GRADE 6 MATHEMATICS

Shape and Space

Grade 6: Shape and Space (Measurement) (6.SS.1, 6.SS.2)

Enduring Understanding(s):

All measurements are comparisons.

The unit of measure must be of the same nature as the property of the object being measured.

Many geometric properties and attributes of shapes are related to measurement.

General Learning Outcome(s):

Use direct or indirect measurement to solve problems.

Specific Learning Outcome(s):	Achievement Indicators:
6.SS.1 Demonstrate an understanding of angles by ▪ identifying examples of angles in the environment ▪ classifying angles according to their measure ▪ estimating the measure of angles using 45^o, and 90^o, and 180^o as reference angles ▪ determining angle measures in degrees ▪ drawing and labelling angles when the measure is specified [C, CN, ME, V]	➛ Provide examples of angles found in the environment. ➛ Classify a set of angles according to their measure (e.g., acute, right, obtuse, straight, reflex). ➛ Sketch 45^o, 90^o, and 180^o angles without the use of a protractor, and describe the relationship among them. ➛ Estimate the measure of an angle using 45^o, 90^o, 180^o as reference angles. ➛ Measure, using a protractor, angles in various orientations. ➛ Draw and label an angle in various orientations using a protractor. ➛ Describe the measure of an angle as the measure of rotation of one of its sides. ➛ Describe the measure of an angle as the measure of an interior angle of a polygon.
6.SS.2 Demonstrate that the sum of interior angles is ▪ 180^o in a triangle ▪ 360^o in a quadrilateral [C, R]	➛ Explain, using models, that the sum of the interior angles of a triangle is the same for all triangles. ➛ Explain, using models, that the sum of the interior angles of a quadrilateral is the same for all quadrilaterals.

Prior Knowledge

Students may have had experience with the following:

- Dividing 3-digit numerals by 1-digit numerals
- Solving one-step single-variable equations, and problems involving these equations, using whole numbers only
- Adding and subtracting 1-, 2-, and 3-digit numerals with answers to 1000

Related Knowledge

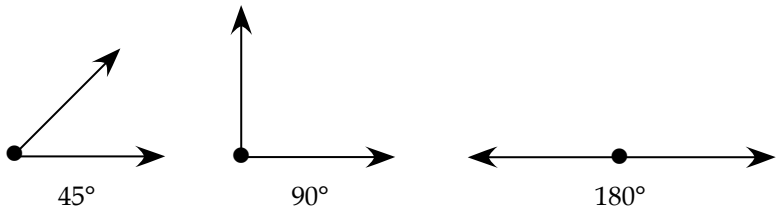
Students should be introduced to the following:

- Demonstrating and explaining the meaning of preservation of equality, concretely, pictorially, and symbolically

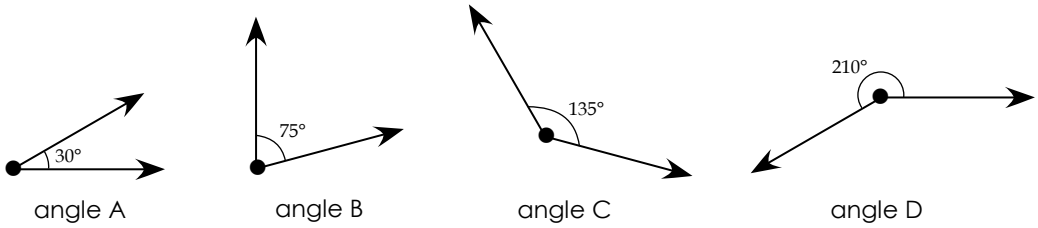
Background Information

An angle is the space between two rays or line segments that are joined at a common point. There are many different sizes of angles, some are small and some are large. Angles can be seen by observing different geometric shapes as well as the environment around us. Angles are measured in degrees using a protractor.

Grade 6 is the first year students are formally learning about angles and angle measures. This year, students will learn to measure angles using a protractor, recognize reference angles (45^o, 90^o, and 180^o), and estimate measures of angles using the reference angles. Look at reference angles shown below:



Based on these angles, students will be able to recognize whether an angle is smaller than 45° (see angle A below), between 45° and 90° (see angle B below), between 90° and 180° (see angle C below), or bigger than 180° (see angle D below).

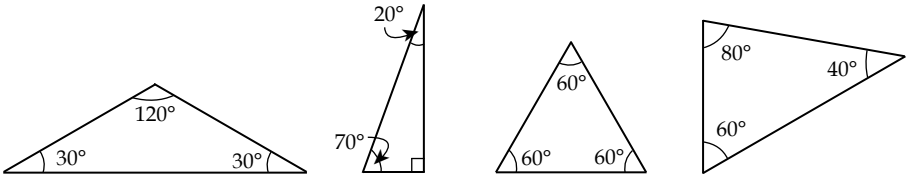


Based on the angle measure, students will learn to name and classify angles as acute (between 0° and 90°, such as angles A, B, and reference angle 45° above), right (90°, such as reference angle 90° above), obtuse (between 90° and 180°, such as angle C above), straight (180°, such as reference 180° above), and reflex (between 180° and 360°, such as angle D above).

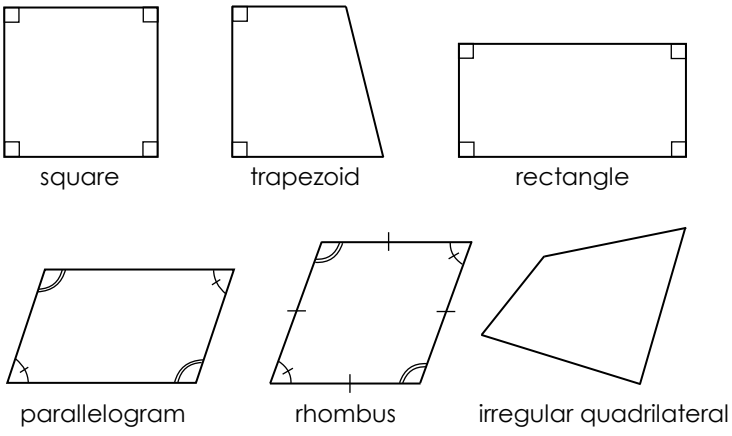
Students will also learn to identify examples of angles found in the environment, such as the space between the ceiling and the wall, the space between the ground and a lamp post, or the space between two branches on a tree.

Measuring interior angles of polygons will reveal some interesting geometric rules. Students will learn that

- the sum of interior angles is 180° in a triangle



- the sum of interior angles is 360° in a quadrilateral



Mathematical Language

angle
acute angle
degree
interior angle
polygon
quadrilateral
reflex angle
straight angle
triangle

Learning Experiences

Assessing Prior Knowledge

Materials:

- BLM 5–8.9: Centimetre Grid Paper

Organization: Individual/whole class

Procedure:

1. Distribute to each student the centimetre grid paper from BLM 5-8.9.
2. Ask students to use the centimetre grid to draw the following:
   a) Four kinds of polygons with 2 cm sides each
   b) Four kinds of polygons with 3 cm sides each
   c) Four kinds of polygons with 4 cm sides each
3. Ask them to write the name of the polygon inside each shape.
4. Ask students to write inside each polygon the number of sides it has.
5. Have one student draw one of his or her polygons on the board.
6. With a piece of paper, cover most of the polygon, leaving only one angle visible, and say to the students: "This is an angle. Who could come to the board and show another angle?"
7. Have students point out different angles inside the polygon.
8. Draw each angle separately, next to the polygon.

9. Discuss angles. Ask students questions such as the following:
   a) Are all of these angles the same?
   b) What is the same?
   c) What is different?
10. Tell students to count the number of angles they find in each polygon, and write the number inside each polygon.
11. Discuss the relationship between the number of sides and the number of angles

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Construct different kinds of polygons (i.e., triangle, quadrilateral, pentagon, and hexagon).
- ❒ Construct polygons given measure of sides.
- ❒ Name polygons according to the sides that they have.
- ❒ Recognize angles.
- ❒ Recognize the relationship between sides and angles of a polygon.

Suggestions for Instruction

- **Provide examples of angles found in the environment.**

Materials:

- BLM 5-8.2: Concept Description Sheet #1

Organization: Whole class/small group/individual

Procedure:

1. Have a class discussion on the concept of angles. Have students answer questions, such as the following:
   a) What are angles?
   b) Where can you find angles?
   c) Are all angles the same?

2 Distribute to each student a copy of BLM 5-8.2: Concept Description Sheet #1.

3 Tell students to write the word "angle" inside the oval, and complete the sheet, doing the following:

   a) Describe the characteristics of an angle.

   b) Provide examples of angles.

   c) Provide examples of non-angles.

   d) Draw pictures of angles.

4 Tell students to discuss with their group members the angles they can find

   a) in the classroom

   b) in their homes

   c) on the playground

5 Have students record their examples of angles in their journals.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Describe the characteristics of an angle.
- ❒ Provide examples of angles.
- ❒ Provide examples of non-angles.
- ❒ Draw pictures of angles.
- ❒ Provide examples of angles found in the classroom.
- ❒ Provide examples of angles found in their homes.
- ❒ Provide examples of angles found on the playground.

Suggestions for Instruction

- **Classify a set of angles according to their measure (e.g., acute, right, obtuse, straight, reflex).**
- **Provide examples of angles found in the environment.**

Materials:

- BLM 6.SS.1.1: Angles

Organization: Whole class/individual

Procedure:

1. Draw a right angle on the board and write "right angle" under it.
2. Tell students to look around the classroom to spot some right angles.
3. Discuss the examples of right angles they spotted in the classroom.
4. Draw a straight angle on the board and write "straight angle" under it.
5. Tell students to look around the classroom to spot some straight angles.
6. Discuss the examples of straight angles they spotted in the classroom.
7. Discuss other angles and name them.
8. Ask students to draw examples of acute, obtuse, and reflex angles on the board.
9. Distribute a copy of BLM 6.SS.1.1, and ask students to write the name of each angle inside it.
10. Discuss their classifications.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Recognize a right angle.
- ❒ Recognize a straight angle.
- ❒ Recognize an acute angle.
- ❒ Recognize an obtuse angle.
- ❒ Recognize a reflex angle.
- ❒ Provide examples of angles found in the environment.

Suggestions for Instruction

- **Sketch 45º, 90º, and 180º angles without the use of a protractor, and describe the relationship among them.**
- **Provide examples of angles found in the environment.**

Materials:

- BLM 6.SS.1.2: Reference Angles

Organization: Whole class/individual

Procedure:

1. Place on the overhead projector a transparency of BLM 6.SS.1.2.
2. Draw two vertical lines on the chalkboard in order to create three sections.
3. Write on the top part of the first section "45º", second section "90º", and third section "180º". Then say to the students "Look around the classroom and find possible examples of each angle."
4. Have a class discussion about the possible examples for each angle.
5. Ask for three volunteers (one for each section) to record possible examples of each angle, and sketch the angle (without a protractor).
6. Discuss the relationship among the three angles and their importance (reference angles).
7. Tell students to sketch a "45º", "90º", and "180º" angle without a protractor, and describe the relationship among these three angles in their journals.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Provide an example of a 45º angle.
- ❒ Provide an example of a 90º angle.
- ❒ Provide an example of a 180º angle.
- ❒ Sketch a 45º angle without a protractor.
- ❒ Sketch a 90º angle without a protractor.
- ❒ Sketch a 180º angle without a protractor.
- ❒ Describe the relationship among these three (45º, 90º, 180º) angles.

Suggestions for Instruction

- **Estimate the measure of an angle using 45º, 90º, 180º as reference angles.**
- **Measure, using a protractor, angles in various orientations.**

Materials:

- BLM 6.SS.1.1: Angles
- BLM 6.SS.1.2: Reference Angles

Organization: Individual

Procedure:

1. Distribute to each student a copy of BLM 6.SS.1.1.
2. Place a transparency of the BLM 6.SS.1.2 on the overhead projector.
3. Tell students to use the reference angles to estimate the measure of each angle on their paper, and write the estimated measure next to the angle.
4. Ask a few students to read out loud their estimated measures of each angle.
5. Say "Take out your protractors, measure each angle, and record the angle measurement next to the angle."
6. When they complete their work, ask students to record the difference between the measured angle and its estimate.
7. Let students know that, with practice, their estimates can get quite close to the actual angle measurement.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Use the reference angles to estimate the measure of an angle.
- ❒ Estimate the measure of an acute angle, using 45^{o} and 90^{o} angles as reference angles.
- ❒ Estimate the measure of an obtuse angle, using 90^{o} and 180^{o} angles as reference angles.
- ❒ Estimate the measure of a reflex angle, using a 90^{o} angle as a reference angle.
- ❒ Use a protractor to measure angles in standard position.
- ❒ Use a protractor to measure angles in various orientations.

Suggestions for Instruction

- **Draw and label an angle in various orientations using a protractor.**

Materials:

- poster-sized paper
- protractor
- pencil

Organization: Groups of four

Procedure:

1. Seat students in groups of four.
2. Distribute a large poster-sized paper to each group.
3. Say to the class:
   a) I want each of you to choose an angle measure. Make sure that the number of degrees you choose is not the same as the angle chosen by any of the other three members in your group.
   b) Use your protractor to draw your angle on the poster-sized paper in front of you.
   c) Then, using the same angle measure, draw the angle in three other orientations.
   d) Label each angle (write the angle measure inside the angle).
4. Have one member of each group place the paper (completed work) on the board.
5. Ask students to have a class discussion on what they have observed about angles in various orientations.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Use a protractor to measure an angle.
- ❒ Draw an angle.
- ❒ Replicate an angle.
- ❒ Draw an angle in various orientations.

Suggestions for Instruction

- **Describe the measure of an angle as the measure of rotation of one of its sides.**

Organization: Whole class

Procedure:

1. Say and demonstrate the following to the class:
   a) Today's activity will be a live demonstration of an angle measure. It will involve some physical activity on your part.
   b) I want everyone to stand up facing me. Stand with your feet together and pretend your feet are the two sides of an angle.
   c) Now, keeping your heels together, rotate your left foot so that you make an angle. (Demonstrate).
   d) How big of an angle can you make without moving your right foot and without falling?
   e) Now, make a zero degree angle by rotating your left foot back to a "feet together" position. (Demonstrate).
   f) Show the person next to you a variety of angles you can make by keeping your heels together and rotating your right foot.
   g) Thanks for the great participation. Now, go back to your seats.
2. Discuss the number of angles and the kinds of angles that students made with their feet.
3. Tell students to describe in their journals what happens to the measure of an angle when you rotate one of its sides.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Follow directions.
- ❒ Make an angle using their feet.
- ❒ Make a variety of angles using their feet.
- ❒ Associate the change in measure of an angle with the rotation of its side.

Suggestions for Instruction

- **Describe the measure of an angle as the measure of rotation of one of its sides.**

Materials:

- a variety of craft materials
- scissors

Organization: Pairs

Procedure:

1. Let students know that they will be making their own protractors using what they know about angles.
2. Have them work in pairs to formulate a plan for how they are going to make their protractor.
3. Ask them to document the steps they take, making specific reference to reference angles and rotation.
4. Allow students to construct a protractor.
5. Have the students all measure the same angles from the classroom using their protractors (i.e., the edge of a desk, the binding and edge of a three-ring binder).
6. Facilitate a class discussion about the students' protractors.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Reason mathematically.
- ❒ Apply their knowledge about reference angles.
- ❒ Apply their knowledge about rotation.
- ❒ Measure angles using a protractor.

Suggestions for Instruction

- **Describe the measure of an angle as the measure of an interior angle of a polygon.**

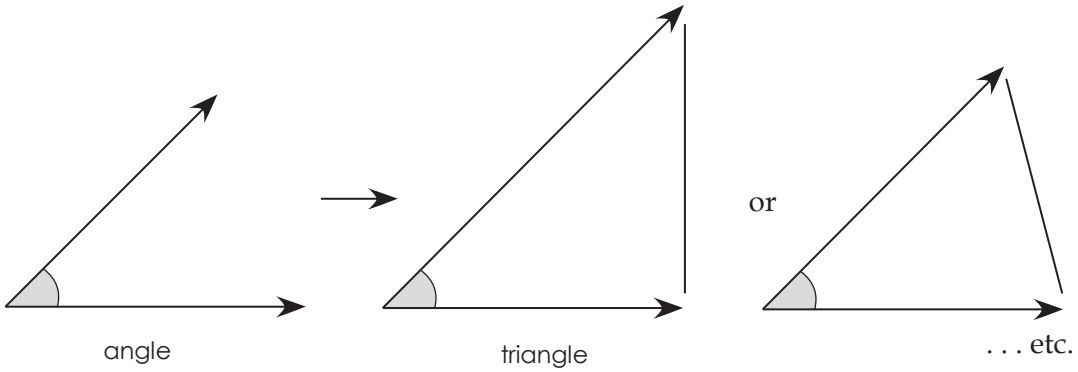
Materials:

- protractor
- ruler
- pencil

Organization: Whole class/individual

Procedure:

1. Draw four distinct angles on the board (e.g., 30^o, 45^o, 90^o, and 120^o).
2. Measure each angle using a protractor.
3. Write the measure of each angle inside it.
4. Have a discussion on angles and angle measures.
5. Draw a triangle (i.e., choose one of the angles you drew on the board and connect its two adjacent sides by a third line segment, creating a side opposite to the angle). See example below.



6. Ask students the following:
   a) What happened to the angle?
   b) Did the angle measure change?
7. Ask a student to measure the angle that was made into an interior angle of a triangle.
8. Discuss interior angles of a polygon.
9. Ask one of the students to come up to the board and draw a polygon, but not a triangle, using another one of the angles.
10. Ask students to predict the measure of the angle, which is now an interior angle.
11. Ask a different student to come up to the board and measure that particular interior angle.

12. Ask students what would happen to the other two angles if they were made into interior angles.
13. Discuss the measure of angles as the measure of an interior angle of a polygon.
14. Tell students to do the following:
    a) Draw an angle in their journals.
    b) Describe the measure of the angle as the measure of an interior angle of a polygon of their choice.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Sketch an angle (approximate size).
- ❒ Measure an angle with a protractor.
- ❒ Know what an interior angle of a polygon is.
- ❒ Describe the measure of an angle as the measure of an interior angle of a polygon.

Suggestions for Instruction

- **Explain, using models, that the sum of the interior angles of a triangle is the same for all triangles.**

Materials:

- BLM 6.SS.1.3: Sum of Interior Angles of a Triangle
- straws
- pipe cleaners
- scissors
- protractor
- pencil

Organization: Whole class/four groups

Procedure:

1. Pre-cut straws to the following lengths: 6 cm, 9 cm, 12 cm, and 15 cm. It would be helpful if you could have a different colour for each length, such as red, green purple, and white.
2. For each group, you will need the following:
   a) two straws of each: 6 cm (red)
      15 cm (white)
   b) four straws of each: 9 cm (green)
      12 cm (purple)
3. Distribute to each **group** the 12 pre-cut straws and 12 pipe cleaners.
4. Distribute to each **student** a copy of BLM 6.SS.1.3.
5. Tell students to do the following:
   a) Separate the straws into four piles according to size (colour).
   b) Slightly bend each pipe cleaner.
   c) Take one red straw, one purple straw, and one white straw
   d) Use the pipe cleaners to join two straws together.
   e) You will need three bent pipe cleaners to join the three straws and form a triangle.
   f) Using three different lengths of straws will make a *scalene* triangle.
   g) Measure each angle of the *scalene* triangle.
   h) On BLM 6.SS.1.3, mark "*scalene*" under "Triangle Name."
   i) Write the measure of each interior angle under "Interior Angle Measures."
   j) Add up the measure of the three interior angles and write the sum under "Sum of Interior Angles of Triangle."
6. Place on the overhead projector a transparency copy of BLM 6.SS.1.3.
7. Ask students for information to record under each heading.
8. Discuss the results they got by measuring the interior angles of the scalene triangle.
9. Tell students to use one green straw, one purple straw, and one white straw to construct a *right* triangle.
10. Tell students to record the name, each angle measure, and sum of interior angles of the right triangle on the BLM sheet.
11. Tell students to use three green straws to make an equilateral triangle, and the two purples and one red straw to make an isosceles triangle.
12. Have them record the results of each triangle.

13. Discuss the results they got by measuring the interior angles of the other three triangles. For example, compare the
    a) interior measures of each triangle to the interior measures of the scalene triangle
    b) sum of the interior measures of each triangle to the sum of the interior measures of the scalene triangle
    c) interior measures of each triangle to the interior measures of the other triangles
    d) sum of the interior measures of each triangle to the sum of the interior measures of the other triangles
14. Tell students to write in their journals what they observed about how different measures of the interior angles of a triangle influence the sum of the interior angles of a triangle.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Follow instructions.
- ❒ Construct a model.
- ❒ Measure interior angles of a triangle accurately.
- ❒ Explain, using models, that the sum of the interior angles of a triangle is 180^{o}.
- ❒ Explain, using models, that the sum of the interior angles of a triangle is the same for all triangles.

Suggestions for Instruction

- **Explain, using models, that the sum of the interior angles of a quadrilateral is the same for all quadrilaterals.**

Materials:

- BLM 6.SS.1.4: Sum of Interior Angles of a Quadrilateral
- BLM 6.SS.2.3: Sides for Flexible Quadrilaterals
- protractor
- scissors
- pencil
- small binder rings (safety pins or paper clips will also work)

Organization: Whole class/small groups

Procedure:

1. Have students seated in small groups.
2. Distribute to each student a copy of BLM 6.SS.2.2 and BLM 6.SS.2.3.
3. Tell students to do the following:
   a) Take BLM 6.SS.2.3.
   b) Cut the strips of paper along the lines. You should have 16 strips of paper.
      1. 5 large
      2. 1 medium
      3. 10 small
   c) Use a hole-puncher to punch holes on each end of the strips of paper.
   d) Separate the strips of paper into four piles according to the following classifications: parallelogram P1, parallelogram P2, trapezoid T1, and trapezoid T2.
   e) Join two strips of paper by lining up the holes and putting a ring through the holes. Then, using the same method, join to them the third and the fourth strip of paper to form the four sides of each quadrilateral.
   f) Measure each angle of the "parallelogram P1" without moving the sides of the model.
   g) On your sheet BLM 6.SS.2.2, mark "parallelogram P1" under the "Name of Quadrilateral."
   h) Write the measure of each interior angle under "Interior Angle Measures."
   i) Add up the measure of the four interior angles and write the sum under "Sum of Interior Angles of Quadrilaterals."
4. Write on the board "*parallelogram P1*."
5. Tell students to discuss with their group members their angle measures and the sum of their interior measures.
6. Ask a few students to record on the board their angle measures and the sum of their interior measures.
7. Discuss with the class the results they got by measuring the interior angles of "*parallelogram P1*" and how some results may be similar or different from other students' results.
8. Tell students to do the following:
   a) Measure the interior angles of the other three quadrilaterals.
   b) Record the measure of each interior angle.
   c) Record the sum of the measures of each quadrilateral.

9. Discuss the results they got by measuring the interior angles of the other three quadrilaterals. For example: compare the following:
   a) Interior measures of each quadrilateral to the interior measures of the *"parallelogram P1"*
   b) Sum of the interior measures of each quadrilateral to the sum of the interior measures of the *"parallelogram P1"*
   c) Interior measures of each quadrilateral to the interior measures of the other quadrilaterals
   d) Sum of the interior measures of each quadrilateral to the sum of the interior measures of the other quadrilaterals
10. Tell students to write in their journals what they observed about how different measures of the interior angles of a quadrilateral influence the sum of the interior angles of a quadrilateral.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Follow instructions.
- ❒ Construct a model.
- ❒ Measure interior angles of a quadrilateral accurately.
- ❒ Explain, using models, that the sum of the interior angles of a quadrilateral is 360^{o}.
- ❒ Explain, using models, that the sum of the interior angles of a quadrilateral is the same for all quadrilaterals.

Grade 6: Shape and Space (Measurement) (6.SS.3)

Enduring Understanding(s):

All measurements are comparisons.

There is no direct relationship between perimeter and area.

Perimeter, area, and volume are measurable properties of objects.

The units of measure must be of the same nature as the property being measured.

General Learning Outcome(s):

Use direct or indirect measurement to solve problems.

Specific Learning Outcome(s):	Achievement Indicators:
6.SS.3 Develop and apply a formula for determining the ▪ perimeter of polygons ▪ area of rectangles ▪ volume of right rectangular prisms [C, CN, PS, R, V]	➛ Explain, using models, how the perimeter of any polygon can be determined. ➛ Generalize a rule for determining the perimeter of polygons. ➛ Explain, using models, how the area of any rectangle can be determined. ➛ Generalize a rule for determining the area of rectangles. ➛ Explain, using models, how the volume of any right rectangular prism can be determined. ➛ Generalize a rule for determining the volume of right rectangular prisms. ➛ Solve a problem involving the perimeter of polygons, the area of rectangles, or the volume of right rectangular prisms.

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of multiplying 2-digit numerals by 2-digit numerals to solve problems
- Demonstrating an understanding of division of 2-digit numerals by 3-digit numerals
- Using perimeter or area or both (whole numbers) to design and construct different rectangles and draw conclusions
- Demonstrating an understanding of measuring length

- Demonstrating an understanding of volume
- Identifying and sorting quadrilaterals
- Describing orally and in writing the rule for pattern
- Demonstrating an understanding of area of regular and irregular 2-D shapes
- Describing and constructing rectangular and triangular prisms
- Demonstrating an understanding of perimeter of regular and irregular shapes
- Adding and subtracting 1-, 2-, and 3-digit numerals with answers to 1000

Related Knowledge

Students should be introduced to the following:

- Explaining and applying the order of operations, excluding exponents
- Representing generalizations arising from number relationships

Background Information

Perimeter, **area**, and **volume** are not of the same nature; therefore, they do not use the same units of measurement.

Perimeter is the distance around a shape, and it is measured in linear units such as kilometres (km), metres (m), centimetres (cm), and millimetres (mm).

Area is the amount of surface a shape covers, and it is measured in square units such as square kilometres (km^2), square metres (m^2), square centimetres (cm^2), and square millimetres (mm^2).

Volume is the amount of space an object occupies or, if the object is hollow, the amount of space inside the object (capacity). Volume is measured in cubic units such as cubic metres (m^3), cubic centimetres (cm^3), and cubic millimetres (mm^3).

When given either perimeter, or area, or both, students in Grade 5 learned to: (a) design and construct different rectangles; (b) draw conclusions. They also demonstrated an understanding of volume by: (a) selecting and justifying referents for cm^3 and m^3 units; (b) estimating volume by using referents for cm^3 and m^3; (c) measuring and recording volume (cm^3 and m^3); and (d) constructing rectangular prisms for a given volume.

In Grade 6, students will learn through their activities to develop and apply a formula for determining the (a) perimeter of polygons; (b) area of rectangles; and (c) the volume of right rectangular prisms.

Mathematical Language

centimetre
millimetre
kilometre
height
length
width
perimeter
area
volume
polygon
rectangle
right rectangular prism

Learning Experiences

Assessing Prior Knowledge

Materials:

- BLM 5-8.9: Centimetre Grid Paper
- ruler
- pencil
- scissors
- tape

Organization: Individual

Procedure:

1. Tell students that you wish to assess their understanding of perimeter and area of rectangles and also their understanding of volume of rectangular prisms.
2. Distribute to each student a copy of BLM 5-8.9.
3. Tell students to draw two rectangles:
   a) One with a 14 cm perimeter and mark P = 14 cm inside it.
   b) One with a 24 cm^2 area and mark A = 24 cm^2 inside it.

4. Circulate to check that students chose a correct rectangle for each scenario.
5. Construct a rectangular prism that has a 60 cm^3 volume.
6. Circulate to check that students chose a correct rectangular prism.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Use a given perimeter to find the dimensions of a rectangle.
- ❒ Use a given area to find the dimensions of a rectangle.
- ❒ Divide two-digit numerals by one-digit numerals.
- ❒ Draw a rectangle.
- ❒ Use a given volume to find the dimensions of a rectangular prism.
- ❒ Construct a rectangular prism.

Suggestions for Instruction

- **Explain, using models, how the perimeter of any polygon can be determined.**

Materials:

- geoboard
- elastic band
- pegs

Organization: Whole class/small group/individual

Procedure:

1. Distribute to each student:
   a) 1 geoboard
   b) 6 pegs
   c) 1 elastic band
2. Say the following to the students:
   a) Use the geoboard, elastic band, and any number of pegs you want to make a polygon.
   b) Analyze your polygon to determine its perimeter.

c) Write in your notebook how you determined the perimeter of your polygon.

d) Use your geoboard, elastic band, and pegs to make a different polygon.

e) Analyze your new polygon to determine its perimeter.

f) Write in your notebook how you determined the perimeter of your new polygon.

g) Use your geoboard, elastic band, and pegs to make two more types of polygons, and go through the same process.

h) Analyze the notes you wrote about how to determine the perimeter of each polygon.

i) Write in your notebook how you would determine the perimeter of any polygon.

j) Repeat the process for regular polygons.

3. Discuss together what students found out about determining the perimeter of any polygon.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Construct a polygon using a geoboard, elastic band, and pegs.
- ❒ Determine the perimeter of a polygon.
- ❒ Add lengths together.
- ❒ Explain how they determined the perimeter of a polygon.

- **Generalize a rule for determining the perimeter of polygons.**

Materials:

- collection of polygons (both regular and irregular)
- BLM 6.SS.3.1: Polygon Collection: Set 1–5, with the polygons cut out and placed in envelopes

Organization: Four or five groups

Procedure:

1. Distribute a different collection of polygons to each group.
2. Tell students to do the following:
   a) Sort the polygons by their number of sides.
   b) Estimate and then measure the perimeter of each polygon.
   c) Can you find a shortcut or rule for finding the perimeter of polygons?
   d) Exchange your polygon collection with another group.
   e) Verify that the shortcut or rule you developed works for other polygons.
   f) Report to the class how your group calculated the perimeters, including the shortcut (or rule) you found.
   g) Discuss similarities or differences among your findings.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Sort polygons by the number of their sides.
- ❒ Estimate the perimeter of a polygon.
- ❒ Measure the perimeter of a polygon.
- ❒ Explain the shortcut or rule for finding the perimeter of a polygon.

Suggestions for Instruction

- **Explain, using models, how the area of any rectangle can be determined.**

Materials:

- geoboard
- elastic band
- pegs

Organization: Whole class/individual

Procedure:

1. Distribute the following to each student:
   a) 1 geoboard
   b) 1 elastic band
   c) 4 pegs
2. Say the following to the students:
   a) Use the geoboard, elastic band, and the four pegs to make a rectangle.
   b) Analyze your rectangle to determine its area.
   c) Write in your notebook how you determined the area of your rectangle.
   d) Use your geoboard, elastic band, and pegs to make a different rectangle.
   e) Analyze your new rectangle to determine its area.
   f) Write in your notebook how you determined the area of your new rectangle.
   g) Use your geoboard, elastic band, and pegs to make two more types of rectangles, and go through the same process.
   h) Analyze the notes you wrote about how to determine the area of each rectangle.
   i) Write in your notebook how you would determine the area of any rectangle.
3. Discuss together what students found out about determining the area of any rectangle.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Construct a rectangle using a geoboard, elastic band, and pegs.
- ❒ Determine the area of a rectangle.
- ❒ Explain how they determine the area of a rectangle.

Suggestions for Instruction

- **Generalize a rule for determining the area of rectangles.**

Materials:

- square tiles or a geoboard

Organization: Groups

Procedure:

1. Distribute square tiles or a geoboard to each group.
2. Tell students to do the following:
   a) Make a collection of rectangles.
   b) Look for shortcuts that can be used in determining area.
   c) Determine the area of each rectangle.
   d) Draw each rectangle on grid paper.
   e) Record each rectangle's dimensions on grid paper.
   f) Record the area of each rectangle inside it.
   g) Exchange collections with another group.
   h) Verify that the shortcut or rule you developed works for other rectangles.
   i) Report to the class how your group calculated the area, including any shortcut (or rule) you may have found.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Construct different rectangles.
- ❒ Determine the area of different rectangles.
- ❒ Measure the sides of a rectangle.
- ❒ Use multiplication facts.
- ❒ Find a shortcut or rule for finding the area of a rectangle.
- ❒ Explain the shortcut or rule for finding the area of a rectangle.

Suggestions for Instruction

- **Explain, using models, how the volume of any right rectangular prism can be determined.**
- **Generalize a rule for determining the volume of right rectangular prisms.**

Materials:

- centicubes

Organization: Small groups

Procedure:

1. Tell students to:
   a) Make right rectangular prisms whose volumes are 48 cm^3, 36 cm^3, 28 cm^3, 21 cm^3, and 12 cm^3.
   b) Make a table, such as the one below.
   c) Record each prism's
      - dimensions
      - area of its base
      - volume

Width in cm	Length in cm	Area of Base in cm^2	Height in cm	Volume in cm^3

   d) Explain in your journals how the volume of each of these right rectangular prisms was found.
   e) Discuss with your group members how a prism's dimensions are related to its area and volume.
   f) Explain in your journals how the volume of any right rectangular prisms can be found.
   g) Write a shortcut method or rule to be used to calculate volume of any right rectangular prism.
   h) Verify your shortcut rules by predicting the volume of a right rectangular prism that is 4-cm wide by 3-cm long by 10-cm high.
   i) Build the prism using centicubes and count the number of cubes. The count should match the volume calculated by your rule.
   j) Share your conclusions with the class.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Construct a right rectangular prism using centicubes, given a specific volume.
- ❒ Calculate dimensions of a right rectangular prism, given a specific volume.
- ❒ Find a shortcut or rule for finding the volume of a right rectangular prism.
- ❒ Explain the shortcut or rule for finding the volume of a right rectangular prism.

Suggestions for Instruction

- **Explain, using models, how the volume of any right rectangular prism can be determined.**
- **Generalize a rule for determining the volume of right rectangular prisms.**

Materials:

- centicubes

Organization: Small groups

Procedure:

1. Have a class discussion about volume and right rectangular prisms. Be sure everyone understands before you move on.
2. Present students with the following problem:

   A grocery store chain is opening up a new store in Stonewall, MB. They are given very specific instructions as to how to place each shelf so that all of the items will fit. The page about juice boxes has a small tear in the bottom and some information seems to be missing.

The volume of juice boxes is 36 cm^3 and 48 cm^3 and they are to be displayed on the same shelf.

The space to the next shelf must be

The manager does not want the owners to doubt his abilities, so he has the stocking clerks try to figure out the height that the shelf should be. They can record their information in the table below:

Width in cm	Length in cm	Area of Base in cm^2	Height in cm	Volume in cm^3

3. Have students explain in their journals how the volume relates to the width, length, area of base, and height of their right rectangular prisms. Ask them to use this relationship to determine the volume of a juice box that is 5 cm long by 3 cm wide by 6 cm tall.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Construct a right rectangular prism using centicubes, given a specific volume.
- ❒ Find a shortcut or rule for finding the volume of a right rectangular prism.
- ❒ Explain the shortcut or rule for finding the volume of a right rectangular prism.

- **Solve a problem involving the perimeter of polygons, the area of rectangles, or the volume of right rectangular prisms.**

Materials:

- BLM 6.SS.3.2: Dolly Made a Garden

Organization: Individual

Procedure:

1. Distribute to each student a copy of BLM 6.SS.3.2.
2. Tell students to do the following:
   a) Read the problem carefully.
   b) Name the polygon.
   c) Explain how they will find the perimeter of Dolly's garden.
   d) Solve the perimeter problem of Dolly's garden.
3. Discuss with the class the polygon created by Dolly.
4. Ask a student to write the name of the polygon on the board.
5. Have a student tell the class how he or she found the perimeter of Dolly's garden.
6. Invite comments by other students.
7. Ask a student to write his or her solution on the board.
8. Discuss the results with the class.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Recognize and name a polygon.
- ❒ Explain how they will find the perimeter of a polygon.
- ❒ Solve the perimeter of a polygon.

Suggestions for Instruction

- **Solve a problem involving the perimeter of polygons, the area of rectangles, or the volume of right rectangular prisms.**

Materials:

- BLM 6.SS.3.3: David's Playroom
- A poster-sized paper

Organization: Small group

Procedure:

1. Distribute to each student a copy of BLM 6.SS.3.3 and a poster-sized paper.
2. Tell students to do the following:
   a) Read the problem carefully.
   b) Name the quadrilateral.
   c) Explain how they will find the area of David's playroom.
   d) Solve the area problem of David's playroom.
3. Discuss with the class the quadrilateral created by David.
4. Ask each group to use the poster-sized paper to do the following:
   a) Write the name of the quadrilateral on top of the poster paper.
   b) Draw the shape of David's playroom.
   c) Write down how they found the area of David's playroom.
   d) Write their solution to the problem involving the area of David's playroom.
5. Place each poster on the board.
6. Discuss the results with the class.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Recognize and name a quadrilateral.
- ❒ Explain how they will find the area of a quadrilateral.
- ❒ Solve the area of a quadrilateral.

- **Solve a problem involving the perimeter of polygons, the area of rectangles, or the volume of right rectangular prisms.**

Materials:

- BLM 6.SS.3.4: Peter's Toy Box

Organization: Individual

Procedure:

1. Distribute to each student a copy of BLM 6.SS.3.4.
2. Tell students to do the following:
   a) Read the problem carefully.
   b) Create a chart based on the problem.
   c) Explain how you will find the volume of Peter's toy box.
   d) Solve the volume problem of Peter's toy box.
3. Discuss with the class the dimensions of Peter's toy box.
4. Write the dimensions on an overhead transparency.
5. Have a student tell the class how he or she found the volume of Peter's toy box.
6. Ask if anyone has a different explanation.
7. Ask a student to write his or her solution on the board.
8. Discuss the results with the class.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Create a chart based on a problem.
- ❒ Explain how they will find the volume of a right rectangular prism.
- ❒ Solve a problem involving the volume of a right rectangular prism.

Grade 6: Shape and Space (3-D Objects and 2-D Shapes) (6.SS.4, 6.SS.5)

Enduring Understanding(s):

Shapes are distinguished by their properties

General Learning Outcome(s):

Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Specific Learning Outcome(s):	Achievement Indicators:
6.SS.4 Construct and compare triangles in different orientations, including ▪ scalene ▪ isosceles ▪ equilateral ▪ right ▪ obtuse ▪ acute [C, PS, R, V]	➛ Sort a set of triangles according to the length of the sides. ➛ Sort a set of triangles according to the measures of the interior angles. ➛ Identify the characteristics of a set of triangles according to their sides or their interior angles. ➛ Sort a set of triangles and explain the sorting rule. ➛ Draw a triangle (e.g., scalene). ➛ Replicate a triangle in a different orientation and show that the two are congruent.
6.SS.5 Describe and compare the sides and angles of regular and irregular polygons. [C, PS, R, V]	➛ Sort a set of 2-D shapes into polygons and non-polygons, and explain the sorting rule. ➛ Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by superimposing. ➛ Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by measuring. ➛ Demonstrate that the sides of a regular polygon are of the same length and that the angles of a regular polygon are of the same measure. ➛ Sort a set of polygons as regular or irregular and justify the sorting. ➛ Identify and describe regular and irregular polygons in the environment.

Prior Knowledge

Students may have had experience with the following:

- Designing and constructing different rectangles
- Identifying and sorting quadrilaterals according to their attributes
- Demonstrating an understanding of area of regular and irregular 2-D shapes
- Describing and constructing rectangular and triangular prisms
- Demonstrating an understanding of line symmetry
- Sorting regular and irregular polygons according to the number of sides

Related Knowledge

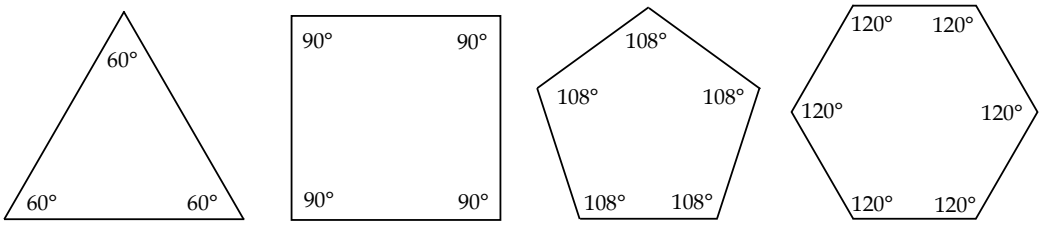
Students should be introduced to the following:

- Demonstrating an understanding of angles
- Performing a combination of transformations on a single 2-D shape, and drawing and describing the image

Background Information

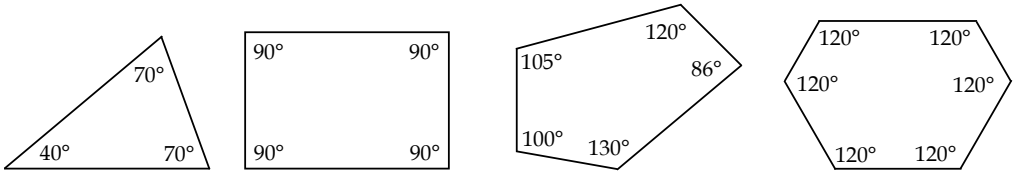
A **polygon** is a closed plane figure formed by three or more line segments. The simplest polygon is a triangle. There are regular polygons and irregular polygons.

A **regular polygon** is a polygon in which all sides and all angles are congruent. See examples below.



triangle quadrilateral pentagon hexagon

An **irregular polygon** is a polygon whose sides and angles are not all congruent. See examples below.

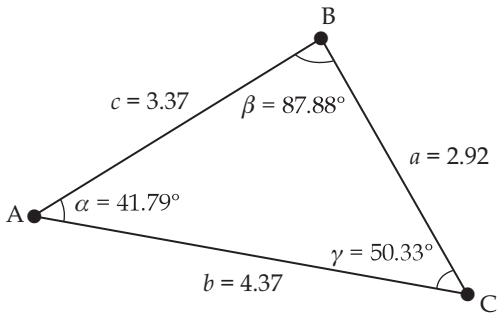


triangle quadrilateral pentagon hexagon

A **triangle** is a polygon with three sides and three angles. Triangles are sorted according to their sides and angles.

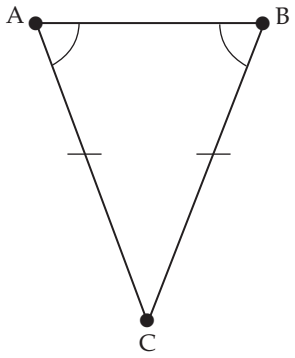
- A **scalene triangle** is a triangle with no congruent sides and no congruent angles.

  Example:



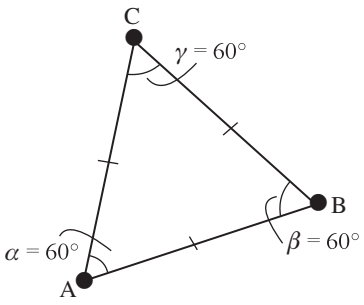
- An **isosceles triangle** is a triangle with at least two congruent sides and two congruent angles.

  Example:



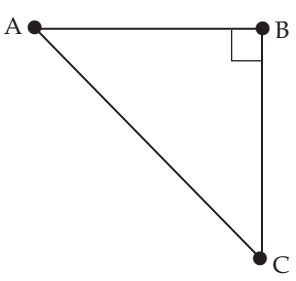
- An **equilateral triangle** is a triangle with three congruent sides and three congruent angles.

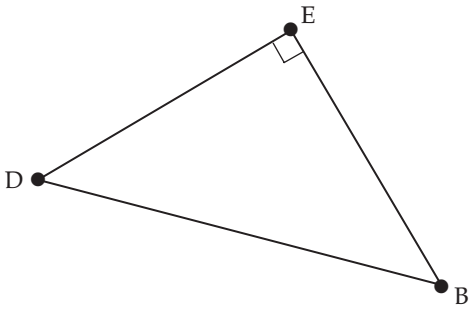
  Example:



- A **right triangle** is a triangle with one right angle.

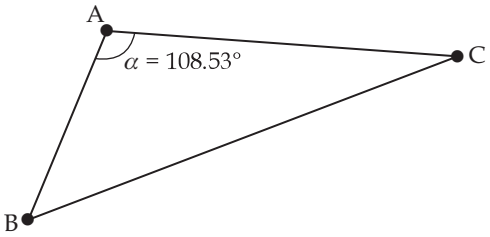
  Examples:





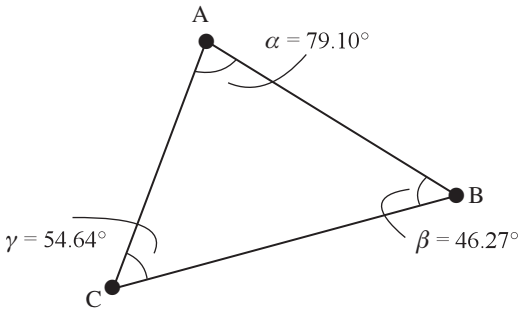
- An **obtuse triangle** is a triangle containing one obtuse angle (greater than 90º and less than 180º).

  Example:



- An **acute triangle** is a triangle in which all three angles are acute (greater than 0º and less than 90º).

  Example:



When two figures have the same shape and size, they are **congruent**.

Examples:

- Two sides (line segments) are congruent if they are the same length.
- Two angles are congruent if they have the same measure.

Mathematical Language

acute
congruent
equilateral
hexagon
isosceles
obtuse
pentagon
polygon
right
scalene
square
triangle

Learning Experiences

Assessing Prior Knowledge

Materials:

- BLM 5-8.9: Centimetre Grid Paper
- ruler
- protractor
- pencil

Organization: Individual

Procedure:

1. Distribute to each student a copy of BLM 5-8.9.
2. Ask students to use the centimetre grid to draw the following:
   a) One triangle using a ruler and protractor.
   b) One quadrilateral using a ruler and protractor.
3. Tell students to do the following:
   a) Mark each interior angle of each shape with a different letter of the alphabet.
   b) Name each interior angle of each polygon according to its measure.

4. Circulate to check that students do the following:
   a) Draw the correct shape.
   b) Write the correct name for each angle.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Draw a triangle using a ruler and protractor.
- ❒ Draw a quadrilateral (e.g., square, rectangle, parallelogram, trapezoid, or other irregular quadrilateral) using a ruler and protractor.
- ❒ Recognize interior angles in a polygon.
- ❒ Mark interior angles in a polygon.
- ❒ Name interior angles of a polygon according to their measure (e.g., acute, right, obtuse, straight, reflex).

Suggestions for Instruction

- **Sort a set of triangles according to the length of the sides.**

Materials:

- scissors
- ruler
- BLM 6.SS.4.1: Cards of Triangles #1
- BLM 6.SS.4.2: Sorting of Triangles According to the Length of the Sides

Organization: Whole class/pairs

Procedure:

1. Distribute to each pair a copy of BLM 6.SS.4.1, and to each student a copy of BLM 6.SS.4.2.
2. Have the students cut the cards of triangles in BLM 6.SS.4.1 so they get eight cards (that is, four cards per student).

3. Tell students to do the following:
   a) Measure all three sides of each triangle.
   b) On your copy of BLM 6.SS.4.2, mark down
      - the name of the triangle
      - length of each side
      - how many sides have the same length—that is,
        - 3 sides have the same length
        - 2 sides have the same length
        - No sides are the same length
   c) Discuss the results with your partner.
   d) Copy your partner's results onto your sheet.
   e) Sort the triangles according to the number of sides that have the same length (3, 2, none).
4. Have a class discussion on sorting triangles according to the length of the sides.
5. Tell students to write in their journals their observations on sorting triangles according to the length of the sides.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Measure the sides of a triangle.
- ❒ Compare triangles according to the length of the sides.
- ❒ Sort triangles according to the length of the sides.

Suggestions for Instruction

- **Sort a set of triangles according to the measures of the interior angles.**

Materials:

- scissors
- protractor
- BLM 6.SS.4.3: Cards of Triangles #2
- BLM 6.SS.4.4: Sorting of Triangles According to the Measure of the Interior Angles

Organization: Whole class/pairs

Procedure:

1. Distribute to each pair a copy of BLM 6.SS.4.3, and to each student a copy of BLM 6.SS.4.4.
2. Have students cut the cards of triangles in BLM 6.SS.4.3 so they get eight cards (that is, four cards per student).
3. Tell students the following:
   a) Measure all three interior angles of each triangle.
   b) On your copy of BLM 6.SS.4.4, mark down
      - the name of the triangle
      - the measure of each interior angle
3. Ask students to use the measure of the interior angles to sort their triangles into groups. Have them explain their sorting rules on the table provided.
4. Discuss the various sorting rules as a class. Through the discussion, encourage students to sort the triangles by
   a) the number of angles that are equal (3 – equilateral, 2 – isosceles, none – scalene)
   b) the type of angles it contains (one obtuse – obtuse, one right – right, all acute – acute)
5. Encourage correct use of mathematical terminology
6. Ask students to show their knowledge of different types of triangles through their choice of
   a) a journal entry
   b) a flip book
   c) an interview
   d) a poem
   e) etc.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Measure interior angles of a triangle.
- ❒ Compare triangles according to the measure of the interior angles.
- ❒ Sort triangles according to the measure of the interior angles.

Suggestions for Instruction

- **Identify the characteristics of a set of triangles according to their sides or their interior angles.**

Materials:

- ruler
- protractor
- BLM 6.SS.4.5: Triangle Identification

Organization: Whole class/individual

Procedure:

1. Distribute to each student a copy of BLM 6.SS.4.5.
2. Tell students to analyze the set of triangles.
3. Say to the students the following:
   a) You need to measure the sides or interior angles of the triangles.
   b) Record their measurements.
   c) Identify the characteristics of the set of triangles based on your measurements.
4. Have a class discussion.
5. Tell students to write their observations regarding the identification of the set of triangles in their journals.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Measure the sides of triangles.
- ❒ Measure the interior angles of triangles.
- ❒ Compare triangles according to the length of their sides.
- ❒ Compare triangles according to their interior angles.
- ❒ Identify the characteristics of a set of triangles according to their sides or their interior angles.

Suggestions for Instruction

- **Sort a set of triangles according to the length of the sides.**
- **Sort a set of triangles according to the measures of the interior angles.**
- **Identify the characteristics of a set of triangles according to their sides or their interior angles.**

Materials:

- paper
- pencil
- ruler
- protractor
- grid paper

Organization: Small groups

Procedure:

1. Seat students in small groups.
2. Hand out a grid paper to each student.
3. Tell students the following:
   a) Each group needs to make a set of triangles (that is, a group of triangles with similar characteristics).
   b) Discuss with group members what kind of triangles each person will make.
   c) Discuss with group members how many triangles each person will make.
   d) Make your triangles.

4. When your set of triangles is complete, change sets with another group.
5. Identify the characteristics of the other group's set of triangles according to their sides or their interior angles.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Work well as a member of a group.
- ❒ Construct a variety of triangles.
- ❒ Identify the characteristics of the other group's set of triangles according to their sides or their interior angles.

Suggestions for Instruction

- **Sort a set of triangles, and explain the sorting rule.**

Materials:

- protractor
- ruler
- BLM 6.SS.4.6: Triangle Page

Organization: Whole class/small groups

Procedure:

1. Distribute to each student a copy of BLM 6.SS.4.6.
2. Tell students to do the following:
   a) Analyze the triangles carefully.
   b) Sort the triangles and record them in your notebooks.
   c) Explain your sorting rule in your notebooks.
   d) Discuss the results with your group members.
3. Have a few students orally present their work.
4. Use the presentations as the basis for a class discussion on the sorting of triangles.
5. Tell students to write their observations on sorting triangles in their journals (e.g., Did everyone use the same sorting rule? Why? What are the sorting methods some students used?).

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Measure the sides of triangles.
- ❒ Measure the interior angles of triangles.
- ❒ Sort triangles according to their sides or interior angles.
- ❒ Explain the sorting of triangles.

Suggestions for Instruction

- **Draw a triangle (e.g., scalene).**
- **Replicate a triangle in a different orientation and show that the two are congruent.**

Materials:

- protractor
- ruler
- a blank sheet of paper
- pencil

Organization: Individual

Procedure:

1. Have a class discussion on triangles (e.g., Can students name some triangles and describe them according to their sides or interior angles?).
2. Distribute to each student a blank sheet of paper (computer paper is fine).
3. Tell students to do the following:
   a) Draw a triangle using a ruler and a protractor.
   b) Replicate the triangle in a different orientation.
   c) Show that the two triangles are congruent.
4. Ask a few students to give an oral presentation to the class.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Draw a triangle using a ruler and protractor.
- ❒ Measure sides of a triangle accurately.
- ❒ Measure inside angles of a triangle accurately.
- ❒ Replicate triangles using a ruler and protractor.

Suggestions for Instruction

- **Sort a set of 2-D shapes into polygons and non-polygons, and explain the sorting rule.**

Materials:

- BLM 6.SS.5.1: Polygons or Non-polygons?

Organization: Whole class/pairs

Procedure:

1. Distribute to each pair a copy of BLM 6.SS.5.1.
2. On the board, create two columns. Write the word "polygons" in one column and the word "non-polygons" in the other column.
3. Tell students to do the following:
   a) Analyze the 2-D shapes in BLM 6.SS.5.1.
   b) Discuss the 2-D shapes with your partners (e.g., Which column does each 2-D shape belong to? Why?).
   c) Copy the chart off the board into your notebooks.
   d) Place each 2-D shape in the appropriate column.
4. Ask a few students to help you complete the chart on the board.
5. Have a class discussion on the results of the sorting of the 2-D shapes.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Recognize polygons.
- ❒ Recognize non-polygons
- ❒ Explain what makes a polygon.
- ❒ Explain what makes a non-polygon.

Suggestions for Instruction

- **Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by superimposing.**

Materials:

- BLM 6.SS.5.2: Equilateral Triangle

Organization: Whole class/small groups

Procedure:

1. Distribute to each student a copy of BLM 6.SS.5.2.
2. Discuss with the class the characteristics of an equilateral triangle.
3. Tell students to do the following:
   a) Fold your equilateral triangle, matching up two sides and two angles.
   b) Rotate the paper, and match up two other sides and angles until all the sides and angles are checked for congruence.
   c) Discuss the congruence by folding and superimposing with your group members.
4. Have a student give a class presentation on what he or she learned about congruence by folding and superimposing.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Superimpose two sides and two interior angles of a triangle by folding.
- ❒ Recognize that all three sides of an equilateral triangle are congruent.
- ❒ Recognize that all three interior angles of an equilateral triangle are congruent.

Suggestions for Instruction

- **Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by measuring.**

Materials:

- BLM 6.SS.5.2: Equilateral Triangle
- ruler
- protractor
- pencil

Organization: Whole class/pairs

Procedure:

1. Distribute to each pair a copy BLM 6.SS.5.2.
2. On the board, create two columns. Write the word "Sides of an Equilateral Triangle" in one column and the word "Angles of an Equilateral Triangle" in the other column.
3. Tell students to do the following:
   a) Mark each side of the triangle with a lower-case letter.
   b) Mark each interior angle of the triangle with an upper-case letter.
   c) Measure each side of the triangle with your ruler.
   d) Record the length of each side of the triangle.
   e) Measure each interior angle of the triangle with your protractor.
   f) Record the measure of each interior angle of the triangle.
4. Ask a few students to help record the results on the board.
5. Have a class discussion on demonstrating congruence in a regular triangle by measuring.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Measure sides of a triangle with a ruler.
- ❒ Recognize congruence of sides by measuring.
- ❒ Measure inside angles of triangle with a protractor.
- ❒ Recognize congruence of interior angles by measuring.
- ❒ Explain congruence by measuring sides and angles.

Suggestions for Instruction

- **Demonstrate that the sides of a regular polygon are of the same length and that the angles of a regular polygon are of the same measure.**

Materials:

- BLM 6.SS.5.3: Regular Pentagon
- ruler
- protractor
- pencil

Organization: Whole class/small group

Procedure:

1. Distribute to each pair a copy BLM 6.SS.5.3.
2. On the board, create two columns. Write the word "Sides of regular pentagon" in one column and the word "Angles of a regular pentagon" in the other column.
3. Tell students to do the following:
   a) Mark each side of the pentagon with a lower-case letter.
   b) Mark each interior angle of the pentagon with an upper-case letter (with the same letter as the side opposite the angle).
   c) Choose a method to demonstrate that the sides are the same length.
   d) Choose a method to demonstrate that the interior angles are of the same measure.
4. Have a student demonstrate to the class that the sides are the same length.
5. Have a student demonstrate to the class that the interior angles are of the same measure.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Demonstrate that the sides of a regular polygon are the same length.
- ❒ Demonstrate that the interior angles of a regular polygon are of the same measure.

Suggestions for Instruction

- **Sort a set of polygons as regular or irregular, and justify the sorting.**

Materials:

- BLM 6.SS.5.4: Polygons: Regular and Irregular
- BLM 6.SS.5.5: Am I a Regular Polygon?

Organization: Individual/whole class

Procedure:

1. Distribute to each student a copy of BLM 6.SS.5.4 and BLM 6.SS.5.5.
2. Tell students to do the following:
   a) Analyze the polygons on BLM 6.SS.5.4.
   b) Complete the chart in BLM 6.SS.5.5 by classifying each polygon and providing a justification.
3. Place on the overhead projector a transparency copy of the chart in BLM 6.SS.5.5.
4. Ask a few students to help you complete the chart.
5. Have a class discussion on the results of sorting the polygons.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Recognize regular polygons.
- ❒ Recognize irregular polygons
- ❒ Explain what makes a polygon regular.
- ❒ Explain what makes a polygon irregular.

- **Identify and describe regular and irregular polygons in the environment.**

Materials:

- ruler
- protractor
- pencil and paper

Organization: Whole class/small group

Procedure:

1. Discuss with the class the characteristics of regular and irregular polygons.
2. Tell students to do the following:
   a) Find polygons in the classroom, hallway, gym floor, or on the playground (e.g., square tiles in the hallway, large square drawn on the gym floor, large, square cement sidewalk tiles).
   b) Identify the polygons as regular or irregular.
   c) Discuss with your group members the characteristics of each polygon.
3. Have a class discussion regarding the polygons students found in the environment.
4. Tell students to do the following:
   a) Describe in your journals the polygons you found
   b) Describe what makes each example a regular or irregular polygon.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Recognize regular polygons in the environment.
- ❒ Recognize irregular polygons in the environment.
- ❒ Explain what makes a polygon regular.
- ❒ Explain what makes a polygon irregular.

PUTTING THE PIECES TOGETHER

What's There?

Purpose: The purpose of this activity is to assess students' knowledge in measuring angles and distance, calculating perimeter, distinguishing between regular and irregular polygons, and justifying choices.

The mathematical processes demonstrated by this task are communication, connections, problem solving, reasoning, and visualization.

In order for students to be able to perform this task, they need to have some prior knowledge related to the task.

Prior Knowledge: Students should be able to do the following:

- Use perimeter or area or both (whole numbers) to design and construct different rectangles and draw conclusions.
- Demonstrate an understanding of measuring length.
- Identify and sort quadrilaterals.
- Describe orally and in writing the rule for pattern.
- Demonstrate an understanding of area of regular and irregular 2-D shapes.
- Sort regular and irregular polygons according to the number of sides.
- Demonstrate an understanding of perimeter of regular and irregular shapes.
- Add and subtract 1-, 2-, and 3-digit numerals with answers to 1000.

Related Knowledge: Students should be able to do the following:

- Demonstrate an understanding of angles.
- Perform a combination of transformations on a single 2-D shape, and draw and describe the image.
- Demonstrate and explain the meaning of preservation of equality, concretely, pictorially, and symbolically.
- Represent generalizations arising from number relationships.

Curricular Links: This task links to other curriculum such as social studies and English language arts.

Materials/Resources:

- BLM 6.SS.5.4: Polygons: Regular and Irregular
- map of Manitoba
- scissors
- ruler
- protractor

Note: During this task, students will use angles and polygons to get familiar with the map of Manitoba. (You may use a different map to fit your social studies lesson.)

Organization:

1. Prior to the students entering the room, place the following on each board:
   a) A map of Manitoba
   b) Images of two angles and two regular polygons (different ones on each board)
   c) A copy of BLM 6.SS.1.2: Reference Angles
2. Have students work in small groups.

Inquiry:

Scenario:

Each group of students will be responsible for naming places on a map that lie:

a) a certain distance and direction from a given point
b) inside or outside of the perimeter of a regular polygon

Procedure:

Before the students enter the room,

a) arrange the desks so they are suitable for working in small groups
b) provide the following for each group:
   - Map
   - Transparent copy of BLM 6.SS.5.4
c) provide each student with one of the following:
   - A pair of scissors
   - Ruler
   - Protractor
   - Looseleaf paper
   - A copy of the directions
   - A copy of BLM 5-8.9: Centimetre Grid Paper
   - A copy of BLM 5-8.4: How I Worked in My Group

Student Directions:

1. Look at the transparency of BLM 6.SS.5.4.
2. Each student needs to choose a polygon. Make sure that each group member chooses a different polygon.

3. Tell students the following:

   On your looseleaf,

   a) identify your polygon as regular or irregular

   b) specify the polygon according to the number of sides and angles, such as triangle, quadrilateral, pentagon, and hexagon

   c) justify your classification by describing the characteristics of your polygon

4. Carefully cut out your polygon, making sure that all sides of your polygon stay intact and none of the other polygons on the transparent BLM copy get damaged. Your group members will need the other polygons.
5. On your centimetre grid paper,

   a) replicate your polygon in two different orientations **and**

   b) inside each polygon, write its name, such as polygon #1 and polygon #2

6. On your looseleaf, describe why polygon #1 and polygon #2 are congruent to each other and to the transparent polygon.
7. Calculate the perimeter of your polygon and record your calculations on your looseleaf.
8. Place your transparent polygon on the map in front of you.
9. On your looseleaf, record the specific places or landmarks (city, river, lake, road) that lie inside the perimeter of your polygon.
10. Choose a city or town that lies inside your polygon (e.g., Winnipeg).
11. Choose a city or town that lies on the left side your polygon (e.g., Brandon).
12. Choose a city or town that lies above your polygon (e.g., Churchill).
13. Look at the reference angles placed on the board.
14. Based on the reference angles, identify the angle you would get if you imagined that a line going from (for example) Brandon to Winnipeg would be one side of an angle and Winnipeg to Churchill would be the other side of the angle.
15. On your looseleaf, record your observations regarding the angle.
16. On your centimetre grid paper, replicate your angle in two different orientations and write the name inside each angle, such as "angle A" and "angle B."
17. Discuss your observations with your group members.

Literature Link:

After students complete this activity, read to the class the book entitled *Hamster Champs* by Stuart J. Murphy and illustrated by Pedro Martin. This is a well written and neatly illustrated book about three hamsters using a protractor and a straight edge to perform stunts for a cat that has nothing to do except to chase them.

Assessment:

Use the following observation checklist to assess student learning.

The student can do the following:	Yes	No	Comment
Identify a polygon as regular or irregular.			
Identify the characteristics of a given polygon.			
Identify a polygon according to the number of its sides and angles (e.g., triangle, quadrilateral, pentagon, hexagon).			
Replicate a polygon in two different orientations.			
Justify that two replicated polygons are congruent to each other and the original polygon.			
Identify an angle based on the reference angles.			
Replicate an angle in two different orientations.			
Communicate observations orally or in writing.			

Extension:

Taking it further

Have students describe in detail how they would design a map of their classroom. Students should be reminded to use correct mathematical terminology.

Grade 6: Shape and Space (Transformations) (6.SS.6, 6.SS.7, 6.SS.8, 6.SS.9)

Enduring Understanding(s):

The position of shapes can be changed by translating, rotating, or reflecting them.

General Learning Outcome(s):

Describe and analyze position and motion of objects and shapes.

Specific Learning Outcome(s):	Achievement Indicators:
6.SS.6 Perform a combination of transformations (translations, rotations, or reflections) on a single 2-D shape, and draw and describe the image. [C, CN, PS, T, V]	➛ Demonstrate that a 2-D shape and its transformation image are congruent. ➛ Model a set of successive translations, successive rotations, or successive reflections of a 2-D shape. ➛ Model a combination of two different types of transformations of a 2-D shape. ➛ Draw and describe a 2-D shape and its image, given a combination of transformations. ➛ Describe the transformations performed on a 2-D shape to produce a given image. ➛ Model a set of successive transformations (translation, rotation, or reflection) of a 2-D shape. ➛ Perform and record one or more transformations of a 2-D shape that will result in a given image.
6.SS.7 Perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations. [C, CN, T, V]	➛ Analyze a design created by transforming one or more 2-D shapes, and identify the original shape and the transformations used to create the design. ➛ Create a design using one or more 2-D shapes and describe the transformations used.

(continued)

Specific Learning Outcome(s):	Achievement Indicators:
6.SS.8 Identify and plot points in the first quadrant of a Cartesian plane using whole-number ordered pairs. [C, CN, V]	➛ Label the axes of the first quadrant of a Cartesian plane and identify the origin. ➛ Plot a point in the first quadrant of a Cartesian plane given its ordered pair. ➛ Match points in the first quadrant of a Cartesian plane with their corresponding ordered pair. ➛ Plot points in the first quadrant of a Cartesian plane with intervals of 1, 2, 5, or 10 on its axes, given whole-number ordered pairs. ➛ Draw shapes or designs, given ordered pairs in the first quadrant of a Cartesian plane. ➛ Determine the distance between points along horizontal and vertical lines in the first quadrant of a Cartesian plane. ➛ Draw shapes or designs in the first quadrant of a Cartesian plane and identify the points used to produce them.
6.SS.9 Perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole-number vertices). [C, CN, PS, T, V]	➛ Identify the coordinates of the vertices of a 2-D shape (limited to the first quadrant of a Cartesian plane). ➛ Perform a transformation on a given 2-D shape and identify the coordinates of the vertices of the image (limited to the first quadrant). ➛ Describe the positional change of the vertices of a 2-D shape to the corresponding vertices of its image as a result of a transformation (limited to first quadrant).

Prior Knowledge

Students may have had experience with the following:

- Performing a single transformation of a 2-D shape and drawing and describing the image
- Identifying a single transformation of 2-D shapes
- Identifying triangles, quadrilaterals, pentagons, hexagons, and octagons according to the number of sides
- Identifying triangles, squares, rectangles, and circles

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of integers, concretely, pictorially, and symbolically
- Demonstrating and explaining the meaning of preservation of equality, concretely, pictorially, and symbolically
- Constructing and comparing triangles in different orientations
- Creating, labelling, and interpreting line graphs to draw conclusions
- Graphing collected data and analyzing the graph to solve problems

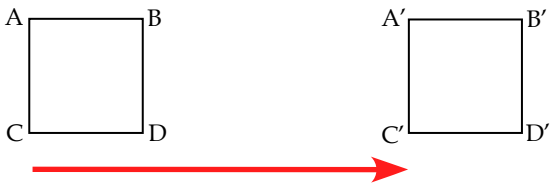
Background Information

Transformations play an important role in the mathematics curriculum. In the Middle Years, the study of transformation can support students' work in patterning, algebra, problem solving, geometry, and statistics. In high school and beyond, the study of transformations helps students recognize the connections between algebra and geometry, and enhances their understanding of other topics such as matrices, scaling, and complex numbers.

A **transformation** can be thought of as a change in the position, size, or shape of a figure.

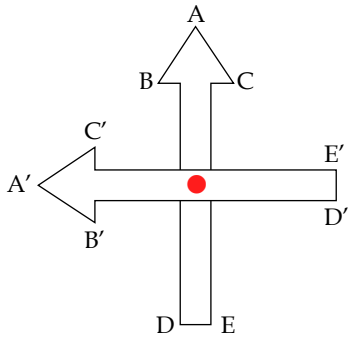
In the learning activities that follow, students are asked to perform three transformations that change the position of a figure. Informally, these transformations are referred to as slides, flips, and turns. Formally, they are known as translations, reflections, and rotations.

A **translation** "slides" a figure a fixed distance in a given direction. The figure and its translation are congruent (same size and shape) and face in the same direction. In the diagram shown below, square ABCD has been translated to a new position represented by square A'B'C'D'.

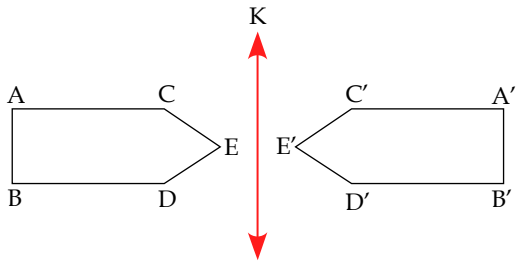


Note that Square A'B'C'D', which is called the image of Square ABCD, is congruent to Square ABCD and faces in the same direction. The arrow indicates the distance and the direction of the translation.

A rotation "turns" a figure any number of degrees around a fixed point called the centre (or point) of rotation. The centre of rotation may be any point within or outside the figure. The figure and its image (the result of the transformation) are congruent, but they may have different orientations (e.g., in the diagram below, the arrow ABCDE has been rotated 90° counter-clockwise about its midpoint). The image arrow A'B'C'D'E' is congruent to Arrow ABCDE, but faces in a different direction.

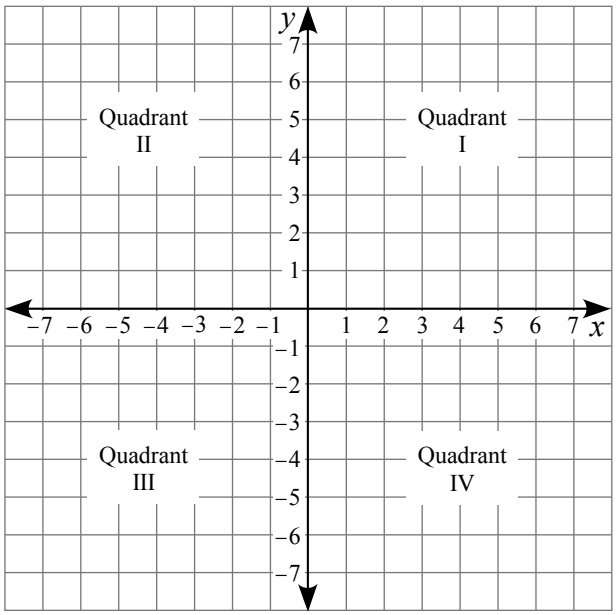


A reflection "flips" the figure over a line, creating a mirror image. The figure and its image are congruent but have different orientations. The line the figure is flipped over is called the line of reflection, and it is the same distance from the figure as its image (e.g., in the diagram below, pentagon ABCDE has been flipped over line K).



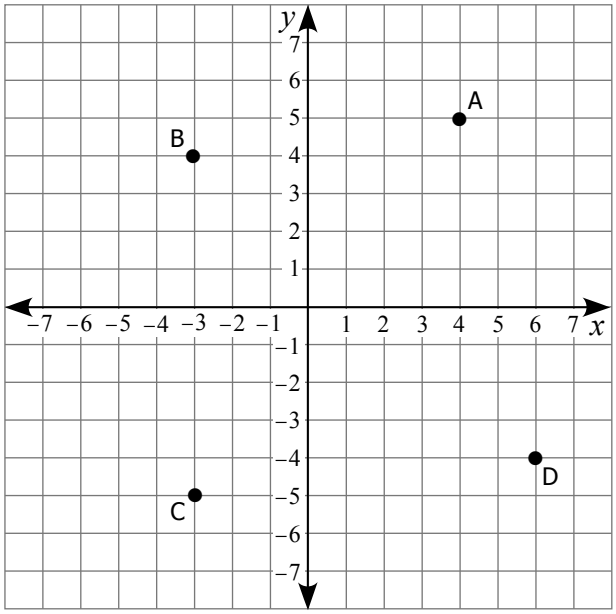
Note that Pentagon A'B'C'D'E' is congruent to Pentagon ABCDE but faces in the opposite direction. Line *k*, the line of reflection, is equidistant from the two pentagons.

In Grade 5, students were introduced to all three above-named transformations using both the formal and informal terminology. In Grade 6, students continue their study of transformations, which gets expanded with the introduction of the Cartesian plane (also called the coordinate plane). The Cartesian plane is formed by a horizontal axis and a vertical axis, often labelled the *x*-axis and *y*-axis respectively. It contains quadrants 1 to 4 (the quadrants are often labelled using Roman numerals I to IV).



Each point on the Cartesian plane is identified by a unique ordered pair (a set of two numbers named in a specific order), represented by (x, y). The first number, x, represents the x-coordinate and the second number, y, represents the y-coordinate. (See points A, B, C, and D below.)

Image of points A, B, C, and D on a Cartesian plane.



On the Cartesian plane above, point A has coordinates (4, 5), point B has coordinates (–3, 4), point C has coordinates (–3, –5), and point D has coordinates (6, –4).

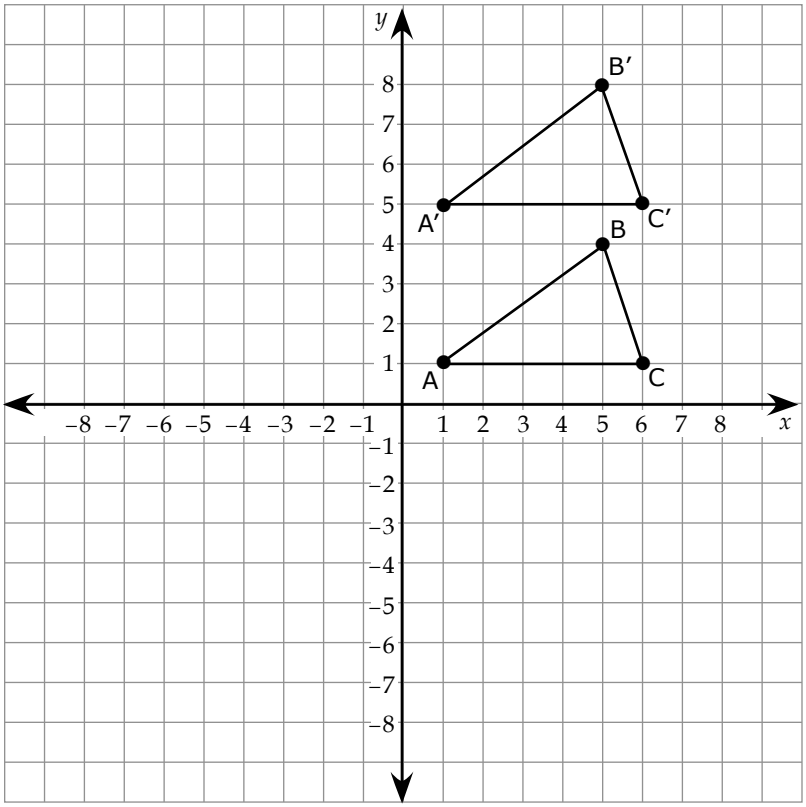
The only point in the above example with both coordinates being positive is point A, which lies in the first quadrant.

In Grade 6, students will explore the first quadrant of the Cartesian plane by

a) identifying and plotting points (such as point A, on a Cartesian plane above)

b) performing and describing single transformations of a 2-D shape (such as shown below)

2-D shape and transformation image on a Cartesian plane.



The triangle shown above has the following vertices: A (1, 1), B (5, 4), and C (6, 1); its image has the following vertices: A′ (1, 5), B′ (5, 8), and C′ (6, 5). This means that the triangle has been translated four units in the vertical upward direction.

Mathematical Language

axes
Cartesian plane
horizontal
image
line of reflection
ordered pairs
origin
point
quadrant
reflection
rotation
transformation
translation
vertical
vertices

Learning Experiences

Assessing Prior Knowledge

Materials:

- carpeted area or floor mats

Organization: Whole class

Procedure:

1. Tell students you want to check what they remember about transformations.
2. Have a class discussion and demonstration about each type of transformation.
3. Have students lie down on a carpet or mat. Ask them to slide a short distance in one direction. Have them repeat the movement several times by asking them to slide up, down, and sideways.

4. After each slide, ask students the following questions:
   a) What changed?
   b) What remained the same?
5. Emphasize that when a slide is made, the direction in which an object is pointing does not change.
6. Ask the students what the mathematical terminology is for this type of transformation.
7. Have students demonstrate flips. At the end of a flip, students should have changed from stomach to back or back to stomach. Discuss the different ways flips can be completed. For example, students may roll to the left or to the right, or over the feet or head. Have students try these different ways.
8. After each flip, ask students the following questions:
   a) What changed?
   b) What remained the same?

   Emphasize that when an object is flipped, its orientation changes.
9. Ask students how this is different from looking at their reflection in the mirror. Emphasize that a true reflection of oneself would have exactly the same image, just in a different orientation.
10. Ask the students what the mathematical terminology is for this type of transformation.
11. Have students demonstrate turns. To perform a turn, students must keep either their feet or their heads (or belly button) at the same location for the duration of the turn. If the feet are the point (centre) of rotation, then the arms and head are used to move the body. If the head is the point (centre) of rotation, the feet are used to make the move.
12. Have students turn all the way around or partway around. Have them turn in either a clockwise or counter-clockwise direction.
13. After each turn ask students the following questions:
   a) What changed?
   b) What remained the same?
14. Discuss the fact that, after a turn, the direction in which the head points is different, except when a complete turn is made.
15. Ask the students what the mathematical terminology is for this type of transformation.
16. Inform students that in the next few lessons they will be learning more about translations, reflections, and rotations.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Perform a slide.
- ❒ Know the mathematical terminology of a slide.
- ❒ Perform a flip.
- ❒ Know the mathematical terminology of a flip.
- ❒ Perform a turn.
- ❒ Know the mathematical terminology of a turn.

Suggestions for Instruction

- **Demonstrate that a 2-D shape and its transformation image are congruent.**

Materials:

- ruler
- protractor
- BLM 6.SS.6.1: Shape and Image #1

Organization: Whole class/small groups

Procedure:

1. Have a class discussion on transformations (translations, rotations, and reflections).
2. Place on the overhead projector a transparency copy of BLM 6.SS.6.1.
3. Ask students to identify which transformation is presented on the overhead projector, and why they believe that.
4. Distribute to each student a copy of BLM 6.SS.6.1.
5. Tell students to
   a) analyze and discuss with their partners the shape and its image
   b) demonstrate that the 2-D shape and its transformation image are congruent
6. Have a student volunteer come up to the overhead projector and using the transparency copy of BLM 6.SS.6.1; demonstrate that the 2-D shape and its transformation image are congruent.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify the transformation.
- ❒ Demonstrate that the 2-D shape and its transformation image are congruent.

Suggestions for Instruction

- **Model a set of successive translations, successive rotations, or successive reflections of a 2-D shape.**

Materials:

- scissors
- grid paper
- BLM 6.SS.6.1: Shape and Image #1

Organization: Groups of three

Procedure:

1. Distribute to each student a sheet of grid paper, a pair of scissors, and a copy of BLM 6.SS.6.1.
2. Have students cut out the 2-D shape.
3. To model a set of successive translations, have student A
   a) draw a set of successive translations in the horizontal or vertical direction
4. Have the rest of the students in the group figure out the translations that took place.
5. After each slide, student A will ask other students in the group the following questions:
   a) In which direction did the 2-D shape move?
   b) What changed?
   c) What remained the same?
6. To model a set of successive rotations, have student B do a few rotations in the clockwise direction and a few rotations in the counter-clockwise direction, always making sure that the corner of the 2-D shape that touches the dot stays touching the dot.

7. After each rotation, student B will ask other students in the group the following questions:
   a) In which direction did the 2-D shape move?
   b) What changed?
   c) What remained the same?
8. To model a set of successive reflections, have student C do a few reflections downward or upward and a few to the left or right, always making sure that the side of the 2-D shape touching the dot remains touching the dot and on the same line as it was before it got flipped.
9. After each reflection, student C will ask other students in the group the following questions:
   a) In which direction did the 2-D shape move?
   b) What changed?
   c) What remained the same?

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Model a successive translation.
- ❒ Model a successive rotation.
- ❒ Model a successive reflection.
- ❒ Describe the transformation performed on a 2-D object.

Suggestions for Instruction

- **Model a combination of two different types of transformations of a 2-D shape.**

Materials:
- scissors
- grid paper
- BLM 6.SS.6.2: Shape and Image #2

Organization: Whole class/small class

Procedure:

1. Distribute to each student a sheet of grid paper, a pair of scissors, and a copy of BLM 6.SS.6.2.
2. To model a combination of two different types of transformations of a 2-D shape, have students do the following:
   a) Draw a vertical line on the grid paper.
   b) Place a dot somewhere in the middle of the line.
   c) Cut out the 2-D shape.
   d) Place the 2-D shape on the left-hand side of the line so that one corner is touching the dot.
   e) Turn the 2-D shape clockwise, making sure that the corner of the 2-D shape that touches the dot stays touching the dot.
   f) Slide the 2-D shape up along the line, making sure they do not change the direction of the 2-D shape.
3. After the two transformations have been completed, ask students the following questions:
   a) What changed?
   b) What remained the same?
4. Have students work in small groups.
5. Let them repeat the activity by doing the following:
   a) Each group member designs an activity using two transformations of his or her own choice.
   b) Let the other members describe the combination of transformations.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Model a combination of two different types of transformations of a 2-D shape.
- ❒ Describe a combination of two different types of transformations of a 2-D shape.

Suggestions for Instruction

- **Draw and describe a 2-D shape and its image, given a combination of transformations.**

Materials:

- grid paper
- ruler
- protractor
- pencil

Organization: Individual

Procedure:

1. Distribute to each student a grid paper
2. Ask students to draw a 2-D shape on the grid paper.
3. Tell students to describe in their journals the 2-D shape that they drew on the grid paper.
4. Tell students to do the following:
   a) Rotate the 2-D shape 90^{o} in the clockwise direction, then make a vertical reflection (flip it over) to the right.
   b) Draw on the grid paper the image that results from the combination of the two transformations.
   c) Describe in their journals the image of the 2-D shape that they drew on the grid paper.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Draw a 2-D shape.
- ❒ Describe a 2-D shape.
- ❒ Draw an image of a 2-D shape given a combination of transformations.
- ❒ Describe an image of a 2-D shape given a combination of transformations.

Suggestions for Instruction

- **Describe the transformations performed on a 2-D shape to produce a given image.**

Materials:

- BLM 6.SS.6.2: Shape and Image #2

Organization: Individual

Procedure:

1. Review the different types of transformations (translations, rotations, and reflections).
2. Distribute to each student a copy of BLM 6.SS.6.2.
3. Tell students to analyze the 2-D shape and its image in order to determine the different types of transformations that were performed to produce the image.
4. Have students describe in their journals the transformations performed on the 2-D shape shown on the BLM to produce the image shown.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Analyze the transformations performed on the 2-D shape to produce a given image.
- ❒ Describe the transformations performed on the 2-D shape to produce a given image.

Suggestions for Instruction

- **Model a set of successive transformations (translation, rotation, or reflection) of a 2-D shape.**

Materials:

- scissors
- grid paper
- BLM 6.SS.6.2: Shape and Image #2

Organization: Whole class

Procedure:

1. Distribute to each student a sheet of grid paper, a pair of scissors, and a copy of BLM 6.SS.6.2.
2. To model a set of successive transformations of a 2-D shape, tell students to do the following:
   a) Cut out the 2-D shape.
   b) Place the 2-D shape on the grid paper near the top so that one side of the 2-D shape lies along a perpendicular line.
   c) Move the 2-D shape down along the line, stopping after every third square until you reach the bottom of the grid paper.
3. Ask students the following questions:
   a) What was the set of successive transformations of a 2-D shape you performed?
   b) What other sets of successive transformations of a 2-D shape can you perform?
4. Ask a student volunteer to come to the front of the class and demonstrate another set of successive transformations of a 2-D shape.
5. Repeat the activity, making sure that students use all three types of transformations (translation, rotation, and reflection).

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

❒ Model a set of successive transformations (translation, rotation, and reflection) of the 2-D shape.

- **Model a set of successive transformations (translation, rotation, or reflection) of a 2-D shape.**

Materials:

- grid paper
- small plastic triangle

Organization: Four groups/whole class

Procedure:

1. Divide the class into four groups.
2. Each group needs to create an activity in order to model a set of successive transformations (translation, rotation, or reflection) of a 2-D shape.
3. Each group will choose a presenter and three recorders.
4. The presenter will use the grid paper and small plastic triangle to demonstrate to the other three groups the original (starting) position of the triangle and the end result of each successive transformation.
5. Each recorder will be responsible to record the correct questions for one designated group (e.g., recorder #1 will record the correct questions asked by members of group B, recorder #2 will record the correct questions asked by members of group C, etc.).
6. To play the game "Guess My Transformation":
   a) The presenter presents (e.g., from group A).
   b) One member from the other three groups will ask a question (e.g., one student from group B asks one question). Members of each group can only ask one question at a time (followed by one student from group D, then back to group B…).
   c) Recorder #1 will record the question for group B only if it is correct.
   d) One member from another group will ask a question (e.g., one student from group C).
   e) Recorder #2 will record the question for group C only if it is correct.
   f) One member from the third group will ask a question (e.g., one student from group D).
   g) Recorder #3 will record the question for group D only if it is correct.
   h) Repeat this form of questioning and recording until all the conditions are satisfied for creating the resultant image. The game is over when all the necessary conditions are filled.
   i) Each question needs to be simple and can request one piece of information only (see sample questions below).
   j) The group with the greatest number of correct questions wins.

7. The following are sample questions:
   a) Is it a translation?
   b) Is it to the right?
   c) Is it three units?
   d) Is it a rotation?
   e) Is it clockwise?
   f) Is it 45°?
   g) Is it a reflection?
   h) Is it vertical?
8. Repeat the game with another group presenting and recording.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Demonstrate understanding of a set of successive transformations (translation, rotation, and reflection) of the 2-D shape.
- ❒ Model a set of successive transformations (translation, rotation, and reflection) of the 2-D shape.
- ❒ Recognize a set of successive transformations (translation, rotation, and reflection) of the 2-D shape.

Suggestions for Instruction

- **Perform and record one or more transformations of a 2-D shape that will result in a given image.**

Materials:

- scissors
- BLM 6.SS.6.3: Envelope Shape

Organization: Individual

Procedure:

1. Distribute to each student a sheet of grid paper, a pair of scissors, and a copy of BLM 6.SS.6.3.
2. Tell students to do the following:
   a) Cut out the shape so that the image and the rest of the grid paper stay as they are.
   b) Place the 2-D shape back on the grid paper in its original position.
   c) Move the 2-D shape on the grid paper, using necessary transformations to obtain the given image.
3. Discuss with the class what transformations of a 2-D shape they performed in order to obtain the given image.
4. Tell students to record in their notebooks the transformations they used to obtain the given image.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Perform the necessary transformations of a 2-D shape to obtain the given image.
- ❒ Record the transformations of a 2-D shape they used to obtain the given image.

Suggestions for Instruction

- **Analyze a design created by transforming one or more 2-D shapes, and identify the original shape and the transformations used to create the design**
- **Create a design using one or more 2-D shapes and describe the transformations used.**

Materials:

- grid paper
- pencil
- ruler
- transparency of BLM 6.SS.7.1: Design

Organization: Whole class/individual

Procedure:

1. Place on the overhead projector a transparency of BLM 6.SS.7.1.
2. Discuss the design.
   a) What do they see?
   b) What type of transformations created the design?
   c) What is the original position of the 2-D shape?
3. Tell students to create and draw on the grid paper their own design using one or more 2-D shapes.
4. Tell students to describe in their notebooks the transformations they used to create the design.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify the original 2-D shape used to create a design.
- ❒ Identify the transformations used to create a design.
- ❒ Create a design using one or more 2-D shapes.
- ❒ Describe the transformations they used to create a design.

Suggestions for Instruction

- **Label the axes of the first quadrant of a Cartesian plane and identify the origin.**
- **Plot a point in the first quadrant of a Cartesian plane given its ordered pair.**

Materials:

- grid paper
- pencil
- ruler
- overhead projector

Organization: Whole class/individual

Procedure:

1. Show a transparency copy of a grid paper on the overhead projector.
2. Have a class discussion regarding a plane and points on a plane.
3. Draw a horizontal line through the middle of your grid paper.
4. Discuss what happens to the plane when you draw a horizontal line through the middle.
5. Draw a vertical line on the grid paper through the middle of the horizontal line.
6. Discuss what happens to the plane when you draw a vertical line through the middle of the horizontal line.
7. Mark the top right quadrant with a Roman numeral, such as *I*.
8. Tell students that this year they are going to be exploring the first quadrant.
9. Label the horizontal line x and the vertical line y.
10. Discuss horizontal and vertical axes.
11. Label the origin.
12. Tell students to do the following:
    a) Draw on their grid paper the two axes and label them.
    b) Label the origin.
    c) Mark quadrant I.
13. Discuss the significance of the origin.
14. Plot a point such as (2, 1), and explain that the first numeral always shows the x-value (horizontal direction) and the second numeral always shows the y-value (vertical direction).
15. Plot another point, explaining the x- and y-coordinates.
16. Ask a student volunteer to come up to the overhead projector and plot a point in the first quadrant. For example: (3, 5).
17. Have several student volunteers come up to the overhead projector and plot a point in the first quadrant (a different point by each student).
18. Provide an ordered pair for several points in Quadrant I, and tell students to plot them.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Label the axes of the first quadrant of a Cartesian plane.
- ❒ Identify the origin.
- ❒ Plot a point in the first quadrant of a Cartesian plane given its ordered pair.

Suggestions for Instruction

- **Label the axes of the first quadrant of a Cartesian plane and identify the origin.**
- **Plot a point in the first quadrant of a Cartesian plane given its ordered pair.**
- **Match points in the first quadrant of a Cartesian plane with their corresponding ordered pair.**

Materials:

- pencil
- BLM 6.SS.8.1: Matching Game

Organization: Whole class/individual

Procedure:

1. Before you play the Matching Game, discuss the Cartesian plane.
   a) Have a student volunteer draw a Cartesian plane on the chalkboard, and label the axes and the origin.
   b) Provide an ordered pair, and have one student plot the required point in the first quadrant of the Cartesian plane.
   c) Provide an ordered pair, and have another student plot the designated point in the first quadrant of the Cartesian plane.
   d) Have students place a few points on the chalkboard, and then tell students that now they can match dots by playing the Matching Game.
2. Distribute to each student a copy of BLM 6.SS.8.1.
3. Tell students to do the following:
   a) Match each point in the first quadrant of a Cartesian plane with its corresponding ordered pair.
   b) Write next to each dot the corresponding letter.
4. Discuss the results of the Matching Game after each student has completed the game.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Label the axes of the first quadrant of a Cartesian plane.
- ❒ Identify the origin.
- ❒ Plot a point in the first quadrant of a Cartesian plane, given its ordered pair.
- ❒ Match points in the first quadrant of a Cartesian plane with their corresponding ordered pair.

Suggestions for Instruction

- **Plot points in the first quadrant of a Cartesian plane with intervals of 1, 2, 5, or 10 on its axes, given whole-number ordered pairs.**

Materials:

- BLM 6.SS.8.2: Cartesian Plane #1
- BLM 6.SS.8.3: Cartesian Plane #2

Organization: Whole class/individual

Procedure:

1. Discuss the Cartesian plane.
2. Place on the overhead projector a transparency of BLM 6.SS.8.2.
3. Choose several whole-number ordered pairs, and plot and label the points. Make sure you choose numbers that are not marked on the interval. For example: (5, 3).
4. Have some students come up and plot a point in the first quadrant of the Cartesian plane after you provide its ordered pair.
5. Distribute to each student a copy of BLM 6.SS.8.2.
6. Write several ordered pairs on the board, and have students plot the points.
7. Repeat the activity with BLM 6.SS.8.3.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Plot points in the first quadrant of a Cartesian plane with intervals of 2 on its axes, given whole-number ordered pairs.
- ❒ Plot points in the first quadrant of a Cartesian plane with intervals of 5 on its axes, given whole-number ordered pairs.

Suggestions for Instruction

- **Plot points in the first quadrant of a Cartesian plane with intervals of 1, 2, 5, or 10 on its axes, given whole-number ordered pairs.**
- **Draw shapes or designs, given ordered pairs in the first quadrant of a Cartesian plane.**

Materials:

- pencil crayons
- ruler
- BLM 6.SS.8.2: Cartesian Plane #1

Organization: Pairs

Procedure:

1. Discuss using ordered pairs to draw shapes or designs in the first quadrant of a Cartesian plane.
2. Distribute to each student a copy of BLM 6.SS.8.2.
3. Tell students that they will be playing with a partner a game called "Guess My Shape."
4. Explain the rules of the game:
   a) Student A will plot points to make a 2-D shape or designs in the first quadrant of a Cartesian plane.
   b) Student A will call out the coordinates of the points he or she used in producing the shape of design.
   c) Student B plots the points using the ordered pairs that student A tells him or her.
   d) When student A has all the points called out, he or she will say "Guess my shape!"

e) Student B will guess the shape.

f) Student A and student B will compare their shapes to make sure they are the same.

5. Repeat the game with student B making the shape and student A guessing.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Plot points in the first quadrant of a Cartesian plane with intervals of two on its axes, given whole-number ordered pairs.
- ❒ Draw shapes or designs, given ordered pairs in the first quadrant of a Cartesian plane.

Suggestions for Instruction

- **Plot points in the first quadrant of a Cartesian plane with intervals of 1, 2, 5, or 10 on its axes, given whole-number ordered pairs.**
- **Draw shapes or designs, given ordered pairs in the first quadrant of a Cartesian plane.**
- **Determine the distance between points along horizontal and vertical lines in the first quadrant of a Cartesian plane.**

Materials:

- pencil crayons
- ruler
- BLM 6.SS.8.2: Cartesian Plane #1

Organization: Small groups/individual/class

Procedure:

1. Distribute to each student a copy of BLM 6.SS.8.2.
2. Tell students that they will be drawing four lines.
   a) The first line will be blue. It will go through points A, B, and C, whose ordered pairs are: A (3, 15), B (8, 15), and C (15, 15)
   b) The second line will be orange. It will go through points D, E, and F, whose ordered pairs are: D (3, 7), E (4, 7), and F (13, 7)
   c) The third line will be black. It will go through points G, H, and I, whose ordered pairs are: G (4, 4), H (8, 4), and I (15, 4)

d) The fourth line will be yellow. It will go through points J, K, and L, whose ordered pairs are: J (3, 0), K (8, 0), and L (13, 0)

3. Discuss the four lines. (What is similar? What is different?)
4. Tell students to determine the distance between points:
   a) A and B
   b) B and C
   c) A and C
5. Students discuss with their group members.
6. Discuss with the whole class the three distances on the first horizontal line.
7. Repeat the process for the three distances for the other three horizontal lines.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Plot points in the first quadrant of a Cartesian plane with intervals of 2 on its axes, given whole-number ordered pairs.
- ❒ Draw horizontal lines given ordered pairs in the first quadrant of a Cartesian plane.
- ❒ Determine the distance between points along horizontal lines in the first quadrant of a Cartesian plane.

Suggestions for Instruction

- **Plot points in the first quadrant of a Cartesian plane with intervals of 1, 2, 5, or 10 on its axes, given whole-number ordered pairs.**
- **Draw shapes or designs, given ordered pairs in the first quadrant of a Cartesian plane.**
- **Determine the distance between points along horizontal and vertical lines in the first quadrant of a Cartesian plane.**

Materials:

- pencil crayons
- ruler
- BLM 6.SS.8.3: Cartesian Plane #2

Organization: Small groups

Procedure:

1. Distribute to each student a copy of BLM 6.SS.8.3.
2. Tell students that they will be playing a game called "How many units did they move?" using dots on vertical lines.
3. Explain the rules of the game:
   a) Student #1 will call out the ordered pairs for three points not on the same vertical line. For example: (2, 3); (5, 1); and (6, 7) for points A, B, and C.
   b) The other students will plot the points and draw the shape.
   c) Student #1 will do the following:
      - Draw a vertical line through each point and move the three points either up or down the same number of units.
      - Say to the rest of the group that the three points each found a new location.
      - Call out the three new ordered pairs for the three new points. For example: (2, 9); (5, 7); and (6, 13).
   d) The other students will do the following:
      - Plot the new points.
      - Draw the new shape.
   e) Student #1 will ask the following:
      - How many units did point A move, and in which direction?
      - How many units did point B move, and in which direction?
      - How many units did point C move, and in which direction?
   f) The rest of the group members will provide the required reply. (In this example, each point moved 6 units up.)
4. Repeat the game with Student #2, and so on.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Plot points in the first quadrant of a Cartesian plane with intervals of 2 on its axes, given whole-number ordered pairs.
- ❒ Draw vertical lines given ordered pairs in the first quadrant of a Cartesian plane.
- ❒ Determine the distance between points along vertical lines in the first quadrant of a Cartesian plane.

Suggestions for Instruction

- **Draw shapes or designs in the first quadrant of a Cartesian plane and identify the points used to produce them.**

Materials:

- grid paper
- pencil
- ruler
- overhead projector

Organization: Whole class/individual

Procedure:

1. Show a transparency copy of a grid paper on the overhead projector.
2. Have a class discussion regarding points and ordered pairs on a plane.
3. Draw and label the axes of the first quadrant of a Cartesian plane, and identify the origin.
4. Draw a capital **N** shape on grid paper without plotting the dots first. For example: draw line segments from (1, 1) to (1, 7), from (1, 7) to (4, 1), and from (4, 1) to (4, 7)
5. Ask students to identify the points used to draw the **N** shape.
6. Draw a square on the grid paper without plotting the dots first. For example, draw line segments from (6, 2) to (6, 7), from (6, 7) to (11, 7), from (11, 7) to (11, 2) and from (11, 2) to (6, 2).
7. Ask students to identify the points used to draw the square.
8. Distribute to each student a grid paper.
9. Tell students to draw and label the axes of the first quadrant of a Cartesian plane, and identify the origin.
10. Tell students to do the following:
    a) Draw one of each:
        - Triangle
        - Rectangle
        - Pentagon
        - Hexagon

    b) Identify the points used to draw each shape.
11. Circulate to make sure they are: (a) drawing the correct shapes, and (b) identifying the correct points.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Label the axes of the first quadrant of a Cartesian plane.
- ❒ Identify the origin.
- ❒ Draw shapes or designs in the first quadrant of a Cartesian plane.
- ❒ Identify the points used to produce the shapes or designs.

Suggestions for Instruction

- **Identify the coordinates of the vertices of a 2-D shape (limited to the first quadrant of a Cartesian plane).**

Materials:

- pencil
- BLM 6.SS.8.2: Cartesian Plane #1
- BLM 6.SS.9.1: Identification Game

Organization: Whole class/individual

Procedure:

1. Discuss drawing shapes in the first quadrant of a Cartesian plane, and identifying the points used to produce the designs.
2. Place on the overhead projector a transparency of BLM 6.SS.8.2.
3. Draw a triangle in the first quadrant. For example, use the following ordered pairs (0, 4), (6, 0), and (5, 8) as the vertices of the triangle.
4. Ask students to identify the coordinates of the vertices of your triangle.
5. Discuss what they had to do to identify the coordinates of the vertices of your triangle.
6. Distribute to each student a copy of BLM 6.SS.9.1.
7. Tell students to do the following:
   a) Identify the coordinates of the vertices of the 2-D shape.
   b) Write the coordinates of each vertex next to it.
8. Circulate to make sure that students correctly identify each vertex.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify the coordinates of the vertices of a 2-D shape (limited to the first quadrant of a Cartesian plane).

Suggestions for Instruction

- **Perform a transformation on a given 2-D shape and identify the coordinates of the vertices of the image (limited to the first quadrant).**

Materials:

- pencil
- BLM 6.SS.8.2: Cartesian Plane #1

Organization: Whole class/individual

Procedure:

1. Discuss transformations (translations, rotations, and reflections).
2. Place on the overhead projector a transparency copy of BLM 6.SS.8.2.
3. Draw a triangle in the first quadrant. For example, use the following ordered pairs (8, 2), (12, 1), and (11, 5) as the vertices of the triangle.
4. Tell students to perform the following transformation:
   a) Horizontal translation 5 units to the left.
5. Ask students to identify the coordinates of the vertices of the image.
6. Repeat the activity with different transformations.
7. Circulate to make sure that students correctly identify each vertex.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Perform a transformation on a given 2-D shape (limited to the first quadrant).
- ❒ Identify the coordinates of the vertices of the image (limited to the first quadrant).

Suggestions for Instruction

- **Describe the positional change of the vertices of a 2-D shape to the corresponding vertices of its image as a result of a transformation (limited to first quadrant).**

Materials:

- pencil
- BLM 6.SS.9.2: Dizzy Pentagon

Organization: Whole class/individual

Procedure:

1. Discuss transformations (translations, rotations, and reflections).
2. Distribute to each student a copy of BLM 6.SS.9.2.
3. Tell students to do the following:
   a) Analyze the Dizzy Pentagon and its image.
   b) Analyze the vertices of the Dizzy Pentagon and its image.
   c) Describe in their journals how the vertices of the Dizzy Pentagon changed to the corresponding vertices of its image.
4. Circulate to make sure that students correctly describe the positional change in each vertex.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

❒ Describe the positional change of the vertices of a 2-D shape to the corresponding vertices of its image as a result of a transformation (limited to first quadrant).

Ten Flags

Introduction: Ten Flags (Based on NCTM article from *Mathematics Teaching in the Middle Schools* 16.2, Sept. 2010, pp. 72-75.)

Purpose: The purpose of this activity is for students to put into practice skills that they acquired inside the mathematics classroom. Students will need to apply their knowledge of the first quadrant of the Cartesian plane. They will also have to rely on some prior knowledge, such as the following:

a) the metric system—in particular, measuring distance

b) angle measures

The processes that are demonstrated by this task are communication, connections, problem solving, reasoning, and visualization. To make the task fun, students will have to collaborate as well as cooperate with each other.

Curricular Links: This task can be linked to the social studies curriculum. The flags can be those of different provinces within Canada, or they can be of different countries of the world.

Materials/Resources:

- 10 flags
- one stand-alone post
- 10 high-stools (with square seats)
- 10 protractors
- 10 boxes

For each student:

- one instruction sheet
- measuring tape
- pencil
- notepad
- assessment sheet
- BLM 5-8.4: How I Worked in My Group

Note: During this task, students will use angles and polygons to get familiar with the map of Manitoba. (You may use a different map to fit your social studies lesson.)

Organization:

- Use the gym for this task, and set it up as a large-scale Cartesian plane.
- Small groups

Inquiry:

Scenario:

Students will be located along two walls: (1) the wall by the entrance of the gym, and (2) the wall that is perpendicular to and to the left of the entrance. The flags will be standing up seemingly scattered but actually strategically placed by the teacher. Teacher will ensure that each flag is clearly visible from all ten marked spots (high stools).

Procedure:

Day 1: Let's Prepare!

Students will be divided into small groups. They will be working in the classroom.

Students will do the following:

1. Number the 10 high-stools as 1, 2, 3, 4, 5, A, B, C, D, E.
2. Make an open box (seat cover) to be placed upside-down on each box so that the box covers the seat of the high stool and the sides of the box hang over the sides of the seat, thus preventing itself (the box) from moving.
3. Tape a large copy of a protractor onto each box, making sure that the straight edge of the protractor is even with a straight edge of the box.
4. Practise measuring angles. Use the taped-on protractor and a straight edge to measure various angles in the classroom.

The teacher will do the following:

1. Take as many pieces of equal-sized paper as there are groups.
2. Mark each paper with a different number set, using two numbers of the 10 high stools such as (1, D), (3, B), (4, A), (5, C), (2, E), and so on.
3. Fold each piece of paper so that the number is not displayed.
4. Place all 10 pieces of marked and folded paper into a paper bag.

Day 2: Let's Get Moving!

Everyone will be in the gym.

Before students enter the gym, the teacher strategically places the flags, ensuring that each is clearly visible from all 10 marked spots (high stools).

1. Students will mark a line parallel to and four metres away from the wall along the entrance, as well as a line parallel to and four metres away from the wall that is perpendicular to, and to the left of, the entrance. The two lines will be the two axes of the "life-size" Cartesian plane. Where the two perpendicular lines meet, students will place a stand-alone post, which will mark the origin.

2. Along each line (each axis), starting at the intersection of the two lines (the origin), students will measure and mark two-metre intervals by placing a high stool on each two-metre mark. Along each line, therefore, there will be five high stools: at the two-metre, four-metre, six-metre, eight-metre, and 10-metre marks. To achieve good results, accuracy in measuring is important.
3. Each group of students will do the following:
   a) Draw a number set out of a paper bag.
   b) Stand by the first specified high stool.
   c) Measure the angles made by all 10 flags from the first location.
   d) Record the angles made by all 10 flags measured from the first location.
   e) Stand by the second specified high stool.
   f) Measure the angles made by all 10 flags from the second location.
   g) Record the angles made by all 10 flags measured from the second location.

Day 3: Let's Get the Record Straight!

The teacher will have a transparency of the correct locations of each flag.

1. Each group of students will do the following:
   a) Design a Cartesian plane.
   b) Use a dot to record the correct spot for each flag.
   c) Describe the location of each flag in terms of the two axes.
   d) Check for accuracy of location for each flag.
   e) Do self-assessment.

Assessment:

Use the following observation checklist to assess student learning.

The student can do the following:	Yes	No	Comment
Design a Cartesian plane.			
Correctly identify the origin.			
Correctly identify each axis.			
Plot points correctly.			
Demonstrate an understanding of an angle.			
Measure angles accurately.			
Replicate an angle.			

Extension:

Taking it further

Have students describe how they would design a task involving the Cartesian plane.

NOTES