Grade 12 Introduction to Calculus

Manitoba Curriculum Framework of Outcomes

Topic: Limits

Big Ideas:

- Limits can describe function values as the input values approach a number or infinity.
- Limits are especially useful when the input value is not part of the domain of a function.

Specific Learning Outcomes: *It is expected the students will…*		Achievement Indicators:
IC.1.1	Demonstrate an understanding of the concept of the limit.	■ Explore the concept of limits by analyzing a function's graph and table of values. ■ Utilize the definition and proper notation of limits to express a function's limit at a specific point. ■ Verify limit theorems: ▪ identity function ▪ constant function ▪ constant times a function ▪ sum/difference of functions ▪ product/quotient of functions ▪ power of a function ■ Utilize limit theorems to determine the limit of functions by direct substitution.
IC.1.2	Evaluate limits to analyze functions.	■ Explain why $\frac{0}{0}$ is called an indeterminate form. ■ Solve limits of indeterminate form by algebraic manipulation. ■ Define one-sided limits. ■ Evaluate one-sided limits of functions (including piecewise) graphically and algebraically. ■ Explain the behaviour of limits in the form $\lim_{x \to 0}\left(\frac{\text{number}}{x}\right)$. ■ Determine limits at infinity. ■ Apply limits to determine the equations of horizontal and vertical asymptotes.
IC.1.3	Apply the concept of limit to the continuity of a function.	■ Determine graphically whether a function is continuous. ■ Determine algebraically whether a function is continuous.

Topic: Derivatives

Big Ideas:

- The derivative extends the concept of slope to the slope of a curve at a point.
- A derivative function can help someone describe the "shape" of the curve with that derivative.

Specific Learning Outcomes: *It is expected the students will…*		**Achievement Indicators:**
IC.2.1	Develop the definition of the derivative as the slope of a curve at a point.	*Note: Prerequisite knowledge includes determining the slope and the equation of a line.* ■ Explain how slopes of secant lines can approximate the slope of a tangent line. ■ Define the derivative as the limit of the difference quotient, which is the slope of the tangent line at a point. ■ Describe the derivative function, $f'(x)$, as a function that determines the slope at any point of the function, $f(x)$. *Note: Students should be exposed to different notations for derivatives* $\left(f', y', \text{and } \frac{dy}{dx}\right)$.

IC.2.2	Develop and apply differentiation rules.	■ Develop and apply differentiation rules:
		  ■ a constant times $f(x)$
		  ■ the power rule with rational exponents
		  ■ sum and difference
		  ■ product
		  ■ quotient
		  ■ chain rule
		■ Apply the derivative rules to determine the equation of a tangent line at a point, given a function equation and a point on the function.
		■ Define and determine higher-order derivatives of a function.
		Note: The functions explored in this introductory course do not include trigonometric, exponential, and logarithmic functions.

IC.2.3	Demonstrate an understanding of implicit differentiation.	■ Determine the derivative of a relation implicitly. ■ Determine the equation of a tangent line to a relation, given a point. ■ Determine higher-order derivatives of a relation using implicit differentiation.

Topic: Applications of Derivatives

Big Idea:

- Applying derivatives can help someone solve problems based on many other function models as accurately and efficiently as those with linear or quadratic models.

Specific Learning Outcomes: *It is expected the students will…*		**Achievement Indicators:**
IC.3.1	Apply derivatives to solve problems involving the motion of particles.	■ Describe the meaning of a displacement function. ■ Determine average and instantaneous velocity given a displacement function. ■ Determine average and instantaneous acceleration given a displacement function. ■ Solve particle motion problems.
IC.3.2	Determine features of a function using derivatives to sketch the function accurately.	*Note: Prerequisite skills for this topic include describing domain using interval notation, set notation, and number line graphs. Teachers may want to review solving linear and non-linear inequalities using a sign diagram.* ■ Determine the critical values of a function. ■ Determine the intervals where a function is increasing and decreasing. ■ Determine relative extremes and absolute extremes graphically and algebraically. ■ Determine intervals where the graph of a function is concave up and concave down. ■ Determine the points of inflection. ■ Sketch a polynomial function accurately using its characteristics, including intercepts, domain, range, maxima, minima, points of inflection, and concavity. *Note: The functions explored in this introductory course do not include trigonometric, exponential, and logarithmic functions.*
IC.3.3	Apply derivatives to solve optimization and related rates problems.	■ Solve optimization problems. ■ Apply the chain rule and implicit differentiation to determine rates of change. ■ Solve problems involving related rates.

Topic: Integrals

Big Ideas:

- Integration extends the area of geometric shapes to the area under a function curve where the height of a region is changing.
- Derivatives and integrals are inversely related.

Specific Learning Outcomes: *It is expected the students will...*		**Achievement Indicators:**
IC.4.1	Demonstrate an understanding of the relationship between anti-differentiation and integration of functions.	■ Describe anti-differentiation as the inverse operation of differentiation. ■ Determine the general antiderivative (family of functions), given the derivative of a function. ■ Define integration in terms of the area bounded by a function curve and the x-axis. ■ Relate anti-differentiation and integration as the fundamental theorem of calculus (first part). ■ Define the indefinite integral.
IC.4.2	Apply integration to solve problems.	■ Determine a specific antiderivative, given the derivative function and the coordinates of a point. ■ Apply integration in a context such as particle motion.
IC.4.3	Demonstrate and apply an understanding of the definite integral.	■ Define the definite integral. ■ Evaluate definite integrals geometrically by calculating area. ■ Evaluate definite integrals using antiderivatives and the fundamental theorem of calculus (second part). ■ Evaluate the definite integral of functions algebraically and geometrically where parts of the function may be below the x-axis. ■ Relate the total area bounded by a function curve, $f(x)$, and the x-axis on interval $[a, b]$ to the definite integral of the absolute value of the function, $\int_a^b \lvert f(x) \rvert dx$. ■ Determine the area between any two functions on a given interval. ■ Determine the area between two functions where intersecting points determine the interval.