Grade 4 Mathematics

Number

Grade 4: Number (4.N.1, 4.N.2)

Enduring Understandings:

Numbers can be represented in a variety of ways (e.g., using objects, pictures, and numerals).

Place value patterns are repeated in large numbers, and these patterns can be used to compare and order numbers.

The position of a digit in a number determines the quantity it represents.

There is a constant multiplicative relationship between the places.

Essential Questions:

How many different ways can a number be represented?

How does changing the order of the digits in a number affect its placement on a number line?

How are place value patterns repeated in numbers?

How does the position of a digit in a number affect its value?

Specific Learning Outcome(s):	Achievement Indicators:
4.N.1 Represent and describe whole numbers to 10 000, pictorially and symbolically. [C, CN, V]	➛ Read a four-digit numeral without using the word "and" (e.g., 5321 is five thousand three hundred twenty-one, NOT five thousand three hundred AND twenty-one). ➛ Write a numeral using proper spacing without commas (e.g., 4567 or 4 567, 10 000). ➛ Write a numeral 0 to 10 000 in words. ➛ Represent a numeral using a place value chart or diagrams. ➛ Describe the meaning of each digit in a numeral. ➛ Express a numeral in expanded notation (e.g., 321 = 300 + 20 + 1). ➛ Write the numeral represented in expanded notation. ➛ Explain the meaning of each digit in a 4-digit numeral with all digits the same (e.g., for the numeral 2222, the first digit represents two thousands, the second digit two hundreds, the third digit two tens, and the fourth digit two ones).

Specific Learning Outcome(s):	**Achievement Indicators:**
4.N.2 Compare and order numbers to 10 000. [C, CN]	➢ Order a set of numbers in ascending or descending order, and explain the order by making references to place value. ➢ Create and order three 4-digit numerals. ➢ Identify the missing numbers in an ordered sequence or between two benchmarks on a number line (vertical or horizontal). ➢ Identify incorrectly placed numbers in an ordered sequence or between two benchmarks on a number line (vertical or horizontal).

Prior Knowledge

Students may have had experience

- representing and describing numbers to 1000, concretely, pictorially, and symbolically
- comparing and ordering numbers to 1000 (999)
- illustrating, concretely and pictorially, the meaning of place value for numerals to 1000 (hundreds, tens, and ones)

Background Information

As a convention, the word *and* is reserved for the reading of decimal numbers. The reading of number words such 625 should be read as "six hundred twenty-five." Many people, especially adults, use *and* inappropriately. Have students listen for and record examples of the misuse of the word *and*.
Note: In some other countries numbers are read using *and*.

Four-digit numbers can be written with or without a space between the hundreds and the thousands digits. Writing numbers that are five or more digits requires a space between the thousands and hundreds place (10 000).

Note: Students will see commas used in many resources and situations.

Meaningful real-life contexts (e.g., population data from a social studies unit) should be explored in order to help students develop an understanding of the relative size (magnitude) of numbers.

According to Kathy Richardson in her book, *How Children Learn Number Concepts: A Guide to the Critical Learning Phases* (145), in order for students to understand the structure of thousands, hundreds, tens, and ones they need to be able to

- count one thousand as a single unit
- know the total instantly when the number of thousands, hundreds, tens, and ones is known
- mentally add and subtract 10 and 100 to/from any four-digit number
- know the number of thousands that can be made from any group of hundreds, and the number of hundreds left over (e.g., 15 hundreds is 1 thousand and 5 hundreds)
- describe any number from 1000 to 10 000 in terms of its value in ones, or tens, or hundreds (e.g., 3400 is 34 hundreds, 3400 ones, and 3 thousand and 4 hundred)
- determine the total value of groups of thousands, hundreds, tens, and ones by reorganizing them into all possible thousands, hundreds, tens with leftover ones (e.g., 6 thousands, 27 hundreds, 45 ones can be reorganized to make 8745)

Preventing Misconceptions: The way we talk about concepts/ideas can create misconceptions for students. For example: Students are shown the number 168 and asked, "How many tens are in this number?" Generally, the expected response is "6" but in fact, there are 16 tens in 168. Rephrasing the question to ask, "How many tens are in the tens place in this number?" may help prevent misconceptions.

Mathematical Language

place value
thousand
hundreds
tens
ones
expanded notation
numeral
digit
benchmark
vertical
horizontal
greatest
least
ascending order
descending order

Assessing Prior Knowledge

Interview:

Give students a 3-digit number such as 264. Have them explain the meaning of each digit using base-10 materials, Digi-Blocks, or teacher/student-made representations, to support their explanation.

The student is able to

- ❐ use materials to represent a 3-digit number
- ❐ explain that the first digit represents 2 hundreds (e.g., two hundred blocks)
- ❐ explain that the second digit represents 6 tens (e.g., six ten blocks)
- ❐ explain that the third digit represents 4 ones (e.g., four single blocks)

Paper-and-Pencil Task:

1. Roll a 0-to-9 die three times. Record the numbers. (If any of the numbers are the same, roll the die again.)

   Make as many 3-digit numbers as you can.

   Order the numbers from greatest to least.
2. Choose one of the numbers you made. Explain the value of each digit. Use pictures and words.
3. Choose another number. Represent it in at least 6 different ways using what you know about place value.

- **Read a four-digit numeral without using the word "and" (e.g., 5321 is five thousand three hundred twenty one, NOT five thousand three hundred AND twenty one).**
- **Write a numeral using proper spacing without commas (e.g., 4567 or 4 567, 10 000).**
- **Write a numeral 0 to 10 000 in words.**
- **Represent a numeral using a place value chart or diagrams.**
- **Describe the meaning of each digit in a numeral.**
- **Express a numeral in expanded notation (e.g., 321 = 300 + 20 + 1).**
- **Write the numeral represented in expanded notation.**
- **Explain the meaning of each digit in a 4-digit numeral with all digits the same (e.g., for the numeral 2222, the first digit represents two thousands, the second digit two hundreds, the third digit two tens, and the fourth digit two ones).**

Representing Numbers

Students should be able to represent numbers in standard form, expanded notation, words, and with models such as tent/arrow cards, base-10 materials, money, and place-value charts.

Standard form is the usual form of a number, where each digit is in its place value.
Example: twenty-nine thousand three hundred four is written as 29 304

Expanded notation is a way to write a number that shows the value of each digit.
Example: 4556 = 4000 + 500 + 50 + 6

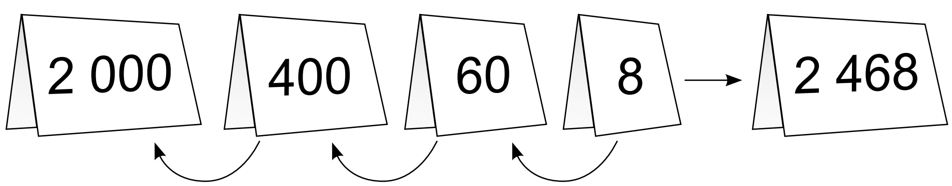
Suggestions for Instruction

BLM 4.N.1.1

- **Standard Form, Expanded Form, and Words:** This can be part of a Number of the Day routine. See BLM 4.N.1.1 for an example of a Number of the Day.
- **Tent Cards:** Place value tents/arrows help students to see the relationship between a digit and its value based on its position in the number.

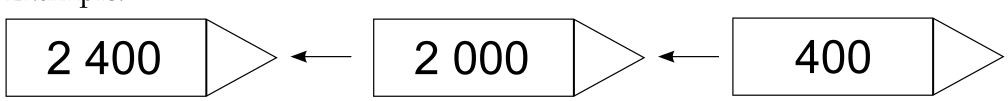
  Tent cards can be used to build numbers from their expanded form. They nest one on top of the other. They can also be used to move from the standard form to the expanded form (pulling apart the number). They can be downloaded from http://www.edu.gov.mb.ca/k12/cur/math/games/index.html.

  Example:



- **Arrow cards** are a set of place value cards with an arrow on the right side. They can be organized horizontally or vertically to represent numbers in expanded notation. Cards can be overlapped by lining up the arrows to form multi-digit numbers.

  Example:



- **Base-10 Materials:** These are proportional materials, which means that each block is 10 times larger than previous one (e.g., the flat is 10 times as large as the long).

Have students use the blocks to solve problems such as the following:

- Make the number that is one less than 1000.
  - If you have ten longs, what is the total value?
  - If you were able to break up the thousands block, how many flats would you have? How many longs? How many ones blocks?
  - Make the number 3468 with the blocks.
  - Make the number 2008.
  - Use five base-10 blocks. Make six different numbers. Each number must have at least one thousand block. Record your answers using pictures and numbers.
  - Problem: Samuel has seven base-10 blocks. The value of these blocks is more than 3000 and less than 3902. Which blocks might Samuel have chosen? Give four possible answers and explain your choices.

Extension: Find all the possible numbers.

- **Place-Value Chart:** Build numbers in the place-value chart. Be sure to include numbers with zeroes.

Example: Show the number 3 057.

Thousands			**Ones**		
hundreds	tens	ones	hundreds	tens	ones
		3	0	5	7

Transfer the information on the place value chart to standard form. (5 902)

Thousands			**Ones**		
hundreds	tens	ones	hundreds	tens	ones
		5	9	0	2

Note: Placing numbers on the place-value chart and transferring them from the chart can become a rote procedure that students can often accomplish without understanding. Using non-standard place value representations can challenge student thinking and allow them to demonstrate their understanding.

Examples:

Show 3 thousands, 46 tens, 8 ones on the chart.

Thousands			**Ones**		
hundreds	tens	ones	hundreds	tens	ones
		3	4	6	8

Write the number shown on the chart in standard form. (1523)

Thousands			**Ones**		
hundreds	tens	ones	hundreds	tens	ones
			14	12	3

- **Money:** Money can be used as a representation. Ask questions such as, "A large swimming pool costs $4 982.00. If you paid for it with hundred dollar bills, how many would you need? If you paid with ten dollar bills, how many would you need? If you paid with loonies, how many would you need?

  Pictures/charts can also be used.

$100	$10	$1
		

- **Make the Number?** Students write numbers following the directions given.

  Example:

  Write two different numbers that match the directions.

  1. 2 in the thousands place and 4 in the hundreds place (Answers may be a variety of numbers such as 2400, 2410, 2456, 2479, etc., but there must be a 2 in the thousands place and 4 in the hundreds place.)
  2. 8 in the tens place and 5 in the hundreds place
  3. 7 in the thousands place and 3 in the ones place
  4. 9 in the tens place and 6 in the thousands place

BLM 4.N.1.2

- **Renaming Numbers:** As a grouping or sorting activity, use a set of cards that have different ways of representing numbers. (If used for grouping, decide on the number of groups needed and then use one number for each group.)

  Example:

  In order to make 4 groups of 5, use a set such as the following:

4230	1305	2087	4387
4000 + 200 +30	1000 +300 + 5	2000 + 80 + 7	4000 + 300 +80 +7
423 tens	130 tens 5 ones	208 tens 7 ones	3 th 13 h 8 t 7 ones
3 th 12 h 3 t	1 th 2 h 10 t 5 ones	1 th 10 h 8 t 7 ones	438 tens 7 ones
4 th 1 h 13 tens	1305 ones	207 tens 17 ones	4387 ones

  Randomly pass out the cards and have students find their group members.

- **Calculator Wipe It Out!** The object of the activity/game is to wipe/zero out one or more digits from the display using subtraction. Initially the digits should all be different.

  Example:

  Students enter the number 3268 on their calculator. Ask them to "wipe out" only the numeral 6 or to make the display show 3208. Explain what they subtracted and why they chose that particular number. Students should also communicate what they will do before they press the buttons, and what number will be gained by removing the digit.

  Variation of the game: Use addition.

Example: How can you use addition to wipe out the 6? (Add 40)

Alternative ways to play the game:

Example: Enter 4537.

- Using addition, turn the 7 into a 2.
- Using subtraction, turn the 5 into a 3.
- Wipe out more than one place value position (e.g., Make the display show 4007).
- Make your display show 2000.
- Make your display show 0.

- **Place Value Game:**

BLM 4.N.1.3

Materials: a spinner with place value positions (BLM 4.N.1.3) and a 0-to-9 spinner (BLM 4.N.1.3) to be shared, and a white board or other erasable surface (page protector) with a place-value chart (BLM 4.N.1.3) for each player

Directions:

1. Each player draws a place-value chart on their board.

   Example:

TH	H	T	O

2. Player 1 spins the 0-to-9 spinner and the place-value spinner and enters the number in the correct position on their board. If the place is already filled, their turn is over.
3. The first person to complete their chart scores 10 points, the second 8 points, and so on.

   Extension: Bonus points can be given to the player with the largest/smallest number.
4. The game ends when a player reaches the point goal (set at the start of the game).

Assessing Understanding: Paper-and-Pencil Task

1. Dictate the numbers and have students record.

    4 651

    2 075

    1 902

    8 364

    5 008

2. Write the following numbers in words.

    7 268 ______________________________

    4 080 ______________________________

    5 921 ______________________________

    6 004 ______________________________

3. Write the following numbers in expanded form.

    1 634 ______________________________

    9 999 ______________________________

    2 100 ______________________________

    7 305 ______________________________

4. Explain the value of each digit in the number 4444.

    ____

    ____

    ____

    ____

5. Fill in the blanks to make these true:

    6070 = ______ hundreds + ______ tens

    3254 = ______ hundreds + ______ ones

    1280 = ______ tens

    2900 = ______ hundreds or ______ tens

- **Order a set of numbers in ascending or descending order, and explain the order by making references to place value.**
- **Create and order three 4-digit numerals.**
- **Identify the missing numbers in an ordered sequence or between two benchmarks on a number line (vertical or horizontal).**
- **Identify incorrectly placed numbers in an ordered sequence or between two benchmarks on a number line (vertical or horizontal).**

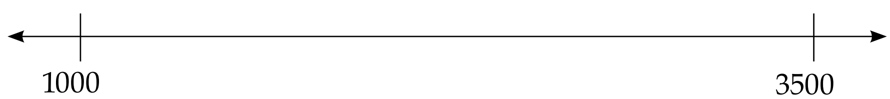
Suggestions for Instruction

Students should be exposed to both vertical (thermometers, measuring cups, etc.) and horizontal number lines. Discussions related to the importance of scale (the distance/difference between the reference points) will assist students in determining the placement of a number in relative position.

- **Number Line:** Provide a number line with end/reference points identified. Have students place given whole numbers on the number line.

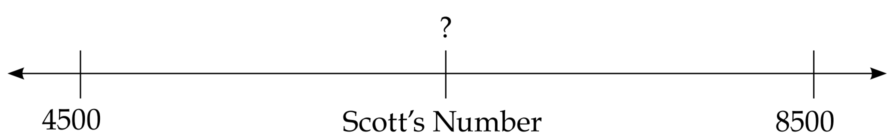
  Examples:

  1.



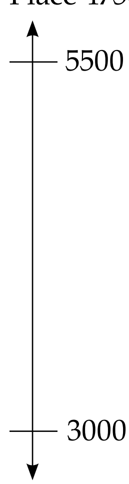
  Where would 2450 be on the number line?

  2.



  Scott placed a number on the number line. What might his number be? Explain your answer.

  3. Place 4750 on the number line.



Suggestion: Set up a clothesline (a string held up by a couple of magnets) in the classroom. Write numbers on tent cards (paper that is folded so that it loops over the string). Identify the end points (reference points). Have students place given numbers on the line and then justify their placement.

- **Roll the Dice:** Students roll a 0-to-9 die four times and record the numbers shown as a 4-digit number on an erasable surface or on paper. Have students use their numbers to arrange themselves from greatest to least (descending order). This can be made more challenging by doing it without talking.

  Note: It is important that students are aware that when comparing two numbers with the same number of digits, the digit with the greatest value should be focused on first. For example, when asked to explain why one number is greater or less than another, they might say that 2541 is less than 3652 because 2541 is less than 3 thousands while 3652 is more than 3 thousands. When comparing 5367 and 5489, students will begin comparing the thousands and move to the right.

- **Find the Error:** Prepare sets of numbers that have been ordered from least to greatest (ascending order) or greatest to least but with one or two errors. Have students identify the error(s) and then write them in the correct order.

  Example:

4000	4004	4040	4404	4044	4400
			X	X	X

  Correct order: 4000 4004 4040 4044 4400 4404

  Have students make of sets for the class to solve.

- **What Number Fits?** Give two reference points and have students write a number that fits between them.

  Example:

  Write a number that lies between

  5100 and 5200

  3199 and 4019

  8490 and 9500

  1250 and 1285

- **More and Less:** Use a double set of 0-to-9 digit cards for each student. Dictate a 4-digit number and have them make it with their digit cards (e.g., 4251). Give directions such as the following:
  - Make the number that is 200 more than 4251.
  - Make the number that is 1000 less than 4251.
  - Make the number that is 7 more than 4251.
  - Make the number that is 40 more than 4251.
  - Make the number that is 900 more than 4251.

  Observe students as they work. Do they have to remake the number from scratch or do they change only the place value position(s) affected?

- **Greater or Less Than:** Have students compare numbers in different ways. The comparisons should reference the understanding of place value in explanations.

  Ask questions such as:

  A. Which number is greater? Why?

  1. 6005 or 6050
  2. 4209 or 4029
  3. 3124 or 3214
  4. 7642 or 6742

  B. Fill in the missing digits so that the first number is greater than the second number.

  1. 5 $\square$ 21 > 5 $\square$ 21
  2. $\square$ 250 > 6368
  3. 20 $\square$ 9 > 2049
  4. 7306 > 7 $\square$ $\square$ 6

  Note: The use of the greater than (>) and less than (<) symbols are not taught formally until Middle Years. However, the symbols can be introduced earlier. The symbols are conventions of mathematics and should be introduced once students have a solid understanding of the concepts of greater than and less than. (Try to have students determine and share their own ways to remember symbols. For example, "I put 2 dots [colon] beside the larger number and 1 dot beside the smaller number and then I join the dots to make the symbol.")

- **Mystery Number:** Have students write Mystery Number riddles for the class to solve.

  Examples:

  1. I am a 4-digit number between 4500 and 6000.
     I am odd.
     I am a multiple of 5.
     The digit in the thousands place is repeated in the ones place.
     The sum of my digits is 17.
     The digit in the tens place is 2 more than the digit in the ones place.
     What number am I? (5075)
  2. I am a 4-digit number.
     I am even.
     The digit in the ones place is 4 times larger than the digit in the thousands.
     The digit in the tens place is 7 less than the digit in the ones place.
     The digit in the hundreds place is 5 more than the digit in the tens place.
     The sum of my digits is 17.
     What number am I? (2618)

- **Twenty Questions:** Think of a 4-digit number. Place dashes on the board to indicate the number of digits. Students ask questions to determine the number. Keep a tally of the number of questions asked. If the number is guessed in less than 20 questions, the students win. If not, the teacher/leader wins. (After modelling by the teacher, students should assume the role of leader for this game.)

  Example:

  □□□□

  Question examples:

  - "Is there a three in the tens place?"
  - "Is the number greater than 5000?"
  - "Is there a 5 anywhere in the number?" (A yes doesn't mean that the 5 is then placed on one of the blanks. Students would still have to determine its position in the number through additional questions.)
  - "Is the number odd?"
  - "Does the number have more than 20 tens?"

- **Higher or Lower:** Students play in groups of three (2 players and 1 leader). The leader secretly writes down a 4-digit number and then gives players the range (e.g., "The number is between 5 000 and 6 000"). Each player draws a number line, marking the reference points.

  The first player gives a possible number, and the leader tells them whether the number is higher or lower than the one chosen. The players record the response on their number lines. The game continues in this manner until one player gives the correct number.

Have students discuss the strategies they used to determine the secret number.

- **Guess My Number:** Prepare a card/piece of paper (a strip of masking tape will work) with a 4-digit number written on it for each student. Tape one card on each student's back. Students ask their classmates questions requiring a "yes" or "no" answer in order to determine their number. Limit the questions they can ask to one per classmate. (e.g., Am I greater than 5000? Am I less than 6000? Am I an even number? Am I a multiple of 10?)

  When all numbers have been identified, have students line up in order (ascending/descending).

Assessing Understanding: Performance Task/Observation/Interview

Materials: a deck of playing cards with the face cards and tens removed (aces count as 1 and the jokers count as 0) or use 4 sets of 0-to-9 numeral cards.

Organization: Work with a small group of students (4 or 5).

Directions:

Player A turns over 4 cards from the deck. Each player then arranges the cards to make a different 4-digit number. Player A records the numbers on individual pieces of paper/cards and keeps them in a pile. Players each take turns turning over four cards and recording the group's 4-digit numbers. Play continues until each student has had a turn.

Have each player order their numbers in ascending or descending order.

Ask students to

- ❒ read each number
- ❒ explain how they know their ordering is correct
- ❒ pick one of their numbers and identify the place value of each numeral
- ❒ pick one of the numbers and identify the number before and after
- ❒ pick one of the numbers and represent it in as many ways as they can (words, expanded form, base 10)*
- ❒ count forwards/backwards by tens/hundreds/thousands from one of the numbers

* Students can be doing this while the teacher interviews individual students.

Extend the activity by combining all of the number cards and have the group order them in ascending or descending order. Observe the process.

Grade 4: Number (4.N.3)

Enduring Understandings:

Quantities can be taken apart and put together.

Addition and subtraction are inverse operations.

There are a variety of appropriate ways to estimate sums and differences depending on the context and the numbers involved.

Essential Questions:

How can symbols be used to represent quantities, operations, or relationships?

How can strategies be used to compare and combine numbers?

What questions can be answered using subtraction and/or addition?

How can place value be used when adding or subtracting?

Specific Learning Outcome(s):	Achievement Indicators:
4.N.3 Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals), concretely, pictorially, and symbolically, by • using personal strategies • using the standard algorithm • estimating sums and differences • solving problems [C, CN, ME, PS, R]	➢ Model addition and subtraction using concrete materials and visual representations, and record the process symbolically. ➢ Determine the sum of two numbers using a personal strategy (e.g., for 1326 + 548, record 1300 + 500 + 74). ➢ Determine the difference of two numbers using a personal strategy (e.g., for 4127 – 238, record 238 + 2 + 60+ 700 +3000 + 127 or 4127 – 27 – 100 – 100 – 11). ➢ Model and explain the relationship that exists between an algorithm, place value, and number properties. ➢ Determine the sum and difference using the standard algorithms of vertical addition and subtraction. (Numbers are arranged vertically with corresponding place value digits aligned.) ➢ Describe a situation in which an estimate rather than an exact answer is sufficient. ➢ Estimate sums and differences using different strategies (e.g., front-end estimation and compensation). ➢ Solve problems that involve addition and subtraction of more than 2 numbers. ➢ Refine personal strategies to increase efficiency when appropriate (e.g., 3000 – 2999 should not require the use of an algorithm).

Prior Knowledge

Students may have an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1-, 2-, and 3-digit numerals) by

- using personal strategies for adding and subtracting with and without the support of manipulatives
- creating and solving problems in contexts that involve addition and subtraction of numbers concretely, pictorially, and symbolically.

They may be able to describe and apply mental math strategies for adding and subtracting two 2-digit numerals including

- adding from left to right
- taking one addend to the nearest multiple of 10 and then compensating
- using doubles
- taking the subtrahend to the nearest multiple of ten and then compensating
- thinking of addition

They may be able to apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem-solving context.

They may be able to recall addition and related subtraction facts to 18.

Background Information

There are many different types of addition and subtraction problems. Students should have experience with all types.

<table>
<tr><th colspan="4">Addition</th><th>Both
+ and -</th></tr>
<tr><th>Result Unknown
(a + b = ?)</th><th>Change Unknown
(a + ? = c)</th><th>Start Unknown
(? + b = c)</th><th>Combine
(a + b = ?)</th><th>Compare</th></tr>
<tr><td>Pat has 8 marbles. Her brother gives her 4. How many does she have now?
(8 + 4 = ?)</td><td>Pat has 8 marbles but she would like to have 12. How many more does she need to get?
(8 + ? = 12)</td><td>Pat has some marbles. Her brother gave her 4 and now she has 12. How many did she have to start with?
(? + 4 = 12)</td><td>Pat has 8 blue marbles and 4 green marbles. How many does she have in all?
(8 + 4 = ?)</td><td rowspan="4">Pat has 8 blue marbles and 4 green marbles. How many more blue marbles does she have?
(8 – 4 = ? or 4 + ? = 8)

Pat has 8 blue marbles and some green marbles. She has 4 more blue marbles than green ones. How many green marbles does she have?
(8 – 4 = ? or 4 + ? = 8)</td></tr>
<tr><th colspan="4">Subtraction</th></tr>
<tr><th>Result Unknown
(a – b = ?)</th><th>Change Unknown
(a – ? = c)</th><th>Start Unknown
(? – b = c)</th><th>Combine</th></tr>
<tr><td>Pat has 12 marbles. She gives her brother 4 of them. How many does she have left?
(12 – 4 = ?)</td><td>Pat has 12 marbles. She gives her brother some. Now she has 8. How many marbles did she give to her brother?
(12 – ? = 8)</td><td>Pat has some marbles. She gives her brother 4 of them. Now she has 8. How many marbles did she have to start with?
(? – 4 = 8)</td><td>Pat has 12 marbles. 8 are blue and the rest are green. How many are green?
(12 – 8 = ?)</td></tr>
</table>

The **standard algorithm** is a procedural method for performing a mathematical computation. It should be introduced after students have demonstrated a conceptual understanding of the operations through the use of concrete materials, visual representations, and personal strategies. The term *traditional algorithm* is used to indicate the symbolic algorithm traditionally taught in North America.

Front-end estimation: A method for estimating an answer to a calculation problem by focusing on the front-end or left-most digits of a number (e.g., 2356 + 1224 is estimated to be 2000 + 1000 = 3000).

Compensation: This strategy involves rounding one quantity up and the other down. For example, 1 752 + 648 would be thought of as 1 700 + 700. The 1 700 is a low estimate for 1 752 so the 648 is estimated as 700 (a high estimate) in order to compensate.

Mathematical Language

Operations:
addition
add
sum
total
more
subtraction
subtract
difference
less
take away
story problem
number sentence
estimate
addition fact
subtraction fact
strategy
standard algorithm
regroup
exchange
front-end estimation
compensation

Instructional Strategies: Consider the following guidelines for teaching addition and subtraction:

- Teach through problem solving.
- Select thought-provoking problems that are meaningful for students (relate to their own lives).
- Ensure that students understand the problem without inadvertently directing them to the way to solve the problem.
- Have students estimate the answer to the problem first.
- Ensure that students have access to manipulatives if they need them.
- Provide time for students to think individually before having them share/ discuss with their partner, group, or whole class.
- Circulate, listen, observe, encourage, and/or question without telling or evaluating strategies. Carefully selected questions can help students move forward when they are "stuck."
- Once a solution has been reached, have students compare the answer with their initial estimate.
- Orchestrate the sharing and critiquing of strategies. Which strategies worked? Which strategy was the most efficient? Have students justify their solutions.
- Have students create their own problems. An addition or subtraction number sentence can be provided or just the answer (e.g., The answer is 1250. What is the question?).

Assessing Prior Knowledge: Paper-and-Pencil Task

A. Solve the problems.

Be sure to show your work.

1. The students in Mrs. Johnson's class collected aluminum cans for recycling. Jana collected 214 cans. Mason collected 206 cans, and Marilyn collected 255 cans. How many cans did they collect altogether?
2. The elementary school has 457 students. If 232 of the students are boys, how many girls are in the school?
3. Simone has two jars of buttons. One jar has 326 buttons and the other jar has 387 buttons. How many buttons does Simone have altogether?
4. The answer is 236. What is the question?
   Write an addition problem that has an answer of 236.
5. The answer is 154. What is the question?
   Write a subtraction problem that has an answer of 154.

B. Show two different ways to solve each question.

218 + 407 = ______ 683 – 364 = ______

- **Model addition and subtraction using concrete materials and visual representations, and record the process symbolically.**
- **Determine the sum of two numbers using a personal strategy (e.g., for 1326 + 548, record 1300 + 500 + 74).**
- **Determine the difference of two numbers using a personal strategy (e.g., for 4127 – 238, record 238 + 2 + 60+ 700 +3000 + 127 or 4127 – 27 – 100 – 100 – 11).**
- **Model and explain the relationship that exists between an algorithm, place value, and number properties.**
- **Determine the sum and difference using the standard algorithms of vertical addition and subtraction. (Numbers are arranged vertically with corresponding place value digits aligned.)**
- **Solve problems that involve addition and subtraction of more than 2 numbers.**
- **Refine personal strategies to increase efficiency when appropriate (e.g., 3000 – 2999 should not require the use of an algorithm).**

Suggestions for Instruction

There are many different strategies that can be used for addition and subtraction.

Possible Strategies for Addition

Each of the following are strategies to calculate 1382 + 126.

Breaking Up Numbers using Place Value (Split Strategy)

This method requires place value understanding.

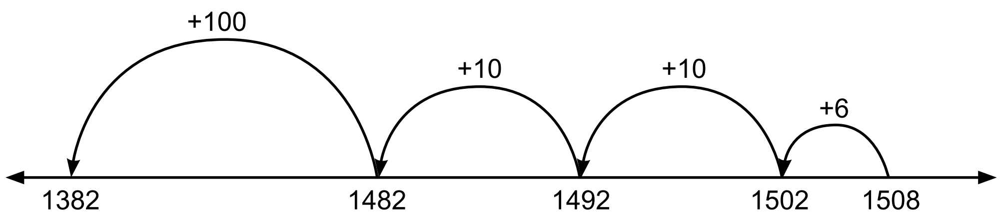
1382 + 126 = 1508

(1000 + 300 + 80 + 2) + (100 + 20 + 6)

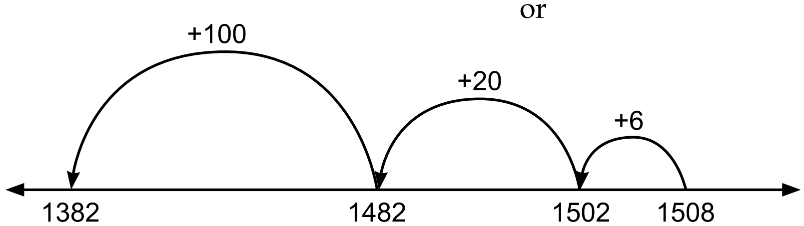
1000 + 400 + 100 + 8 = 1000 + 500 + 8 = 1508

Note: As the size of the numbers increases, it is more difficult for students to use this method mentally. This strategy is easily demonstrated with base-10 blocks.

Empty Number Line (Jump Strategy)



or



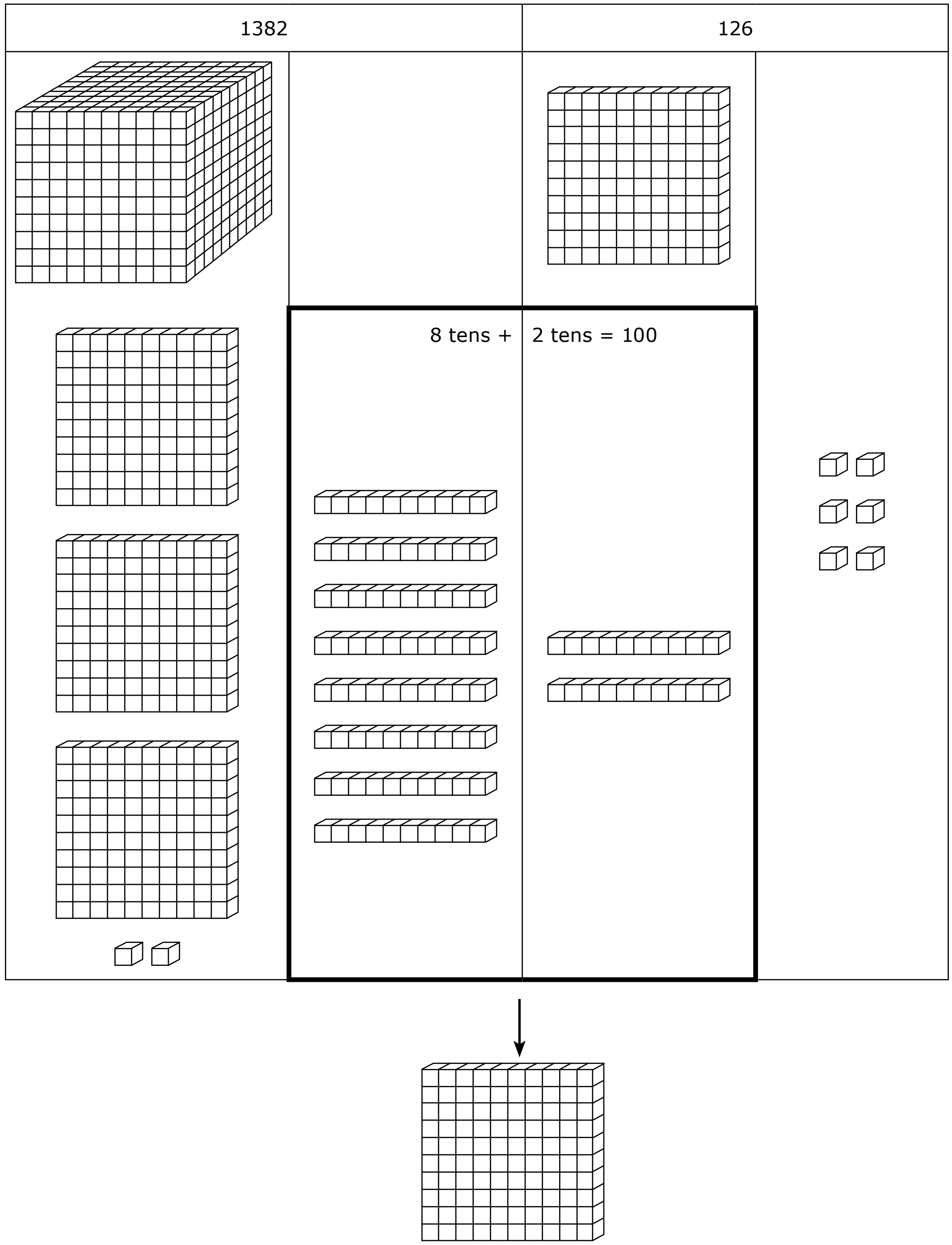
There are other possibilities.

Making "nice" or "friendly" numbers

1382 + 126 = 1382 + 18 + 108 = 1400 + 108 = 1508
because 1382 needs 18 more to get to 1400 and then only 108 are left to add on.

Note: Students need to use their knowledge of compatible number pairs for 10 and be able to extend this knowledge to pairs for 100 in order to be able to use this strategy.

Use representations of materials such as base-10 blocks.



(1 x 1000) + (5 x 100) + (8 x 1) = 1508

Therefore 1382 + 126 = 1508

Modified Standard Algorithm

$$
\begin{array}{rl}
1382 & \\
\underline{+\ \ 126} & \\
8 & = 2 + 6 \\
100 & = 80 + 20 \\
400 & = 300 + 100 \\
\underline{1000} & = 1000 + 0 \\
1508 &
\end{array}
$$

Standard Algorithm

$$
\begin{array}{r}
\overset{1}{1}382 \\
\underline{+\ \ 126} \\
1508
\end{array}
$$

Possible Strategies for Subtraction

Each of the following are strategies to calculate 1382 – 126.

Breaking Up Numbers Using Place Value

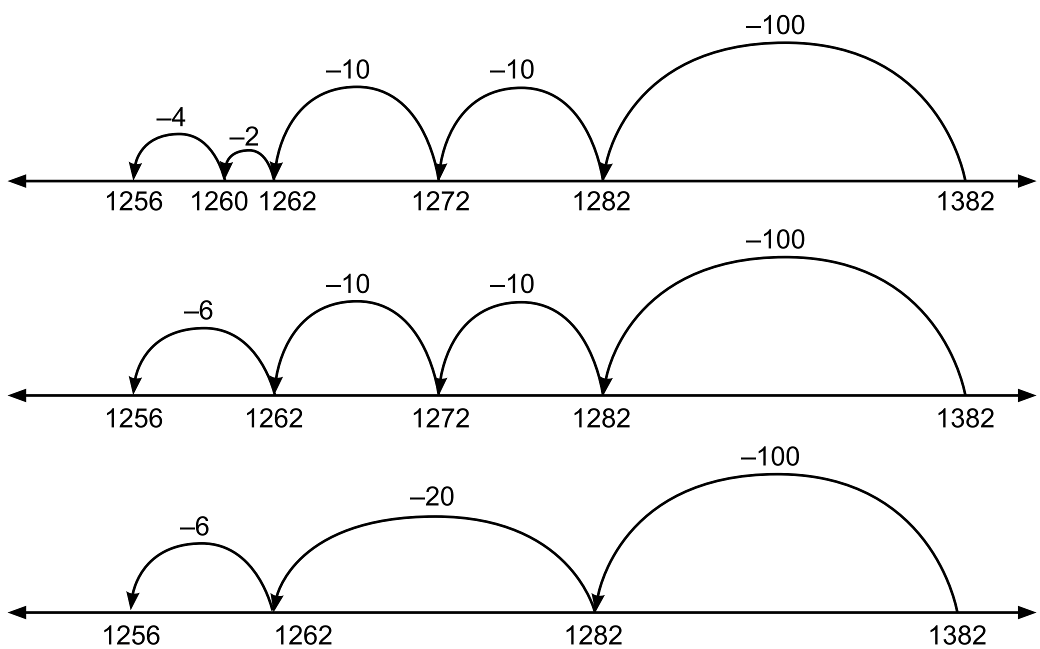
This method requires place value understanding.

1382 – **126 =**

(1000 + 300 + 80 + 2) – 100 + 20 + 6 =

1000 + (300 – 100) + (70 – 20) + (12 – 6) = 1000 + 200 + 50 + 6 = 1256

Empty Number Line (Jump Strategy)



There are other possibilities.

Making "nice" or "friendly" numbers

Add 4 to both numbers.

(1382 + 4) – (126 + 4) ⟶ 1386 – 130 = 1256

Renaming

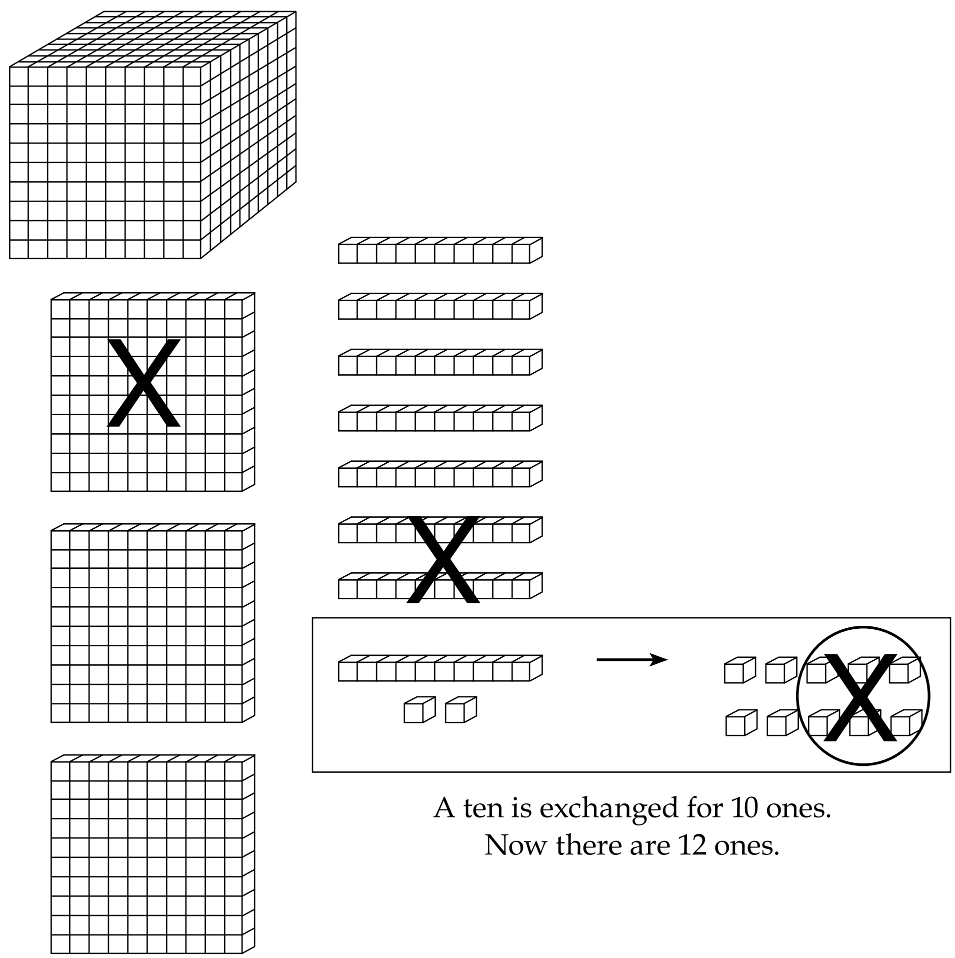
This strategy relies on the student's sense of number.

5 000 – 2 674 **=**
4 999 + 1 (renamed the 5000)
<u>– 2 674</u>
2 325 + 1 = 2 326

or

One can be subtracted from each number before subtracting (4 999 – 2 673).

Use representations of materials such as base-10 blocks.



A ten is exchanged for 10 ones.
Now there are 12 ones.

1000 + (300 – 100) + (70 – 20) + (12 – 6)

Therefore 1382 – 126 = 1256

Standard Algorithm

$$
\begin{array}{r}
13\,{}^{7}\cancel{8}\,{}^{1}2 \\
\underline{-\ \ 126} \\
1256
\end{array}
$$

Note: Students will also develop their own strategies.

Example: Use negative numbers

$$
\begin{array}{rl}
1382 & \\
\underline{-\ 126} & \\
-\ 4 & (2 - 6) \\
60 & (80 - 20) \\
200 & (300 - 100) \\
\underline{1000} & \\
1256 & (1000 + 200 + 60 - 4)
\end{array}
$$

Assessing Understanding: Paper-and-Pencil Task

Jonah and Savana were given the question 1382 + 126 to solve.

Jonah solved it this way.

$$
\begin{array}{r}
1382 \\
\underline{+\ 126} \\
8 \\
100 \\
400 \\
\underline{1000} \\
1508
\end{array}
$$

Savana solved it this way.

$$
\begin{array}{r}
1^{1}382 \\
\underline{+\ 126} \\
1508
\end{array}
$$

Both students got the correct answer.

How are their methods the same? How are they different?

Note: It is important to model correct vocabulary. In looking at the standard algorithm, the terms *carrying* and *borrowing* are no longer used because they have no real mathematical meaning with respect to the operations. Instead, the terms *regroup, exchange,* or *trade* should be used in their place.

- **Multi-step Problems:** Students should have opportunities to solve problems that involve the addition or subtraction of more than one number.

  Examples:

  1. On Monday, there were 4128 visitors to the Lower Fort Gary. 2709 of them were adults and the rest were children. How many children visited the fort? The first 2890 visitors received a small Canadian flag. How many visitors did not receive a flag?
  2. There were 3670 bags of cotton candy sold at the fair. 1565 of them were pink and 1005 were blue. The rest were green. How many green bags were sold?
  3. In July, 3889 people flew from Winnipeg to Vancouver. In August, the number of people who flew from Winnipeg to Vancouver was 1335 more than in July. How many people flew from Winnipeg to Vancouver in the July and August altogether?
  4. Mark was downloading apps to his phone. The first app he downloaded was 177 kb, the second was 446 kb, and the last was 207 kb. What was the total size (in kb) of all the apps he downloaded?

Using the Bar Model to Support Part-Whole Understanding for Addition and Subtraction

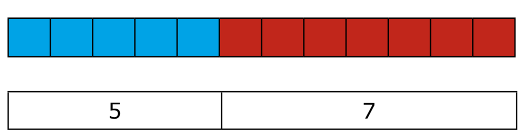
The Bar Model is a problem-solving strategy that originates from Singapore. It is used to help students convert the data from a word problem into concrete visual images. These images can then be converted into relevant mathematical expressions or number sentences.

This model also helps students see that different problems in a variety of contexts share the same mathematical structure therefore they can be visualized in the same way.

If students have not had prior experience with model drawing, it is recommended that they begin by using physical objects. The objects can be organized in a linear fashion and then eventually be replaced by a model drawing (bar).

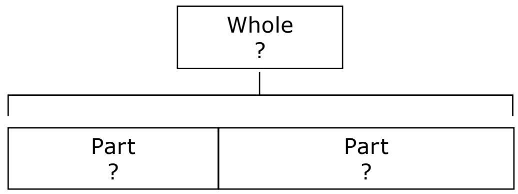
Example:

John had 5 blue candies and 7 red candies. How many candies did he have altogether?



5 + 7 = 12

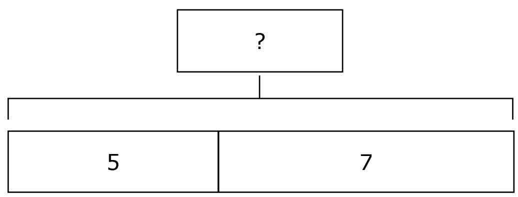
In problems involving addition and subtraction, there are three possible unknowns. When the value of two of them is known the third can be found.



Addition Types

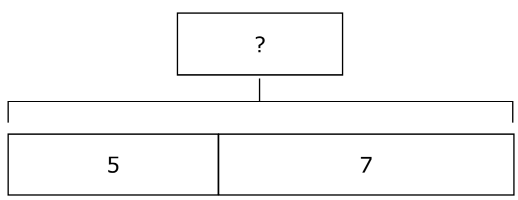
Two Quantities Combined

I have 5 green candies and 7 red candies. How many candies do I have?



A Quantity Is Increased

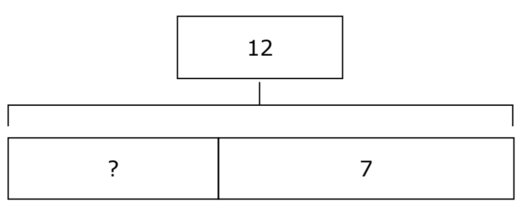
I have 5 green candies and I buy 7 red candies. How many candies do I have?



Subtraction Types

Take Away

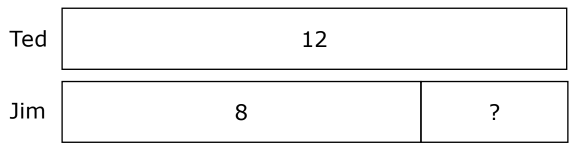
I had 12 candies. I gave 7 away. How many candies do I have now?



The whole is known along with one of the parts. The whole is partitioned and one of the parts removed to identify the missing part.

Comparison or Difference

Ted has 12 candies. Jim has 8 candies. How many more does Ted have?



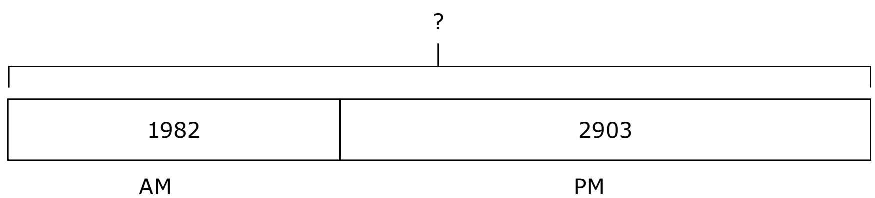
Example Using a Multi-step Problem:

At the fair, 1982 hotdogs were sold in the morning and 2903 were sold in the afternoon.

How many hotdogs were sold altogether?

How many more hotdogs were sold in the afternoon than in the morning?

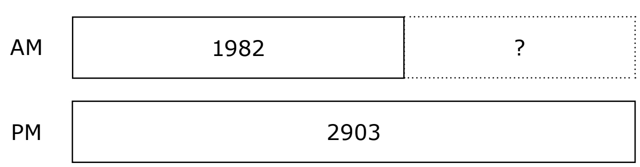
Part A



1982 + 2903 = 4885

There were 4885 hotdogs sold all together.

Part B



2903 – 1982 = 921

There were 921 more hotdogs sold in the afternoon than were sold in the morning.

- **Describe a situation in which an estimate rather than an exact answer is sufficient.**
- **Estimate sums and differences using different strategies (e.g., front-end estimation and compensation).**

Suggestions for Instruction

- **Estimation Strategies:** Students need to be introduced to a variety of estimation strategies such as the following:
  - **Front-end estimation:** In this strategy, only the digit with the largest place value is used even though the estimate may be low. For example, 127 + 238 is estimated to be 100 + 200 = 300.
  - **Compensation:** This strategy involves rounding one quantity up and the other down. In doing so, one number might be overestimated in order to compensate for the other number being underestimated. For example, 1 752 + 648 would be thought of as 1 700 + 700. The 1 700 is a low estimate for 1 752 so the 648 is estimated at 700 (a high estimate) in order to compensate.
  - **Rounding:** 1 439 + 352 is estimated to be 1440 + 350 = 1790 or 1400 + 400 = 1800. Note: This is not to be taught in a formal/structured manner.

  Students should have multiple opportunities to estimate not only in mathematics but in other subject areas as well.

- **Estimate or Exact Answer?** Students should be able to determine when an exact answer is required or when an estimate is sufficient based on the situation. Give students problems. Have them decide if an exact answer or an estimate is needed, and then justify their choice.

  Examples:

  1. Lee has 482 hockey cards, 173 baseball cards, and 198 football cards. Does Lee have more than 1000 cards in all? (estimate)
  2. Amy empties her piggy bank. She counts $104.50 in quarters, $75.10 in dimes and $27.75 in nickels. How much money was in the piggy bank? (exact)
  3. The 120 Grade 4 students are going on a field trip. The teachers have collected 87 permission slips so far. How many permission slips still need to be returned? (exact)
  4. The school concert was held on two nights. On Wednesday, there were 652, and on Thursday there were 571 people. About how many people attended the concert? (estimate due to the way the question is asked)

- **Modelling Estimation Strategies:** Present students with the following problem:

  You read 175 pages on the first day, 198 pages on the second day, and 150 pages on the third day. About how many pages did you read over the three days?

  - Have students represent the numbers using base-10 materials.
  - Use the base-10 materials to model front-end rounding e.g., 175 → 100, 198 → 100, and 150 → 100. Using this strategy the estimation would be 300 pages. Ask students if this is a good estimate for the answer.
  - Focus their attention on the remaining base-10 blocks. Point out that the remaining (75 + 98 + 50) blocks would together make at least 200. Therefore, adding on another 200 would compensate for the values that were dropped off when using the front-end strategy. An estimate of 500 (100 + 100 + 100 + 200) is a better estimate.
  - Make sure that students understand that the front-end strategy and compensation used together enable them to make a more reasonable estimate.

  Model the same process for subtraction where compensation is used to subtract more from the initial front-end estimate.

  Example:

  Estimate the answer to 410 – 395.

  - Represent the numbers using base-10 blocks. Using front-end estimation—510 → 500 and 395 → 300 therefore the estimate (500 – 300) is 200.
  - The students should see that there are still 95 blocks remaining after the hundreds are compared that were initially to be subtracted from the 510. Therefore, since 95 is close to 100, an additional 100 should be subtracted from the initial estimate. 500 – 300 – 100 = 100.

- **Writing Problems:** Have students write their own addition and subtraction problems. Some of the problems should require an estimate only and others should require both an estimate and a calculation (exact answer).

Assessing Understanding: Paper-and-Pencil Task

Present students with the following problems.

1. Your family is going to visit friends in Calgary 1320 kilometres away. On the first day they travel 650 km and stop in Moose Jaw. Will you have to travel more or less than 700 km to reach your destination on the second day?
2. The book you are reading has 525 pages. If you read 220 pages the first day and 230 pages the second day, how many more pages do you have to read in order to finish the book?
3. A jogger jogs 1300 m the first day and 1800 m the second day. About how far did she jog in all?
4. On Saturday, 4012 people registered to run in the marathon. If 1278 of them were males, about how many were females?
5. In order to move to a new level in the video game, you need at least 2000 points. If you have 1254 points, how many more points do you need before you can move to the next level?
6. On Saturday, 1500 people visited the zoo, and on Sunday 2800 people visited. Approximately how many people visited the zoo over the weekend?
   a. Have students decide which problems can be answered with an estimate only and which problems require calculation as well as an estimate. (Note: Estimating is necessary for every problem because estimates help to determine the reasonableness of the calculated answer.)
   b. Have students answer the problems independently.
   c. Have students share their solutions and strategies with one another or in an interview.

The student

- ❒ explains the meaning of the problem and justifies why only an estimate is needed or why a calculated answer is necessary as well
- ❒ uses compensation as well as front-end rounding to estimate the sum or difference
- ❒ explains clearly the strategies used in estimating and how he or she knows that the resulting estimate is reasonable

Grade 4: Number (4.N.4, 4.N.5)

Enduring Understandings:

Multiplication and division are inverse operations.

Multiplication is repeated addition.

Division is repeated subtraction.

Essential Questions:

How can skip-counting and arrays be used to demonstrate multiplication and division?

How are addition and multiplication related?

How are subtraction and division related?

Specific Learning Outcome(s):	Achievement Indicators:
4.N.4 Explain the properties of 0 and 1 for multiplication, and the property of 1 for division. [C, CN, R]	➛ Explain the property for determining the answer when multiplying numbers by one. ➛ Explain the property for determining the answer when multiplying numbers by zero. ➛ Explain the property for determining the answer when dividing numbers by one.
4.N.5 Describe and apply mental mathematics strategies, such as • skip-counting from a known fact • using halving/doubling • using doubling and adding one more group • using patterns in the 9s facts • using repeated doubling to develop an understanding of basic multiplication facts to 9 × 9 and related division facts. [C, CN, ME, PS, R] Recall of the multiplication and related division facts up to 5 × 5 is expected by the end of Grade 4.	➛ Provide examples for applying mental mathematics strategies: • skip-counting from a known fact (e.g., for 6 × 3, think 5 × 3 = 15, then 15 + 3 = 18) • halving/doubling (e.g., for 4 × 3, think 2 × 6 = 12) • using a known double and adding one more group (e.g., for 3 × 7, think 2 × 7 = 14, then 14 + 7 = 21) • repeated doubling (e.g., for 4 × 6, think 2 × 6 = 12 and 2 × 12 = 24) • use ten facts when multiplying by 9 (e.g., for 9 × 6, think 10 × 6 = 60, and 60 – 6 = 54; for 7 × 9, think 7 × 10 = 70, and 70 – 7 = 63) • halving (e.g., for 30 ÷ 6, think 15 ÷ 3 = 5) • relating division to multiplication (e.g., for 64 ÷ 8, think 8 × ☐ = 64).

Prior Knowledge

Students may have

- represented and explained multiplication (to 5 x 5) using equal groups and arrays
- modelled multiplication using concrete and visual representations, and recorded the process symbolically
- related multiplication to repeated addition
- related multiplication to division and division to multiplication
- created and solved problems in context that involve multiplication
- represented and explained division using equal sharing and equal grouping
- modelled equal sharing and equal grouping using concrete and visual representations, and recorded the process symbolically
- related division to repeated subtraction
- created and solved problems in context that involve equal sharing and equal grouping

Background Information

Terminology

Multiplication: A mathematical operation of combining groups of equal amounts; repeated addition; the inverse of division.

Product: The number obtained when two or more factors are multiplied (e.g., in $6 \times 3 = 18$, 18 is the product).

Division: A mathematical operation involving two numbers that tells how many groups there are or how many are in each group.

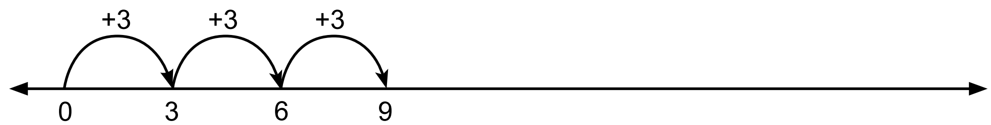
Quotient: The answer to the division of two numbers (in $12 \div 3 = 4$, the quotient is 4).

Array: A set of objects or numbers arranged in an order, usually in rows and/or columns.

Meanings of Multiplication at the Grade 4 Level

1. Repeated addition

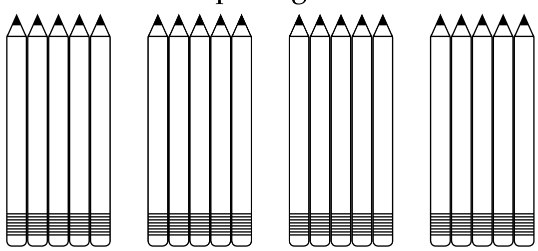
   For example: 3 + 3 + 3 = 9



2. Equal groups or sets

   For example:

   Pencils come in packages of 5.

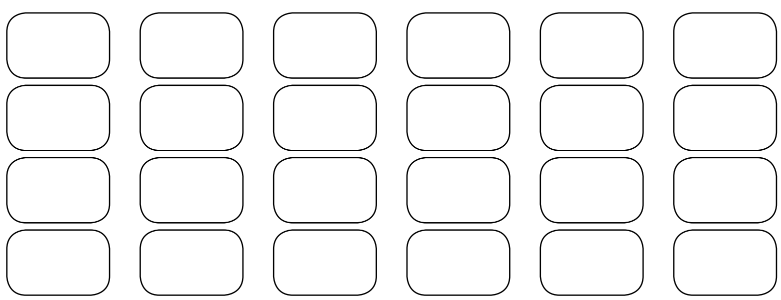


   How many pencils are in 4 packages?

3. An array

   For example:

   A classroom has 4 rows with 6 desks in each row.



   How many desks are in the classroom?

BLM 4.N.5.1

BLM 4.N.5.1 is a picture of a 10 x 10 dot array that can be used in a student's tool kit to help them with problem solving. Print off the page and place it in a plastic sheet protector. Students can then use a white board marker to show different problems.

Multiplication Problems

In a multiplication problem both the number of objects in each group and the number of groups are given. The total number of objects is the unknown.

Meanings of Division at the Grade 4 Level:

1. Repeated Subtraction

   For example: $6 \div 2 = 3$ is the number of times you can subtract groups of 2 before you get to 0.

   $6 - 2 - 2 - 2 = 0$

   $6 \div 2 = 3$

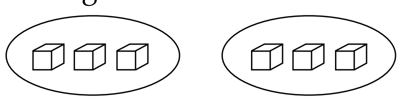
2. Equal Sharing (Partitive)

   For example: $6 \div 3 = 2$ is the amount each person gets if you share 6 things equally among 3 people.

   $6 \div 3 = 2$

3. Equal Grouping (Quotative)

   For example: $6 \div 3 = 2$ is the number of equal groups of 3 you can make with 6 things.



   $6 \div 3 = 2$

Note: Division should be taught together with multiplication so that students can see the inverse relationship between the two operations.

Preventing Misconceptions: Common misconceptions students may develop include multiplication makes bigger and division makes smaller. This is not true when working with fractions or decimals less than one.

Example: $\frac{1}{2} \times 8 = 4$, $8 \div \frac{1}{2} = 16$, $8 \times 0.25 = 2$ or $8 \div 0.25 = 32$

Mathematical Language

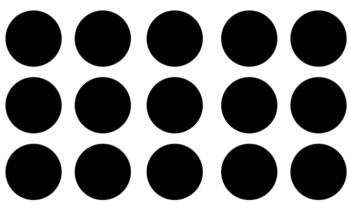
sets of
groups of
multiply
multiplication
product
quotient
divide
division
equal groups
sharing
array
times
skip-counting
halving
doubling
property
properties

Learning Experiences

Assessing Prior Knowledge: Paper-and-Pencil Task

1. What does this array show?

   Write two multiplication number sentences and two division number sentences.

2. The Grade 4 class is playing a game.

   The teacher wants them to be in equal groups with no remainders.

   If there are 20 students in the class, how many different sizes of equal groups can you make?

3. The answer to a multiplication question is 12.

   What might the question be?

4. My friend said that multiplication is repeated addition and division is repeated subtraction. Explain what s/he means. Use words, pictures, number lines, and/or numbers and symbols in your explanation.

Look for evidence that the student understands that

❐ an array can represent both operations
❐ multiplication is repeated addition
❐ division is repeated subtraction
❐ multiplication and division are inverse operations

- **Explain the property for determining the answer when multiplying numbers by one.**
- **Explain the property for determining the answer when multiplying numbers by zero.**
- **Explain the property for determining the answer when dividing numbers by one.**

Note: It is important that students have multiple opportunities to solve problems, create their own problems, and interact with concrete and visual models related to multiplication and division by 1 and multiplication by 0. Through these experiences, students will develop their understanding of the properties and then be able to come to their own generalizations.

Identity Property of Multiplication: Any number multiplied by one is equal to the original number.

Identity Property of Division: Any number divided by 1 is equal to the original number.

Zero Property of Multiplication: Any number multiplied by zero is equal to zero.

This approach is far more effective than just giving students arbitrary rules.

Suggestions for Instruction

- **Exploring Multiplication by 1:** Have students represent 5 x 1 using materials, an array, and a number line. Have students represent 1 x 5 using materials, an array, and a number line. Ask students what they notice about their representations (answers). (The answers are always the same as the number being multiplied by 1.)

  Extension: Will this be true for larger numbers? Have students justify their thinking.

- **Is there a difference?** Have students use materials to show 1 group of 6 objects and 6 groups of 1 object. What is the same about their models and what is different?

  Exploring Division by 1:

  Materials: 1-to-6 or 1-to-9 die.

  Procedure: Students roll the die and record the number shown. Have them model with materials and a number line the division of their number by 1. Repeat two more times. What do they notice about their answers?

  Extension: Do they think that this will be true for larger numbers? Have students explain/justify their thinking.

- **Similarities and Differences:** Have students represent the following problems using materials.
  - Paul has 8 cookies. He puts them in a bag. How many cookies are in the bag?
  - Paul has 8 cookies. If he puts 1 cookie in each bag, how many bags can he make?

  How are their representations the same? How are they different?

- **Problem Writing:** Have students create and share their own problems that involve multiplying or dividing by 1.
- **Multiplication by 0:** Have students represent 5 x 0 and 0 x 5 using materials such as paper plates to represent groups and/or a number line. What do they notice about their answers? Will the result be the same for any number multiplied by 0? Explain your thinking.

BLM 4.N.4.1

- **Equation Match:** Prepare a set of cards from BLM 4.N.4.1 for each pair of students. Have students find the matching representations and justify their thinking.

Assessing Understanding: Paper-and-Pencil Task

1. Write a note to your parent(s)/guardian(s)/caregiver(s) explaining what you have learned about multiplying and dividing any number by 1. Use pictures, number lines, and words.
2. Is she correct? Explain/show how you know.

The student is able to

- ❒ give a generalization for multiplying by 1
- ❒ give a generalization for dividing by 1
- ❒ support their generalizations using pictures and/or number lines
- ❒ explain why multiplication by zero always results in an answer of zero

- **Provide examples for applying mental mathematics strategies:**
  - **skip-counting from a known fact (e.g., for 6 x 3, think 5 x 3 = 15, then 15 + 3 = 18)**
  - **halving/doubling (e.g., for 4 x 3, think 2 x 6 = 12)**
  - **using a known double and adding one more group (e.g., for 3 × 7, think 2 × 7 = 14, then 14 + 7 = 21)**
  - **repeated doubling (e.g., for 4 x 6, think 2 x 6 = 12 and 2 x 12 = 24)**
  - **use ten facts when multiplying by 9 (e.g., for 9 × 6, think 10 × 6 = 60, and 60 – 6 = 54; for 7 × 9, think 7 × 10 = 70, and 70 – 7 = 63)**
  - **halving (e.g., for 30 ÷ 6, think 15 ÷ 3 = 5)**
  - **relating division to multiplication (e.g., for 64 ÷ 8, think 8 × $\square$ = 64).**

Mental Math

Note: The development of mental math strategies is greatly enhanced by sharing and discussion. Students should be given the freedom to adapt, combine, and invent their own strategies.

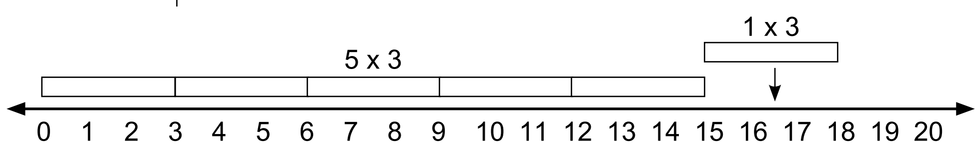
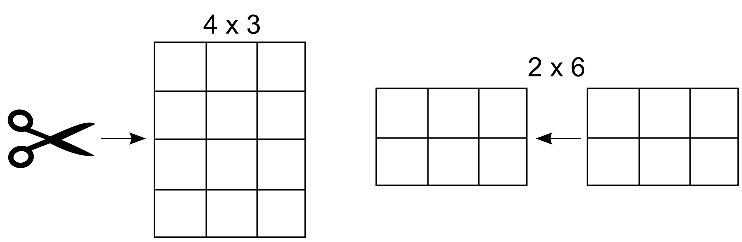
Students should be able to apply these strategies to larger numbers.

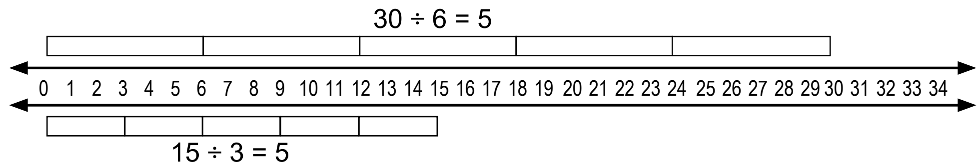
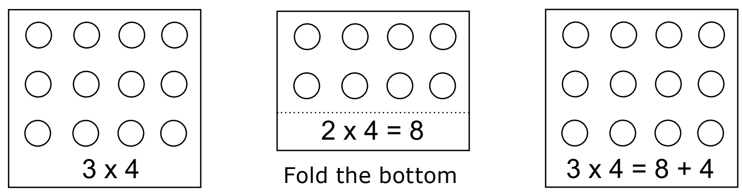
Background Information

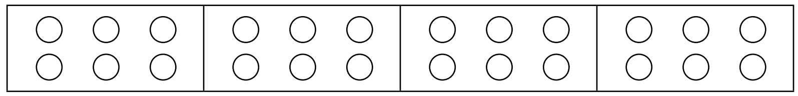
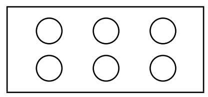
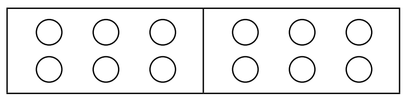
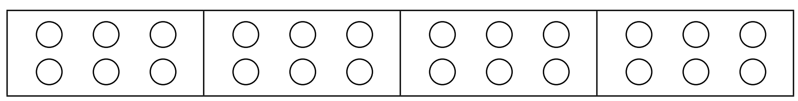
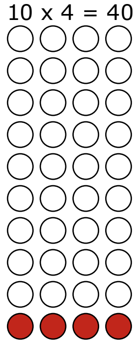
An understanding of the multiplication and division properties is needed in order for students to be able to develop and use mental math strategies. These properties include:

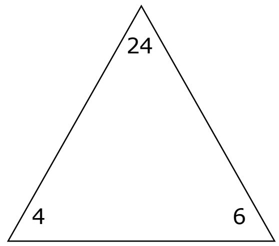
- **Commutative property of multiplication:** Numbers can be multiplied in any order. (e.g., 3 × 4 = 4 × 3). An array model can help to demonstrate this property.
- **Associative Property:** When three or more numbers are multiplied together, it doesn't matter in which order they are grouped or associated. For example, 5 × 2 × 4 = (5 × 2) × 4 = 5 × (2 × 4).
- **Distributive Property:** The distributive property refers to the idea that one or both of the factors in a multiplication question can be decomposed into two or more parts and each part multiplied separately and then added [e.g., 9 x 7 is equivalent to (9 × 5) + (9 × 2)].

Suggestions for Instruction

Strategy	Teaching Strategies
Skip-Counting from a known fact	Prerequisite knowledge: Students should be able to skip-count forward and backward by 2s, 3s, 4s, 5s, and 10s. Cuisenaire rods can help to make this strategy visible for students. (**Note:** Use a metre stick as a number line. The rods are 1 to 10 cm in length, so they will match the centimetre markings on the metre stick.) Example: For 6 x 3: "I know that 5 x 3 is 15 so for 6 x 3 I just need to add one more rod/group."  Have students connect the strategy to larger numbers. Example: For 6 x 30: Think "5 x 30 is 150, so 6 x 30 is 150 + 30 = 180."
Halving/Doubling	Halving and doubling can make multiplication calculations easier. Using grid paper to make arrays and then cutting them apart and reassembling them can help make this strategy visible for students. Example: For 4 x 3: Use grid paper to make the 4 x 3 array and cut it out. Cut the array in half and reassemble it to show 2 x 6. Students will be able to see that the total number of squares did not change; therefore, the two representations are equal.  Have students apply this strategy to larger numbers. Example: 6 x 50 can be thought of as 3 x 100.

Strategy	Teaching Strategies
Halving	Using the strategy of halving both the dividend and the divisor can help make division calculations easier. Using a double number line and Cuisenaire rods can help students understand/see that dividing both the dividend and the divisor by two results in the same answer that they would get if no changes were made to the original question. Example:  Have students apply this strategy to larger numbers. Example: 160 ÷ 4 can be thought of as 80 ÷ 2.
Using a known double and adding one more group BLM 4.N.5.1	This strategy is similar to using the doubles plus 1 strategy in addition except that it is one more group that is being added. Using array cards (see BLM 4.N.5.1) can help to make this strategy visible for students.  Initial array card Fold the bottom flap up to cover the last row of the array. Unfold to show one more group.

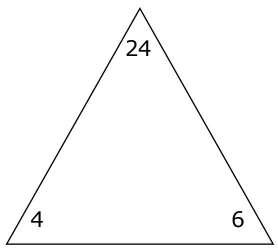
Strategy	Teaching Strategies
Repeated Doubling	This strategy is useful when multiplying by 4 or by 8. Using paper strips and arrays can help to make this strategy visible for students. Use a strip of paper. Fold it in half and then in half again. Unfold the paper and make dot arrays for the number being multiplied by 4 or 8. Example: For 4 x 6 the original strip looks like this.  Fold the strip so that one section is showing.  Open it up to show the double.  2 x 6 = 12 Finally, open it up to show all four sections.  2 x 12 = 24 Have students make their own representations of the repeated doubling strategy.
Use ten facts when multiplying by 9	Although not considered basic facts, most Grade 4 students know how to multiply by 10. This knowledge can be applied when multiplying by 9. Two-colour counters can be used to help make this strategy visible. Example: For 9 x 4: Have students use the counters to make 10 groups of 4, all of one colour. Then have them turn over last counter in each column to represent the extra group of four that needs to be subtracted. 10 x 4 = 40  40 – 4 = 36 Have students extend this strategy to larger numbers. For 40 x 9: Think "40 x 10 = 400, but I need to take away the extra group of 40 so 400 – 40 = 360. 40 x 9 = 360."

Strategy	**Teaching Strategies**
Relating division to multiplication BLM 4.N.5.2	Thinking multiplication is often an easier way of solving a division question. Students need to be able to understand the relationship between the two operations. Triangle flash cards can support this understanding. Example: For this flashcard students can see that  4 x 6 = 24 6 x 4 = 24 24 ÷ 4 = 6 24 ÷ 6 = 4 Using triangular flashcards: Display the card with one of the numbers covered. Students have to figure out the hidden number. Example: If the 24 (product) is covered students need to multiply 4 x 6 (factors) to find the answer. If the 4 or the 6 is covered students need to either divide or to “think multiplication” to find the answer. For example, if the 4 is covered, students can think “24 ÷ 6 = ?” or “6 x ? = 24” **Match Game:** See BLM 4.N.5.2.

Assessing Understanding: Interview

Ask the student to

- explain how knowing 4 × 6 helps find the product/answer for 8 × 6.
- explain how knowing 7 × 10 can help someone find the answer for 7 × 9.
- write 25 ÷ 5 = ? as a multiplication question/number sentence.
- show how they can solve 5 x 64 using the doubling and halving strategy.
- use counters to show why 7 × 6 is the same as (5 × 6) + (2 × 6).
  Note: Students are not expected to use parentheses.
- explain why 2 × 7 is equal to 7 × 2.
- write two multiplication number sentences and two division number sentences for the following flashcard



The student understands and can use the following mental math strategies and/or properties:

- ❒ repeated doubling
- ❒ using ten facts when multiplying by 9 (build down)
- ❒ relate multiplication to division
- ❒ doubling and halving
- ❒ distributive property
- ❒ commutative property

- **Recall of the multiplication and related division facts up to 5 x 5 is expected by the end of Grade 4.**

Background Information

Stages of Basic Fact Acquisition

Learning math facts is a developmental process where the focus of instruction is on thinking and building number relationships. Facts become automatic for students through repeated exposure and practice.

Arthur Baroody identifies three stages through which students typically progress in acquiring basic facts:

- **Counting Strategies:** The student uses objects (e.g., blocks, counters, fingers) or verbal counting to determine the answer. For example, for 3 x 7, the student starts with 7 and skip counts saying, "7, 14, 21."
- **Reasoning:** The student uses known information (i.e., known facts and relationships) to logically determine the answer of an unknown fact. For example, for 3 x 7, the student says, "2 x 7 or double 7 is 14, and add one more set of seven is 21."
- **Automaticity or Mastery:** Student produces efficient (fast and accurate) answers. For example, for 3 x 7 the student quickly answers, "It is 21; I just know it."

Assessing the Facts

Basic facts should be assessed through observation, interviews, games, self-assessment, and strategy-focused paper/pencil practice. Although the goal is to have students recall facts in a reasonable time frame, researchers strongly caution against the use of timed tests.

Timed tests may lead to math anxiety in some students. "Timed tests as well as other speed-related materials (such as flash cards) cause slow, strong mathematical thinkers to become discouraged in class, develop math anxiety, and turn away from the subject" (Boaler 471). Math anxiety, in turn, causes a negative impact on students who use higher-level strategies, ones that rely on working memory, because the anxiety interferes with the working memory (Ramirez, Gunderson, Levine, and Beilock). This indicates that some of the best mathematical thinkers are often those most negatively affected by timed testing.

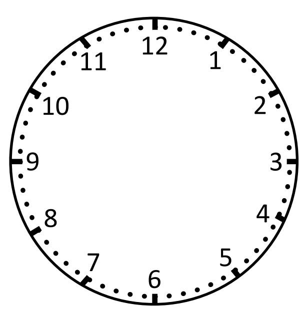
Basic Facts for Grade 4

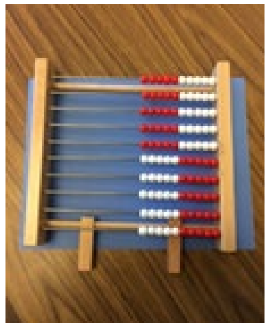
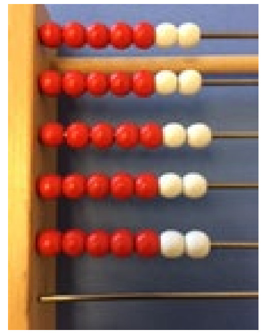
Multiplication facts to 81

End of grade expectations: Grade 4 and Grade 5									
0 x 0	1 x 0	2 x 0	3 x 0	4 x 0	5 x 0	6 x 0	7 x 0	8 x 0	9 x 0
0 x 1	1 x 1	2 x 1	3 x 1	4 x 1	5 x 1	6 x 1	7 x 1	8 x 1	9 x 1
0 x 2	1 x 2	2 x 2	3 x 2	4 x 2	5 x 2	6 x 2	7 x 2	8 x 2	9 x 2
0 x 3	1 x 3	2 x 3	3 x 3	4 x 3	5 x 3	6 x 3	7 x 3	8 x 3	9 x 3
0 x 4	1 x 4	2 x 4	3 x 4	4 x 4	5 x 4	6 x 4	7 x 4	8 x 4	9 x 4
0 x 5	1 x 5	2 x 5	3 x 5	4 x 5	5 x 5	6 x 5	7 x 5	8 x 5	9 x 5
0 x 6	1 x 6	2 x 6	3 x 6	4 x 6	5 x 6	6 x 6	7 x 6	8 x 6	9 x 6
0 x 7	1 x 7	2 x 7	3 x 7	4 x 7	5 x 7	6 x 7	7 x 7	8 x 7	9 x 7
0 x 8	1 x 8	2 x 8	3 x 8	4 x 8	5 x 8	6 x 8	7 x 8	8 x 8	9 x 8
0 x 9	1 x 9	2 x 9	3 x 9	4 x 9	5 x 9	6 x 9	7 x 9	8 x 9	9 x 9

http://www.edu.gov.mb.ca/k12/cur/math/facts/chart.pdf

Specific Fact Strategies

Multiplication by	Strategies
2	Connect to addition—doubling
3	Double and add one more group
4	Double, double
5	■ Relate to an analog clock—skip-counting by 5s  ■ Multiply by 10 and then divide by 2.

Multiplication by	Strategies
6	■ Multiply by 5 and then add one more group. ■ Multiply by 3 and then double.
7	Split the 7 into 5 + 2. Multiply by 5 and then add the multiplication by 2. For example, 7 x 4 → (5 x 4) + (2 x 4) The 100-bead abacus can help students see how this strategy works.   5 x 7 The red beads show 5 x 5. The white beads show 5 x 2. Students can clearly see the multiplication by 5 and by 2.
8	■ Double, double, and double ■ Multiply by 4 and then double
9	■ Multiply by ten and then subtract one group. ■ Students might use the patterns in the nine times table as a strategy. 1 x 9 = 9 2 x 9 = 18 3 x 9 = 27 4 x 9 = 36 5 x 9 = 40 6 x 9 = 54 7 x 9 = 63 8 x 9 = 72 9 x 9 = 81 10 x 9 = 90 ■ Patterns: ▪ Looking at the products in the column, the ones are decreasing by one and the tens are increasing by 1. ▪ The sum of the digits in the product always add up to 9. ▪ The tens digit is always one less than the number being multiplied by 9. For example, for 8 x 9, the product will have a 7 in the tens place and a two (7 + □ = 9) in the ones place.

- The book *The Best of Times* by Greg Tang can be used to help students develop the basic facts strategies. Each two-page spread deals with a specific times table. There is a poem to introduce the strategy, and then examples that include both the basic facts as well as the application of the strategy to larger numbers.

Grade 4: Number (4.N.6, 4.N.7)

Enduring Understandings:

Flexible methods of calculation in multiplication and division involve decomposing and composing numbers in a wide variety of ways.

Flexible methods of calculation in multiplication and division require a strong understanding of the operations and the properties of the operations.

There are a variety of appropriate ways to estimate products and quotients depending on the context and the numbers involved.

Essential Questions:

How can materials be used to model multiplication and division?

How can arrays be used to model multiplication and division?

How are multiplication and division related? How can this relationship help with calculations?

Specific Learning Outcome(s):	Achievement Indicators:
4.N.6 Demonstrate an understanding of multiplication (2- or 3-digit numerals by 1-digit numerals) to solve problems by • using personal strategies for multiplication with and without concrete materials • using arrays to represent multiplication • connecting concrete representations to symbolic representations • estimating products [C, CN, ME, PS, R, V]	➛ Model a multiplication problem using the distributive property [e.g., 8 × 365 = (8 × 300) + (8 × 60) + (8 × 5)]. ➛ Use concrete materials, such as base-10 blocks or their pictorial representations, to represent multiplication, and record the process symbolically. ➛ Create and solve a multiplication problem that is limited to 2 or 3 digits by 1 digit. ➛ Estimate a product using a personal strategy (e.g., 2 × 243 is close to or a little more than 2 × 200, or close to or a little less than 2 × 250). ➛ Model and solve a multiplication problem using an array, and record the process. ➛ Solve a multiplication problem and record the process.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
4.N.7 Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by • using personal strategies for dividing with and without concrete materials • estimating quotients • relating division to multiplication [C, CN, ME, PS, R, V]	(It is not intended that remainders be expressed as decimals or fractions.) ➢ Solve a division problem without a remainder using arrays or base-10 materials. ➢ Solve a division problem with a remainder using arrays or base-10 materials. ➢ Solve a division problem using a personal strategy, and record the process. ➢ Create and solve a word problem involving a 1- or 2-digit dividend. ➢ Estimate a quotient using a personal strategy (e.g., 86 ÷ 4 is close to 80 ÷ 4 or close to 80 ÷ 5).

BACKGROUND INFORMATION

Students should be encouraged to use a variety of strategies for multiplication and division for the following reasons:

- Sometimes a particular strategy makes more sense to one student than to another.
- Sometimes a strategy works better for a particular set of numbers.
- Being familiar with a variety of strategies allows students to use one strategy to calculate, and then use another strategy to check the answer (justify their answer).

Vocabulary

Terms for Multiplication

$$\underbrace{3 \times 4}_{\text{factors}} = \underset{\text{product}}{\underset{\downarrow}{12}}$$

Terms for Division

$$25 \div 6 = 4^{r1}$$

dividend divisor quotient remainder

Distributive Property: The distributive property refers to the idea that one or both of the factors in a multiplication question can be decomposed into two or more parts and each part multiplied separately and then added [e.g., 9 x 7 is equivalent to (9 × 5) + (9 × 2)].

Mathematical Language

multiply	remainder
multiplication	divisor
factor	expanded form
product	array
divide	base 10
division	estimate
quotient	estimation

Learning Experiences

Assessing Prior Knowledge

Materials: Math journals

Organization: Individual/Whole class

Procedure:

1. Ask students to solve each of the following problems in two different ways:
   a. Rosa is planning to arrange 48 books on six shelves. If she puts an equal number of books on each shelf, how many books will she put on each shelf?
   b. Mark has a six-page photo album. How many pictures does Mark have if each page holds eight pictures?
2. Have students share their solutions with the other members of the class. Encourage students to explain their reasoning by asking questions, such as the following:
   - Which strategy did you use to solve the problem?
   - What is another strategy you could use to solve the problem?
   - Will the strategy work for other problems involving division (multiplication)? Show me.
   - Which strategy do you prefer to use? Why?

Observation Checklist

Use students' responses to determine which strategies students know. Also, examine their responses to determine whether they can do the following:

- ❒ identify problem situations that call for the operation of multiplication
- ❒ identify problem situations that call for the operation of division
- ❒ describe and apply a thinking strategy to determine the product or quotient of two whole numbers
- ❒ describe and apply more than one thinking strategy to determine the product or quotient of two whole numbers

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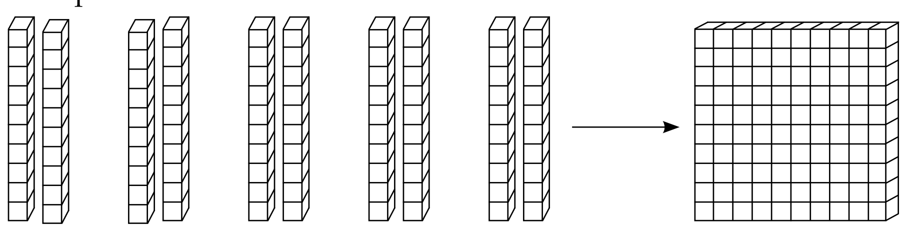
- **Use concrete materials, such as base-10 blocks or their pictorial representations, to represent multiplication, and record the process symbolically.**

Suggestions for Instruction

- Have students use base-10 blocks to model 5 × 20 (five groups of 20)

  Example:

  5 × 20 = 100

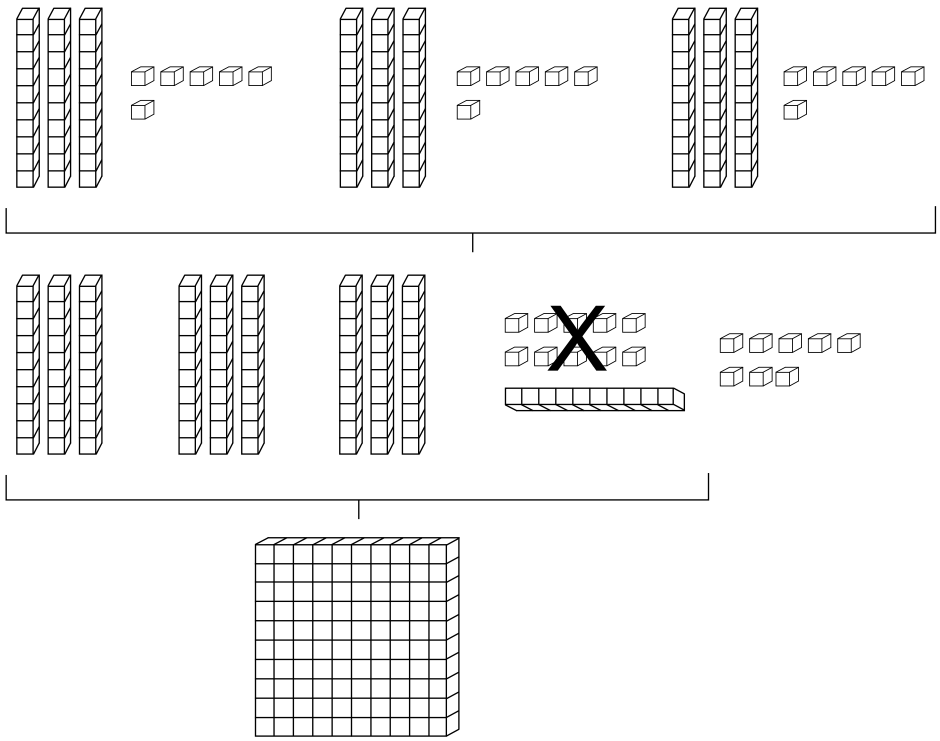
  Have students model 4 × 10, 3 × 10, 6 × 10.

  What do they observe? *All of the answers end in a zero. If the zero is covered the number remaining is the product of the multiplier and the numeral in the tens place (basic facts).*

  Note: It is important that teachers avoid telling students that when multiplying by multiples of ten you just add a zero. They need to recognize the patterns to understand that when multiplying by 10 there is always a zero in the one's place.

  Repeat the activity using 100. Students will see that when multiplying by 100, there is always a zero in both the one's place and the ten's place.

- Have students use base-10 blocks to model 3 x 36.



3 groups of 36 → 9 groups of ten and 18 ones → (9 tens + 1 ten) + 8 ones = 108

1 ten and 8 ones

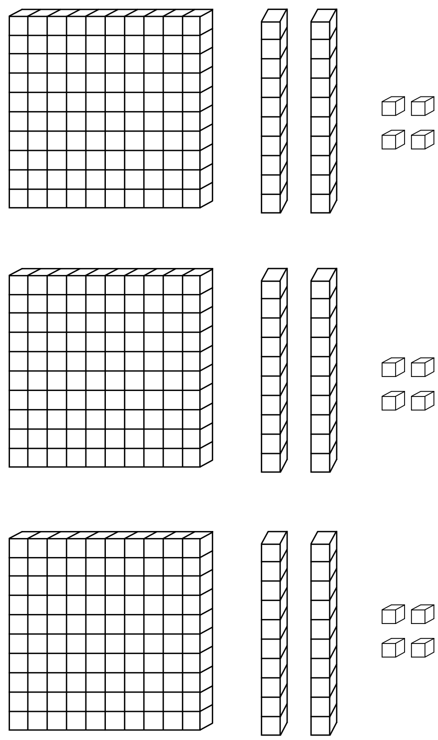
- Have students solve problems using the base-10 blocks. Provide opportunities for students to explain the process orally as well as symbolically.

  Examples:

  - Mark swims 280 laps in the pool each week. How many laps does he swim in 4 weeks?
  - Julie has a stamp collection. She puts 25 stamps on each page of her binder. If she has filled 7 pages of her binder, how many stamps has she collected?

Assessing Understanding

- Use base-10 blocks to show how to multiply 3 × 207. Use pictures and numbers to record your work.
- What multiplication problem does this picture show?



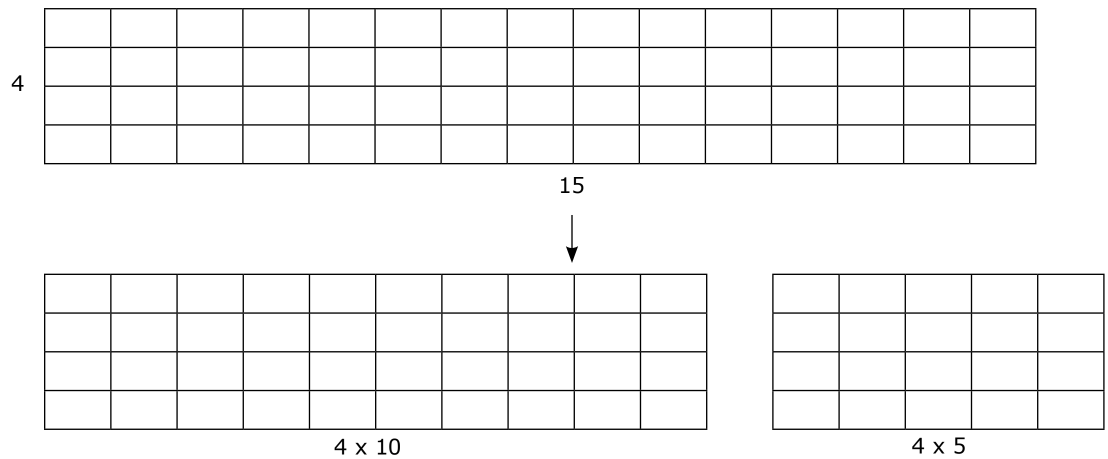
Explain how to use the picture to solve the problem.

- **Model and solve a multiplication problem using an array, and record the process.**

Suggestions for Instruction

Arrays can be made using grid paper or colour (square) tiles.

Example: 4 × 15



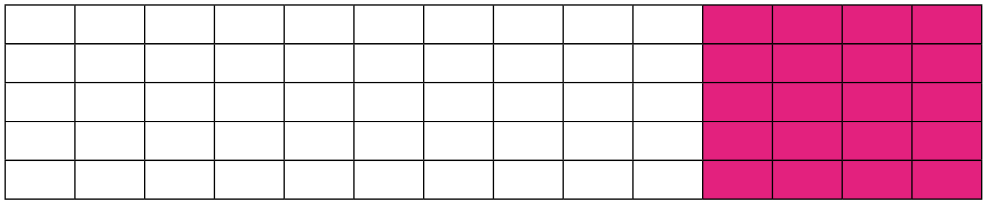
It is easier to multiply if the 15 is decomposed into 10 and 5.

(4 × 10) + (4 × 5) ⟶ 40 + 20 = 60

Decomposing numbers using arrays serves as an introduction to using the distributive property.

Assessing Understanding

- What multiplication problem does this array show? (5 × 14)



- Explain how you can use the array to solve the problem.

  Use the problem 6 x 18. Explain why you might chose to split (decompose) the 18 into 10 and 8.

  Is there another way that you might chose to use this strategy for this problem? (Students might break the problem down into (6 x 9) + (6 x 9).)

- **Model a multiplication problem using the distributive property [e.g., 8 × 365 = (8 × 300) + (8 × 60) + (8 × 5)].**
- **Create and solve a multiplication problem that is limited to 2 or 3 digits by 1 digit.**
- **Solve a multiplication problem and record the process.**

Prior Knowledge

In order to be successful using the distributive property students need to be able to represent 2-digit and 3-digit numbers in expanded form.

Note: Students should have a good understanding of 2-digit × 1-digit multiplication before introducing 3-digit × 1-digit multiplication.

The distributive property can be recorded in several ways.

Examples:

4 × 52
4 × 52 = (4 × 50) + (4 × 2) = 208
4 × 52 = (2 × 52) + (2 × 52) = 208

Suggestions for Instruction

- **Solving Multiplication Problems:** Ask students to solve problems using expanded form.

  Examples:

  - Troy has $34 in his bank. Sarah has 3 times as much in her bank. How much money does Sarah have in her bank?
  - There are 28 students in each of the four Grade 4 classes in the school. How many Grade 4 students are in the school?
  - Health Canada says that children should have 60 minutes of exercise each day. How many minutes should a child have in one week?
  - Eva's pedometer shows that it takes 428 steps to go around the perimeter of the gym. If Eva goes around the gym five times, how many steps will her pedometer show?
  - The school is having a pancake breakfast. Three hundred seventy-six tickets have been sold. If each person will be served 3 pancakes, how many pancakes do they need to prepare?

- **What is the problem?** Have students create and solve problems (using the distributive property) for given factors.

  Examples:

  - Create and solve a problem that includes the factors 5 and 231.
  - Create and solve a problem that includes the factors 6 and 345.

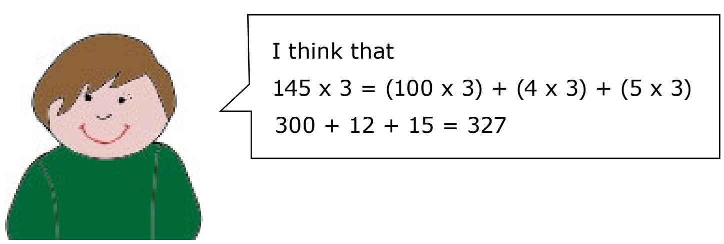
  Extension: Use dice (1 to 6 or 1 to 9) to determine the factors.

  Example:

  Students roll one die to determine the multiplier and then two or three dice to determine the multiplicand.

  Note: Place value dice could also be used.

Assessing Understanding: Paper-and-Pencil Task or Interview



Do you agree with his solution? Explain your thinking.

- **Estimate a product using a personal strategy (e.g., 2 × 243 is close to or a little more than 2 × 200, or close to or a little less than 2 × 250).**

Suggested Estimation Strategies:

- Make "friendly" numbers by rounding (one or both factors) to the nearest multiple of 10 or 100 (e.g., 4 × 62 is about 4 × 60 = 240).
- Round one factor up and one down (e.g., 33 × 8 is about 30 × 10 = 300).
- Using front-end estimation (e.g., 219 × 9 is about 200 × 9 = 1800).

Note: Students should estimate before solving a problem/question.

Learning Experiences

- **Is it reasonable?** Present students with a scenario along with estimations that have been done using different strategies. Have students determine which estimate is the most reasonable based on the scenario.

  Example:

  - Zoe has 4 pieces of ribbon. Each piece is 38 cm long. About how many centimetres of ribbon does she have?

    Possible estimates:

    Using front-end estimation: 4 x 30 = 120

    Using "friendly" numbers: 4 x 40 = 160

    Which estimate is more reasonable? Justify your thinking.

Assessing Understanding: Paper-and-Pencil Task or Interview

- Give an estimate for each of the following and explain the strategy you used.
  - 3 x 87
  - 211 x 9
  - 59 x 4
- Jacob drives 425 km each day. About how far will he have travelled after four days? Explain your thinking.
- Explain the strategy used for each of these estimations.
  - 431 x 4 = 400 x 4 or 1600
  - 68 x 8 = 70 x 8 or 560
  - 43 x 8 = 40 x 10 or 400

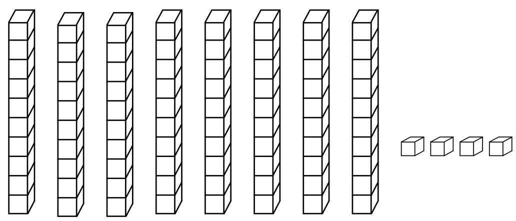
- **Solve a division problem without a remainder using arrays or base-10 materials.**
- **Solve a division problem with a remainder using arrays or base-10 materials.**

Suggestions for Instruction

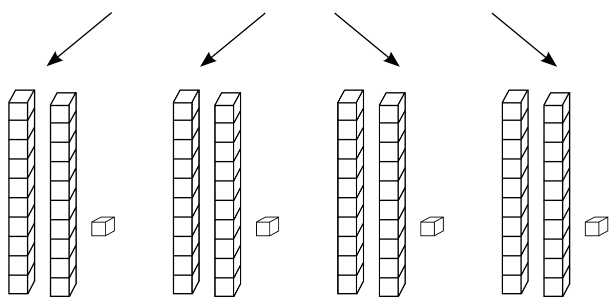
- **Using Base-10 Materials**

  Example:

  84 ÷ 4



Make the 84.



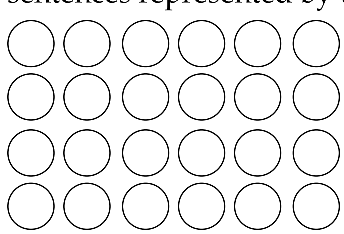
Divide it into four equal groups.

84 = 80 + 4

(80 ÷ 4) + (4 ÷ 4) = 20 + 1

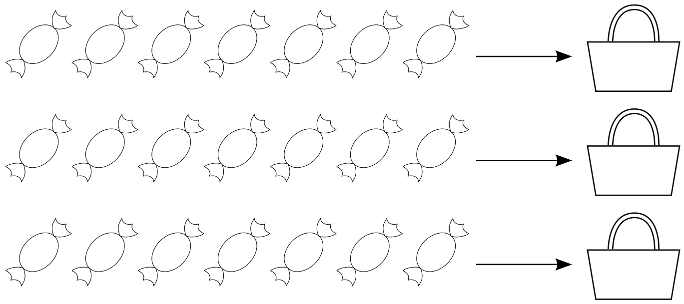
20 + 1 = 21

- **Using an Array:** It is important that students understand the relationship between multiplication and division. Have students write the number sentences represented by this array.



4 × 6 = 24 6 × 4 = 24 24 ÷ 4 = 6 24 ÷ 6 = 4

- **Using an Array to Solve a Problem:** You have 21 candies to share equally into 3 treat bags. How many candies does each bag get? Use an array to solve the problem.



Show the candies in a 3 x 7 array.

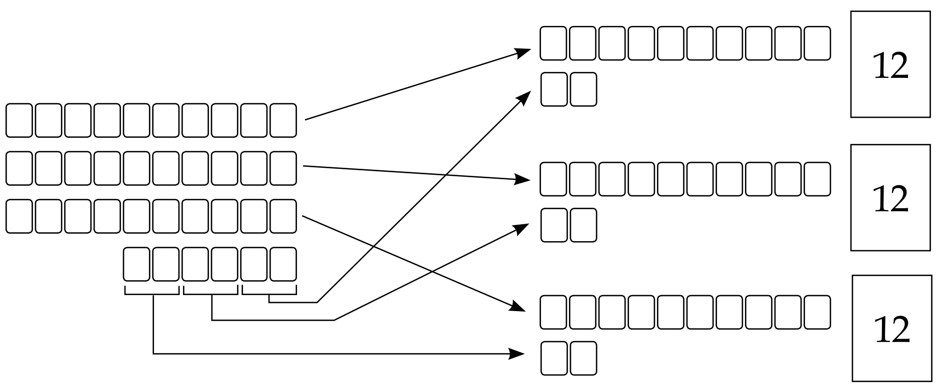
Demonstrate the division using sharing—one for the first bag, one for the second, and one for the third, et cetera.

Represent the action with a number sentence.

21 ÷ 3 = 7 where 21 represents the number of candies, 3 represents the number of bags and 7 represents the number of candies in each bag.

- **Arrays and 2-Digit Division:** Jonas has 36 hockey cards on 3 pages of his binder. How many cards are on each page?

Demonstrate the solution.



Represent the 36 cards using counters.

Demonstrate that the cards can be arranged in an array using the equal sharing process.

Record the process symbolically.

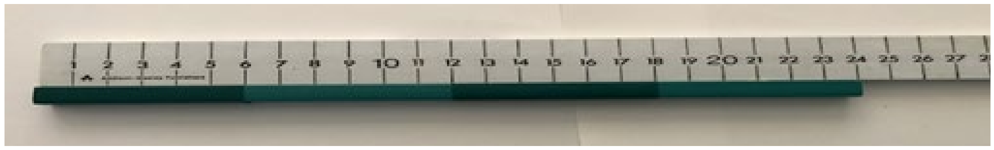
30 ÷ 3 = 10 and 6 ÷ 3 = 2

10 + 2 = 12 cards per page

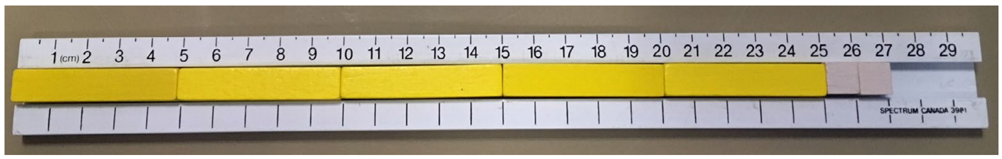
Extension: Ask student to represent this problem using a multiplication number sentence (3 × ? = 36)

- **Using Cuisenaire Rods:** Cuisenaire rods can help make the division process visible.

  Example: 24 ÷ 6 = 4



  Showing remainders: 27 ÷ 5 = 5 R2



Assessing Understanding: Performance

1. Give students the following problems. Have them use manipulatives to solve them. Ask them to explain their strategies.
   - There are 48 chocolates in a box. If Martha eats 4 chocolates each day, how many days will the box last?
   - The baker puts 6 cookies in each package. If there are 45 cookies, how many packages can the baker make? Will there be cookies left over?
2. Ask students to use a model to explain to their partner how to share 65 gumballs among 4 friends.
3. Roll two dice (1 to 9) to create a 2-digit dividend. Arrange the order of the digits so that when divided by 7, it will give you the lowest remainder. Record the remainder after each roll. Total the score (reminders) after 5 rolls. The player with the lowest score wins.

   Example:

   If you roll a 4 and a 5, decide if the dividend will be 45 or 54.

   45 ÷ 7 = 6 R3 and 54 ÷ 7 = 7 R5 so it is better to chose 45 as the dividend. 3 is recorded as the score for that roll.

   Extension: Try the game using different divisors.
4. Present the following problem.

   Mrs. Wong bought a package of 52 pencils to be shared equally among her 4 children. Mr. Rodriquez bought a package of 40 pencils to be shared equally among his 3 children. Which children received the most pencils—the Wong family or the Rodriquez family?

   Use an array to solve the problem.

   Are there any pencils left over?

5. Ask students to model three different division questions of their choice using base-10 blocks. Have them write the division sentence for each.

- **Solve a division problem using a personal strategy, and record the process.**

Suggestions for Instruction

- **Personal Strategies:** Encourage students to come up with their own strategies. Have them explain how their strategies work.

  Ask students if they used place value in their strategies. If so, have them explain how it was used.

  Note: Students are not expected to use the standard algorithm. Some students, however, may have seen it used outside of the classroom. If it is suggested acknowledge it as a strategy but encourage students to find other ways to solve the problems.

  Strategy examples: $36 \div 6 =$ ____

  a. $$36 \div 6 = (\overset{5}{30 \div 6}) + (\overset{1}{6 \div 6})$$
  $$5 + 1 = 6$$

  b. $$\begin{array}{r} 5 + 1 = 6 \\ 6\overline{)\quad 30 + 6} \end{array}$$

  c. $$\begin{array}{r} 5 + 1 = 6 \\ 6\overline{)\,36\qquad} \\ \underline{-\,30}\qquad \\ 6\qquad \\ \underline{-6}\qquad \end{array}$$

  d. Repeated subtraction

  36 – 6 = 30

  30 – 6 = 24

  24 – 6 = 18

  18 – 6 = 12

  12 – 6 = 6

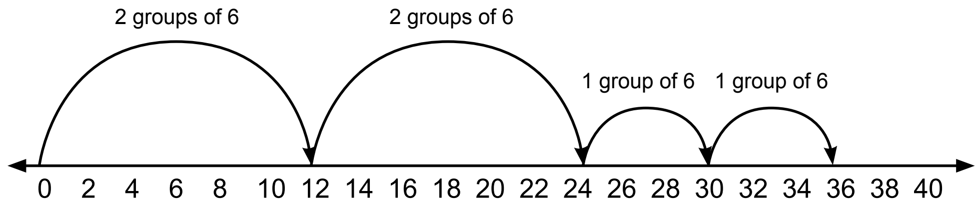
  6 – 6 = 1

  There are 6 sixes in 36.

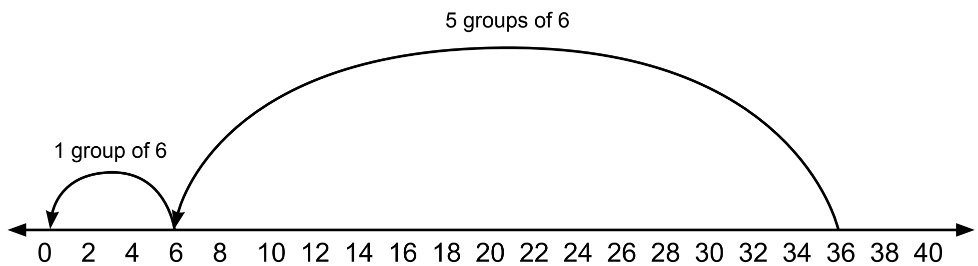
e. I know that two sixes are 12 and that 12 and 12 are 24. That is 4 sixes.
36 – 24 = 12 and that is another two sixes so there are 4 + 2 or 6 sixes in 36.

f. Number Line

Building Up—Adding



Building Back—Subtracting



g.
$$\begin{array}{r|l}
6\overline{)36} & \\
\underline{-30} & 5 \\
6 & 1 \quad 5 + 1 = 6 \\
\underline{-6} & \\
0 &
\end{array}$$

- ***A Remainder of One*** by Elinor J. Pinczes, illustrated by Bonnie MacKain: This is the story of the Queen's 25th marching corps. Joe, one of the bugs, wants to march in the parade but every time they group the 25 troupe members he seems to be the odd one out—the remainder of one. The troupe keeps regrouping until there is no remainder.

  Use this book to have students model the division for each regrouping.
- Extension: Students could create their own story involving remainders.

Assessing Understanding

Paper-and-Pencil Task:

Have students use two different strategies to calculate 54 ÷ 6.

Interview:

Ask students to explain how to use 92 = 30 + 30 + 30 + 2 to calculate 92 ÷ 5.

- **Create and solve a word problem involving a 1- or 2-digit dividend.**

Suggestions for Instruction

- **Number Draw:** Prepare number tickets or a spinner with the digits 1 to 9. Prepare number tickets or a spinner (could also use tens and ones place value dice) with 2-digit numbers. Have students select one 1-digit number and one 2-digit number and then create and solve a division problem using personal strategies. Repeat.
- **Multiplication, Division, or Both?** Present students with problems. Have students determine whether the problem can be solved using multiplication, division, or both operations.

  Examples:

  - Joey places five rows of 24 chairs in the gym. Tracy places twice as many chairs as Joey. How many chairs are placed in the gym?
  - Ninety-six apples are shared equally into 4 baskets. If one of the baskets is shared equally among 8 people, how many apples does each person get?
  - Sarah and Jim buy three pieces of chocolate fudge that each cost 28¢. If they share the cost of the fudge equally, how much does each person pay?

  Have students work in pairs to classify the problems as multiplication, division, or both multiplication and division. Have the students represent each problem with appropriate number sentences and then solve the problem.

- **Make It Multiplication:** Give students a set of problems. Have them write both a division number sentence and a multiplication number sentence that could be used to solve the problem.

  Examples:

  1. Kate has 32 beads to share equally with her two friends. How many beads will each friend get? (e.g., 32 ÷ 2 = ?, 2 x ? = 32 or ? x 2 = 32)
  2. There are 91 stickers to be shared equally with 7 people. How many stickers will each person get?
  3. You travel 84 km in 3 days. If you travel the same distance each day, how far did you travel each day?
  4. Sixty students are going on a bus to the park. If 3 students can fit on each seat, how many seats are needed for the whole group?

Assessing Understanding

- **Paper and Pencil:** The answer is 6. What is the question? Have students create a division problem with a quotient of 6.
- **Interview:** Ask the student to explain how to use multiplication to solve the following: Ms. Hardy's 4 children have 52 stickers to share equally. How many stickers will each child get?

- **Estimate a quotient using a personal strategy (e.g., 86 ÷ 4 is close to 80 ÷ 4 or close to 80 ÷ 5).**

Note: Using facts and fact strategies can help students make more reasonable estimates for division problems.

Background Information

When estimating students might need to change one or both numbers so that familiar multiplication and division facts can be used.

Examples:

- 43 ÷ 5 is about 45 ÷ 5 = 9, or 43 ÷ 5 is about 40 ÷ 5 = 8
- 33 ÷ 8 is about 36 ÷ 9 = 4, or 33 ÷ 8 is about 28 ÷ 7 = 4

Possible Estimation Strategies for Division:

- Nearest Multiple of 10: Present the following problem:

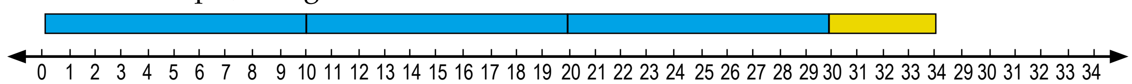
  Mandy has 34 cm of string. About how many pieces of string, each 5 cm long, can be cut from this string?

  Draw attention to the word *about.* This indicates that an estimation rather than an exact answer is required.

  Base-10 materials can be used to focus on the place values of the numbers. Cuisenaire rods and a centimetre ruler could also be used.

  Model your thinking (think aloud) as you decide how to estimate.

  Example, using Cuisenaire rods:



  "I cannot make 34 multiplying by 5. I see that 34 is close to 30 and I can make 30 multiplying by 5. 34 is also close to 40 and I can make 40 multiplying by 5. I am going to use 30 because it is closer to 34 (only 4 away) than 40 is (6 away).

  30 ÷ 5 = 6 so about 6 pieces of string can be cut from the string."

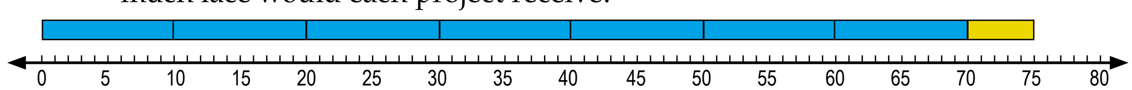
  When does this strategy work? Present other examples and ask students to decide if the nearest multiple of 10 strategy will work (e.g., 41 ÷ 7, 68 ÷ 9, 23 ÷ 5).

  Students should understand that this is a good strategy if the divisor divides evenly into the multiple of 10.

- Compatible Numbers with Compensation: This strategy that can be used when the divisor in the problem does not divide evenly into multiples of ten.

  Present the following problem:

  You have 75 cm of lace to share equally among 9 craft projects. About how much lace would each project receive?



  Think aloud:

  "Nine does not divide evenly into 70 or 80. I can see that 75 is halfway between 70 and 80 but 9 does not divide evenly into 75. What number close to 75 is divisible by 9? I know that 72 is divisible by 9 so I will use 72 ÷ 9 to estimate the quotient. 72 ÷ 9 = 8 so each project will get about 8 cm of lace."

Suggestions for Instruction

- **Estimate or Calculate?** Present students with problems and have them determine if the problem requires an estimate or an exact answer (calculation).

  Examples:

  - Mason rode his bike every day for 7 days. He cycled 47 km altogether. About how far did he cycle each day?
  - Eva put $8 in her bank each week. How long did it take her to save $64?
  - Ian has $80. About how many books could he buy if each book costs $9?
  - Pencils come in packages of 24. If they are shared equally among 4 children, how many pencils will each child get?
  - The tennis ball factory puts 3 balls in each package. If there are 42, how many packages can they make?

Assessing Understanding: Performance and Interview

- Give pairs of students the following questions:

  43 ÷ 6 = ____

  67 ÷ 5 = ____

  Have them estimate a quotient and then explain their strategy to their partner. Explain why their estimate is high or low.

- Give the student the following division problems:
  - The football team's score was 38. If touchdowns are worth 7 (including the convert), about how many touchdowns might they have scored?
  - Over the past 6 days Mark has eaten a total of 27 cookies. About how many cookies did he eat each day?

  Have the student estimate and explain their reasoning.

- Ask the student to describe a situation in which
  - the quotient is determined using an estimate
  - the quotient is determined using an exact calculation

Putting the Pieces Together: Math Information Night

Organization:

Individual or small group activity

Materials:

Large poster paper, markers, et cetera

Directions:

Your class is planning a math information night for parents. In order to help the parents understand what you have been learning about multiplication and division, you are going to create a poster showing the strategies that you have learned.

Criteria:

- Each strategy is labelled.
- An example of the strategy and an explanation of how it works is included.
- Examples include both 2-digit and 3-digit numbers.
- Examples of estimation strategies are included.

Look for the following:

- ❒ The student/group has included
  - an array
  - base-10 representations
  - distributive property
- ❒ Examples used are appropriate for the strategy
- ❒ Explanations are organized and easy to follow
- ❒ Estimation strategies are included

Grade 4: Number (4.N.8)

Enduring Understandings:

Fractions are numbers with magnitudes.

A fraction represents a part of a whole or a set.

Fractions can be compared using a variety of models.

The size of the fractional part depends on the size of the whole.

Equal parts do not have to look the same, but they must be the same size or have the same amount of the whole.

Essential Questions:

What is a fraction?

Where do you use fractions in everyday life?

What is the numerator?

What is the denominator?

How are the numerator and denominator related?

Specific Learning Outcome(s):	**Achievement Indicators:**
4.N.8 Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to • name and record fractions for the parts of a whole or a set • compare and order fractions • model and explain that for different wholes, two identical fractions may not represent the same quantity • provide examples of where fractions are used [C, CN, PS, R, V]	➢ Represent a fraction using concrete materials. ➢ Identify a fraction from its concrete representation. ➢ Name and record the shaded and non-shaded parts of a set. ➢ Name and record the shaded and non-shaded parts of a whole. ➢ Represent a fraction pictorially by shading parts of a set. ➢ Represent a fraction pictorially by shading parts of a whole. ➢ Explain how denominators can be used to compare two unit fractions. ➢ Order a set of fractions that have the same numerator, and explain the ordering. ➢ Order a set of fractions that have the same denominator, and explain the ordering. ➢ Identify which of the benchmarks 0, $\frac{1}{2}$, or 1 is closest to a fraction. ➢ Name fractions between two benchmarks on a number line (vertical or horizontal). ➢ Order a set of fractions by placing them on a number line (vertical or horizontal) with benchmarks. ➢ Provide examples where two identical fractions may not represent the same quantity (e.g., half of a large apple is not equivalent to half of a small apple; half of ten berries is not equivalent to half of sixteen berries). ➢ Provide an example of a fraction that represents part of a set, and a fraction that represents part of a whole, from everyday contexts.

Prior Knowledge

Students may have had experience in Grade 3 exploring parts of a whole that has been divided into "fair shares" or equal-sized pieces. They have also described situations in which fractions were used and compared fractions of the same whole with like denominators.

Background Information

In Grade 4, the focus continues to be on developing a solid understanding of fractions less than or equal to one. Students will continue to build their understanding of fractional parts of a whole. They will also work with fractions that represent parts of a set.

Terminology

- **Fraction:** A number that represents part of a whole, part of a set, or a quotient in the form $\frac{a}{b}$, which can be read as *a* divided by *b*.
- **Numerator:** The number above the line in a fraction that can state one of the following:
  - the number of equal parts in a set to be considered
  - the number of equal parts of a whole to be considered
- **Denominator:** The number below the line in a fraction that can state one of the following:
  - the number of elements in a set
  - the number of equal parts into which the whole is divided
- **Unit Fraction:** A fraction with a numerator of 1.
- **Set:** Any collection of things, without regard to their order. The members (or elements) of a set could be numbers, names, shapes, and so on.

Suggested Manipulatives:

Fraction circles, fraction pieces, square (colour) tiles, Cuisenaire Rods, number lines, geoboards, paper (for folding), and egg cartons.

Mathematical Language

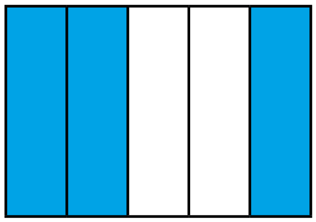
fraction	third
numerator	quarter
denominator	fifth
fair share	sixths
whole	eighths
one whole	tenths
half	one of ____ equal parts, set

Assessing Prior Knowledge

- Draw a picture for these fractions:

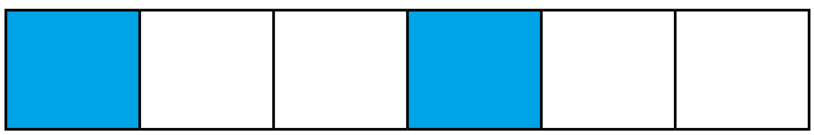
  $\frac{3}{4}$ $\frac{2}{6}$

- What fraction of each shape is shaded? What fraction is not shaded?



shaded ______

unshaded ______



shaded ______

unshaded ______

- Sam says that his fraction has a denominator of 8 and a numerator of 3. Draw a picture to match Sam's fraction.
- Order these fractions from the smallest to the largest:

  $\frac{7}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{5}{8}$ $\frac{4}{8}$ $\frac{6}{8}$

The student understands that

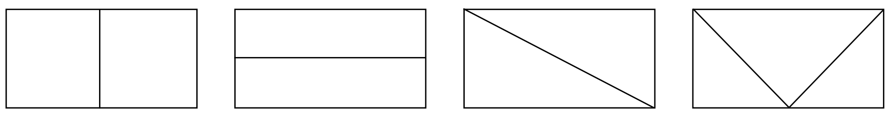
- ❒ the denominator represents the total number of parts the whole has been divided into
- ❒ the numerator represents the number of shaded parts or the parts being focused on
- ❒ when ordering fractions with the same denominator the larger the numerator the larger the fraction

- **Represent a fraction using concrete materials.**
- **Identify a fraction from its concrete representation.**
- **Name and record the shaded and non-shaded parts of a set.**
- **Name and record the shaded and non-shaded parts of a whole.**
- **Represent a fraction pictorially by shading parts of a set.**
- **Represent a fraction pictorially by shading parts of a whole.**

Suggestions for Instruction

- **Paper Folding:** Give students a rectangular piece of paper and have them fold the paper to show $\frac{1}{2}$. Compare the representations.

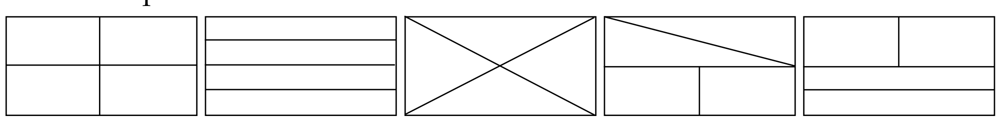
  Possible representations:

  Ask students to prove that their representation shows $\frac{1}{2}$ of the whole. (Students can cut out the shapes and overlap them to prove that they are congruent.)

  Repeat the activity having students show $\frac{1}{4}$.

  Possible representations:

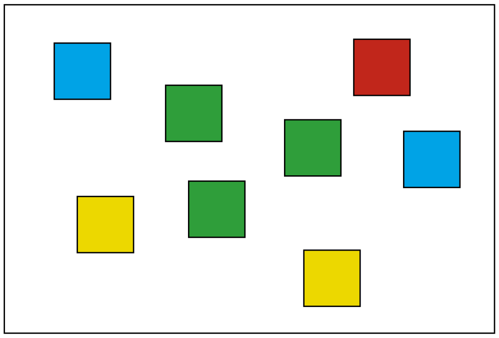
  

  Have students explain how they know that the non-congruent shapes represent $\frac{1}{4}$ of the whole.

- **Square (Colour) Tile Sets:** Have students choose 8 tiles. Have them identify the fraction of the set each colour represents.

  Note: Begin the activity by limiting students to use two colours only.

  Example:

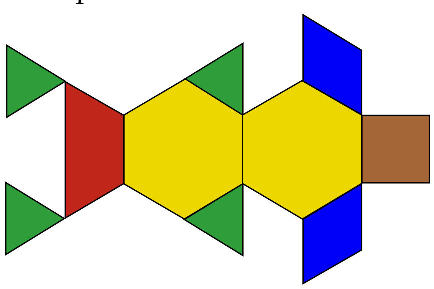
  

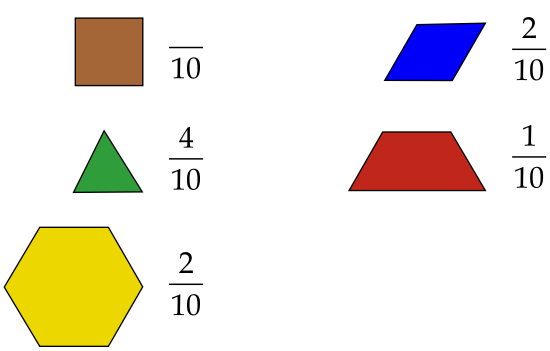
  Repeat using a different number of tiles.

- **Pattern Block Sets:** Have students make a simple design with the pattern blocks. Have them identify the fraction of the design/set that each shape represents.

  * Initially have students use only 2 shapes for their designs.

  Example:

Extension: Ask students to create designs that are

- $\frac{1}{4}$ yellow and $\frac{3}{4}$ green
- $\frac{2}{6}$ blue and $\frac{4}{6}$ red
- $\frac{1}{6}$ green, $\frac{2}{6}$ yellow, and $\frac{3}{6}$ blue

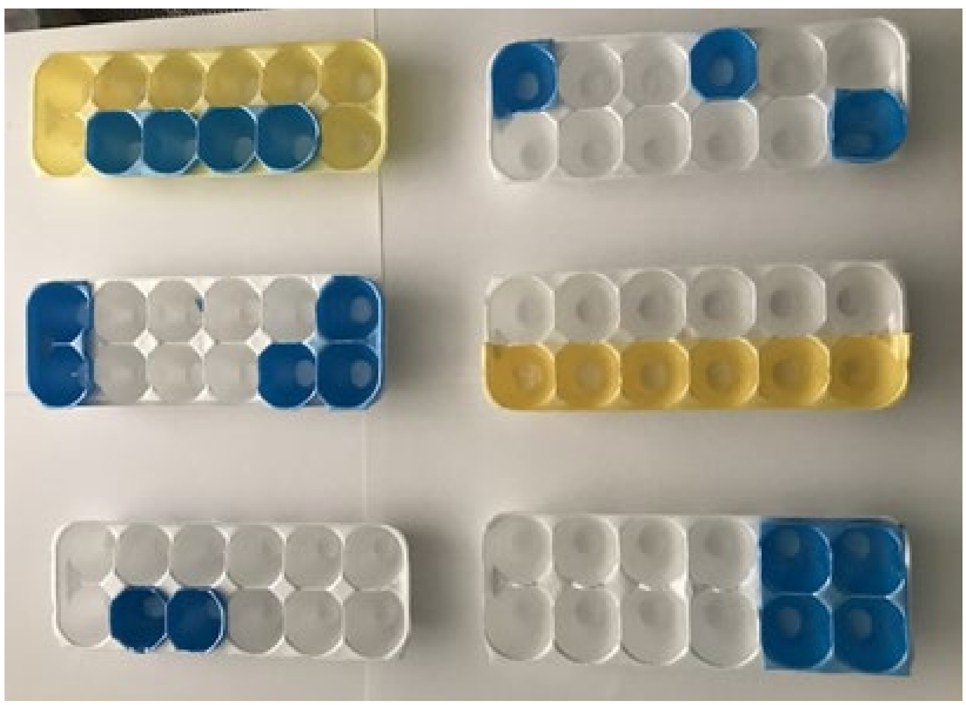
Have students make a design of their choice and then share their designs with a partner. Have the partner identify the fraction of the design/set that each shape represents.

- **Egg Carton Fractions:** Egg cartons are easy to obtain and easy to cut apart.

  Have students write a fraction for both the coloured/shaded part of the egg carton and the uncoloured/unshaded part of the carton.

  Note: Students may describe the coloured section in more than one way (e.g., for the carton with 6 coloured parts, students might record the fraction as $\frac{6}{12}$).

  Examples:



Extension: Cut out multiples of 2, 3, 4, and 6 cups so that students manipulate them to see that, for example, 1 group of three cups is $\frac{3}{12}$ but is also $\frac{1}{4}$ because 4 of the groups of 3 fit into the 12 cups in the carton.

BLM 4.N.8.1

- **Show the Fraction:** Give students a fraction. Have them represent it pictorially as part of a whole and as part of a set.

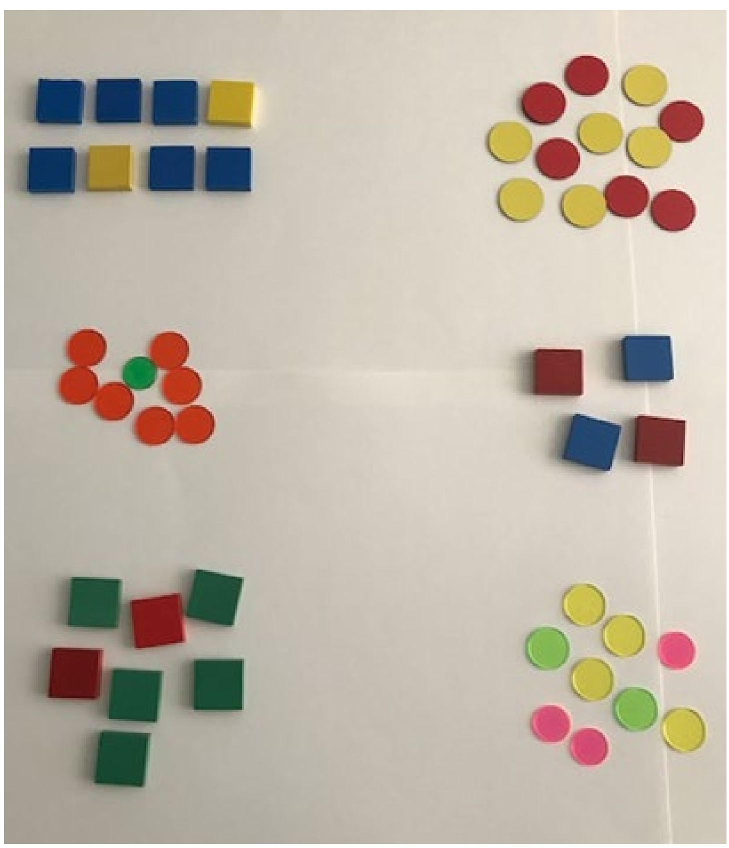
  Example:

  Directions: Draw a picture of each fraction as part of a whole and as part of a set.

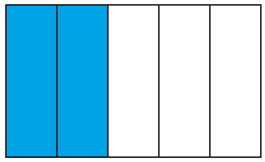
Fraction	of a whole	of a set
$\frac{3}{4}$		
$\frac{1}{3}$		
$\frac{2}{5}$		
$\frac{4}{8}$		

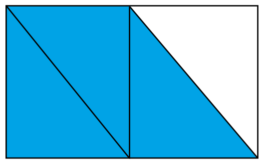
Assessing Understanding: Paper and Pencil

- What fraction does each colour represent in these sets?



- Write the fraction representing
    - the shaded part of the diagram
    - the unshaded part of the diagram

a. 

b. 

- Draw a diagram to show $\frac{3}{4}$ as
    - part of a whole
    - part of a set
- Sam has a dozen (12) eggs. He used 3 for his cookie recipe. What fraction of the eggs did he use?

Assessing Understanding: Performance Task

- In this task, students will be able to demonstrate their understanding of fractions of a set. Have paper squares available to represent cheese slices. Students can cut the squares to help them solve the following problems:
    - Show how 4 people can share 3 cheese slices.
    - Show how 3 people can share 2 cheese slices.
    - Show how 12 people can share 6 cheese slices.
    - Show how 6 people can share 4 cheese slices.

Recording Sheet Example:

Draw a picture of the divided cheese.   ___ cheese slices shared by ___ people. Each person's share is ___ (fraction).

- **Explain how denominators can be used to compare two unit fractions.**
- **Order a set of fractions that have the same numerator, and explain the ordering.**
- **Order a set of fractions that have the same denominator, and explain the ordering.**

Suggestions for Instruction

- **Paper Folding:** Give students a strip of paper. Give the following directions:
  1. Fold the paper in half. Open it up. How many sections do you see? (2) What fraction is represented by each section/part? ($\frac{1}{2}$) Record the fraction. Refold the paper.
  2. Fold the paper in half again. Open it up. How many sections do you see? (4) What fraction is represented by each section/part? ($\frac{1}{4}$) Record the fraction. Refold the paper.
  3. Repeat two more times.

  Ask students what they noticed about the sections as they were doing the folding.

  Each time the paper was folded the number of sections/parts increased but the size of the sections/parts got smaller.

  Extension: Fold paper strips separately and label the fraction.

$\frac{1}{1}$							
$\frac{1}{2}$				$\frac{1}{2}$			
$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$	
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

  The visual representation can help students conclude that the larger the denominator the smaller the fraction.

  Exit Slip: Would you rather have $\frac{1}{5}$ of a chocolate bar or $\frac{1}{10}$ of a chocolate bar? Explain your thinking.

- **Ordering Fractions:** Same Numerator

  The students in Mr. Grove's class were colouring fractions of paper strips. Josh coloured $\frac{2}{7}$ of his strip, Anna coloured $\frac{2}{5}$ of her strip, Levi coloured $\frac{2}{3}$ of his strip, and Sara coloured $\frac{2}{8}$ of her strip. Order the fractions from least to greatest. Explain your thinking using pictures and words.

  (Paper strips should be made available.)

- **Ordering Fractions:** Same Denominator

  Ms. Ang is baking four different items for the school bake sale. Each recipe calls for eggs. Ms. Ang has one dozen (12) eggs.

  - Cookies use $\frac{4}{12}$ of the eggs.
  - Cupcakes use $\frac{1}{12}$ of the eggs.
  - Brownies use $\frac{2}{12}$ of the eggs.
  - Lemon tarts use $\frac{6}{12}$ of the eggs.

  Order the fractions from the greatest number of eggs used to the least number of eggs used. Explain your thinking using pictures and words.

  (Egg cartons should be available, if needed, to help support student thinking.)

BLM 4.N.8.2

Assessing Understanding: Interview

1. Prepare a set of cards with fractions that have the same numerator and different denominators or use BLM 4.N.8.2.

$\frac{3}{8}$	$\frac{3}{4}$	$\frac{3}{5}$
$\frac{3}{10}$	$\frac{3}{9}$	$\frac{3}{12}$
$\frac{3}{7}$	$\frac{3}{3}$	$\frac{3}{6}$

Place the cards face down on the table. Have the student select 4 of the cards and order them from least to greatest. Ask them to explain their thinking.

BLM 4.N.8.3

2. Prepare a set of cards with fractions that have the same denominator and different numerators or use BLM 4.N.8.3.

$\frac{6}{10}$	$\frac{4}{10}$	$\frac{1}{10}$
$\frac{8}{10}$	$\frac{2}{10}$	$\frac{7}{10}$
$\frac{3}{10}$	$\frac{5}{10}$	$\frac{10}{10}$

Place the cards face down on the table. Have the student select 5 of the cards and order them from least to greatest. Ask them to explain their thinking.

The student understands that

- ❐ the greater the denominator the smaller the fraction
- ❐ when ordering fractions with a common denominator, the larger the numerator the larger the fraction

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- **Identify which of the benchmarks 0, $\frac{1}{2}$, or 1 is closest to a fraction.**
- **Name fractions between two benchmarks on a number line (vertical or horizontal).**
- **Order a set of fractions by placing them on a number line (vertical or horizontal) with benchmarks.**

Suggestions for Instruction

Note: The most important benchmarks or referents for fractions are 0, $\frac{1}{2}$, and 1. These benchmarks help students understand the relative size of fractions. Noticing the relationships between the numerators and the denominators can help students determine the placement of a fraction related to the benchmarks.

- **What Am I Thinking?** Tell students that you are thinking of a fraction between 0 and 1. What might my fraction be? Record student responses.

  Repeat having students suggest possible fractions:

  - between 0 and $\frac{1}{2}$
  - between $\frac{1}{2}$ and 1

  Have students explain how they know that their fraction belongs between the given benchmarks.

BLM 4.N.8.4

- **Less than $\frac{1}{2}$ or Greater than $\frac{1}{2}$:** Have students use fraction bars to determine whether a given fraction is less than or greater than $\frac{1}{2}$.

  Example:

  Is $\frac{5}{8}$ less than or greater than $\frac{1}{2}$? Students can compare the $\frac{5}{8}$ to $\frac{1}{2}$ to see that $\frac{5}{8}$ is greater than $\frac{1}{2}$.

$\frac{1}{2}$	$\frac{1}{2}$

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

$\underbrace{\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}}_{\frac{5}{8}}$

BLM 4.N.8.5

- **Less than $\frac{1}{2}$ or Greater than $\frac{1}{2}$ Sort:** Prepare a set of fraction cards or use BLM 4.N.8.5. Have students work with a partner or in small groups to sort them into two groups—Less than $\frac{1}{2}$ and Greater than $\frac{1}{2}$. Have students explain their placement.

$\frac{5}{8}$	$\frac{3}{4}$	$\frac{4}{10}$
$\frac{6}{9}$	$\frac{3}{7}$	$\frac{2}{3}$
$\frac{4}{12}$	$\frac{2}{5}$	$\frac{2}{4}$

BLM 4.N.8.6

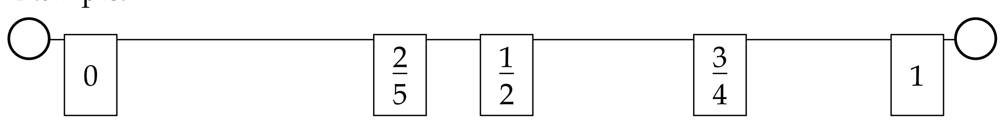
- **Close to 0 or Close to 1?** Prepare a set of fraction cards or use BLM 4.N.8.6. Have students work with a partner or small group to sort them into two groups—Close to 0 and Close to 1. Have them explain their placement.

  Example:

$\frac{1}{8}$	$\frac{9}{10}$	$\frac{3}{4}$
$\frac{2}{7}$	$\frac{2}{10}$	$\frac{7}{9}$
$\frac{5}{6}$	$\frac{4}{12}$	$\frac{1}{2}$

- **"Clothesline" Fractions:** Set up a "clothesline" number line using 2 strong magnets and a long piece of string. Make tent cards with 0, $\frac{1}{2}$, 1 along with other fractions less than 1. Place the benchmarks (0, $\frac{1}{2}$, and 1) on the line. Have students place a few of the remaining fractions on the number line and justify their placement.

  Example:



  Note: Having every student in the class place a fraction is very time consuming and can result in a lack of focus on the part of the students. Instead, select two or three students and have them place a fraction.

  This activity could become a class routine with different students selected each day.

  This also provides formative assessment data.

Assessing Understanding

Performance:

Have students place fractions on the clothesline number line or paper number line (benchmarks included) and explain their placement.

Interview:

Use cards from the learning experiences above. Have the student sort the fractions into three groups—Close to 0, Close to $\frac{1}{2}$, Close to 1—and justify their placement.

Paper-and-Pencil Task:

Matt sorted fractions into two groups.

Here is his sort.

$\frac{1}{5}$ $\frac{1}{3}$ $\frac{2}{8}$

$\frac{9}{10}$ $\frac{6}{7}$ $\frac{10}{12}$

What might his sorting rule be? Explain your thinking.

- **Provide examples where two identical fractions may not represent the same quantity (e.g., half of a large apple is not equivalent to half of a small apple; half of ten berries is not equivalent to half of sixteen berries).**

Suggestions for Instruction

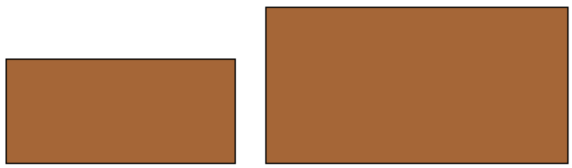
Note: It is important that students understand that a fraction represents the relationship between the part and the whole but that it does not provide information about the size of the whole or the size of the parts.

- **When Does Size Matter?** Show students two fruits or vegetables that have an obvious size difference (e.g., an apple and a grape, an orange and a watermelon, a pea and a potato). Cut each one in half. Point out that although the fraction we write for each piece is $\frac{1}{2}$, the halves are not the same because of the difference in the size of the fruits/vegetables.
  Have students suggest other examples.

- **Pizza Dilemma:** Kim and her friend Erin went out to eat, and each one ordered a pizza. After eating $\frac{1}{4}$ of each of their pizzas, Kim noticed Erin had a lot more pizza left. How could this happen? Explain using pictures and words. (Erin's pizza is larger than Kim's.)

- **Chocolate Bar Challenge:** James has 2 chocolate bars to share with his 6 friends.

  Here are the chocolate bars.

  James decides to divide each chocolate bar into thirds and then give $\frac{1}{3}$ to each friend.

  Do you agree with his plan? Why or why not?

Assessing Understanding: Journal Entry

- Ian saw a poster in the classroom that said:

  When comparing fractions, the whole matters.

  Explain what the poster means.

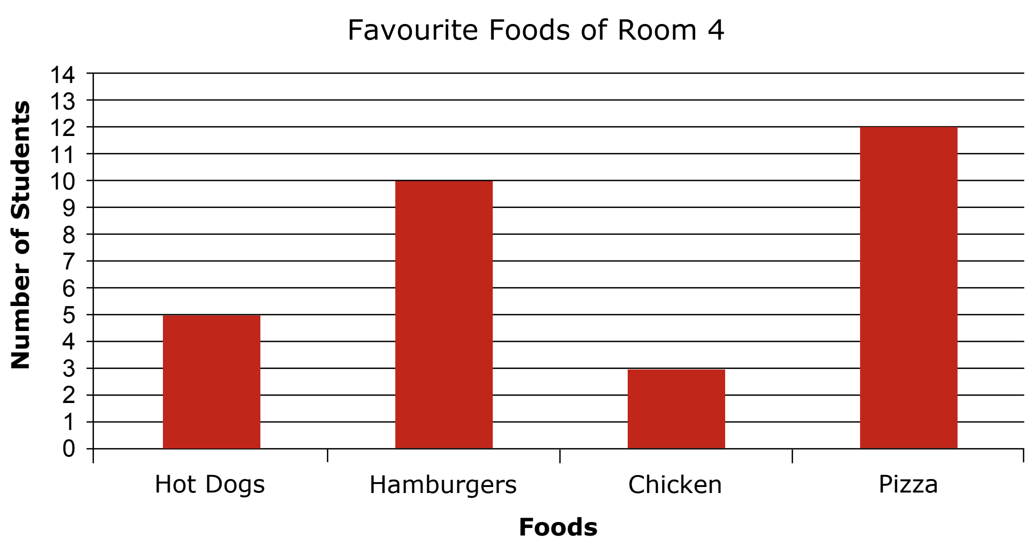
  The student understands that, when comparing fractions, the whole must be the same size for each fraction.

- **Provide an example of a fraction that represents part of a set, and a fraction that represents part of a whole, from everyday contexts.**

Suggestions for Instruction

- **Fraction Hunt:** Have students find examples of both fractions of a set and fractions of a whole from real-world contexts. Have them present their findings using pictures and symbols.

  Possible examples might include
  - Food—measuring for recipes
  - Time—fractions of an hour, day, month, year, etc.
  - Measurement tools
  - Classroom contexts—using student attributes (e.g., fraction of students with dark hair, wearing runners, etc.)
- **Classroom Routine(s):**
  - Have the daily attendance recorded as a fraction (e.g., fraction of students absent, fraction of students present).
  - Although Grade 4 students might not celebrate 100 Day, counting the number of days in school could be recorded as a fraction, either counting to 100 day (e.g., $\frac{40}{100}$) or using the total number of school days for the school year (e.g., $\frac{40}{186}$).
- **Connecting with Statistics:** Show students this graph. Have them summarize the findings using fractions of the set of 30 students (e.g., $\frac{5}{30}$ of the students like hot dogs).



Assessing Understanding: Performance

- Have the students
  - decide on a survey questions to ask the students in the classroom
  - gather the data
  - represent the data in graph form
  - summarize their findings using fractions of the set (number of students surveyed)

  Criteria (connect to Statistics and Probability):
  - survey question is clear
  - categories/choices are appropriate
  - graph is complete (labels, title, scale, bars)
  - data is accurately represented on the graph
  - summary of the data is accurately represented in fraction form

Putting the Pieces Together: The Birthday Party

Present the following problems related to a birthday party.

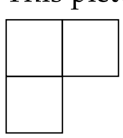
1. Alex's birthday is in September. Instead of telling his friends the date, he gives these clues:

   My birthday is $\frac{1}{2}$ the way through the month.

   What is the date of Alex's birthday? (15th)

2. Alex helped his mother bake the birthday cake. The recipe called for 3 eggs. If there were 7 eggs left in the egg carton, what fraction of the remaining eggs did they use? ($\frac{3}{7}$)

3. This picture represents $\frac{1}{4}$ of the cake.

   Draw the whole cake.

4. Alex is turning 9. He uses blue and yellow candles for the cake. What combinations of candles are possible? Record your answers in fraction form.

   Example:

Possible Combinations	
Yellow	**Blue**
$\frac{1}{9}$	$\frac{8}{9}$
	Etc.

5. Alex has 10 friends coming to the party. $\frac{2}{5}$ of his guests are girls, the rest are boys. How many girls are coming to the party? (4) How many boys are coming? (6)
6. The party starts at 12:00 p.m. and ends at 4:00 p.m. What fraction of the whole day will the party take up? ($\frac{4}{24}$)
7. Ten different games were played at the party.

   May won $\frac{4}{10}$. Sal won $\frac{2}{10}$, Mark won $\frac{1}{10}$, and Jonas won $\frac{3}{10}$.
   Order the children from the greatest number of wins to the least number of wins. (May $\frac{4}{10}$, Jonas $\frac{3}{10}$, Sal $\frac{2}{10}$, Mark $\frac{1}{10}$)
   What fractions of the guests did not win any of the games? ($\frac{6}{10}$)
8. Alex puts 12 treats in each goodie bag.

   In each bag,

   - $\frac{1}{4}$ of the treats are chocolates (3 × 10 = 30)
   - $\frac{7}{12}$ of the treats are candies (7 × 10 = 70)
   - $\frac{2}{12}$ of the treats are small toys (2 × 10 = 20)

   How many of each treat does Alex need to make 1 goodie bag?

   How many of each treat does he need to make 10 goodie bags?

   Recording sheet:

Treats	Number needed for 1 bag	Number needed for 10 bags
Chocolates		
Candies		
Toys		

The student

- is able to apply their understanding of fractions of a whole in problem-solving contexts
- is able to apply their understanding of fractions of a set in problem-solving contexts
- is able to make connections between fractions and real-world contexts

Grade 4: Number (4.N.9, 4.N.10, 4.N.11)

Enduring Understandings:

Decimals are an extension of our place value system.

Fractions and decimals are related.

Essential Questions:

How are decimals connected to place value?

How are fractions and decimals related?

Specific Learning Outcome(s):	Achievement Indicators:
4.N.9 Describe and represent decimals (tenths and hundredths) concretely, pictorially, and symbolically. [C, CN, R, V]	➛ Write the decimal for a concrete or pictorial representation of part of a set, part of a region, or part of a unit of measure. ➛ Represent a decimal using concrete materials or a pictorial representation. ➛ Explain the meaning of each digit in a decimal with all digits the same. ➛ Represent a decimal using money values (pennies and dimes). ➛ Record a money value using decimals. ➛ Provide examples of everyday contexts in which tenths and hundredths are used. ➛ Model, using manipulatives or pictures, that a tenth can be expressed as hundredths (e.g., 0.9 is equivalent to 0.90 or 9 dimes is equivalent to 90 pennies).
4.N.10 Relate decimals to fractions (to hundredths). [CN, R, V]	➛ Read decimals as fractions (e.g., 0.5 is zero and five-tenths). ➛ Express orally and in written form a decimal in fractional form. ➛ Express orally and in written form a fraction with a denominator of 10 or 100 as a decimal. ➛ Express a pictorial or concrete representation as a fraction or decimal (e.g., 15 shaded squares on a hundred grid can be expressed as 0.15 or $\frac{15}{100}$). ➛ Express orally and in written form the decimal equivalent for a fraction (e.g., $\frac{50}{100}$ can be expressed as 0.50).

Specific Learning Outcome(s):	Achievement Indicators:
4.N.11 Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by • using compatible numbers • estimating sums and differences • using mental math strategies to solve problems. [C, ME, PS, R, V]	➣ Predict sums and differences of decimals using estimation strategies. ➣ Solve problems, including money problems, which involve addition and subtraction of decimals, limited to hundredths. ➣ Determine the approximate solution of a problem not requiring an exact answer. ➣ Estimate a sum or difference using compatible numbers. ➣ Count back change for a purchase.

Prior Knowledge

Students have had no formal instruction related to decimals prior to Grade 4.

Background Information

Possible misconceptions:

- Students may mistakenly believe that the place value places to the right of the decimal point have the same names as those on the left (e.g., the first place to the right is the tens not the tenths).
- Students think that the larger the decimal the larger the number because they are applying what they understand about whole numbers to decimals.
- Students may ignore the decimal point altogether when reading decimal numbers.

Reading Decimal Numbers: It is important that students are able to read decimal numbers correctly. Avoid using the term *point* because it has no mathematical meaning to students. Instead, use *and* to represent the decimal (e.g., 2.5 should be read as two and five tenths, not two point five).

When writing numbers less than one use a zero in the one’s place. This helps emphasis that the decimal is less than 1.

Suggested Manipulatives: money, metre stick, ruler, egg cartons, hundred chart, hundred bead frame (Rekenrek), ten frames, base-10 blocks, Cuisenaire rods, number lines, measuring tape, and grid paper.

Mathematical Language

fraction(s)
decimals
tenths
hundredths
decimal point

Learning Experiences

Assessing Prior Knowledge

- Ask students what they know about decimals. Where have they seen them before?
- Show students this number $2.35. Ask them to read it. What does the 2 represent? What does the 35 represent?

- **Write the decimal for a concrete or pictorial representation of part of a set, part of a region, or part of a unit of measure.**
- **Represent a decimal using concrete materials or a pictorial representation.**
- **Explain the meaning of each digit in a decimal with all digits the same.**
- **Represent a decimal using money values (pennies and dimes).**
- **Record a money value using decimals.**
- **Provide examples of everyday contexts in which tenths and hundredths are used.**
- **Model, using manipulatives or pictures, that a tenth can be expressed as hundredths (e.g., 0.9 is equivalent to 0.90 or 9 dimes is equivalent to 90 pennies).**
- **Read decimals as fractions (e.g., 0.5 is zero and five-tenths).**
- **Express orally and in written form a decimal in fractional form.**
- **Express orally and in written form a fraction with a denominator of 10 or 100 as a decimal.**
- **Express a pictorial or concrete representation as a fraction or decimal (e.g., 15 shaded squares on a hundred grid can be expressed as 0.15 or $\frac{15}{100}$).**
- **Express orally and in written form the decimal equivalent for a fraction (e.g., $\frac{50}{100}$ can be expressed as 0.50).**

Suggestions for Instruction

Note: Begin decimals by working with tenths. Students come to understand that the unit (1) can be divided into 10 equal parts (tenths) and, to be able to record these parts, another place value has been added to the right of the one's place separated by a decimal point to show that it is a fractional part.

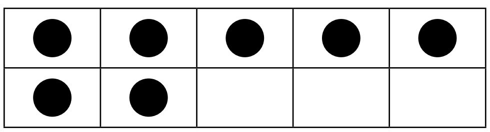
- **Ten Frames:** Ten frames are a familiar manipulative material and, as a result, students can easily apply their prior experience to their work with decimals.

  Use a set of ten frames. Select a ten frame. Have students identify the number of filled/shaded spaces as a fraction. Explain how the fraction can be written as a decimal.

  Have students record the number shown as a fraction and as a decimal.

  Have students record both the fraction and decimal for both the shaded and unshaded portions of each ten frame.

  Example:



Shaded/filled in	**Unshaded/not filled in**
Fraction: $\frac{7}{10}$	Fraction: $\frac{3}{10}$

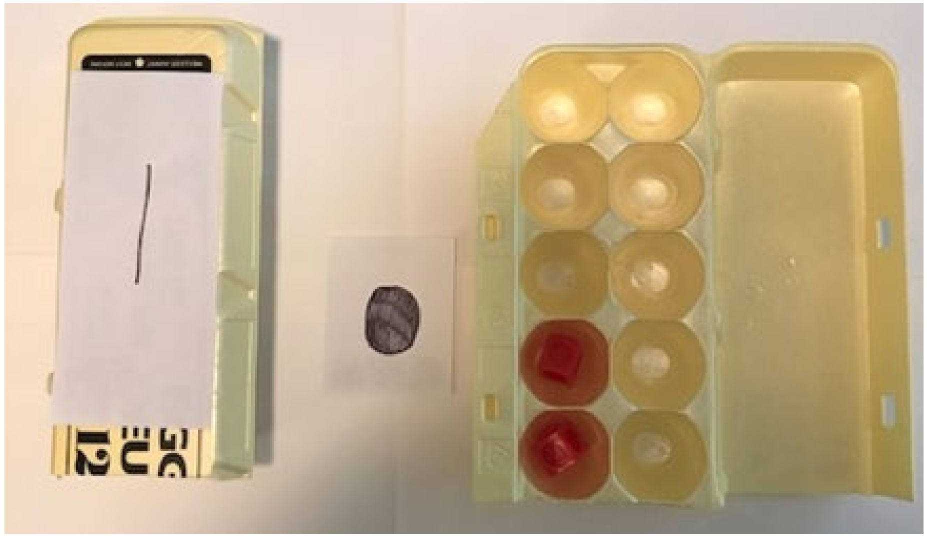
- **Egg Carton Ten Frames:** Keep the egg carton intact (lid included). Cut off two of the cups of the egg carton. Cut off the portion of the lid covering the two cups. Label the remaining portion of the lid with the number 1. The cup portion serves as a ten frame. When the ten frame is full the lid can be closed and students will see it as one whole.

  Example:

  The ten frame shows 0.7.

The student rolls a die and adds 5 more cubes.

3 cubes fill the ten frame, and the lid is closed and moved to the left of the decimal. The remaining 2 cubes are placed in a new ten frame.



Students can see that they now have one whole and 2 tenths. The visual representation supports the recording as 1.2.

Extension: Have students play a Race to Two or Three game.

Organization:

Students play with a partner.

Materials:

Each person needs egg carton ten frames and cubes, die (1 to 6 or 1 to 9), and a recording sheet.

Directions:

Students take turns rolling the die and adding cubes to match the number rolled to the ten frame. When a ten frame is filled, the lid is closed, and it is moved to the left of the decimal. Students record after each roll.

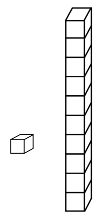
The first person to reach two or three (or more) is the winner.

Example:

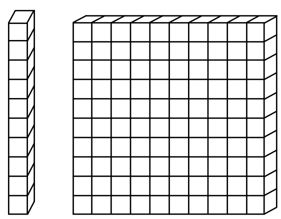
Roll #	Amount rolled	Total
1	0.7	0.7
2	0.5	0.7 +0.5 = 1.2
3	0.2	1.2 + 0.2 = 1.4

- **Tenths as Parts of Different Wholes:** Tenths can be represented using different materials or wholes. Students need experiences with a variety of these materials.

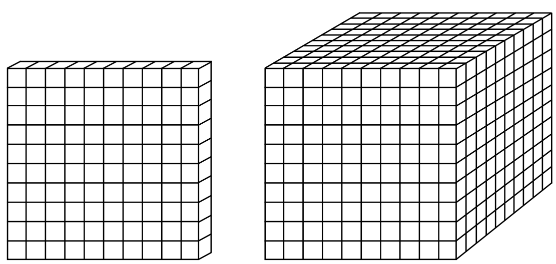
  Base-10: Note: Using base-10 materials for decimals can be confusing for some students because they are used to working with these materials for whole numbers (e.g., If the flat (100) is the whole, it is important that students understand that the flat is now 1 (or the whole) and not 100 as it is with whole numbers).



If the ten is the whole, the ones cube is one tenth of the ten.



If the hundred is the whole, the ten is one tenth of the hundred.



If the thousand is the whole, the hundred is one tenth of the thousand.

  Metre Stick: When exploring a metre stick students can see the following:

  - If the centimetre is the whole, 1 millimetre is one tenth of the centimetre.
  - If the decimetre is the whole, 1 centimetre is one tenth of the decimetre.
  - If the metre is the whole, 1 decimetre is one tenth of the metre.

  Money: Money is a non-proportional model for decimal numbers. Some students may find this model more difficult.

  Note: Although the penny is no longer in circulation in Canada, it is still used in electronic transactions. Most sets of "play" money include pennies. Pennies can also be made using labelled counters or paper/foam discs.

  As students work with money, they will see the following:

  - A penny is one tenth of a dime or one hundredth of a loonie/dollar (0.01).
  - A dime is one tenth of a loonie/dollar or 0.1.
  - A loonie/dollar is 1 whole.

- **Clothesline Number Line:** Place a zero at one end of the line and 1 at the other end. Have students place tent cards with numbers such as 0.7, 4 tenths, 0.5, et cetera, on the line and justify their placement.

BLM 4.N.9.1

BLM 4.N.9.2

- **Hundredth Squares/Grids:** Hundredth squares/grids are excellent tools to use to help develop an understanding of hundredths.

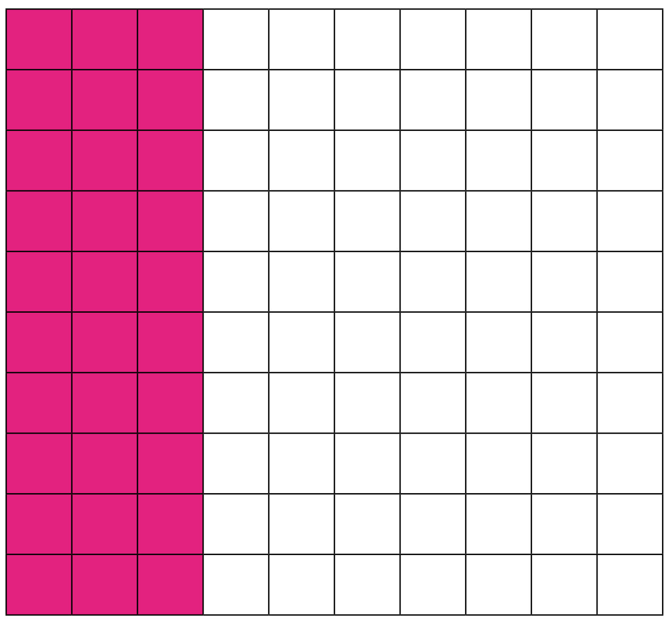
  Begin by having students shade in only tenths on grids. Have them identify the fraction and the decimal for both the shaded and unshaded parts of the grid.

  Example:

  Show 0.3 on the hundredth grid.

  Write the fraction and the decimal for the shaded part of the grid.

  Write the fraction and the decimal for the unshaded part of the grid.



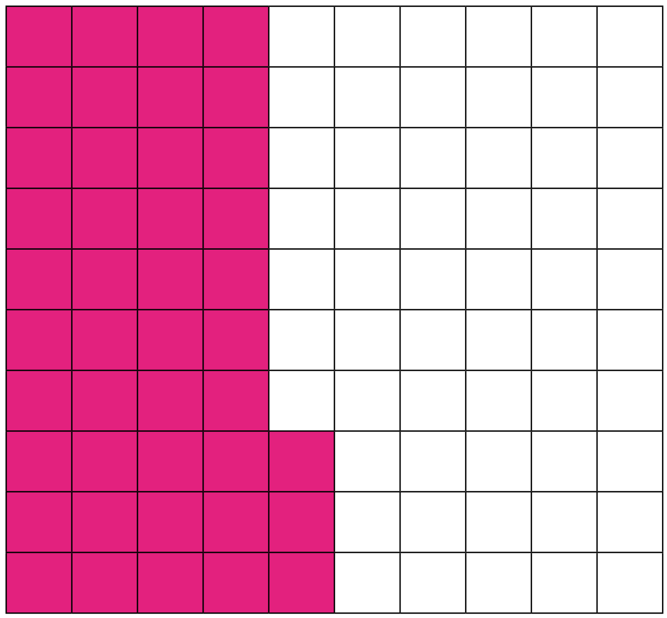
$\frac{3}{10}$ or 0.3 of the grid is shaded.

$\frac{7}{10}$ or 0.7 of the grid is unshaded.

Once students are confident with the tenths, introduce the hundredths place value.

Prepare cards with tenths and hundredths and have students select a card, and shade in the number shown. Have them label the fraction and the decimal for both the shaded and unshaded portions of the grid.

Example: Show 0.43 on the hundredth grid.



$\frac{43}{100}$ or 0.43 of the grid is shaded.

$\frac{57}{100}$ or 0.57 of the grid is unshaded.

Note: It is important that students understand that tenths can be written as hundredths (e.g., 0.8 can also be written as 0.80; 80 cents is written as $0.80 in decimal form).

- **Representing Hundredths with a Variety of Materials**

  Money: Have students represent decimal values with money and money values as decimals.

  Examples:

  a. How much money is shown in each picture? Record using decimals.

  ($1.35)

  ($2.73)

  b. Show each money value using coins or pictures of coins.

  $1.90 $3.46 $2.07

  Extension: Have students bring in grocery store flyers. Have students choose two items from the flyer and show and record at least two different ways (using cash) that they could pay for each item.

  Example:

Item and Cost	First Way	Second Way
a litre of milk for $1.89	1 loonie, 3 quarters, 1 dime, 4 pennies	4 quarters, 8 dimes, 1 nickel and 4 pennies

- **What's My Value?** Give students decimals in which all digits are the same. Have students identify the value of each digit.

  Example: In the number 4.44 what is the value of each of the digits? Explain using base-10 blocks, money, or hundredth grids, numbers, and words.

- **Decimals Scavenger Hunt:** Have students find examples of decimal use in their everyday life (home, school, TV, newspapers, other). Have them shared with the class. Prepare a class poster or book with the examples.

  Possible examples: items packaged in tens, fingers and toes, statistics for sports, gas prices, prices in flyers, et cetera

Assessing Understanding

Performance:

Prepare a set of tent cards with decimals representing tenths and hundredths. Have the student select 4 cards and place them on the clothesline number line (or paper number line) and justify the placement.

Example:

0.60	0.54	0.3
0.98	0.26	0.05
0.17	0.83	0.77

Performance:

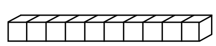
Provide a collection of play money. Ask the student to take a small handful of coins. Have the student count the money and then record the amount as a decimal.

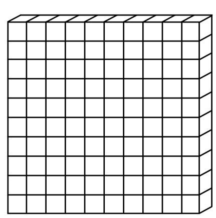
Have the student show $2.56 using the coins in two different ways.

Performance:

Give students the following problem.


represent hundredths


represent tenths


represent wholes

If you use ten base-10 blocks, what number can you make? Give at least 8 possible answers.

Paper-and-Pencil Task/Journal Entry:

How are fractions and decimals related? Explain using pictures and words.

Interview:

Have the student explain where decimals are used in everyday life.

Paper-and-Pencil Task

Show the student a ten frame. Have them write a fraction and decimal for both the shaded and unshaded parts.

Show the student a hundredth grid with 0.64 shaded in. Have the student write a fraction and a decimal for both the shaded and unshaded portions.

- **Predict sums and differences of decimals using estimation strategies.**
- **Solve problems, including money problems, which involve addition and subtraction of decimals, limited to hundredths.**
- **Determine the approximate solution of a problem not requiring an exact answer.**
- **Estimate a sum or difference using compatible numbers.**
- **Count back change for a purchase.**

Background Information

Mental math strategies used for the addition and subtraction of whole numbers can be used with decimal numbers.

Examples:

- Compatible (Friendly) Numbers: For example, for 0.73 + 0.24, think 0.73 is close to 0.75 and 0.24 is close to 0.25, so my estimate is 0.75 + 0.25 = 1.
- Front-End Addition: For example, for 42.5 + 31.8, think the nearest multiple of ten to 42.5 is 40 and the nearest multiple of ten to 31.8 is 30, so my estimate is 40 + 30 = 70.
- Front-End Subtraction: For example, for 5.38 – 2.41, think 5.38 is close to 5 and 2.41 is close to 2, so my estimate is 5 – 2 = 3.
- Rounding: For example, for 5.38 + 2.87, think 5.38 is close to 5 and 2.87 is close to 3, so my estimate is 5 + 3 = 8.

Suggestions for Instruction

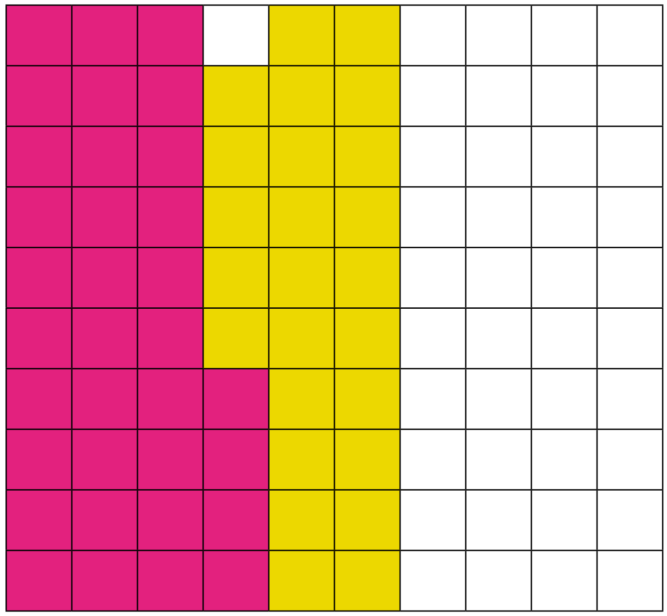
- **Adding and Subtracting from $1.00:** Provide students with hundredth squares. Have them model addition and subtraction from 1 using the squares.

  Note: The hundredth square can also be used to model addition and subtraction from $1.00. (Students should estimate before solving each problem.)

Examples:

Addition

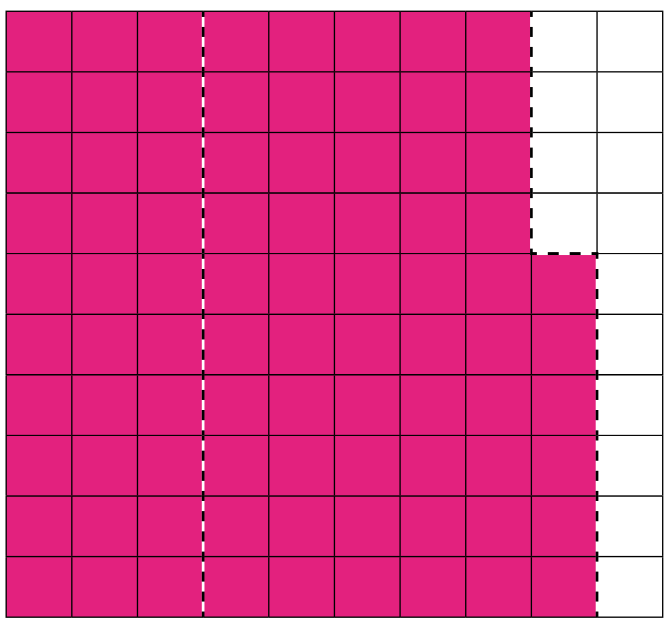
0.34 + 0.25



Shade in the 0.34. Shade in the 0.25. It is easy for students to see that the total is 0.59.

Subtraction

0.86 – 0.56



Shade in the 0.86. Cut off the unshaded portion. Using the shaded portion, cut out the 0.56. The remaining portion is the solution.

- **Base-10 Modelling:** Give students addition and subtraction questions and have them use base-10 blocks to model the process. (Have students estimate first.)

  Examples:

  - Marg has 1.35 m of red material and 2.67 m of blue material. How much material does she have altogether?
  - John has $25 to spend. If he buys a game for $19.56, how much money will he have left?
  - Ken ran 13.4 km on Monday, 12.82 km on Tuesday, and 14.07 km on Wednesday. How many kilometres did he run over the three days?

    The following week Ken ran a total of 35.2 km. In which week did he run the farthest? How much farther?
  - Ms. Allan's grocery bill came to $70.06. If she paid with a $100 bill, how much change did she receive?

- **Spending Spree:** Have students bring in flyers. Ask students to do the following:

  - Pick a flyer.
  - Choose 3 or 4 items that they might like to buy from the flyer.
  - Find the total cost of the items.

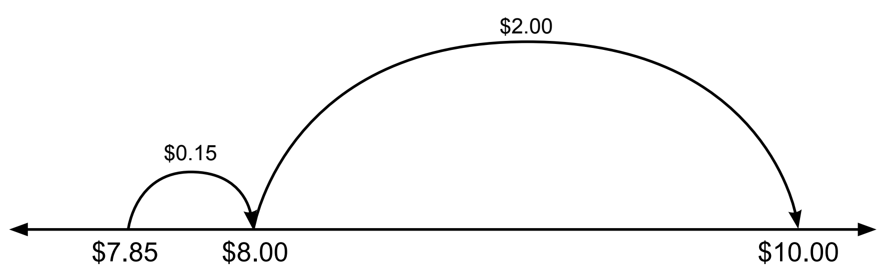
  Have students record their work. (Pictures of the items can be cut out and put in their notebooks/journals.)

- **Classroom Store:** Have students bring in food boxes/containers, small toys, et cetera. Have students price the items. Students can take turns being the customers and the store clerks. (Be sure that students and clerks estimate before calculating an exact answer.) Play money should be used for payment.

  Have students practise counting back change.

- **Counting Back Change:** Use a number line to model counting on as you give back change.

  Example: Making change from $10 for a $7.85 purchase.



Assessing Understanding

Interview:

Ask the student to estimate the answers to these problems and explain the strategy they used.

- 3.32 – 2.15
- 42.55 + 23.07
- $20 – $16.78

Performance:

Prepare cards with money values less than $20.

Example:

$12.75	$9.98	$15.35
$13.65	$11.40	$8.05

Place the cards face down on the table. Tell the student that they have a $20 bill. Have them select a card and then show how they would count back the change. Repeat one more time.

Performance:

Jenna bought a new pair of jeans for $28.97. She paid with a $50 bill.

- What was her change? Explain your thinking using models, words, and pictures.
- What other bills could Jenna have used to pay for the jeans? What change would she receive?

Paper-and-Pencil Task:

Create an addition problem and a subtraction problem that use decimal numbers. Show the solution.