Grade 12 Advanced Mathematics

Manitoba Curriculum
Framework of Outcomes

Topic: Complex Numbers and Polar Coordinates

Big Ideas:

- All other number systems are a subset of the complex number system.
- The operations and properties that apply to other number systems also apply to the complex number system.
- Complex numbers can be represented on a two-dimensional plane in rectangular or polar form.

Overview: The set of complex numbers were developed to describe all the roots of polynomial functions including the real number roots. Unlike the real number system, which can be represented on a one-dimensional number line, the complex number system is represented on a two-dimensional plane. Often "i" is referred to as an imaginary unit that is a symbol with the property that $i = \sqrt{-1}$ or $i^2 = -1$. Complex numbers apply to electrical engineering, aircraft design, medicine, and graphic design. Polar coordinates allow us to sketch some relations that are not functions with much more ease. Some applications of polar coordinates include guiding vessels and guiding industrial robots.

Specific Learning Outcomes: *It is expected the students will…*		**Achievement Indicators:**
AM.1.1	Define and perform operations on complex numbers.	■ Define the complex number system and describe its history. ■ Determine the absolute value of a complex number. ■ Determine the geometrical representation of a complex number. ■ Compare complex numbers. ■ Perform operations on complex numbers. ■ Define complex conjugates and apply them to the division of complex numbers.
AM.1.2	Make connections between complex numbers and quadratic equation solutions.	■ Solve quadratic equations with complex roots. ■ Solve quadratic equations with complex coefficients. ■ Given the roots, real or complex, determine the corresponding quadratic equation.
AM.1.3	Demonstrate an understanding of polar coordinates and their graphs.	■ Demonstrate how to read polar coordinates. ■ Sketch a point expressed in polar coordinates. ■ Convert rectangular coordinates to polar coordinates and vice versa. ■ Convert equations in polar form to rectangular form and vice versa. ■ Sketch a variety of polar equations. ■ Apply symmetry to sketch a polar graph.
AM.1.4	Make connections between complex numbers and polar coordinates.	■ Determine the argument of a complex number. ■ Represent a complex number in polar form. ■ Convert a complex number in polar form to rectangular form and vice versa.

Topic: Statistics

Big Ideas:

- Using a data sample, statistics can be used to describe a set of data or allow us to predict, based on probability, things about a set of data.
- Statistics allow someone to explore, describe, model, and explain data.
- Statistics can describe the central tendency of a set of data and how the data is spread out.

Overview: Statistics is the study of data and how to represent it. By analyzing the data, statistics can help us understand data and make inferences about it. Data can be described in terms of measures of central tendency and measures of spread. Statistics connects to the concept of probability when applied to probability distributions such as the binomial distribution and the normal distribution. Binomial distributions build on previous knowledge students gained about the binomial theorem. Data analysis is prevalent and integral to the work in a broad array of fields including social sciences, sports teams, business, scientific research, and data analytics.

Specific Learning Outcomes: *It is expected the students will…*		**Achievement Indicators:**
AM.2.1	Demonstrate an understanding of the concepts of measures of central tendency and spread.	■ Define vocabulary used in statistics (including measures of central tendency and spread). ■ Represent data in histograms, frequency tables, and grouped frequency tables. ■ Calculate quartiles, the interquartile range, and the range of data. ■ Determine whether a data point is an outlier. ■ Represent data with a box and whisker plot. ■ Demonstrate an understanding of the properties of variance and standard deviation. ■ Calculate the variance and standard deviation of data.
AM.2.2	Demonstrate an understanding of probability distributions including the binomial distribution.	■ Define discrete random variables and probability distributions. ■ Recognize that the probabilities in a probability distribution always add to 1. ■ Represent data with tree diagrams and probability distribution charts. ■ Use probability distributions to solve problems. ■ Define binomial distribution. ■ Use the formula for calculating probabilities within a binomial distribution. ■ Find the mean and standard deviation of a binomial distribution. *Note: Teachers are encouraged to connect the binomial expansion formula from Pre-Calculus Mathematics 40S to the formula for calculating binomial distributions.*
AM.2.3	Develop and apply the properties of a normal distribution.	■ Describe the properties of the normal distribution and the standard normal distribution. ■ Analyze the normal distribution to show probabilities can be estimated with the 68-95-99.7 rule. ■ Apply the formula for calculating z-scores and use z-scores to compare data. ■ Calculate probabilities in normal distributions, given scores. ■ Calculate scores in normal distributions, given probabilities. ■ Use a normal distribution to estimate a binomial distribution when appropriate. ■ Determine confidence intervals. ■ Calculate the margin of error of a confidence interval.

Topic: Number Theory

Big Ideas:

- Many ideas are true about all integers, some about subsets of integers, and some apply to only one integer.
- It is necessary to provide some type of proof that a conjecture is true for all integers, since we cannot test every possibility.

Overview: Number theory is a branch of mathematics and is, in large part, devoted to the study of integers. Important number theory topics include primes, prime factorization, and properties of numbers made out of integers such as rational numbers. Proofs are examples of deductive or inductive reasoning, and they demonstrate that a statement is always true under the given conditions. Many unanswered questions in mathematics originate in number theory. Applications of number theory are modular arithmetic, cryptography, and topics of computer science; however, number theory is often studied just for fun.

Specific Learning Outcomes: *It is expected the students will…*		**Achievement Indicators:**

AM.3.1	Apply proof techniques to prove mathematical theorems or statements.	■ Demonstrate an understanding of proof techniques:
		  ■ direct proof (proof by construction)
		  ■ proof by contradiction
		  ■ proof by induction
		Note: Other proof techniques may be introduced.

AM.3.2	Explore, develop, and apply the properties of integers.	■ Illustrate and explain the divisibility of integers and the Archimedean property. ■ Demonstrate an understanding of modular arithmetic. ■ Define, develop, and apply the greatest common divisor (GCD). ■ Apply the Euclidean algorithm to find the GCD. ■ Express the GCD as a linear equation. ■ Define prime and composite numbers. ■ Demonstrate an understanding of the sieve of Eratosthenes. ■ Prove there are an infinite number of primes. ■ Demonstrate an understanding of the fundamental theorem of arithmetic. ■ Define the least common multiple (LCM). ■ Apply prime factorization to determine the LCM and GCD. *Note: Teachers are encouraged to demonstrate the formal proof of the Fundamental Theorem of Arithmetic*
AM.3.3	Represent numbers in different bases.	■ Define decimal representation (base 10). ■ Represent an integer in bases other than 10. ■ Convert a number from one base to another. ■ Demonstrate an understanding of binary and hexadecimal notation.

Topic: Matrices and Systems of Equations

Big Ideas:

- The intrinsic relationship of the four operations when applied to numbers is the same when they are applied to matrices.
- Writing the coefficients and constants in a linear system in the form of a matrix is a way to represent the system and can assist in determining solutions to that system.

Overview: A matrix is a rectangular array of numbers with algebraic properties and geometric connections to linear systems and vectors in 2-space, 3-space, and *n*-space (with any number of dimensions). These topics are studied in depth in post-secondary courses involving linear algebra. There are many matrix applications in computer science.

Specific Learning Outcomes: *It is expected the students will…*		Achievement Indicators:
AM.4.1	Demonstrate an understanding of matrices.	■ Define the following terms: ▫ matrix ▫ dimension or order of a matrix ▫ entry of a matrix ▫ equal matrices ▫ square matrix ▫ zero matrix ▫ row and column matrix ▫ transpose of a matrix ■ Give examples of where matrices are used.
AM.4.2	Perform operations on matrices.	■ Perform addition and subtraction on matrices where possible. ■ Perform scalar multiplication of a matrix. ■ Apply matrix operations to solve simple matrix equations. ■ Multiply matrices where possible. ■ Explain why matrix multiplication is not commutative. ■ Determine the inverse of a 2×2 matrix using a formula. ■ Explain the three operations used when row-reducing a matrix. ■ Explain the difference between a row-echelon matrix and a reduced row-echelon matrix. ■ Apply matrix row reduction to find the inverse of a square matrix.
AM.4.3	Solve systems of equations using matrices.	■ Define a determinant. ■ Determine the determinant of a matrix using: ▫ row reduction ▫ cofactor expansion ▫ the arrow method for a 3×3 matrix ■ Solve a system of equations using: ▫ row reduction ▫ inverse ▫ Cramer's rule *Note: Solving a system of equations using either the inverse or Cramer's rule is not expected beyond a 2×2 system.*

Topic: 3-Dimensional Geometry

Big Ideas:

- It takes three pieces of information to describe a point in 3-space.
- Algebraic concepts developed for geometry in two dimensions can be extended to three dimensions.

Overview: Geometry in three dimensions is an extension of two-dimensional Euclidean geometry studied in Middle and Senior Years. Applications of three-dimensional geometry include computer graphics, computer assisted design, architecture, interior design, and 3-D modelling. The topic of three-dimensional geometry has connections with the topics of matrices (3×3), vectors, and calculus.

Specific Learning Outcomes: *It is expected the students will…*		**Achievement Indicators:**

AM.5.1	Demonstrate an understanding of 3-space.	■ Define the following terms in 3-space:
		  ■ coordinate planes
		  ■ coordinates of a point
		  ■ octant
		■ Sketch the coordinate axes and points given the coordinates.
		■ Sketch a rectangular prism or a plane in 3-space.
		■ Determine the distance between two points in 3-space.

AM.5.2	Represent and analyze lines, planes, and surfaces algebraically and graphically in 3-space.	■ Determine the point-normal equation of a plane. ■ Prove two planes are parallel. ■ Determine the general equation of a plane. ■ Determine the equation of a sphere. ■ Sketch the curve of intersection of surfaces. ■ Determine the points of intersection of non-parallel planes. ■ Determine the equation of a sphere. ■ Determine the equation of a solid of revolution.

Topic: Vectors

Big Ideas:

- Quantities that have both a magnitude and a direction can be efficiently represented by a vector.
- Vectors can be represented both geometrically and algebraically and some ideas about vectors are easier to see with one representation than the other.

Overview: A vector is a quantity that can be represented with a directed line segment. The study of vectors in mathematics helps us understand how vectors interact with each other, and it can be applied to the study of kinematics and forces in physics. Vector mathematics connects to analytic geometry in two dimensions and helps extend the concept of an equation of a line to three dimensions. This topic has connections to matrices and three-dimensional geometry. Vectors are commonly used in physics and have applications in branches of engineering.

Specific Learning Outcomes: *It is expected the students will…*		**Achievement Indicators:**
AM.6.1	Develop an understanding of vectors and perform basic vector operations.	■ Define a vector and a scalar. ■ Determine the magnitude of a vector. ■ Determine the unit vector. ■ Add and subtract vectors. ■ Apply vector addition and subtraction to geometrical situations. ■ Multiply vectors by a scalar.
AM.6.2	Demonstrate an understanding of the dot product and cross product of vectors to solve problems.	■ Define the dot product and explore its properties. ■ Apply the dot product to find the angle between two vectors. ■ Define the cross product and explore its properties. ■ Apply the right-hand rule to determine the direction of the cross product vector. ■ Represent the cross product in Cartesian form. ■ Apply the cross product to problems involving contexts such as the area of a parallelogram and torque.

AM.6.3	Develop and apply the vector equation of a line.	■ Write the vector equation of a line using the direction vector.
		■ Write the parametric and Cartesian forms of the vector equation of a line.
		■ Determine whether a point is on a line.
		■ Determine the point of intersection of two lines in two and three dimensions using vectors.
		■ Apply the vector equation of a line to kinematics such as:
		  ■ determining the speed of an object given its equation
		  ■ determining whether objects will collide given the paths of the objects
		  ■ determining the distance between objects at given times

Topic: Conic Sections

Big Idea:

- Thinking of conic sections geometrically makes certain properties of conic sections easier to see than thinking of them algebraically, and vice versa.

Overview: The curves associated with conic sections are the circle, the ellipse, the parabola, and the hyperbola. Conics can be defined in terms of distance between points or distance to a line and a point. This topic extends students' learning of circles and parabolas. Conics are an aspect of analytic geometry and have many applications in calculus and the sciences, especially physics and astronomy.

Specific Learning Outcomes: *It is expected the students will...*		Achievement Indicators:
AM.7.1	Represent and analyze conic sections algebraically and geometrically.	■ Define conic sections algebraically and geometrically: ■ circle ■ parabola ■ ellipse ■ hyperbola ■ Analyze the conics in terms of their characteristics (as applicable), such as centre, vertices, axes of symmetry, length of major and minor axes, and asymptotes. *Note: Teachers are encouraged to relate the conic sections to cross-sectional slices of a pair of cones (one inverted). The conics are restricted to those with equations of the form:* $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$*, where* $B = 0$*.*
AM.7.2	Demonstrate an understanding of focal points in a conic section.	■ Determine the focal points of any conic section. ■ Determine the equation of a conic section given its focal point(s). ■ Determine the directrix of a parabola and its relationship to the focal point.
AM.7.3	Analyze a conic section in terms of its eccentricity.	■ Define eccentricity and its relationship to each of the conic sections. ■ Given the eccentricity, identify the conic section. ■ Evaluate the eccentricity of a conic section. ■ Determine the equation of a conic, given eccentricity and other characteristics.