APPENDIX 3: ELECTRICITY ▬▬▬▬

Appendix 3.1: The Historical Development of Ohm's Law

Tapping into Prior Knowledge

What are some characteristics of this new phenomena we call electricity?

• The forces of attraction and repulsion depend on separation.

• Energy is dissipated as heat and shock.

• It can be conducted from one place to another.

• In Senior 1 Science, a particle model electricity was extended to a qualitative study of electric circuits. More particles, more current; more current, more brightness in the bulbs.

Some points to consider:

1. Now we want to develop a quantitative model of current electricity.

2. Note, at this time, Ohm had no model of electricity in mind. His interest was in formulating a mathematical law. This is a form of induction.

3. Ohm knew that electricity could be conducted from one place to another through a wire (see Gray's experiment below).

4. Ohm did not have electrical meters as we know them today. What did he do?

Demonstration: Gray's Experiment

Figure 1 is Gray's initial experiment. He used thread, feathers, and an electroscope. Figure 2 is a classroom demonstration using a tinfoil bit and ordinary copper wire. You can demonstrate that electricity can be conducted over large distances through the wire.
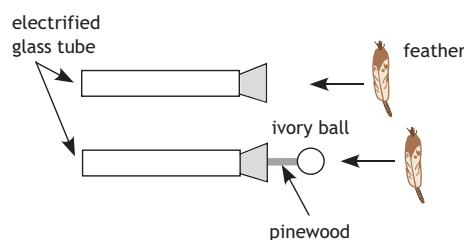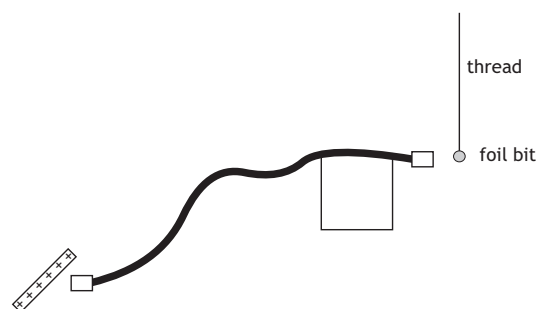

Figure 1

Figure 2

Henry Cavendish and the First Electric Meter

In 1799, Cavendish designed an investigation to qualitatively describe the conductivity of different metals. He used a very simple technique by discharging an electrical apparatus through a wire to his own body. Cavendish rotated a frictional generator with a fixed number of turns to produce the same amount of charge for each trial, and he was able to correctly order the conductivity of the metals by the intensity of shock he received from the generator. Cavendish, whose results were never published in his time, was qualitatively comparing the resistance of different materials to the effects of current.

This type of qualitative analysis was an entertaining pastime in some 18th-century kitchens and parlours—small boys and women have been depicted in paintings passing shocks from electrostatic machines through the familiar "human chain" of resistance. In addition to the obvious risk, resistance, from the earliest times, was demonstrated to be dependent on the length of the conductor. In addition, the spark gap and physiological shock could be related to the intensity of the charge distribution. For example, consider a sphere charged by a fixed number of turns. When discharged, it could produce a spark across a measurable gap. If we use a smaller sphere charged with the same number of turns, we observe that the spark jumps a larger gap. In other words, the same quantity of charge in a smaller volume has a greater "intensity."

This intensity, or "tension," could be more accurately measured by an electroscope and, after Volta's invention of the electric pile in 1800, attempts were made to relate static electricity with other types of electricity. Volta's battery marks a significant conceptual change in our understanding of the electrical phenomenon. The battery remains inactive until an external conductor provides a path through which the electricity is conveyed. Thus, it became imperative to reveal the characteristics and influence of the external conductor. However, at this time, no instrument (other than human sensation) existed that could measure or ascertain the phenomena associated with the conductor.

Fortunately, Cavendish's role as a human meter was no longer necessary by 1820 with Oersted's discovery of the deflection of a compass needle by an electric current. Shortly afterwards, Schweigger used a coil to pass a current repeatedly over a compass to fabricate a more sensitive instrument to detect and compare the electromagnetic effects of various currents.

The Tangent Galvanometer

The first reference to a tangent galvanometer appeared in an 1837 paper by Claude Servais Mathias Pouillet (1790–1868). Pouillet used the tangent galvanometer to investigate Ohm's Law and later, James Joule, in 1841, immersed different lengths of wire into cylinders of water to investigate the relationship between the rate of heat dissipation and current.

The tangent galvanometer was modified by Hermann von Helmholtz (1821–94) in 1849. He suggested that two, identical, current-carrying coils be placed parallel to each other to form the arrangement now known as Helmholtz coils. In this arrangement the magnetic field is essentially uniform. The instrument at the left is in the collection at Wesleyan University.

To measure current, Joule used a tangent galvanometer, which was aligned with the North-South meridian such that the magnetic field of the coil was perpendicular to the magnetic field of the Earth. The deflection of the compass needle is the vector sum of the magnetic effects of the Earth and the field of the loop. Therefore,

$$\tan\theta = \frac{B_{loop}}{B_{earth}}$$

Since B_{earth} is constant, it follows that $\tan\theta \; \alpha \; B_{loop}$. By increasing the number of loops, and therefore the current past any given point, we can also establish that $I \; \alpha \; B_{loop}$, and therefore $\tan\theta \; \alpha \; I$ and $\tan\theta$ can be used as a measure of current. Joule's tangent galvanometer proved to be a reliable measure of current, which we can still use today. The use of a tangent galvanometer has the additional advantage of providing an evidential base for the modern galvanometer and ammeter.
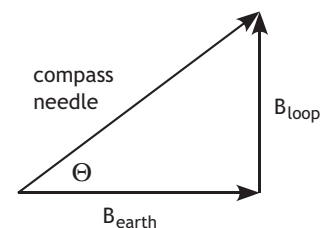
Ohm's Experiment
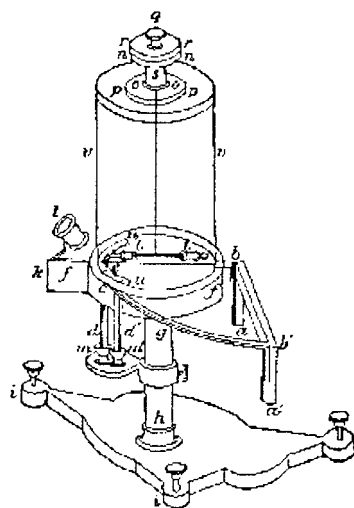
A reliable tangent galvanometer can easily be constructed by looping a wire around a platform that supports a compass. Different lengths of resistance wire are connected to the tangent galvanometer in a manner similar to Ohm's actual experiments. For convenience, a nichrome resistance wire can be laid across a meter stick, and using a sliding contact, different lengths of wire can easily be obtained. The magnetic field of the loop is aligned in a North-South direction and the tangents of the deflections of the compass needle (current) are graphed versus the resistance (measured by length) to confirm that the current is inversely proportional to the resistance of the circuit.
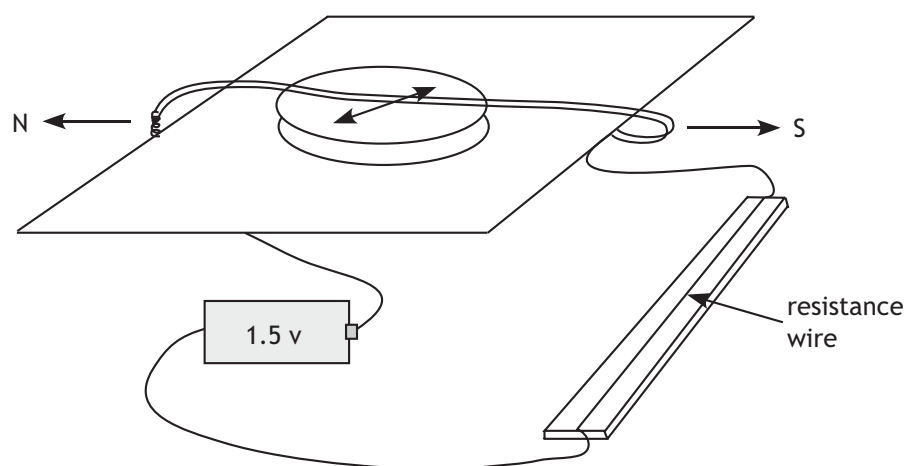
Ohm's Experiment

1. Set up the apparatus as shown. Leave one connection at the battery open (or use a switch to turn the current on and off).

2. Prepare several lengths of resistance wire or use a sliding connection to a resistance wire 1 metre long.

3. Beginning with the longest wire first, connect the circuit and measure the deflection of the compass. Be sure your compass needle moves freely (you may lightly tap the platform to check). Note: disconnect the battery as soon as you record the deflection of the compass needle. Leaving the battery connected could heat the wires and change the resistance.

Data Table

Resistance Length (m)	0.20	0.40	0.60	0.80	1.00
θ **(degrees)**					
tan θ					

4. Graph Current (tan2) versus Resistance (length of wire).

5. "Straighten" the curve to determine the relationship between current and resistance.

6. What does the constant represent?

7. Repeat the experiment using an ammeter instead of a tangent galvanometer.

Conclusion

We have demonstrated that

$$I = \frac{a}{R},$$

where I is current, R is the total resistance, and a is a constant. $R_T = b + x$, where x is the resistance wire and b is the fixed resistance of the circuit. Can you calculate b in terms of length? Proportion of total resistance?

Ohm repeated the experiment with a different temperature difference and found a different value for a. The constant a must be associated with the battery, but what it meant was left for Kirchoff to determine 25 years later. Kirchoff synthesized Coulomb's law, electric potential energy between two charges, Joule's experiment, and Ohm's experiments into a coherent mathematical theory.

# Appendix 3.2: Power, Resistance, and Current

Introduction

James Prescott Joule first explored the relationships among power, resistance, and electric current in 1841. We shall attempt to duplicate his efforts with this experiment.

As current passes through a coil of wire, it generates heat. The heat is transferred to water in a simple calorimeter (Styrofoam™ cup). You will calculate the heat gained by the water and the power of the heating coil. You may then determine the relationships graphically.

Part 1: Power and Resistance

Apparatus

- 6V or 12V battery
- nichrome heating coils
- Styrofoam™ cups and lids
- thermometers
- watch
- plastic straws
- 100 mL graduate
- patch cords
- ohmmeter
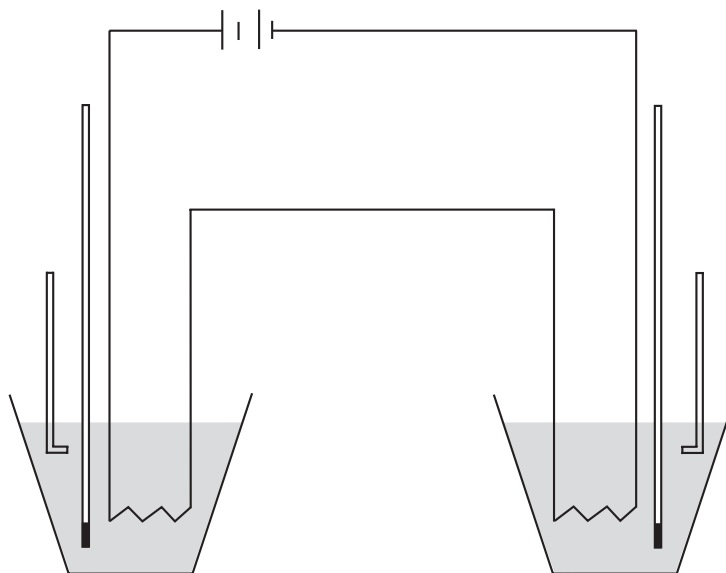
Procedure

1. Prior to the experiment, allow sufficient water to reach room temperature.

2. Pour exactly 100 mL of water into each Styrofoam™ cup. This will provide 100 g of water to be heated.

3. Measure the resistance of each coil. Record in the data table.

4. Assemble the battery and heating coils in series to provide an identical current through each coil. Leave one connection open until you are ready to begin the experiment.

5. Record the initial temperature, to the nearest 0.1° C, for each water sample. Record in the table.

6. Complete the circuit to allow current to pass through the coils. At one-minute intervals, gently stir the water samples with their straws. Observe the temperatures.

7. After sufficient heating has occurred, record the time elapsed and the final temperatures of each sample.

8. Calculate the heat developed from:

   Heat = mass (g) x specific heat x temperature difference (°C)

   $H = mc\Delta T$

   Water has a specific heat of 4.2 J/g°C

   Determine power:

   Power = Energy (heat)/time

   (Time must be measured in seconds.)

9. Plot a graph of power vs. resistance and graphically determine the relationship.
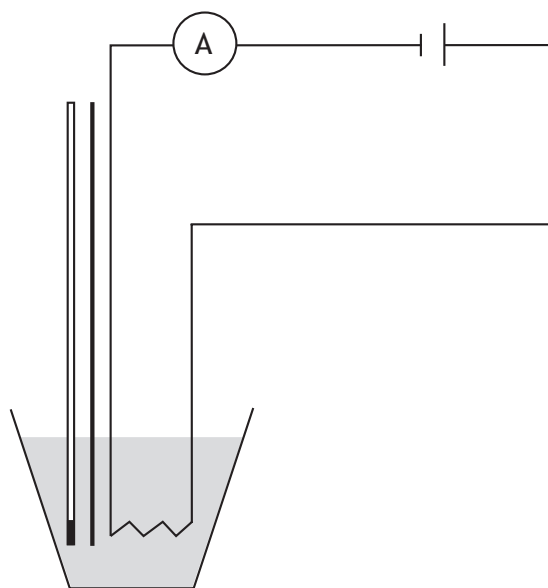
Resistance (Ω)	Mass of Water (g)	Time (s)	Temperature (°C)			Heat (J)	Power (W)
			T_i	T_f	ΔT		

Part 2: Power and Current

Apparatus

- 6V battery
- heating coils of identical resistance
- resistance coil
- patch cords
- ammeter
- thermometer
- watch (or digital thermometer and LabPro)
- 100 mL graduated cylinder
- Styrofoam™ cups and lids

Procedure

1. Prior to the experiment, allow about one litre of water to reach room temperature.

2. Pour exactly 100 mL of water into each Styrofoam™ cup. Place the heating coil and thermometer arrangement into the water and obtain an initial temperature reading to the nearest 0.1° C.

3. Prepare the circuit to deliver current to the heating coil and resistance coil in parallel. Be sure to connect the ammeter in series with the heating coil only.

4. Connect the D cell and begin recording time, current, and temperature readings every minute. Gently stir the water for a few seconds prior to reading the thermometer. Collect data until a temperature rise of at least a few degrees is noticed. Record in the data table.

5. Use a fresh sample of water and another heating coil. Adjust the resistance coil to a different value so a different current will pass through the heating coil. Repeat Step 4 above. Perform several different trials, using very different currents in identical heating coils.

6. Calculate the heat developed from:

   Heat = mass (g) x specific x temperature heat difference (°C)

   H = mcΔT

   Water has a specific heat of 4.2 J/g °C

7. Determine power:

   Power = Energy (heat)/time (Time must be measured in seconds.)

8. Plot a graph of power vs current. Perform graphical manipulations as required to determine the relationship between power and current.

Data Table: Power and Current

Current (A)	Mass (g)	Temperature (°C)			Heat (J)	Time (s)	Power (W)
		T_i	T_f	ΔT			

Appendix 3.3: Kirchoff's Contribution

By experimentation Ohm found that in an electric circuit

$$I = \frac{a}{R},$$

where *a* is a constant of proportionality. Later, Joule demonstrated that $P = I^2R$. Both experiments are examples of scientific laws developed inductively from observations of the behaviour of electric circuits. Additionally, both experiments are concerned with charges in motion. Kirchoff started with the energy considerations of static charges and showed the following.

Electric Potential

Electric potential is the electric potential energy ***per unit charge***.

$$V = \frac{E_e}{Q}$$

Therefore, $E_e = QV$

We also know that the work done moving a charge in a field is the change in energy.

$$W = \Delta E_e$$

Therefore, the change in energy between points *a* and *b*

$$E_b - E_a = QV_b - QV_a$$

or

$$\Delta E_e = QV_b - QV_a$$
$$\Delta E_e = Q(V_b - V_a)$$
$$\Delta E_e = Q\Delta V$$

where ΔV is called **Electric Potential Difference (voltage)**. Since we only deal with changes in energy, this is the most important term. Now, if we divide both sides of the equation by time

$$\frac{\Delta E_e}{t} = \frac{Q}{t}\Delta V$$
$$P = I\Delta V$$

That is, the power delivered (dissipated) in a circuit is the current times the voltage. (In terms of charged particles, why does this make sense?) Recall Joule,

$$P = I^2R$$

Therefore, $I^2R = I\Delta V$, and

$$I = \frac{\Delta V}{R}$$

Thus, Kirchoff demonstrated that if the constant in Ohm's Law was voltage, that everything we knew about in terms of static and dynamic electricity fit together in a coherent theoretical and practical system (in other words, it is a good theory).

Determine the resistance, current, voltage, and power for series, parallel, and combined networks.

Circuit analysis would begin with simple series and simple parallel circuits. The following is an example of the analysis of a combined network or complex circuit.

In the circuit below, determine the current, the voltage drop, and the power consumed by each resistor.
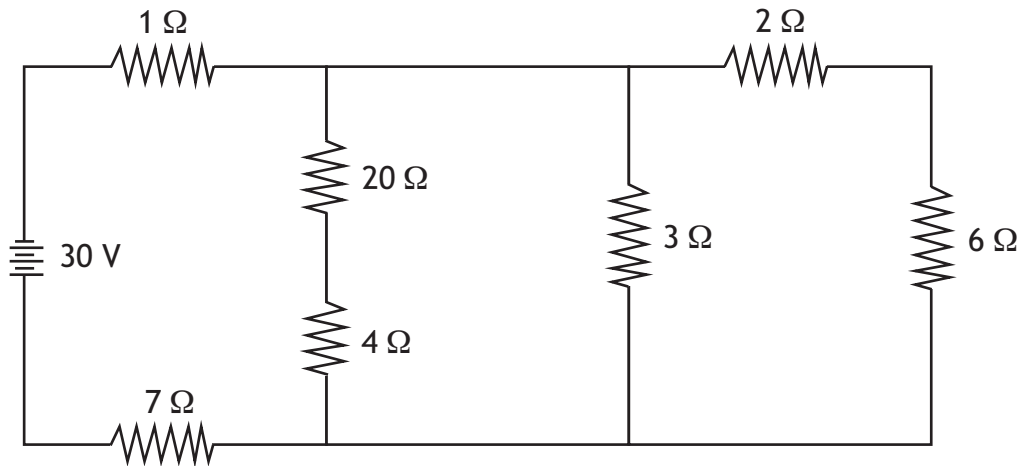

The circuit is neither a simple series nor a simple parallel connection of resistors. It contains both groupings, so it is an example of a complex circuit.

Generally, it is necessary to determine the total or equivalent resistance of the circuit before other electrical quantities can be calculated. In such a complex circuit, it is necessary to identify the resistors that are in series with each other and those that are in parallel with each other. These groups of resistors can be added together to reduce the number of resistors in the circuit. This process continues until the circuit is reduced to a single resistor.

Usually, begin by looking at resistors furthest from the source. In this case, the 2 Ω and 6 Ω resistor are in series and may be combined to a single value of 8 Ω. At the same time, 20 Ω and 4 Ω may be combined to give 24 Ω.
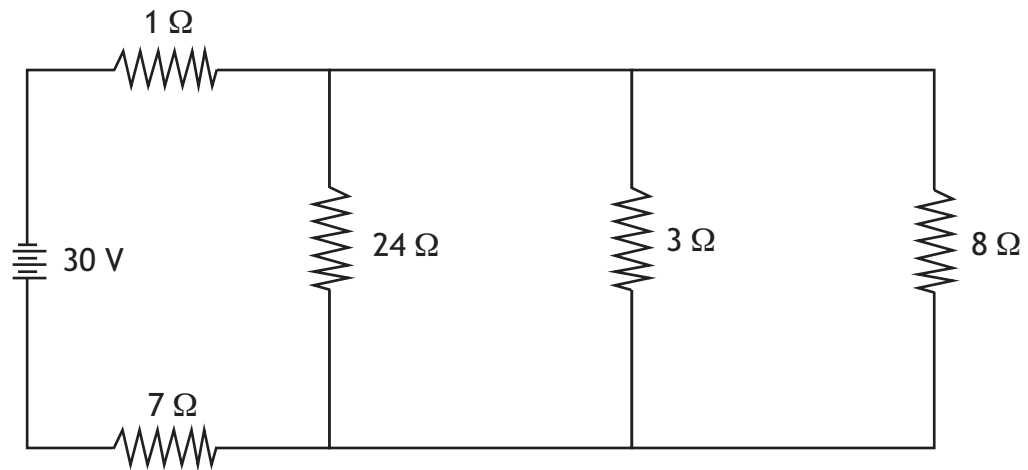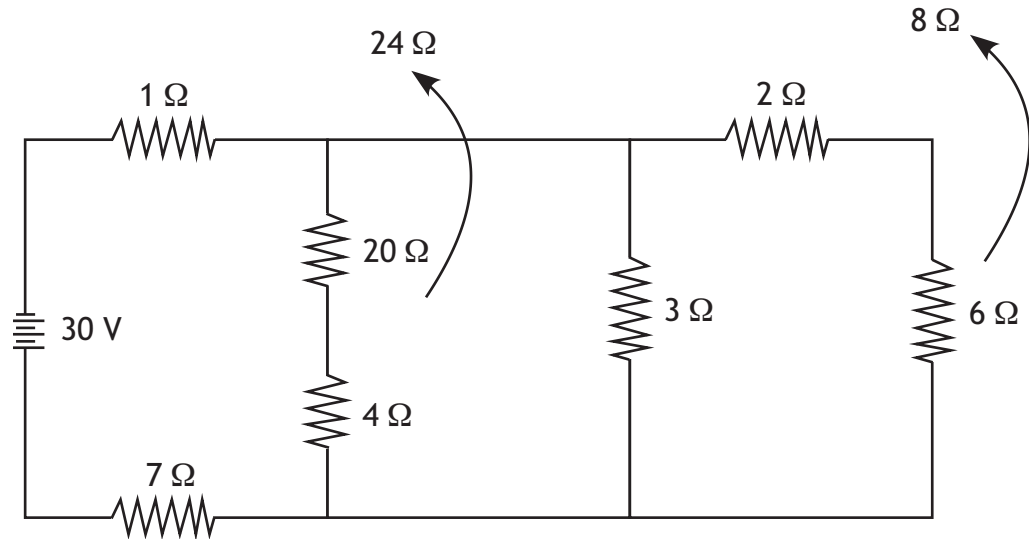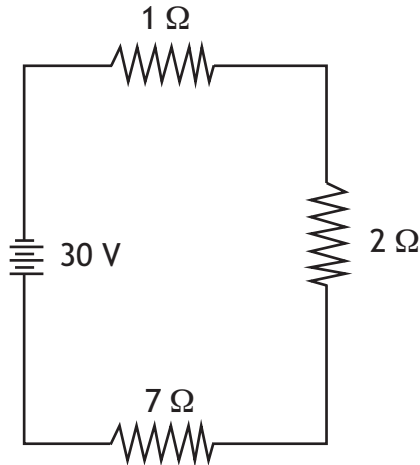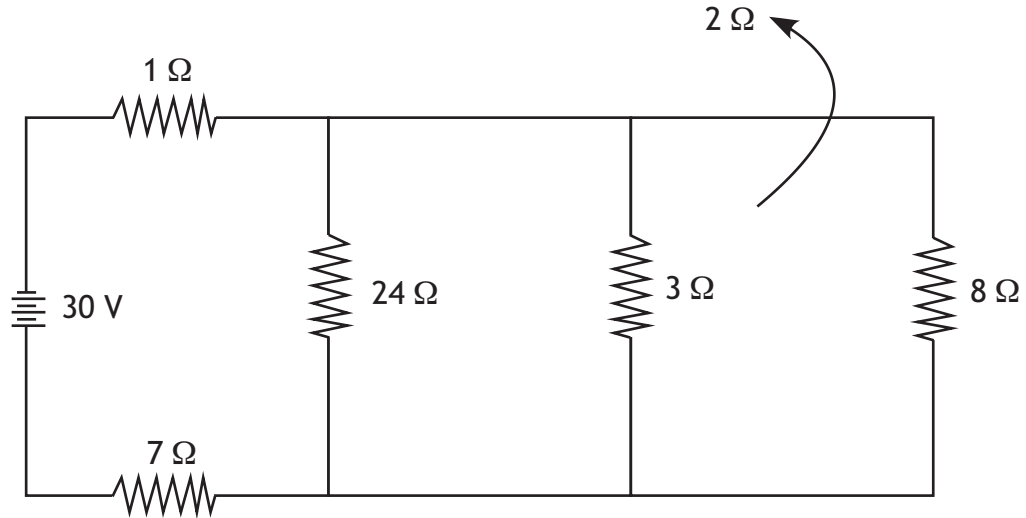
This results in three resistors of 24 Ω, 3 Ω, and 8 Ω in parallel. Combining these in parallel:
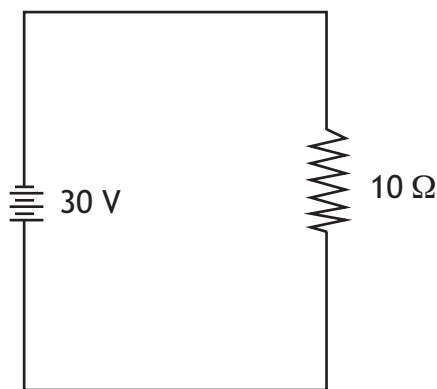
$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{24} + \frac{1}{3} + \frac{1}{8} = \frac{1}{24} + \frac{8}{24} + \frac{3}{24} = \frac{12}{24}$$

$$\therefore\ R_T = \frac{24}{12} = 2\ \Omega$$

2 Ω

1 Ω

30 V

24 Ω   3 Ω   8 Ω

7 Ω

1 Ω

30 V

2 Ω

7 Ω

The circuit now consists of three resistors in series. The total resistance for the entire circuit is $R_T = 1 + 2 + 7 = 10$ Ω.

30 V

10 Ω

This is often called the equivalent resistance, and this simple circuit, consisting of a single source and a single resistor, is known as the equivalent circuit.

The equivalent resistance of 10 Ω draws the same amount of current from the power supply as the original complex network.

To complete the analysis, we work backwards to the original circuit, applying Kirchoff's laws:

• Kirchoff's Current Law: The sum of currents entering a junction must equal the sum of currents leaving that junction.

• Kirchoff's Voltage Law: The sum of potential drops around a circuit must equal the sum of potential rises around the circuit.

Once the total resistance is calculated, find the total current that leaves and returns to the source $I = \dfrac{V}{R} = \dfrac{30\ V}{10\ \Omega}$.

This is called the main line of the circuit since it has the total current in it. It is helpful to draw this in the circuit.
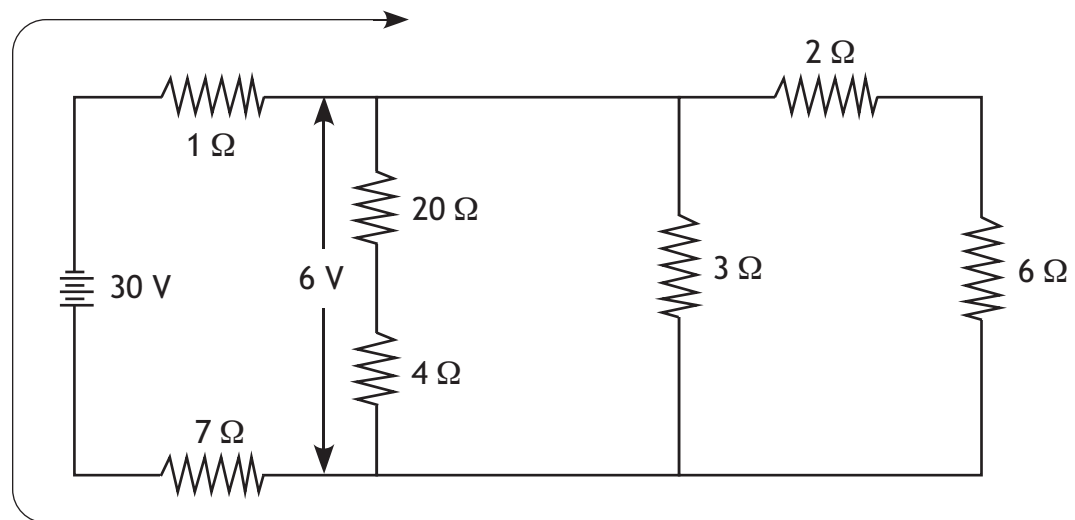
This total current passes through the 1 Ω and 7 Ω resistors. This results in a voltage drop across each of them.

$$V_{drop} = IR = (3\ A)\,(1\ \Omega) = 3\ V \text{ and}$$

$$V_{drop} = (3\ A)(7\ \Omega) = 21\ V$$

The voltage drop left for the remainder of the circuit is 30 *V* – 24 *V* = 6 *V*.

This 6 *V* appears across the three parallel branches so the current in each branch can be determined. Since voltages across parallel resistances are the same:

$$I = \frac{V}{R} = \frac{6\ V}{24\ \Omega} = 0.25\ A$$

$$I = \frac{V}{R} = \frac{6\ V}{3\ \Omega} = 2\ A$$

$$I = \frac{V}{R} = \frac{6\ V}{8\ \Omega} = 0.75\ A$$

The voltage drop across the 20 Ω and 4 Ω resistors may be calculated.

$$V_{drop} = IR = (0.25\ A)(20\ Ω) = 5\ V \text{ across the 20 Ω resistor}$$

The voltage drop across the 4 Ω resistor can be found the same way or by using Kirchoff's Voltage Law. Since 6 *V* appears across both resistors and the 20 Ω resistor has a drop of 5 *V* across it, then the 4 Ω resistor has a drop of 6 *V* – 5 *V* = 1 *V*.

A similar method can be used to find the voltage drops across the 2 Ω and 6 Ω resistors. For the 2 Ω resistor: $V_{drop} = IR = (0.75\ A)(2\ Ω) = 1.5\ V$. The voltage drop across the 6 Ω resistor is 6 *V* – 1.5 *V* = 4.5 *V*.

Use Watt's Law or its two variations to find the power consumed by each resistor:

For the 1 Ω and 7 Ω series resistors in the main line of the circuit:

$$P = IV = (3\ A)(1\ Ω) = 3\ W$$
$$P = IV = (3\ A)(7\ Ω) = 21\ W$$

For the 20 Ω and 4 Ω resistors that are in series with each other, but part of the parallel group:

$$P = I^2R = (0.25\ A)^2(20\ Ω) = 1.25\ W$$
$$P = I^2R = (0.25\ A)^2 (4\ Ω) = 0.25\ W$$

The 3 Ω resistor has a voltage drop of 6 *V* across it.

$$P = \frac{V^2}{R} = \frac{(6\ V)^2}{3\ Ω} = 12\ W$$

The 2 Ω and 6 Ω resistors are in series and have the same current through them.

$$P = I^2R = (0.75\ A)^2(2\ Ω) = 1.125\ W$$
$$P = I^2R = (0.75\ A)^2 (6\ Ω) = 3.375\ W$$

Note that these values may be obtained in different ways by using different variations of Watt's Law, Ohm's Law, or Kirchoff's Laws. This provides the students with excellent problem-solving practice.

Also note that conventions regarding significant figures have not been adhered to in this example. This allows one to verify results obtained by different methods of calculation.

Appendix 3.4: Electromagnetic Induction
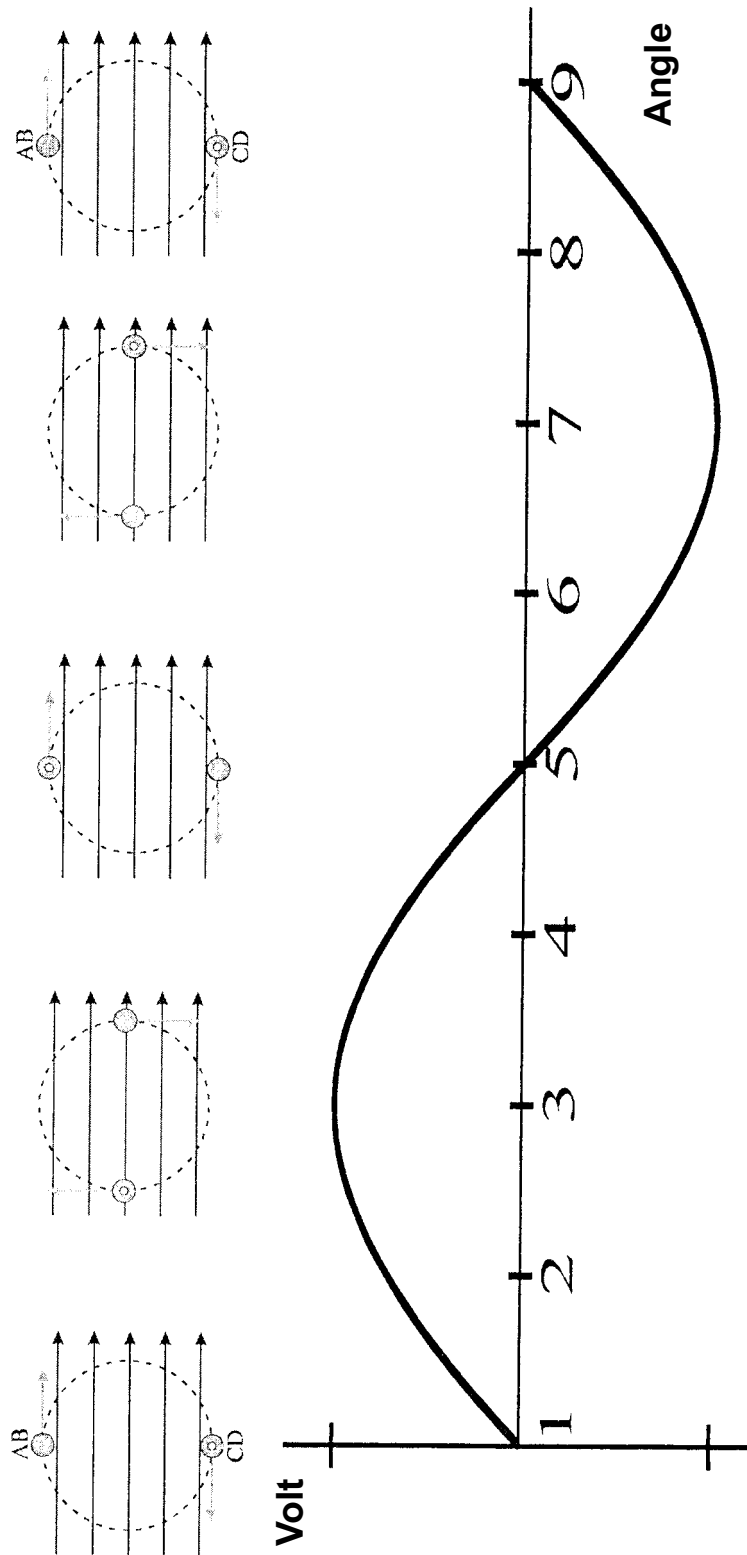
Appendix 3.5: Faraday's Law

$$V = \frac{N\Delta\Phi}{\Delta t}$$

Illustrative Example 1

The magnetic flux through a flat coil of 20 turns changes by 9×10^{-4} webers in 3 milliseconds. Determine the magnitude of the voltage induced in the coil.

$$V = \frac{N\Delta\Phi}{\Delta t} = \frac{(20)(9 \times 10^{-4}\ Wb)}{3 \times 10^{-3}\ s} = 6\ V$$

Illustrative Example 2

A coil of 120 turns and radius 12 cm is rotating in a 0.055 T magnetic field at 3200 revolutions per minute. What is the magnitude of the voltage produced during the quarter turn of the coil from B being parallel to the normal of the coil to B being perpendicular to the normal of the coil?

For B being parallel to the normal, then $\sin\theta = 1$ and $B_\perp = 0.055\ T$. For B being perpendicular to the normal, $\sin\theta = 0$ and $B_\perp = 0$. So, for this quarter turn, $|\Delta B_\perp| = 0.055\ T$.

The area of the loop is: $A = \pi r^2 = \pi(0.12)^2 = 0.0452\ m^2$

The time for a quarter revolution is: $\dfrac{1\ min}{3200\ rev} \times \dfrac{60\ s}{1\ min} \times 0.25\ rev = 0.00469\ s$
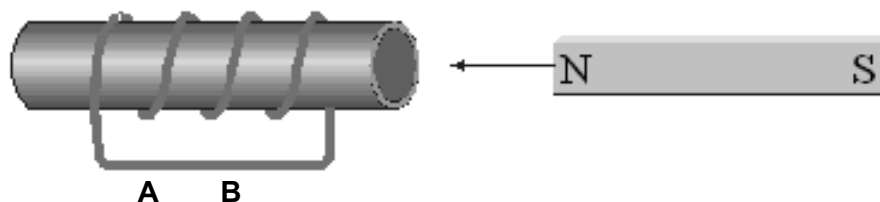
Finally, $V = \dfrac{N\Delta\Phi}{\Delta t} = \dfrac{N(\Delta B)A}{\Delta t} = \dfrac{(120)(0.055\ T)(0.0452\ m^2)}{0.00469\ s} = 64\ V$

Illustrative Example 3

1. A bar magnet is thrust into a solenoid as shown below. Find the direction of current between points A and B.

From Lenz's Law, the induced current in the solenoid establishes a magnetic field that opposes the original changing flux. Therefore, a North pole is induced at the right side of the solenoid and South will be on the left side. Using the right-hand rule for coils, extend the thumb of the right hand in the direction of the magnetic field (remember that inside a solenoid, the magnetic field points from South to North). Point the thumb right and your fingers will curl out of the page on top of the coil and into the page on the bottom of the coil. Therefore, the current will flow from A to B.

2. The switch in solenoid circuit 1 is closed. Find the direction of current between A and B in solenoid 2.
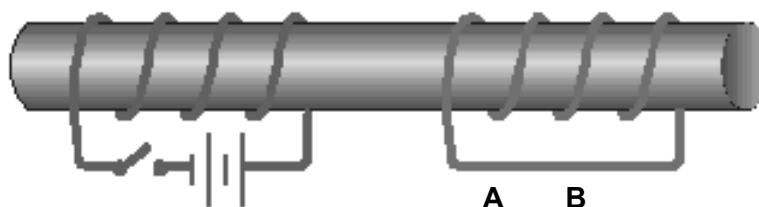
When the switch is closed, the current in solenoid 1 causes a magnetic field to be created with force lines pointing to the right (use the right-hand rule). This field (momentarily changing from 0 to B) induces a current in solenoid 2 such that the magnetic field created by that current opposes the field in solenoid 1. Therefore, solenoid 2 must have field lines that point to the left and the current in solenoid 2 must flow from B to A.

Note: The examples shown use conventional current and right-hand rules. The electron current is actually in the opposite direction.

NOTES