Grade 3 Mathematics

Patterns and Relations

Grade 3: Patterns and Relations (3.PR.1, 3.PR.2)

Enduring Understandings:

Patterns can grow and repeat.

Patterns can be found in many different forms.

Essential Questions:

What is the increasing or decreasing unit in the pattern?

What strategies can be used to continue an increasing or decreasing pattern?

What strategies can be used to continue a numerical sequence?

How is the pattern increasing or decreasing?

Specific Learning Outcome(s):	Achievement Indicators:
3.PR.1 Demonstrate an understanding of increasing patterns by ■ describing ■ extending ■ comparing ■ creating patterns using manipulatives, diagrams, and numbers (to 1000). [C, CN, PS, R, V]	➛ Describe an increasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues. ➛ Identify the pattern rule of an increasing pattern, and extend the pattern for the next three terms. ➛ Identify and explain errors in an increasing pattern. ➛ Identify and describe various increasing patterns found on a hundred chart, such as horizontal, vertical, and diagonal patterns. ➛ Compare numeric patterns of counting by 2s, 3s, 4s, 5s, 10s, 25s, and 100s. ➛ Create a concrete, pictorial, or symbolic representation of an increasing pattern for a pattern rule. ➛ Create a concrete, pictorial, or symbolic increasing pattern, and describe the pattern rule. ➛ Solve a problem using increasing patterns. ➛ Identify and describe increasing patterns in the environment. ➛ Identify and apply a pattern rule to determine missing elements for a pattern. ➛ Describe the strategy used to determine missing elements in an increasing pattern.

Specific Learning Outcome(s):	Achievement Indicators:
3.PR.2 Demonstrate an understanding of decreasing patterns by ■ describing ■ extending ■ comparing ■ creating patterns using manipulatives, diagrams, and numbers (starting from 1000 or less). [C, CN, PS, R, V]	➢ Describe a decreasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues. ➢ Identify the pattern rule of a decreasing pattern, and extend the pattern for the next three terms. ➢ Identify and explain errors in a decreasing pattern. ➢ Identify and describe various decreasing patterns found on a hundred chart, such as horizontal, vertical, and diagonal patterns. ➢ Compare decreasing numeric patterns of counting backward by 2s, 3s, 4s, 5s, 10s, 25s, and 100s. ➢ Create a concrete, pictorial, or symbolic decreasing pattern for a pattern rule. ➢ Create a concrete, pictorial, or symbolic decreasing pattern, and describe the pattern rule. ➢ Solve a problem using decreasing patterns. ➢ Identify and describe decreasing patterns in the environment. ➢ Identify and apply a pattern rule to determine missing elements for a pattern. ➢ Describe the strategy used to determine missing elements in a decreasing pattern.

Prior Knowledge

Students may have worked with a wide variety of repeating patterns.

They may have reproduced, extended, and created increasing patterns using manipulatives, diagrams, sounds, and actions (numbers to 100).

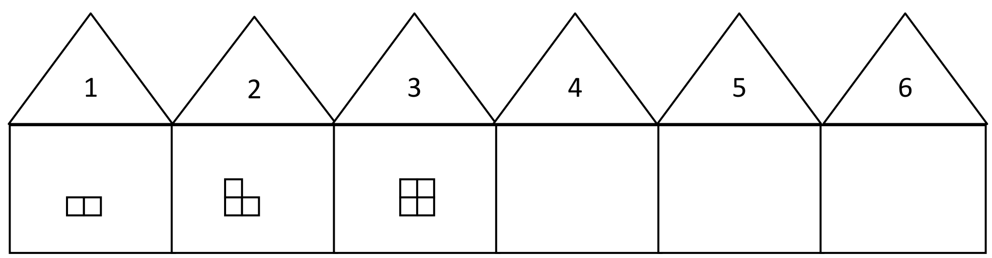
Background Information

Patterns are found throughout mathematics and should be taught throughout the year through problem solving. Our number system is based on an increasing pattern.

Simple repeating and growing/increasing/decreasing patterns consist of a series of related elements—each new element is related to the previous in some manner. Students must be able to identify the relationship in order to understand the pattern.

Students who develop the ability to **identify, reproduce, extend,** and **create** patterns are able to make generalizations and see relationships with numbers. This is why identifying and extending patterns is an important process in algebraic reasoning. To work with increasing and decreasing patterns, it is important that Grade 3 students have a sound foundation with repeating patterns, as well as some abilities with increasing patterns.

In Grade 3, students are expanding their knowledge of increasing patterns. They are beginning to explore decreasing patterns and making more in-depth connections with number concepts. When presenting a pattern, encourage students to verbalize the rule for the patterns they are working with. To encourage students to make connections with numbers, present the pattern with numerical term positions. For example:



Increasing and decreasing patterns are patterns in which the basic core pattern grows/shrinks or changes in a predictable way.

Mathematical Language

pattern
decreasing pattern
increasing pattern
element
extend
reproduce
rule
hundred chart

Assessing Prior Knowledge

Present students with a variety of increasing patterns. Have them identify the missing elements and explain the rule used to create and extend the pattern.

Examples:

1.

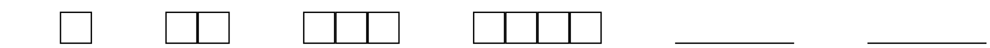


Figure 1 Figure 2 Figure 3 Figure 4 Figure 5 Figure 6

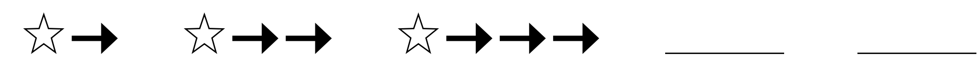


Figure 1 Figure 2 Figure 3 Figure 4 Figure 5

2. 2, 4, 6, 8, ______, 12, 14, ______, ______

3. Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23		25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63		65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The student is able to

- ❒ explain how the pattern increases/grows
- ❒ extend the pattern
- ❒ fill in the missing element

- **Describe an increasing/decreasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues.**
- **Identify the pattern rule of an increasing/decreasing pattern, and extend the pattern for the next three terms.**
- **Create a concrete, pictorial, or symbolic increasing/decreasing pattern, and describe the pattern rule.**
- **Identify and explain errors in an increasing/decreasing pattern.**

Suggestions for Instruction

- Use the book *Rooster's Off to See the World* by Eric Carle to review increasing patterns and introduce decreasing patterns. The pattern is graphically displayed to follow the action in the text. Ask students to describe how the pattern changes from page to page.

  Note: The pattern images could be photocopied so students can build the pattern as they retell the story.

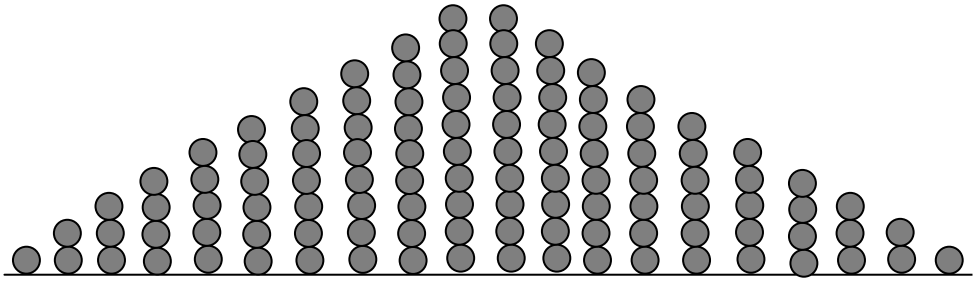
- Use a book such as *Mrs. McTats and Her Houseful of Cats* by Alyssa Satin Capucilli, illustrated by Joan Rankin. Read the book through first. Reread the book. Use pictures or counters to represent the changes in the number of cats. Have students identify and describe the increasing pattern.

  Note: The pattern changes at the end of the story.

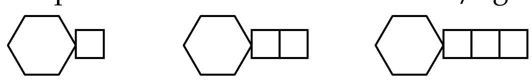
  

  The pattern is +2, +3, +4, +5, and +6.

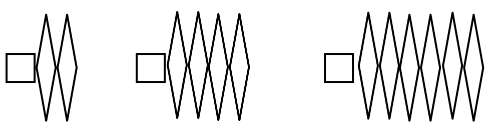
- Use a book such as *Counting Crocodiles* by Judy Sierra, illustrated by Will Hillenbrand, to explore both increasing and decreasing patterns. The first half of the book counts the number of crocodiles from 1 to 10. The second half counts the crocodiles from 10 to 1. Students can use pictures or counters to represent the actions in the story. For example,



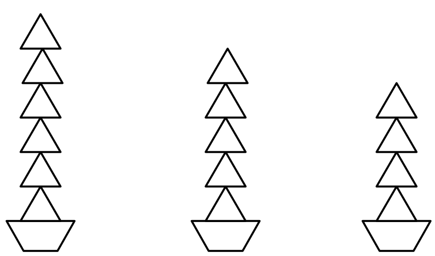
- **Pattern Problem:** Use two different pattern block shapes. Make an increasing pattern. Use the same shapes to make a decreasing pattern. Record your patterns in your journal. Explain the pattern rules.
- Use pattern blocks. Have students identify the pattern rule and then extend the pattern for the next three terms/figures.



Term 1 Term 2 Term 3 Term 4 Term 5 Term 6



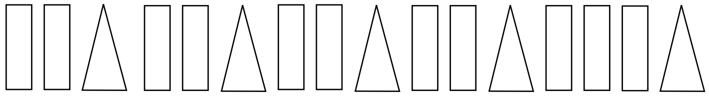
Term 1 Term 2 Term 3 Term 4 Term 5 Term 6



Term 1 Term 2 Term 3 Term 4 Term 5 Term 6

- **Pattern Errors:** Students work with a partner. Each student creates a decreasing or increasing pattern that contains an error. Students exchange patterns and identify the errors in each others' pattern.

Example:



- **Counting Errors:** The teacher or student leader recites a counting sequence. When an error is detected students raise their hands (or use some other agreed upon signal). A student is selected to identify and correct the error.
- **Mystery Rule:** The teacher decides on a pattern rule. Students give a number (input), the teacher applies the rule and gives the result (output). This continues until someone can identify the pattern rule.

  Students can take turns being the leader. Have the rule written down and shown to the teacher first.

Assessing Understanding: Performance Task

Student Directions:

Work with a partner.

1. Use colour tiles to make an increasing or decreasing pattern. Your pattern should have at least 4 terms.
2. Draw your pattern.
3. Explain your pattern rule.
4. Carefully describe your pattern so that someone else can make it increase or decrease three more times.
5. Look at your partner's pattern. Draw your partner's pattern.
6. Make it increase or decrease three more times.
7. What is your partner's pattern rule?
8. Compare your pattern with your partner's. How are they the same? How are they different?

Paper-and-Pencil Task: Answer the following questions in your journal/ notebook:

1. What is an increasing pattern? Give an example.
2. What is a decreasing pattern? Give an example.

- **Locate and describe various increasing/decreasing patterns found on a hundred chart, such as horizontal, vertical, and diagonal patterns.**

Suggestions for Instruction

BLM 3.PR.1.1 & 3.PR.2.1

- **Patterns on the Hundred Chart:** Have students explore patterns on a hundred chart. Possible increasing or decreasing patterns include the following:
  - for each row, left to right, numbers increase by 1
  - for each row, right to left, numbers decrease by 1
  - for each column, top to bottom, numbers increase by 10
  - for each column, bottom to top, numbers decrease by 10
  - numerous skip counting patterns
  - on the diagonal (starting at the top) from left to right numbers increase by 11
  - on the diagonal (starting at the bottom) from right to left numbers decrease by 11
  - on the diagonal (starting at the top) from right to left numbers increase by 9
  - on the diagonal (starting at the bottom) from left to right numbers decrease by 9

Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Note: Extend student thinking by

- changing the numbers on the chart to start at a larger number
- having the numbers of the chart increase by a multiple rather than by 1s (e.g., start at 0 and count up by 2s or start at 0 and count up by 10s)
- change the orientation of the chart by having the 1 to 10 row at the bottom

Assessing Understanding: Missing Numbers

This is a piece from a hundred chart. Fill in the missing numbers. Explain how you figured out each answer.

	26			
	36		38	
45				

- **Compare numeric patterns of counting by 2s, 3s, 4s, 5s, 10s, 25s, and 100s.**
- **Compare decreasing numeric patterns of counting backward by 2s, 3s, 4s, 5s, 10s, 25s, and 100s.**

Suggestions for Instruction

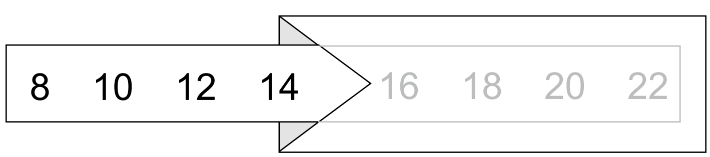
- Students compare counting patterns on the 100 chart.
  - Count by 2s. Colour the numbers on the hundred chart. Describe the patterns you see.
  - Count by 5s on the same 100 chart. Describe the patterns you see.
  - How are these patterns alike? How are they different?

  Use additional charts to compare counting by 3s, 4s, 10s, 25s, and 100s.

- Prepare a "pattern slider" to use with numeric patterns. Use a legal size sheet of paper and fold it to form a flat tube. Tape it together. Cut a V on one side.

  Use pattern strips. Slide a strip through the slider. Gradually pull the pattern through until three or four numbers can be seen. Students predict the next number(s) in the pattern. Have students justify their prediction by identifying the pattern rule.

  For example,



Assessing Understanding: Paper-and-Pencil Task

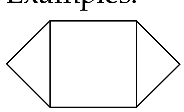
- Mark started at 60 and counted backward by 3s to 0.
  Pam started at 60 and counted backward by 4s to 0.
  Hans started at 60 and counted backward by 2s to 0.
  What numbers will each person say?
  How are the counting patterns the same?
  How are they different?
- If you start at 465 and count by 5s to 550, and your friend starts at 410 and counts by 10s to 550, what numbers would you both say? Explain how you know.

- **Create a concrete, pictorial, or symbolic representation of an increasing/decreasing pattern for a pattern rule.**
- **Identify and apply a pattern rule to determine missing elements for a pattern.**
- **Describe the strategy used to determine missing elements in an increasing/decreasing pattern.**
- **Solve a problem using increasing/decreasing patterns.**

Suggestions for Instruction

- **The Rule Is _____:** Give the first term of an increasing or decreasing pattern along with the pattern rule. Have students create a representation of the pattern.
  Note: The representation should have at least 4 terms.

  Examples:

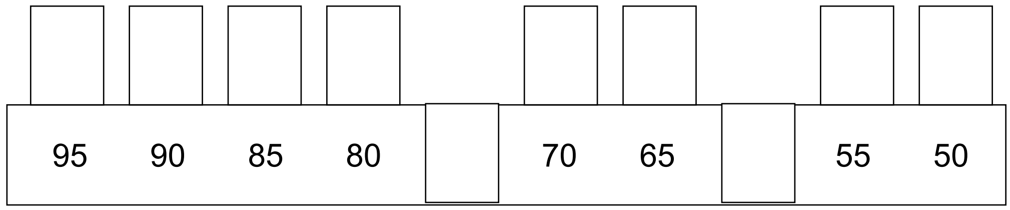


  Term 1
  Rule: Add 2

  24
  Term 1
  Rule: Minus 2

- Use a Hidden Numeral Board for missing element problems.

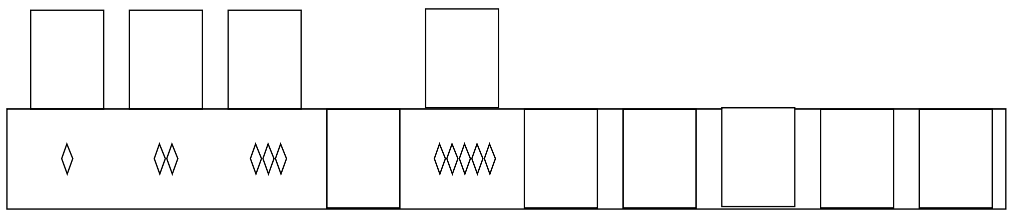
  Examples:



  How is the pattern changing?

  What is the pattern rule? How did you figure it out?

  Use the pattern rule to find the missing numbers.



  How is the pattern changing?

  What is the pattern rule? How did you figure it out?

  Use the pattern rule to find the missing terms.

- Have students share the strategies they use to determine the missing elements in an increasing or decreasing pattern. Strategies can be recorded on a chart and put up for student reference.

Sample Problems:

- How many stars are in Term 3?

  How many stars are in Term 5?

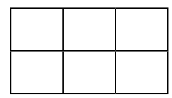
  What strategy did you use to solve the problem?

Term 1	Term 2	Term 3	Term 4	Term 5
☆☆☆☆☆☆☆ ☆☆☆☆☆☆☆ ☆☆☆☆☆☆☆ ☆☆☆☆☆☆☆ ☆☆☆☆☆☆☆ ☆☆☆☆☆☆☆	☆☆☆☆☆☆☆ ☆☆☆☆☆☆☆ ☆☆☆☆☆☆☆ ☆☆☆☆☆☆☆ ☆☆☆☆☆☆☆		☆☆☆☆☆☆☆ ☆☆☆☆☆☆☆ ☆☆☆☆☆☆☆	

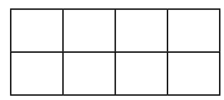
- Mara wants to find out the number of tires on 7 cars. Use a pattern to help Mara solve this problem.

- Sam has a problem. He is putting new tile in his house. The book he is using shows the first 3 terms of the pattern but he is not sure how to continue the pattern. Complete the next two terms for him. Write a note to Sam explaining how he should continue the pattern.

Term 1



Term 2



Term 3

- The Grade 3 class is having their picture taken. The photographer put them in rows. She put 1 person in the first row, 2 in the second, 3 in the third, and so on. If there were 7 rows of students in the picture, how many students were there in the class? Use a pattern to find your answer.

- Mrs. Allen has a box of chocolates. There are 24 chocolates in the box. She decides to eat 2 chocolates each day. How many days will the chocolates last? Use a pattern to explain your answer.

- Henri baked cookies over the weekend. Each day during the week he took three cookies to school for his lunch. On Saturday he had 18 cookies left. How many cookies had he baked? Use a pattern to solve the problem.

- Alex loves cookies. He ate 10 cookies on Monday, 9 on Tuesday, 8 on Wednesday, and so on through Sunday. How many cookies did he eat in all from Monday through Sunday? Use a pattern to help solve the problem.

- Write a problem that can be solved using an increasing or decreasing pattern. Share your problem with a partner.

Note: Problems could be compiled into a class book.

Assessing Understanding

Observe students as they work on problems.

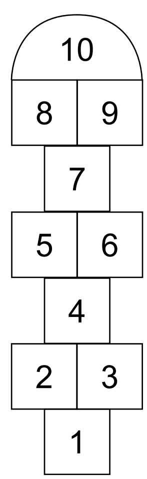
The student is able to

- ❒ identify the pattern rule and apply it to solve the problem
- ❒ use mathematical language related to increasing and decreasing patterns
- ❒ use an increasing pattern to solve a problem
- ❒ use a decreasing pattern to solve a problem
- ❒ discuss the strategies used to find missing elements

- **Identify and describe increasing/decreasing patterns in the environment.**

Suggestions for Instruction

- Have students use a digital camera to take pictures of increasing and decreasing patterns in the environment.
- **Home Pattern Hunt:** Students look for examples of increasing and decreasing patterns (concrete, pictorial, or symbolic) at home and bring them (or a replica) to school to share with the class.
- **Science Observation Centre:** Students bring in objects from nature that have increasing or decreasing patterns and place them at the science centre. Students identify and record the patterns observed.
- **Patterns in Hopscotch Exploration:** Draw a hopscotch grid on the board or outside on the tarmac. Ask students to describe the patterns they see.



Focus the exploration with questions such as the following:

- What is happening to the numbers in the single rectangles?
- What is happening to the numbers in the left-hand double rectangles?
- What is happening to the numbers in the right-hand double rectangles?
- What is the pattern rule?
- If the hopscotch grid was continued, what number would be in the seventh single rectangle? In the eighth left-hand double rectangle? In the tenth right-hand double rectangle?
- Would the number 14 be in a single rectangle? Explain your thinking.

Extending student thinking:

Design a hopscotch grid made up of single and double rectangles so that 6, 11, and 16 are in single rectangles.

Putting the Pieces Together: Performance Task

Increasing and/or Decreasing Pattern Story

Introduce the task by reading an increasing/decreasing pattern book. Revisit a book used earlier or read a new one such as *Ten Little Ladybugs* by Melanie Gerth, illustrated by Laura Huliska-Beith.

Tell students that they are going to work with a partner to write and illustrate an increasing/decreasing pattern story to share with a younger class (reading buddy).

The activity can be differentiated by having students

- write an increasing story
- write a decreasing story
- write a story that has both an increasing and decreasing pattern
- write a story that increases or decreases by numbers greater than one

Grade 3: Patterns and Relations (3.PR.3)

Enduring Understandings:

"Equals" indicates equivalent sets.

Unknown quantities can be found by using the balance strategy.

Essential Questions:

How is a number sentence like a balance scale?

What does the equal sign mean?

Specific Learning Outcome(s):	Achievement Indicators:
3.PR.3 Solve one-step addition and subtraction equations involving symbols representing an unknown number. [C, CN, PS, R, V]	➢ Explain the purpose of the symbol, such as a triangle or a circle, in an addition or a subtraction equation with one unknown. ➢ Create an addition or a subtraction equation with one unknown to represent a combination or separation action. ➢ Provide an alternative symbol for the unknown in an addition or a subtraction equation. ➢ Solve an addition or a subtraction equation that represents combining or separating actions with one unknown, using manipulatives. ➢ Solve an addition or a subtraction equation with one unknown using a variety of strategies including guess and test. ➢ Explain why the unknown in an addition or a subtraction equation has only one value.

Prior Knowledge

Students may be able to

- demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100)
- record equalities and inequalities symbolically using the equal symbol or the not-equal symbol

The equal symbol represents a relation between two equal quantities. In other words, the expression on the left-hand side of the equal symbol represents the same quantity as the expression on the right-hand side of the equal symbol. The equal symbol means "is the same as."

Many students have misconceptions about the equal symbol. Many think that the equal symbol means "give answer." As a result they have difficulty with questions such as the following:

4 + ___ = 7	Students will add across the equal sign and fill the blank with 11.
___ = 2 + 5	Students will say that the question itself is incorrect because the blank is on the wrong side.
3 + 4 = 5 + ___	Students will add all the numbers and put 12 in the blank.

Exploring Relationship between Terms

Exploring the relationship between expressions/terms on either side of an equal symbol enables students to develop the understanding of the relationship that exists between the expressions/terms.

Note: It is important that students discover this relationship on their own.

Ask students what they notice about the expressions on both sides of the equal sign. If the students have difficulty seeing a relationship, provide additional examples using smaller numbers.

Examples:

16 + 18 = 18 + 16	(Think: "The order of the addends is reversed but the numbers are the same so they are equal.")
13 + 9 = 15 + 7	(Think: "15 is 2 more than 13 and 7 is 2 less than 9 so they are equal.")
16 + 26 = 8 + 34	(Think: "8 is 8 less than 16 and 34 is 8 more than 26 so they are equal.")
2 + 8 = 1 + 9	(Think: "1 is 1 less than 2 and 9 is 1 more than 8 so they are equal.")

Once students see the relationship between expressions you can introduce unknowns.

Examples:

$21 + 56 = \square + 50$	(Think: "50 is 6 less than 56. I need to add 21 + 6 to the right side in order to make them equal so 21 + 56 = 27 + 50.")
$16 + 12 = \square + 17$	(Think: "17 is one more than 16. I need to take 1 away from the 12 to make them equal so 16 + 12 = 11 + 17.")
$\square + 48 = 36 + 42$	(Think: "48 is 6 more than 42. I need to subtract 6 from the 36 in order to make them equal so 30 + 48 = 36 + 42.")

Mathematical Language

same	equal sign
more	equal symbol
less	inequality
equal	equality
not equal	symbol
balance	unknown
match	equation

Learning Experiences

Assessing Prior Knowledge

Present the following equations. Have students fill in the missing numbers.

a. ___ + 5 = 11

b. 8 + 1 = 6 + ___

c. 6 = 10 – ___

d. ___ + 5 = 2 + 4

e. ___ = 10 + 5

f. 4 + 5 = ___ + 3

Ask students to explain the strategies they used to solve the problems.

Students

❒ correctly fill in the missing numbers

❒ demonstrate an understanding of the equal sign

❒ explain their strategies using correct mathematical language

- **Explain the purpose of the symbol, such as a triangle or a circle, in an addition or a subtraction equation with one unknown.**

Suggestions for Instruction

Note: Students have been solving equations with one unknown in previous grades. The unknown has been represented by a line or question mark. The use of a symbol should be an easy transition for them.

- Give students the following equations and ask them to solve them.

  a. $14 + 7 = \triangle$

  b. $20 - \bigcirc = 15$

  c. $\square + 2 = 18$

  d. 16 – 4 = ⬡

  Ask the following:

  - How did you know what the questions were asking?
  - How did you know where to put the answers?
  - What is the purpose of the symbols (triangle, circle, square, and hexagon) in each equation?

- Create an addition or a subtraction equation with one unknown to represent a combination or separation action.
- Provide an alternative symbol for the unknown in an addition or a subtraction equation.
- Solve an addition or a subtraction equation that represents combining or separating actions with one unknown using manipulatives.
- Solve an addition or a subtraction equation with one unknown using a variety of strategies including guess and test.
- Explain why the unknown in an addition or a subtraction equation has only one value.

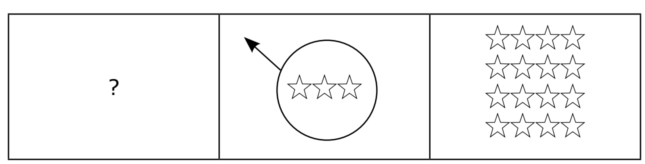
Suggestions for Instruction

- Provide pictorial representations of addition and subtraction. Have students write an equation representing the unknown with a symbol of their choosing.

  Examples:

  1. 

     18 + 10 = ____

  2. 

     ____ – 3 = 16

- Have students create their own pictorial representations and the matching equations. Use the representations and equations to play a matching game such as concentration.
- **Equation Solving Stations:** Set up stations with different manipulative materials.

  Materials:
  - four equations for each station
  - balance scale(s) and unifix cubes
  - base-10 blocks
  - counters
  - ten frames

Directions for Each Station:

1. Solve the equations using the [balance scale and unifix cubes, base-10 blocks, counters, ten frames].
2. Record your solutions in your journal/notebook.

As students are working, rotate through the stations and ask questions such as the following:

- What is the equation asking you to find?
- How do you know that the materials you are using match the equation?
- What does the equal sign mean in the equation?

- Have students solve the following equations:

  a. $16 + \square = 17 + 5$

  b. $\triangle - 6 = 17 - 5$

  c. $32 + 19 = \bigcirc + 20$

  d. 100 = 64 + ⬡

  Ask students to explain the strategies they used to find the unknown.

- Mr. Nelson put this equation on the board.
  $21 - 8 = \triangle$
  Tom said the answer was 12. Mark said the answer was 13.
  Is it possible that both boys have the correct answer? Why or why not?

- **Classroom Routine—"Free Facts"**

  Free facts are facts that are derived from a given equation. In order to identify these facts students must use their understanding of number relationships.

  Example:

Free Facts for 16 + 18 = 34		
16 + 8 = 24	34 - 16 = 18	14 + 20 = 34
34 - 18 = 16	160 + 180 = 340	15 + 19 = 34
Etc.		

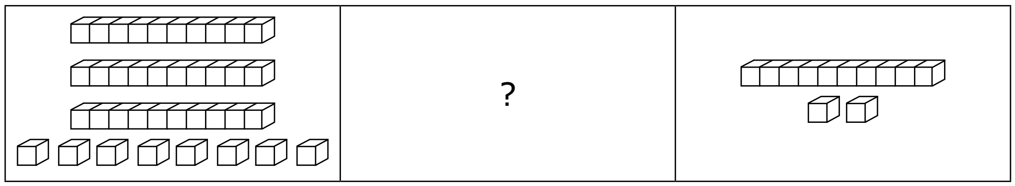
Assessing Understanding: Paper-and-Pencil Task

1. Solve these equations. Explain the strategies you used to figure them out in your journal.

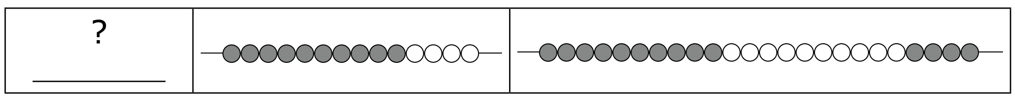
   a. $17 + 10 = \bigcirc + 4$

   b. $26 - \triangle = 12$

2. Write an equation for the picture below.



3. Write two possible equations for this picture.



   ($\triangle + 14 = 24$ or $\triangle - 14 = 24$)

4. Give at least eight “Free Facts” for the following equation:

   $25 + 32 = 57$

Notes