Grade 7 Mathematics

Number

Number (7.N.1)

Enduring Understanding(s):

Number sense and mental mathematics strategies are used to estimate answers and lead to flexible thinking.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
7.N.1 Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10, and why a number cannot be divided by 0. [C, R]	➛ Determine if a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10, and explain why. ➛ Sort a set of numbers based upon their divisibility using organizers, such as Venn or Carroll diagrams. ➛ Determine the factors of a number using the divisibility rules. ➛ Explain, using an example, why numbers cannot be divided by 0.

Prior Knowledge

Students may have had experience with the following:

- Determining addition facts and related subtraction facts (to 18).
- Explaining the properties of 0 and 1 for multiplication, and the property of 1 for division.
- Describing and applying mental mathematics strategies, such as
  - skip-counting from a known fact
  - using doubling or halving
  - using doubling and adding one more group
  - using patterns in the 9s facts
  - using repeated doubling

  to develop recall of basic multiplication facts to 9 × 9 and related division facts.

- Demonstrating an understanding of multiplication to solve problems by
  - using personal strategies for multiplication with and without concrete materials
  - using arrays to represent multiplication
  - connecting concrete representations to symbolic representations
  - estimating products
- Demonstrating an understanding of division to solve problems by
  - using personal strategies for dividing with and without concrete materials
  - estimating quotients
  - relating division to multiplication
  - interpreting remainders
- Identifying and explaining mathematical relationships using charts and diagrams to solve problems.
- Applying mental mathematics strategies for multiplication, such as
  - annexing, then adding zeros
  - halving and doubling
  - using the distributive property
- Demonstrating an understanding of place value for numbers
  - greater than one million
  - less than one-thousandth
- Demonstrating an understanding of factors and multiples by
  - determining multiples and factors of numbers less than 100
  - identifying prime and composite numbers
  - solving problems involving factors or multiples

For more information on prior knowledge, refer to the following resource:

Manitoba Education and Advanced Learning. *Glance Across the Grades: Kindergarten to Grade 9 Mathematics*. Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/cur/math/glance_k-9/index.html.

BACKGROUND INFORMATION

Divisibility

A dividend is considered to be divisible by a divisor if it can be divided by that divisor to make a quotient that is a whole number (with no remainders).

Example:

36 is divisible by 4 because it gives 9 sets, with no remainders.

If a dividend is divisible by a divisor, that divisor is a factor of the dividend.

Example:

Since 36 is divisible by 4, 4 is a factor of 36.

Since the divisor is a factor of the dividend, the dividend is a multiple of the divisor.

Example:

Since 4 is a factor of 36, 36 is a multiple of 4.

If a number is divisible by more than two factors, it is also divisible by the product of any combination of its prime factors.*

Example:

36 is divisible by both 2 and 3.

2 and 3 are both prime factors, so 36 is also divisible by 6, the product of those two prime factors ($2 \times 3 = 6$).

*** Note:**
In Grade 7, students are not formally exposed to prime factorization. It is an achievement indicator in Grade 8 Mathematics in the study of squares and square roots, and a learning outcome in Grade 10 Introduction to Applied and Pre-Calculus Mathematics.

A clear grasp of divisibility is fundamental to achieving many other learning outcomes. It helps students to identify factors and understand relationships between numbers. It makes it easier for them to solve problems, sort numbers, work with fractions, understand percents and ratios, and work with algebraic equations. When students can identify factors with ease, they can readily identify prime and composite numbers, identify common factors and multiples, and find both the greatest common factors and the least common multiples. Understanding divisibility enhances students' ability to rename fractions with common denominators and to represent fractions in lowest terms, thereby making it easier for them to compare fractions and to perform operations with fractions.

If students understand place value and have facility in using mental mathematics strategies and facts, it will be easier for them to find patterns in multiples of factors, to add the digits of multiples, and to recognize numbers that are divisible by a particular factor. Proficiency with these skills will help students to discover divisibility rules, understand and explain why divisibility rules work, and use divisibility rules effectively to determine divisibility.

Understanding divisibility rules and the reasons why the rules work increases students' number sense and their understanding of our number system and patterns within the system. Exploring these relationships and developing divisibility rules or explanations for the rules can be challenging and time-consuming, but will provide students with rich opportunities to practise the mathematical processes of problem solving, reasoning, making connections, and communicating. When selecting learning experiences, verify that students have the required background knowledge and skills, clearly outline the tasks and expectations, provide a warm-up learning activity with the simpler factors (e.g., 2, 5, 10), and provide appropriate hints to guide student discovery without being prescriptive.

Divisibility Rules

Below are some possible divisibility rules for common factors, along with explanations and examples. Provide students with opportunities to make their own discoveries and to develop their understanding through learning experiences, rather than asking them to memorize the divisibility rules.

Divisibility Rules for Common Factors			
Divisible by	**Rule**	**Explanation**	**Examples and Non-examples**
2	The number is even. OR The final digits are 2, 4, 6, 8, or 0.	Even numbers are composed of groups of 2. Therefore, it is necessary only to examine the units (or ones) place when determining divisibility by 2.	▪ 238 is divisible by 2 because the digit in the units place (8) is even. ▪ 89 is not divisible by 2 because the digit in the units place (9) is odd.
3	The sum of the digits is divisible by 3. Continually adding the digits until you end up with a single digit will ultimately result in a total of 3, 6, or 9.	Use place value and the logic of remainders. ▪ Each hundred can be divided into 33 groups of 3 and leaves 1 unit remaining. ▪ Each ten divides into three groups of 3 and leaves 1 unit remaining. ▪ The ones are already individual units. Add all the remaining units (or remainders). If this sum divides evenly by 3, the original number is divisible by 3.	▪ 351 is divisible by 3 because dividing 3 hundreds by 3 leaves 3 units remaining, 5 tens leaves 5 units remaining, and 1 unit is the remainder in the units place. Add up the remainders: 3 + 5 + 1 = 9. Since the remainders are divisible by 3, the entire number is divisible by 3. ▪ 238 is not divisible by 3 because dividing 2 hundreds by 3 leaves 2 units remaining, 3 tens leaves 3 units remaining, and 8 units are the units remainder. Add up the remainders: 2 + 3 + 8 = 13; 1 + 3 = 4. Since 4 is not divisible by 3, the entire number is not.

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Divisibility Rules for Common Factors (continued)			
Divisible by	**Rule**	**Explanation**	**Examples and Non-examples**
4	The number formed by the final two digits is divisible by 4. OR The number formed by the final two digits is divisible by 2 twice.	Use place value logic. 100 is the smallest place value position divisible by 4 (100 ÷ 4 = 25). Any number greater than 100 can be expressed as *x* number of hundreds. Therefore, only the number formed by the digits in the tens and units places must be examined.	▪ 524 is divisible by 4 because 100 is divisible by 4, and so 5 × 100 is divisible by 4, and 24 is divisible by 4. ▪ 490 is not divisible by 4. Although 4 × 100 is divisible by 4, 90 is not divisible by 2 twice (not divisible by 4).
5	The final digit is 0 or 5.	Use place value logic. Every group of 10 forms two groups of 5. Therefore, it is necessary only to examine the units (or ones) place when determining divisibility by 5.	▪ 130 is divisible by 5 because the digit in the ones place (0) is 0 or 5. ▪ 89 is not divisible by 5 because the digit in the ones place (9) is not 0 or 5.
6	The number is even and divisible by 3. OR The number has both 2 and 3 as factors (is divisible by both 2 and 3).	If the number is divisible by both the prime factors 2 and 3, it must also be divisible by 6 because two groups of 3 make a group of 6.	▪ 426 is divisible by 6 because it is divisible by both 2 (it is an even number) and 3 (the sum of its digits is divisible by 3). ▪ 153 is not divisible by 6 because it is not divisible by 2 (it is an odd number), but it is divisible by 3 (the sum of its digits is divisible by 3).
8	The number formed by the final three digits is divisible by 8. OR The number formed by the final three digits is divisible by 2 three times.	Use place value logic. 1000 is the smallest place value position divisible by 8 (1000 ÷ 8 = 125). Any number larger than 1000 can be expressed as *x* number of thousands. Therefore, only the number formed by the digits in the hundreds, tens, and units places must be examined.	▪ 7480 is divisible by 8 because 480 is divisible by 2 three times (480 ÷ 2 = 240; 240 ÷ 2 = 120; 120 ÷ 2 = 60). ▪ 3220 is not divisible by 8 because 220 is not divisible by 8.

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Divisibility Rules for Common Factors (continued)			
Divisible by	**Rule**	**Explanation**	**Examples and Non-examples**
9	The sum of the digits is divisible by 9. OR The number is divisible by 3 twice.	Use place value and the logic of remainders. ▪ Each hundred can be divided into 11 groups of 9 and leaves 1 unit remaining. ▪ Each ten divides into one group of 9 and leaves 1 unit remaining. ▪ The ones are already individual units. Add all the remaining units (or remainders). If this sum divides evenly by 9, the original number is divisible by 9.	▪ 351 is divisible by 9 because dividing 3 hundreds by 9 leaves 3 units remaining, 5 tens leaves 5 units remaining, and 1 unit is the remainder in the units place. Add up the remainders: 3 + 5 + 1 = 9. Since the remainders are divisible by 9, the entire number is. ▪ 418 is not divisible by 9 because dividing 4 hundreds by 9 leaves 4 units remaining, 1 ten leaves 1 unit remaining, and 8 units are the remainder in the units place. Add up the remainders: 4 + 1 + 8 = 13. Since 13 is not divisible by 9, the entire number is not.
10	The final digit is 0.	All written multiples of 10 end in 0.	▪ 130 is divisible by 10 because the digit in the ones place (0) is 0. ▪ 89 is not divisible by 10 because the digit in the ones place (9) is not 0.

The following are ways to show a number cannot be divided by zero:

- Using a calculator to divide a number by zero results in an error message.
- Applying the action of division results in an impossible situation.

Example:

If you have a quantity *x*, how many groups of zero can you make? You would be trying to make groups of zero forever. If you had to share a quantity into zero groups, you would have no groups to share the quantity with. Both scenarios are impossible.

Using the pattern and logic of related facts provides no solution to dividing by zero. Multiplication and division are inverse operations. Think of related statements such as the following:

$4 \times 2 = 8$ and $8 \div 4 = 2$

$0 \times ? = 8$ and $8 \div 0 = ?$ (There is no answer.)

MATHEMATICAL LANGUAGE

addend
Carroll diagram
difference
dividend
divisibility
divisible
divisor
factor
multiple
prime
product
quotient
sum
Venn diagram

LEARNING EXPERIENCES

Assessing Prior Knowledge

Materials:

- BLM 7.N.1.1: Math Language Crossword Puzzle
- grid paper (optional)

Organization: Individual, pairs, whole class

Procedure:

1. Distribute copies of BLM 7.N.1.1: Math Language Crossword Puzzle. Have students read the clues and complete the puzzle. Students may consult references for assistance in using mathematical terms.
2. After giving students sufficient time to complete the puzzle, have them share their answers. Discuss responses as a class. Encourage students to ask questions, explain their responses, or extend the concepts.

Variations:

- Include or omit the word bank found on BLM 7.N.1.1: Math Language Crossword Puzzle.
- Supply students with mathematical terms and have them create clues and puzzles using online puzzle generators or grid paper.

Sample Website:

Crossword Puzzle Games. *Create a Crossword Puzzle.* 2003. www.crosswordpuzzlegames.com/create.html.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Respond correctly to the mathematical terms when hearing or reading them.
- ❒ Use the terms appropriately in comments.

Suggestions for Instruction

- **Determine if a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10, and explain why.**
- **Sort a set of numbers based upon their divisibility using organizers, such as Venn or Carroll diagrams.**

Materials:

- BLM 7.N.1.2: Divisibility Questions
- math journals or notebooks
- BLM 5–8.11: Multiplication Table (optional)
- calculators (optional)

For additional problems, refer to the following books:

Sachar, Louis. *More Sideways Arithmetic from Wayside School*. New York, NY: Scholastic Inc., 1994.

———. *Sideways Arithmetic from Wayside School*. New York, NY: Scholastic Inc., 1989.

Organization: Individual, pairs or small groups, whole class

Procedure:

1. Distribute copies of BLM 7.N.1.2: Divisibility Questions for students to complete. Students will need to use mental mathematics strategies, as well as Venn and Carroll diagrams.
2. Have students work alone at first, answering as many questions as they can.
3. Then have students work in pairs or in small groups to see whether they can answer more questions together, making sure each student is able to explain the solutions.
4. After students have had sufficient time to work on the questions, have them gather as a class to share responses and reasoning.
5. Have each student create a question similar to the questions on the BLM.
6. Have students select one problem to solve, and ask them to explain their solution in their math journals or notebooks.

Variation:

- If necessary, allow some students to use multiplication charts (e.g., BLM 5–8.11: Multiplication Table), array charts, or calculators. Allow students to use the constant key on a calculator to generate multiples.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Use a repertoire of mental mathematics strategies.
- ❒ Multiply and divide numbers.
- ❒ Understand place value and rename numbers (e.g., the number 65 432 has 654 hundreds and 32 units).
- ❒ Identify multiples and factors of a number.
- ❒ Use Venn and Carroll diagrams to represent relations between number sets.
- ❒ Communicate mathematically.

Suggestions for Instruction

- **Determine if a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10, and explain why.**
- **Sort a set of numbers based upon their divisibility using organizers, such as Venn or Carroll diagrams.**

Materials:

- display board
- calculators

Organization: Whole class

Procedure:

1. Challenge the class to a teacher-versus-students contest to see who can determine whether a particular number is divisible by a designated factor (1 to 10).
2. Name a factor.
3. Have a student secretly write a number (two to six digits) on a display board.
4. Designate two students to use calculators to verify the correct response. (Yes, it is divisible, or no, it is not divisible.)
5. Reveal the number and begin the contest.
6. Whoever replies correctly first, wins.
7. Keep score if you wish (teacher versus students). Before long, you will dazzle them with your speed, and pique their curiosity. Stop the game when it becomes evident to students that there is a "trick" to this.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❐ Sort numbers according to their divisibility.
- ❐ Apply reasoning skills and knowledge of number properties and operations to develop a personal meaning for the divisibility of a number by 2, 3, 4, 5, 6, 8, 9, or 10.
- ❐ Communicate mathematically.

Suggestions for Instruction

- **Determine if a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10, and explain why.**
- **Sort a set of numbers based upon their divisibility using organizers, such as Venn or Carroll diagrams.**

Materials:

- 100 Board, available from the following website:
  Manitoba Education. "Middle Years Activities and Games." *Mathematics*.
  www.edu.gov.mb.ca/k12/cur/math/my_games/index.html.
- BLM 5–8.6: Blank Hundred Squares
- counters
- pencil crayons or markers
- math journals or notebooks
- calculators
- Venn diagram or Carroll diagram templates (optional)

Organization: Whole class, individual, pairs, small groups

Guided Discovery:

As some of the explorations in this three-part learning experience may not be intuitive for students, the following suggestions are meant to help you prepare for this guided discovery:

- Students can use patterns and relations to create generalizations about numbers that are divisible by the factor, and then test the universality of their predictions.
- Challenge students to explain why their divisibility rules work.
- Lead students to discover simple practical tests that can be applied to any number to determine easily whether the number is divisible by the factors 2, 3, 4, 5, 6, 8, 9, or 10.
- List multiples of a factor. Begin with the first 10 multiples of the factor, and look for patterns. If no patterns appear, extend the list of multiples.
- Circle multiples on either BLM 5–8.6: Blank Hundred Squares or a 100 Board to reveal patterns.
- Look for further patterns or relations that may include place values, even or odd final digits, similarities in the final two or final three digits, and similarities in the sums of digits and their common factors.

Procedure:

The following procedure will take place over the course of several classes.

Part A: Divisibility by 10, 5, and 2

1. As a class, begin with the following question:

   How can we know which numbers are divisible by 10?

   Hints:
   - Choose one colour marker, work with BLM 5–8.6: Blank Hundred Squares, and circle all the multiples of 10. Examine the multiples. What do you notice?
   - Test your idea on larger numbers. Divide numbers ending in zero by 10 to ensure there are no remainders. Then try dividing numbers that do not end in zero by 10 to prove there are remainders.

2. Discuss students' responses to the question. (Using the Think-Pair-Share strategy with a partner may help students prepare for whole-class discussion. Students think about the question individually, discuss their ideas with a partner, and then share their responses with the class.) If students reply that all the ones digits are 0, ask them how they know this or how they could prove it to someone who did not agree.

   The test results could be shown in a Carroll diagram to illustrate the test's reliability.

   Example:

	Divisible by 10	Not Divisible by 10
Numbers with 0 units	All examples will be here	
Numbers with other than 0 units		All examples will be here

3. Ask students why the test works.
4. Following the discussion, have students record their findings in their math journals, using words, pictures, diagrams, or symbols, or create a Divisibility Study Booklet.

   Divisibility rule for the factor ______

   This rule works because . . .

5. Repeat steps 1 to 4 for the following question:

   How can we know a number is divisible by 5?

   Aim for less teacher guidance and more student control of the discussion. Remind students to write a summative journal entry.

6. Repeat steps 1 to 4 for the following question:

   How can we know a number is divisible by 2?

Again, aim for less teacher guidance and more student control of the discussion. Remind students to write a summative journal entry.

Part B: Divisibility by 4, 8, 3, 9, and 6

After working through the examples in Part A, students will have a general idea of how to determine whether a number is divisible by a specific factor. Further ideas on how to proceed are provided below, in a suggested order.

1. How can we know which numbers are divisible by 4?

   This is a subset of the numbers divisible by 2, so students can continue their findings about factors of 2. Every second multiple of 2 is a factor of 4. It may be helpful to consider that 100 is divisible by 4.

2. Determine whether a number is divisible by 8.

   Simply apply and extend the same strategies. 1000 is divisible by 8. Show multiples of 2 and 4 and 8 in a Venn diagram.

3. Determine whether a number is divisible by 3.

   Consider making groups of 3 for each place value in each multiple, and then include multiples with three or more digits. Repeat the process with numbers that are not multiples of 3. Consider what happens to the remaining units as you group each place value. Next, examine the sums of the digits in the multiples. Arrange the multiples according to the sums. Look for patterns. Put patterns into charts, or sort them with Venn or Carroll diagrams.

4. Determine whether a number is divisible by 9.

   Apply the strategies used for the factor 3.

5. Determine whether a number is divisible by 6.

   This set contains numbers divisible by both 2 and 3. Consider using a Venn diagram to show the intersection.

Ensure students record their rules and explanations in their math journals or notebooks, or in their Divisibility Study Booklets, and that they make any desired additions or changes during whole-class discussion.

Part C: Communicating Ideas

1. Inform students that to celebrate and consolidate the learning resulting from this inquiry, they will create a display of their divisibility rules, with brief explanations of why their rules work.
2. Allow displays to be small and personal. They may be created individually or in small groups, with tasks divided among group members. Create a larger wall display.
3. The class could organize a divisibility challenge event.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Sort numbers according to their divisibility.
- ❒ Apply reasoning skills and knowledge of number properties and operations to develop a personal meaning for the divisibility of a number by 2, 3, 4, 5, 6, 8, 9, or 10.
- ❒ Communicate mathematically.

Suggestions for Instruction

- **Explain, using an example, why numbers cannot be divided by 0.**

Materials:

- calculators
- math journals or notebooks
- counters

Organization: Individual

Procedure:

1. Let students know that they will explore divisibility by zero.
2. Ask students to divide numbers by zero on their calculators and record the results in their math journals. Have them explain why they think there is an error message.
3. Ask students to review the meaning of *division* and to model division by some factor using concrete materials (e.g., counters) or a diagram. Next, ask students to model division using zero as a factor (divisor) and to record their discovery in their math journals.
4. Ask students to create a table of multiplication facts for a given product, and then to write the related division statements. Next, ask students to find 0 × ___ = that product and the related division fact. (There is no answer.)

Example:

Multiplication Facts	Related Division Facts
4 × *3* = 12	12 ÷ 4 = *3*
3 × *4* = 12	12 ÷ 3 = *4*
2 × *6* = 12	12 ÷ 2 = *6*
1 × *12* = 12	12 ÷ 1 = *12*
0 × *not possible* = 12	12 ÷ 0 = *not possible*

5. Have students record in their math journals what they have discovered about dividing by zero, and then ask them to answer the following question:

   Based on what you have discovered, do you think there should be a divisibility rule for zero? Explain your response.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Determine that a number cannot be divided by zero.
- ❒ Explain, using examples, that a number cannot be divided by zero.

Suggestions for Instruction

- **Determine if a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10, and explain why.**
- **Determine the factors of a number using the divisibility rules.**

Materials:

- BLM 7.N.1.3: Applying Divisibility Rules
- display board
- pens or markers of different colours
- math journals or notebooks

Organization: Individual, pairs or small groups, whole class

Procedure:

1. Distribute copies of BLM 7.N.1.3: Applying Divisibility Rules.
2. Write 10 numbers on the display board.
3. Allow students to choose any five of the numbers and complete the table provided on BLM 7.N.1.3: Applying Divisibility Rules.
4. After giving students time for individual work, assign pairs or small groups of students one of the questions to present to the class. Have them meet to discuss their response and to determine how they arrived at their answer, how they know their answer is correct, and how they will present their question and response.
5. During the class presentations, the audience members may ask questions of the presenters, or share their opinions. They may make desired adjustments to their own papers as they participate in the discussion. (Using writing instruments of different colours reveals what new knowledge or connections were acquired during that time.)
6. Have students respond to the following in their math journals:

    Determine which digit(s) could be placed in the following number to make it divisible by 2, 3, and 9:

        23 8__4

    Explain your thinking.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Demonstrate proficiency with a variety of divisibility rules.
- ❒ Use divisibility rules to determine factors of numbers.

Number (7.N.2)

Enduring Understanding(s):

The principles of operations and algorithms used with whole numbers also apply to operations with decimals, fractions, and integers.

Number sense and mental mathematics strategies are used to estimate answers and lead students to develop personal algorithms.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
7.N.2 Demonstrate an understanding of the addition, subtraction, multiplication, and division of decimals to solve problems (for more than 1-digit divisors or 2-digit multipliers, technology could be used). [ME, PS, T]	➛ Solve a problem involving the addition of two or more decimal numbers. ➛ Solve a problem involving the subtraction of decimal numbers. ➛ Solve a problem involving the multiplication or division of decimal numbers (for more than 1-digit divisors or 2-digit multipliers, technology could be used). ➛ Place the decimal in a sum or difference using front-end estimation (e.g., for 4.5 + 0.73 + 256.458, think 4 + 256, so the sum is greater than 260). ➛ Place the decimal in a product using front-end estimation (e.g., for $12.33 × 2.4, think $12 × 2, so the product is greater than $24). ➛ Place the decimal in a quotient using front-end estimation (e.g., for 51.50 m ÷ 2.1, think 50 m ÷ 2, so the quotient is approximately 25 m). ➛ Check the reasonableness of answers using estimation. ➛ Solve a problem that involves operations on decimals (limited to thousandths), taking into consideration the order of operations. ➛ Explain, using an example, how to use mental mathematics for products or quotients when the multiplier or the divisor is 0.1 or 0.5 or 0.25.

Prior Knowledge

Students may have had experience with the following:

- Describing and applying mental mathematics strategies for adding two 2-digit numerals, such as
  - adding from left to right
  - taking one addend to the nearest multiple of 10 and then compensating
  - using doubles
- Describing and applying mental mathematics strategies for subtracting two 2-digit numerals, such as
  - taking the subtrahend to the nearest multiple of 10 and then compensating
  - thinking of addition
  - using doubles
- Describing and applying mental mathematics strategies, such as
  - skip-counting from a known fact
  - using doubling or halving
  - using doubling and adding one more group
  - using patterns in the 9s facts
  - using repeated doubling

  to develop recall of basic multiplication facts to 9×9 and related division facts.
- Applying estimation strategies, including
  - front-end rounding
  - compensation
  - compatible numbers

  in problem-solving contexts.
- Applying mental mathematics strategies for multiplication, such as
  - annexing, then adding zeros
  - halving and doubling
  - using the distributive property
- Demonstrating an understanding of multiplication (2-digit numerals by 2-digit numerals) to solve problems.
- Demonstrating an understanding of division (3-digit numerals by 1-digit numerals) with and without concrete materials, and interpret remainders to solve problems.
- Describing and representing decimals (tenths, hundredths, thousandths) concretely, pictorially, and symbolically.
- Demonstrating an understanding of addition and subtraction of decimals (limited to thousandths).

- Demonstrating an understanding of place value for numbers
  - greater than one million
  - less than one-thousandth
- Solving problems involving large numbers, using technology.
- Demonstrating an understanding of percent (limited to whole numbers) concretely, pictorially, and symbolically.
- Demonstrating an understanding of multiplication and division of decimals involving
  - 1-digit whole-number multipliers
  - 1-digit natural number divisors
  - multipliers and divisors that are multiples of 10
- Explaining and applying the order of operations, excluding exponents (limited to whole numbers).
- Developing and applying a formula for determining the
  - perimeter of polygons
  - area of rectangles
  - volume of right rectangular prisms

For more information on prior knowledge, refer to the following resource:

Manitoba Education and Advanced Learning. *Glance Across the Grades: Kindergarten to Grade 9 Mathematics*. Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/cur/math/glance_k-9/index.html.

Background Information

Grade 7 provides an opportunity for students to review decimal concepts, to estimate, and to solve problems using operations with decimal numbers. In Grade 5, students added and subtracted decimals, and in Grade 6, they multiplied decimals by whole numbers and powers of 10 and divided decimals by whole numbers and powers of 10. In Grade 7, students extend multiplication and division to include decimal numbers with 2-digit multipliers and 1-digit divisors. They also explain mental mathematics strategies for multiplying and dividing by 0.1, 0.5, and 0.25.

For many practical situations involving decimals (e.g., calculating tips on restaurant bills, averaging numbers of points or people, purchasing goods sold by area or volume), estimated answers are often preferred over precise calculations. In other instances, such as when numbers are very large or very small, scientific notation is used.*

***Note:**
Scientific notation is no longer formally taught in the mathematics classroom.

Situations that involve many decimal points and require precise answers are often technical in nature, and technology is used to calculate these answers. Because estimates are common in everyday use, they provide an effective approach to teach operations with decimals.

The same principles that students have previously used with operations, estimations, models, and algorithms for whole numbers also apply to decimal numbers. When you choose learning activities, focus on number sense before addressing procedural operations. If students have difficulty with decimal operations, ensure that they have a good understanding of place value and that they understand the role of the decimal point. For example, certain statements (e.g., dividing by 100 means you move the decimal point two places to the left) can actually lead to misconceptions and emphasizes memorization of rules rather than understanding of concepts.

In learning activities that require estimating, provide students with opportunities to develop methods of determining where to position the decimal point in their estimates, and then have them attempt to calculate precise answers. As they work through this process, using their understanding of place value, expanded notation, and operations, they will develop a personal method or algorithm based on meanings. To be reliable, their algorithms must apply to all situations. Invite students to share, explain, and evaluate each other's methods. Then introduce traditional algorithms and explain their methodology and how these link to the students' personal methods. Avoid favouring one method over another; students should be free to select a method, providing it is mathematically sound and allows students to be efficient and accurate.

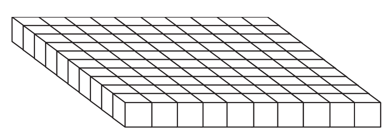
If students understand operations as actions (e.g., addition as a combining operation, subtraction as a taking away operation, multiplication as repeated addition or x groups of, and division as repeated subtraction, or partitioning a number into groups, or how many groups of x are in this number), they can represent the operations concretely or pictorially.

Manipulatives can be used to represent decimal numbers. Operations can be visually represented using number lines, place value mats, and/or grid paper (e.g., BLM 5–8.10: Base-Ten Grid Paper).

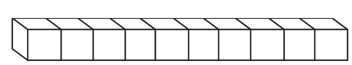
When representing decimals with base-10 blocks, it is necessary to establish which manipulatives represent the value of 1.

Example 1:

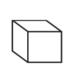
If a flat represents 1 unit,



then a rod can represent $\frac{1}{10}$ or 0.1 unit,

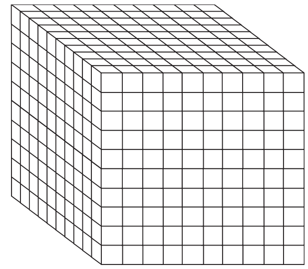


and a cube can represent $\frac{1}{100}$ or 0.01 unit.

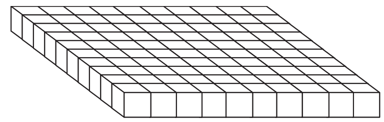


Example 2:

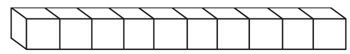
If a large block represents 1 unit,



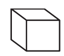
then a flat can represent $\frac{1}{10}$ or 0.1 unit,



and a rod can represent $\frac{1}{100}$ or 0.01 unit,



and a cube can represent $\frac{1}{1000}$ or 0.001 unit.



For illustrations, refer to the Appendix at the end of this document.

MATHEMATICAL LANGUAGE

addend
addition
array
brackets
decimals
difference
dividend
division
divisor
estimation
exponents*
front-end estimation
hundredths

***Note:**
The term *exponent* may arise during discussion of the order of operations, but students are not required to know the term. Students will study exponents formally in Grade 9.

mental mathematics

multiplicand

multiplication

multiplier

order of operations

parentheses

product

quotient

subtraction

sum

tenths

thousandths

LEARNING EXPERIENCES

Assessing Prior Knowledge

Materials:

- list of mathematical terms
- paper for personal or class posters
- reference books

Organization: Individual or pairs

Procedure:

1. Have student create posters that illustrate the terms in the mathematical language list provided.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Illustrate and use mathematical terms correctly.

Assessing Prior Knowledge

Materials:

- BLM 5–8.17: Number Fans
- BLM 7.N.2.1: Whole and Decimal Number Cards
- BLM 7.N.2.2: Operation Cards
- brass tacks
- card stock (paper)
- display board (optional)

Organization: Whole class, a caller and a verifier (teacher or student)

Procedure:

1. Copy BLM 5–8.17: Number Fans onto card stock and attach two copies of each digit with a brass tack at the base of the fan.
2. Give each student a number fan.
3. Inform students that for this learning activity, either the teacher or students will take turns calling out whole and decimal numbers.
4. Ensure that the caller has access to BLM 7.N.2.1: Whole and Decimal Number Cards.
5. Inform students that they will proceed as follows:
   a) The caller chooses a number from the list of cards, and says it slowly three times.
   b) Players arrange the number using their respective number fans and hold it in front of their chests.
   c) The verifier checks whether or not students' responses are correct.
   d) Students perform some operation on their number using BLM 7.N.2.2: Operation Cards.
   e) The verifier checks whether the responses are correct.

Variation:

- Form teams. Team members take turns writing numbers on the display board, or each team member records the numbers individually, and one team member is named to hold up the group's answer.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Represent decimal numbers to thousandths correctly.
- ❒ Understand place value.
- ❒ Use mental mathematics strategies.
- ❒ Perform mathematical operations on decimal numbers.

Assessing Prior Knowledge

Materials:

- BLM 7.N.2.3: Equivalent Percent, Fraction, and Decimal Cards (sets of cards)

Organization: Small groups (two to five students)

Procedure:

1. The objective of this game is to create a set of four matching cards. A set consists of
   - a percent card
   - the decimal number expressed as tenths or hundredths
   - the decimal number expressed as thousandths
   - the number expressed as a fraction in lowest terms
2. Have students form small groups to play the game according to the general rules of Go Fish.
3. Students shuffle the cards, and deal five cards to each player in a group. They place the remaining cards face down in a pile.
4. One player asks another for a specific card. The asker must have at least one card of the set requested.
5. If the asked player has the card, he or she must give it to the asker.
6. If the asked player does not have the card, he or she says, "Go fish."
7. The asker then draws a card from the pile. If the asker is successful (picks up the card requested), he or she takes another turn. If not, play passes to the next player.

Note:
This learning experience could be used within a body of evidence to report on the following competency on the Grade 7 Numeracy Assessment:

Student understands that a given number may be represented in a variety of ways.

Reference:
Manitoba Education and Advanced Learning. *Middle Years Assessment: Grade 7 Mathematics: Support Document for Teachers: English Program.* Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/assess/support/math7/.

Variations:

- Use the cards to play Concentration. Arrange a number of matching cards face down. Players take turns turning over two cards to create sets. If the cards don't match, they are turned face down again.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Create sets of equivalent fractions, decimals, and percents.

Assessing Prior Knowledge

Materials:

- BLM 7.N.2.4: Order of Operations and Skill-Testing Questions (or similar questions)
- calculators

Organization: Individual or pairs, whole class

Procedure:

1. Review the order of operations with the class. Comment on how the order of operations is required in many skill-testing questions for contests (e.g., a draw at a store to win a bicycle or an electronic device).
2. Distribute copies of BLM 7.N.2.4: Order of Operations and Skill-Testing Questions, and have students answer the questions individually or in pairs.
3. After a set amount of time has passed, review responses as a class by having students share and justify their answers.
4. Have students respond to the questions using calculators to check whether or not their calculators are programmed to follow the order of operations.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❐ Follow the order of operations.
- ❐ Find solutions using mental mathematics strategies.

Suggestions for Instruction

- **Solve a problem involving the addition of two or more decimal numbers.**
- **Solve a problem involving the subtraction of decimal numbers.**
- **Place the decimal in a sum or difference using front-end estimation (e.g., for 4.5 + 0.73 + 256.458, think 4 + 256, so the sum is greater than 260).**
- **Check the reasonableness of answers using estimation.**

Materials:

- BLM 7.N.2.5: Money Problems
- BLM 7.N.2.6: Restaurant Bills and Biking
- BLM 5–8.16: Place-Value Mat—Decimal Numbers (optional)
- base-10 blocks (optional)

Organization: Individual or pairs, whole class

Procedure:

1. Distribute copies of BLM 7.N.2.5: Money Problems and/or BLM 7.N.2.6: Restaurant Bills and Biking.
2. Have students estimate answers before they do the calculations.
3. Students may model the addition and subtraction using BLM 5–8.16: Place-Value Mat—Decimal Numbers.
4. Circulate among students while they are working to assess their progress, and supply guidance if necessary. After students have had sufficient time to work on the problems, hold a debriefing discussion with the class, or collect and assess the completed papers.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Add decimal numbers correctly.
- ❒ Subtract decimal numbers correctly.
- ❒ Apply appropriate strategies to solve problems.

Suggestions for Instruction

- **Solve a problem involving the addition of two or more decimal numbers.**
- **Solve a problem involving the subtraction of decimal numbers.**
- **Solve a problem involving the multiplication or division of decimal numbers (for more than 1-digit divisors or 2-digit multipliers, technology could be used).**
- **Place the decimal in a sum or difference using front-end estimation (e.g., for 4.5 + 0.73 + 256.458, think 4 + 256, so the sum is greater than 260).**
- **Place the decimal in a product using front-end estimation (e.g., for $12.33 × 2.4, think $12 × 2, so the product is greater than $24).**
- **Place the decimal in a quotient using front-end estimation (e.g., for 51.50 m ÷ 2.1, think 50 m ÷ 2, so the quotient is approximately 25 m).**
- **Check the reasonableness of answers using estimation.**

Materials:

- BLM 5–8.10: Base-Ten Grid Paper
- base-10 blocks
- coloured pencils
- math journals
- BLM 7.N.2.7: Sample Scenarios 1 (optional)

Organization: Individual, pairs, whole class

Procedure:

1. Present the class with scenarios that require multiplying by decimals, and ask students to estimate reasonable solutions. Samples are provided on BLM 7.N.2.7: Sample Scenarios 1. Exact solutions are not required at this point. These scenarios involve percent values that need to be converted to decimal numbers. Present one scenario at a time, or present several scenarios together as a set, depending on the needs of the class.
2. Ask students to record their estimates and the strategies they used to arrive at them.
3. Multiplication and division are inverse operations. Have students investigate the related division statements that match the scenarios. Compare the relations, and explore division strategies.

4. Begin with a brief Think-Pair-Share strategy to help students prepare for a whole-class discussion. Ask students to share their estimates, explain the strategies they used, justify where they placed the decimal, and explain why they think their solutions are reasonable. Possible responses may include
   - front-end estimation
   - rounding to the nearest whole
   - expanded notation

   Encourage students to agree with, question, or challenge responses respectfully.
5. Following the discussion, have students return to the problems and calculate exact solutions, record the solutions in their math journals, and prepare to articulate the strategies they used and to explain why they think their solutions are correct. In a later learning experience, students will devise a method for multiplying by decimals, and apply it to other situations.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:
- ❒ Create multiplication estimates with the decimal in the correct position.
- ❒ Calculate exact solutions.
- ❒ Solve a problem involving the multiplication of decimal numbers.
- ❒ Place the decimal in a product using front-end estimation.
- ❒ Check the reasonableness of answers using estimation.
- ❒ Communicate mathematically.

Suggestions for Instruction

- **Solve a problem involving the multiplication or division of decimal numbers (for more than 1-digit divisors or 2-digit multipliers, technology could be used).**
- **Place the decimal in a product using front-end estimation (e.g., for $12.33 × 2.4, think $12 × 2, so the product is greater than $24).**
- **Place the decimal in a quotient using front-end estimation (e.g., for 51.50 m ÷ 2.1, think 50 m ÷ 2, so the quotient is approximately 25 m).**
- **Check the reasonableness of answers using estimation.**

Materials:

- BLM 5–8.10: Base-Ten Grid Paper
- BLM 5–8.24: Number Line
- BLM 7.N.2.8: Sample Scenarios 2
- base-10 blocks
- coloured pencils
- math journals

Organization: Individual, pairs, whole class

Procedure:

1. Ask students to define *division* and to provide an illustration of the operation. Division may be understood as repeated subtraction, or partitioning into groups of a particular size or into a particular number of groups. Repeated subtraction can be shown on a number line, and partitioning can be shown with base-10 blocks. Division may be demonstrated by forming an array with base-10 blocks or drawing on base-10 grid paper (e.g., BLM 5–8.10: Base-Ten Grid Paper). For some examples, refer to the Appendix at the end of this document.
2. Present the class with scenarios requiring dividing by decimals, and ask students to estimate reasonable solutions. Samples are provided on BLM 7.N.2.8: Sample Scenarios 2. Present one scenario at a time, or present several scenarios together as a set, depending on the needs of the class.
3. Ask students to record their estimates and the strategies they used to arrive at them. Have students record their thinking.
4. Division and multiplication are inverse operations. Have students investigate the related multiplication statements that match the scenarios. Compare the relations and multiplication strategies.
5. Allow a few minutes for the use of a Think-Pair-Share strategy to help students prepare for a whole-class discussion in which they share their estimates and explain the strategies they used, how they decided where to place the decimal, and why they think the solutions are reasonable. Encourage students to agree with, question, or challenge responses respectfully. Record the reasonable estimates in a chart.

6. Following the discussion, have students return to the problems and calculate exact solutions, again recording their solutions in their math journals, and prepare to articulate the strategies they used and explain why they think their solutions are correct.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Create estimates for division with the decimal in the correct position.
- ❒ Calculate exact solutions.
- ❒ Solve a problem involving the division of decimal numbers.

Suggestions for Instruction

- **Solve a problem involving the multiplication or division of decimal numbers (for more than 1-digit divisors or 2-digit multipliers, technology could be used).**
- **Place the decimal in a product using front-end estimation (e.g., for \$12.33 × 2.4, think \$12 × 2, so the product is greater than \$24).**
- **Place the decimal in a quotient using front-end estimation (e.g., for 51.50 m ÷ 2.1, think 50 m ÷ 2, so the quotient is approximately 25 m).**
- **Check the reasonableness of answers using estimation.**

Materials:

- BLM 5–8.10: Base-Ten Grid Paper
- BLM 5–8.25: My Success with Mathematical Processes
- base-10 blocks
- coloured pencils or highlighters
- math journals
- calculators

Organization: Individual, pairs, whole class

Procedure:

Part A: Multiplying and Dividing by Powers of Ten

1. Remind students that by applying their present knowledge of mathematics, they have successfully solved multiplication questions with 1- and 2-digit decimal multipliers, and have explored strategies for division involving decimal numbers.
2. Challenge students to articulate a method that will allow them to solve any such multiplication and division questions, and to share those methods with others.
3. Give students a set of multiplication questions that will help them to see an emerging pattern.

   Examples:

0.5×10	0.6×10	0.7×10	0.8×10
0.5×1	0.6×1	0.7×1	0.8×1
0.5×0.1	0.6×0.1	0.7×0.1	0.8×0.1
0.5×0.01	0.6×0.01	0.7×0.01	0.8×0.01

4. Have students choose a few of the questions from above and vary the multiplier, the multiplicand, and both. Have them note anything interesting they discover.
5. Give students a set of division questions that will help them to see an emerging pattern.

   Examples:

$0.5 \div 10$	$0.6 \div 10$	$0.7 \div 10$	$0.8 \div 10$
$0.5 \div 1$	$0.6 \div 1$	$0.7 \div 1$	$0.8 \div 1$
$0.5 \div 0.1$	$0.6 \div 0.1$	$0.7 \div 0.1$	$0.8 \div 0.1$
$0.5 \div 0.01$	$0.6 \div 0.01$	$0.7 \div 0.01$	$0.8 \div 0.01$

6. Have students choose a few of the questions from above and vary the divisor, the dividend, and both. Have them note anything interesting they discover.

Part B: Multiplying and Dividing Decimals

1. Remind students that by applying their present knowledge of mathematics, they have successfully solved multiplication questions with 1- and 2-digit decimal multipliers, and have explored strategies for division involving decimal numbers.
2. Challenge students to articulate a method that will allow them to solve any such multiplication and division questions, and to share those methods with others.

3. Give students a set of multiplication questions that will help them to see an emerging pattern.

   Examples:

1.2 × 3	12 × 3	0.12 × 3	0.12 × 0.3
5.64 × 2.5	56.4 × 2.5	564 × 2.5	5.64 × 0.25

4. Have students create other sets of multiplication questions where the digits in the multiplier and multiplicand are the same, but the decimal shifts. Have them note anything interesting they discover.
5. Give students a set of division questions that will help them to see an emerging pattern.

   Examples:

67.3 ÷ 4.8	6.73 ÷ 4.8	67.3 ÷ 0.48	0.673 ÷ 0.48
937.54 ÷ 2.56	9.3754 ÷ 25.6	9375.4 ÷ 256	93.754 ÷ 25.6

6. Have students create other sets of division questions where the digits in the divisor and dividend are the same, but the decimal shifts. Have them note anything interesting they discover.
7. Have students reflect on and record their success with mathematical processes, using BLM 5–8.25: My Success with Mathematical Processes.

Note:
When determining the placement of the decimal, it is important to explore a variety of methods, such as

- temporarily disregarding the decimal(s), using any previous algorithms to determine the digits in the solution, and then deciding where to place the decimal in the product based on estimation strategies
- removing the decimal(s) by multiplying by a power of 10, and, after finding the product, compensating by dividing by that power of 10
- using knowledge of place value and annexing zeros when multiplying and dividing by powers of 10, and noticing and applying generalizations (e.g., the number of digits to the right of the decimal point in the product is equal to the number of digits to the right of the decimal point in the factors)

It is imperative that students have ample exploration time to discover these generalizations without merely being told what they are. Remain open to hearing all discoveries students make and encourage students to discuss them.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❐ Understand problem-solving strategies.
- ❐ Apply appropriate problem-solving strategies.
- ❐ Communicate ideas clearly.
- ❐ Question ideas.
- ❐ Describe an effective method for solving multiplication with decimal multipliers.
- ❐ Describe an effective method for solving division questions with decimal multipliers.

Suggestions for Instruction

- **Solve a problem involving the multiplication or division of decimal numbers (for more than 1-digit divisors or 2-digit multipliers, technology could be used).**
- **Place the decimal in a product using front-end estimation (e.g., for $12.33 × 2.4, think $12 × 2, so the product is greater than $24).**
- **Place the decimal in a quotient using front-end estimation (e.g., for 51.50 m ÷ 2.1, think 50 m ÷ 2, so the quotient is approximately 25 m).**
- **Check the reasonableness of answers using estimation.**

Materials:

- BLM 5–8.10: Base-Ten Grid Paper
- BLM 5–8.24: Number Line
- base-10 blocks
- coloured pencils
- calculators
- math journals

Organization: Individual, pairs, whole class

Procedure:

1. Ask students to define *division* and to provide an illustration of the operation. Division may be understood as repeated subtraction, or partitioning into groups of a particular size or into a particular number of groups. Repeated subtraction can be shown on a number line (see BLM 5–8.24: Number Line), and partitioning can be shown with base-10 blocks. Division may be demonstrated by forming an array with base-10 blocks or drawing on base-10 grid paper (see BLM 5–8:10: Base-Ten Grid Paper). Illustrations using base-10 blocks are outlined in the Appendix at the end of this document.
2. Ask students to illustrate the following division questions. After giving students sufficient time to consider the questions, ask them to discuss their definitions, illustrations, and discoveries.
   a) $8 \div 4$
   b) $8 \div 0.4$
   c) $0.8 \div 0.4$
   d) $0.8 \div 4$
3. Have students create other division questions and vary the place value position of the digits. They may use technology for divisors with two or more digits. Students record the equations in charts, and develop a personal method or algorithm for dividing with decimals. They prepare to discuss findings with the class.
4. Students may make the following observations or conclusions:
   - If the digits in the division questions remain the same, the digits in the quotients are the same. As with multiplication, the difference is in the place value position of the digits.
   - Students can use known division algorithms and either ignore the decimal place or find the digits in the solution, and then place the decimal according to their estimation.
   - They can rename the decimal numbers so they have the same place value (e.g., $4.0 \div 0.5$ is equivalent to 40 tenths divided by 5 tenths, which is 8).
   - They can use a balance scale principle and will get the same quotient if they multiply both numbers in the question by the same power of 10 (e.g., $4.5 \div 0.09$ is equivalent to multiplying both numbers by 100, which becomes $450 \div 9 = 50$).
5. Have students record in their math journals how to divide decimal numbers.
6. Ask students to write related multiplication statements for each of their division statements, and to compare the responses and procedures for dividing and multiplying decimal numbers.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❐ Understand problem-solving strategies.
- ❐ Apply appropriate problem-solving strategies.
- ❐ Communicate ideas clearly.
- ❐ Question ideas.
- ❐ Describe an effective method for solving division with decimals.
- ❐ Solve a problem involving the division of decimal numbers.
- ❐ Place the decimal in a quotient using front-end estimation.
- ❐ Check the reasonableness of answers using estimation.

Suggestions for Instruction

- **Solve a problem involving the multiplication or division of decimal numbers (for more than 1-digit divisors or 2-digit multipliers, technology could be used).**
- **Place the decimal in a product using front-end estimation (e.g., for \$12.33 × 2.4, think \$12 × 2, so the product is greater than \$24).**
- **Place the decimal in a quotient using front-end estimation (e.g., for 51.50 m ÷ 2.1, think 50 m ÷ 2, so the quotient is approximately 25 m).**
- **Check the reasonableness of answers using estimation.**

Materials:

- BLM 7.N.2.9: Sample Scenarios 3

Organization: Individual

Procedure:

1. When students have a method for solving multiplication problems with decimal numbers, give them opportunities to solve a variety of problems, such as those provided on BLM 7.N.2.9: Sample Scenarios 3.
2. Remind students that estimating answers before calculating solutions is important to verify that answers are reasonable. This strategy is important when using technology.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Place decimal points in the correct position.
- ❒ Solve problems correctly.
- ❒ Solve a problem involving the multiplication or division of decimal numbers.
- ❒ Place the decimal in a product using front-end estimation.
- ❒ Place the decimal in a quotient using front-end estimation.
- ❒ Check the reasonableness of answers using estimation.

Suggestions for Instruction

- **Solve a problem involving the multiplication or division of decimal numbers (for more than 1-digit divisors or 2-digit multipliers, technology could be used).**
- **Place the decimal in a quotient using front-end estimation (e.g., for 51.50 m ÷ 2.1, think 50 m ÷ 2, so the quotient is approximately 25 m).**
- **Check the reasonableness of answers using estimation.**
- **Solve a problem that involves operations on decimals (limited to thousandths), taking into consideration the order of operations.**

Materials:

- BLM 7.N.2.10: Decimal Problems
- index cards (optional)

Organization: Individual

Procedure:

1. Students solve division and order of operation questions such as those found on BLM 7.N.2.10: Decimal Problems.

Variation:

- Have students create their own scenarios and solutions on index cards. Students can exchange their scenarios and solutions with a classmate, or add them to a question bank to use as Exit Slips or for in-class challenges.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Place the decimal correctly while estimating.
- ❒ Use the order of operations to answer multi-step problems correctly.
- ❒ Solve a problem involving the multiplication or division of decimal numbers.
- ❒ Place the decimal in a quotient using front-end estimation.
- ❒ Check the reasonableness of answers using estimation.
- ❒ Solve a problem that involves operations on decimals, taking into consideration the order of operations.

Suggestions for Instruction

- **Explain, using an example, how to use mental mathematics for products or quotients when the multiplier or the divisor is 0.1 or 0.5 or 0.25.**

Materials:

- BLM 5–8.10: Base-Ten Grid Paper
- base-10 blocks
- math journals

Organization: Pairs or small groups, whole class, individual

Procedure:

1. Present students with a mission: Discover a quick mental mathematics strategy to use when multiplying any number by 0.1. Later, the problem will be extended to finding the quotient for dividing by 0.1, and then to finding products and quotients for multiplying and dividing by 0.5 and 0.25.
2. If students need guidance to organize their investigation, assist them with the following hints. Encourage them to use a problem-solving strategy, rather than follow a set of steps.

Example:

What must you do? What must you know in order to do what you have determined you need to do? How can you get that information? What does it mean to multiply? What does it mean to multiply by 0.1? What is 0.1? What do you need in order to find a pattern?

a) First, ensure that students understand the problem and can articulate the meaning of multiplication by 0.1.

b) Next, have them explore the patterns created by multiplying a set of numbers by 0.1.

c) Finally, have them examine the patterns they see, and create a mental mathematics strategy to find the product when multiplying by 0.1. Test the strategy to ensure it works for all numbers. Try numbers such as 120 or 122, or smaller numbers such as 0.1×0.5 or 0.1×0.01.

3. After students (working in pairs or in small groups) have had sufficient time to explore and develop a strategy, have them reassemble as a class to debrief, giving them an opportunity to communicate their ideas and questions and revise their strategies as necessary. Then have them use their math journals to record their strategies and explain why the strategies work.

4. Extend the problem to finding the quotient for dividing by 0.1, and then products and quotients for multiplying and dividing by 0.5 and 0.25.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Use a mathematically correct mental mathematics strategy for each operation.
- ❒ Provide an explanation that supports the strategy.
- ❒ Use effective problem-solving strategies.
- ❒ Explain, using an example, how to use mental mathematics for products or quotients when the multiplier or the divisor is 0.1 or 0.5 or 0.25.

School-Supply Kits

Introduction:

Students receive a budget for creating 20 school-supply kits to be given as welcome gifts to new students in need.

Purpose:

In this investigation, students will demonstrate the ability to do the following (connections to learning outcomes are identified in parentheses):

- Perform operations with decimals to solve problems. (7.N.2)
- Solve problems involving percents from 1% to 100%. (7.N.3)
- Compare and order decimals and integers. (7.N.7)

Students will also demonstrate the following mathematical processes:

- Communication
- Connections
- Mental Mathematics and Estimation
- Problem Solving
- Reasoning
- Technology

Materials/Resources:

- office- and school-supply flyers and print and online catalogues, and/or access to office and school-supply stores
- calculators
- shopping list
- purchase order

Organization: Individual or small group

Procedure:

Student Directions

1. Several new students have been arriving at your school without any school supplies or any means to obtain them. This has added to the students' difficulties in trying to adjust to their new school experience. The student council has held a fundraising drive to help future new students adjust to their new school experience. They have raised $500 to create 20 school-supply kits. In the future, a kit will be given as a welcome gift to a new student in need. The student council has chosen your group to prepare the kits.

2. Create a list of school-supply items and quantities required for each kit.
3. Calculate the total number of each item required for all the kits.
4. Shop to find prices for the items. Use estimations to help guide your choices. Remember to add PST and GST and any applicable shipping or handling costs. Stay as close to budget as possible.
5. Prepare a clearly written letter to inform the store of your project and request their assistance. You receive a response to your letter from the store offering a 25% discount to help with your project.
6. Make any desired adjustments to your shopping list.
7. Finalize the items to purchase. Remember that PST and GST will be added and shipping or handling charges may be added. Stay as close to budget as possible.
8. Prepare a purchase order for the store. List the items from least expensive to most expensive, the quantities to order, unit price, extended price, discounted or adjusted price, subtotal, taxes, shipping and handling charges (if applicable), and the grand total.
9. Prepare a clearly written letter to the student council regarding your decision. Include a list of the items to be included in each school-supply kit, identify any extra items that will be left over, compare the cost to the budget, and include the purchase order.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❐ Create an adequate list of school supplies to include in each kit.
- ❐ Make reasonable estimates to guide decision making.
- ❐ Perform operations with decimal numbers correctly.
- ❐ Calculate percents accurately (discount, GST, PST).
- ❐ Make decisions to match a budget within 5%.
- ❐ Order and calculate the purchase order correctly.
- ❐ Prepare a clear letter requesting assistance.
- ❐ Prepare a clear summary letter outlining actions and comparing cost to budget.

Shopping List for School-Supply Kits

Item	Quantity Required for Each Kit	Quantity for 20 Kits	Purchase Price	Cost Per Item	Cost Per 20 Kits

Purchase Order for School-Supply Kits

To: ______________________________ From: ______________________________

______________________________ ______________________________

______________________________ ______________________________

Item Description	Quantity Required	Unit Price	Extended Price	Adjusted Price
Subtotal				
GST (5%)				
PST (7%)				
Shipping and Handling Charges				
Grand Total				

Number (7.N.3)

Enduring Understanding(s):

Percents can be thought of as a ratio comparing to 100 or a fraction out of 100.

Circle graphs show a comparison of each part to a whole using ratios.

Percents, fractions, decimals, and ratios are different representations of the same quantity.

Number sense and mental mathematics strategies are used to estimate answers and lead to flexible thinking.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
7.N.3 Solve problems involving percents from 1% to 100%. [C, CN, ME, PS, R, T]	➛ Express a percent as a decimal or fraction. ➛ Solve a problem that involves finding a percent. ➛ Determine the answer to a percent problem where the answer requires rounding, and explain why an approximate answer is needed (e.g., total cost including taxes).

Prior Knowledge

Students may have had experience with the following:

- Applying estimation strategies, including
  - front-end rounding
  - compensation
  - compatible numbers

  in problem-solving contexts.
- Applying mental mathematics strategies for multiplication, such as
  - annexing, then adding zeros
  - halving and doubling
  - using the distributive property

- Demonstrating an understanding of fractions by using concrete and pictorial representations to
  - create sets of equivalent fractions
  - compare fractions with like and unlike denominators
- Describing and representing decimals (tenths, hundredths, thousandths) concretely, pictorially, and symbolically.
- Relating decimals to fractions (tenths, hundredths, thousandths).
- Comparing and ordering decimals (tenths, hundredths, thousandths) by using
  - benchmarks
  - place value
  - equivalent decimals
- Demonstrating an understanding of place value for numbers
  - greater than one million
  - less than one-thousandth
- Demonstrating an understanding of factors and multiples by
  - determining multiples and factors of numbers less than 100
  - identifying prime and composite numbers
  - solving problems involving factors or multiples
- Demonstrating an understanding of ratio, concretely, pictorially, and symbolically.
- Demonstrating an understanding of percent (limited to whole numbers) concretely, pictorially, and symbolically.

For more information on prior knowledge, refer to the following resource:

Manitoba Education and Advanced Learning. *Glance Across the Grades: Kindergarten to Grade 9 Mathematics*. Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/cur/math/glance_k-9/index.html.

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of the addition, subtraction, multiplication, and division of decimals to solve problems (for more than 1-digit divisors or 2-digit multipliers, technology could be used).
- Demonstrating an understanding of the relationship between repeating decimals and fractions, and terminating decimals and fractions.

Background Information

People regularly encounter practical situations requiring them to understand and solve problems related to percent. These situations include problems related to sports statistics, discounts, price increases, taxes, polls, social changes and trends, the likelihood of precipitation, and so on. The media provide sources of contextual data for creating problems involving percent.

Learning outcome 7.N.3 builds on understandings related to fractions, ratios, decimals, percents, and problem solving that students have developed in previous grades.

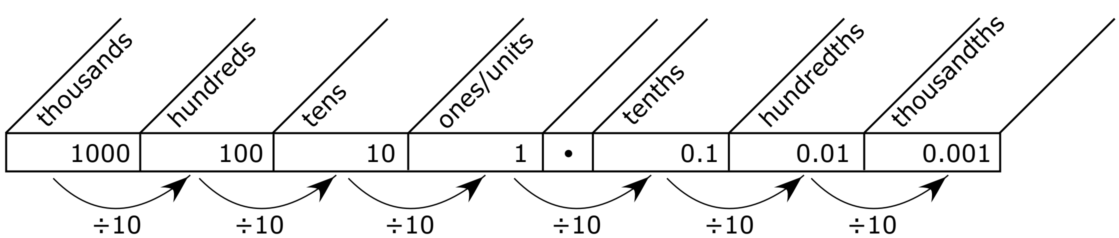
Fractions and Decimals

Before students become skilful at solving problems involving percent, they must have a strong conceptual understanding of fractions, decimals, and percents, and they must be able to interchange equivalent names to represent the concepts.

The term *fraction* has several meanings. An expert blends and separates these meanings for convenience, but this blending can confuse students who lack fluency in applying the different meanings of fraction. *Fraction notation* is used to represent a "cut" or a part of a unit, a part of a group or set, a measurement, or a point on a number line. It is also used to represent a ratio or a portion of a turn, and to indicate the division operation.

Decimals are a convenient means of representing fractional quantities using a place value system. Fractions may be converted to decimals by using the division operation meaning of fraction and dividing the numerator by the denominator (e.g., $\frac{3}{4}$ may be viewed as $3 \div 4 = 0.75$). Fractions may also be converted to decimals by finding an equivalent fraction with a denominator of any power of 10 (e.g., 100), and then writing the fraction in standard notation $\left(\text{e.g., } \frac{7}{50} = \frac{14}{100} = 0.14\right)$. It is useful to commit to memory some common fraction decimal equivalents, such as halves, quarters, and tenths.

A *decimal point* is used to separate whole units from parts of units. Each position to the right of the decimal represents a tenth part of one of the previous units. In standard notation, the first position following the decimal represents tenth parts of one whole unit, and the second place represents tenth parts of a tenth, or hundredth parts of one unit.



Problems Involving Percent

When translating standard notation to percent, the decimal point indicates where to read the hundredths in a number. The word *percent* means per hundred and may be substituted for the word *hundredths* when reading a number. Therefore, $\frac{7}{100}$ or 0.07 may be read as 7 hundredths and also as 7 percent.

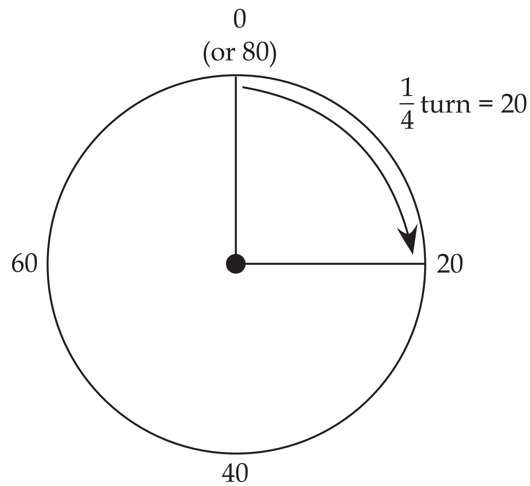
Percent may also be used to represent fractional quantities that are a little larger than a hundredth. Place value positions to the right of the decimal represent the "cut" meaning of a fraction, and each successive position represents one of the previous units cut into 10. In standard notation, the third position represents thousandth parts of one unit, but it may also be viewed as tenths of a hundredth.

An understanding of place value allows us to express any number as a number of selected units. Just as 141 can represent 14 tens and 1 one, 0.141, which represents a number that is a little larger than one tenth of one whole, may be expressed as 1.41 tenths, 14.1 hundredths, or 141 thousandths. Substituting the word *percent* as another word for *hundredth*, the decimal number 0.141 may be considered as 14.1 hundredths, or 14.1 percent (%).

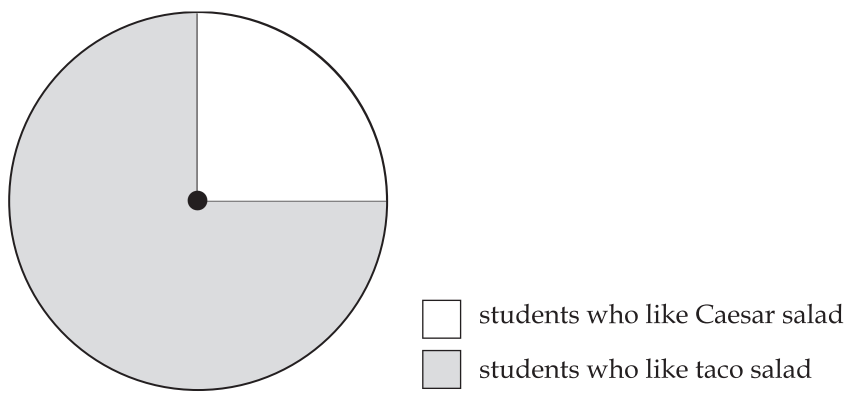
In Grade 7, students need to work only with numbers from 1% to 100%. The percents may represent a part of one whole item or a part of one whole group. Percent represents a special type of fraction with a denominator of 100. The quantity represented by the percent depends on the amount in the whole. For example, 1% may be a large or small quantity, depending on the whole. Consider 1% of the money in an individual's piggy bank, versus 1% of the money in the bank's books. The same quantity may also represent different percent values. For example, 20 is 20% of 100, but it is also 100% of 20. Identifying which number in a situation represents the whole and which number represents the part is important when solving problems involving percent.

When students have developed multiple views of fractions and percents, they will benefit from having multiple meaningful approaches to find percent values.

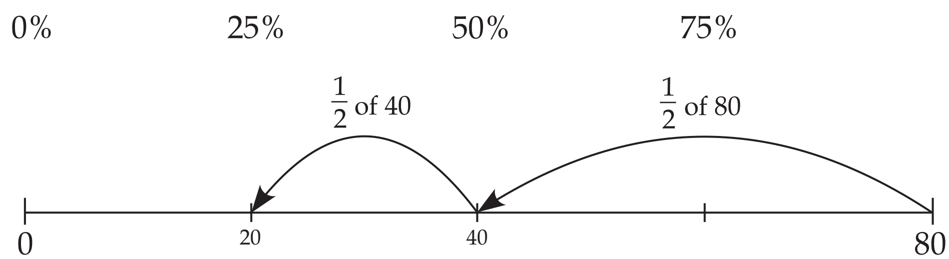
- To find 25% of 80, students may approach the problem as follows:
  - Think of 25% as $\frac{25}{100}$, and the equivalent fraction $\frac{1}{4}$, and then find $\frac{1}{4}$ of 80 by dividing 80 into 4 groups: 80 ÷ 4 = 20, so 25% of 80 is 20.
  - Think of an equivalent fraction $\frac{25}{100} = \frac{1}{4} = \frac{}{80}$. This view is less convenient in this situation, but more convenient in other situations.
  - Think of a part-to-whole ratio and proportion: $\frac{25}{100} = \frac{}{80}$.
  - Think of a circle with beginning and end points 0 and 80, and think of 25% as $\frac{1}{4}$ of a turn.



- Think of a circle graph that shows 25% of the students in a class like Caesar salad and 75% like taco salad. If there are 80 students in the class, how many like Caesar salad?



- Think of a number line with end points of 0 and 80, and corresponding points 0% and 100%. Students may think 25% is half of 50%. Half of 80 is 40, and half of 40 is 20.



- Think of the decimal equivalent, and change the word expression into a number expression.

  25% of 80 is _____

  $0.25 \times 80 =$ _____

- Students may also use mental mathematics and the distributive property to solve problems involving percent.
  - To find 35% of 80, think of 35% as 25% + 10%.
  - In the above problem, 25% of 80 is 20, and 10% of 80 is 8.

    20 + 8 = 28, so 35% of 80 is 28.
- Students may also use related fractions to solve percent problems.
  - If $\frac{1}{4}$ of 80 is 20, then $\frac{3}{4}$ of 80 must be 3×20 or 60.

With multiple approaches to finding percents, students can choose the most convenient approach for each problem.

When setting up examples and creating problems for students, frequently choose numbers that are convenient to work with, so that students will be able to concentrate on the processes involved, rather than on the arithmetic.

Also encourage students to use a variety of approaches and not to over-rely on one specific method. For example, they may develop a habit of using factors of 10, and forget to use equivalent fractions or the commonly used, very effective part-to-whole ratio approach. To find 25% of a number, you may think 25% is 10% + 10% + half of 10%, but it may be much more convenient to think of 25% as $\frac{1}{4}$, and divide the whole by 4.

Mathematical Language

decimal

equivalent

factor

fraction

multiple

percent

proportion

ratio

simplify

Assessing Prior Knowledge

Materials:

- markers or pens of two different colours (for each pair of students)
- two regular number cubes (providing factors 1 to 12) or a multi-sided cube
- grid paper or tic-tac-toe grids or frames (of various sizes), such as the following:
  - BLM 7.N.3.1A: Tic-Tac-Toe Frames
  - BLM 7.N.3.1B: Tic-Tac-Toe Frames (Medium Challenge)
  - BLM 7.N.3.1C: Tic-Tac-Toe Frame (Ultimate Challenge)

Organization: Pairs

Procedure:

1. Have pairs of students take turns filling in the squares of their tic-tac-toe grids with numbers 1 to 99, or multiples that correspond to the numbers on their number cubes.
2. Students choose a colour and an X or an O mark, and determine who will play first.
3. Students take turns rolling the number cube(s), and use their colour markers to mark an X or an O on a multiple of the number they rolled. Encourage them to practise using mathematical language with statements such as the following:
   - 27 is a multiple of 9 because $3 \times 9 = 27$.
   - 9 and 3 are factors of 27 because $3 \times 9 = 27$.
   - 17 is a prime number. Its only factors are 1 and 17.
4. Students will need to agree about what to do if someone makes an error.
5. The first student who creates a horizontal, vertical, or diagonal line with his or her marks wins.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Use vocabulary for multiples, factors, and primes correctly.
- ❒ Identify multiples of various numbers correctly.
- ❒ Identify prime numbers correctly.

Assessing Prior Knowledge

Materials:

- BLM 7.N.3.2: Equivalent Fraction Challenge
- a pair of six-sided number cubes, or a multi-sided number cube, or a spinner (for each pair of students)

Organization: Whole class, pairs

Procedure:

1. As a class, review procedures for creating equivalent fractions by multiplying or dividing by a fraction name for 1, or by multiplying or dividing each term in the fraction by the same factor.
2. Demonstrate one round of the game, following the procedures outlined on BLM 7.N.3.2: Equivalent Fraction Challenge, and using the game cards provided on the BLM. In summary, students create a target fraction, take turns rolling the number cube(s) to determine a change factor, and then create an equivalent fraction. The player who returns the fraction to its original target name wins.
3. Distribute game cards.
4. Have students play the game in pairs.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Create equivalent fractions correctly and simplify fractions with ease.

Assessing Prior Knowledge

Materials:

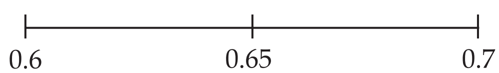
- BLM 7.N.3.3: It's Between: Rounding Decimal Numbers
- demonstration board

Organization: Individual, or pairs, or whole class

Procedure:

1. Review the place value positions for decimal numbers, and review how to read and write decimal numbers.
2. Review strategies for rounding decimal numbers to a given place value position by identifying which numbers the given number is between and then determining which number it is closest to.

   Example: Round 0.6521 to the nearest tenth.
3. Identify the value of the required place value. In the above example, it is tenths (0.652 has 6 tenths and a little more).
4. Identify one unit higher for the same place value. One unit higher than 6 tenths is 7 tenths (0.652 is between 6 tenths and 7 tenths).
5. Determine whether 0.652 is closer to 6 tenths or to 7 tenths by thinking of the numbers on a line, with the midway point between the numbers being 0.65. Since 0.652 is a little more than 0.65, it is closer to 0.7 than it is to 0.6. So, 0.652 rounded to the nearest tenth is 0.7.



6. Repeat the same process for rounding to the nearest hundredth, and then to the nearest thousandth.
7. Have students verify their skill by rounding a set of numbers such as those provided on BLM 7.N.3.3: It's Between: Rounding Decimal Numbers.

Variation:

- Using the demonstration board, present one question at a time and have students solve it. Call upon one student to identify the numbers between which a given number lies, and another student to identify which number it is closest to, and therefore rounds to.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❐ Identify place values correctly.
- ❐ Determine the numbers between which the number being rounded lies.
- ❐ Round a number to the required decimal place correctly.

Suggestions for Instruction

- **Express a percent as a decimal or fraction.**
- **Determine the answer to a percent problem where the answer requires rounding, and explain why an approximate answer is needed (e.g., total cost including taxes).**

Materials:

- BLM 7.N.3.4: Choose Your Question (Point Sheet, Game Sheet 1 and Game Sheet 1 Answer Key, Blank Game Sheet)
- scissors
- low-tack glue
- corrugated board
- pins, tacks, or staples

Organization: Whole class, small groups (three to five students)

Procedure:

1. Review the place value positions for decimal numbers, and review reading and writing decimal numbers.
2. Review strategies for rounding decimal numbers to a given place value position, as described in the preceding learning activity.
3. Review reading decimal numbers as percents, as outlined in the Background Information for learning outcome 7.N.3 (e.g., 0.456 may be read as 45.6%).

4. Play the game outlined in BLM 7.N.3.4: Choose Your Question, using the following procedure:
   a) Form student groups, each containing one quizmaster and three to five contestants.
   b) Distribute copies of BLM 7.N.3.4: Choose Your Question (Point Sheet, Game Sheet 1 and Answer Key, Blank Game Sheet), along with other required materials, and have student groups make game boards.
      - The contestants receive the category Point Sheet and cut up the sections, which they give to the quizmaster.
      - The quizmaster receives the Game Sheet and Answer Key. He or she hides the Answer Key, and uses the Game Sheet to create a game board by tacking the Point Sheet sections over the questions with temporary low-tack glue, or by placing the Game Sheet on the corrugated board and tacking the point cards in place with pins, tacks, or staples.
   c) Students decide which contestant will go first.
   d) That contestant chooses a category and point value.
   e) The quizmaster uncovers the question. If the contestant responds correctly, he or she receives the point card.
   f) Play then moves to the next contestant. The player with the most points at the end wins. The winner becomes the quizmaster for additional rounds. A Blank Game Sheet is included for additional rounds.

Variation:

- Alter the categories to include any skills you wish to review (Game Sheet 2 includes part-to-whole ratios and percents as decimals and fractions).

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:
- ❒ Round decimal numbers to the designated place value correctly.
- ❒ Express a decimal number as a percent.

- **Express a percent as a decimal or fraction.**

Materials:

- BLM 7.N.3.4: Choose Your Question (Point Sheet, Game Sheet 2, Blank Game Sheet) (optional)
- scissors

Organization: Small groups (three to five students)

Procedure:

1. Review expressing a situation as a part-to-whole ratio, and vice versa.
2. Review expressing a percent as a decimal number, and vice versa (include percents such as $5\frac{3}{4}\%$ or $2\frac{1}{2}\%$), which are common in interest rates or pay-increase rates).
3. Review expressing a percent as a decimal and as a fraction, and vice versa.
4. Follow the directions for Game Sheet 1 in the previous learning activity.

Note:

This learning experience could be used within a body of evidence to report on the following competency on the Grade 7 Numeracy Assessment:

Student understands that a given number may be represented in a variety of ways.

Reference:

Manitoba Education and Advanced Learning. *Middle Years Assessment: Grade 7 Mathematics: Support Document for Teachers: English Program.* Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/assess/support/math7/.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Express situations as ratios and percents correctly.
- ❒ Express a percent as a decimal correctly.
- ❒ Express a percent as a fraction correctly.
- ❒ Express a fraction as a percent correctly

- **Express a percent as a decimal or fraction.**

Materials:

- various card sets for each group from the following website:
  Manitoba Education. "Middle Years Activities and Games." *Mathematics*.
  www.edu.gov.mb.ca/k12/cur/math/my_games/index.html.
- BLM 7.N.2.3: Equivalent Percent, Fraction, and Decimal Cards

Organization: Small groups (three or four students)

Procedure:

1. Have students use decks of cards created from BLM 7.N.2.3: Equivalent Percent, Fraction, and Decimal Cards. Tell students they will play a game according to the rules of Rummy. The objective of the game is to create sets of four matching cards (a percent, a fraction, a decimal, and an illustration) and to be the first person to dispose of all his or her cards.
2. Have students form small groups and choose a dealer. The dealer shuffles the cards and deals seven cards to each player, who then secretly sorts them into sets. The dealer places the remaining cards face down on the table to form a draw pile, and turns the top card of the draw pile face up to form a discard pile.
3. The first player begins by drawing a card from either the draw pile or the discard pile. The player may lay any sets of three or four equivalent cards face up on the table in front of him or her. In the same turn, the player may also play equivalent cards on top of other players' equivalent sets. The player completes a turn by playing a card on the discard pile. This card may not be the same card the player selected from the discard pile at the beginning of the turn.
4. Play passes to the next player.
5. The round is over when the first player discards his or last card on the discard pile.

Note:

This learning experience could be used within a body of evidence to report on the following competency on the Grade 7 Numeracy Assessment:

Student understands that a given number may be represented in a variety of ways.

Reference:

Manitoba Education and Advanced Learning. *Middle Years Assessment: Grade 7 Mathematics: Support Document for Teachers: English Program.* Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/assess/support/math7/.

Variations:

- Use the cards to play Go Fish, using rules outlined in the suggested learning experiences for learning outcome 7.N.2 for fraction, decimal, and percent equivalents.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Express a percent as a decimal or fraction correctly.

Suggestions for Instruction

- **Express a percent as a decimal or fraction.**

Materials:

- BLM 7.N.3.5: Designing to Percent Specifications
- coloured pencils or markers

Organization: Individual

Procedure:

1. Create a design on 100-grid paper that meets set percent specifications. Grid paper is provided on BLM 7.N.3.5: Designing to Percent Specifications.
2. Express each percent as a decimal and as a fraction.
3. Simplify the fractions.

   Example:

   10% of the design is red.

   40% of the design is blue.

   15% of the design is green.

   The remainder of the design is yellow.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Express a percent as a decimal or fraction correctly.

Suggestions for Instruction

- **Express a percent as a decimal or fraction.**
- **Solve a problem that involves finding a percent.**

Materials:

- BLM 7.N.3.6: Determining the Whole, the Part, and the Percent
- math journals
- demonstration board
- marking pen

Organization: Whole class

Procedure:

Part A

1. Inform students that percents are a special type of fractions out of 100. Therefore, each percent problem is a part-to-whole relation and may be represented as a fraction problem.
2. Present situations that involve finding the whole, the part, and the percent. Present one situation at a time, recording it on the board.
3. For each situation presented, ask students to identify the whole, the part, and the percent, either on BLM 7.N.3.6: Determining the Whole, the Part, and the Percent or in their math journals, and then highlight what they are to find. Next, they write a word phrase or a number expression to represent the situation. Students do not find the solutions at this time. The problem will be solved in the next step.
4. Ask a student to share his or her response and record it on the board. Encourage students to confirm or question the response. During this discussion time, address any questions students have, and correct any errors with a marking pen.
5. Continue with the next situation, until sufficient examples have been explored. Include several examples of each of the following three types of situations students will encounter in solving problems with percent:
   - A designated percent of a designated number is what number?

     Example:

     There are 80 cars in a shipment, and 40% of them are silver. How many are silver?

     40% of 80 is ____.

- A designated number is what percent of another designated number?

  Example:

  There are 60 cars in a shipment, and 15 of them are red. What percent are red?

  15 is ___% of 60.

- A designated number is a designated percent of what number?

  Example:

  25% of the cars in a shipment are blue. There are 50 blue cars. How many cars are in the shipment?

  50 is 25% of ___.

Part B

1. Return to the first situation presented, and ask students to find the solution.
2. After students have had sufficient time to find the solution, ask individual students to share their response and describe the strategy they used to find the solution. In the class discussion, encourage students to confirm or question the shared responses, and to suggest alternative strategies. Address any questions students have, and correct any errors with a marking pen.
3. Solve all the situations in this manner.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Express a percent as a decimal or fraction.
- ❒ Solve a problem that involves finding a percent.

Suggestions for Instruction

- **Express a percent as a decimal or fraction.**
- **Solve a problem that involves finding a percent.**

Materials:

- BLM 7.N.3.7: Finding the Missing Numbers in the Percent (Scenarios)

Organization: Individuals, small groups or whole class (for sharing responses)

Procedure:

1. Distribute copies of BLM 7.N.3.7: Finding the Missing Numbers in the Percent (Scenarios).
2. Ask students to complete three of the percent problems presented on the BLM, one problem requiring them to find the whole, one to find the part, and one to find the percent. Remind students to identify the whole, the part, and the percent of each problem, before attempting a solution (as they did in the previous learning activity). They may find it helpful to create an expression to summarize the problem. Having them show two strategies to solve each problem will help them stay flexible in their problem-solving strategies.
3. When students have had sufficient time for their individual work, have them meet in small groups or as a whole class to share their strategies and answers. Have students discuss their preferred strategy for each problem.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Express a percent as a decimal or fraction.
- ❒ Solve a problem that involves finding a percent.

Suggestions for Instruction

- **Determine the answer to a percent problem where the answer requires rounding, and explain why an approximate answer is needed (e.g., total cost including taxes).**

Materials:

- BLM 7.N.3.8: Percent Problems
- demonstration board
- math journals or notebooks
- marking pens

Organization: Whole class (for demonstrations), individual or small groups (for practice)

Procedure:

1. Explain to students that some problems contain situations that require the solution to be rounded for practical reasons.

2. Review decimal, fraction, and percent equivalents with a quick oral quiz or a contest (e.g., a spelling bee), with teams lined up perpendicular to the board. Each team sends a representative to the board. The representative writes the decimal, fraction, or percent equivalents on the board. If the response is correct, the player returns to the end of the line; if the response is incorrect, the player steps out of the line and watches. The last team with a player at the board wins.
3. Present the class with a problem, which may or may not require rounding.

   Example:

   > Your grandmother is going to buy a hooded sweatshirt for your birthday. She heads down to the Pretty Trendy Clothing Store, and finds that the store has an anniversary special. All regular prices are reduced by 20%. She selects a sweatshirt that is regularly priced at $59.99 and pays GST of 5%. How much does she pay for the sweatshirt?

4. Ask students to solve the problem in their math journals or notebooks.
5. After students have had sufficient time to work on the problem, ask individuals to share the strategies they used to solve the problem. Compare students' solutions, noting whether or not students used rounding, and discuss reasons for their decisions. Discuss which strategies students prefer for this problem. Ensure that students consider the option of calculating the remaining cost versus calculating the value of the sale and subtracting it from the original price (80% of $59.99 versus $59.99 – 20% of $59.99). During the discussion, have students make changes or add comments to their work with a marking pen. They may make a math journal entry suggesting hints for solving problems with percents.
6. Follow the same procedure for a few more problems such as the ones listed on BLM 7.N.3.8: Percent Problems.

Variation:

- After presenting situations requiring rounding and providing a few examples, have students create their own problems and solution keys. Then have students exchange problems with group members, solve the problems, and later reassemble as a group to discuss solutions.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Express a percent as a decimal or fraction.
- ❒ Solve a problem that involves finding a percent.
- ❒ Determine the answer to a percent problem where the answer requires rounding, and explain why an approximate answer is needed.

Number (7.N.4)

Enduring Understanding(s):

Percents, fractions, decimals, and ratios are different representations of the same quantity.

Number sense and mental mathematics strategies are used to estimate answers and lead to flexible thinking.

Circle graphs show a comparison of each part to a whole using ratios.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
7.N.4 Demonstrate an understanding of the relationship between repeating decimals and fractions, and terminating decimals and fractions. [C, CN, R, T]	➛ Predict the decimal representation of a fraction using patterns $\left(\text{e.g., } \frac{1}{11} = 0.\overline{09}, \frac{2}{11} = 0.\overline{18}, \frac{3}{11} = ?\ldots\right)$. ➛ Match a set of fractions to their decimal representations. ➛ Sort a set of fractions as repeating or terminating decimals. ➛ Express a fraction as a terminating or repeating decimal. ➛ Express a repeating decimal as a fraction. ➛ Express a terminating decimal as a fraction. ➛ Provide an example where the decimal representation of a fraction is an approximation of its exact value.

Prior Knowledge

Students may have had experience with the following:

- Applying estimation strategies, including
  - front-end rounding
  - compensation
  - compatible numbers

  in problem-solving contexts.

- Applying mental mathematics strategies for multiplication, such as
  - annexing, then adding zeros
  - halving and doubling
  - using the distributive property
- Demonstrating an understanding of division (3-digit numerals by 1-digit numerals) with and without concrete materials, and interpreting remainders to solve problems.
- Describing and representing decimals (tenths, hundredths, thousandths) concretely, pictorially, and symbolically.
- Relating decimals to fractions (tenths, hundredths, thousandths).
- Comparing and ordering decimals (tenths, hundredths, thousandths) by using
  - benchmarks
  - place value
  - equivalent decimals
- Demonstrating an understanding of place value for numbers
  - greater than one million
  - less than one-thousandth
- Demonstrating an understanding of factors and multiples by
  - determining multiples and factors of numbers less than 100
  - identifying prime and composite numbers
  - solving problems involving factors or multiples

For more information on prior knowledge, refer to the following resource:

Manitoba Education and Advanced Learning. *Glance Across the Grades: Kindergarten to Grade 9 Mathematics*. Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/cur/math/glance_k-9/index.html.

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of the addition, subtraction, multiplication, and division of decimals to solve problems (for more than 1-digit divisors or 2-digit multipliers, technology could be used).
- Comparing and ordering fractions, decimals (to thousandths), and integers by using
  - benchmarks
  - place value
  - equivalent fractions and/or decimals
- Demonstrating an understanding of oral and written patterns and their corresponding relations.

- Demonstrating an understanding of circles by
  - describing the relationships among radius, diameter, and circumference of circles
  - relating circumference to pi
  - determining the sum of the central angles
  - constructing circles with a given radius or diameter
  - solving problems involving the radii, diameters, and circumferences of circles
- Expressing probabilities as ratios, fractions, and percents.

Background Information

In previous grades, students learned that *fractions* and *decimals* are interchangeable names for the same quantity. In Grade 7 (learning outcome 7.N.3), students review multiple meanings for the term *fraction* and learn strategies for finding decimal and fraction equivalents. For learning outcome 7.N.4, students will sometimes use calculators and the understanding that a fraction also represents division to find the decimal equivalents for fractions that are not conveniently renamed in other ways (e.g., $\frac{1}{8}$ may be read as $1 \div 8$, which equals 0.125).

Fraction and Decimal Equivalents

All fractions have equivalent decimal names. The decimal names may have a definite number of digits. These are *terminating decimals*. A terminating decimal can be easily renamed as a fraction with a denominator that is a power of 10 (e.g., 0.125, read as 125 thousandths, and written as a fraction $\frac{125}{1000}$, which can be simplified to $\frac{1}{8}$).

When some fractions are renamed as decimals, the decimal number contains one or more digits that repeat in a continuous pattern indefinitely (e.g., $\frac{1}{3} = 0.333\ldots$). These are *repeating decimals*. The three dots indicating the digits continue without end are called an *ellipsis*. In North America, the common representation for repeating decimals is to write the number with one set of the repeating digits, and then draw a bar over the digits that form the repeating pattern $(0.\overline{3})$. The series of digits that repeat may be called a *period*. The bar is called a *vinculum*. Other notations include placing a dot over the digits at each end of the repeating sequence $(0.\dot{3},)$ or enclosing the repeating sequence in parentheses [0.(3)]. Repeating decimals may also be renamed as fractions $\left(\frac{1}{3}\right)$. Characteristic patterns may be used to predict the decimal representation of these fractions and to predict the fraction representation of repeating decimals.

Note:
Students from varying cultural backgrounds may have different conventions for representing the repeating sequence. Students should be aware of all conventions, but should not be required to memorize the representations.

Provide students with learning activities that lead them to discover interesting patterns in the relationships between fraction and decimal equivalents. Investigating the patterns provides an opportunity to explore number sense. The situation with ninths may lead to a question of $0.\overline{9}$ versus 1. (Some patterns are listed for teacher reference in a table on the next page.)

> **Note:**
> These are interesting patterns for students to discover, but students are not required to memorize these relationships.

To determine whether a fraction will result in a terminating or repeating decimal, simplify the fraction and consider the prime factors of the denominator. If the only prime factors of the denominator are 2s and/or 5s, the fraction will have a terminating decimal equivalent. The number of 2s and 5s may be used to predict the number of place value positions in the decimal equivalent. The relationship is described below. If the denominator contains prime factors other than 2 or 5, the decimal number will be a repeating decimal. The maximum number of digits that may repeat will be one less than the denominator. This is sometimes the case when the denominator is a prime number. The decimal equivalents for fractions with the prime denominator 7 form a cyclic repeating decimal pattern with six repeating digits (142857).

To predict the number of digits in a terminating decimal number, simplify the fraction and express the denominator as a product of prime factors. Count the number of 2s and the number of 5s in the product. Determine whether there are more 2s or more 5s. The number of times the most frequently occurring digit occurs in the prime product equals the number of place value positions in the decimal number.

Example:

$\left(\frac{1}{8}\right)$

8 written as a product of prime factors = 2 × 2 × 2.

There are three 2s in the product and three decimal places in the decimal equivalent for $\frac{1}{8}$, which is 0.125.

$\left(\frac{2}{8}\right)$

$\frac{2}{8} = \frac{1}{4}$

4 written as a product of prime factors = 2 × 2.

There are two decimal places in the decimal equivalent 0.25.

Some fractions and their repeating decimal equivalents are listed in the following table for reference. Give students opportunities to discover these patterns.

Fractions and Their Repeating Decimal Equivalents		
Denominator of the Fraction	**Pattern in the Repeating Decimal**	**Example**
7ths	■ six repeating digits ■ digits are 142857 in a cyclic pattern	$\frac{2}{7} = 0.\overline{285714}$
9ths	■ single repeating digit ■ the numerator is the repeating digit	$\frac{7}{9} = 0.\overline{7}$
99ths	■ two repeating digits ■ the numerator is the repeating sequence	$\frac{20}{99} = 0.\overline{20}$
999ths	■ three repeating digits ■ the numerator is the repeating sequence	$\frac{1}{999} = 0.\overline{001}$
11ths	■ two repeating digits that are a multiple of 9 ■ the numerator is the factor × 9 that equals the repeating sequence	$\frac{3}{11} = 0.\overline{27}$

Rounding a decimal equivalent results in an approximation of the value of the fraction. Contexts or circumstances may dictate that a decimal number must be rounded to a specified number of digits. Many calculators round to the final digit in their display (e.g., a student's score of $\frac{12}{18}$ may be reported as 67%, $\frac{2}{3}$ of a metre may be measured as 67 cm). Each of these situations represents an approximation of the true value of the number. To express an exact value for a repeating decimal, indicate the repeating section with a vinculum, or write the fraction equivalent. To indicate that the number is an approximation of the true value, use a tilde mark (~), an approximately equal sign (≈), or an equal sign with a dot over it (≐).

Note:
Similar to the notation for repeating decimals, students may have had prior exposure to one of these representations. All representations should be considered correct.

Mathematical Language

decimal equivalent
decimal number
denominator
factor
fraction
fraction equivalent
multiple
numerator
prime number
product of prime factors
repeating decimal
simplify a fraction to lowest terms
terminating decimal

Optional language that may be used by teachers, but is not required of students

- ellipsis
- tilde
- unit fraction
- vinculum

Assessing Prior Knowledge

Materials:

- math notebooks
- calculators (optional)

Organization: Individual, pairs

Procedure:

1. Introduce and/or review the meaning of a prime number, factors, prime factors, and writing a number as the product of prime factors.*
2. Have each student use each of the digits 0 to 9 once to write five 2-digit numbers in his or her math notebook.
3. Next, have students write each of their numbers as a product of prime factors.
4. When students are finished with their individual work, ask them to exchange notebooks with a partner. Have them verify that the numbers in the product are all prime numbers and that the product is equal to the original number.

*** Note:**
In Grade 7, students are not formally exposed to prime factorization. It is an achievement indicator in Grade 8 Mathematics in the study of squares and square roots, and a learning outcome in Grade 10 Introduction to Applied and Pre-Calculus Mathematics. Provide students with guided support when prime factorization is required.

Variations:

- Supply numbers and a template for finding the prime factors.
- Have students create 3-digit numbers, using each digit no more than twice.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Identify factors and multiples of a number correctly.
- ❒ Identify prime factors correctly, with guidance.
- ❒ Use mental mathematics strategies, including divisibility rules, and using calculators when required.

Assessing Prior Knowledge

Materials:

- demonstration board, chalk or markers, erasers
- a list of numbered division questions specifying the format of the answers (e.g., fraction remainder, number of decimal places) and the answers
- copies of each numbered question on individual papers (the number of copies equals the number of teams)
- a reward for the winners (optional)

Organization: Teams—groups of three (of mixed ability) are ideal, but the number depends on the size of the board space available and the number of students in the class.

Procedure:

1. Have teams line up perpendicular to the board. The first players on all teams position themselves at the board and draw a point box at the top of the board, while the other players remain about two metres behind (or whatever distance works). Remind players to write large enough and high enough so you can see their work.
2. Provide the first player on each team with a division question, and state the form for expressing the answer (e.g., fraction remainder, number of decimal places). Students record their respective questions on the board, find the solutions, and draw a box around the quotients when their responses are complete. Establish a signal that students can use to get your attention for assessing their responses.
3. Respond to a student's signal and verify his or her response. If the response is correct, the student records a point in the team's point box. This player then comes to get the next question from you, dictates the question to the next team member in line, and then returns to the end of the line for his or her team.
4. The next team member goes to the board, and repeats the process.
5. When students have had sufficient time to work on the division questions, end the game, and declare the winning team to be the one with the most points. Provide a reward (optional).

Variations:

- Establish any desired rules about obtaining help from teammates.
- Have all teams work on the same question at the same time. You decide the time available to complete the question, call out the time, and provide the answer. Students award their team with a point if their answer is correct. Players return to the end of their team line, and you call out the next question.
- Have students create the questions and answer keys.
- Have students call out the questions and the answers, giving you more time to observe.
- Have teams complete the work on large pieces of scrap paper at table groups and show their work on request.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Apply an appropriate division algorithm.
- ❒ Recognize that when a remainder repeats, the quotient has entered a repeating pattern.
- ❒ Express a fraction as a terminating or repeating decimal.
- ❒ Sort a set of fractions as repeating or terminating decimals.
- ❒ Provide an example where the decimal representation of a fraction is an approximation of its exact value.

Suggestions for Instruction

- **Predict the decimal representation of a fraction using patterns $\left(\text{e.g., } \frac{1}{11} = 0.\overline{09}, \frac{2}{11} = 0.\overline{18}, \frac{3}{11} = ? \ldots\right)$.**
- **Sort a set of fractions as repeating or terminating decimals.**
- **Express a fraction as a terminating or repeating decimal.**
- **Express a repeating decimal as a fraction.**
- **Express a terminating decimal as a fraction.**
- **Provide an example where the decimal representation of a fraction is an approximation of its exact value.**

Materials:

- BLM 5–8.10: Base-Ten Grid Paper
- base-10 blocks (10 flats, 10 rods, and 10 units for each student)
- three-dimensional or paper models
- small strips to represent a unit block divided into 10 columns and no rows
- three food items (e.g., candy bars, granola bars), of uniform size, that can easily be cut into equal portions (or enough to share with the whole class)
- demonstration board
- math notebooks
- calculators (at least one of which rounds repeating decimals)

Organization: Whole class

Procedure:

1. Lead a class demonstration and discussion about representing a fraction quantity with a base-10 place value system. Use base-10 blocks, and include the concepts of terminating and repeating decimals, using division to represent a fraction, notations that represent exact quantities, and approximation.
2. The purpose of numbers is to provide a way to communicate about quantity. Share a candy bar equally between two students. Ask how much of the candy bar each one received. Write $\frac{1}{2}$ on the board. One-half of the candy bar describes exactly what each student received. It's an exact number. Ask whether it is possible to name that number with the base-10 place value system.

3. Have students demonstrate the action with base-10 models. The flat represents one candy bar. The place value system dictates that if the flat is cut, it must be cut into 10 equal pieces. Review that there are 10 pieces (use the digits 0 to 9) in each place value position. If students have not used base-10 blocks to represent decimals or fractions before, you may need to make it very clear that they are using the blocks differently here than when they used them for whole-number operations. They are using them because they are nicely "cut" into tenths. If one flat represents one candy bar, and individuals are going to share it, the one flat must first be "cut" into 10 pieces. Don't actually cut the block; instead, swap it for 10 rods. Each rod represents one-tenth of one. Now share the candy bar block with two imaginary people. How much does each one receive? Each one receives five rods, which are named five-tenths, and written as 0.5. Record 0.5 on the board. It is an exact number too. Five-tenths describes the exact quantity here.
4. Repeat the process (steps 2 and 3) with the second candy bar, sharing it among three students. Ask how much candy bar each one received. Record $\frac{1}{3}$ on the board. One-third of the candy bar describes exactly what each student received. It's an exact number. Ask whether we are able to name that number with the base-10 place value system.
5. Have students model sharing their "flat" candy bar with three imaginary people. Each will get three-tenths, and one-tenth will be left over. Explore options for writing the number. Can we write 0.3 as $\frac{1}{3}$? Record this question on the board with a question mark. That would be mixing two languages in the same word, which would be confusing. Go back and "cut" up the tenth rod and share the pieces. Remember, the only possible "cut" is into 10 equal pieces. Swap the rod for 10 small cubes. This results in 0.33 and one-tenth of a tenth left to share among three. You need to cut that piece into 10 again in order to share it. You are out of blocks, so use the grid paper to represent the action. The grid represents one little block cut into 10 pieces. Share the little pieces, and one will be left again. Students should realize this is going to go on forever. A similar situation was encountered when students explored divisibility rules for 3. Explore possibilities for writing the number. Introduce the term *repeating decimal number* and the concept of drawing a bar over the 3 as a notation that this 3 repeats forever.
6. Repeat the process (steps 2 and 3), with the third candy bar shared among four students. Write $\frac{1}{4}$ on the board, and then 0.25. Both are exact numbers. They describe the exact quantity.
7. Note that $\frac{1}{3}$ is named a *repeating decimal* because the sharing is never complete, as the value 3 repeats in every place value position indefinitely. A repeating decimal may have more than one repeating digit, but the pattern will never stop. Observe that $\frac{1}{2}$ and $\frac{1}{4}$ were shared completely in the place value model. The numerals had a definite number of digits. Decimal numbers with a definite number of digits are called *terminating decimals*. Terminating decimals may have many digits, but there is no repeating pattern, and they stop, or there is zero repeating.

8. In this learning activity, the actions of cutting and sharing the bars denote division. We can read fractions as division operations and obtain names for the fraction in our place value system. Demonstrate long division on the board, or have students use their math notebooks to perform long division for 1 ÷ 2, 1 ÷ 3, and 1 ÷ 4, and compare their results to the models. Ask students to obtain a decimal name for $\frac{1}{6}$. Ask whether these are exact names for the fractions.
9. Ask students to use calculators to convert the unit fractions from $\frac{1}{2}$ to $\frac{1}{6}$ to decimal numbers, and have them record the equivalents in a table in their math notebooks. Ask students to share the results. Some students will likely have calculators that round $\frac{1}{6}$ to 0.16666667.

Note:
Rounding a decimal number, or failing to provide an indication that the decimal repeats, is an approximation, not an exact value, and it ought to be noted as such. It also shows students whether the number of digits equals or exceeds the capacity of the calculator display. Students cannot rely on the calculator to verify whether a fraction is represented as a terminating or repeating decimal.

Variation:

- Use smaller decimal examples, sharing with fifths, sixths, or eighths. With these variations, the model is a little more tedious to use and the candy bar pieces are small.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Predict the decimal representation of a fraction using patterns $\left(\text{e.g., } \frac{1}{11} = 0.\overline{09}, \frac{2}{11} = 0.\overline{18}, \frac{3}{11} = ?\ldots\right)$.
- ❒ Sort a set of fractions as repeating or terminating decimals.
- ❒ Express a fraction as a terminating or repeating decimal.
- ❒ Express a repeating decimal as a fraction.
- ❒ Express a terminating decimal as a fraction.
- ❒ Provide an example where the decimal representation of a fraction is an approximation of its exact value.

- **Express a fraction as a terminating or repeating decimal.**
- **Provide an example where the decimal representation of a fraction is an approximation of its exact value.**

Materials:

- coloured counters
- chart paper

Organization: Pairs, whole class

Procedure:

1. Have pairs of students make up one multiple-choice question to ask the class, or a target group within the class.

   Sample Questions:

   - **For all students:** If you could have a superpower, would it be time travel, rocket speed, or immortality?
   - **For the girls:** As a lunchtime activity, do you prefer intramurals, school governance, free time, homework help, or games club?
   - **For the boys:** Is your favourite season spring, summer, fall, or winter?

2. Have students collect the data from the class or target group.
3. Provide students with access to coloured counters, and have them represent the collected data in the form of a circle graph. Once you have had a chance to look at their circle graphs, ask students to represent their circle graphs on chart paper, labelling the titles of each piece of the pie on the front of the chart paper and recording their responses to the following questions on the reverse side of the chart paper for future reference:

   a) What percent of this class or target group is represented by each piece of the pie?

   b) What fraction would best represent each piece of the pie?

   c) What decimal would best represent each piece of the pie?

   d) Make a statement about the largest section of the pie or circle graph.

Note:
Students' work in this learning activity is meant to address repeating and terminating decimals and their fractional equivalents, and should remain general with respect to circle graphs. Learning outcome 7.SP.3 relates specifically to circle graphs. You might find this to be a natural opportunity to make connections between the Number and Statistics and Probability strands.

4. Have students rotate, in pairs, to the circle graphs created by the rest of the class and answer the following questions:
   a) What percent of this class or target group is represented by each piece of the pie?
   b) What fraction would best represent each piece of the pie?
   c) What decimal would best represent each piece of the pie?
   d) Make a statement about the largest section of the pie or circle graph.
5. Gather as a class and discuss the following questions:
   a) What can be said about representing the data on a circle graph?
   b) Which is the best way to view this data (as fractions, decimals, or percents)?
   c) Are exact or approximate values needed on the circle graph to represent the fraction of students? Give examples of when an exact representation is needed and when an approximate representation is needed.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:
- ❒ Express a fraction as a repeating decimal.
- ❒ Express a fraction as a terminating decimal.
- ❒ Represent a number in a variety of ways.
- ❒ Estimate a value based on a pictorial representation.
- ❒ Communicate mathematically.

Suggestions for Instruction

- **Predict the decimal representation of a fraction using patterns** $\left(\text{e.g., } \frac{1}{11} = 0.\overline{09}, \frac{2}{11} = 0.\overline{18}, \frac{3}{11} = ?\ldots\right)$**.**
- **Sort a set of fractions as repeating or terminating decimals.**
- **Express a fraction as a terminating or repeating decimal.**

Materials:

- BLM 7.N.4.1: Table for Recording Fractions and Their Decimal Equivalents (or another chart for recording fraction-decimal equivalents, such as the one students began in their math notebooks in the previous learning activity)
- demonstration board
- calculators
- Venn diagram or T-chart (optional)

Organization: Whole class, individual, small groups

Procedure:

In the previous learning activity, students identified the decimal equivalents for the unit fractions $\frac{1}{2}$ to $\frac{1}{6}$ and classified them as repeating or terminating decimals. They used calculators and long division to confirm their classifications and noted the difference between exact and approximate representations. In this learning activity, students investigate patterns in the unit fractions $\frac{1}{2}$ to $\frac{1}{20}$ to develop generalizations useful for predicting terminating or repeating decimals and expressing fraction and decimal equivalents.

1. Review the concepts addressed in the previous learning activity. Stimulate student curiosity by asking if they think it is possible to predict whether a decimal representation will terminate or repeat.
2. Launch an inquiry task to find an answer to the question of predictability. Looking at examples and discovering patterns gives a basis for making predictions that can then be confirmed and generalized into rules.

   Possible steps that students can follow for the inquiry task are listed below.

   a) Specify the question to investigate. (How do you predict whether a fraction number is represented by a terminating or repeating decimal number?)
   b) Select an element that you think may be the determining factor (e.g., numerator or denominator). It is not possible to examine all elements at once, so select one to investigate. The only elements in a fraction are the numerator and the denominator. Look at the list created so far for a hint as to which one to choose. The fraction $\frac{1}{3}$ repeats, as does $\frac{1}{6}$.

      What about $\frac{2}{3}$ or $\frac{2}{6}$? Fraction names for 1 need not be considered because they all equal 1.0. The fraction $\frac{1}{4}$ terminates. What about $\frac{2}{4}$ and $\frac{3}{4}$?

      Note:
      The important thing here is to have students investigate the denominator as the determining factor.

   c) Generate a list of examples that isolate the chosen factor. (Suggest beginning with unit fractions with denominators 2 to 10 and numerators of 1.)
   d) Examine the list of examples for a common element or patterns that describe the relation. Use a Venn diagram or T-chart to organize findings. (These repeating decimals have denominators 3, 6, 7, and 9. The terminating decimals have denominators 2, 4, 5, 8, and 10. Notice 2, 4, and 8 are all divisible by 2, and 5 and 10 are divisible by 5. To add more examples, extend the list to denominators to 20. A group effort is advised for 17 and 19, as these denominators have a long repeating period. Students may be excited to note that the denominators of all the terminating decimals have 2 and/or 5 as factors. See the $2 \times 5 = 10$ base-10 connection.)

e) Create a descriptive phrase on which to base predictions. (Terminating decimals have denominators that are multiples of 2 and/or 5. The denominators have no prime factors other than 2 and/or 5. Repeating decimals have denominators with prime factors other than 2 or 5.)

f) Test your predictions with several new examples. (Any fractions will do. Consider including various numerators such as $\frac{8}{32}$ and $\frac{11}{55}$, as well. Students should note that equivalent fractions have the same decimal representations.)

g) If all predictions are correct, it's time to create a rule. Be open to the fact that some examples may disprove your rule, and you will need to begin at step (d) again, or note exceptions to your rule. (Ensure that, at some point, students realize a fraction must be simplified to apply their rule.)

3. Record and celebrate students' inquiry findings by playing a terminating or repeating decimal game, such as the one suggested in the next learning activity.

Variations:

- Guide the whole class as a group, or have students work individually or in small groups based on students' ability to conduct inquiries.
- Provide templates to guide sample selections, and provide questions to prompt generalizations for inquiries.
- Supply students with the rule, and explain how it is based on factors. Have them use prime factor denominators in several predetermined fractions to determine whether the fractions are represented by terminating or by repeating decimals.
- Following the inquiry, present students with a list of fractions, and ask them to sort the fractions according to whether they have terminating or repeating decimal equivalents. Have students supply prime factors as evidence of their decision.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Express a fraction as a terminating or repeating decimal.
- ❒ Sort a set of fractions as repeating or terminating decimals.
- ❒ Express a fraction as a terminating or repeating decimal.
- ❒ Make connections among repeating decimals, terminating decimals, and place value.

Suggestions for Instruction

- **Sort a set of fractions as repeating or terminating decimals.**
- **Express a fraction as a terminating or repeating decimal.**

Materials:

- tic-tac-toe frames or grids (of various sizes), such as the following:
  - BLM 7.N.3.1A: Tic-Tac-Toe Frames
  - BLM 7.N.3.1B: Tic-Tac-Toe Frames (Medium Challenge)
  - BLM 7.N.3.1C: Tic-Tac-Toe Frame (Ultimate Challenge)
- calculators
- notebook paper
- pens of two different colours

Organization: Pairs

Procedure:

Pairs of students play a tic-tac-toe game on a grid of any size. The object is to create a line of fractions whose equivalents are either terminating or repeating decimals and to monitor the fractions played by the opponent to verify whether they represent terminating or repeating decimal numbers.

1. The players each choose a colour and decide who will play fractions represented by terminating decimals, and who will play fractions represented by repeating decimals. They decide on the size of grid on which to play, and who will go first.
2. On the first move, a player selects which square to play in, creates a fraction represented by her or his type of decimal representation, and writes the fraction clearly in the selected square.
3. The opponent verifies the play, using a calculator if necessary. If the player has played a fraction represented by a terminating decimal instead of a repeating decimal, or vice versa, the opponent may capture the play by circling or re-colouring the fraction with his or her own colour.
4. The next player plays, repeating steps 2 and 3.
5. Play continues until one player wins the round by connecting a horizontal, vertical, or diagonal line of repeating or terminating decimals. If a player fails to notice a connecting line on his or her turn, the opponent may draw the connection and declare himself or herself the winner. If the challenger is incorrect, the other player is a double winner.

6. Students have two options to end the game:
   a) The game is over after a specified amount of time has passed.
   b) Play stops when one player reaches a target number of wins.

Variations:

- Vary the size of the grid the players use for the game.
- Have students design their own game.
- Prepare a list of fractions. Have students sort the fractions into two groups, repeating decimals or terminating decimals.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:
- ❒ Sort a set of fractions as repeating or terminating decimals.
- ❒ Express a fraction as a terminating or repeating decimal.

Suggestions for Instruction

- **Predict the decimal representation of a fraction using patterns** $\left(\text{e.g., } \frac{1}{11} = 0.\overline{09}, \frac{2}{11} = 0.\overline{18}, \frac{3}{11} = ?\ldots\right)$.
- **Sort a set of fractions as repeating or terminating decimals.**
- **Express a fraction as a terminating or repeating decimal.**
- **Express a repeating decimal as a fraction.**
- **Express a terminating decimal as a fraction.**

Materials:

- BLM 7.N.4.1: Table for Recording Fractions and Their Decimal Equivalents (or another chart for recording fraction-decimal equivalents, such as the one students began in their math notebooks in a previous learning activity)
- demonstration board
- calculators
- Venn diagrams or T-charts (optional)
- paper (small size), colours, glue, scissors, or computer technology for posters (optional)

Organization: Individual, small group, or whole class (depending on the interest and inquiry ability of students in the class)

Procedure:

1. Ask students to suggest other predictions that may be interesting or useful to investigate.
   - Can we predict the number of digits that will appear in the decimal, or in the repeating period?
   - Can the identity of the digits in a fraction–decimal equivalent be predicted?
   - Be sure to investigate fractions with denominators of 9, 99, 999, and 11.
   - The patterns of denominators 90 and 7 are also interesting investigations.
2. Have students conduct inquiries to find patterns on which to base predictions. General rules for each investigation are listed below.
   a) Specify the question to investigate.
   b) Select an element that you think may be the determining factor.
   c) Generate a list of examples that isolate the chosen factor.
   d) Examine the list of examples for a common element or patterns that describe the relation.
   e) Create a descriptive phrase on which to base predictions.
   f) Test your prediction with several new examples. This may lead back to step (d).
   g) Create a rule or generalization.
3. When students have a set of rules or generalizations, ask whether they could make predictions about the fraction represented by a decimal number. From all their observations, students should note the reverses. For repeating decimals, if $0.\overline{7}$ is an equivalent representation of $\frac{7}{9}$, then $\frac{7}{9}$ is the equivalent representation of $0.\overline{7}$.
4. Have students share their generalizations and create posters that outline general rules about converting fraction and decimal equivalents.

Variations:

- Provide students with templates to guide sample selections, and questions to prompt generalizations for inquiries.
- Provide students with a list of generalizations. Guide them through examples that prove each generalization.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Predict the decimal representation of a fraction using patterns $\left(\text{e.g., } \frac{1}{11} = 0.\overline{09}, \frac{2}{11} = 0.\overline{18}, \frac{3}{11} = ? \ldots\right)$.
- ❒ Sort a set of fractions as repeating or terminating decimals.
- ❒ Express a fraction as a terminating or repeating decimal.
- ❒ Express a repeating decimal as a fraction.
- ❒ Express a terminating decimal as a fraction.
- ❒ Communicate mathematical understanding.
- ❒ Reason in order to make connections to prior understanding.

Suggestions for Instruction

- **Predict the decimal representation of a fraction using patterns** $\left(\text{e.g., } \frac{1}{11} = 0.\overline{09}, \frac{2}{11} = 0.\overline{18}, \frac{3}{11} = ? \ldots\right)$.
- **Match a set of fractions to their decimal representations.**
- **Sort a set of fractions as repeating or terminating decimals.**
- **Express a fraction as a terminating or repeating decimal.**
- **Express a repeating decimal as a fraction.**
- **Express a terminating decimal as a fraction.**

Materials:

- calculators
- a list of generalizations about converting fraction and decimal equivalents (created in previous learning activities)
- BLM 7.N.3.4: Choose Your Question
- paper (small size), colours, glue, scissors, or computer technology for posters (optional)
- computers or other technology (optional)

Organization: Individual or small groups

Procedure:

As a culminating activity, have students create and participate in a variety of games that will help them to demonstrate an understanding of the relationship between repeating decimals and fractions, and terminating decimals and fractions.

1. Have students use the general rules about converting fraction and decimal equivalents that they created previously to create a Fraction and Decimal Expressions Trivia game, with players having to collect designated points from each category. Alternatively, have students use the templates from BLM 7.N.3.4: Choose Your Question to create a Choose the Question and Points game in an effort to score the highest points. Students may also wish to play some other game of their choice.
2. The game categories for the selected game are as follows:
   - fraction/decimal patterns
   - sorting fractions as repeating or terminating decimals
   - expressing a fraction as a terminating or repeating decimal
   - expressing a repeating decimal as a fraction
   - expressing a terminating decimal as a fraction
3. The class could host a grand competition.

Variations:

- Have students create a challenge quiz sheet and an answer key with an assortment of terminating and repeating decimals and fraction representations. Create criteria for the quiz, including questions about
  - fraction/decimal patterns
  - sorting fractions as repeating or terminating decimals
  - expressing a fraction as a terminating or repeating decimal
  - expressing a repeating decimal as a fraction
  - expressing a terminating decimal as a fraction

  Students may use computers or other technology to create their quiz. Photocopy the quiz sheets, and ask students to exchange challenges with a partner. The partners complete and correct each other's challenges.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Predict the decimal representation of a fraction using patterns $\left(\text{e.g., } \frac{1}{11} = 0.\overline{09},\ \frac{2}{11} = 0.\overline{18},\ \frac{3}{11} = ?\ \ldots\right)$.
- ❒ Sort a set of fractions as repeating or terminating decimals.
- ❒ Express a fraction as a terminating or repeating decimal.
- ❒ Express a repeating decimal as a fraction.
- ❒ Express a terminating decimal as a fraction.

Number (7.N.5)

Enduring Understanding(s):

The principles of operations used with whole numbers also apply to operations with decimals, fractions, and integers.

Number sense and mental mathematics strategies are used to estimate answers and lead to flexible thinking.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
7.N.5 Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially, and symbolically (limited to positive sums and differences). [C, CN, ME, PS, R, V]	➛ Model addition and subtraction of positive fractions or mixed numbers using concrete representations, and record symbolically. ➛ Determine the sum of two positive fractions or mixed numbers with like denominators. ➛ Determine the difference of two positive fractions or mixed numbers with like denominators. ➛ Determine a common denominator for a set of positive fractions or mixed numbers. ➛ Determine the sum of two positive fractions or mixed numbers with unlike denominators. ➛ Determine the difference of two positive fractions or mixed numbers with unlike denominators. ➛ Simplify a positive fraction or mixed number by identifying the common factor between the numerator and denominator. ➛ Simplify the solution to a problem involving the sum or difference of two positive fractions or mixed numbers. ➛ Solve a problem involving the addition or subtraction of positive fractions or mixed numbers, and determine if the solution is reasonable.

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of fractions by
  - explaining that a fraction represents a portion of a whole divided into equal parts
  - describing situations in which fractions are used
  - comparing and ordering fractions with like and unlike denominators
  - naming and recording fractions for the parts of a whole or a set
  - modelling and explaining that for different wholes, two identical fractions may not represent the same quantity
  - creating sets of equivalent fractions
- Relating improper fractions to mixed numbers.
- Describing and applying mental mathematics strategies, such as
  - skip-counting from a known fact
  - using doubling or halving
  - using doubling and adding one more group
  - using patterns in the 9s facts
  - using repeated doubling

  to develop recall of basic multiplication facts to 9 × 9 and related division facts.
- Applying mental mathematics strategies for multiplication, such as
  - annexing, then adding zeros
  - halving and doubling
  - using the distributive property
- Demonstrating an understanding of factors and multiples by
  - determining multiples and factors of numbers less than 100
  - identifying prime and composite numbers
  - solving problems involving factors or multiples
- Demonstrating an understanding of ratio, concretely, pictorially, and symbolically.
- Demonstrating an understanding of percent (limited to whole numbers) concretely, pictorially, and symbolically.

For more information on prior knowledge, refer to the following resource:

Manitoba Education and Advanced Learning. *Glance Across the Grades: Kindergarten to Grade 9 Mathematics*. Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/cur/math/glance_k-9/index.html.

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of the relationship between repeating decimals and fractions, and terminating decimals and fractions.
- Comparing and ordering fractions, decimals (to thousandths), and integers by using
  - benchmarks
  - place value
  - equivalent fractions and/or decimals

Background Information

Topic Overview

Fractions are commonly used in a variety of contexts in daily life. They are used to indicate quantities other than a whole. Fractions can be used to measure money, time, and distance. Fraction measurements are used in construction and cabinetry, in cooking, in art and design projects, and in sports. They are used to conduct tests and experiments, to gauge liquids, to keep track of periods at sporting events, and so on. Fractions are important for sharing anything from chocolate bars and pies to splitting restaurant bills.

In Grade 7, students develop their mathematical literacy by extending their conceptual understanding of fractions to combining and comparing fractional quantities using addition and subtraction.

Fractions name quantities between whole numbers. The part between the wholes can be divided into any number of equal parts. The number of equal parts in one whole is the denominator of the fraction, and the number of parts being referred to forms the numerator. Fractions are an extension of the whole number system, and the same principles for adding and subtracting whole numbers apply to adding and subtracting fractions. Understanding operations with whole numbers and having a good conceptual understanding of fractions provides an important foundation for both adding and subtracting fractions. When teaching operations with fractions, encourage students to focus on meaning and to make sense of a variety of contextual problems, regardless of whether they are working with proper fractions or with mixed numbers.

Many people, both students and adults, have unfriendly relationships with fractions. This indicates the importance of emphasizing number sense and concepts when working with fractions.

Conceptual Understandings

Before students perform operations, it is important to verify their conceptual understandings.

The term *fraction* has several meanings. *Fraction notation* is used to represent a "cut" or a part of a whole unit or region, a part of a group or set, a measurement, or a point on a number line. It is also used to represent a ratio or a portion of a turn, and to indicate the division operation. Ensure that students can record and interpret these different meanings.

The quantity represented by a fraction depends on the size of the whole. For example, $\frac{1}{4}$ of Prince Edward Island represents a different area than $\frac{1}{4}$ of Quebec, and $\frac{1}{5}$ of 100 represents a different quantity than $\frac{1}{5}$ of 10.

The numerator, or the top number of the fraction, indicates the number of parts in the fraction, and the denominator, or the bottom number, represents the type of part or unit size of the fraction. When adding or subtracting any numbers, the value or size of units must be the same. If the distance from a given point to your house is 2 km, and 500 m more to the park, it is incorrect to combine the numbers for a total distance of 502. It is necessary to convert the measurements to common units before combining them. For example, 2 km plus 0.5 km totals 2.5 km, or 2000 m plus 500 m totals 2500 m. These distances are equivalent. Fractions that have the same denominator, such as $\frac{1}{5}+\frac{3}{5}$, can easily be combined as $\frac{4}{5}$, but a collection of fractions might have different denominators. The different denominators indicate different types or units of measure that must be related to common reference points, or converted to common units, before adding or subtracting them. Ensure that students are able to create and identify equivalent names for fractions.

Some equivalent fractions, such as $\frac{1}{2}$ and $\frac{2}{4}$, are easily recognizable. Proficiency in identifying common factors and multiples facilitates renaming less recognizable fractions. Student will have a much easier time identifying factors and multiples if they have ready access to multiplication and division facts, and if they can apply divisibility rules.

Fraction notations may represent different meanings, and not all fraction meanings can be combined in the same manner. When considering the "cut" meaning of a fraction, it is quite clear that $\frac{1}{2}$ of a pizza and $\frac{1}{2}$ of a pizza can be combined to form the equivalent of 1 whole pizza, as long as all the pizzas are the same size. However, if you wrote a test, and answered $\frac{1}{2}$ of the questions in Part A correctly and $\frac{1}{2}$ of the questions in Part B correctly, you cannot combine the two parts and say you answered 1 test correctly. In this case, the fractions are parts of sets. When you combine them, you increase both the number of selected parts and the number of parts in the set. The score totals $\frac{2}{4}$.

Likewise, if you represent $\frac{1}{6}$ of the members in your family, and your friend represents $\frac{1}{3}$ of the members in her family, and you are asked to combine these fractions, you may be tempted to say $\frac{1}{3}+\frac{1}{6}=\frac{1}{2}$. Ask, $\frac{1}{2}$ of what? You and your friend do not represent $\frac{1}{2}$ of the members of both families. These fractions represent part of a set. The sets in the addends are not the same set referred to in the answer. Your friend and you would represent $\frac{2}{9}$ of both families. You need to combine the members composing the set, as well as the numerators in each set. If you wanted to view the fractions as ratios, and you wanted to find the average portion of each family you represent, you could add the fractions and divide by 2 to obtain $\frac{1}{4}$. This illustrates the importance of students having both number sense and an understanding of the different meanings of fractions in order to add and subtract them correctly.

Adding and subtracting whole numbers sometimes requires regrouping, or carrying and borrowing. Conversions and regrouping between mixed numbers and improper fractions provide regrouping opportunities for operations with fractions. Students require the ability to make these conversions.

Focus of Instruction

Some mathematics teaching resources begin instruction with common denominators and addends with sums that are less than one. They then move to subtraction with common denominators, with the stipulation that the smaller fraction is removed from the larger fraction. Next, they progress to questions in which one denominator is a multiple of the other, and then to questions in which both denominators must be changed. Then they move to sums larger than one, and finally to mixed numbers. The progression is logical and increases in complexity. Students may, however, have difficulty with fraction operations if they focus or depend on remembering a sequence of steps they must follow to complete the operation. Students are more likely to internalize concepts if they have the opportunity to apply number sense to solving problems.

If students have a strong understanding of fraction concepts and whole number operations, and if teaching is focused on making meaning in a problem-solving context, it is not necessary to begin instruction with common denominators, and follow a set progression. Instead, present problems as realistic scenarios, and encourage students to use manipulatives and informal methods to arrive at solutions. Use friendly fractions that can easily be represented with manipulatives or drawings, and fractions that can easily be related to one another, such as quarters and eighths. An example of a friendly fraction combination is thirds and sixths. An example of an unfriendly fraction combination is fifths and twelfths. Using friendly fractions makes it easier for students to find equivalent units and helps them build confidence in the strategies they are developing. Highlight the generalizations that students make by connecting them to symbolic models for adding and subtracting fractions.

Encourage students to use rounding and benchmarks to make estimates. Estimates help students to focus on meaning and to create target zones for their solutions. The benchmarks of 0, $\frac{1}{2}$, and 1 (or 5, $5\frac{1}{2}$, 6, and so on) are useful when adding and subtracting fractions. Include problems relating to the various meanings of subtraction, such as take away or compare, and problems relating subtraction to addition by finding the missing addend. Provide students with opportunities to share and to assess one another's strategies.

Concrete and Pictorial Models

Concrete materials include fraction circles, fraction bars, fraction strips combined with number lines, metric rulers, and metre sticks, a collection of equivalent numbers lines with different unit divisions, base-10 blocks, Cuisenaire rods, pattern blocks, clocks, and money. Use commercially available products, or have students construct the materials by measuring them or by using blackline masters provided (e.g., BLM 5–8.12: Fraction Bars).

Students can generate simple pictures and diagrams to represent fractions and their combinations. Be aware that inaccurate drawings can lead to inaccurate results, especially when using circles. Minimize inaccuracies by supplying grid paper for drawing rectangles. Templates to represent circles, rulers, or number lines with equal intervals are also useful.

Equivalent fractions and combinations of fractions with unlike denominators can be represented by using grid drawings and by placing counters on the specified numbers of squares, or by colouring them. The denominators of the two fractions determine the number of rows and columns in the grid, and the numerators determine the number of squares in the grid that must be coloured or covered with counters.

Example:

Represent $\frac{2}{5}+\frac{1}{3}$ with a grid having five columns and three rows. Cover two columns to represent $\frac{2}{5}$, and cover one row to represent $\frac{1}{3}$. Rearrange the counters so there is only one counter per square. Of the 15 squares, 11 are covered, so the sum is $\frac{11}{15}$.

F	F			
F	F			
FT	FT	T	T	T

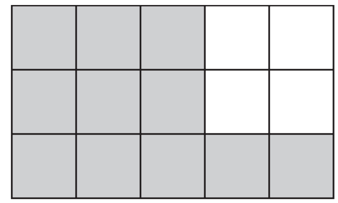
$\frac{2}{5}+\frac{1}{3}$

=

F	F	T		
F	F	T		
F	F	T	T	T

$\frac{11}{15}$

or



$\frac{6}{15}+\frac{5}{15}=\frac{11}{15}$

Equivalent fractions with common denominators are also evident in the drawing.

$\frac{2}{5}$ covers six squares or $\frac{6}{15}$ of the grid, so $\frac{2}{5}=\frac{6}{15}$.

$\frac{1}{3}$ covers five squares or $\frac{5}{15}$ of the grid.

Therefore, $\frac{1}{3}=\frac{5}{15}$. Combined, $\frac{6}{15}$ plus $\frac{5}{15}$ total $\frac{11}{15}$.

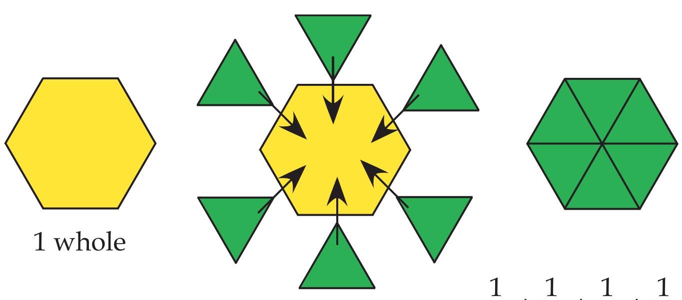
Similar grids can also be created by folding a piece of paper horizontally x number of times to represent one denominator, and then vertically y number of times to represent the other denominator. The numerators can be represented by placing counters in the rectangles of the grid or by colouring them.

Examples with Pattern Blocks:

Use pattern blocks for fractions of halves, thirds, and sixths.

- Students may cover the hexagon with six triangles.

  So, $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{6}{6}=1$.

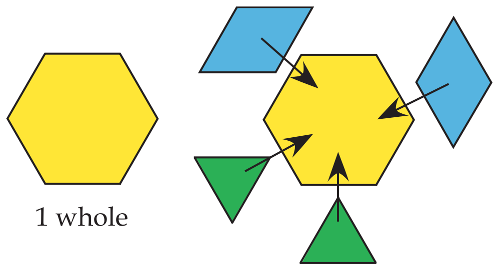


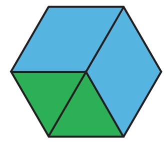
$\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{6}{6}=1$

This is also $\frac{2}{6}+\frac{2}{6}+\frac{2}{6}=1$ or $\frac{6}{6}$, or $\frac{3}{6}+\frac{3}{6}=1$ or $\frac{6}{6}$.

Also include related subtraction possibilities.

- Models may also include combinations of fractions, such as $\frac{1}{3}+\frac{1}{3}+\frac{1}{6}+\frac{1}{6}=1$.





$\frac{1}{3}+\frac{1}{3}+\frac{1}{6}+\frac{1}{6}=1$ or $\frac{2}{3}+\frac{2}{6}=\frac{6}{6}=1$

Also include related subtraction possibilities.

Examples with Rods:

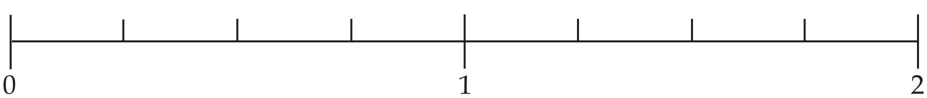
- The use of rods allows for various equivalent fraction representations, depending on which rod represents the whole (e.g., halves and quarters of 4 and 8, halves and thirds of 6, thirds and ninths of 9, halves and fifths of 10), and allows for connections to common factors and multiples.
- With Cuisenaire rods, students can choose any rod to represent a whole. If they choose the rod that is eight white squares long, then the 8 cm brown rod is the whole. The 8 cm rod can be covered with four red 2 cm rods or with two purple 4 cm rods, or a combination of 2 cm and 4 cm rods. Note that fractions are equal parts of a particular whole.

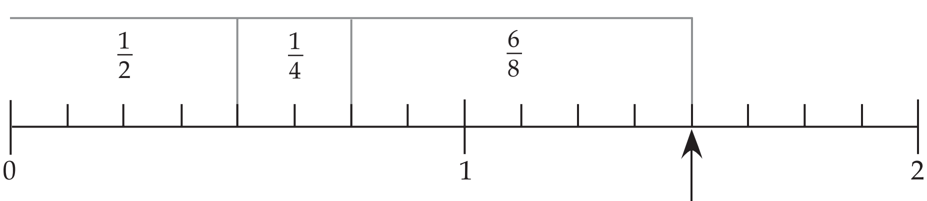
Examples with Circles:

- Circles can be divided into any fractional segments. For example, a blank clock face with minute divisions can represent multiple fractions and many equivalents, such as halves, thirds, fourths, fifths, sixths, tenths, twelfths, fifteenths, twentieths, and thirtieths.

Examples with Fraction Strips:

- Fraction strips are convenient models because there are multiple fraction sizes for the same size of a whole. They represent fractions as parts of whole numbers. Also, these strips can be joined together on the coordinating number lines to match sums that are greater than 1. Demonstrate how this works.





$\frac{1}{2}+\frac{1}{4}+\frac{6}{8}=1\frac{4}{8}$ or $1\frac{1}{2}$

- Students may wish to make their own fraction strip models and number lines using BLM 5–8.12: Fraction Bars.

Developing Algorithms

Allow students to use models as long as they need them. As students model adding and subtracting fractions, and discuss the strategies they use, they will develop algorithms. Multiple models help students to focus on meaning and encourage them to be flexible in their thinking. They provide opportunities to create equivalent fractions and to rename improper fractions and mixed numbers. By using models, students learn that renaming fractions makes statements easier to solve and that the equivalent statements are merely different names for the same action. (As with algorithms for whole-number and decimal operations, introduce algorithms for fractions after students have had time to develop their understanding.)

Mathematical Language

denominator

difference

equivalent fractions

factor, greatest common factor

fraction

improper fraction

mixed number

multiple, least common multiple

numerator

proper fraction

simplify

sum

Venn diagram

Assessing Prior Knowledge

Materials:

- BLM 7.N.5.1: Interpreting and Recording Different Meanings of Fractions
- demonstration board
- marking pens
- projector (optional)
- poster paper (optional)

Organization: Individual or pairs

Procedure:

1. Distribute copies of BLM 7.N.5.1: Interpreting and Recording Different Meanings of Fractions, and have students complete them.
2. Correct the responses together with students, and review various meanings of fractions. During the review, encourage discussion, questions, and additional scenarios, and clear up any misunderstandings. If students have errors on their sheets, have them make changes and add notes with a marking pen. A fraction may represent a "cut" or a part of a whole, a part of a set, a ratio, a portion of a turn, a measurement, a point on a number line, or a division statement.

Variations:

- Vary the complexity of the questions.
- Have students create, model, and name their own situations.
- Have students record representations of stated fractions on paper or at the board.
- Have students create posters or collages to represent various meanings of fractions.
- Use a projector to present visuals of various fractional representations, and have students record matching fraction names.
- Have students create fraction spiders. Draw a circle to represent the spider body and record the name of the fraction on the body. Draw eight legs coming out from the body. At the end of each leg, include some representation of the fraction. The feet may be illustrations, equivalent fractions, equivalent number sentences, word sentences, and so on.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❐ Interpret and record various meanings of fractions correctly.

Assessing Prior Knowledge

Materials:

- number cubes (10-sided or regular) or spinners
- recording paper
- index cards (optional)
- grid paper and circle templates (optional)

Organization: Whole class, individual, pairs

Procedure:

1. As a class, review definitions of *proper fractions*, *improper fractions*, and *mixed numbers*. Also review procedures for converting improper fractions to mixed numbers, and vice versa.
2. Have students, working individually, roll a number cube or spin a spinner to generate a list of 15 fractions. The first roll or spin determines the numerator and the second roll or spin determines the denominator. Using recording paper, students number and record each fraction neatly in a column, double spacing between fractions. Some fractions will be proper fractions and some will be improper fractions.
3. Have students convert the improper fractions to mixed numbers and record the new names in an adjacent column (about 3 cm away). They may also choose to simplify any fractions not in lowest terms.
4. Students then fold their papers vertically, so that the original fractions are not visible, but the mixed numbers are. They exchange papers with a partner. The partner neatly records the improper fraction equivalents for each mixed number in another column.
5. Students return the papers to the original owners, who compare the responses in all three columns. The partners discuss any discrepancies in the conversions. For example, someone may have simplified or failed to simplify a proper fraction, resulting in a different answer. Discuss the reason for the differences, and whether these differences are really differences in numbers, or in name only.

Variations:

- Have students add one whole to each of the fractions. Record the sum, and repeat the preceding process (e.g., $\frac{6}{4}$ becomes $\frac{10}{4}$, $1\frac{1}{2}$ becomes $2\frac{1}{2}$).
- Have students record the fractions on one face of an index card and use the reverse side to illustrate the fraction and rename it as a mixed number, and perhaps simplify it. Provide students with grid paper and circle templates to ensure their drawings are accurate. Cards can be saved and used for future addition and subtraction learning activities.
- Create a human number line. Ask students to record one of their fractions on paper with large writing. Call small groups to the front of the class. Students in the group hold their fractions in front of their chests, and stand in order from smallest to greatest fraction. Call the next group to fit into the line. Discuss students' strategies for ordering fractions and for what to do with equivalent fractions.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Correctly convert mixed numbers to improper fractions, and vice versa.
- ❒ Create and identify equivalent fractions.

Assessing Prior Knowledge

Materials:

- BLM 7.N.5.2: Improper Fraction and Mixed Number Cards
- various card sets (for each group) from the following website: Manitoba Education. "Middle Years Activities and Games."
  Mathematics. www.edu.gov.mb.ca/k12/cur/math/my_games/index.html.
- manipulatives (optional)
- grid paper or circle templates (optional)

Organization: Pairs or groups of three

Procedure:

1. Have students form pairs or groups of three to play Concentration.
2. Choose four sets of matching cards (e.g., from BLM 7.4.5.2: Improper Fraction and Mixed Number Cards).
3. Shuffle the cards and arrange them face down in a rectangle.
4. Have players take turns turning over two cards. If the cards represent the same quantity, the players keep the cards. Decide whether a match set warrants another turn.
5. The game ends when all the cards have been matched. The player with the most cards wins.

Variations:

- Students vary the number of card sets, or place the cards face up and match the sets.
- Shuffle a random number of fraction cards and place the pile face down. Players draw a card from the pile and build the fraction using manipulatives, or illustrate that fraction using grid paper or circle templates. They write both the improper fraction and mixed number names for the quantity.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Correctly convert mixed numbers to improper fractions, and vice versa.

Assessing Prior Knowledge

Materials:

- markers or pens of two different colours (for each pair of students)
- two regular number cubes (providing factors 1 to 12) or a multi-sided number cube (for each pair of students)
- grid paper or tic-tac-toe frames (of various sizes), such as the following:
  - BLM 7.N.3.1A: Tic-Tac-Toe Frames
  - BLM 7.N.3.1B: Tic-Tac-Toe Frames (Medium Challenge)
  - BLM 7.N.3.1C: Tic-Tac-Toe Frame (Ultimate Challenge)
- spinners (optional)

Organization: Pairs

Procedure:

1. Students draw a tic-tac-toe grid and take turns filling in the squares with numbers 1 to 99, or with multiples that correspond to the numbers on their number cube(s).
2. Explain the procedure for this learning activity to students:
   - Students choose a colour and an X or an O mark, and determine who will play first.
   - Students take turns rolling the number cube(s), and use their colour marker to mark an X or an O on a multiple of the number they rolled. Encourage them to practise using mathematical language with statements such as the following:
     - 27 is a multiple of 9 because $3 \times 9 = 27$.
     - 9 and 3 are factors of 27 because $3 \times 9 = 27$.
     - 17 is a prime number. Its only factors are 1 and 17.
   - Students will need to agree about what to do if someone makes an error. They may lose a turn, forfeit their play to their opponent, or just accept the correction.
   - The first student who creates a horizontal, vertical, or diagonal line with his or her marks wins.

Variations:

- Extend the grid to 4 × 4 or 5 × 5, or whatever dimensions students are able to handle.
- Vary the shape or size of the winning line.
- Include multiples of 12, 15, and 25, or target specific factors with custom-labelled number cubes or spinners.
- Prepare boards with selected numbers for students to practise.
- Encourage students to practise strategies when selecting their multiples.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Use vocabulary for multiples, factors, and prime numbers correctly.
- ❒ Identify multiples of various numbers correctly.
- ❒ Identify prime numbers correctly.

Suggestions for Instruction

- **Simplify a positive fraction or mixed number by identifying the common factor between the numerator and denominator.**

Materials:

- BLM 7.N.3.2: Equivalent Fraction Challenge
- a pair of six-sided number cubes, or a multi-sided cube, or a spinner (for each pair of students)
- calculators or multiplication charts (optional)

Organization: Whole class, pairs

Procedure:

1. As a class, review procedures for creating equivalent fractions by multiplying or dividing by a fraction name for 1, or by multiplying or dividing each term in the fraction by the same factor.

2. Demonstrate one round of the game, following the procedures outlined on BLM 7.N.3.2: Equivalent Fraction Challenge, and using the game cards provided on the BLM. In summary, students create a target fraction, take turns rolling the number cube(s) to determine a change factor, and then create an equivalent fraction. The player who returns the fraction to its original target name wins.
3. Distribute game cards.
4. Have students play the game in pairs.

Variations:

- Vary the complexity of the arithmetic by controlling the options on the type of number cubes. Use basic six-sided number cubes for numbers 1 to 6, a pair of number cubes for numbers 1 to 12, custom-labelled number cubes, or multi-sided number cubes. If you use a number cube with a zero, make a rule pertaining to zero (e.g., the player who rolls zero forfeits his or her turn, or forfeits the game). A spinner may also be used.
- Allow students who have difficulty with multiplication and division facts to use a calculator, a multiplication chart, or some other aid. Continue to work on developing students' understanding of multiplication and division facts so that they may develop recall.
- The game could be played with larger groups, or as a class.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Create equivalent fractions correctly and simplify fractions with ease.
- ❒ Use mathematical language to communicate about fractions.

Suggestions for Instruction

- **Simplify a positive fraction or mixed number by identifying the common factor between the numerator and denominator.**

Materials:

- presentation board
- math journals or notebooks

Organization: Whole class, individual

Procedure:

1. Present the class with a fraction that can be easily simplified $\left(\text{e.g., } \frac{2}{4}, \frac{2}{6}, \frac{6}{8}\right)$. Record the fraction on the board, ask for its simplified form, and record that fraction on the board. Continue recording and simplifying fractions, increasing the demand of the task. Ask students to explain the reasons behind their simplifications, and ask whether the new fractions can be simplified further. Eventually, it should become evident that it would be desirable to have a reliable, simple procedure to simplify less obvious fractions $\left(\text{e.g., } \frac{24}{36}, \frac{14}{63}\right)$.
2. Solicit fraction suggestions from students. Someone may suggest that if you knew the largest factor of both numbers, you would need to divide each number only once. Tell students this strategy is called *finding the greatest common factor.*
3. Write a fraction $\left(\text{e.g., } \frac{24}{36}\right)$ on the board. List the numerator and the denominator, and ask students to identify the factors for each number in a systematic progression.

   Example:

   Begin with the smallest factor of 24 (which is 2) and record it toward the left end of the row. Record its corresponding factor (12) toward the right end of the row. Continue working toward the centre until both factors begin to repeat.

   24: 2, 3, 4, 6, 8, 12

   36: 2, 3, 4, 6, 9, 12, 18

   Look for the largest factor that is common to both numbers. In this case, the largest factor is 12. Use this factor to simplify the fraction by dividing by $\frac{12}{12}$ as a name for 1. Or divide both the numerator and the denominator by 12.

   $$\frac{24}{36} \div \frac{12}{12} = \frac{2}{3}$$

4. Have students generate a list of obvious and less obvious fractions for which they will find the greatest common factor, and which they will simplify. Include proper fractions, improper fractions, and mixed numbers.
5. Record the list of fractions on the board, and have students record it in their math journals or notebooks. Complete one more sample together with the class, and then have students find the factors and simplify the fractions on their own. When they have completed the task, correct responses as a class. Solicit questions and comments, and have students record corrections and notes in their math journals or notebooks.

Variation:

- Supply students with a sheet containing a collection of fractions and proper, improper, and mixed numbers. Students list the factors for the numerator and the denominator, indicate the greatest common factor, and simplify the fraction.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Simplify a positive fraction or mixed number by identifying the common factor between the numerator and denominator.

Suggestions for Instruction

- **Model addition and subtraction of positive fractions or mixed numbers using concrete representations, and record symbolically.**
- **Determine a common denominator for a set of positive fractions or mixed numbers.**

Materials:

- presentation board
- math journals or notebooks
- rulers
- pens or markers of different colours
- manipulatives to represent fractions (e.g., pattern blocks, fraction circles, Cuisenaire rods, grids, counters)
- templates copied on card stock (e.g., fraction bars, circles, pattern blocks, rods, grid paper) for students to copy or cut and glue into their math notebooks
- scissors, glue (optional)

Organization: Small groups, whole class, individual

Procedure:

Part A: Modelling Fractions Equalling a Whole

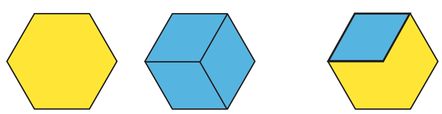
1. Have students, working in small groups, use different types of manipulatives to explore different ways to represent a whole. Consider giving each group different types of manipulatives, depending on the number of resources available. Decide whether to include only manipulatives that represent the "cut" (part of a whole) meaning of a fraction, or whether to include a point on a line. Have students talk about their models within their groups, and draw illustrations (or cut and paste templates) of two or more models in their math journals or notebooks.
2. Have students reassemble as a class. Ask a few students to share their models with the class. Show their illustrations on the presentation board.
3. Solicit ideas from students about how to write an addition statement to match each model of fractions equalling a whole.

   Example:

   Here is one example using pattern blocks:

   - A hexagon can represent one whole.
   - Three rhombuses cover the hexagon and occupy the same space, so each rhombus represents $\frac{1}{3}$ of the whole.

     1 whole $= \frac{3}{3}$ $\frac{1}{3}$

     $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$ or $\frac{3}{3}$

   - Combine fractional pieces if you wish. (See Concepts to Review during Discussion, following Parts A to C of the procedures.)

     $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$ and $\frac{2}{3}+\frac{1}{3}=\frac{3}{3}$ and $\frac{3}{3}=1$

   - Subtraction can be represented with the same model.
   - Demonstrate a take-away action. Also, remove a fractional part and find the difference by comparing what is left of the whole. Ask what part is missing.

- Write the matching subtraction statements:

  $1 \text{ or } \frac{3}{3}-\frac{1}{3}=\frac{2}{3}$

- Take away some more if you wish:

  $\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$

- Find the difference between two of the fractional parts:

  $\frac{1}{3}-\frac{1}{3}=0$

- When the appropriate model arises, extend the example to the following:

  $\frac{1}{3}-\frac{1}{6}=\frac{1}{6}$

4. Have students return to their math journals or notebooks and write addition and subtraction statements to match the models they illustrated.

Part B: Modelling Fractions, Including Proper Fractions

5. Have the groups change manipulatives and repeat the process outlined in Part A as many times as seems useful. If students are ready, have them consider equivalent representations, simplified fractions, and statements involving proper fractions less than a whole. Remind students to talk with their groups about their models and the matching of addition and subtraction statements. Have students record two or more new models, and write matching addition and subtraction statements.
6. Have students reassemble as a class to share interesting discoveries and to verify responses.

Part C: Modelling Proper Fractions, Improper Fractions, and Mixed Numbers

7. When appropriate, invite students to include combinations that represent more or less than a whole (e.g., mixed numbers such as $1\frac{3}{8}$ pizzas, $2\frac{1}{2}$ cans of juice, $\frac{3}{4}$ of a chocolate bar, $\frac{5}{8}$ of an inch). Remind students to verify that the fractional pieces are equal parts of the particular whole. Identifying the whole is important to understanding the fractional relations. Have students talk about their models and statements within their groups. Remind them to draw illustrations (or cut and paste templates) of two or more of the models in their math journals or notebooks.
8. Reassemble as a class, and have a few students share their new models and addition or subtraction statements. This sharing process provides both you and the students with an opportunity to verify responses. When modelling subtraction, compare two fractions and find the difference or missing part between them. Also model subtraction as taking away a fractional part from a mixed number. Have students write or verify their addition and subtraction statements for the models.

9. Have the groups change manipulatives and repeat the process as many times as seems useful. Have students discuss their models and statements within their groups, and record two or more models and matching addition and subtraction statements.

Concepts to Review during Discussion

A. The fraction models provide opportunities for rewriting addition and subtraction statements by combining fractions with like denominators, and renaming unlike denominators with equivalent fractions to obtain common denominators.

B. As you write equations for the models, discuss with students why these fractions with different names can be combined. The $\frac{1}{3}$ sections represent three equal parts of a whole, and the $\frac{1}{6}$ sections represent six equal parts of the same whole. They are all parts of the same whole.

C. Not all fraction sizes of the same whole can be combined to equal one whole. For example, combining halves and fifths will always result in either more or less than a whole.

D. The same fraction name can be used to represent different quantities. For example, $\frac{1}{2}$ of the water in my glass is different than $\frac{1}{2}$ of the water in my bathtub. Furthermore, $\frac{1}{3}$ of my whole pattern block is not the same as $\frac{1}{3}$ of your Cuisenaire rod, nor the same as $\frac{1}{3}$ of someone's fraction block. When using a "cut" (part of a whole) meaning of a fraction, the parts must be a fraction of the same whole in order to combine them. Revisit this topic frequently when discussing combining different types of fractions. When using Cuisenaire rods, students can explore to find that the red 2 cm blocks represents $\frac{1}{4}$ of the brown 8 cm rod, or $\frac{1}{5}$ of the orange 10 cm rod, but it represents neither $\frac{1}{4}$ nor $\frac{1}{5}$ of the blue 9 cm rod.

E. If a fraction represents a division situation, then it is a name for a number in our number system. It represents a portion of one unit. For example, $\frac{4}{2}$ represents $4 \div 2$, or the number 2, which is two whole units, and $\frac{1}{2}$ represents one divided by 2, $\frac{1}{2}$, or 0.5. If the fractions represent numbers, then any fractions can be combined, because they all represent parts of the same whole number unit, and not parts of different wholes, regions, or sets. When fractions are presented without a stated context, they represent this name for a number meaning, and can be added or subtracted freely. Fraction strips are a convenient representation of this concept. (See the next learning activity.)

Variations:

- For more direct instruction, guide students through specific models and matching addition and subtraction sentences.
- Provide scaffolding by supplying students with templates to complete.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Model addition and subtraction of positive fractions or mixed numbers using concrete representations, and record symbolically.

Suggestions for Instruction

- **Model addition and subtraction of positive fractions or mixed numbers using concrete representations, and record symbolically.**

Materials:

- BLM 5–8.12: Fraction Bars (copied on heavy paper or card stock)
- prepared sample of fraction strips and matching number lines
- scissors
- resealable bags for storing pieces
- math journals or notebooks
- magnetic tape (optional)

Organization: Whole class, individual

Procedure:

1. Demonstrate how fraction strips can serve as convenient models, as there are multiple fraction sizes for the same size of a whole. They can be used to find equivalent fractions.
2. Fraction strips are useful models for representing fractions as numbers because they are parts of the unit number 1. They represent fractions as points on a number line, and can be joined together on the coordinating number line to match fraction combinations with corresponding sums or differences. Demonstrate how to use the model.

3. Provide students with card stock copies of BLM 5–8.12: Fraction Bars. Have students make a set of fraction strips and corresponding number lines to use in various learning activities. Store the products in resealable bags.
4. Have students generate a list of addition and subtraction questions, and ask them to use their models to write several addition and subtraction statements, recording them in their math journals or notebooks.

Variations:

- Fasten the fraction strips to magnetic tape before cutting them. This adds greater durability, and the strips can be stored and used on a magnetic surface to minimize difficulty managing so many loose pieces.
- Prepare a list of addition and subtraction statements for students to practise using the fraction strips.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Model addition and subtraction of positive fractions or mixed numbers using concrete representations, and record symbolically.
- ❒ Use models to aid in the visualization of adding and subtracting with fractions.

Suggestions for Instruction

- **Model addition and subtraction of positive fractions or mixed numbers using concrete representations, and record symbolically.**
- **Determine the sum of two positive fractions or mixed numbers with like denominators.**
- **Determine a common denominator for a set of positive fractions or mixed numbers.**
- **Determine the sum of two positive fractions or mixed numbers with unlike denominators.**
- **Simplify the solution to a problem involving the sum or difference of two positive fractions or mixed numbers.**
- **Solve a problem involving the addition or subtraction of positive fractions or mixed numbers, and determine if the solution is reasonable.**

Materials:

- BLM 7.N.5.3A: Ace Aviation: Adding Fractions
- BLM 7.N.5.4A: Representing Recognizable Fractions and Writing Addition Statements (optional)
- presentation board
- manipulatives to represent fractions (e.g., BLM 5–8.12: Fraction Bars, pattern blocks, fraction circles, Cuisenaire rods, grids, counters)
- fraction strips and number lines (made by students)
- a reference list or a handout of model illustrations, scenarios, and problems involving adding and subtracting fractions (optional)
- index cards (optional)

Organization: Small groups, whole class, individual

Procedure:

1. Provide students with copies of BLM 7.N.5.3A: Ace Aviation: Adding Fractions, and have them respond to questions 1 to 4.
2. Discuss students' responses to questions 1 to 3.
3. Discuss students' thinking regarding the addition in question 4. Include the following ideas:
   - Discuss the models students used to combine the tourists and the vacationers with those visiting family or friends $\left(\frac{1}{3} + \frac{1}{6}\right)$. These numbers can be added together, because each fraction represents parts of the same whole. That whole is all the airline passengers. The principles of adding apply.
   - Discuss the benefits of making an estimate before adding the fractions. The estimate helps establish a target zone for the solution, and verifies whether or

not the answer is reasonable. Benchmarks of close to, more or less than, 0, $\frac{1}{2}$, or wholes are helpful. Students may use their model of a whole, or a number line, as a reference point for benchmarks (e.g., $\frac{1}{3}$ is less than $\frac{1}{2}$, and $\frac{1}{6}$ is a little more than 0, and together they must be close to $\frac{1}{2}$).

- In the models $\left(\text{for } \frac{1}{3} + \frac{1}{6}\right)$, it is evident the pieces cover $\frac{1}{2}$ of the whole, but how to count these pieces to equal $\frac{1}{2}$ is not as clear. As in combining any measure, before we count, the pieces must all have the same unit of measure. When counting fractions, converting the units to the same measure is called *finding a common denominator.* For example, $\frac{1}{3}$ is equivalent to $\frac{2}{6}$, $\frac{2}{6} + \frac{1}{6} = \frac{3}{6}$, and $\frac{3}{6}$ is equivalent to $\frac{1}{2}$. To represent this conversion with a concrete model, replace the $\frac{1}{3}$ piece with two $\frac{1}{6}$ pieces, and then replace the three $\frac{1}{6}$ pieces with a $\frac{1}{2}$ piece. The parts are different names for the same numbers. Model writing the addition statements with the conversions.
- Ensure students are comfortable with these concepts. If the numbers represent different types of parts, such as $\frac{1}{3}$ of the passengers and $\frac{1}{6}$ of the crew, they cannot be added together, because they are parts of different wholes. See Conceptual Understandings in the Background Information for learning outcome 7.N.5.

4. Have students represent the addition of other recognizable fractions and write representative addition statements. Include improper fractions and mixed numbers. Discuss how to rename the resulting improper fractions as mixed numbers. Solicit the addends from the class, or have a list prepared as a handout, such as BLM 7.N.5.4A: Representing Recognizable Fractions and Writing Addition Statements.

Variations:

- Have students invent a passenger survey for a different airline, or prepare data for other scenarios, and write questions and answers based on their data.
- Supply students with a number of scenarios requiring them to find the total. Use friendly fractions and include mixed numbers and improper fractions.
- Have students generate a number of scenarios requiring adding friendly fractions, including mixed numbers and improper fractions. Have them find the sums. These can be recorded on index cards, with the scenario on one side and the answer on the reverse. The cards can be used later for review and drill exercises.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Model addition and subtraction of positive fractions or mixed numbers using concrete representations, and record symbolically.
- ❒ Determine the sum of two positive fractions or mixed numbers with like denominators.
- ❒ Determine a common denominator for a set of positive fractions or mixed numbers.
- ❒ Determine the sum of two positive fractions or mixed numbers with unlike denominators.
- ❒ Simplify the solution to a problem involving the sum or difference of two positive fractions or mixed numbers.
- ❒ Solve a problem involving the addition or subtraction of positive fractions or mixed numbers, and determine if the solution is reasonable.

Suggestions for Instruction

- **Model addition and subtraction of positive fractions or mixed numbers using concrete representations, and record symbolically.**
- **Determine the sum of two positive fractions or mixed numbers with like denominators.**
- **Determine the difference of two positive fractions or mixed numbers with like denominators.**
- **Determine a common denominator for a set of positive fractions or mixed numbers.**
- **Determine the sum of two positive fractions or mixed numbers with unlike denominators.**
- **Determine the difference of two positive fractions or mixed numbers with unlike denominators.**
- **Simplify the solution to a problem involving the sum or difference of two positive fractions or mixed numbers.**
- **Solve a problem involving the addition or subtraction of positive fractions or mixed numbers, and determine if the solution is reasonable.**

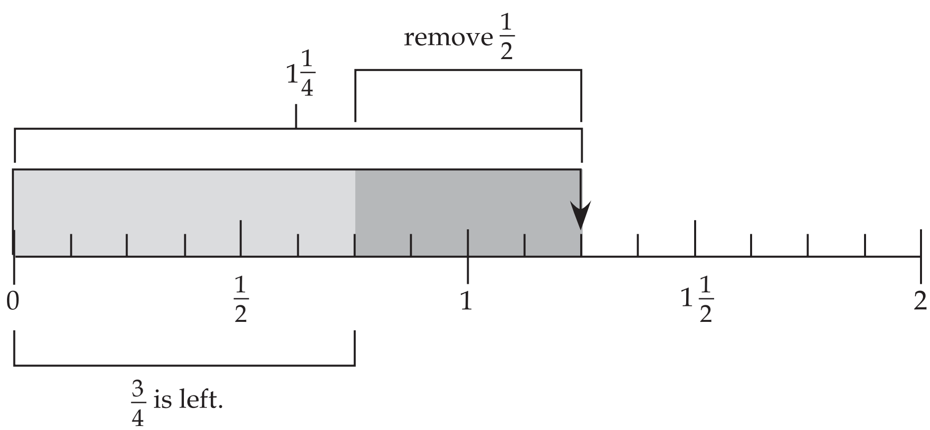
Materials:

- BLM 7.N.5.3B: Ace Aviation: Subtracting Fractions
- BLM 7.N.5.4B: Representing Recognizable Fractions and Writing Subtraction Statements (optional)
- BLM 7.N.5.5: Adding and Subtracting Fractions (Scenarios) (optional)
- presentation board
- manipulatives to represent fractions (e.g., BLM 5–8.12: Fraction Bars, pattern blocks, fraction circles, Cuisenaire rods, grids, counters)
- fraction strips and number lines (made by students)
- math journals or notebooks
- a reference list or a handout of model illustrations, scenarios, and problems involving adding and subtracting fractions (optional)
- index cards (optional)

Organization: Small groups, whole class, individual

Procedure:

1. Provide students with copies of BLM 7.N.5.3B: Ace Aviation: Subtracting Fractions, and have them respond to questions 1 to 4.
2. Have students share their responses to the subtraction questions. Examine students' models for representing removing the business travellers.
3. Model different subtraction scenarios using number lines and fraction strips.



4. Have students make any necessary revisions to their work, or add any notes they consider useful.

5. Once again, discuss the benefits of making an estimate before subtracting the fractions. The estimate helps establish a target zone for the solution, and verifies whether or not the answer is reasonable. Benchmarks of close to, more or less than, 0, $\frac{1}{2}$, or wholes are helpful. Students may use their model of a whole, or a number line, as a reference point for benchmarks.
6. Have students model and record subtraction of other recognizable fractions and mixed numbers, such as those on BLM 7.N.5.4B: Representing Recognizable Fractions and Writing Subtraction Statements.
7. Ask students to create scenarios to match the fractions. Have them write subtraction statements for each model. Ensure they understand that the fractional part must be equal to or less than the fractional part it is being taken away from. Solicit and reinforce ideas about borrowing from the wholes and cutting up the new piece to form an improper fraction from which to subtract. Include related addition statements if you wish.
8. Reassemble as a class and have students present a few models and subtraction statements to ensure everyone is on the right track. Present models and subtraction scenarios, and ask students to write subtraction statements to represent the models and scenarios, recording them in their math journals or notebooks. Include related addition statements if you wish. Discuss the responses.
9. Have students complete a selection of addition and subtraction problems and arithmetic questions. Include improper fractions and mixed numbers in the problems and questions. Solicit the addends from the class, or have a list prepared on a handout, such as those presented on BLM 7.N.5.5: Adding and Subtracting Fractions (Scenarios).

Variations:

- Have students invent a passenger survey for a different airline, or prepare data for other scenarios, and write questions and answers based on their data.
- Supply students with a number of scenarios requiring them to find the total or the difference. Use friendly fractions and include mixed numbers and improper fractions.
- Have students generate a number of scenarios requiring adding or subtracting friendly fractions, including mixed numbers and improper fractions. Have them find the sums and differences. These can be recorded on index cards, with the scenario on one side and the answer on the reverse. The cards can be used later for review and practice exercises.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Model addition and subtraction of positive fractions or mixed numbers using concrete representations, and record symbolically.
- ❒ Determine the sum of two positive fractions or mixed numbers with like denominators.
- ❒ Determine the difference of two positive fractions or mixed numbers with like denominators.
- ❒ Determine a common denominator for a set of positive fractions or mixed numbers.
- ❒ Determine the sum of two positive fractions or mixed numbers with unlike denominators.
- ❒ Determine the difference of two positive fractions or mixed numbers with unlike denominators.
- ❒ Simplify the solution to a problem involving the sum or difference of two positive fractions or mixed numbers.
- ❒ Solve a problem involving the addition or subtraction of positive fractions or mixed numbers, and determine if the solution is reasonable.

Suggestions for Instruction

- **Determine a common denominator for a set of positive fractions or mixed numbers.**

Materials:

- manipulatives
- presentation board
- a list of fractions for students to add and subtract, some with obvious solutions and some with solutions that require finding common denominators
- grids and counters (optional)
- paper, markers, and other art supplies (for making posters or brochures)

Organization: Whole class, individual

Procedure:

1. Present students with sets of fractions to add and subtract. Ask them what makes some types of questions involving fractions easier to answer than other types. The questions that have friendly fractions with common denominators, or fraction sizes that relate easily to each other, are easier to model and to solve. If the more difficult questions could be renamed to have common denominators, they would be easier to answer.
2. Ask students to explore finding a way to rename fractions with common denominators. *Finding a common denominator* is the term used for converting fractions to common units. Students may use grids and counters (as explained in the Background Information for learning outcome 7.N.5), or generate a list of equivalent fractions, and look for a pattern. All denominators in the equivalent fractions are multiples of the original denominators. Therefore, the common denominator must be a multiple of both the original denominators. Students can use their prior knowledge of finding multiples and the lowest common multiple, and their ability to simplify fractions, to develop a strategy to find common denominators.

Extension:

- Have students generate a list of the best hints and strategies for adding and subtracting fractions, and present them as small posters or brochures.

Variations:

- Guide students through a series of steps to find common denominators.
- Provide students with handouts containing sets of fractions. Have students show how they generated a common denominator for the set, and how they renamed the fractions.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❐ Determine a common denominator for a set of positive fractions or mixed numbers.
- ❐ Make connections between determining a common denominator and their prior knowledge regarding factors and multiples.

- **Simplify the solution to a problem involving the sum or difference of two positive fractions or mixed numbers.**
- **Solve a problem involving the addition or subtraction of positive fractions or mixed numbers, and determine if the solution is reasonable.**

Materials:

- BLM 7.N.5.6: Problems Involving Fractions (or other sample fraction problems)
- marking pens

Organization: Individual, whole class

Procedure:

1. Distribute copies of BLM 7.N.5.6: Problems Involving Fractions or other fraction problems.
2. Ask students to solve the problems, provide estimates, and simplify answers.
3. After students have had sufficient time to complete the tasks, discuss their solutions to the problems.
4. Have students use a marking pen to make any necessary corrections or add any notes to their work.

Variation:

- Have students set criteria for problems involving fractions, and ask them to create a set number of problems and an answer key. Photocopy their problem sheets. Students trade sheets, solve the problems, and then reassemble as a group and review the solutions together.

Observation Checklist

☑ Listen to and observe students’ responses to determine whether students can do the following:

- ❒ Simplify the solution to a problem involving the sum or difference of two positive fractions or mixed numbers.
- ❒ Solve a problem involving the addition or subtraction of positive fractions or mixed numbers, and determine if the solution is reasonable.

NOTES

Number (7.N.6)

Enduring Understanding(s):

The principles of operations used with whole numbers also apply to operations with decimals, fractions, and integers.

Number sense and mental mathematics strategies are used to estimate answers and lead to flexible thinking.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
7.N.6 Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially, and symbolically. [C, CN, PS, R, V]	➛ Explain, using concrete materials such as integer tiles and diagrams, that the sum of opposite integers is equal to zero. ➛ Illustrate, using a horizontal or vertical number line, the results of adding or subtracting negative and positive integers (e.g., a move in one direction followed by an equivalent move in the opposite direction results in no net change in position). ➛ Add two integers using concrete materials or pictorial representations, and record the process symbolically. ➛ Subtract two integers using concrete materials or pictorial representations, and record the process symbolically. ➛ Solve a problem involving the addition and subtraction of integers.

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals) by
  - using personal strategies for adding and subtracting
  - estimating sums and differences
  - solving problems involving addition and subtraction
- Describing and applying mental mathematics strategies, such as
  - skip-counting from a known fact
  - using doubling or halving
  - using doubling and adding one more group
  - using patterns in the 9s facts
  - using repeated doubling

  to develop recall of basic multiplication facts to 9×9 and related division facts.
- Applying estimation strategies including
  - front-end rounding
  - compensation
  - compatible numbers

  in problem-solving contexts.
- Demonstrating an understanding of integers, concretely, pictorially, and symbolically.

For more information on prior knowledge, refer to the following resource:

Manitoba Education and Advanced Learning. *Glance Across the Grades: Kindergarten to Grade 9 Mathematics*. Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/cur/math/glance_k-9/index.html.

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of oral and written patterns and their corresponding relations.
- Constructing a table of values from a relation, graphing the table of values, and analyzing the graph to draw conclusions and solve problems.
- Demonstrating an understanding of preservation of equality by
  - modelling preservation of equality, concretely, pictorially, and symbolically
  - applying preservation of equality to solve equations

- Evaluating an expression, given the value of the variable(s).
- Modelling and solving problems that can be represented by one-step linear equations of the form $x + a = b$, concretely, pictorially, and symbolically, where a and b are integers.
- Demonstrating an understanding of central tendency and range by
  - determining the measures of central tendency (mean, median, mode) and range
  - determining the most appropriate measures of central tendency to report findings
- Determining the effect on the mean, median, and mode when an outlier is included in a data set.

Background Information

Integers: Definition and Notation

Integers are the set of numbers consisting of the natural numbers (1, 2, 3, . . .), their opposites (–1, –2, –3, . . .), and zero. They are also referred to as the *whole numbers* and their opposites.

Integers indicate both a quantity and a direction from zero. Positive integers are greater than zero. They are represented by a positive symbol (+) before the integer, such as (+5). Negative integers are less than zero. They are represented by a negative symbol (–) before the integer, such as (–3). There are two common notations for integers. The symbols are written either as superscripts preceding the integer, as in $^{+}5$, $^{-}3$, or the symbol and the integer are both enclosed within parentheses, as in (+5), (–3). The parentheses are commonly used in student materials to avoid any confusion between the integer sign and the notations for addition and subtraction. In the equation (+5) – (–3) = (+8), the parentheses indicate the numbers inside are integers and distinguish the integer symbols from the subtraction symbol.

Integer Use

Understanding and working with integers is important in daily life. Integers are regularly encountered in contexts such as finances, investments, temperatures, elevations, time relevant to events, and sports. Proficiency with adding and subtracting integers will be important in students' future algebra work, and is a useful mental mathematics strategy for multi-digit subtraction. Knowledge of integers provides a language for students to express their thinking when they use numbers less than zero.

Example:

To subtract 526 – 379,

Think: (500 – 300) + (20 – 70) + (6 – 9)

(+200) + (–50) + (–3)

(+150) + (–3)

Equals: (+147)

Representing Integer Operations with Models

In Grade 7, students extend their understanding of integers acquired in Grade 6 as they learn to add and subtract positive and negative numbers. Provide students with many opportunities to represent integers (concretely, pictorially, and symbolically) to develop their understanding. Encouraging students to use a variety of manipulatives and strategies will help them to develop confidence in determining and applying general rules for both adding and subtracting integers.

Concrete Models

Concrete models include the following:

1. **Algebra tiles:** One face of an algebra tile is one colour, and the opposite side is a different colour. Use one side to represent positive integers, and the reverse side to represent negative integers.
2. **Sets of counters in two different colours:** Choose one colour to represent positive integers, and the other to represent negative integers. Matching sets of both colours represent zero. For example, blue chips represent negative integers, while red chips represent positive integers. To solve the problem of (+3) + (–7), set out three red chips and seven blue ones. Physically match up pairs of red and blue chips to equate them to zero, and remove the remaining chips. The remaining four blue chips represent the solution, (–4).
3. **Computer models:** Many computer simulations allow students to pull the representative positive and negative counters into a collection bin. Students match opposite representations to represent zero, and the counters disappear. The answer remains in the bin to be counted.

   Sample Website:

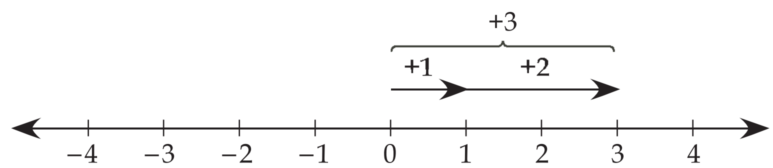
   For an example of a computer model, refer to the following website:

   Utah State University. "Number and Operations (Grades 3–5)." *National Library of Virtual Manipulatives.* 1999–2010. http://nlvm.usu.edu/en/nav/category_g_2_t_1.html.

   Select Color Chips—Addition or Color Chips—Subtraction from the list of virtual manipulatives provided.
4. **Number lines:** A thermometer is a natural number line, and can be viewed vertically or horizontally. The distance of an integer from zero represents the quantity of the integer, and the direction from zero represents whether the integer is positive or negative.
   - On a *vertical number line*, the distance above zero represents positive integers. Distances below zero represent negative integers. Values always increase up the line, and decrease down the line.

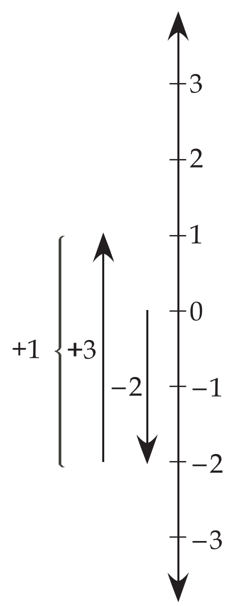
- On a *horizontal number line*, positive integers are represented to the right of zero, and negative integers are represented to the left of zero. Values always increase from left to right, and decrease from right to left.
- An integer's quantity may also be represented with vector models or the length of an arrow from zero to the integer.
  - An arrow pointing to the right indicates a positive value, and an arrow to the left indicates a negative value. The length of the combined arrows indicates the combined value. These arrows are typically placed end to end.

    Example:



  - Arrows pointing in opposite directions are laid on top of (on a horizontal number line) or beside (on a vertical number line) each other. The beginning of one arrow is matched with the end of the other arrow.

    Example:



- The combination of integers may also be represented by jumps on a number line. Jumps to the right represent addition, and jumps to the left represent subtraction. A negative integer moves in the opposite direction. Use both vertical and horizontal number lines to represent changes in temperatures, elevations, and distances travelled.

Generalizations about Integers

As students work with different manipulatives and use different strategies, they will likely come to the following generalizations about integers. Rather than explicitly teaching the generalizations as rules, provide students with opportunities to discover these generalizations.

The Zero Principle

- The sum of opposite integers (sometimes called the zero pairs) is always zero.
- Adding equal positive and negative numbers to a quantity does not change the net value of the quantity.

Adding Integers

- The sum of two positive integers is always positive (e.g., (+2) + (+3) = (+5)).
- The sum of two negative integers is always negative (e.g., (–2) + (–3) = (–5)).
- The sum of one negative integer and one positive integer may be either negative or positive, depending on the sign of the number that is farthest from zero (i.e., subtract the absolute values of the integers and use the sign of the integer with the greater absolute value) (e.g., (+2) + (–3) = (–1)).

Subtracting Integers

- Subtracting an integer is equivalent to adding its opposite (e.g., (+4) – (–2) = (+4) + (+2) = (+6)).
- If both integers have the same sign and the minuend is further away from zero than the subtrahend, find the difference and keep the sign (e.g., (+7) – (+3) = (+4) or (–7) – (–3) = (–4)).
- If both integers have the same sign and the subtrahend is further away from zero than the minuend, find the difference and use the opposite sign (e.g., (+2) – (+6) = (–4) or (–2) – (–6) = (+4)). (Equivalent to adding the opposite.)
- If the signs are different, add the values and use the sign of the minuend (e.g., (–5) – (+3) = (–8) or (+3) – (–5) = (+8)). (Equivalent to adding the opposite.)

Mathematical Language

absolute value*
integer
minuend
negative integer
positive integer
sign
subtrahend
zero principle

*** Note:**
Teachers may model correct use of absolute value, but it is not an expectation for Grade 7 students.

Assessing Prior Knowledge

Materials:

- research resources, such as
  - magazines, newspapers, pamphlets
  - almanacs
  - online databases
  - Statistics Canada. www.statcan.gc.ca/.
- scissors, glue, markers, and poster board (for making posters)

Organization: Whole class, small groups

Procedure:

1. As a class, brainstorm contexts in which integers are used in daily life. Situations may include the following:
   - depths/levels of oceans, lakes, and rivers
   - levels of tides, mountains, and cities
   - levels of tall buildings, underground parking garages, and mine shafts
   - temperatures
   - finances, savings and spending, loans and debts
   - value of investments (e.g., share prices, stocks, mutual funds)
   - sports and player statistics
2. Divide the class into small groups, and have each group research and compare statistics related to a selected theme. Themes may include elevations of various cities or mountains, depths of lakes or oceans, river levels in times of flood and drought, temperature extremes in various cities, and player statistics in various sports leagues (e.g., +/– differentials in hockey, and par in golf).
3. Each group then finds a creative way to present their research findings in a collage or on a poster with appropriate titles. For example, students may create an illustration of cities in order of lowest to highest elevation, or coldest to warmest cities, or highest and lowest point on each continent, or worst to best performing stocks for a given period, or the performance of sports teams or players.

Variation:

- Each group has a general thematic focus. Students cut out headlines, diagrams, charts, graphs, and illustrations of contexts in which integers are represented in daily life, and use the clippings to create a collage. They add appropriate headings.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Describe contexts in which integers are used.
- ❒ Order integers correctly.

Assessing Prior Knowledge

Materials:

- paper and pens or demonstration board (to keep track of points)
- space for each pair of teams to use as a pitching mound, home plate, and bases

Organization: One to four teams, depending on class size

Procedure:

1. Have students form teams to play integer baseball.
2. Choose one team to bat, one team to pitch, and a scorekeeper.
3. Each team lines up at the home plate or on the pitching mound.
4. The first pitcher "pitches" to the first batter a situation or an action phrase that may be represented by an integer.
5. The batter replies with the representative integer. The waiting pitchers determine whether the answer is correct. If the batter is correct, he or she has a "hit," and begins rotating through the bases. An error counts as an "out." The bases are cleared, and the players return to the end of the batting line.
6. The previous pitcher goes to the end of the pitching line, and the next pitcher pitches a new situation, and play continues.
7. When a player returns to home plate, a "run" is scored. At three "outs," the teams switch places.

8. **Suggested rules:**
    - No situations may be repeated.
    - Pitchers and batters must respond within five seconds.
    - No players can steal bases; they must be "hit" to the next base.
    - If the pitchers make an error in judgment, the batting team scores a "home run," and any players on bases are "hit" home, each scoring a run.

Variations:

- Have small teams or pairs sit in groups and rotate bases on a paper "field."
- Switch or vary the actions. Pitchers pitch integers, and batters describe a matching contextual situation.
- Supply a list of situations for pitchers to use.
- Use paper-and-pencil or student-generated tasks requiring students to write the integer that matches a situation and/or describe a situation that may match an integer.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Describe contexts in which integers are used.
- ❒ Use integers to represent contexts.

- **Explain, using concrete materials such as integer tiles and diagrams, that the sum of opposite integers is equal to zero.**
- **Illustrate, using a horizontal or vertical number line, the results of adding or subtracting negative and positive integers (e.g., a move in one direction followed by an equivalent move in the opposite direction results in no net change in position).**

Materials:

- a long line on the floor or playground (e.g., use a pattern in the floor tiles, use lines painted on the gymnasium floor or sports field, or create a line with tape, chalk, or coloured dots)
- a measuring tool to create intervals
- low-tack stickers to label intervals (use a special marker for zero)
- a pile of cards labelled with integers
- two arrows, one labelled "increasing in value," and the other labelled "decreasing in value"
- paper to create a number line (10 cm by 55 cm) (optional)

Organization: Small groups

Procedure:

1. Create a number line on the classroom floor, the gymnasium floor, or outdoors, using tape, chalk, or coloured dots. Number the line (–20) to (+20). The line can be used later for other learning experiences.
2. When the number line is complete, have students draw an integer card from the pile, show where the number would be on the line by pacing the distance from zero, and then stand at the spot on the number line that represents the integer.
   - The first student compares his or her number to zero: "___ is *(greater* or *less)* than zero."
   - The next person compares the size of his or integer to that of a neighbour already on the line: "___ is *(greater* or *less)* than ______."
3. Note that the farther a number is from zero, the larger or smaller its value is, depending on its direction from zero. Place arrows on the line to label increasing values or decreasing values.

Variations:

- When students state their comparison responses, they could add a value to the "greater or less than" statement. For example, instead of saying, "(–8) is less than (-6)," they could say, "(–8) is 2 less than (–6)."
- Have students create and use both vertical and horizontal number lines.
- Make a personal number line (10 cm wide and 55 cm long) on paper, label the integers (–20) to (+20), and use small cards to represent the above actions. This line may be used for other learning experiences.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Create an accurate number line.
- ❒ Order numbers correctly on a number line.

Suggestions for Instruction

- **Explain, using concrete materials such as integer tiles and diagrams, that the sum of opposite integers is equal to zero.**
- **Illustrate, using a horizontal or vertical number line, the results of adding or subtracting negative and positive integers (e.g., a move in one direction followed by an equivalent move in the opposite direction results in no net change in position).**

Materials:

- demonstration board
- BLM 7.N.6.1: Centimetre Number Line
- BLM 5–8.9: Centimetre Grid Paper
- coloured paper or markers (Decide whether to use the same colour of manipulatives to represent positive and negative integers for every activity.)
- scissors
- a directional toy (e.g., a person, animal, car) (for each student or group)
- 20 counters of two different colours (for each student or group)
- math journals or notebooks

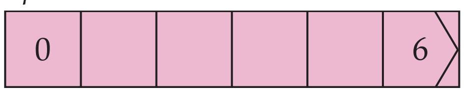
Organization: Whole class, individual or small groups

Procedure:

The purpose of this learning activity is to determine the sum of opposite integers.

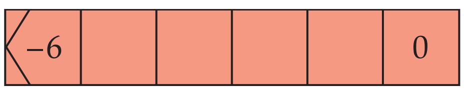
1. Remind students that integers measure quantity and direction from zero.
2. Ask students to use BLM 5–8.9: Centimetre Grid Paper to measure and cut five strips of paper the length of five different positive integers. The strips should be of the same colour and the same height. Have students mark zero at the left end of the strip, the integer at the right end, and the vector arrow the length of the strip and pointing to the right, indicating that the strip represents a positive integer.

   Example:

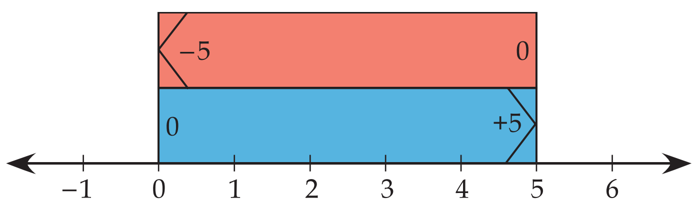
   

   Ask students to use another colour to create strips representing the negative or opposite of each chosen integer. This time, have students label zero at the right end of the strip, the integer at the left end, and the vector arrow pointing to the left, indicating that the strip represents a negative integer.

   Example:

3. Demonstrate how to combine the opposite strips on the number line to represent adding opposite integers: (+5) + (–5) = ___ . Place the beginning or zero point of the vector arrow of the strip representing the first integer at zero, and place the beginning or zero point of the vector arrow of the strip representing the second integer at the end or integer value of the first strip. The resulting end point of the second strip can be read on the number line. In this case, it is zero.

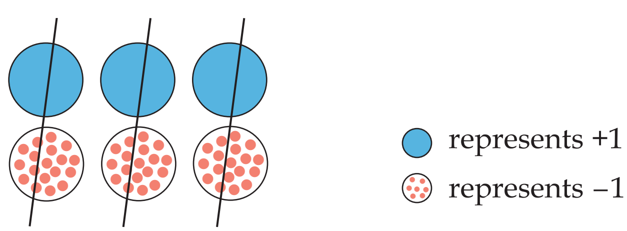
   

4. Ask students to use their strips to model five combinations of different integers and their opposites, and to write a general statement about the sum of an integer and its opposite, recording their work in their math journals or notebooks.
5. After giving students sufficient time to work on their combinations, ask students what they discovered. Record each of their combinations on the demonstration board as equations. Ask students to make a general statement about the sum of an integer and its opposite.

6. Ask students to imagine that the number line is an elevator shaft. Use of a vertical number line is realistic for this situation. Have a student demonstrate entering at level zero, go up three floors, and then come back down three floors. The elevator will be back where it started. There has been zero change.
7. Demonstrate the combination (+5) + (–5) = ___ as jumps on a number line. If you are using the large line, have a student begin at zero and take 5 positive jumps, facing to the positive right. The next action is adding or combining, so the student continues to face right, ready to jump on. The next integer is negative though. It is the opposite of 5, so the student must face the same direction and jump backwards to show the opposite of 5. The number to which the student jumps indicates the sum. Have students demonstrate several of their opposite combinations. Be sure to act out some combinations beginning with negative integers. Also act out the same combinations beginning at any floor in the elevator scenario, or beginning at any number on the number line. Adding one value and then its opposite results in zero or no net change to the original position or number. If you do not have access to a large number line, have students use a directional toy with a front (e.g., a car, an animal) to act out the situations on their individual number lines. Number lines measuring 10 cm × 55 cm are handy and can be used for many learning experiences. Review the generalizations about adding integers and their opposites, as discussed in the Background Information for learning outcome 7.N.6. Inform students this is called the *zero principle*.
8. Distribute two colours of counters to students. State the colour that will represent positive integers, and the colour that will represent negative integers. Remind students of the zero principle they have just established, and ask them how they could use the counters to illustrate that the sum of an integer and its opposite is zero. Circulate among students and, after sufficient time, have students share their ideas. Listen for the idea that matching a positive and a negative counter equals zero, so the pair can be withdrawn. If all the integers match up as opposite pairs, there is nothing remaining, and the value of the leftovers is zero.

   Example:

   This example represents (+3) + (–3), and each (+1) + (–1) pair can cancel, leaving 0.



   Computer applets may also be used to illustrate that the sum of an integer and its opposite is zero.

Sample Website:

Computer applets are available on the following website:

Utah State University. *National Library of Virtual Manipulatives*. 1999–2010. http://nlvm.usu.edu/en/nav/vlibrary.html.

Select Number and Operations (Grades 3–5), and then select Color Chips—Addition.

9. Students record the zero principle in their math journals or notebooks and draw diagrams and the corresponding number sentences to illustrate the generalization. Encourage them to draw both horizontal and vertical number lines.

Variations:

- Supply number lines and cut strips for students to work with.
- Supply templates on which students can record the zero principle.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Explain, using concrete materials such as integer tiles and diagrams, that the sum of opposite integers is equal to zero.
- ❒ Illustrate, using a horizontal or vertical number line, the results of adding or subtracting negative and positive integers (e.g., a move in one direction followed by an equivalent move in the opposite direction results in no net change in position).

Suggestions for Instruction

- **Illustrate, using a horizontal or vertical number line, the results of adding or subtracting negative and positive integers (e.g., a move in one direction followed by an equivalent move in the opposite direction results in no net change in position).**
- **Add two integers using concrete materials or pictorial representations, and record the process symbolically.**
- **Subtract two integers using concrete materials or pictorial representations, and record the process symbolically.**
- **Solve a problem involving the addition and subtraction of integers.**

Materials:

- demonstration board
- number lines (a large physical number line such as the one used in the previous learning activity, or a personal number line)
- small directional toy (e.g., a person, an animal, a car)
- 20 counters of two different colours (for each student or group)
- containers or paper boundaries in which to place the counters (for each student or group)
- index cards
- rulers, pencils, and colours
- display area (to post completed scenarios)
- math journals or notebooks

Organization: Whole class (for demonstration), small groups (for investigations)

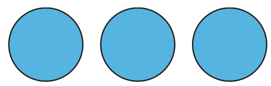
Procedure:

1. Remind students that in a previous learning experience they modelled adding opposite integers using a number line by comparing distances, and representing moves or jumps on a number line. They also modelled making zero pairs using two colours of counters. In this learning activity, students will extend their modelling to represent adding or subtracting any integers (–20) to (+20). Ask students to use their number lines or counters to model scenarios, and use integers to write corresponding addition and subtraction statements to represent the action in the scenarios. Encourage students to use both the number lines and the counters to model scenarios. Include a subtraction scenario that requires adding more integers using zero pairs.

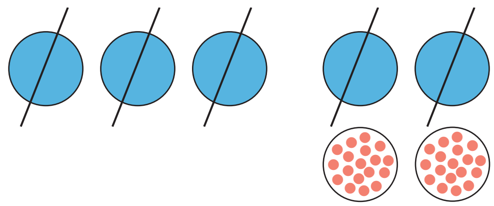
Example:

In this example, the dotted chip represents negative and the solid chip represents positive.

To model (+3) – (+5), start with +3.



Then remove +5 (but there are only 3 to remove, so it is necessary to add enough zero pairs so that there are 5 positives to take away).



You are left with –2, so (+3) – (+5) = (–2).

2. Present sample scenarios, such as those suggested below. In the samples, have students model the action both on the number lines and with counters to gain experience with both models. Record the representative equation using integers.

Sample Scenarios:

- Lucienne put $8 in an envelope in the morning. Later in the day, he put $2 in the envelope. How much money is in the envelope?

  (+8) + (+2) = (+10)

- Ricki was in a cycling derby. She rode 5 km, and realized she missed the turn by the oak grove, which was 2 km back. How much of the course has she completed?

  (+5) + (–2) = (+3)

  Here, students may begin to notice that subtracting the positive integer and adding the negative integer are equivalent.

- It was a dry summer in Okitown. The river was 2 m below its normal level. During August, there was no rain, and the water level went down another metre. How far is the river below the normal level now?

  (–2) + (–1) = (–3)

- Ainsley owed her dad $12. Her dad cancelled $5 of the debt. How much debt remains?

  (–12) – (–5) = (–7)

- Ravi had a collection of model cars. He sold three cars to friends at school, and used the money to purchase a new model. What is the resulting change in the number of cars in his collection?

  (–3) + (+1) = (–2)

  Remind students to use integers to represent the quantities.

3. When students have developed sufficient proficiency in modelling scenarios, have them work in small groups to write scenarios, act them out, and identify the corresponding equations using integers. Remind students to vary the action models they use, sometimes using number lines and sometimes using counters. When a scenario is completed, they record the situation on one side of an index card. On the reverse side, they draw a pictorial representation of the solution, and write the corresponding integer equation(s). Aim for a variety of addition and subtraction scenarios combining positive and negative integers. As students work together in groups, ask them to look for generalizations or rules they can apply when adding or subtracting integers. As groups complete cards, have them verify their correctness, write their group name on each card, and post the cards in the designated area so classmates have access to them.
4. When groups have completed five cards, they can select a few scenarios from their classmates and write the solutions in their math journals or notebooks. Ask them to include diagrams of manipulatives used, and to write an applicable equation.
5. Meet together as a class and have students share any generalizations or methods that were helpful to them. Students may record useful generalizations in their math journals or notebooks.

Variations:

- Provide students with scenarios instead of having them create their own.
- Provide a handout with necessary supports for solving the scenarios.
- Include online computer applets of integer counters as manipulatives for students to use while solving their scenarios.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Illustrate, using a horizontal or vertical number line, the results of adding or subtracting negative and positive integers (e.g., a move in one direction followed by an equivalent move in the opposite direction results in no net change in position).
- ❒ Add two given integers using concrete materials or pictorial representations, and record the process symbolically.
- ❒ Subtract two given integers using concrete materials or pictorial representations, and record the process symbolically.
- ❒ Solve a given problem involving the addition and subtraction of integers.
- ❒ Visualize the integers to assist in symbolically adding and subtracting integers.

- **Solve a problem involving the addition and subtraction of integers.**

Materials:

- a deck of regular playing cards
- number lines
- counters
- paper and pencils (for finding solutions)

Organization: Pairs or small groups (of three or four students)

Procedure:

In this learning activity, students play a game requiring them to calculate the value of integer cards.

1. Decide which cards (red or black) will represent positive integers, and which cards will represent negative integers. Aces will have a value of 1. Jokers may be included as zero cards. Decide whether to include the face cards as values 11, 12, and 13, or whether to remove them and work with integers 0 to 10.
2. Have students form pairs or small groups. Then deal all the cards evenly among the players. Players put their cards in a pile face down. On the dealer's signal, all players flip over their top cards, making them easily visible to all. All players calculate the value of the cards. The first person to say the correct value takes the up-turned cards and puts them in his or her win pile. The dealer signals for the next round, and play continues. When someone's pile is depleted, the player shuffles his or her win pile and continues playing with it. Once someone has no remaining cards, that player becomes a referee. Play continues until a set time is called, or until only one player has all the cards. The player with the most cards wins.

Variation:

- Use the cards to play integer baseball, using the process outlined in the Assessing Prior Knowledge learning experience. This time, the pitcher has two card piles. The top cards are turned over, and the batter returns the combined value. An error results in an "out."

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❐ Solve a given problem involving the addition and subtraction of integers.
- ❐ Apply mental mathematics strategies when adding and subtracting integers.

Suggestions for Instruction

- **Illustrate, using a horizontal or vertical number line, the results of adding or subtracting negative and positive integers (e.g., a move in one direction followed by an equivalent move in the opposite direction results in no net change in position).**
- **Solve a problem involving the addition and subtraction of integers.**

Materials:

- BLM 7.N.6.2: Integer Football
- blank game cards
- markers and tokens
- paper or card stock (for a football field)
- scoreboard
- rulers, scissors, and tape
- word processor and printer (optional)

Organization: Pairs or small groups

Procedure:

1. Ask students to work in pairs or in small groups to develop a football game requiring players to answer integer problems. Have them prepare rules of play and any required materials, such as those listed below. In designing their games, students may wish to refer to BLM 7.N.6.2: Integer Football.

   Game Suggestions:

   - Draw a paper football field with the yards and end zones marked off.
   - Provide a token (for each team) and a "ball."
   - Prepare sets of game cards, including run cards and pass cards, each with appropriate integer statements or problems, and the solutions on the reverse or under a fold. (The solutions would be the yards gained or lost on the play.)

- Identify rules of play, including penalties or interceptions for incorrect challenges.
- Provide a scoreboard.

2. Have students play each other's games.

Variations:

- Supply the materials and cards and have students play the game.
- Have students develop any other game requiring players to answer integer problems to score points or to advance a play.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Solve a given problem involving the addition and subtraction of integers.

Number (7.N.7)

Enduring Understanding(s):

Percents, fractions, decimals, and ratios are different representations of the same quantity.

Number sense and mental mathematics strategies are used to estimate answers and lead to flexible thinking.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
7.N.7 Compare and order fractions, decimals (to thousandths), and integers by using ▪ benchmarks ▪ place value ▪ equivalent fractions and/or decimals. [CN, R, V]	➛ Order the numbers of a set that includes fractions, decimals, or integers in ascending or descending order, and verify the result using a variety of strategies. ➛ Identify a number that would be between two numbers in an ordered sequence or on a horizontal or vertical number line. ➛ Identify incorrectly placed numbers in an ordered sequence or on a horizontal or vertical number line. ➛ Position fractions with like and unlike denominators from a set on a horizontal or vertical number line, and explain strategies used to determine order. ➛ Order the numbers of a set by placing them on a horizontal or vertical number line that contains benchmarks, such as 0 and 1 or 0 and 5. ➛ Position a set of fractions, including mixed numbers and improper fractions, on a horizontal or vertical number line, and explain strategies used to determine position.

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of fractions by using concrete and pictorial representations to
  - name and record fractions for the parts of a whole or a set
  - compare and order fractions with like and unlike denominators
  - model and explain that for different wholes, two identical fractions may not represent the same quantity
  - provide examples of where fractions are used
  - create sets of equivalent fractions
  - relate improper fractions to mixed numbers
- Describing and representing decimals (tenths, hundredths, thousandths) concretely, pictorially, and symbolically.
- Relating decimals to fractions (tenths, hundredths, thousandths).
- Comparing and ordering decimals (tenths, hundredths, thousandths) by using
  - benchmarks
  - place value
  - equivalent decimals
- Demonstrating an understanding of place value for numbers
  - greater than one million
  - less than one-thousandth
- Demonstrating an understanding of percent (limited to whole numbers), concretely, pictorially, and symbolically.

For more information on prior knowledge, refer to the following resource:

Manitoba Education and Advanced Learning. *Glance Across the Grades: Kindergarten to Grade 9 Mathematics*. Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/cur/math/glance_k-9/index.html.

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of the addition, subtraction, multiplication, and division of decimals to solve problems (for more than 1-digit divisors or 2-digit multipliers, technology could be used).
- Solving problems involving percents from 1% to 100%.

- Demonstrating an understanding of the relationship between repeating decimals and fractions, and terminating decimals and fractions.
- Demonstrating an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially, and symbolically (limited to positive sums and differences).
- Demonstrating an understanding of addition and subtraction of integers, concretely, pictorially, and symbolically.
- Expressing probabilities as ratios, fractions, and percents.

Background Information

Comparing and Ordering Fractions, Decimals, and Integers

To be efficient at comparing and ordering fractions, decimals, and integers, students must understand the values of these numbers in our number system and their various representations. They must realize that fractions and decimals are interchangeable names for the same quantity and must be able to convert one to the other. They must be proficient at renaming and simplifying fractions and use multiple strategies for comparing them.

Fractions

Fractions are used to name quantities between whole numbers. The part between whole numbers can be divided into any number of equal parts. The number of equal parts in one whole becomes the *denominator* of the fraction, and the number of parts referred to forms the *numerator* of the fraction. The larger the denominator is, the smaller the fraction pieces are. The smaller the denominator is, the larger the fraction pieces are. For example, $\frac{1}{15}$ is less than $\frac{1}{10}$. As the numeric value of the numerator approaches the numeric value of the denominator, the number gets closer to one whole. For example, $\frac{14}{15}$ is larger than $\frac{9}{10}$. Numerators with numeric values larger than their denominators represent *improper fractions* with values greater than one whole. *Equivalent fractions* have different numerators and denominators, but represent the same portion of a whole. (The learning experiences suggested for learning outcomes 7.N.3, 7.N.4, and 7.N.5 contain information about, and strategies for, renaming equivalent fractions and mixed numbers. They are recommended for students who need to review these skills.)

Decimals

Decimal numbers represent fraction quantities using the base-10 number system. Each successive decimal place represents a tenth of the previous place value. All fractions can be renamed as decimals.

- One way to rename a fraction as a decimal is to find an equivalent fraction with a denominator that is a power of 10.

  Examples:

  $\frac{1}{2}$ is equal to $\frac{5}{10}$, written as 0.5

  $\frac{12}{25}$ is equal to $\frac{48}{100}$, written as 0.48

- Another way to rename a fraction as a decimal is to divide the numerator by the denominator.

  Example:

  $\frac{12}{40} = 12 \div 40 = 0.3$

Likewise, most decimal numbers can be renamed as fractions. *Terminating decimals* have a definite number of digits and can easily be renamed as fractions with denominators that are powers of 10. The digits in the decimal number form the numerator of the fraction, and the denominator is 1, followed by a number of zeros equal to the number of digits to the right of the decimal number $\left(\text{e.g., } 0.623 = \frac{623}{1000} \text{ and } 3.4 = 3\frac{4}{10}\right)$. *Repeating decimals* can be renamed as fractions according to characteristic patterns explored in relation to learning outcome 7.N.4 (a single repeating digit has a denominator of 9, so $0.\overline{7} = \frac{7}{9}\bigg)$. Decimals that are both non-repeating and non-terminating are irrational numbers, and, therefore, cannot be renamed as fractions $\left(\text{e.g., } \pi, \sqrt{2}\right)$.

Integers

Integers comprise positive numbers, negative numbers, and zero. Positive integers refer to the regular counting numbers, and negative integers refer to numbers less than zero. Negative integers are the opposite of their positive counterparts. The greater the numeric value of the negative integer is, the farther it is from zero, and, therefore, the smaller is the value of the number. For example, (–9) is smaller than (–1).

Strategies for Comparing Relative Size

Strategies for comparing the relative size of fractions, decimals, and integers include the following:

- Associate the numbers with *benchmarks* such as 0, $\frac{1}{2}$, and 1, and place them on a number line. For closer comparisons, use benchmarks of 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1.
- All negative integers are less than 0. The greater the numeric value is, the smaller the number is.
- If fractions have the same numerator, then it is necessary to compare only the denominators. The larger the denominator is, the smaller each piece is. If the numerator is 1, then the larger the denominator is, the closer the fraction is to zero (e.g., $\frac{1}{9}$ is closer to zero than $\frac{1}{5}$ is).
- If fractions have the same denominator, then it is necessary to compare only the numerators. The larger the numerator is, the more pieces there are, and so the larger the fraction is.
- If the numerator is close to half of the denominator, then the fraction is close to one-half (e.g., $\frac{3}{8}$ is a little less than $\frac{1}{2}$, and $\frac{5}{8}$ is a little more than $\frac{1}{2}$).
- If the numerator is close to the denominator, then the number is close to one whole. A larger denominator indicates smaller pieces. The smaller the piece is, the closer it is to 1, and the larger the fraction is.
- Compare decimal numbers to the decimal equivalents for the benchmark fractions, 0.25, 0.5, 0.75, and 1.0.
- Use an understanding of place value to compare decimal numbers. First, compare the units in the largest place value. Tenths are larger than hundredths, which are larger than thousandths (e.g., 0.543 is a little larger than 0.54, which is a little larger than 0.5). Rewriting decimal numbers as equivalents with the same number of digits helps make this concept clear (e.g., 0.543 is larger than 0.540, which is larger than 0.500).
- Move flexibly between the various representations (e.g., $\frac{7}{10}$ is larger than $\frac{17}{25}$ because $\frac{7}{10}$ is 0.7, and $\frac{17}{25}$ is $\frac{68}{100}$ or 0.68).

Mathematical Language

ascending
benchmark
denominators
descending
equivalent fractions
horizontal
improper fractions
mixed numbers
numerators
proper fractions
repeating decimal
sequence
terminating decimal
unlike denominators
verify
vertical

Learning Experiences

Assessing Prior Knowledge

Materials:

- BLM 7.N.7.1: Equivalent Fractions and Decimals
- BLM 7.N.7.2: Equivalent Fractions, Decimals, and Percents
- manipulatives for representing fractions and decimals (e.g., counters, fraction bars, number lines, base-10 blocks)
- math journals or notebooks
- calculators (optional)

Organization: Individual or pairs, whole class

Procedure:

1. Select a BLM for students to work with (e.g., BLM 7.N.7.1: Equivalent Fractions and Decimals or BLM 7.N.7.2: Equivalent Fractions, Decimals, and Percents).
2. Provide students with copies of the selected BLM, and have them complete the tasks individually or in pairs, using manipulatives or calculators as needed.
3. After a designated time has passed, reassemble as a class, and have students share their responses and the strategies they used to arrive at their answers.
4. Have students use their math journals or notebooks to record helpful strategies for converting fraction and decimal numbers. They can add to these strategies in future learning activities.

Variations:

- Have students create pictorial, equivalent fraction, decimal, and percent representations for fractions of their choice. Provide template squares, and combine the different representations as a class display.
- Have students create their own scenarios and word problems involving converting decimal and fraction notations.

Note:
Many of the learning experiences for learning outcome 7.N.7 could be used within a body of evidence to report on the following competencies on the Grade 7 Numeracy Assessment:

Student orders fractions.
Student orders decimal numbers.
Student understands that a given number may be represented in a variety of ways.

Reference:
Manitoba Education and Advanced Learning. *Middle Years Assessment: Grade 7 Mathematics: Support Document for Teachers: English Program.* Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/assess/support/math7/.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Convert decimal numbers to fractions (to thousandths), and vice versa.
- ❒ Relate improper fractions to mixed numbers.

Assessing Prior Knowledge

Materials:

- blank paper
- pens of different colours

Organization: Individual, small groups or whole class

Procedure:

1. Ask students to create a brainstorming web entitled Everything I Know about Fractions.
2. Have students participate in a group or class discussion to share information from their webs.
3. Have students use pens of different colours to add new ideas to their own webs as they listen to the shared ideas of classmates.

Variation:

- Students keep the webs so that they can revisit and add to them as their conceptual understanding of fraction grows.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Represent a variety of fractions in various ways, such as
  - pictorially, as parts of a whole or a set
  - a ratio
  - a division statement
  - improper fractions and mixed numbers
  - equivalent fractions
  - expressed as a decimal
  - expressed as a percent

Suggestions for Instruction

- **Order the numbers of a set that includes fractions, decimals, or integers in ascending or descending order, and verify the result using a variety of strategies.**
- **Position a set of fractions, including mixed numbers and improper fractions, on a horizontal or vertical number line, and explain strategies used to determine position.**

Materials:

- BLM 7.N.7.3: Comparing Fraction and Decimal Equivalents
- BLM 5–8.10: Base-Ten Grid Paper
- BLM 5–8.12: Fraction Bars
- manipulatives for representing fractions and decimals (e.g., counters, number lines, base-10 blocks)
- calculators (optional)
- math journals or notebooks

Organization: Individual or pairs

Procedure:

1. Distribute copies of BLM 7.N.7.3: Comparing Fraction and Decimal Equivalents, and have students find solutions to the given problems, using manipulatives or calculators as needed.
2. After a designated time has passed, have students share their responses and the strategies they used to arrive at their answers.
3. Have students use their math journals or notebooks to record helpful strategies for converting and comparing fraction and decimal numbers. They can add to these strategies in future learning activities.

Variations:

- Ask students to create their own scenarios and word problems that involve converting decimal and fraction notations or that require comparing fractions and decimal numbers. Have students share their scenarios with either the class or a small group. Have individuals present and explain their solutions, and have other group members verify the solutions.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Convert decimal numbers to fractions (to thousandths), and vice versa.
- ❒ Relate improper fractions to mixed numbers.
- ❒ Compare and order fractions and decimals.

Suggestions for Instruction

- **Order the numbers of a set that includes fractions, decimals, or integers in ascending or descending order, and verify the result using a variety of strategies.**
- **Identify a number that would be between two numbers in an ordered sequence or on a horizontal or vertical number line.**
- **Identify incorrectly placed numbers in an ordered sequence or on a horizontal or vertical number line.**

Materials:

- a collection of library books labelled with decimal numbers on the spine (If possible, have three books per student. Choose books with whole numbers relatively close together, and a variety of decimal numbers in between.)
- math journals or notebooks
- card stock (optional)

Organization: Small groups (two to four students), whole class

Procedure:

1. Choose a variety of ways to form groups quickly (e.g., combinations of colours worn, beginning letters in first names).
2. Have two or three students combine their library books and sort them in ascending order according to the decimal numbers on the spines. (Groups of four can work as two groups of two.) Have group members verify that their order is correct and explain their reasoning to one another.
3. Have students reclaim their books, form new groups, sort the books, and verify the results once again. Repeat as often as the learning activity seems useful.
4. Meet as a class and have students share strategies for ordering decimal numbers.
5. Have students use their math journals or notebooks to record strategies for ordering decimal numbers.

Variations:

- Have one or two students order the books incorrectly, and have a third student identify the misplaced books and explain why the books belong elsewhere.
- Use fewer books. Designate the learning activity as one station of a set of rotating learning activities.
- Instead of using library books, use card stock to create a set of book spines labelled with titles and decimal numbers, or provide students with a handout containing images of book spines labelled with decimal numbers.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Order the numbers of a set that includes fractions, decimals, or integers in ascending or descending order, and verify the result using a variety of strategies.
- ❒ Identify a number that would be between two numbers in an ordered sequence or on a horizontal or vertical number line.
- ❒ Identify incorrectly placed numbers in an ordered sequence or on a horizontal or vertical number line.

Suggestions for Instruction

- **Order the numbers of a set that includes fractions, decimals, or integers in ascending or descending order, and verify the result using a variety of strategies.**

Materials:

- BLM 7.N.7.4: Ordering Decimal Numbers

Organization: Individual

Procedure:

1. Define a*scending order* and *descending order.*
2. Distribute copies of BLM 7.N.7.4: Ordering Decimal Numbers, and have students complete the tasks. Students place the given decimal numbers in ascending order. They then choose six of the numbers and write them in descending order.

Variations:

- Have students add decimal numbers that would fit between the given numbers.
- Create other number sets. Consider restricting the type of numbers presented in each set (e.g., use numbers containing only hundredths or only thousandths).
- Have students create their own number sets, exchange sets with a partner, order their partner's set, and then verify their partner's ordering of the sets.
- Repeat the learning activity with fractions or combinations of fractions and decimals.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Order the numbers of a set that includes fractions, decimals, or integers in ascending or descending order, and verify the result using a variety of strategies.

Suggestions for Instruction

- **Identify a number that would be between two numbers in an ordered sequence or on a horizontal or vertical number line.**
- **Identify incorrectly placed numbers in an ordered sequence or on a horizontal or vertical number line.**
- **Position fractions with like and unlike denominators from a set on a horizontal or vertical number line, and explain strategies used to determine order.**
- **Order the numbers of a set by placing them on a horizontal or vertical number line that contains benchmarks, such as 0 and 1 or 0 and 5.**

Materials:

- BLM 7.N.7.5: Sequential Fractions and Their Decimal Equivalents
- calculators
- 45 small blank cards
- markers
- large area to create a number line

Organization: Individual, whole class

Procedure:

1. Distribute copies of BLM 7.N.7.5: Sequential Fractions and Their Decimal Equivalents. Have students follow the directions to find the decimal equivalents for the sequential fractions, compare sizes using decimal and fraction notation, indicate equivalent fractions, and make generalizations about comparing fractions.
2. Distribute all the blank cards to students. Assign fractions to students, and ask them to write the given fractions on the blank cards. Specify a size of font so that the cards look similar. Ensure that each card contains a fraction.
3. Have students create a number line by ordering the fraction cards. Ask them to explain why they have placed numbers in a particular order.
4. Discuss strategies students used to determine the order of the fractions.
5. Choose two of the fractions on the number line, and ask students to suggest a number between the two fractions. Discuss the strategies used to determine that number.

Variations:

- Prepare number cards ahead of time, distribute them to students, and ask students to order the cards.
- Call upon groups of students to order the fraction cards they have.
- Call for all students with cards near benchmarks or between particular numbers to order their cards.
- Have students play games with the cards (e.g., call two to four students, and the student with the largest or smallest fraction wins).
- Have students write the decimal equivalents on the other side of the fraction cards. Have them alternate between the fraction and the decimal when ordering the numbers.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Identify a number that would be between two numbers in an ordered sequence or on a horizontal or vertical number line.
- ❒ Identify incorrectly placed numbers in an ordered sequence or on a horizontal or vertical number line.
- ❒ Position fractions with like and unlike denominators from a set on a horizontal or vertical number line, and explain strategies used to determine order.
- ❒ Order the numbers of a set by placing them on a horizontal or vertical number line that contains benchmarks, such as 0 and 1 or 0 and 5.

Suggestions for Instruction

- **Identify a number that would be between two numbers in an ordered sequence or on a horizontal or vertical number line.**

Materials:

- grid paper
- rulers (to draw number lines with benchmarks)
- math journals or notebooks
- bolts of various sizes and a socket set (optional)

Organization: Pairs, whole class, individual

Procedure:

1. Select the type of numbers to work with (e.g., fractions, mixed numbers, decimals, integers, combinations).
2. Select the size and the orientation of the number line (e.g., 0 – 1, (–5) – (+5), horizontal, vertical). Draw the number line and mark some benchmarks.
3. Assign roles to pairs of students. For example, player A will write above a horizontal number line, or to the left of a vertical number line, and player B will write below a horizontal number line, or to the right of a vertical number line. Choose which player will play first.
4. Player A marks a point on the line, and, writing above the line, draws an arrow to the point and labels the point with an approximate value.
5. Player B marks another point on the line, and, writing below the line, draws an arrow to the point and labels the point with an approximate value.
6. Player A then marks and labels a point that can be found between the last two marked points.
7. Player B then marks and labels a point between the last two marked points.
8. Play continues until one of the players can no longer place points on the number line.
9. When students have had sufficient time for the learning activity, have them reassemble as a class and discuss strategies they used to find a number between two fractions, decimals, or integers. (See Background Information for strategies.)
10. Have students use their math journals or notebooks to record strategies for finding a number between two fractions, decimals, or integers.

Variations:

- Provide students with pre-marked number lines.
- Play the game as a class, selecting students to place numbers on a demonstration number line.
- Provide students with two numbers $\left(\text{e.g., } \frac{1}{3} \text{ and } \frac{3}{5}, \text{ or } 1.4 \text{ and } 1\frac{3}{4}\right)$ and ask them to identify a number between the given numbers. Discuss the strategies they used to identify the number.
- Designate the learning activity as one station of a set of rotating learning activities. Use an assortment of bolt sizes and a socket set as an alternative to a number line. Have students find a bolt between two specified bolt sizes. Use the sockets to verify the order.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❐ Identify a number that would be between two numbers in an ordered sequence or on a horizontal or vertical number line.

Suggestions for Instruction

- **Order the numbers of a set by placing them on a horizontal or vertical number line that contains benchmarks, such as 0 and 1 or 0 and 5.**
- **Position a set of fractions, including mixed numbers and improper fractions, on a horizontal or vertical number line, and explain strategies used to determine position.**

Materials:

- BLM 7.N.7.6: Relating Numbers to Benchmarks
- pens or pencils of different colours

Organization: Individual, small groups, whole class

Procedure:

1. Distribute copies of BLM 7.N.7.6: Relating Numbers to Benchmarks, and have students complete the sheet individually. They place numbers (e.g., words, pictures, symbols, proper fractions, improper fractions, mixed numbers, decimals, integers, percents) in the appropriate boxes labelled as follows: less than one-half, equal to one-half, and greater than one-half. They then place eight selected numbers on a number line by drawing a point and a label for each number, explaining their placement choices.

2. Have students form small groups of two to four. Ask them to share examples of their numbers, number lines, and strategies. If they wish, they may make revisions to their sheets, using a different coloured pen or pencil.
3. Meet as a class and have students share examples and strategies.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Order the numbers of a set by placing them on a horizontal or vertical number line that contains benchmarks, such as 0 and 1 or 0 and 5.
- ❒ Position a set of fractions, including mixed numbers and improper fractions, on a horizontal or vertical number line, and explain strategies used to determine position.

Suggestions for Instruction

- **Order the numbers of a set that includes fractions, decimals, or integers in ascending or descending order, and verify the result using a variety of strategies.**
- **Position a set of fractions, including mixed numbers and improper fractions, on a horizontal or vertical number line, and explain strategies used to determine position.**

Materials:

- BLM 7.N.7.7: Ordering Numbers and Verifying the Order
- sets of six to ten numbers, including combinations of proper and improper fractions, mixed numbers, integers, decimal numbers, and percents
- presentation board
- blank paper
- blank cards (optional)

Organization: Whole class, individual

Procedure:

1. Present students with a set of numbers and ask them to place the numbers in order. Specify whether you would like ascending or descending order.
2. Choose students to demonstrate the order and to explain strategies they used to place the numbers.

3. Repeat the process with other number sets, or ask students to suggest numbers. Each time, discuss strategies students used to place the numbers.
4. Distribute copies of BLM 7.N.7.7: Ordering Numbers and Verifying the Order. Have students draw a vertical or a horizontal number line, place a set of numbers on the line, and explain the strategies they used.

Variation:

- Write number sets on cards, distribute the cards, and have students line up with their cards in ascending or descending order. Classmates can share how they know a number is in the correct order, or why it is out of place.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Order the numbers of a set that includes fractions, decimals, or integers in ascending or descending order, and verify the result using a variety of strategies.
- ❒ Position a set of fractions, including mixed numbers and improper fractions, on a horizontal or vertical number line, and explain strategies used to determine position.

NOTES