TOPIC 1.6: WORK AND ENERGY

S4P-1-25 Define work as the product of displacement and the component of force parallel to the displacement when the force is constant.

S4P-1-26 Determine work from the area under the force-position graph for any force.

Include: positive or negative force, uniformly changing force

S4P-1-27 Describe work as a transfer of energy.

Include: positive and negative work, kinetic work, conservation of energy

S4P-1-28 Give examples of various forms of energy and describe qualitatively the means by which they can perform work.

S4P-1-29 Derive the equation for kinetic energy using $W = \vec{F}\ \Delta\vec{d}\ \cos\theta$ and kinematics equations.

S4P-1-30 Derive the equation for gravitational potential energy near the surface of the Earth ($E_p = mgh$).

S4P-1-31 Experiment to determine Hooke's Law $\left(\vec{F} = -k\,\vec{x}\right)$.

S4P-1-32 Derive an equation for the potential energy of a spring, using Hooke's Law and a force-displacement graph.

S4P-1-33 Solve problems related to the conservation of energy.

Include: gravitational and spring potential, and kinetic energy

General Learning Outcome Connection

Students will…

Recognize that scientific knowledge is based on evidence, models, and explanations, and evolves as new evidence appears and new conceptualizations develop (GLO A2)

Specific Learning Outcome

S4P-1-25: Define work as the product of displacement and the component of force parallel to the displacement when the force is constant.

Suggestions for Instruction

Entry Level Knowledge

In Senior 3 Physics, students developed an understanding of net force and of concepts of force of friction, normal force, gravitational force, and applied forces.

Notes to the Teacher

Newton's laws provide a useful means for analyzing motion in terms of force, mass, velocity, and acceleration. There are other ways to look at motion. In this topic, motion will be analyzed using the concepts of work and energy.

In physics, work is defined as a **force** acting upon an object, which results in a displacement of the object. Work is a familiar everyday concept. For example, it takes work to push a stalled car, lift a book above the table, or open a door. Force and displacement are the two essential elements of work. Mathematically, the work done on an object by a constant force (constant in both magnitude and direction) is defined as the product of the magnitude of the displacement times the component of the force in the direction of the displacement.

Work = $F_{net}\,\Delta d \cos\theta$, where F_{net} is the magnitude of the constant force, Δd is the magnitude of the displacement of the object, and θ is the angle between the direction of the force and the displacement. Note that if the displacement is zero, the work is zero, even if a force is applied. The θ factor is present because $F_{net}\cos\theta$ is the component of the force in the direction of the displacement. If the angle between the force and the displacement is zero, $\cos\theta = 1$.

Work is a scalar quantity—therefore, it has only a magnitude. The SI unit of work is a Newton-meter called the Joule (J), in honour of James Prescott Joule (1818–1889). However, note that work can be positive or negative, depending on whether work is gained or lost by the system.

Illustrative examples are in Appendix 1.8: Force-Work Relationships.

SKILLS AND ATTITUDES OUTCOME

S4P-0-2c: Formulate operational definitions of major variables or concepts.

GENERAL LEARNING OUTCOME CONNECTION

Students will…

Recognize that characteristics of materials and systems can remain constant or change over time, and describe the conditions and processes involved (GLO E3)

SUGGESTIONS FOR INSTRUCTION

Teaching Notes

SUGGESTIONS FOR ASSESSMENT

Pencil-and-Paper Tasks

Students solve various types of problems involving:

1. single force acting horizontally and at an angle;
2. multiple forces acting horizontally and at angles; or
3. using either/both methods.

General Learning Outcome Connection

Students will…

Demonstrate appropriate scientific inquiry skills when seeking answers to questions (GLO C2)

Specific Learning Outcome

S4P-1-26: Determine work from the area under the force-position graph for any force.

Include: positive or negative force, uniformly changing force

Suggestions for Instruction

Entry Level Knowledge

Students should be able to calculate the areas of a rectangle, triangle, and trapezoid and be familiar with resolving units (i.e., $m/s \times s = m$, displacement quantity unit).

Notes to the Teacher

If the force acting on an object is constant, the work done by that force can be calculated using the equation established in S4P-1-25. If the force varies in magnitude or direction, the work done must be determined by calculating the area under the curve of a force vs. displacement graph. Note, in general, work can always be calculated using the area method, but the product of force and displacement can only be used if the force is constant.

W_1 = Area of a rectangle (Ll)

$= F_1 d_1$ (+ value)

W_2 = Area of a trapezoid ($(B + b)h/2$)

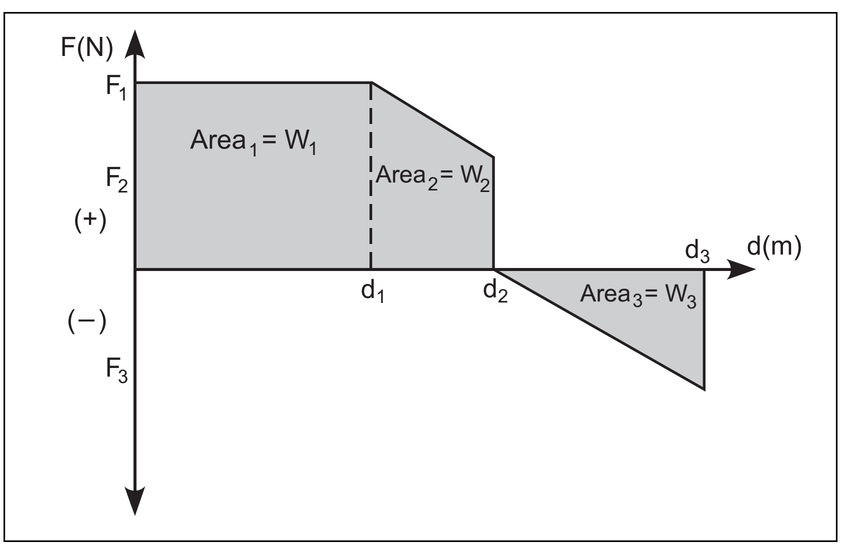
$$= \frac{(F_1 + F_2)}{2}(d_2 - d_1)$$ (+ value)

W_3 = Area of a triangle ($bh/2$)

$$= \frac{F_3(d_3 - d_2)}{2}$$ (having—value)

$W_t = W_1 + W_2 + W_3$

Example:



SKILLS AND ATTITUDES OUTCOMES

S4P-0-2a: Select and use appropriate visual, numeric, graphical, and symbolic modes of representation to identify and represent relationships.

S3P-0-2d: Estimate and measure accurately using SI units.

GENERAL LEARNING OUTCOME CONNECTION

Students will…

Understand how stability, motion, forces, and energy transfers and transformations play a role in a wide range of natural and constructed contexts (GLO D4)

SUGGESTIONS FOR INSTRUCTION

Teaching Notes

SUGGESTIONS FOR ASSESSMENT

Pencil-and-Paper Tasks

Students evaluate total work done given a graph with constant and uniformly changing forces, positive as well as negative.

Collaborative Teamwork

Students exchange stories that describe some form of motion and change in motion. Each group draws free-body diagrams and then qualitatively describes the work done throughout the motion.

General Learning Outcome Connection

Students will…

Describe and appreciate how the natural and constructed worlds are made up of systems and how interactions take place within and among these systems (GLO E2)

Specific Learning Outcomes

S4P-1-27: Describe work as a transfer of energy.

Include: positive and negative work, kinetic energy, conservation of energy

S4P-1-28: Give examples of various forms of energy and describe qualitatively the means by which they can perform work.

Suggestions for Instruction

Entry Level Knowledge

Students should understand that the +/– sign assigned to work does not imply direction since work is a scalar quantity. Learning outcome S4P-1-1 explains that positive work results if work is gained, and negative work results if work is lost.

Notes to the Teacher

In all cases, an object must possess some form of energy to do work. Commonly, an agent doing the work (a student, a car, a batter, a motor) will have **chemical potential energy** stored in food or fuel, which is transformed into work. In the process of doing work, the objects doing the work exchange energy—one gains and one loses. A force doing positive work on an object will transfer its energy to the object and increase the energy of that object. A force doing negative work will decrease the energy of that object and will have energy transferred from the object to the agent exerting the force. Energy is conserved by simply being transferred from the agent to the object or vice versa, depending on the direction of the force relative to the displacement. We call energy that is possessed by an object, due to its motion or its stored energy of position, **mechanical energy**.

For example, when you lift an object, potential chemical energy in the glucose molecules in your muscles allow you to do work on the object. Initially, the work done by you is converted into gravitational potential energy as the object rises, as well as kinetic energy as the object is put in motion. In the end, the gain in energy equals the loss in potential chemical energy in your muscles. A curling rock once released has a certain speed and therefore kinetic energy, which will gradually be converted into thermal energy as friction acts in the opposite direction of the movement of the curling rock.

Different types of energy have one thing in common: they imply a system capable of doing work. Additionally, all types of energy may be subdivided into two different categories: potential energy (E_p or U) or kinetic energy (E_k).

Potential energy is energy stored in a system, which could be used to do work. For example, energy behind a dam has potential gravitational energy (E_g or U_g), which can be used to do work to turn the turbines. A stretched spring or elastic or a compressed spring has potential elastic energy, which can be used to do work to propel an object. A gas has heat of vaporization, which can be released to do work when it condenses, as in a steam engine. A combustible such as propane or octane has potential chemical energy, which can be released to do work once it is burned in an engine.

SKILLS AND ATTITUDES OUTCOMES

S4P-0-2a: Select and use appropriate visual, numeric, graphical, and symbolic modes of representation to identify and represent relationships.

S3P-0-4b: Work co-operatively with a group to identify prior knowledge, initiate and exchange ideas, propose problems and their solution, and carry out investigations.

GENERAL LEARNING OUTCOME CONNECTION

Students will…

Recognize that energy, whether transmitted or transformed, is the driving force of both movement and change, and is inherent within materials and in the interactions among them (GLO E4)

SUGGESTIONS FOR INSTRUCTION

Kinetic energy is energy by virtue of being in motion. Kinetic energy can be used to do work when one thing interacts with something else. For example, at the macroscopic level (visible): a swinging bat has kinetic energy and does work hitting a ball transferring the kinetic energy to the ball. At the microscopic or invisible level: heat contained in matter is due to the sum of the kinetic energies of all the particles that compose that matter. In gaseous matter, the particles are continually in motion; even in a solid, particles have a vibrational motion. The greater the motion of these particles, the greater is the heat content of that matter. The electric current in a wire is simply the motion of electrons in that wire. The kinetic energy of the electrons is called electrical energy, which can be used to do work as in an electric motor. Electromagnetic radiation (radio waves, IR, visible light, UV, X-rays, γ-rays) are photons in motion at a speed of 3.0×10^8 m/s in a vacuum.

SUGGESTIONS FOR ASSESSMENT

Visual Display

Students create posters to describe situations that depict the transfer of energy from one object to another.

General Learning Outcome Connection

Students will…

Recognize that scientific knowledge is based on evidence, models, and explanations, and evolves as new evidence appears and new conceptualizations develop (GLO A2)

Specific Learning Outcome

S4P-1-29: Derive the equation for kinetic energy using

$W = \vec{F}\ \Delta\vec{d}\ \cos\theta$

and kinematics equations.

S4P-1-30: Derive the equation for gravitational potential energy near the surface of the Earth ($E_p = mgh$).

Suggestions for Instruction

Entry-Level Knowledge

Kinematic equations in S4P-1-1 are used to derive the equation for kinetic energy.

Notes to the Teacher

In the section on momentum, a new variable was introduced from the product FΔt, which we called **impulse**. This variable is equivalent to the change in momentum, $m\Delta v$; i.e., F$\Delta t = m\Delta v = m(v_f - v_i)$. Since the mass travels Δd during the time t, not only is an impulse given to the mass, but work equivalent to FΔd is also done on the mass. We know from the previous section that this work becomes kinetic energy if the mass is subjected to a constant force on a horizontal surface with no friction present. As with momentum, we will derive an expression in terms of m and v, equivalent to the work done on the mass.

Since a constant force acting on a mass results in a uniformly accelerated motion, and we want to include Δd in our equation instead of Δt, let us begin our derivation with the following kinematic equation:

$$v_f^{\ 2} = v_i^2 + 2a\Delta d$$

or $$v_f^{\ 2} - v_i^2 = 2a\Delta d$$

or $$a\Delta d = \frac{v_f^{\ 2}}{2} - \frac{v_i^2}{2} \qquad (1)$$

Multiply both sides of the equation (1) by m,

$$ma\Delta d = \frac{mv_f^{\ 2}}{2} - \frac{mv_i^2}{2} \qquad (2)$$

Since F = ma, rewrite (2) as

$$F\Delta d = \frac{mv_f^{\ 2}}{2} - \frac{mv_i^2}{2} \qquad (3)$$

where FΔd represents the work (W) done on mass, and m and 1/2 mv^2 corresponds to kinetic energy (E_k).

Equation (3) can therefore be rewritten as

$$W = E_{cf} - E_{ci} = \Delta E_c$$

For an object of mass, m, being pulled a distance, Δd, by a force, F, on a horizontal surface where no friction is present, the work done on the object is equivalent to a change in its kinetic energy due to a change in its speed from v_i to v_f.

In terms of units:

$$W = N \cdot m = J$$

$$E_c = kg \cdot \left(\frac{m}{s}\right)^2 = kg \cdot \frac{m^2}{s^2} = \frac{kg \cdot m}{s^2} \cdot m = N \cdot m = J$$

SKILLS AND ATTITUDES OUTCOME

S4P-0-2g: Develop mathematical models involving linear, power, and/or inverse relationships among variables.

GENERAL LEARNING OUTCOME CONNECTION

Students will…

Understand the composition of the universe, the interactions within it, and the impacts of humankind's continued attempts to understand and explore it (GLO D6)

SUGGESTIONS FOR INSTRUCTION

Kinetic energy was at the centre of a debate that took place at the beginning of the 18th century. Descartes insisted that momentum was the most important quantity in dynamics, while Leibniz believed that the important quantity was kinetic energy. The question was, what is more important: the time a force acts or the distance through which a force acts?

Near the surface of the Earth, the gravitational force can be considered to be constant. Therefore, the amount of work done to lift an object of mass, m, a height, h, is $W = F\Delta d = Fh$ where F is the average force to lift the object; i.e., its weight, F_g. That work becomes gravitational potential energy (E_g or U_g).

Therefore, $E_g = F_g h$

$$E_g = mgh, \text{ where } g = 9.81 \text{ N/kg}.$$

E_g represents the potential gravitational energy of mass, m, from a height, h.

In terms of units,

$$E_g = kg \cdot \frac{m}{s^2} \cdot m = \frac{kg \cdot m}{s^2} \cdot m = N \cdot m = J$$

SUGGESTIONS FOR ASSESSMENT

Students use process notes to outline the steps for the derivation of the energy equations.

Students compare/contrast work-energy to impulse momentum.

Students make simple calculations for the energy of various objects in motion or at rest above the surface of the Earth.

General Learning Outcome Connection

Students will…

Demonstrate appropriate scientific inquiry skills when seeking answers to questions (GLO C2)

Specific Learning Outcome

S4P-1-31: Experiment to determine Hooke's law

$\left(\vec{F} = -k\,\vec{x}\right)$.

Skills and Attitudes Outcomes

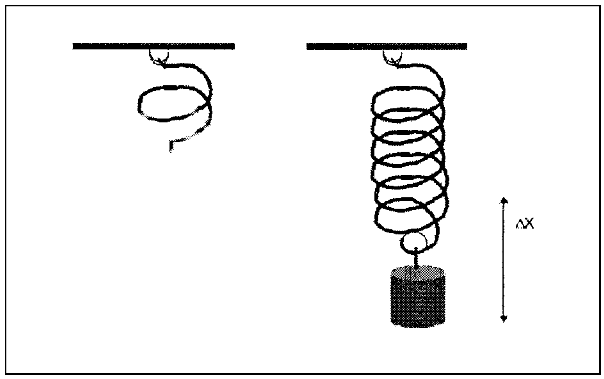
S4P-0-1e: Differentiate between how scientific theories explain natural phenomena and how scientific laws identify regularities and patterns in nature.

S4P-0-2a: Select and use appropriate visual, numeric, graphical, and symbolic modes of representation to identify and represent relationships.

Suggestions for Instruction

Notes to the Teacher

Hooke's experiment is very common and very simple to perform. Suspend a spring from a fixed support. Add mass to the spring in equal increments (to increase the force). Record the stretch of the spring and graph force versus stretch (x). The slope of the line is the "k" spring constant. The spring constant is large when you are using very stiff springs (like springs in a car) and small for springs that stretch very easily.



Skills and Attitudes Outcomes

S4P-0-2b: Propose problems, state hypotheses, and plan, implement, adapt, or extend procedures to carry out an investigation where required.

S4P-0-2c: Formulate operational definitions of major variables or concepts.

S4P-0-2d: Estimate and measure accurately using SI units.

S4P-0-2e: Evaluate the relevance, reliability, and adequacy of data and data-collection methods.
Include: discrepancies in data and sources of error

S4P-0-2f: Record, organize, and display data using an appropriate format.
Include: labelled diagrams, tables, graphs

S4P-0-2g: Develop mathematical models involving linear, power, and/or inverse relationships among variables.

General Learning Outcome Connection

Students will…

Demonstrate curiosity, skepticism, creativity, open-mindedness, accuracy, precision, honesty, and persistence, and appreciate their importance as scientific and technological habits of mind (GLO C5)

Suggestions for Instruction

Teaching Notes

Suggestions for Assessment

Students collect and analyze data, and submit a lab report.

Suggested Learning Resources

Lab 5.3: Hooke's Law, p. 192, *Physics: Concepts and Connections*, Irwin Publishing Ltd., 2003

General Learning Outcome Connection

Students will…

Demonstrate appropriate critical thinking and decision-making skills when choosing a course of action based on scientific and technological information (GLO C4)

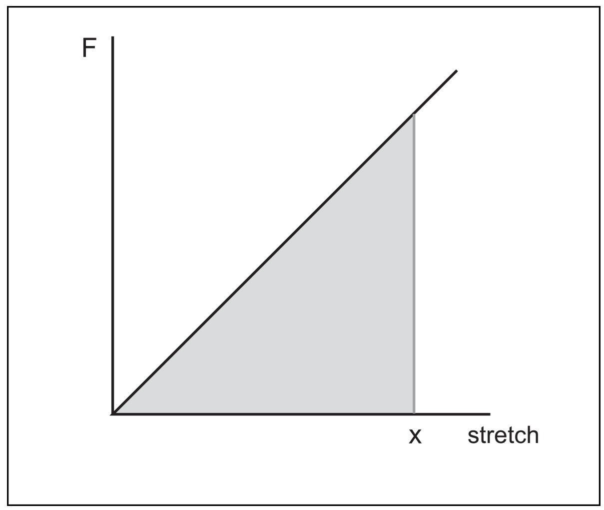
Specific Learning Outcome

S4P-1-32: Derive an equation for the potential energy of a spring, using Hooke's Law and a force-displacement graph.

Suggestions for Instruction

Notes to the Teacher

If you apply a force to a spring to stretch it, then you must be doing work since you are applying the force over a displacement. Since work is the transfer of energy, we say that energy is transferred into the spring. The work becomes stored potential energy in the spring. A spring can be stretched or compressed.



A linear restoring force stretches a spring in direct proportion to the amount of stretch. That is, the force vs. stretch graph forms a straight line that passes through the origin. The work is the area between the line and the axis.

$$\text{Work} = \text{area of triangle}\left(\frac{1}{2}bh\right)$$

$$= \frac{1}{2}Fx$$

Since $F = kx$

$$\text{Work} = \frac{1}{2}kx^2$$

$$\text{Spring Potential}\left(E_s \text{ or } U_s\right) = \frac{1}{2}kx^2$$

SKILLS AND ATTITUDES OUTCOME

S4P-0-2g: Develop mathematical models involving linear, power, and/or inverse relationships among variables.

GENERAL LEARNING OUTCOME CONNECTION

Students will…

Evaluate, from a scientific perspective, information and ideas encountered during investigations and in daily life (GLO C8)

SUGGESTIONS FOR INSTRUCTION

Teaching Notes

SUGGESTIONS FOR ASSESSMENT

Students use process notes to outline the steps for the derivation of spring potential energy.

Students compare/contrast spring and gravitational potential energy.

Students make simple calculations for the energy of various objects in motion, at rest above the surface of the Earth, or in terms of the stretch or compression of a spring.

General Learning Outcome Connection

Students will…

Demonstrate appropriate scientific inquiry skills when seeking answers to questions (GLO C2)

Specific Learning Outcome

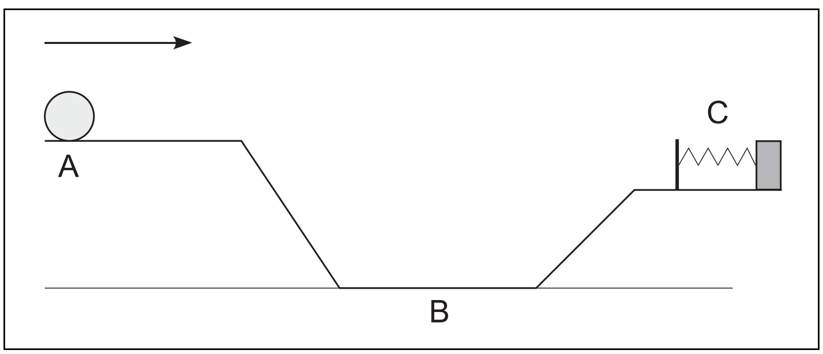
S4P-1-33: Solve problems related to the conservation of energy.

Include: gravitational and spring potential, and kinetic energy

Suggestions for Instruction

Notes to the Teacher

Various combinations of gravitational, kinetic, and spring potential can be found in a problem. Roller-coaster problems can illustrate changes in energy. "Pinball" types of problems can also combine all three forms of energy. For example, find the speed of the ball at point B and the compression of the spring at point C. At point A the energy is gravitational potential and kinetic, at point B the energy is all kinetic, and at point C, for maximum compression of the spring, the energy is gravitational and spring potential.



SKILLS AND ATTITUDES OUTCOME

S4P-0-2g: Develop mathematical models involving linear, power, and/or inverse relationships among variables.

GENERAL LEARNING OUTCOME CONNECTION

Students will…

Demonstrate appropriate scientific inquiry skills when seeking answers to questions (GLO C2)

SUGGESTIONS FOR INSTRUCTION

Teaching Notes

SUGGESTIONS FOR ASSESSMENT

Pencil-and-Paper Tasks

Students solve problems with various combinations of gravitational, spring, and kinetic energies.

Collaborative Teamwork

Students design a roller-coaster and calculate the energy and speed of the coaster at different points. They can also investigate the loop de loop.

Students exchange "pinball" problems. Each group provides the appropriate information and calculates the energy at various points.

Notes