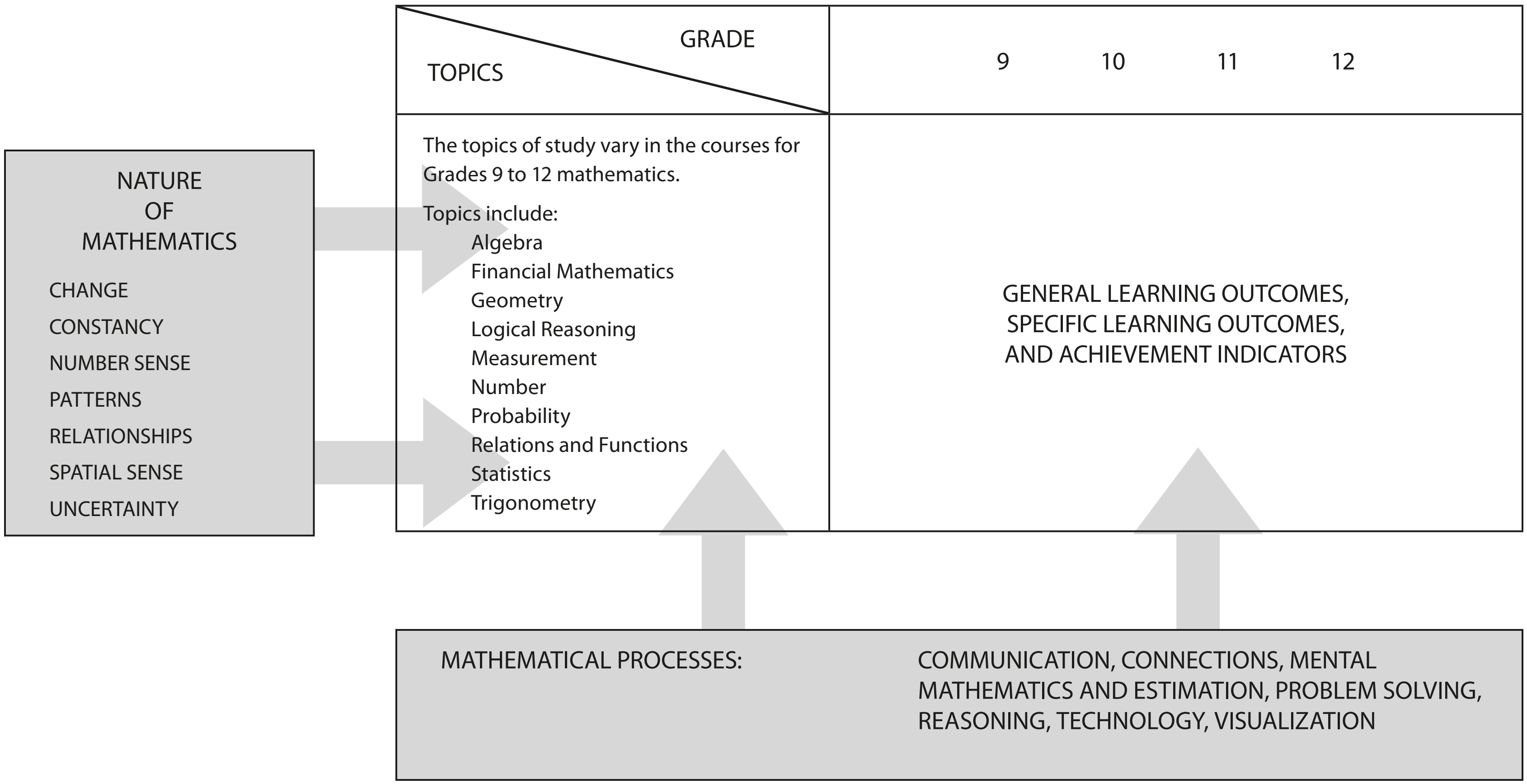
Conceptual Framework for Grades 9 to 12 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



Mathematical Processes

The seven mathematical processes are critical aspects of learning, doing, and understanding mathematics. Students must encounter these processes regularly in a mathematics program in order to achieve the goals of mathematics education.

The common curriculum framework incorporates the following interrelated mathematical processes. It is intended that they permeate the teaching and learning of mathematics.

Students are expected to

- Communication [C]
- Connections [CN]
- Mental Mathematics and Estimation [ME]
- Problem Solving [PS]
- Reasoning [R]
- Technology [T]
- Visualization [V]

- use *communication* in order to learn and express their understanding
- make *connections* among mathematical ideas, other concepts in mathematics, everyday experiences, and other disciplines
- demonstrate fluency with *mental mathematics and estimation*
- develop and apply new mathematical knowledge through *problem solving*
- develop mathematical *reasoning*
- select and use *technology* as a tool for learning and solving problems
- develop *visualization* skills to assist in processing information, making connections, and solving problems

All seven processes should be used in the teaching and learning of mathematics. Each specific learning outcome includes a list of relevant mathematical processes. All seven processes should be incorporated into learning experiences but the identified processes are to be used as a primary focus of instruction and assessment.

Communication [C]

Students must be able to communicate mathematical ideas in a variety of ways and contexts.

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links among their own language and ideas, the language and ideas of others, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing, and modifying ideas, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can play a significant role in helping students make connections among concrete, pictorial, graphical, symbolic, verbal, written, and mental representations of mathematical ideas. Explanation of ideas should use the various representations as appropriate.

Emerging technologies enable students to engage in communication beyond the traditional classroom to gather data and share mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students begin to view mathematics as useful, relevant, and integrated.

Through connections, students begin to view mathematics as useful and relevant.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. "Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding…Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching" (Caine and Caine 5).

Mental Mathematics and Estimation [ME]

Mental mathematics and estimation is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves using strategies to perform mental calculations.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility in reasoning and calculating.

"Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math" (National Council of Teachers of Mathematics, May 2005).

Mental mathematics and estimation are fundamental components of number sense.

Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein 442).

Mental mathematics "provides a cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers" (Hope v).

Estimation is used for determining approximate values or quantities, usually by referring to benchmarks or referents, or for determining the reasonableness of calculated values. Estimation is also used to make mathematical judgments and to develop useful, efficient strategies for dealing with situations in daily life. When estimating, students need to learn which strategy to use and how to use it.

To help students become efficient with computational fluency, students need to develop mental math skills and recall math facts automatically. Learning math facts is a developmental process where the focus of instruction is on thinking and building number relationships. Facts become automatic for students through repeated exposure and practice. When a student recalls facts, the answer should be produced without resorting to inefficient means, such as counting. When facts are automatic, students are no longer using strategies to retrieve them from memory.

Problem Solving [PS]

"Problem solving is an integral part of all mathematics learning" (NCTM, *Problem Solving*). Learning through problem solving should be the focus of mathematics at all grade levels. Students develop a true understanding of mathematical concepts and procedures when they solve problems in meaningful contexts. Problem solving is to be employed throughout all of mathematics and should be embedded throughout all the topics.

When students encounter new situations and respond to questions of the type, *How would you...? or How could you..?*, the problem-solving approach is being modelled. Students develop their own problem-solving strategies by listening to, discussing, and trying different strategies.

Learning through problem solving should be the focus of mathematics at all grade levels.

In order for an activity to be based on problem solving, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. Students should not know the answer immediately. A true problem requires students to use prior knowledge in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement. Students will be engaged if the problems relate to their lives, cultures, interests, families, or current events.

Both conceptual understanding and student engagement are fundamental in moulding students' willingness to persevere in future problem-solving tasks.

Problems are not just simple computations embedded in a story, nor are they contrived. They are tasks that are rich and open-ended, so there may be more than one way of arriving at a solution or there may be multiple answers. Good problems should allow for every student in the class to demonstrate their knowledge, skill, or understanding. Problem solving can vary from being an individual activity to a class (or beyond) undertaking.

In a mathematics class, there are two distinct types of problem solving: solving contextual problems outside of mathematics and solving mathematical problems. Finding the maximum profit given manufacturing constraints is an example of a contextual problem, while seeking and developing a general formula to solve a quadratic equation is an example of a mathematical problem.

Problem solving can also be considered in terms of engaging students in both inductive and deductive reasoning strategies. As students make sense of the problem, they will be creating conjectures and looking for patterns that they may be able to generalize. This part of the problem-solving process often involves inductive reasoning. As students use approaches to solving the problem they often move into mathematical reasoning that is deductive in nature. It is crucial that students be encouraged to engage in both types of reasoning and be given the opportunity to consider the approaches and strategies used by others in solving similar problems.

Problem solving is a powerful teaching tool that fosters multiple creative and innovative solutions. Creating an environment where students openly look for, and engage in, finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk-takers.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. Questions that challenge students to think, analyze, and synthesize help them develop an understanding of mathematics. All students need to be challenged to answer questions such as *"Why do you believe that's true/correct?"* or *"What would happen if…?"*.

Mathematical reasoning helps students think logically and make sense of mathematics.

Mathematical experiences provide opportunities for students to engage in inductive and deductive reasoning. Students use inductive reasoning when they explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Students use deductive reasoning when they reach new conclusions based upon the application of what is already known or assumed to be true. The thinking skills developed by focusing on reasoning can be used in daily life in a wide variety of contexts and disciplines.

When explaining ideas, students should be encouraged to use concrete, pictorial, symbolic, graphical, verbal, and written representations of their mathematical ideas.

Technology [T]

Technology can be used effectively to contribute to and support the learning of a wide range of mathematical learning outcomes. Technology enables students to explore and create patterns, examine relationships, test conjectures, and solve problems. Students in Grades 9 to 12 are expected to have consistent access to technology for their mathematics courses.

Technology has the potential to enhance the teaching and learning of mathematics. It can be used to

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- generate and test inductive conjectures
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems

Technology contributes to the learning of a wide range of mathematical outcomes, and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

- increase the focus on conceptual understanding by decreasing the time spent on repetitive procedures
- reinforce the learning of basic facts
- develop personal procedures for mathematical operations
- model situations
- develop number and spatial sense
- create geometric figures

Technology contributes to a learning environment in which the curiosity of students can lead to rich mathematical discoveries at all grade levels.

Students need to know when it is appropriate to use technology such as a calculator and when to apply their mental computation, reasoning, and estimation skills to predict and check answers. The use of technology can enhance, although it should not replace, conceptual understanding, procedural thinking, and problem solving.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and spatial reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate and involves knowledge of several estimation strategies (Shaw and Cliatt 150).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations. It is through visualization that abstract concepts can be understood concretely by the student. Visualization is a foundation to the development of abstract understanding, confidence, and fluency.

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Nature of Mathematics

Mathematics is one way of understanding, interpreting, and describing our world. There are a number of characteristics that define the nature of mathematics, including change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.

- Change
- Constancy
- Number Sense
- Patterns
- Relationships
- Spatial Sense
- Uncertainty

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

Change is an integral part of mathematics and the learning of mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12… can be described as

- skip-counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain (Steen 184)

Constancy

Many important properties in mathematics do not change when conditions change. Examples of constancy include

- the conservation of equality in solving equations
- the sum of the interior angles of any triangle
- theoretical probability of an event

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems such as those involving constant rates of change, lines with constant slope, or direct variation situations.

Some problems in mathematics require students to focus on properties that remain constant.

Number Sense

Number sense, which can be thought of as deep understanding and flexibility with numbers, is the most important foundation of numeracy (British Columbia Ministry of Education 146). Continuing to foster number sense is fundamental to growth of mathematical understanding.

Number sense is an awareness and understanding of what numbers are.

Number sense is an awareness and understanding of what numbers are, their relationships, their magnitude, and the relative effect of operating on numbers, including the use of mental mathematics and estimation (Fennell and Landis 187).

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all of the mathematical topics, and it is through the study of patterns that students can make strong connections between concepts in the same and different topics. Working with patterns also enables students to make connections beyond mathematics. The ability to analyze patterns contributes to how students understand their environment.

> Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns.

Patterns may be represented in concrete, visual, auditory, or symbolic form. Students should develop fluency in moving from one representation to another.

Students need to learn to recognize, extend, create, and apply mathematical patterns. This understanding of patterns allows students to make predictions and justify their reasoning when solving problems.

Learning to work with patterns helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics.

Relationships

Mathematics is used to describe and explain relationships. Within the study of mathematics, students look for relationships among numbers, sets, shapes, objects, variables, and concepts. The search for possible relationships involves collecting and analyzing data, analyzing patterns, and describing possible relationships visually, symbolically, orally, or in written form. Technology should be used to aid in the search for relationships.

> Mathematics is used to describe and explain relationships.

Spatial Sense

Spatial sense involves the representation and manipulation of 3-D objects and 2-D shapes. It enables students to reason and interpret among 3-D and 2-D representations.

> Spatial sense offers a way to interpret and reflect on the physical environment.

Spatial sense is developed through a variety of experiences with visual and concrete models. Some of these experiences should involve the use of technology. These experiences offer a way to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions.

Spatial sense is also critical in students' understanding of the relationship between the equations and graphs of functions and, ultimately, in understanding how both equations and graphs can be used to represent physical situations. Graphing calculators or graphing software can aid students in developing this understanding.

Uncertainty

In mathematics, interpretations of data and the predictions made from data inherently lack certainty.

Uncertainty is an inherent part of making predictions.

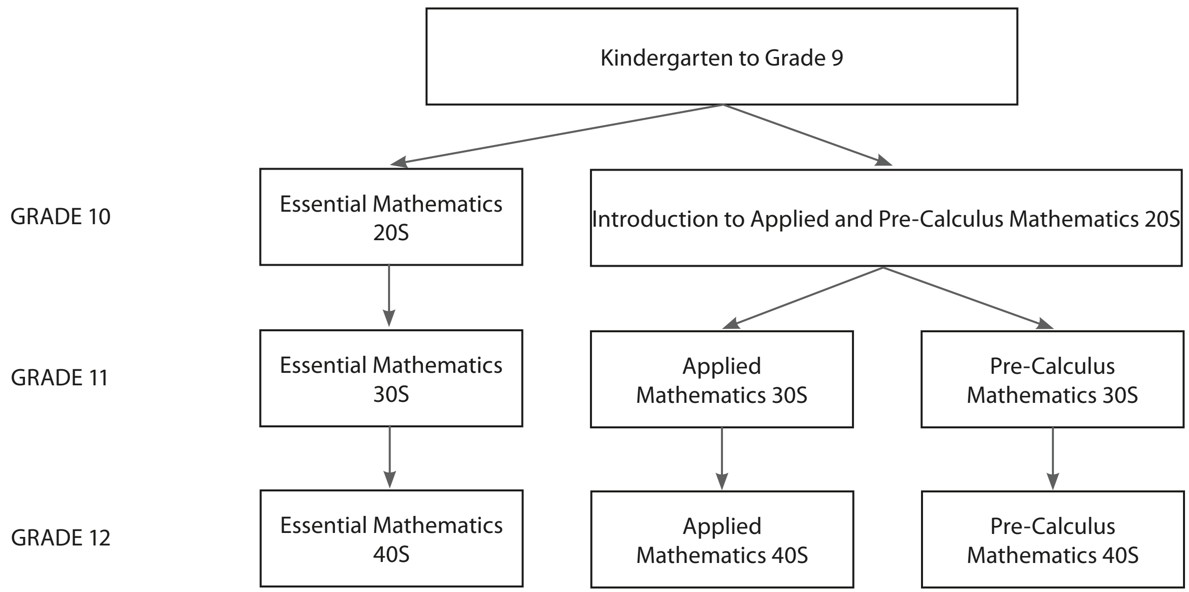
Events and experiments generate statistical data that can be used to make predictions. It is important that students recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of an interpretation or conclusion is directly related to the quality of the data it is based upon. An awareness of uncertainty provides students with an understanding of why and how to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately. This language must be used effectively and correctly to convey valuable messages.

Pathways and Topics

Unlike *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes, Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes* includes topics rather than strands. Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings.

There is one course available for students in Grade 9. In Grade 10, students may choose between two courses or may choose to take both courses. In Grades 11 and 12, students have three choices for courses and may take one or multiple courses.



Goals of Pathways

The goals of all pathways are to provide prerequisite attitudes, knowledge, skills, and understandings for specific post-secondary programs or direct entry into the workforce. All three pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. When choosing a pathway, students should consider their interests, both current and future. Students, parents, and educators are encouraged to research the admission requirements for post-secondary programs of study, as they vary by institution and by year.

Design of Pathways

Each pathway is designed to provide students with the mathematical understandings, rigour, and critical-thinking skills that have been identified for specific post-secondary programs of study and for direct entry into the workforce.

The content of each pathway has been based on the *Western and Northern Canadian Protocol (WNCP) Consultation with Post-Secondary Institutions, Business and Industry Regarding Their Requirements for High School Mathematics: Final Report on Findings* and on consultations with mathematics teachers.

Applied Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require the study of theoretical calculus. Topics include financial mathematics, geometry, logical reasoning, measurement, number, relations and functions, and statistics and probability.

Essential Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require further study in advanced mathematics. Topics include algebra, geometry, measurement, number, statistics and probability, and financial mathematics.

Pre-Calculus Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Topics include algebra and number, measurement, permutations, combinations and binomial theorem, relations and functions, and trigonometry.

Learning Outcomes and Achievement Indicators

The common curriculum framework is stated in terms of general learning outcomes, specific learning outcomes, and achievement indicators.

General learning outcomes are overarching statements about what students are expected to learn in each course. They remain consistent throughout several years of schooling.

Specific learning outcomes are statements that identify the specific knowledge, skills, and understandings that students are required to attain by the end of a given course. Some learning outcomes will be revisited several times during a course to allow for connections to be made to other learning outcomes in the course.

Achievement indicators are samples of how students may demonstrate their achievement of the goals of a specific learning outcome. The range of samples provided is meant to reflect the depth, breadth, and expectations of the specific learning outcome. While they provide some examples of student achievement, they are not meant to reflect the sole indicators of success. They are not presented in any particular order and need not be explicitly addressed in the classroom. However, students need to understand the outcomes at least to the depth indicated by the indicators. Therefore, the achievement indicators are sufficient as a basis for instructional design and assessment, and will form the basis for provincial assessment as appropriate.

- In the specific learning outcomes and achievement indicators, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome.
- In the specific learning outcomes and achievement indicators, the words *such as* indicate that the ensuing items are provided for clarification and are not requirements that must be addressed to fully meet the learning outcome.
- The word *and* used in a learning outcome indicates that both ideas must be addressed to fully meet the learning outcome, although not necessarily at the same time or in the same question.
- The word *and* used in an achievement indicator implies that both ideas should be addressed at the same time or in the same question.
- The word *or* used in an achievement indicator implies that both ideas should be addressed but not at the same time or in the same question.
- The word *concretely* implies students would use physical materials that they manipulate.
- The word *pictorially* implies students would create pictures of concrete materials.
- The word *symbolically* implies students would use variables, numbers, and mathematical signs.

Summary

The Conceptual Framework for Grades 9 to 12 Mathematics describes the nature of mathematics, the mathematical processes, the pathways and topics, and the role of learning outcomes and achievement indicators in Grades 9 to 12 mathematics. Activities that take place in the mathematics classroom should be based on a problem-solving approach that incorporates the mathematical processes and leads students to an understanding of the nature of mathematics.