3, 4)? | $y = 4$ |
| 9. What is the equation of the straight line passing through (17, 2) and (17, 3)? | $x = 17$ |
| 10. Will a straight line passing through (1, 3) and (3, 5) also pass through (6, 7)? | No |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

**A–40**

| | |
|---|---|
| **Learning Target** | Algebraic Representations with Equations (Patterns and Relations: 8.PR.2) |
| **Strategies of Focus** | Visualization |

| **Prior Learning** | **Answers** |
|---|---|
| Use the diagrams shown on the right to answer questions 1 and 2.   | |
| 1. Which object is the heaviest? | Circle |
| 2. Which object is the lightest? | Star |
| Use the diagram shown on the right to answer questions 3 to 6.  | |
| 3. If the rectangle was removed from both sides, would the scale remain balanced? | Yes |
| 4. If four stars were added to each side, would the scale remain balanced? | Yes |
| 5. Which two objects weigh the same? | Star/Circle |
| 6. Is the rectangle heavier or lighter than the star? | There is no way to know. |
| **Grade 8 Questions** | |
| Use the diagrams shown on the right to answer questions 7 to 10.   | |
| 7. If a rectangle is equal to 3, how much is a circle equal to? | Circle = 2 |
| 8. If a rectangle is equal to 12, how much is a star equal to? | Star = 16 |
| 9. If a star is equal to 24, how much is a rectangle equal to? | Rectangle = 18 |
| 10. If a star is equal to 12, how much is a rectangle equal to? | Rectangle = 9 |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–41

| | |
|---|---|
| **Learning Target** | Length (Shape and Space: 8.SS.1) |
| **Strategies of Focus** | Estimation and Rounding |

| **Prior Learning** | **Answers** |
|---|---|
| 1. The sides of this piece of paper are $8\frac{5}{8}$ by 11 inches. I round $8\frac{5}{8}$ down to 8 and find an approximate perimeter of 38 inches. Will the actual perimeter be greater or less than 38 inches? | Greater |
| 2. If I round $8\frac{5}{8}$ to 9, I find an approximate perimeter of 40 inches. Will the actual perimeter of this paper be greater or less than 40 inches? | Less than |
| 3. What would be a reasonable estimation for the area of a circle with a radius of 3 cm? | $\approx 27\ cm^2$ |
| 4. What is the approximate volume of a 21 cm × 21 cm × 30 cm shoebox? Is this an overestimate or an underestimate? | $\approx 12\,000\ cm^3$ underestimate |
| 5. To the nearest metre, how wide is a car? | 2 metres |
| 6. Create a formula for finding the perimeter of a regular dodecagon (a 12-sided polygon). | Perimeter = 12 × side length |
| **Grade 8 Questions** | |
| 7. A computer screen is 7 inches by 9 inches. Is the diagonal greater or less than 10 inches? | Greater than 10 inches |
| 8. A front yard is 10 metres by 8 metres. Is the diagonal across it greater or less than 12 metres? | Greater than 12 m |
| 9. A TV screen is 50 cm by 80 cm. Is the diagonal greater or less than 100 cm? | Less than 100 cm |
| 10. A quilt is 2 metres by 2 metres. Is the diagonal greater or less than 3 metres? | Less than 3 metres |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–42

| | |
|---|---|
| **Learning Target** | Length (Shape and Space: 8.SS.1) |
| **Strategies of Focus** | Various |

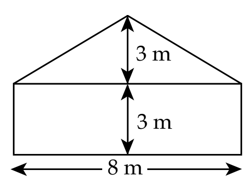
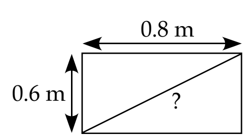
| **Prior Learning** | **Answers** |
|---|---|
| 1. What is the perimeter of a regular triangle if one side is 5 cm long? | 15 cm |
| 2. What is the name of a regular rectangle? | Square |
| 3. Is a regular triangle right, obtuse, or acute? | Acute |
| 4. Is a regular triangle isosceles, equilateral, or scalene? | Equilateral |
| 5. How many mm are in 15.3 cm? | 153 mm |
| 6. How many mm are in 205.64 cm? | 2056.4 mm |
| **Grade 8 Questions** | |
| 7. Two short sides of a right triangle are 3 and 4 units. How long is the hypotenuse? | 5 units |
| 8. If the lengths of the sides of the above triangle are all doubled, what happens to the measurements of the angles? | They remain the same. |
| 9. Two short sides of a right triangle are 6 and 8 units. How long is the hypotenuse? | 10 units |
| 10. The hypotenuse of a right triangle is 15 cm long, and one of the sides is 9 cm. How long is the missing side? | 12 cm |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

**A–43**

| | |
|---|---|
| **Learning Target** | Length (Shape and Space: 8.SS.1) |
| **Strategies of Focus** | Memorization and Visualization |

| **Prior Learning** | **Answers** |
|---|---|
| 1. What is a formula to find the area of a circle? | $A = \pi r^2$ or $A = \pi \times r \times r$ |
| 2. A circle has a diameter of 10 cm. What is the radius? | 5 cm |
| 3. What is the approximate value of pi to two decimal places? | 3.14 |
| 4. If you add the interior angles of any triangle, how many degrees will there be? | 180° |
| 5. What will the ratio of the circumference of a circle to its diameter always be equal to? | $\pi$ |
| 6. If you add the interior angles of any rectangle, how many degrees will there be? | 360° |
| **Grade 8 Questions** | |
| 7. A kite is 80 cm tall and 60 cm across. All of its sides are equal. What is its perimeter? | 200 cm |
| 8. Find the perimeter.   | 24 m |
| 9. The length of a hypotenuse of a right triangle is $\sqrt{5}$ cm. How long might the sides be? | 2 cm and 1 cm |
| 10. Find the diagonal.   | 1 m |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–44

| | |
|---|---|
| **Learning Target** | Area (Shape and Space: 8.SS.3) |
| **Strategies of Focus** | Place-Value Partitioning and Compensation |

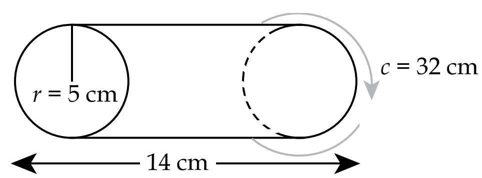
| **Prior Learning** | **Answers** |
|---|---|
| 1. What is the area of a square with sides of 12 m? | 144 $m^2$ |
| 2. What is the area of a rectangle with sides of 16 mm and 19 mm? | 304 $mm^2$ |
| 3. What is the approximate area of a circle with a radius of 10 cm? | ≈ 300 $cm^2$ (314 $cm^2$) |
| 4. What is the approximate area of a circle with a radius of 20 cm? | ≈ 1200 $cm^2$ (1257 $cm^2$) |
| 5. What is the volume of a 21 × 22 × 1 cm rectangular box? | 462 $cm^3$ |
| 6. What is the volume of a 15 × 9 × 2 unit rectangular prism? | 270 $units^3$ |
| **Grade 8 Questions** | |
| 7. What is the surface area of a 3 × 3 × 3 unit cube? | 54 $units^2$ |
| 8. If a cube has a surface area of 35 $cm^2$ on one side, what is its total surface area? | 210 $cm^2$ |
| 9. Fifteen sugar cubes each have a surface area of 5.5 $cm^2$. What is the total surface area of the sugar cubes? | 82.5 $cm^2$ |
| 10. What is the surface area of a 15 cm × 15 cm cube? | 1350 $cm^2$ |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–45

| | |
|---|---|
| **Learning Target** | Area (Shape and Space: 8.SS.3) |
| **Strategies of Focus** | Halving and Doubling |

| **Prior Learning** | **Answers** |
|---|---|
| 1. What is the area of a rectangle that has sides of 24 cm and 15 cm? | 360 $cm^2$ |
| 2. What is the area of a rectangle that has sides of 16 cm and 4 cm? | 64 $cm^2$ |
| 3. What is the area of a right triangle that has lengths of 5.2 cm and 4 cm for its two shortest sides? | 10.4 $cm^2$ |
| 4. The area of a rectangle is 128 $cm^2$. One of its sides is 8 cm. What is the missing side length? | 16 cm |
| 5. The sides of a rectangle are 160 cm and 250 cm. What is the area? | 40 000 $cm^2$ |
| 6. What is the volume of a $3 \times 3 \times 3$ unit cube? | 27 $units^3$ |
| **Grade 8 Questions** | |
| Use the diagram shown on the right to answer questions 7 to 10.   | |
| 7. Find the area of one of the two circles for the cylinder above. Use 3 for an approximation of pi. | ≈ 75 $cm^2$ |
| 8. What is the area of the curved surface? | 448 $cm^2$ |
| 9. What is the surface area of the cylinder? | ≈ 598 $cm^2$ |
| 10. What would the area of the curved surface be if the circumference were doubled and the length were halved? | 448 $cm^2$ |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–46

| | |
|---|---|
| **Learning Target** | Area (Shape and Space: 8.SS.3) |
| **Strategies of Focus** | Using Compatible Numbers |

| **Prior Learning** | **Answers** |
|---|---|
| 1. Distances walked in a day were 3.4 km, 2.9 km, 8.7 km, 2.1 km, and 1.3 km. What is the total distance walked? | 18.4 km |
| 2. The lengths of sides on an irregular pentagon are 2.6 cm, 3.4 cm, 6.7 cm, 9.0 cm, and 3.3 cm. What is the perimeter? | 25 cm |
| 3. The lengths of sides on a regular pentagon are each 7.5 cm. What is the perimeter? | 37.5 cm |
| 4. The lengths of sides on a regular hexagon are each 7.5 cm. What is the perimeter? | 45 cm |
| 5. The lengths of the sides on an irregular octagon are 61, 42, 54, 46, 39, 25, 58, and 15 cm. What is the perimeter? | 340 cm |
| 6. There are 32 patches on a soccer ball. Each has an area of 30 $cm^2$. What is the total area? | 960 $cm^2$ |
| **Grade 8 Questions** | |
| 7. A box with a surface area of 987 $cm^2$ has its top removed. Its top was 13 cm × 11 cm. What is the new exterior surface area? | 844 $cm^2$ |
| 8. A rectangular prism has faces with areas of 126 $cm^2$, 134 $cm^2$, and 140 $cm^2$. What is the total surface area? | 800 $cm^2$ |
| 9. A rectangular prism has faces with areas of 13.4 $mm^2$, 2.2 $mm^2$, and 6.6 $mm^2$. What is the total surface area? | 44.4 $mm^2$ |
| 10. A triangular prism has faces with areas of 60 $cm^2$, 60 $cm^2$, 60 $cm^2$, 15.6 $cm^2$, and 15.6 $cm^2$. What is the total surface area? | 211.2 $cm^2$ |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–47

| | |
|---|---|
| **Learning Target** | Volume (Capacity) (Shape and Space: 8.SS.4) |
| **Strategies of Focus** | Various |

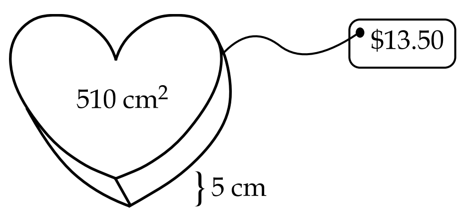
| **Prior Learning** | **Answers** |
|---|---|
| 1. How many mL of water will fit in a 23.5 L tub? | 23 500 mL |
| 2. How many litres are equal to 35 750 mL? | 35.75 L |
| 3. What is $\frac{3}{4}$ of a litre in mL? | 750 mL |
| 4. A cube has all of its sides doubled. What happens to the volume? | It increases eight times. |
| 5. What is the volume of a $4 \times 4$ unit cube? | 64 units$^3$ |
| 6. What is the area of a square that is $10 \times 14$ m? | 140 m$^2$ |
| **Grade 8 Questions** | |
| 7. The face of a can has an area of 65 cm$^2$. It is 10 cm tall. What is the volume? | 650 cm$^3$<br>650 mL |
| 8. A box has a lid with an area of 175 cm$^2$. The box is 20 cm deep. What is the volume? | 3500 cm$^3$ |
| 9. A triangular prism has a triangular face with an area of 61.1 cm$^2$ and is 400 cm deep. What is the volume? | 24 440 cm$^3$ |
| 10. A rectangular fish tank has a lid with dimensions of 20 cm and 40 cm. It is 25 cm deep. What is its volume? | 20 000 cm$^3$ or<br>20 000 mL or<br>20 L |
| **Other Questions** | |
| 11. | |
| 12. | |

# Mental Math

## Grade 8 Mathematics

A–48

| | |
|---|---|
| **Learning Target** | Volume (Capacity) (Shape and Space: 8.SS.4) |
| **Strategies of Focus** | Distribution and Compensation |

| **Prior Learning** | **Answers** |
|---|---|
| 1. Solve: $21 \times 15$ | 315 |
| 2. Solve: $99 \times 5$ | 495 |
| 3. True or False: $l \times w \times h = h \times l \times w$ | True |
| 4. True or False: $35 \times 19 = (35 \times 20) - 35$ | True |
| 5. True or False: $43 \times 15 = (43 \times 10) + (43 \times 5)$ | True |
| 6. True or False: $43 \times 15 = (15 \times 40) + (15 \times 3)$ | True |
| **Grade 8 Questions** | |
| 7. A hexagonal prism has a face of 32 mm$^2$, a length of 11 mm, and a weight of 4.2 grams. What is its volume? | 352 mL or 352 cm$^3$ |
| 8. A heart-shaped box of candy costs \$13.50. The lid has an area of 510 cm$^2$. It is 5 cm deep. What is the volume?   | 2550 cm$^3$ |
| 9. A top of a can is 99 cm$^2$. The can is 12 cm tall. What is the volume? | 1188 cm$^3$ |
| 10. Dimensions of a rectangular box are 11 cm, 10 cm, and 21 cm. Find the volume. | 2310 cm$^3$ |
| **Other Questions** | |
| 11. | |
| 12. | |

# Mental Math

## Grade 8 Mathematics

A–49

| | |
|---|---|
| **Learning Target** | Volume (Capacity) (Shape and Space: 8.SS.4) |
| **Strategies of Focus** | Using Compatible Numbers |

| Prior Learning | Answers |
|---|---|
| Solve questions 1 to 6 and indicate which numbers are easiest to start with. | |
| 1. $3 \times 25 \times 4 \times 7$ | 2100 (start with $25 \times 4$) |
| 2. $12.5 \times 7 \times 2$ | 175 ($12.5 \times 2$) |
| 3. $103 + 89.7 + 3 - 89.7$ | 106 ($89.7 - 89.7$) |
| 4. $84 \times 15 \times 419 \times 0 \times 12$ | 0 (anything $\times$ 0) |
| 5. $6 \times 19 + 1$ | 115 (don't forget order of operations) |
| 6. $1 + 100 \div 4 \div 2$ | 13.5 (order of operations) |
| **Grade 8 Questions** | |
| 7. Eleven identical boxes of nails have dimensions of 10 cm × 15 cm × 10 cm. What is the total volume of the boxes? | 16 500 $cm^3$ |
| 8. Four identical snowmobile trailers have dimensions of 3 m × $1\frac{1}{2}$ m × 2 m. What is the total volume of the trailers? | 36 $m^3$ |
| 9. How much water would it take to fill 5 identical fish tanks with dimensions of 25 cm × 50 cm × 40 cm? | 250 000 mL (or $cm^3$) or 250 L |
| 10. There are two Grade 8 classrooms at George Waters Middle School. There are 20 students in each class. How much hot chocolate needs to be prepared in order to give each student 250 mL? | 10 000 mL or 10 L |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

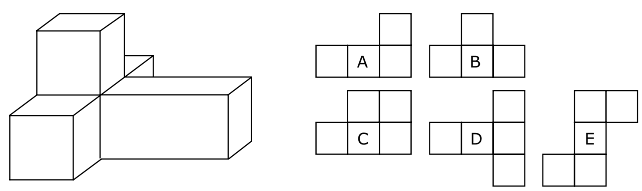
**A–50**

| | |
|---|---|
| **Learning Target** | Identifying, Sorting, Comparing, and Constructing (Shape and Space: 8.SS.2, SS.5) |
| **Strategies of Focus** | Visualization |

| **Prior Learning** | **Answers** |
|---|---|
| 1. How many faces does a cube have? | 6 faces |
| 2. How many edges does a triangular prism have? | 9 edges |
| 3. How many vertices does a cube have? | 8 vertices |
| 4. How many edges does a triangular pyramid have? | 6 edges |
| 5. How many vertices does a hexagon have? | 6 vertices |
| 6. How many faces does a square pyramid have? | 5 faces |

**Grade 8 Questions**

Use the diagram shown on the right to answer questions 7 to 10.




| | |
|---|---|
| 7. Which of the above lettered pictures represents a top view of the 3-D object? | D |
| 8. Which of the above lettered pictures represents a side view of the 3-D object? | A and B |
| 9. Which of the above lettered pictures does not represent a possible view of the 3-D object? | C and E |
| 10. Which of the above lettered pictures represents an underside view of the 3-D object? | D |

**Other Questions**

11.

12.

# Mental Math

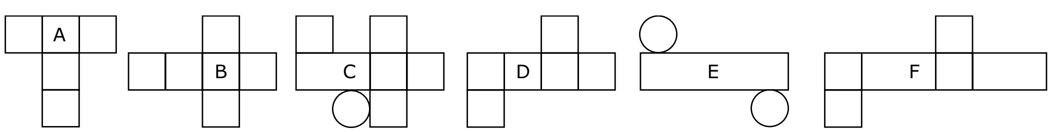
## Grade 8 Mathematics

A–51

| | |
|---|---|
| **Learning Target** | Identifying, Sorting, Comparing, and Constructing (Shape and Space: 8.SS.2, SS.5) |
| **Strategies of Focus** | Visualization |

| **Prior Learning** | **Answers** |
|---|---|
| 1. Which of the following are quadrilaterals?<br>rectangle, octagon, rhombus, triangle | Rectangle and rhombus |
| 2. Are all squares rectangles? | Yes |
| 3. Are all rectangles squares? | No |
| 4. Are all rhombuses squares? | No |
| 5. Are all squares rhombuses? | Yes |
| 6. Are all trapezoids quadrilaterals? | Yes |
| **Grade 8 Questions** | |
| 7. Are all cubes rectangular prisms? | Yes |

Use the diagrams below to answer questions 8 to 10.



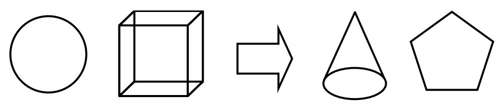

| | |
|---|---|
| 8. Which of the above nets can form a cube? | B and D |
| 9. Which of the above nets can form a rectangular prism? | F |
| 10. Which of the above nets can form a cylinder? | E |
| **Other Questions** | |
| 11. | |
| 12. | |

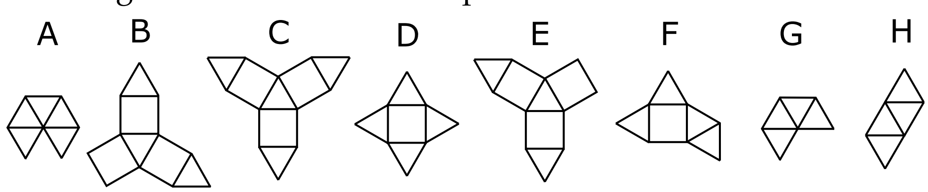
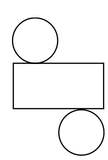
# *Mental Math*

## Grade 8 Mathematics

**A–52**

| | |
|---|---|
| **Learning Target** | Identifying, Sorting, Comparing, and Constructing (Shape and Space: 8.SS.2, SS.5) |
| **Strategies of Focus** | Memorization |

| **Prior Learning** | **Answers** |
|---|---|
| 1. Does the triangle formed by the letter "A" form a scalene, isosceles, or equilateral triangle? | Isosceles |
| 2. Are the four triangles formed by cutting a square across both of its diagonals acute, obtuse, or right triangles? | Right |
| 3. What will be the measure of each angle in an equilateral triangle? | 60° |
| Use the diagrams shown on the right to answer questions 4 to 6.  | |
| 4. Which of the above objects is an irregular polygon? | The arrow (heptagon) |
| 5. Which of the above is a prism? | The cube |
| 6. Which of the above objects contains only obtuse angles? | The pentagon |

| **Grade 8 Questions** | |
|---|---|
| Use the diagrams below to answer questions 7 to 9.   | |
| 7. Which of the above nets can create a square pyramid? | D and F |
| 8. Which of the above nets can create a triangular prism? | None |
| 9. Which of the above nets can create a triangular pyramid? | H |
| 10. Will this net fold to form a cylinder? Why or why not?  | No, the circumference of the circle is much longer than the side of the rectangle. |

| **Other Questions** | |
|---|---|
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

**A–53**

| | |
|---|---|
| **Learning Target** | Position and Motion (Shape and Space: 8.SS.6) |
| **Strategies of Focus** | Visualization |

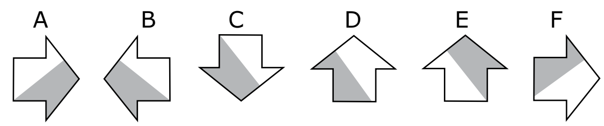
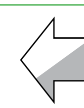
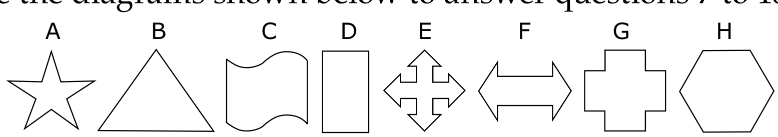
| **Prior Learning** | **Answers** |
|---|---|
| Point B is (3, 1). Find the coordinates of point B′ after completing the following individual transformations. | |
| 1. 90° clockwise rotation around the origin | B′ = (1, −3) |
| 2. 90° counter-clockwise rotation around the origin | B′ = (−1, 3) |
| 3. 180° rotation around the origin | B′ = (−3, −1) |
| 4. Reflection on the $x$-axis | B′ = (3, −1) |
| 5. Reflection on the $y$-axis | B′ = (−3, 1) |
| 6. Translation 3 units left and one unit down | B′ = (0, 0) |
| **Grade 8 Questions** | |
| 7. Will an equilateral triangle tessellate the plane? | Yes |
| 8. Will an isosceles triangle tessellate the plane? | Yes |
| 9. Will a square tessellate the plane? | Yes |
| 10. Will a pentagon tessellate the plane? | No |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

**A–54**

| | |
|---|---|
| **Learning Target** | Position and Motion (Shape and Space: 8.SS.6) |
| **Strategies of Focus** | Visualization |

| **Prior Learning** | **Answers** |
|---|---|
| Use the diagrams shown on the right to answer questions 1 to 6.   | |
| 1. Which image represents a reflection of A? | B |
| 2. Which image represents a rotation of 90° counter-clockwise for A? | E |
| 3. Which image represents a rotation of 90° clockwise for A? | C |
| 4. Which image represents a transformation of F? | D |
| 5. Sketch a 180° rotation of F. |  |
| 6. Which images are not a transformation of B? | D and F |
| **Grade 8 Questions** | |
| Use the diagrams shown below to answer questions 7 to 10.   | |
| 7. True or False: A and B can be combined to tessellate the plane. | False |
| 8. True or False: Both C and D can individually tessellate the plane. | True |
| 9. True or False: E and F can be combined to tessellate the plane. | False |
| 10. True or False: G and H tessellate the plane because the measure of each of their angles is a factor of 360. | True |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

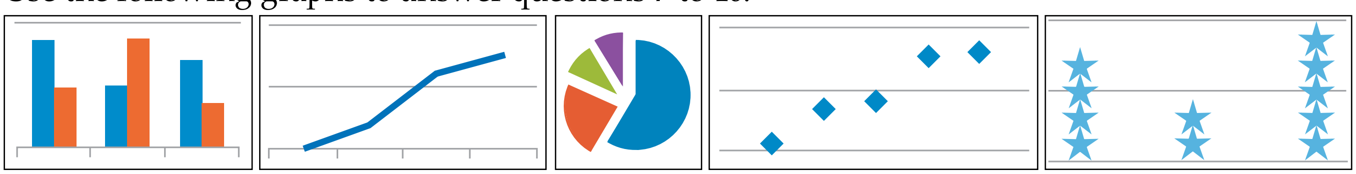
A–55

| | |
|---|---|
| **Learning Target** | Collection, Organization, and Analysis of Data (Statistics and Probability: 8.SP.1) |
| **Strategies of Focus** | Various |

| Prior Learning | Answers |
|---|---|
| The following weights were bench-pressed by a group of Grade 8 students: 32 kg, 41 kg, 15 kg, 22 kg, and 15 kg. Answer questions 1 to 4. | |
| 1. Find the mean. | 25 kg |
| 2. Find the mode. | 15 kg |
| 3. Find the median. | 22 kg |
| 4. What would be an appropriate graph to use to show this information? | Bar graph or pictograph |
| 5. Is the height of a child, measured every month for the course of three years, continuous or discrete data? | Continuous |
| 6. A survey asked the question, “Do you prefer eating fresh food at Jimmy’s Restaurant or eating stale food at home?” Is this a fair survey question? | No, it is biased and provides limited survey options. |

**Grade 8 Questions**

Use the following graphs to answer questions 7 to 10.



| Question | Answers |
|---|---|
| 7. Why would a dot graph better show money earned at a lemonade stand every day than a line graph? | Dots show discrete data. |
| 8. Which type of graph would best show height comparisons between boys and girls among different grades? | Double-bar graph |
| 9. Which graph would best show percentage of daily time used for different activities? | Pie chart |
| 10. Which graph would best show the distance someone travelled over the course of a day? | Line graph |

| Other Questions | |
|---|---|
| 11. | |
| 12. | |

# *Mental Math*

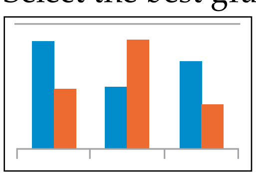
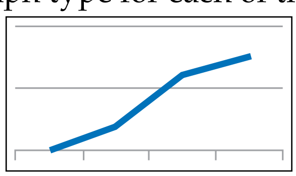
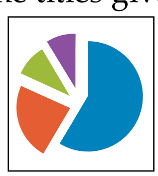
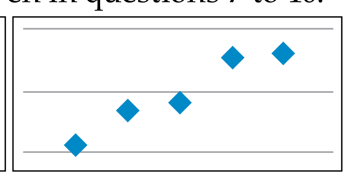
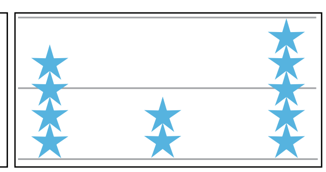
## Grade 8 Mathematics

**A–56**

| | |
|---|---|
| **Learning Target** | Collection, Organization, and Analysis of Data (Statistics and Probability: 8.SP.1) |
| **Strategies of Focus** | Various |

| **Prior Learning** | **Answers** |
|---|---|
| For questions 1 to 3, answer the following question: "Is the following information first-hand or second-hand data?" | |
| 1. You circulate a survey to find out what movies your friends would like to see. | First-hand |
| 2. You look up your favourite hockey player's stats online. | Second-hand |
| 3. You use information from a graph published in the *Daily Graphic* newspaper. | Second-hand |
| For questions 4 to 6, answer the following question: "What would be the most appropriate method of collecting the following data?" | |
| 4. You want to know the average income of Canadian adults. | Consult a database |
| 5. You want to know the average running speed of the students in your class. | Experiment |
| 6. You want to know the favourite foods of your friends. | Survey |
| **Grade 8 Questions** | |

Select the best graph type for each of the titles given in questions 7 to 10.

| | |
|---|---|
| 7. "Goals Scored by the Top Three Grade 8 Soccer Players This Month" | Pictograph |
| 8. "Favourite Soccer Positions in Grade 8: Forward, Mid, Defence, or Goal" | Pie chart |
| 9. "Season Wins and Losses for Three Teams" | Double-Bar graph |
| 10. "Number of Victories Over the Last Five Months" | Dot graph |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–57

| | |
|---|---|
| **Learning Target** | Probability (Statistics and Probability: 8.SP.2) |
| **Strategies of Focus** | Various |

| **Prior Learning** | **Answers** |
|---|---|
| 1. A bag has 8 blue marbles, 5 red marbles, and 7 green marbles. What is the probability, as a percent, of drawing a green marble? | 35% |
| 2. What is the probability, as a reduced fraction, of drawing a red marble? | $\frac{1}{4}$ |
| 3. What is the probability, as a ratio to all possibilities, of drawing a blue marble? | 2 : 5 |
| 4. Can the probability of something occurring ever be greater than 100%? | No |
| 5. If the probability of rain in Brandon is 40%, what is the probability that it won't rain? | 60% |
| 6. What is the probability of rolling a seven on a regular six-sided die? | 0% |
| **Grade 8 Questions** | |
| 7. A coin is tossed and a six-sided die is rolled. What is the probability of tossing a heads and rolling a 4? | 1 out of 12 |
| 8. What is the probability of rolling a 6 twice in a row on a six-sided die? | 1 out of 36 |
| 9. What is the probability of getting 2 true or false questions correct with random guesses? | 1 out of 4 |
| 10. What is the probability of getting 2 true or false questions both wrong with random guesses? | 1 out of 4 |
| **Other Questions** | |
| 11. | |
| 12. | |

# *Mental Math*

## Grade 8 Mathematics

A–58

| | |
|---|---|
| **Learning Target** | Probability (Statistics and Probability: 8.SP.2) |
| **Strategies of Focus** | Distribution |

| **Prior Learning** | **Answers** |
|---|---|
| 1. Calculate: 64% of 25 | 16 |
| 2. Calculate: 25% of 64 | 16 |
| 3. Calculate: 42% of 20 | 8.4 |
| 4. Calculate: 20% of 42 | 8.4 |
| 5. Calculate: 41% of 50 | 20.5 |
| 6. A baseball player has a 20% probability of hitting the ball. This player has 15 opportunities to hit. How many hits should be expected? | 3 hits |
| **Grade 8 Questions** | |
| 7. What is the probability of rolling an even number twice in a row on a six-sided die? | 1 out of 4 |
| 8. There is an 80% chance of rain in Winnipeg, a 10% chance of rain in Portage, and a 25% chance of rain in Thompson all on the same day. What is the probability of rain occurring in all three cities? | 2% chance of rain in the three cities |
| 9. A multiple choice test has questions with options of a, b, c, or d. What is the probability of getting two questions correct with random guesses? | 1 out of 16 |
| 10. What is the probability of getting these same two questions wrong? | 9 out of 16 |
| **Other Questions** | |
| 11. | |
| 12. | |

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