GRADE 3 MATHEMATICS

Number

Grade 3: Number (3.N.1)

Enduring Understanding:

Counting is a strategy for finding the answer to how many.

Essential Question:

Is there a quicker way to find the answer than counting by ones from one?

Specific Learning Outcome(s):	Achievement Indicators:
3.N.1 Say the number sequence between any two given numbers forward and backward ■ from 0 to 1000 by 　■ 10s or 100s, using any starting point 　■ 5s, using starting points that are multiples of 5 　■ 25s, using starting points that are multiples of 25 ■ from 0 to 100 by 　■ 3s, using starting points that are multiples of 3 　■ 4s, using starting points that are multiples of 4 [C, CN, ME]	➤ Extend a skip-counting sequence by 10s or 100s, forward and backward, using a given starting point. ➤ Extend a skip-counting sequence by 5s, forward and backward, starting at a given multiple of 5. ➤ Extend a skip-counting sequence by 25s, forward and backward, starting at a given multiple of 25. ➤ Extend a given skip-counting sequence by 3s, forward, starting at a given multiple of 3. ➤ Extend a given skip-counting sequence by 4s, starting at a given multiple of 4. ➤ Identify and correct errors and omissions in a skip-counting sequence. ➤ Determine the value of a set of coins (nickels, dimes, quarters, loonies) by using skip counting. ➤ Identify and explain the skip-counting pattern for a number sequence.

Prior Knowledge

Students may be able to say the number sequence from 0 to 100 by

■ 2s, 5s, and 10s, forward and backward, using starting points that are multiples of 2, 5, and 10 respectively

■ 10s, using starting points from 1 to 9

■ 2s, starting from 1

BACKGROUND INFORMATION

Students in Grade 3 are expanding their experiences with numbers to 1000 and may struggle with the increase in numbers. It is important to provide many opportunities for students to bridge the decades through the hundreds (e.g., 98, **99, 100, 101,** 102, 103, 104, 105, 106, 107, 108, **109, 110, 111,** 112…). When students are working with larger numbers the goal is to have students understand that there is a pattern within our number system that enables us to predict numbers. Have students recognize and explain errors and omissions in a given skip-counting sequence to help to reinforce the development of counting, number relationships, and place value.

When skip-counting with students, the focus should be on looking for patterns. Understanding patterns can support children's use of invented strategies and prepare students for working with money. Exploring the patterns should strengthen children's understanding of number relationships and properties. Asking children, "What did you observe about the pattern?" can help facilitate children's sense making about number relationships with the patterns and other math concepts.

Counting on and counting back by 5s, 10s, and 100s are important mental math strategies for addition and subtraction. Skip-counting by 2s, 3s, and 4s is a foundation for multiplicative understanding.

Counting a mixed collection of coins can be difficult for students because they are expected to shift how they are skip counting several times (e.g., counting by loonies [1s], then by quarters [25s], and then by dimes [10s]). They need practice switching the count using a set of like coins before counting mixed collections of money.

MATHEMATICAL LANGUAGE

Counting numbers:

one to one thousand	count back
count on	penny
skip count	nickel
set	dime
number	quarter
numeral	loonie
multiple	money

LEARNING EXPERIENCES

Assessing Prior Knowledge: Interview

Ask the student to

❑ start at 42 and count by 2s (stop at 60)

❑ start at 13 and count by 2s (stop at 31)

❑ start at 78 and count backward by 2s (stop at 64)

❑ start at 30 and count by 10s (stop at 100)

❑ start at 7 and count by 10s (stop at 57)

❑ start at 100 and count backward by 10s (stop at 40)

❑ start at 15 and count by 5s (stop at 60)

❑ start at 85 and count backward by 5s (stop at 55)

❑ start at 3 and count by 3s (stop at 24)

❑ start at 4 and count by 4s (stop at 28)

❑ count a set of counters by 2s, 5s, or 10s, and count on

❑ determine the value of 5 quarters

The student is able to

■ count by 2s
  - ❑ forward on the multiple
  - ❑ forward off the multiple
  - ❑ backward on the multiple

■ count by 10s
  - ❑ forward on the multiple
  - ❑ forward off the multiple
  - ❑ backward on the multiple

■ count by 5s
  - ❑ forward on the multiple
  - ❑ backward on the multiple

■ count by 3s
  - ❑ forward on the multiple

■ count by 4s
  - ❑ forward on the multiple

- count a set in groups and count on
  - ☐ by 2s
  - ☐ by 5s
  - ☐ by 10s
- determine the value of set of coins
  - ☐ nickels
  - ☐ dimes
  - ☐ quarters

- **Extend a skip-counting sequence by 10s, or 100s, forward and backward, using a given starting point.**
- **Extend a skip-counting sequence by 5s, forward and backward, starting at a given multiple of 5.**
- **Extend a skip-counting sequence by 25s, forward and backward, starting at a given multiple of 25.**

Suggestions for Instruction

- **Count around the Circle:** Have the class sit in a circle. Give the start and finish for the count. Before counting ask students questions such as the following:
  - We're going to start at 0 and stop at 110. Who do you think will say 110?
  - We're going to count by 25s to $10.00. How many times will we have to go around the circle?

  Have students (each student saying one number in turn) count to confirm their predictions.

- **Problem Solving:** Have students think about counting by asking questions such as the following:
  - If you start with 126, how many 10s do you need to add to get to more than 200?
  - If you start with 345, how many 10s do you need to add to get to 415?
  - If you count on in 100s from 350 to 850, how many 100s do you count?
  - If you count back in 10s from 934 to 854, how many 10s do you count?
  - If you count back in 100s from 978 to 178, how many 100s do you count?
  - If you start at 175 and count on eight 25s, what number will you say last?

**BLM
3.N.1.1**

  For students in group situations, use a set of number cards to select the starting point.

- **Change the Count:** Start counting together from a given starting point. After a short time clap your hands, ring a bell, or use another signal to change the skip-counting sequence. Change the sequence several times.

  Example: Start counting together in 5s from 50. After reaching 150, clap your hands and announce that they will now be counting by 10s. When 290 is reached clap again and announce that the counting will be by 100s. When the class reaches 990, clap once more and announce that they will be counting backwards by 5s, and so on.

- **Extend a skip-counting sequence by 3s, forward, starting at a given multiple of 3.**
- **Extend a skip-counting sequence by 4s, starting at a given multiple of 4.**

Suggestions for Instruction

BLM
3.N.1.2

- **Patterns in Hundred Chart:** Have students place counters on a hundred chart on multiples of 3 and 4. Have students describe the pattern made by the counters.

- **Calculator Exploration:** Students use different starting points (multiples of 3 or 4) and the constant feature on a calculator to check counting by 3s and 4s.

  > Constant feature on the calculator (Not all calculators have the feature.):
  >
  > Example:
  >
  > Counting by 5s:
  >
  > Press the following keys:
  > ⓪ ⊕ ⑤ ⊜ ⊜ ⊜ 15
  >
  > Counting by 3s starting at 9:
  >
  > Press the following keys:
  > ⑨ ⊕ ③ ⊜ ⊜ ⊜ 18
  >
  > Challenge the students by having them predict the next number before they press the equal button.

- Present problems that have students skip-counting and comparing them.

  Example:

  Jan counted the blocks 3 at a time.

  Gill counted the blocks 4 at a time.

  Write the first 10 numbers in Jan's and Gill's pattern.

  How are the patterns the same and how are they different?

Note: Counting can be built in to classroom routines. Students can use the "Number of the Day" and count by 3s, 4s, 5s, 10s, 100s, and 25s (if applicable). It can also be done when there are a few minutes of class time available (e.g., just before dismissal, waiting for class changes).

Assessing Understanding: Interview

Have the student

- ❐ count on by 10s from 460 to 510
- ❐ count on by 5s from 185 to 230
- ❐ count on by 100s from 138 to 838
- ❐ count on by 25s from 125 to 250
- ❐ count back by 10s from 773 to 683
- ❐ count back by 5s from 530 to 485
- ❐ count back by 100s from 996 to 106
- ❐ count back by 25s from 625 to 500
- ❐ count on by 3s starting at 18 to 36
- ❐ count on by 4s starting at 24 to 48

The student is able to

- ❐ count on by 10s
- ❐ count on by 5s
- ❐ count on by 100s
- ❐ count on by 25s
- ❐ count back by 10s
- ❐ count back by 5s
- ❐ count back by 100s
- ❐ count back by 25s
- ❐ count by 3s on multiples of 3
- ❐ count by 4s on multiples of 4

- **Identify and correct errors and omissions in a skip-counting sequence.**
- **Identify and explain the skip-counting pattern for a number sequence.**

Suggestions for Instruction

- Prepare a set of cards containing both correct and incorrect skip-counting sequences. Students sort them into two groups (Correct/Incorrect) and then correct the incorrect sequences.

- **What's the Pattern?** Present students with number sequences such as
  - 298, 398, 498, 598, 698
  - 175, 200, 225, 250, 275
  - 327, 337, 347, 357, 367

  Have students identify the skip-counting pattern (e.g., counting by 10s) and then give the next three numbers in the sequence.

Assessing Understanding: Paper-and-Pencil Task 1

Give students the set of number sequences. Have them identify the skip counting pattern and then give the next four numbers in the sequence.

1. 275, 300, 325, 350, _____, _____, _____, _____
   Counting forward/backward by _____.

2. 456, 446, 436, 426, _____, _____, _____, _____
   Counting forward/backward by _____.

3. 660, 665, 670, 675, _____, _____, _____, _____
   Counting forward/backward by _____.

4. 123, 223, 323, 423, _____, _____, _____, _____
   Counting forward/backward by _____.

5. 708, 718, 728, 738, _____, _____, _____, _____
   Counting forward/backward by _____.

Paper-and-Pencil Task 2

If you start at 465 and count by 5s to 550, and your friend starts at 410 and counts by 10s to 550, what numbers would you both say? Explain how you know.

> ■ **Determine the value of a set of coins (nickels, dimes, quarters, loonies) by using skip counting.**

Suggestions for Instruction

- Initially have students count sets of like coins (e.g., all dimes or all nickels). Then have them count sets with two different coins. Make sure that students start with the coins that have the greatest value first. Keep adding coins until they are able to count a mixed collection.

- **Money Counting Centre:** Prepare bags/containers with coins of varying amounts. Have students select a bag of coins, count the money in the bag, and record the answer. Vary the number and types of coins in each bag/container in order to meet the needs of your students.

Note: When possible involve students in counting money for classroom events such as special lunches, field trips, and so on.

The following site contains BLMs of Canadian coins:
http://lrt.ednet.ns.ca/PD/BLM/table_of_contents.htm

Assessing Understanding: Observation Checklist

Give the student a collection of coins. Observe them as they count the collection.

The student

- ☐ sorts the coins
- ☐ starts with the coins that have the greatest value
- ☐ is able to continue the count as coin values change (e.g., quarters to dimes)
- ☐ counts the collection accurately

Grade 3: Number (3.N.2)

Enduring Understanding:

Quantities can be represented in a variety of ways with objects, pictures, and numerals.

Essential Questions:

How can quantities be shown?

How many different ways can you represent a number?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
3.N.2 Represent and describe numbers to 1000, concretely, pictorially, and symbolically. [C, CN, V]	➤ Read a 3-digit numeral without using the word "and" (e.g., 321 is three hundred twenty-one, NOT three hundred AND twenty-one). ➤ Read a number word (0 to 1000). ➤ Represent a number as an expression (e.g., 300 – 44 for 256 or 20 + 236). ➤ Represent a number using manipulatives, such as base-10 materials. ➤ Represent a number pictorially. ➤ Write number words for multiples of ten to 90. ➤ Write number words for multiples of a hundred to 900. ➤ Determine compatible number pairs for 100.

PRIOR KNOWLEDGE

Students may have had experience representing and describing numbers to 100, concretely, pictorially, and symbolically. They may have represented numbers using

- concrete materials (e.g., ten frames, base-10 materials)
- coins
- tallies
- pictures
- expressions
- words
- symbols
- place value

To develop a good sense of number, students have to develop an intuition about numbers and their relationships. Flexible intuitive thinking about numbers develops gradually as a result of exploring numbers and visualizing in a variety of contexts. Provide the use of concrete materials and models such as base-10 materials, hundred charts, number lines, place value charts, and money to help students make connections between the concrete and pictorial to the symbolic representations of the numbers.

The reading of number words such 625 should be read as "six hundred twenty-five." When reading numbers the word *and* denotes the decimal. When writing four-digit numbers symbolically, there is usually no space or comma between the thousands and hundreds place. Writing numbers that are five or more digits requires a space between the thousands and hundreds place.

When students are representing numbers in a variety of ways, they demonstrate their understanding of the use of a number (e.g., my house number is 34), how a number compares to another number (e.g., 34 is 1 less than 35), how a number can be broken into parts (e.g., 34 is 32 + 2), and place value (e.g., 34 is 30 + 4 or 20 + 14 or 10 + 24).

The ability to represent numbers in a variety of ways will benefit students when doing operations and mental mathematics problems. Present number sentences horizontally as well as vertically, to encourage students to use different representations of numbers and part-part-whole thinking.

Example:

$$25 \;+\; 26 \qquad \text{or} \qquad 25 \;+\; 26$$
$$25 \;+\; 25 + 1 \qquad\quad 20 + 5 + 20 + 6$$

Developing part-part-whole relationships allows students to think of a number as a composition of other numbers. This includes knowing the parts and being able to find the whole, knowing the whole and finding the parts, knowing a part and a whole, and finding the missing part. Students who develop a deep understanding of numbers will be able to partition numbers in flexible ways. This learning outcome can be connected to Specific Learning Outcome 3.N.5, which involves the development of place value.

Encourage students to represent numbers in a variety of ways (e.g., using manipulatives, words and pictures, number sentences, place value, money, ten frames, horizontal and vertical number lines, connections to other strands, and real-life situations).

Students need something to remind them of the different ways in which they can represent numbers. Use a chart or table/desk file folders. Brainstorm with the class different representations in order to build the chart/file folder. As new representations are introduced add them to the chart/file folder.

Example:

Ways to Represent Numbers	
words	pictures
materials—base-10 blocks, etc.	story problems
place value	expanded notation/form
comparisons—greater/less than	1, 10, 100 more or less than
half of/double/twice as many as	number line
number sentences/expressions	money
odd/even	real-world connections (house number, date, etc.)

It would be beneficial to set criteria related to the number of ways expected when students are asked to represent a number. Criteria might also specify specific ways required as well (e.g., place value).

An important facet to building representations of numbers is to provide opportunities for students to discuss and reflect upon their thinking and the connections they are making through the representations. Allow time to discuss these connections during daily routines.

MATHEMATICAL LANGUAGE

represent

place value

base-10 blocks

coins

penny

nickel

dime

quarter

loonie

number words to 1000

number sentence

expression

greatest

least

money

Assessing Prior Knowledge: Performance Task

Student Directions:

38   65   99

1. Choose one of the numbers above.

2. Represent the number in at least 6 different ways.

The student is able to represent a 2-digit number using

❐ concrete materials (ten frames, base-10 materials)

❐ tallies

❐ pictures

❐ words

❐ expressions/number sentences

❐ number line

❐ comparisons (greater/less than, 1 more/less, 10 more/less, etc.)

❐ other

■ **Read a 3-digit numeral without using the word "and" (e.g., 321 is three hundred twenty-one, NOT three hundred AND twenty-one).**

■ **Read a number word (0 to 1000).**

■ **Write number words for multiples of ten to 90.**

■ **Write number words for multiples of a hundred to 900.**

Suggestions for Instruction

■ **Gotcha!** Place cards with a set of numbers between 101 and 1000 in a container. Include ten (or more) cards with the word "Gotcha!" written on them. Have students sit in a circle. Pass the container around the circle. Each student draws a card and reads the number. If read correctly the student keeps the card. If a "Gotcha!" card is selected the student must return all of his/her cards to the container. The game continues until all cards are played. The student with the most (or least) cards is the winner.

Note: The game can also be played with number words.

- **Concentration**

  Materials: a set of 12 number cards along with a matching set of number word cards

  Players: 2

  Directions: Shuffle the cards together and lay them face down in four rows of six. Players take turns drawing 2 cards. The player reads both cards. If the cards match, the player keeps the pair and plays again. If the cards do not match, the cards are returned to the grid. Play continues until all cards have been matched. The winner is the player with the most cards.

- Over a period of time, have students collect number words from newspapers and/or magazines and write the corresponding numerals beside them. Students can sort and graph the numbers they have found.

- Have students practise writing the number words for the multiples of ten to 90 and multiples of a hundred to 900. This will enable them to write any number from 1 to 1000 in words.

- **"Multiple" Stories:** Have students write a story that uses as many of the number words for the multiples of 10 (to 90) or the multiples of 100 (to 900) as possible. Share the stories with the class and/or compile them into a class book.

- **Daily Routine—Number of the Day**

  Organize students into teams of two. Assign the routine to a different team each day. This can be used as assessment for learning. It also helps to keep the concepts fresh in the minds of the students over the course of the year.

  Note: The Number of the Day can be done on a laminated chart (although after a while it becomes difficult to erase). Some teachers have put words/phrases from the chart on individual strips of paper with a magnetic strip on the back and for use on a white board. This enables the teacher to differentiate for groups of students by adding or deleting representations.

Number of the Day

Write the number in words:

Show the number using:

Pictures/Models	Base-10 Materials
Expanded Form	Comparisons (more than/less than, etc.)
Money (in two different ways)	
Number Sentences/Expressions	

Assessing Understanding: Interview

Show the following numbers. Have the student read them.

671   904   297   880   536   355

Show the following number words. Have the student read them.

- eight hundred six
- two hundred thirty-seven
- nine hundred forty-one
- six hundred fifty
- four hundred ninety-eight
- one hundred sixty-three

The student is able to

- read three-digit numerals
  - ❑ confidently
  - ❑ with hesitation
- read number words to 999
  - ❑ confidently
  - ❑ with hesitation

Student Self-Assessment

Have students add to the chart several times during the year (perhaps at reporting times).

Name: _____

Represent and describe numbers to 1000, concretely, pictorially, and symbolically.				
	September	**November**	**March**	**June**
I can				
I can				
I can				
My goal(s) for next term is/are				

Sample:

Name: _____

Represent and describe numbers to 1000, concretely, pictorially, and symbolically.				
	September	**November**	**March**	**June**
I can	represent numbers to 100	represent numbers to 500	represent numbers to 1000	work with numbers larger than 1000
I can	represent numbers using pictures, tallies, ten frames, cubes, counters, money, and other math materials	represent numbers using base-10 materials, a number line, and comparisons	represent numbers using regular and irregular place value	represent numbers in multiple ways
I can	write addition and subtraction number sentences for numbers to 100	write addition and subtraction number sentences for numbers to 500	write addition and subtraction number sentences for numbers to 1000	write addition and subtraction number sentences and some multiplication and division number sentences for numbers to 1000 (e.g., x 1, x 0, ÷ 1)
My goal(s) for next term is/are	■ to be able to represent larger numbers ■ to find new ways to represent these numbers			

Suggestions for Instruction

- **Compatible Number Pairs to 100:** A pair of numbers that is easy to work with mentally (also known as friendly or nice numbers) are said to be **compatible or complementary**. Compatible numbers can help students understand how numbers work. If you know parts of the number, you can put the number together to find the whole. If you know the whole and one of the parts, you can take away the part you know to find the other part.

  Part-whole relationships refers to the idea that numbers can be broken down into parts, and that these parts can be compared to the whole. According to John Van de Walle, to conceptualize a number as being made up of two or more parts is the most important understanding that can be developed about number relationships.

  Have the students brainstorm the list of compatible numbers to 100 and find the patterns.

  Example:

0 + 100	99 + 1	89 + 11
10 + 90	98 + 2	88 + 12
20 + 80	97 + 3	87 + 13
30 + 70	96 + 4	86 + 14
40 + 60	95 + 5	85 + 15
50 + 50	94 + 6	84 + 16
	93 + 7	83 + 17
	92 + 8	82 + 18
	91 + 9	81 + 19 . . .

  Students can make connections with the patterns and build a better sense of number by listing the compatible number.

- Make a two-digit number with ten frames. Students work together to determine what goes with the ten frame to make 100.

- Post the numeral 100 on a chart. Place numbers less than 100 on sticky notes under the number. Students must choose a number from the wall and give its compatible number to 100.

  Have students discuss the strategy to find the compatible number to 100.

| 100 |

26

63   85

34

42

16

36

20
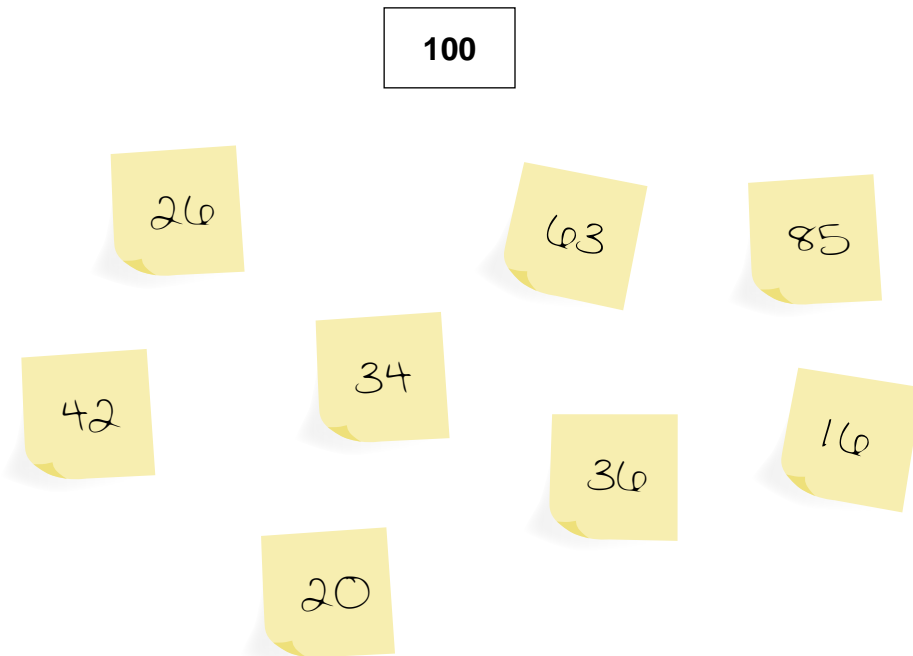
Grade 3: Number (3.N.3, 3.N.4)

Enduring Understandings:

Place value patterns are repeated in large numbers and these patterns can be used to compare and order numbers.

The position of a digit in a number determines the quantity it represents.

Estimation is a way to get an approximate answer.

Essential Questions:

How does changing the order of the digits in a number affect its placement on a number line?

How are place value patterns repeated in numbers?

How does the position of a digit in a number affect its value?

What are strategies to make reasonable estimates?

Does the use of referents help us make a more reasonable estimate?

Specific Learning Outcome(s):	Achievement Indicators:
3.N.3 Compare and order numbers to 1000. [CN, R, V]	➤ Place a set of numbers in ascending or descending order, and verify the result by using a hundred chart (e.g., a one hundred chart, a two hundred chart, a three hundred chart), a number line, or by making references to place value. ➤ Create as many different 3-digit numerals as possible, given three different digits. Place the numbers in ascending or descending order. ➤ Identify errors in an ordered sequence. ➤ Identify missing numbers in parts of a hundred chart. ➤ Identify errors in a hundred chart.

Specific Learning Outcome(s):	Achievement Indicators:
3.N.4. Estimate quantities less than 1000 using referents. [ME, PS, R, V]	➤ Estimate the number of groups of ten in a quantity using 10 as a referent (known quantity). ➤ Estimate the number of groups of a hundred in a quantity using 100 as a referent. ➤ Estimate a quantity by comparing it to a referent. ➤ Select an estimate for a quantity by choosing among three possible choices. ➤ Select and justify a referent for determining an estimate for a quantity.

PRIOR KNOWLEDGE

Students may have had experience comparing and ordering numbers to 100 and estimating quantities to 100 using referents.

BACKGROUND INFORMATION

Students need experiences naming numbers that are greater than, less than, or between numbers. As students place numbers in ascending or descending order, have them explain and justify the result using a number line or number chart. As students become confident with this concept, they should progress to using place value positional names to determine size. An understanding of place value (SLO 3.N.5) is important for students to compare and order larger numbers.

It is not necessary to introduce the symbols "<" or ">" in Grade 3 but more specifically students should not be assessed using the symbols. The focus should be on using and understanding what *greater than* and *less than* mean.

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Estimation is also used to make mathematical judgments and to develop useful, efficient strategies for dealing with situations in daily life. When estimating, students need to know which strategy to use and how to use it.

Students use reasoning skills to help estimate. It is important to provide referents for estimation. Through the process of choosing and using referents, students will be able to justify a referent for determining an estimate.

Referent: A known quantity used to estimate or compare.

Examples:

- Use one layer of marbles from a jar to estimate the number of marbles in the entire jar.
- The height of the room is about twice as tall as a student.
- Know the width of your pinky finger is approximately 1 cm.
- The length of a piece of paper is approximately 30 cm.

MATHEMATICAL LANGUAGE

order	least
compare	three-digit number
greater than	estimate
less than	referent
greatest	approximate

LEARNING EXPERIENCES

Assessing Prior Knowledge

1. Give each student a hundred chart. Ask them to put a marker on
   - ❐ a number greater than 45
   - ❐ a number less than 93
   - ❐ a number between 28 and 55
2. Have each student select eight number cards (from a random collection 1 to 100) and order them from least to greatest.
3. Ask students, "Which is greater: 87 or 78? Explain/show how you know."
4. Have students take a handful of centimetre cubes, bingo chips, or other small objects such as buttons or beans, estimate the quantity, and then explain how they decided on the answer.

The student is able to

- ❐ identify and compare numbers to 100 correctly
- ❐ order numbers to 100
- ❐ use a referent to make a reasonable estimate

- Place a set of numbers in ascending or descending order, and verify the result by using a hundred chart (e.g., a one hundred chart, a two hundred chart, a three hundred chart), a number line, or by making references to place value.
- Create as many different 3-digit numerals as possible, given three different digits. Place the numbers in ascending or descending order.
- Identify errors in an ordered sequence.
- Identify missing numbers in parts of a hundred chart.
- Identify errors in a hundred chart.

Suggestions for Instruction

- Grade 3 students do not have a good understanding of what a 1000 of something looks like. Use a book such as *How Much, How Many, How Far, How Heavy, How Long, How Tall Is 1000?* by Helen Nolan, illustrated by Tracy Walker, to help give students a sense of the magnitude of 1000.

BLM
3.N.3.1

- Use a collection of number cards with values shown to 1000. Have students choose 10 cards and order them from least to greatest. You could also have 10 different students each select a card and then order themselves.

BLM
3.N.3.2

- **Missing Pieces:** Use parts of hundred charts and have students fill in the missing numbers. Ask students to explain how they know what number to write.

  Examples:

  These were taken from larger charts. Fill in the missing numerals.

	826			
835				
			848	
		857		
				869

436	
	457

  Have students make their own charts with an empty chart.

■ **Roll and Order**

Objective: to use number sense to order three-digit numbers

Materials: 3 (0 to 9) dice, a game board for each player

Directions:

1. Determine the Start and End numbers (e.g., start on 1 and end on 1000).

2. Player 1 rolls the three dice and arranges them into a three-digit number. He or she writes the number where he or she feels it belongs on his or her game board.

3. Player 2 takes a turn.

4. Play continues until one player has filled his or her board (numbers are in sequence).

Note: Differentiate the game by

■ changing the Start and End numbers (e.g., narrow the range [200 to 300])

■ having students play in partners

■ reducing or increasing the number of dice

■ increasing or decreasing the number of spaces on the game board

■ **Largest or Smallest?**

Objective: to make the largest/smallest number

Materials: one die (0 to 9), game board for each player

Directions:

1. Decide whether players are making the largest or smallest number.

2. Player 1 rolls the die and calls out the number.

3. Both players choose to place the number (digit) in the hundreds, tens, or ones place on their game boards.

4. The die is rolled two more times with players filling in their game boards each time until the number is complete.

5. Players compare their numbers. The player with the largest/smallest number scores 1 point. In the event of a tie both players score a point.

6. Player 2 then rolls the die. Play continues until one player scores 10 points.

Hundreds	Tens	Ones

Observe students as they play.

Ask

❒ How do you decide where to place each number?

❒ Does your strategy help you score points?

- **Finding Errors:** Provide students with portions of hundred charts and number sequences that contain errors. Have students identify and correct the errors.

Example:

	123			
	133	134		
141				145
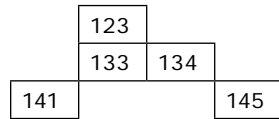
- **Ordering on a Number Line:** Use a clothesline/piece of string and a set of cards with numbers between 1 and 1000. (Tent cards—cards that are folded over so that they sit on the line without flipping or moving—work best.) Label the start and finish of the line. Pass out number cards to several students. Have them place their number on the line. Ask students to explain their placement choice.

   Note: This activity could be used as a classroom routine. Have one or two students each day place two or three numbers on the line explaining their placement choice.

Assessing Understanding: Checklist

Use a checklist to record observations from the learning activities above.

BLM 3.N.3.5

Students	Order numbers from 1 to 200	Order numbers from 300 to 500	Order numbers from 600 to 1000

- **Estimate the number of groups of ten in a quantity using 10 as a referent (known quantity).**
- **Estimate the number of groups of a hundred in a quantity using 100 as a referent.**
- **Estimate a quantity by comparing it to a referent.**
- **Select an estimate for a quantity by choosing among three possible choices.**
- **Select and justify a referent for determining an estimate for a quantity.**

Suggestions for Instruction

- Use a book such as *Great Estimations* by Bruce Goldstone to help students understand how referents are used for estimation.

- Fill a container with cereal, popcorn, or manipulative materials (up to 1000). Ask students to estimate the number of individual pieces in the container, using a referent. The referent can be a set of 10, 25, or 50—this will assist them in making reasonable estimates. Refill the container with other materials to improve the students' estimation skills.

- **Investigation:** Pairs of students open a small box of raisins and count the visible (top layer) of raisins. They use this referent to estimate the number of raisins in a box. Graph the estimates on the chalkboard. Have students spill out and count the actual number of raisins. Graph the actual number counts on the chalkboard. Discuss the differences.

  Repeat the investigation using the same empty raisin boxes filled with puffed rice or using different size boxes of raisins.

- **Estimation Station:** Set up a weekly estimation station with a quantity to be estimated and an appropriate referent (e.g., 2 containers—one filled with centimetre cubes and the other with ten/one hundred cubes [depending on the number of cubes]). Have students record their estimate and then count to find the actual.

Objects	Estimate	Actual	My estimate is too low just right too high

- Show students a quantity of an object (e.g., beans, bingo chips, craft sticks). Provide students with three possible choices and have them choose the best estimate and record it. Show them a referent and allow students to change their estimate.

Assessing Understanding: Paper-and-Pencil Tasks

1. Estimate the number of objects in each box. Explain how you decided on your estimate.
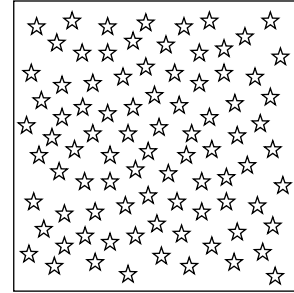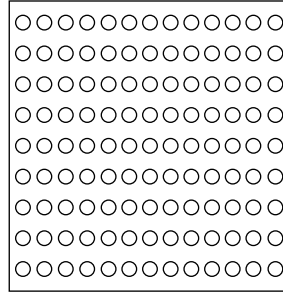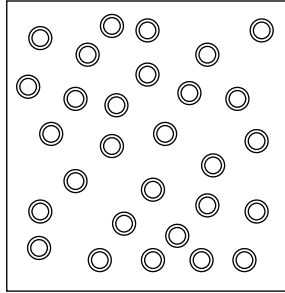
2. **Journal/Learning Log Entry**

   Display a transparent container with small objects such as pennies in it and a matching container with 10 (or 100) of the same objects to use as a referent (depending on the quantity in the container). Ask students to record their estimate in their journal and then write about how they arrived at their estimate.

   For both tasks, the student is able to

   ❐ select and use an appropriate referent (10, 25, 100)

   ❐ make a reasonable estimate

   ❐ explain his or her thinking

Grade 3: Number (3.N.5)

Enduring Understandings:

   The position of a digit in a number determines the quantity it represents.

   The groupings of 1s, 10s, and 100s can be taken apart in different ways.

Essential Question:

   How many ways can a number be taken apart?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
3.N.5 Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000. [C, CN, R, V]	➔ Record in more than one way the number represented by proportional and non-proportional concrete materials. ➔ Represent a number in different ways using proportional and non-proportional concrete materials, and explain how they are equivalent (e.g., 351 can be represented as three 100s, five 10s, and one 1, or two 100s, fifteen 10s, and one 1, or three 100s, four 10s, and eleven 1s). ➔ Explain, and show with counters, the meaning of each digit for a 3-digit numeral with all digits the same (e.g., for the numeral 222, the first digit represents two hundreds [two hundred counters], the second digit represents two tens [twenty counters], and the third digit represents two ones [two counters]).

PRIOR KNOWLEDGE

Students may have illustrated, concretely and pictorially, the meaning of place value for numbers to 100 in Grade 2 using proportional base-10 materials.

BACKGROUND INFORMATION

In order to understand and use place value students need to be able to "think in groups" or to unitize. They need to see ten as a unit and not as a collection of 10 individual parts.

In 1989, Sharon Ross (cited in Van de Walle, Karp, Lovin, and Bay-Williams 157) identified five distinct levels of understanding of place value based on responses to the following task:

1. Place 36 blocks on the table and have the students count them.

2. Have them write the number that tells how many there are.

3. Circle the 6. Ask, "Does this part of your 36 have anything to do with how many blocks there are?"

4. Circle the 3. Ask, "Does this part of your 36 have anything to do with how many blocks there are?"

Do not give clues.

Levels of Understanding of Place Value

1. **Single numeral:** The student writes 36 but views it as a single numeral. The individual digits 3 and 6 have no meaning by themselves.

2. **Position names:** The student identifies correctly the tens and ones/units position but still makes no connections between individual digits and the counters.

3. **Face value:** The student matches 6 counters with the 6 and 3 counters with the 3.

4. **Transition to place value:** The 6 is matched with the 6 counters and the 3 with the remaining 30 counters but they are not seen as 3 groups of 10.

5. **Full understanding:** The 3 is correlated with 3 groups of 10 counters and the 6 with 6 single counters.

In proportional models for base 10, such as craft (popsicle) sticks, Digi-Blocks, and base-10 blocks, the representation for 10 is ten times the size of the representation for 1, 100 is ten times the size of 10, and so on.

Non-proportional models, such as money, do not have any size relationship.

Students need opportunities to construct their own understanding of our place value system. In order to do this, they should work with materials such as craft sticks that they have to group themselves. Base-10 blocks and Digi-Blocks are useful materials once their understanding is in place, but they are adult- rather than student- or child-developed constructs.

Preventing Misconceptions: The way we talk about concepts/ideas can create misconceptions for students. For example: Students are shown the number 68 and asked, "How many ones are in this number?" Generally the expected response is "8" but in fact, there are sixty-eight ones in 68. Rephrasing the question to ask how many extra ones/remaining ones (the number left over after grouping in tens) or how many ones are in the ones/unit place in this number may help prevent misconceptions.

MATHEMATICAL LANGUAGE

place value

expanded form/notation

hundreds

tens

ones

units

LEARNING EXPERIENCES

Assessing Prior Knowledge: Interview

Use the Sharon Ross assessment (see background information) to identify the student's understanding.

Note: At the end of Grade 2 students should be at Stage 4: Transition to place value.

- **Record in more than one way the number represented by proportional and non-proportional concrete materials.**

- **Represent a number in different ways using proportional and non-proportional concrete materials, and explain how they are equivalent (e.g., 351 can be represented as three 100s, five 10s and one 1, or two 100s, fifteen 10s, and one 1, or three 100s, four 10s, and eleven 1s).**

- **Explain, and show with counters, the meaning of each digit for a 3-digit numeral with all digits the same (e.g., for the numeral 222, the first digit represents two hundreds [two hundred counters], the second digit represents two tens [twenty counters], and the third digit represents two ones [two counters]).**

Suggestions for Instruction

- **Toothpick 1000:** Use toothpicks, glue and cardstock/manila tag to make a visual representation of 1000.

Directions:

1. Cut 100 cards (don't use white paper) large enough to hold 10 toothpicks with small spaces between them. Have students glue 10 toothpicks on each card. It is important the toothpicks are oriented the same way on the cards (e.g., the "hotdog" way).

2. Cut 10 strips of cardstock/manilla tag (in a contrasting colour) wide enough to fit the toothpick cards leaving a small space on either side and long enough to hold 10 toothpick cards with a small space between each one. Staple the toothpick cards on each strip to represent a hundred.
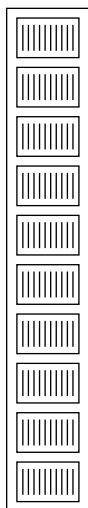
3. Put the 10 hundred strips together (with a small space between each one) to represent a thousand.
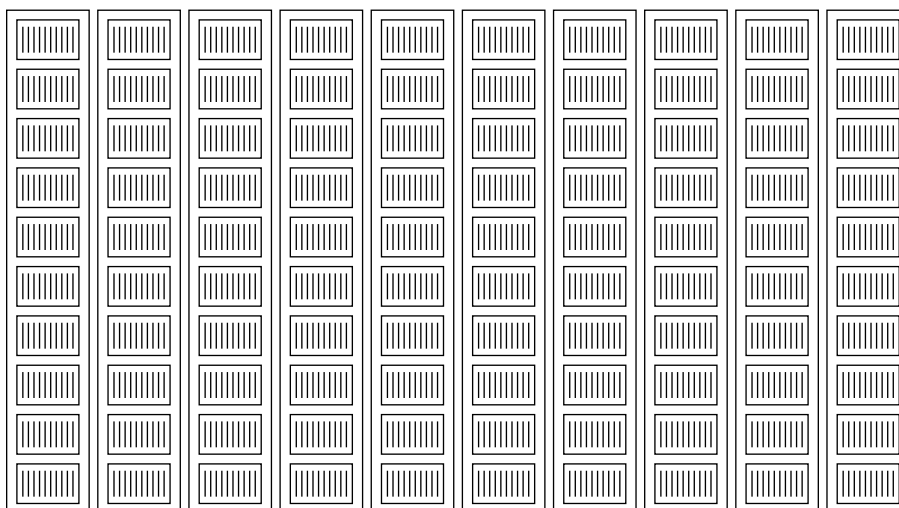
   Students are now able to identify a particular toothpick by counting the hundred strips, the ten cards, and the individual toothpicks on the cards.

Students can play "Guess My Number" or "Find the Number" using this representation.

Examples:

- **Guess My Number:**
  1. "I am thinking of a number between 325 and 650. Guess my number."
  2. Students ask questions that can be answered with a "yes" or a "no":
     - Does the number have 5 hundreds?
     - Is the number greater than 500?
     - Does the number have 6 tens?

     And so on.
  3. Count the number of questions it takes to find the number.
  4. Each time they play, students try to decrease the number of questions needed.

- **Find the Number:**
  - Find the number that has 3 hundreds, 6 tens, and 2 ones.
  - Find the number that is 100 less than 976.
  - Find the number that has 4 hundreds, 15 tens, and 9 ones.

  And so on.

- Use bagged or bundled manipulatives (e.g., beans, straws) to represent numbers concretely. Have students record the number in more than one way.

  Example:

  What number is shown in the picture below?

  Record the number in three different ways.
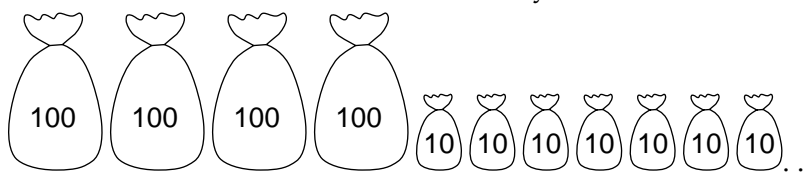
  Possible answers:
  - four hundred seventy-three
  - 4 hundreds, 7 tens, 3 ones
  - 100 +100 100 +100 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 1 + 1 + 1
  - 400 + 70 + 3

- It is very important that students have experience using zero as a place holder.

Have students trade hundreds, tens, and ones in order to represent a number in a variety of ways.

Examples:
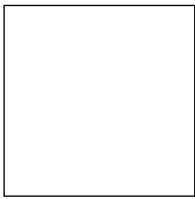
473 = 3 hundreds + 17 tens + 3 ones

473 = 4 hundreds + 6 tens + 13 ones

209 = 2 hundreds + 0 tens + 9 ones

209 = 1 hundred + 10 tens + 9 ones

Students might find that using a place-value grid helps them to organize their thinking and to begin to see patterns in their answers. For example:

Hundreds	Tens	Ones
4	7	3
3	17	3
2	27	3
1	37	3
4	6	13
4	5	23
4	4	33

- **Place Value Tents or Arrows Cards:** Place value tent/arrow cards help students to see the relationship between a digit and its value based on its position in the number. Using the cards, illustrate how numbers are built. The cards are made so that the tens card is twice as long as the ones cards, and the hundreds card is three times as long as the ones cards.

Have students use place value tent/arrow cards to make numerals.

Examples:

- Make a three-digit number between 350 and 500.
- Make an even three-digit number that is greater than 400 but less than 475.
- Make a three-digit number that is 100 more than 652.
- Make a three-digit number that has is 10 more than 492.
- Make a three-digit number in which the ones digit is greater than the tens digit.

Starting with the hundreds card, layer the others on top, right aligned. This approach will show how the number is built, while allowing students to see the individual components of the number.
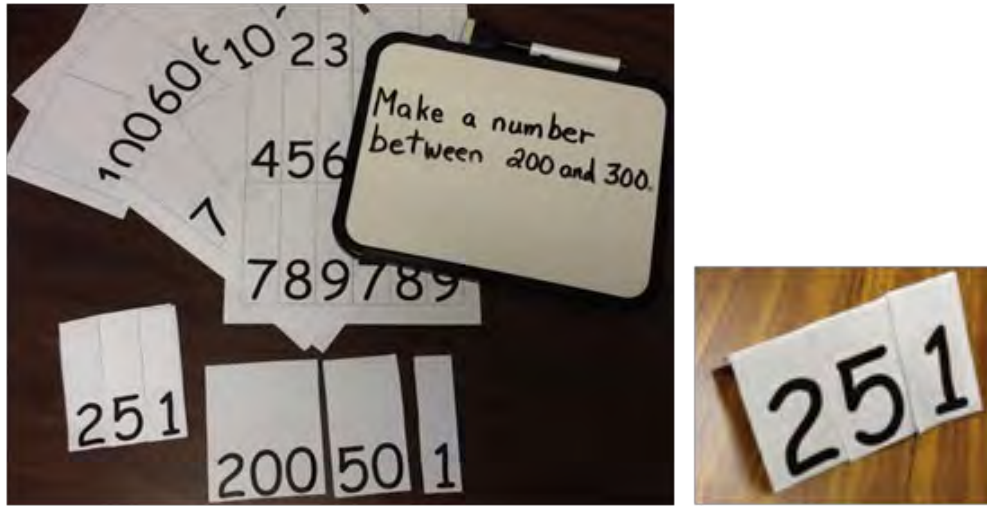
- Create and share number riddles using the place value tent or arrow cards.

    Example:

    I am thinking of a number less than 300. The ones digit is greater than the tens digit. The sum of the digits is 12. What could the number be?

- **What's the Value?** Give students a three-digit number with all three digits the same. Ask them to explain what each of the digits represents using both manipulatives and words.

- **Calculator Challenge:** Have students enter a three-digit number with three identical digits (e.g., 555).

    Ask the following:

    - Can you change the number so that it has a zero in the ten's place? What did you enter? (– 50)

    - Can you change the number so that it has a zero in the hundred's place? What did you enter? (– 500)

    - Can you change the number so that it has a zero in the one's place? What did you enter? (– 5)

    This activity can be used for numbers without identical digits.

- **Classroom Routine: "Stretch It!–Place Value"** Have students work in partners. Give them a two- or three-digit number or have them select one. Using a strip of chart paper, students represent the number (using place value) in as many different ways as they can.

Example:

58
5 tens, 8 ones
4 tens, 18 ones
3 tens, 28 ones
2 tens, 38 ones
1 ten, 48 ones
58 ones

Assessing Understanding: Interview

Give the student a three-digit number such as 264. Have them explain the meaning of each digit using materials to support their explanation.

The student is able to

- ❐ use materials to represent a three-digit number
- ❐ explain that the first digit represents 2 hundreds (two hundred counters)
- ❐ explain that the second digit represents 6 tens (sixty counters)
- ❐ explain that the third digit represents 4 ones (four counters)

Paper-and-Pencil Task

1. Roll a die (0 to 9) three times. Record the numbers. (If any of the numbers are the same roll the dice again.) Make as many three-digit numbers as you can. Order the numbers from greatest to least.

2. Choose one of the numbers you made. Explain the value of each digit. Use pictures and words.

3. Choose another number. Represent it in at least six different ways, using what you know about place value.

Making a Step Book

Tell students that they are going to make their own Step Book for representing a number.

Directions:

1. Take four pieces of letter-sized paper. Stagger the sheets by placing each sheet of paper about 2.5 cm (1 inch) lower than the one before. You will have four steps.
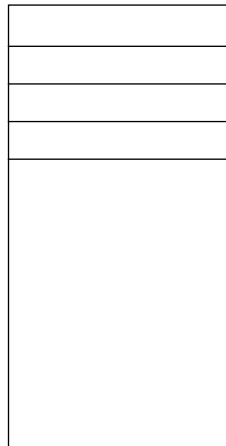
2. Hold the papers together. Turn them over so that the overlapping sections are on the bottom. Fold the top over so that you now have eight steps. Check to see if the steps are even and adjust as necessary. Open the top four steps and put two staples along the crease.

Fold
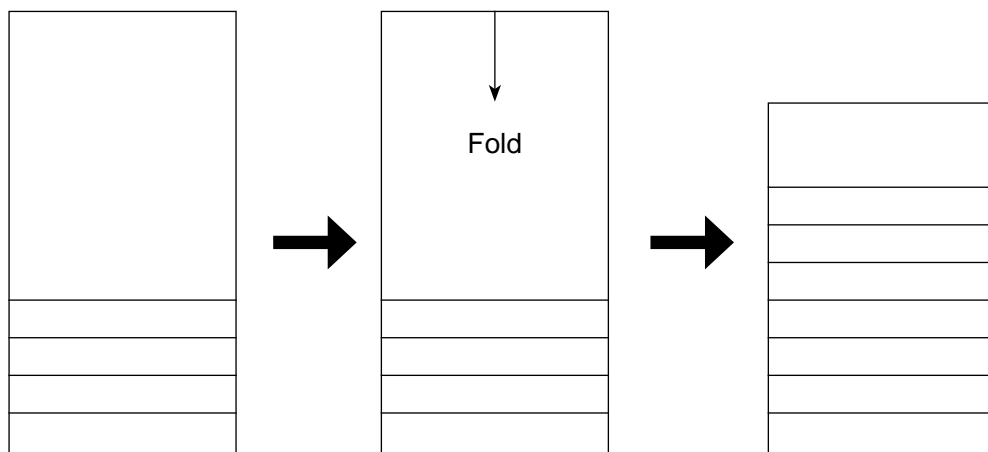
3. Brainstorm the different ways you/the students want to represent their number. Write one method on each of the second through seventh flaps. The last flap can be used for self-reflection/assessment.

4. Assign or have students select a three-digit number to represent.

Example:

Include regular and non-regular representations. → **Place Value**

Place the number on a blank 0-to-1000 number line. Students should give a reason for their placement. → **Number Line**

Equations/number sentences can be used instead of expressions. → **Expressions**

Where/What might this number be found/represent in the real world? (e.g., house number, pages in a book, number of students in our school, money) → **Connections**

Students do a reflection/assessment on how well they feel they have represented their number. See sample. → **Self-Reflection**

Sample Reflection/Assessment (done on a separate sheet and glued/taped in):

I represented my number using
- words
- pictures
- place value-regular and non-regular
- number line
- expressions
  - addition
  - subtraction
- a story problem
- connections

Next time I will

_____

_____

Note: Differentiate the assignment by

- having students select their own number
- assigning numbers, varying the number size and range based on student needs
- having students pick the ways to represent their number
- having students work with a partner

Grade 3: Number (3.N.6, 3.N.7, 3.N.8, 3.N.9, 3.N.10)

Enduring Understandings:

Quantities can be taken apart and put together.

Addition and subtraction are inverse operations.

Essential Question:

How can symbols be used to represent quantities, operations, or relationships?

How can strategies be used to compare and combine numbers?

What questions can be answered using subtraction and/or addition?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
3.N.6 Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as ■ adding from left to right ■ taking one addend to the nearest multiple of ten and then compensating ■ using doubles [C, ME, PS, R, V]	➤ Add two 2-digit numerals using a mental mathematics strategy, and explain or model the strategy. ➤ Explain how to use the "adding from left to right" strategy (e.g., to determine the sum of 23 + 46, think 20 + 40 and 3 + 6). ➤ Explain how to use the "taking one addend to the nearest multiple of ten" strategy (e.g., to determine the sum of 28 + 47, think 30 + 47 – 2 or 50 + 28 – 3). ➤ Explain how to use the "using doubles" strategy (e.g., to determine the sum of 24 + 26, think 25 + 25; to determine the sum of 25 + 26, think 25 + 25 + 1 or doubles plus 1). ➤ Apply a mental mathematics strategy for adding two 2-digit numerals.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
3.N.7 Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as ■ taking the subtrahend to the nearest multiple of ten and then compensating ■ thinking of addition ■ using doubles [C, ME, PS, R, V]	➤ Subtract two 2-digit numerals using a mental mathematics strategy, and explain or model the strategy. ➤ Explain how to use the "taking the subtrahend to the nearest multiple of ten and then compensating" strategy (e.g., to determine the difference of 48 – 19, think 48 – 20 + 1). ➤ Explain how to use the "thinking of addition" strategy (e.g., to determine the difference of 62 – 45, think 45 + 5, then 50 + 12 and then 5 + 12). ➤ Explain how to use the "using doubles" strategy (e.g., to determine the difference of 24 – 12, think 12 + 12). ➤ Apply a mental mathematics strategy for subtracting two 2-digit numerals.
3.N.8 Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem-solving context. [C, ME, PS, R]	➤ Estimate the solution for a story problem involving the sum of two 2-digit numerals (e.g., to estimate the sum of 43 + 56, use 40 + 50; the sum is close to 90). ➤ Estimate the solution for a story problem involving the difference of two 2-digit numerals (e.g., to estimate the difference of 56 – 23, use 50 – 20; the difference is close to 30).
3.N.9 Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1-, 2-, and 3-digit numerals) by ■ using personal strategies for adding and subtracting with and without the support of manipulatives ■ creating and solving problems in contexts that involve addition and subtraction of numbers concretely, pictorially, and symbolically. [C, CN, ME, PS, R]	➤ Model the addition of two or more numbers using concrete or visual representations, and record the process symbolically. ➤ Model the subtraction of two numbers using concrete or visual representations, and record the process symbolically. ➤ Create an addition or subtraction story problem for a solution. ➤ Determine the sum of two numbers using a personal strategy (e.g., for 326 + 48, record 300 + 60 + 14). ➤ Determine the difference of two numbers using a personal strategy (e.g., for 127 – 38, record 38 + 2 + 80 + 7 or 127 – 20 – 10 – 8). ➤ Solve a problem involving the sum or difference of two numbers.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
3.N.10 Apply mental math strategies to determine addition facts and related subtraction facts to 18 (9 + 9). [C, CN, ME, R, V] Recall of addition and related subtraction facts to 18 is expected by the end of Grade 3.	➤ Describe a mental mathematics strategy that could be used to determine a given basic fact, such as ■ doubles (e.g., for 6 + 8, think 7 + 7) ■ doubles plus one (e.g., for 6 + 7, think 6 + 6 + 1) ■ doubles take away one (e.g., for 6 + 7, think 7 + 7 – 1) ■ doubles plus two (e.g., for 6 + 8, think 6 + 6 + 2) ■ doubles take away two (e.g., for 6 + 8, think 8 + 8 – 2) ■ making 10 (e.g., for 6 + 8, think 6 + 4 + 4 or 8 + 2 + 4) ■ commutative property (e.g., for 3 + 9, think 9 + 3) ■ addition to subtraction (e.g., for 13 – 7, think 7 + □ = 13). ➤ Provide a rule for determining answers for adding and subtracting zero.

PRIOR KNOWLEDGE

Students may have an understanding of addition and subtraction (limited to one- and two-digit numerals) to 100 using personal strategies.

They may be able to apply the following mental math strategies:

■ making ten

■ using doubles

■ using one more, one less

■ using two more, two less

■ building on a known double

■ using addition for subtraction

BACKGROUND INFORMATION

One goal of mathematics education is to have students calculate using number sense, which in part means having students learn flexible and accurate methods of computation. We do not want students to rely on one single method of computing but rather to use methods that are efficient depending on the context and the numbers used. Mental math and estimation is one of the math processes that needs to be incorporated throughout the year.

As students progress in their understanding of operations, they continue to build their understanding of place value concepts. Place value understanding is a vital concept that students must possess as they learn multi-digit computations.

Students will develop, apply, and describe strategies to add and subtract. The strategies listed in the learning outcomes should be explored with students. The strategies listed in the learning outcomes are not an exclusive list, however. Students may have their own strategies they use on a regular basis. Students need to be given regular opportunities to ensure that they have ample practice and application with mental math strategies. Instruction needs to focus on a balanced approach that provides students with opportunities to develop their conceptual understanding, procedural thinking, and problem solving that mutually reinforce student learning.

In order for students to be computationally fluent with basic facts, they must exhibit efficiency, accuracy, and flexibility with numbers. Students must be able to develop quick and accurate recall of the basic facts, but this skill is most effective only after achieving conceptual understanding. Recall is a developmental process that improves computational fluency by developing efficiency, accuracy, and flexibility with numbers. It is more than just producing a quick answer to a computation.

Educational math resources define quick recall of math facts as the ability to solve a basic number computation in 3 seconds or less (Van de Walle, Karp, Lovin, and Bay-Williams). However, math educators have found that a premature demand for a quick performance could cause anxiety and undermine understanding for some students. The 3-second recall is an assessment guideline for teachers and does not need to be shared with students because it may cause undue anxiety or may also interfere with students' mathematical thinking.

The recall of basic math facts develops through procedural fluency and proficiency in the other strands of mathematics and leads to higher-order math skills. Recall consists of and is taught through

- number sense
- strategy development
- reasoning skills
- effective practice (practice with meaning)

When teaching for understanding, we require students to learn certain prerequisite skills and knowledge to achieve recall. Students develop their number sense over a period of time. Final recall of math facts is a consolidation of understanding and purposeful practice developed starting in Kindergarten.

Addend: One of the numbers in a designated sum of two or more numbers (e.g., in 3 + 5 + 1 = 9; 3, 5, and 1 are addends).

Minuend: In a subtraction problem, the number from which another number is to be subtracted (e.g., in 5 – 3 = 2, the 5 is the minuend).

Subtrahend: In subtraction, the number being subtracted from a given number (e.g., in 5 – 2 = 3, 2 is the subtrahend).

Note: Addition and subtraction should be taught together. This will enable students to see the relationships between the two operations.

There are many different types of addition and subtraction problems. Students should have experience with all types.

Addition				Both + and -
Result Unknown (a + b = ?)	Change Unknown (a + ? = c)	Start Unknown (? + b = c)	Combine (a + b = ?)	Compare
Pat has 8 marbles. Her brother gives her 4. How many does she have now? (8 + 4 = ?)	Pat has 8 marbles but she would like to have 12. How many more does she need to get? (8 + ? = 12)	Pat has some marbles. Her brother gave her 4 and now she has 12. How many did she have to start with? (? + 4 = 12)	Pat has 8 blue marbles and 4 green marbles. How many does she have in all? (8 + 4 = ?)	Pat has 8 blue marbles and 4 green marbles. How many more blue marbles does she have? (8 – 4 = ? or 4 + ? = 8) Pat has 8 blue marbles and some green marbles. She has 4 more blue marbles than green ones. How many green marbles does she have? (8 – 4 = ? or 4 + ? = 8)
Subtraction				
Result Unknown (a - b = ?)	Change Unknown (a – ? = c)	Start Unknown (? – b = c)	Combine	
Pat has 12 marbles. She gives her brother 4 of them. How many does she have left? (12 – 4 = ?)	Pat has 12 marbles. She gives her brother some. Now she has 8. How many marbles did she give to her brother? (12 – ? = 8)	Pat has some marbles. She gives her brother 4 of them. Now she has 8. How many marbles did she have to start with? (? – 4 = 8)	Pat has 12 marbles. 8 are blue and the rest are green. How many are green? (12 – 8 = ?)	

MATHEMATICAL LANGUAGE

Operations:

addition	take away
add	compatible number pairs
sum	story problem
total	number sentence
more	multiple
subtraction	estimate
subtract	addition fact
difference	subtraction fact
less	strategy

LEARNING EXPERIENCES

Assessing Prior Knowledge: Paper-and-Pencil Task or Interview

Give students the following questions:

34 + 42   28 + 57   89 – 65   73 – 18

Solve the problems. Be sure to show your thinking.

The student is able to use

❐ expanded form or adding from left to right [34 + 42 ⟶ (30 + 40) +(4 + 2)]
❐ compensating [28 + 57 ⟶ (28 + 2) + 55]
❐ number line
❐ other _____

- **Add two 2-digit numerals using a mental mathematics strategy, and explain or model the strategy.**

- **Explain how to use the "adding from left to right" strategy (e.g., to determine the sum of 23 + 46, think 20 + 40 and 3 + 6).**

In the "adding from left to right" strategy, each number is mentally broken into its place value components (expanded notation) and then they are added, starting with the largest place value.

Examples:

43 + 26 =   38 + 15 =
40 + 20 = 60   30 + 10 = 40
3 + 6 = 9   8 + 5 = 13
60 + 9 = 69   40 + 13 = 53

- **Explain how to use the "taking one addend to the nearest multiple of ten" strategy (e.g., to determine the sum of 28 + 47, think 30 + 47 − 2 or 50 + 28 − 3).**

This strategy uses the idea of making "friendly" or "nice" numbers. Friendly numbers are numbers that are easier to work with (e.g., multiples of ten).

Example:

58 + 17
58 is close to 60
60 + 17 = 77
58 + 17 is 2 less
77 – 2 = 75

- **Explain how to use the "using doubles" strategy (e.g., to determine the sum of 24 + 26, think 25 + 25; to determine the sum of 25 + 26, think 25 + 25 + 1 or doubles plus 1).**

In this strategy students use a double fact they know or can determine easily to help them find the total.

Example:

43 + 40
40 + 40 = 80
43 is 3 more so 43 + 40 = 83

> ■ **Apply a mental mathematics strategy for adding two 2-digit numerals.**

Suggestions for Instruction

■ **Classroom Routine—"What's My Strategy?"**

Prepare cards with expressions adding two 2-digit numbers. Have students work in partners. Each student draws a card, determines the strategy he/ she will use to solve it, and then finds the answer. When students have a solution, they take turns explaining to their partner the strategy used.

Both students could work with the same expression and solve it separately. Once the problem is solved, students can compare the strategies used.

In both cases, providing an opportunity for students to share their strategies and solutions with the whole class will help all students further develop their "strategies vocabulary."

BLM
3.N.6.1

■ **Race to 100**

Materials:

2 (1 to 6) dice, recording sheet, and pencil

Directions:

1. Each player, in turn, rolls the two dice, makes a two-digit number, and records it on their recording sheet.

2. On the second roll, the new two-digit number is added to the first and the total is recorded. The player explains the strategy used.

3. Play continues in this manner until one player gets to 100.

Note: Before playing the game, students need to decide if the winner is the first person to go over 100 or if it is the person closest to 100 without going over.

Example:

Race to 100	
Roll	**Total**
0	
25	25
16	41
32	73 and so on

Assessing Understanding: Paper-and-Pencil Task/Portfolio Entry

Using doubles
25 + 28

Using
friendly numbers
49 + 27

I can use these strategies
to add two-digit numbers.

Adding from
left to right
74 + 23
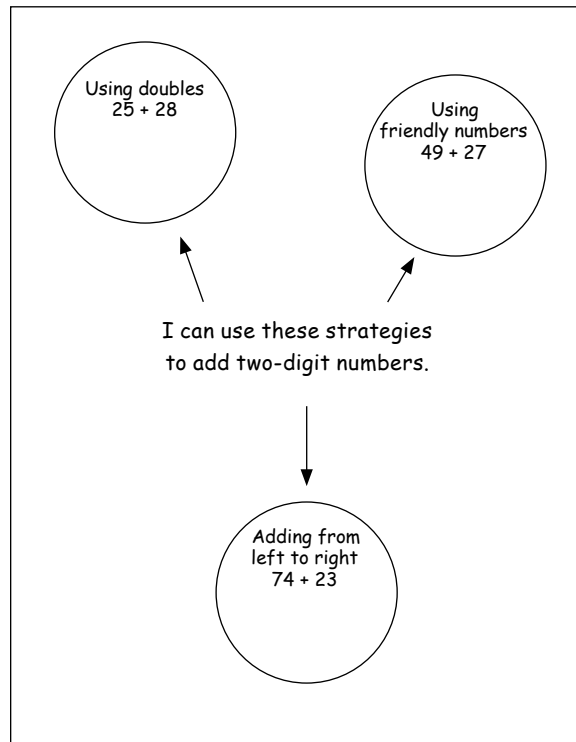
- **Subtract two 2-digit numerals using a mental mathematics strategy, and explain or model the strategy used.**
- **Explain how to use the "taking the subtrahend to the nearest multiple of ten and then compensating" strategy (e.g., to determine the difference of 48 − 19, think 48 − 20 + 1).**

This strategy uses the idea of making "friendly" or "nice" numbers. Friendly numbers are numbers that are easier to work with (in this case, multiples of ten).

Example:

74 – 28
28 is close to/two away from 30
Think 74 – 30 + 2
74 – 30 = 44
44 + 2 = 46

Using a number line:
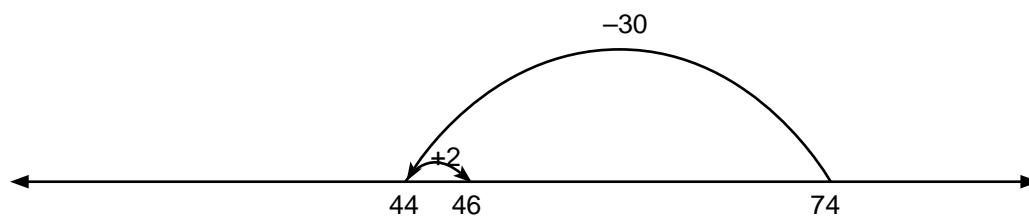
■ **Explain how to use the "thinking of addition" strategy (e.g., to determine the difference of 62 – 45, think 45 + 5, then 50 + 12 and then 5 + 12).**

"Think addition" is an efficient strategy for subtraction.

Example:

83 – 56 (think "56 and how many more to make 83")
56 + 4 = 60 (think "60 and 23 more to make 83")
60 + 23 = 83
Therefore, the difference between 83 and 56 is 4 + 23 or 27.

Using a number line:
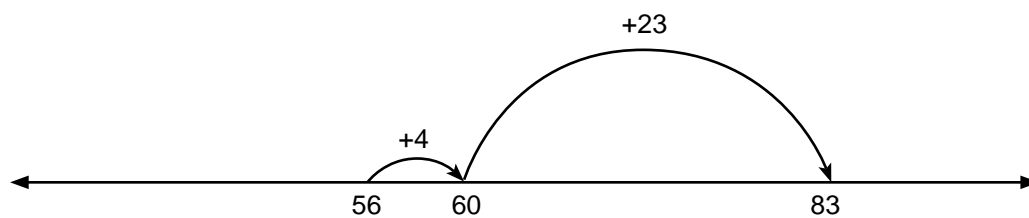
■ **Explain how to use the "using doubles" strategy (e.g., to determine the difference of 24 – 12, think 12 + 12).**

In this strategy students use a double fact they know or can determine easily to help them find the difference.

Example:

28 – 14, think "14 + 14 is 28, so 28 minus 14 is 14"

■ **Apply a mental mathematics strategy for subtracting two 2-digit numerals.**

Suggestions for Instruction

■ **Classroom Routine—"What's My Strategy?"**

Prepare cards with expressions subtracting two 2-digit numbers. Have students work in partners. Each student draws a card, determines the strategy he/she will use to solve it, and then finds the answer. When students have a solution they take turns explaining the strategy used to their partner.

Both students could work with the same expression and solve it separately. Once solved, students can compare the strategies used.

In both cases, providing an opportunity for students to share their strategies and solutions with the whole class will help all students further develop their "strategies vocabulary."

■ **Race to Zero**

Materials:

2 (1 to 6) dice, recording sheet, and pencil

Directions:

1. Each player, in turn, rolls the two dice, makes a two-digit number, and records it on their recording sheet. This number is subtracted from 100.

2. On the second roll the new two-digit number is subtracted from the difference and is recorded. Each time the player should explain the strategy used.

3. Play continues in this manner for four rolls. The player with a difference closest to zero is the winner.

BLM 3.N.7.1

Example:

Race to Zero	
Roll	**Difference**
Start 100	
25	75
16	59
32	27

Assessing Understanding: Paper-and-Pencil Task/Portfolio Entry

Using doubles
50 – 25

Using
friendly numbers
74 – 27

I can use these strategies
to add two-digit numbers.

Thinking
addition
62 – 35
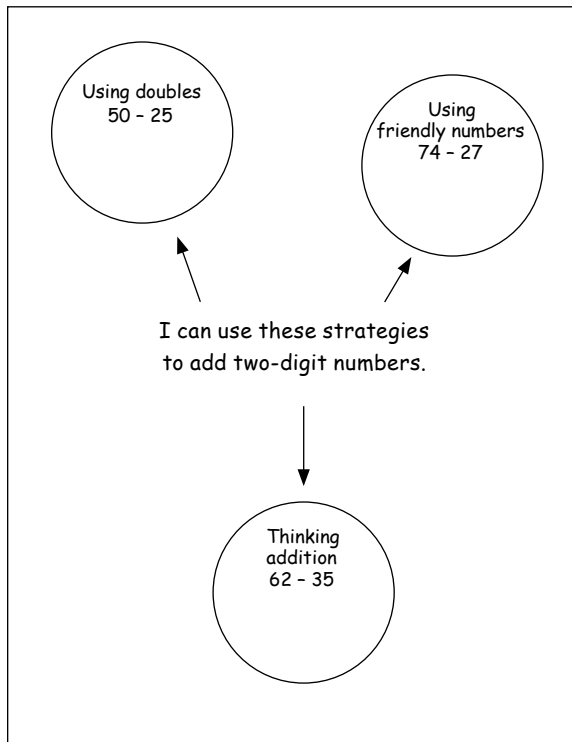
- **Estimate the solution for a story problem involving the sum of two 2-digit numerals (e.g., to estimate the sum of 43 + 56, use 40 + 50; the sum is close to 90).**
- **Estimate the solution for a story problem involving the difference of two 2-digit numerals (e.g., to estimate the difference of 56 − 23, use 50 − 20; the difference is close to 30).**

Suggestions for Instruction

Note: The rules for rounding numbers to the nearest ten are not intended to be taught. Students should be given the freedom to determine the multiples for the estimation.

- Provide multiple opportunities for students to estimate the solutions to a variety of addition and subtraction story problems. Have them explain the strategy used.

- **Model the addition/subtraction of two or more numbers using concrete or visual representations, and record the process symbolically.**
- **Determine the sum of two numbers using a personal strategy (e.g., for 326 + 48, record 300 + 60 + 14).**
- **Determine the difference of two numbers using a personal strategy (e.g., for 127 − 38, record 38 + 2 + 80 + 7 or 127 − 20 − 10 − 8).**

Suggestions for Instruction

Strategies for Addition

382 + 26

Breaking up Numbers (Split Strategy)

This method requires place value understanding.

```
    382   +   26
   ↙ ↓      ↓ ↘
300  80  2  20   6
  ↓    ↘  ↘ ↘ ↓
300 + 100 + 8 = 408
    (80+20)(2+6)
```

Note: As the size of the numbers increases it is more difficult for students to use this method mentally.

Empty Number Line (Jump Strategy)

There are many other possibilities.

Use representations of materials such as base-10 blocks.

382	26

Compensating (making "nice" or "friendly" numbers)

$$382 + 26 \longrightarrow \overset{390}{(382 + 8)} + 18 \longrightarrow 408$$

$$8 + 18$$
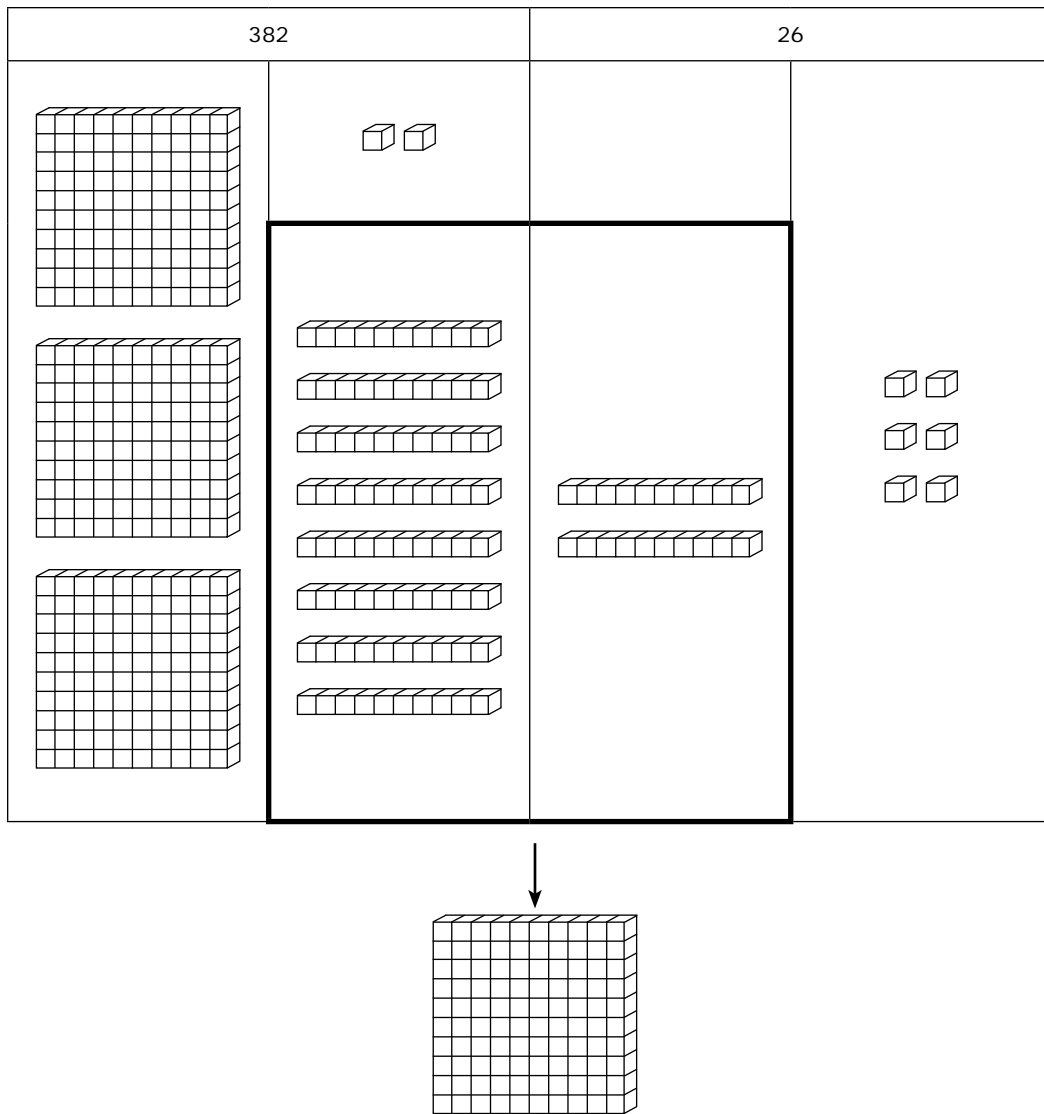
Note: Students need to use their knowledge of compatible number pairs for 10 to use this strategy.
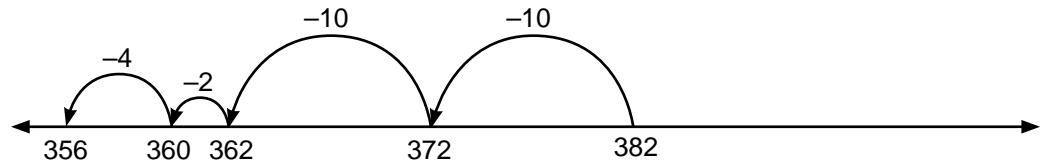
Strategies for Subtraction

382 − 26

Breaking up Numbers (Split Strategy)

This method requires place value understanding.

382   −   26

300 80 2   20   6

70 12

(10 + 2)
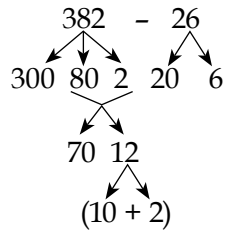
300 + (70 − 20) + (12 − 6) = 356

Empty Number Line (Jump Strategy)

There are many other possibilities.

Use representations of materials such as base-10 blocks.
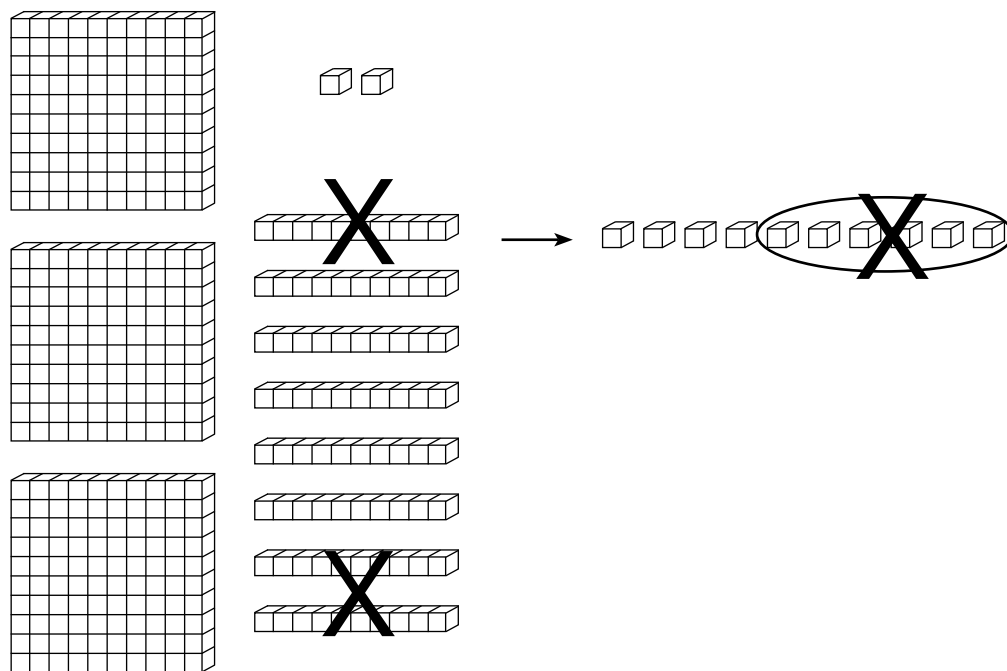
Compensating (making "nice" or "friendly" numbers)

Add 4 to both numbers:

$(382 + 4) - (26 + 4) \longrightarrow 386 - 30 = 356$

- Create an addition or subtraction story problem for a solution.
- Solve a problem involving the sum or difference of two numbers.

Suggestions for Instruction

- **"The Answer Is..." Books**: Make a booklet with 6 to 8 pages. On the front cover write "The answer is 376 [or any other number]. What is the question?" Have students create their own addition and subtraction word problems that would result in an answer of 376 and write them on a page in the booklet.

- Build problem writing into class routines by having two students each day responsible for writing an addition and a subtraction problem that would result an answer equal to a given number. Students could also roll three dice to make their number.

- **What Lies Between?** Present students with a section of a number line.

  Example:

  ```
  ◄─────────────────────────────────────────────►
        125                              140
  ```
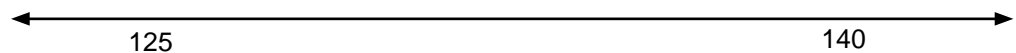
  Have students find different addition and subtraction number sentences that will result in a solution that falls between the two given numbers.

  Example: 100 + 35 = 135 (135 is between 125 and 140.)

  175 – 42 = 133

  And so on.

  Extend the activity by increasing the number size and/or the range.

  Modify the activity by decreasing the size of the numbers.

- Present students with a set of data.

  Example:

Davidson Elementary School Attendance for the Week of January 11 to 15		
Date	Number of Boys (147)	Number of Girls (138)
January 11	145	136
January 12	139	135
January 13	142	134
January 14	137	130
January 15	141	133

  Have students use the data to create story problems.

Extension: Give students an answer and ask them to decide the question using the data.

Examples:

- The answer is 9. What is the question? (How many students were absent on January 13?)
- The answer is 274. What is the question? (How many students were present on January 15?)

Assessing Understanding: Paper-and-Pencil Task
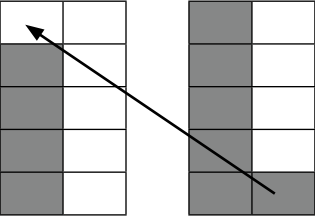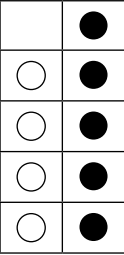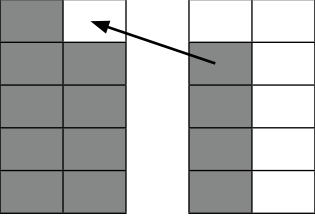
Solve the problems.

Be sure to show your work.

1. The students in Mrs. Johnson's class collected aluminum cans for recycling. Jana collected 124 cans, Mason collected 204 cans, and Marilyn collected 235 cans. How many cans did they collect altogether?

2. The elementary school has 457 students. If 232 of the students are boys, how many girls are in the school?

3. Manjeet has two jars of pennies. One jar has 326 pennies, and the other jar has 387 pennies. How many pennies does Manjeet have altogether?

4. The answer is 245. What is the question? Write an addition problem that has an answer of 245.

5. The answer is 136. What is the question? Write a subtraction problem that has an answer of 136.

- **Describe a mental mathematics strategy that could be used to determine a given basic fact, such as**
  - **doubles (e.g., for 6 + 8, think 7 + 7)**
  - **doubles plus one (e.g., for 6 + 7, think 6 + 6 + 1)**
  - **doubles take away one (e.g., for 6 + 7, think 7 + 7 − 1)**
  - **doubles plus two (e.g., for 6 + 8, think 6 + 6 + 2)**
  - **doubles take away two (e.g., for 6 + 8, think 8 + 8 − 2)**
  - **making 10 (e.g., for 6 + 8, think 6 + 4 + 4 or 8 + 2 + 4)**
  - **commutative property (e.g., for 3 + 9, think 9 + 3)**
  - **addition to subtraction (e.g., for 13 − 7, think 7 + ? = 13).**

Suggestions for Instruction

Mental Math

Note: The development of mental math strategies is greatly enhanced by sharing and discussion. Students should be given the freedom to adapt, combine, and invent their own strategies. Students should be able to apply these strategies to larger numbers.

Strategy	Teaching Strategies
Using doubles: ■ for 4 + 6, think 5 + 5	Use ten frames to help students visualize the strategies. Example: 4 + 6 Students can see that moving the one square (counter) to the other ten frame will make the addition easier by adding 5 + 5.
Using doubles plus one or two: ■ for 4 + 5, think 4 + 4 + 1 ■ for 4 + 6, think 4 + 4 + 2	Use two-colour counters (beans). Example: 4 + 5
Using doubles take away one or two: ■ for 4 + 5, think 5 + 5 − 1 ■ for 4 + 6, think 6 + 6 − 2	Students can see that they can either add 4 + 4 + 1 or 5 + 5 − 1.
Making 10: ■ for 7 + 5, think 7 + 3 + 2	Use a double ten frame to help students visualize the strategy. When adding 9 + 4 students can see that moving one from the 4 to make 10 makes adding easier. This is a practical application of part-part-whole understanding. Eventually students will be able to show the steps without the ten frames. Example: 9 + 4 9 + 1 + 3 10 + 3 = 13

Strategy	Teaching Strategies
Using the commutative property: ■ for 3 + 9 think 9 + 3	These are sometimes referred to as "turn-around facts." Reversing the order of the addends can make it easier for students to apply one of the other mental math strategies (count on, make 10, etc.).
Using addition for subtraction: ■ for 7 − 3, think 3 + ? = 7	Note: Thinking addition is an efficient strategy for subtraction. Teaching addition and subtraction at the same time helps students to see this relationship between the operations. Example: for 9 − 5 think "5 and how many more to make 9?" (5 + ☐ = 9)

■ Use a set of double nine dominoes. Have students sort them into five groups:

- ■ Doubles

- ■ Doubles +/– 1

- ■ Doubles +/– 2

- ■ Make 10

- ■ Other

■ **Provide a rule for determining answers for adding and subtracting zero.**

Suggestions for Instruction

■ Act out problems involving the addition and subtraction of zero.

Examples:

- ■ There are 12 candies in the box. Mark looks at them but doesn't eat any. How many candies are left in the box?

- ■ Samantha is playing a game of Yahtzee. She rolls the dice and gets three sixes. She rolls again but does not get another six. How many sixes does she have altogether?

Have students write their own problems to act out.

■ Present the class with the following equations (one at a time):

- ■ 14 – 0 =

- ■ 19 + 0 =

- ■ 62 + 0 =

- ■ 43 – 0 =

- ■ 18 + 0 =

- ■ 75 – 0 =

After the equations have been solved, ask students if they notice anything about the start number and the sum/difference in each equation. Is there a pattern?

From student observations, develop a class "rule" for adding and subtracting zero (property of zero).

Assessing Understanding: Checklist

Show students a number sentence. Have individual students explain the strategy used to solve the problem. Record the strategy used on the chart.

Note: Some facts will be known by students. Asking them to have to explain a strategy will provide more information about their reasoning strategies and about whether the student understands the fact.

BLM
3.N.10.1

Student	Doubles	Doubles ± 1	Doubles ± 2	Make 10	Commutative Property	Addition for Subtraction	Adding and Subtracting 0	Known Fact	Other

Assessing Understanding

Observe students as they play a game involving doubles.

Doubles

Players: 2

Materials:

- one game board for each player
- four dice (or spinners) labelled with the digits 5, 6, 7, 8, 9, 10
- counters for each player

BLM
3.N.10.2

Doubles			
12	20	18	16
14	18	12	10
10	20	16	14

Directions:

Players take turns to

- roll all four dice
- look for doubles on the dice (e.g., 5, 9, 5, 6 gives double 5)
- cover the sum on their board

If a player throws 2 doubles at once, they can use both of them.

If a player rolls 3 of one number, they make only one double.

The first player to fill in a row is the winner.

Look for:

The student

- ☐ calculates the double mentally
- ☐ calculates the double using manipulatives (number line, counters, fingers, etc.)

■ **Recall compatible number pairs for 5 and 10.**

Assessing Understanding

Use five and ten frames. Flash each frame and have students give the compatible number to make 5 or 10.

Assessing Understanding

Observe students as they play a game involving compatible numbers.

Example: Let's Make 5/10

Players:

■ 2 to 4 for observation purposes
■ whole class for practice

Materials:

■ one game board for each player (either the "Let's Make 5" board or the "Let's Make 10" board)
■ a spinner with the numbers 0 to 5 (0 to 10), or a die labelled 0 to 5 (10-sided die)
■ counters for each player

BLM
3.N.10.3

Let's Make 5				
4	2	0	1	5
3	1	5	2	4
0	3	4	1	2
5	4	2	3	1
1	5	3	4	0

Let's Make 10					
10	5	7	2	9	4
1	8	3	0	6	5
4	3	6	10	0	7
9	2	1	8	5	3
7	0	5	6	1	9
3	8	2	4	10	6

Directions:

1. The teacher spins the spinner or rolls the die and calls out the number shown.
2. Students cover the compatible number to make 5 or 10.
3. The first player to fill in a row, column, or diagonal is the winner.

Look for:

The student

❐ calculates the compatible number mentally within a reasonable length of time
❐ calculates the compatible number using manipulatives (number line, counters, fingers, etc.)

Grade 3: Number (3.N.11, 3.N.12)

Enduring Understandings:

Multiplication and division are inverse operations.

Multiplication is repeated addition.

Division is repeated subtraction.

Essential Questions:

How can skip counting and arrays be used to demonstrate multiplication and division?

How are addition and multiplication related?

How are subtraction and division related?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
3.N.11 Demonstrate an understanding of multiplication to 5 x 5 by ■ representing and explaining multiplication using equal grouping and arrays ■ creating and solving problems in context that involve multiplication ■ modelling multiplication using concrete and visual representations, and recording the process symbolically ■ relating multiplication to repeated addition ■ relating multiplication to division [C, CN, PS, R]	(It is intended that students show their understanding of strategies using manipulatives, pictorial representations, and/or patterns when determining products.) ➤ Identify events from experience that can be described as multiplication. ➤ Represent a story problem (orally, shared reading, written) using manipulatives or diagrams, and record in a number sentence. ➤ Skip-count by 2s, 3s, 4s, and 5s to determine the answer to a multiplication problem represented as equal groups. ➤ Represent a multiplication expression as repeated addition. ➤ Represent a repeated addition as multiplication. ➤ Create and illustrate a story problem for a number sentence. ➤ Represent, concretely or pictorially, equal groups for a number sentence. ➤ Represent a multiplication expression using an array. ➤ Create an array to model the commutative property of multiplication. ➤ Relate multiplication to division by using arrays and by writing related number sentences. ➤ Solve a problem in context involving multiplication.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
3.N.12 Demonstrate an understanding of division by ■ representing and explaining division using equal sharing and equal grouping ■ creating and solving problems in context that involve equal sharing and equal grouping ■ modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically ■ relating division to repeated subtraction ■ relating division to multiplication (limited to division related to multiplication facts up to 5 x 5). [C, CN, PS, R]	(It is intended that students show their understanding of strategies using manipulatives, pictorial representations, and/or patterns when determining quotients.) ➤ Identify events from experience that can be described as equal sharing. ➤ Identify events from experience that can be described as equal grouping. ➤ Illustrate, with counters or a diagram, a story problem involving equal sharing, presented orally or through shared reading, and solve the problem. ➤ Illustrate, with counters or a diagram, a story problem involving equal grouping, presented orally or through shared reading, and solve the problem. ➤ Listen to a story problem, represent the numbers using manipulatives or a sketch, and record the problem with a number sentence. ➤ Create, and illustrate with counters, a story problem for a number sentence. ➤ Represent a division expression as repeated subtraction. ➤ Represent a repeated subtraction as a division expression. ➤ Relate division to multiplication by using arrays and writing related number sentences. ➤ Solve a problem involving division.

PRIOR KNOWLEDGE

Students may have

- skip-counted by 2s, 5s, and 10s, forward and backward, using starting points that are multiples of 2, 5, and 10 respectively
- represented a number in a variety of equal groups with and without singles
- grouped a set of counters into equal groups with and without singles in more than one way and explained which grouping makes counting easier
- worked with doubles to 20

BACKGROUND INFORMATION

Terminology

Multiplication: A mathematical operation of combining groups of equal amounts; repeated addition; the inverse of division.

Product: The number obtained when two or more factors are multiplied (e.g., in $6 \times 3 = 18$, 18 is the product).

Division: A mathematical operation involving two numbers that tells how many groups there are or how many are in each group.

Quotient: The answer to the division of two numbers (in $12 \div 3 = 4$, the quotient is 4).

Array: A set of objects or numbers arranged in an order, usually in rows and/or columns.

The Grade 3 focus for multiplication and division is on having students understand the meaning of multiplication and division as well as on understanding the inverse relationship between the two operations.

Meanings of Multiplication

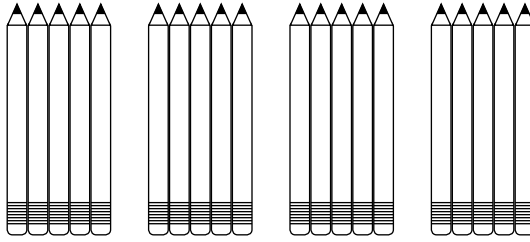1. Repeated addition

    For example: $3 + 3 + 3 = 9$
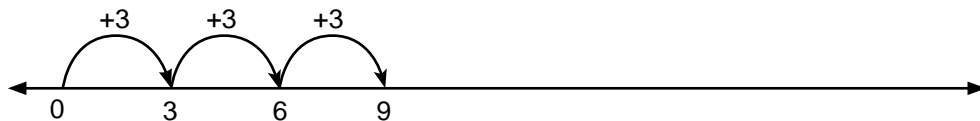
    Note: Repeated addition is helpful to understand multiplication, but students need to move beyond this strategy as their knowledge develops and other strategies become more efficient.

2. Equal groups or sets
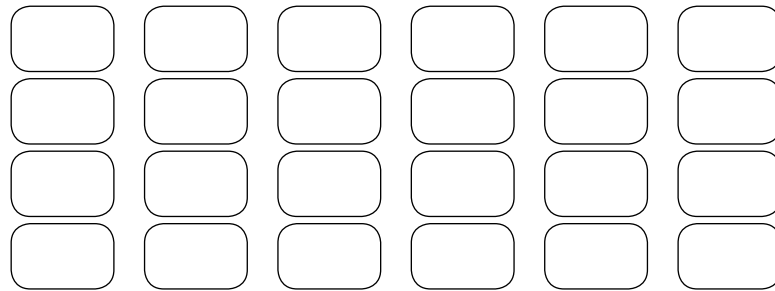
    For example:

    Pencils come in packages of 5.

    How many pencils are in 4 packages?

3. An array

    For example:

    A classroom has 4 rows with 6 desks in each row.

    How many desks are in the classroom?

Multiplication Problems

In a multiplication problem, both the number of objects in each group and the number of groups are given. The total number of objects is the unknown.

Types of Division Problems

1. Partitive or equal sharing division:

    In this type of problem, the total number of groups is known. The unknown is the number of items in each group. The quotient tells the amount for each share.

    For example:

    ▪ Jonas has 24 pieces of candy.

       He wants to share them equally among his 4 friends.

       How many candies will each friend get?

2. Quotative, equal grouping, or measurement division:

In this type of problem, the number of items in each group is known. The unknown is the number of groups that can be made from the given quantity. The quotient tells how many groups can be made from the given quantity.

For example:

■ Martha bought cookies for 10¢ each.

She spent 90¢.

How many cookies did she buy?

Preventing Misconceptions

Common misconceptions students may develop include "multiplication makes bigger" and "division makes smaller." This is not true when working with fractions or decimals less than one, as students will learn in later years.

For example:

$8 \times 0.25 = 2$ or $8 \div 0.25 = 32$

Note: Division should be taught together with multiplication so that students can see the inverse relationship between the two operations.

MATHEMATICAL LANGUAGE

sets of	divide
groups of	division
multiply	equal groups
multiplication	sharing
product	array
quotient	times

Assessing Prior Knowledge: Individual Interview

Skip Counting

Ask the student to

- start at 2 and skip-count by 2s to 24
- start at 5 and skip-count by 5s to 100
- start at 10 and skip-count by 10s to 100
- start at 20 and skip-count backwards by 2s to 8

Thinking in Groups

Give the student at collection of 30 objects/counters. Ask them to count the collection by

- 2s
- 5s
- 10s

Does the student arrange the counters into groups as they are counting?

The student is able to

- ❐ orally skip-count forward by 2s
- ❐ orally skip-count forward by 5s
- ❐ orally skip-count forward by 10s
- ❐ orally skip-count backward by 2s
- ❐ count a collection by 2s using groups of 2
- ❐ count a collection by 5s using groups of 5
- ❐ count a collection by 10s using groups of 10

■ **Identify events from experience that can be described as multiplication.**

Suggestions for Instruction

■ **Thinking about Groups:** Have students brainstorm real-life objects that are sold in groups (juice boxes, eggs, crayons, tires on cars, boxes of granola bars, etc.). If possible bring in some actual packages of objects. These objects can be used for problem solving later on.

   Create a class poster using pictures of objects that are sold in groups.

■ *What Comes in 2s, 3s, and 4s?* by Suzanne Aker, illustrated by Bernie Karlin. This book shows everyday objects that come in groups of 2, 3, or 4. Read the book and make a list/chart of the things that come in 2s, 3s, or 4s. Use these examples to ask questions such as

   ■ If there are 5 children, how many eyes are there?

   ■ There are 4 cars. How many tires are there?

   Initially, have students write repeated addition number sentences to solve these problems. After the multiplication symbol has been introduced, go back and rewrite the number sentences as multiplication.

   Extend the activity by having students make a poster or write a page for a class book about things that come in different sized groups (e.g., What Comes in 5s or 6s?)

■ **Skip Counting/Repeated Addition on a Number Line:** Reinforce the idea of thinking in groups using a number line. For example:
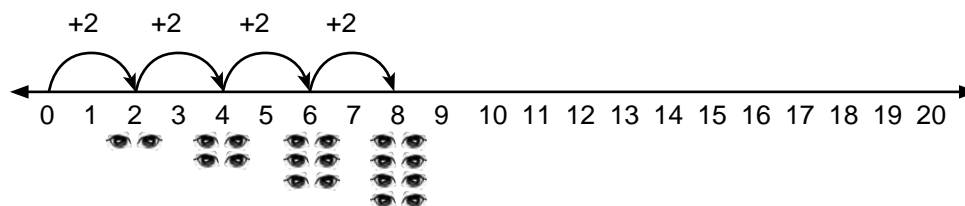
Assessing Understanding

Give students the following problems:

Solve the problems. Be sure to show your work.

- There are four tricycles at the playground. How many wheels are there?
- Nine children are lined up for recess. How many legs are there?
- Marie bakes cookies. Each cookie has 5 chocolate chips on top. How many chocolate chips are on five cookies?

Look for:

The student is able to solve the problems using

- ❑ skip counting
- ❑ repeated addition
- ❑ other (multiplication)

- **Represent a story problem (orally, shared reading, written) using manipulatives or diagrams, and record in a number sentence.**
- **Skip count by 2s, 3s, 4s, and 5s to determine the answer to a multiplication problem represented as equal groups.**
- **Represent a multiplication expression as repeated addition.**
- **Represent a repeated addition as multiplication.**
- **Create and illustrate a story problem for a number sentence.**
- **Represent, concretely or pictorially, equal groups for a number sentence.**

Suggestions for Instruction

- **Introducing the Concept of Multiplication:** Read the book *Amanda Bean's Amazing Dream: A Mathematical Story* by Cindy Neuschwander. This is the story of a girl who counts "everything and anything" by 1s, 2s, 5s, and 10s. Her teacher tries to convince her that multiplication is a faster way of counting, but it is not until Amanda has a dream that overwhelms her counting skills that she decides multiplication is something she wants to learn.

  Have students compare skip counting, adding, and multiplication. Do they all arrive at the same answer? Which one do you think is faster (more efficient)?

Note: Before introducing the symbol for multiplication, it is important that students understand its meaning. Many students will be able to tell you that the "x" is "times" but have no idea what that actually means.

■ **Interpreting a Diagram/Picture or Sets of Manipulatives as Multiplication:** Show students pictures of groups of equal sets and/or groups of manipulative materials and have them describe what they see. If they do not use the language of groups and sets, model it for them.
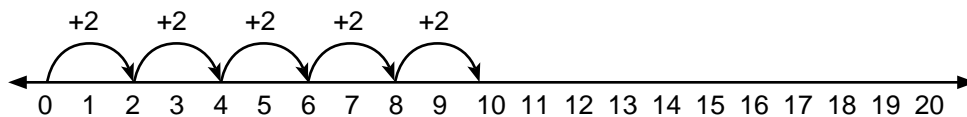
Examples:
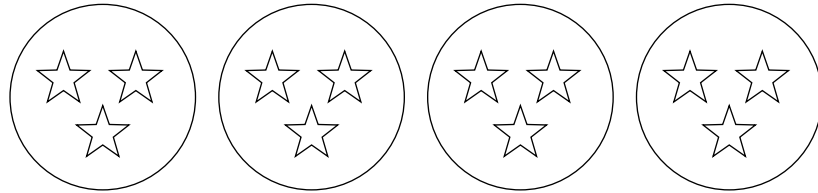
- 4 circles with 3 stars in each
- 4 circles with 3 stars per circle
- 4 sets of 3 stars
- 4 groups of 3 stars
- 3 + 3 + 3 + 3
- 3, 6, 9, 12 stars
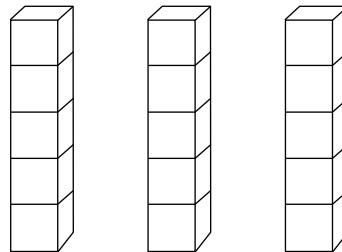
- 3 towers with 5 cubes in each
- 3 towers with 5 cubes per tower
- 3 groups of 5 cubes
- 3 sets of 5 cubes
- 5 + 5 + 5
- 5, 10, 15 cubes

$$+2 \quad +2 \quad +2 \quad +2 \quad +2$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

- 5 jumps of 2
- 5 groups of 2
- 5 sets of 2
- 2 + 2 + 2 + 2 + 2 = 10
- 2, 4, 6, 8, 10

Assessing Understanding

Give students a story problem, and have them represent it using materials and pictures. Have them describe their representations using groups/sets language.

Example:

Rudy puts his stickers into a sticker book.
So far he has filled 3 pages.
Each page has 6 stickers.
How many stickers does Rudy have in his book?

Look for:

❒ a correct representation using materials

❒ a correct representation using a picture

❒ group/set language

    ❒ 3 pages with 6 stickers on each page

    ❒ 3 groups of 6 stickers

    ❒ 3 sets of 6 stickers

    ❒ 6 + 6 + 6

    ❒ 6, 12, 18 stickers

Suggestions for Instruction

- **Introducing the Multiplication Symbol:** Explain to students that we have a symbol we use in place of writing out the words "groups of" or "sets of." For some students it might help if this is represented pictorially. For example:

       ×   ×

3 ~~groups of~~ 6 = 18   4 ~~sets of~~ 2 = 8

Note: It is important that students read/interpret the multiplication number sentence in the same way. The first number (factor) gives the number of equal groups and the second number (factor) tells the number of items within each group. For example the number sentence $5 \times 4 = 20$ should be read/interpreted as 5 groups/sets with 4 objects in each group/set.

Have students practise reading and representing multiplication number sentences.

Example:

$5 \times 2 = 10$

☆☆ ☆☆ ☆☆ ☆☆ ☆☆

5 groups of 2 equal 10

$2 + 2 + 2 + 2 + 2 = 10$

- ■ **Manipulatives/Materials to Support Students' Understanding**
  - ■ **Cuisenaire Rods:** For example: $6 \times 2 = \square$

    Students get six 2-cm rods and place them in a row end to end.

    Have them find the rod(s) that is (are) the same length as the row containing the six twos.

2	2	2	2	2	2

10					2
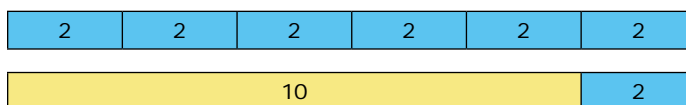
    Note: A centimetre ruler/number line could also be used.

2	2	2	2	2	2

    1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
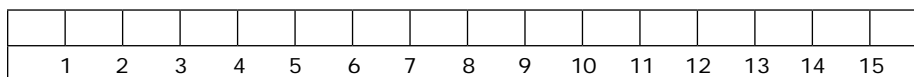
   Therefore, 6×2 is the same as 12.

  - ■ **Egg Cartons:** Egg cartons (or ice cube trays) can be used to make/hold groups of objects.

  - ■ **Multiplication/Division Templates:** The templates can be used for both multiplication and division. For multiplication, students use the template that has the number of groups asked for in the question/problem. They make the groups with objects or a dry erase marker and then find the product using repeated addition or skip counting.

    For division, students use the template that has the number of groups asked for in the question (divisor). They begin with the whole and then divide it among the groups to determine the quotient.

For example: Template for 5 × ? or ? ÷ 5
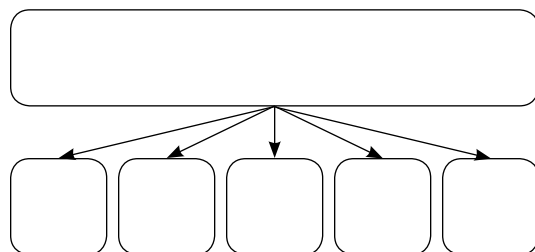
- **Problem Writing:** Prepare a set of multiplication number sentences using index cards. Have students choose one of the number sentences, and write a story problem to match (on an index card).

  The number sentence cards and problem cards can be used as a **Concentration/Matching** game. Put an N on the back of the number sentence cards and a *P* on the back of the problem cards.

  Turn cards up-side-down in an array. Students pick up a number card and a problem card. If they match, they keep the pair. If they don't match, the cards are returned to the array. Play continues until all cards have been matched. The player with the greatest number of cards is the winner.

Assessing Understanding: Performance Task

**BLM
3.N.11.2**

Multiplication Performance Task
Use the number sentence 4 x 3 =
Show different ways to find the answer (product).
Picture
Repeated addition
Number line
Skip counting
Story problem

The student is able to solve/represent the number sentence using

- ❑ a picture
- ❑ repeated addition
- ❑ number line
- ❑ skip counting

The student is able to

- ❑ create a story problem to match the number sentence

- **Represent a multiplication expression using an array.**
- **Create an array to model the commutative property of multiplication.**
- **Solve a problem in context involving multiplication.**

Suggestions for Instruction

- **Introducing Arrays:** Show students a variety of arrays (pictures or actual objects).

  Ask:

  - What do you notice about how the arrays are arranged?
  - How can we find out how many are in each array (repeated addition, skip counting, and multiplication)?

  Explain to students that an array is a way of organizing a set of objects into rows and columns. Use an egg carton (or another array) to help students understand the difference between rows and columns.

  2 rows of 6
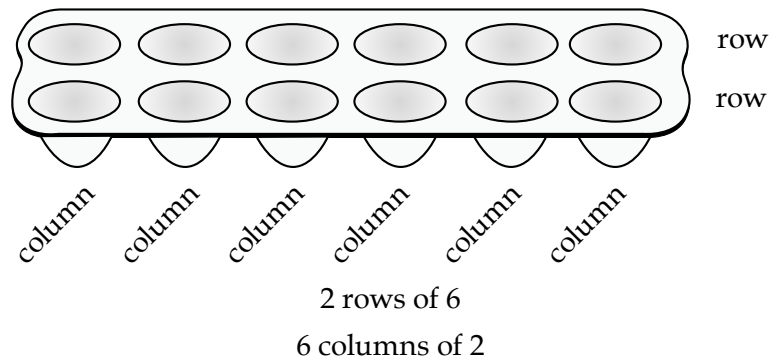
  6 columns of 2

  Turn the carton on end and show them that now we have 6 rows of 2 and 2 columns of 6.

  Have students brainstorm examples of arrays in the environment.

  Examples:
  - desks/tables in a classroom
  - eggs in an egg carton
  - muffins in a muffin tin
  - cookies on a cookie sheet
  - pictures in a photo album
  - cereal boxes in a grocery store
  - dots on a ten frame
  - windows/window panes
  - calendar

If possible have students find actual arrays or pictures of arrays that they can bring in to share with the class.

Note: Students could use a digital camera to take pictures of arrays they find in the school or neighbourhood.
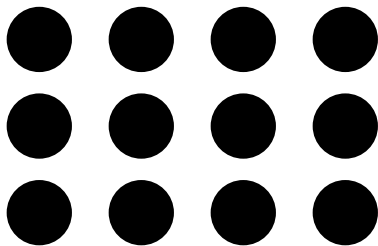
- **Number Sentences and Arrays:** Show students an array and ask them to describe what they see. For example:

  There are 3 rows with 4 dots in each row.
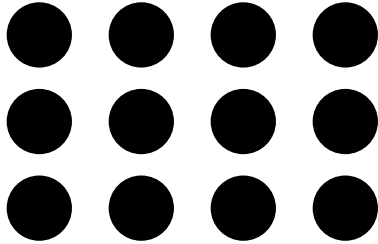
  Ask: "How can we write what we see as a multiplication number sentence?"

  Guide students to see that the multiplication sign can be read as "rows of."

   can be written as 3 rows of 4 or $3 \times 4 = 12$.

  Show additional arrays and have students give a multiplication number sentence for each. If students brought in pictures of arrays, have them write matching multiplication number sentences for them.

  Show the class multiplication number sentences, and have groups of students line up to show the matching array.

- **Revisiting *Amanda Bean's Amazing Dream*:** Pages 1 to 9 have examples of arrays in the illustrations. Have students find the arrays. Use sticky notes to add multiplication number sentences to the pages. For example: On the top left-hand side of page 1 there are windows arranged in 2 rows with 3 windows in each ($2 \times 3 = 6$). Each window is divided into 6 rows with 3 panes in each ($6 \times 3 = 18$).

  Students can do their own pictures to show arrays.

- **Array Game**

  Materials: centimetre grid paper, 2 (0 to 5 or 1 to 6) dice preferably different colours (spinners could be used), markers (a different colour for each player)

  Players: 2

  Establish game **rules**:

  - If different coloured dice (spinners) are used, decide which colour will represent the number of rows and which colour will represent the number in each row.
  - If the dice are the same colour (or one spinner in used), roll them separately. The first die represents the number of rows, and the second die represents the number in each row.
  - Arrays cannot overlap.

  Directions:

  1. The first player rolls the dice, determines the number of rows and the number in each row, and then outlines the matching array on the grid paper. The player writes the corresponding number sentence inside the array.
  2. Player 2 takes his/her turn.
  3. If the array will not fit on the grid paper, the player loses a turn.
  4. Play continues until each player has had 5 turns.

  The player with the most arrays on the board is the winner. In the event of a tie, total up the number of squares outlined by each player. The player with the largest number is the winner.

Assessing Understanding: Paper-and-Pencil Task

1. Write a multiplication number sentence for each array.
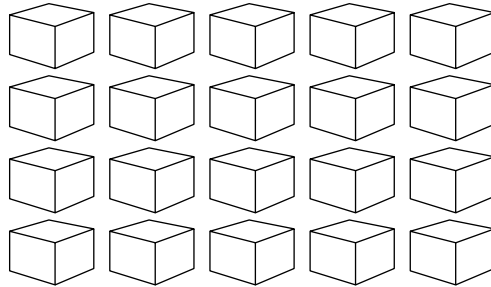
2. Draw an array for each multiplication number sentence.
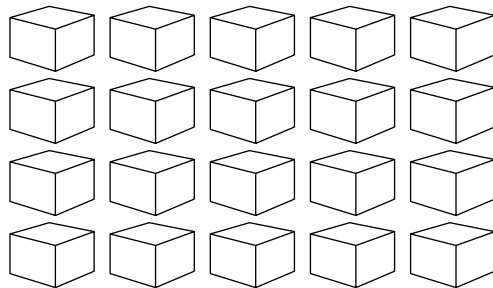
$$2 \times 5 = 10 \qquad\qquad 6 \times 1 = 6$$

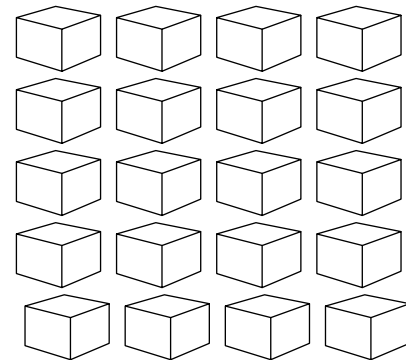3. Give two examples of arrays in the real world.

Suggestions for Instruction

- **Commutative Property of Multiplication:** Show students an array. Have them give the matching multiplication number sentence. Now, turn the array and have students give the matching multiplication number sentence.

  Ask:
  - Did the number of objects change? Why or why not?

$$4 \times 5 = 20 \qquad\qquad 5 \times 4 = 20$$

  - Do you think that this will be true for all arrays?

  Have students make their own arrays from grid paper to investigate. Have them explain their findings in their math journals or with the class.

- **Relate multiplication to division by using arrays and writing related number sentences.**

Suggestions for Instruction

- **Arrays and Division:** Show students an array, and explain that this is a page of baseball cards.
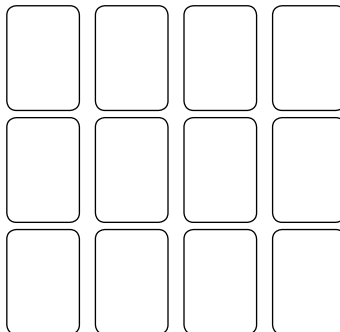
Ask:

- How many cards are there altogether?
- What multiplication sentence matches the array?
- Can you use the array to answer these questions?
  - If the cards are shared with 3 people, how many cards does each person get?
    Students should make a connection to the 3 rows with 4 cards in each row.
  - If the cards are shared with 4 people, how many cards does each person get?
    Students should make a connection to the 4 columns (4 rows if the array is rotated) with 3 cards in each.
  - Do you think that this will work for all arrays?

Students work with a partner or small group. Give each group an array. Ask them to write two "sharing" problems that can be answered using their array.

Have groups share their problems with the class. Problems and arrays could be assembled into a class book.

Assessing Understanding: Paper-and-Pencil Task

Solve the following problems.

Write a multiplication number sentence and draw an array for each problem.

1. Margo planted 3 rows of flower seeds. She put 5 seeds in each row. How many seeds did Margo plant altogether? Number sentence:	Array
2. There are 4 bags of apples. There are 4 apples in each bag. How many apples are there altogether? Number sentence:	Array
3. Juan invited 7 friends for ice cream cones. Juan and his friends each had 2 scoops of ice cream. How many scoops of ice cream did they have altogether? Number sentence:	Array
4. Pencils come in packages of 6. Mrs. Fast buys 3 packages for her class. How many pencils did Mrs. Fast get? Number sentence:	Array
5. Hans buys a sheet of stickers. On the sheet there are 4 rows with 3 stickers in each row. How many stickers did Hans buy? Number sentence:	Array

Use the book *Each Orange Had 8 Slices: A Counting Book* by Paul Giganti, Jr., illustrated by Donald Crews. Each page or double-page spread has a picture of a group of objects (flowers, tricycles, horses, etc.). The pictures are described and then multiplication questions are asked.

For example: On page 1 the illustration shows 3 flowers with the following description and set of questions:

> On my way to the playground
> I saw 3 red flowers
> Each flower had
> pretty petals.
> Each petal had 2 tiny black bugs.
>
> How many red flowers were there?
> How many pretty petals were there?
> How many tiny black bugs were there in all?

Read the book to the students. Go back and explore each page. (This could be done over a period of several days.) Discuss the format of each page.

As a class, write multiplication number sentences and an answer for each question.

Task Directions:

We are going to make a class book like *Each Orange Had 8 Slices: A Counting Book.*

1. Choose an object that can be described using groups.
2. Draw a picture using the object you have chosen.
3. Write a description of your picture. Include 3 numbers in your description.
4. Write three questions that could be solved using multiplication.
5. On the back of your picture, write a multiplication number sentence and an answer for each of your questions.

Assessment Criteria: Student Self-Assessment

- ❏ My picture has objects that can be described using groups.
- ❏ I have included a description of my picture.
- ❏ My description includes 3 numbers.
- ❏ I have included 3 questions for my picture.
- ❏ I have written a multiplication number sentence and an answer for each of my questions on the back of my picture.

- **Identify events from experience that can be described as equal sharing.**
- **Identify events from experience that can be described as equal grouping.**
- **Illustrate, with counters or a diagram, a story problem involving equal sharing, presented orally or through shared reading, and solve the problem.**
- **Illustrate, with counters or a diagram, a story problem involving equal grouping, presented orally or through shared reading, and solve the problem.**
- **Listen to a story problem, represent the numbers using manipulatives or a sketch, and record the problem with a number sentence.**

Suggestions for Instruction

- **Equal Sharing Division:** Have students share examples of times when they had to share something so that everyone got the same amount/number.

  Tell students that you are going to read them a story all about sharing equally. Read the book *The Doorbell Rang* by Pat Hutchins. Ma bakes a dozen cookies and her two children are excited about sharing them until the doorbell rings. Each time the doorbell rings the children's share of the cookies becomes smaller until they are down to one each. In the end, Grandma comes with more cookies to save the day.

  Reread the book, and have students act out the story. Use paper cookies and paper plates to show the sharing. Each time, record the sharing using words instead of the division symbol. For example: 12 shared by 2 = 6 cookies each.

Number of cookies	Number of children	Sharing (picture)	Number of cookies per child
12	2	○○○ ○○○ ○○○ ○○○ 12 cookies shared by 2 children	6
12			
12			
12			

Extension: Students work with a partner to write a story patterned after *The Doorbell Rang*. The story should include at least 3 division situations. Have them read their story to the class while acting out the division problems. Record the division number sentences.

■ **Introducing the Division Symbol:** Before introducing the symbol for division, it is important that students understand its meaning. Have students use the following vocabulary to describe division situations:

　■ shared by

　■ is how many groups of

Explain to students that we have a symbol to use for division in place of writing out the words *shared by* or *is how many groups of.* For some students it might help if this is represented pictorially. For example:

　　　　÷　　　　　　　　　　　　　　　　　÷
12 ~~shared by~~ 2 = 6　　　　　12 ~~is how many groups of~~ 4 = 3

■ **Equal Grouping Division:** Use the book *A Remainder of One* by Elinor J. Pinczes, illustrated by Bonnie MacKain. In the story, Joe is part of a group of 25 bugs that is marching in a parade before the queen. When the bugs divide themselves up, Joe becomes the odd bug out.

Before reading the book, ask the students what they know about the word *remainder*. For the first reading, read to enjoy the language and the overall concept of the book.

As you reread the story, stop each time the insects decide on a different formation ("The troop had divided by two for the show.") and ask students to predict how many insects will be in each line and how many (if any) insects will be left over. Act out the lineup using students (if numbers permit) or manipulatives.

The insects are organized in arrays in the illustrations. Use the arrays to support the predictions. Have division number sentences written for each formation either in words (25 divided into 2 lines = 12 in each line with one leftover) or in symbols (25 ÷ 2 = 12 r1).

Note: The National Council of Teachers of Mathematics (NCTM) Illuminations website (https://illuminations.nctm.org/Lesson.aspx?id=3751) provides a detailed lesson plan for the story as well as a way to create a similar story about lining up that you can use if you do not have the book.

- **Equal Grouping Problems:** Give students equal grouping division problems to solve. Provide objects and bags so that students can act them out. Have a number sentence written for each question.

  1. Marc has 20 trinkets for his birthday bags. How many children can he invite if he

     — puts 5 trinkets in each bag?

     — puts 2 trinkets in each bag?

     — puts 4 trinkets in each bag?

  2. Hannah has 12 apples. How many bags can she make if she

     — puts 6 apples in each bag?

     — puts 3 apples in each bag?

     — puts 2 apples in each bag?

     — puts 4 apples in each bag?

  3. Groups of Grade 3 students are packaging cookies for a bake sale.
     Each group has 18 cookies to package.
     How many packages can they make if they

     — put 9 cookies in each package?

     — put 6 cookies in each package?

     — put 3 cookies in each package?

  Ask students to give examples of situations where they have had to use equal grouping. Have students use one of the examples to write their own equal grouping problems for the class to solve.

- **Create, and illustrate with counters, a story problem for a number sentence.**

Suggestions for Instruction

BLM 3.N.12.3

- **Writing Problems:** Prepare a set of cards (at least one per student) with a division number sentence written on each one. Have students select a card, write a story problem to match, and then act out the problem using counters/objects.

---

10 ÷ 5 = 2	7 ÷ 1 = 7
6 ÷ 2 = 3	20 ÷ 4 = 5
15 ÷ 3 = 5	12 ÷ 4 = 3
10 ÷ 2 = 5	16 ÷ 4 = 4
8 ÷ 4 = 2	9 ÷ 3 = 3
12 ÷ 6 = 2	20 ÷ 5 = 4
8 ÷ 2 = 4	12 ÷ 3 = 4
14 ÷ 7 = 2	5 ÷ 5 = 1
15 ÷ 5 = 3	6 ÷ 3 = 2
4 ÷ 2 = 2	14 ÷ 2 = 7

Assessing Understanding: Journal Entry

1. Use the numbers 12, 3, and 4.
2. Write a division number sentence.
3. Make up a story problem to match your number sentence.
4. Use counters or draw a diagram to act out your story problem.

Extension

Is there another division number sentence you could write using the same numbers?

Can you change your story problem to match the new number sentence?

Look for evidence that the student

❏ sees the relationship between the three numbers
❏ is able to write a division number sentence
❏ can create a story problem to match the number sentence
❏ is able to act out the problem (with counters or a diagram)

Extension (if done)

❏ is able to write a second division number sentence
❏ is able to change the story problem to reflect the new number sentence

> ■ **Represent a division expression as repeated subtraction.**
>
> ■ **Represent a repeated subtraction as a division expression.**

Suggestions for Instruction

■ **Investigating Division as Repeated Subtraction:** Present students with this problem.

Anna is thinking about multiplication and division.

She knows that multiplication can be shown as repeated addition. She wonders if division can be shown as repeated subtraction.

Can you help answer her question?

Use materials (counters, hundred chart, etc.) and number lines to help with your investigation.

Have students work with a partner to investigate this question.

Support students as needed by asking questions such as the following:

- What number sentence are you using for your investigation?
- What number will you have to start with on your number line?
- How do you show subtraction on a number line?
- If you are using cubes, how many do you need to start with?
- How can you show subtraction using the cubes?
- What number in the number sentence tells you how many cubes to take away?
- How can you record what you are doing each time?

Have students present their findings to the class.

■ **Division and the Number Line:** Tell students that Greg is walking on a number line. He starts at 16 and jumps back two numbers each time. How many jumps will do until he gets to zero?

Show the jumps on a number line (or get students to act it out using a large floor number line).

How can we write this as a repeated subtraction number sentence?

$(16 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 = 0)$

What division number sentence will match what is shown on the number line? $(16 \div 2 = 8)$
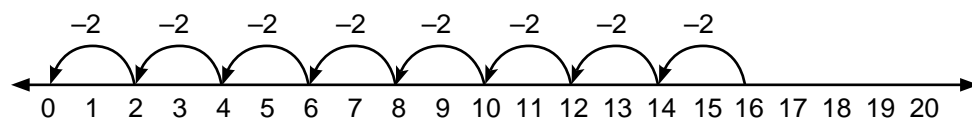
Have students make up their own division problems to represent (act out) on a number line. Use both a repeated subtraction and a division number sentence to show the solution for each problem.

- **Repeated Subtraction and Division:** Give students a set of repeated subtraction number sentences and have them write the matching division number sentence.

$12 - 3 - 3 - 3 - 3 = 0$	$15 - 5 - 5 - 5 = 0$
$20 - 4 - 4 - 4 - 4 - 4 = 0$	$10 - 2 - 2 - 2 - 2 - 2 = 0$
$18 - 6 - 6 - 6 = 0$	$14 - 7 - 7 = 0$

Assessing Understanding: Paper-and-Pencil Task

Fun on the Farm

Use repeated subtraction and a division number sentence to show how to solve the following problems:

1. There are 20 legs in the pigpen.

   How many pigs are there?

2. There are 16 eyes in the chicken coop.

   How many chickens are in the chicken coop?

3. There are 10 hands in the farmhouse.

   How many people are in the house?

The student is able to

❏ read and interpret the problem

❏ write a repeated subtraction number sentence for each problem
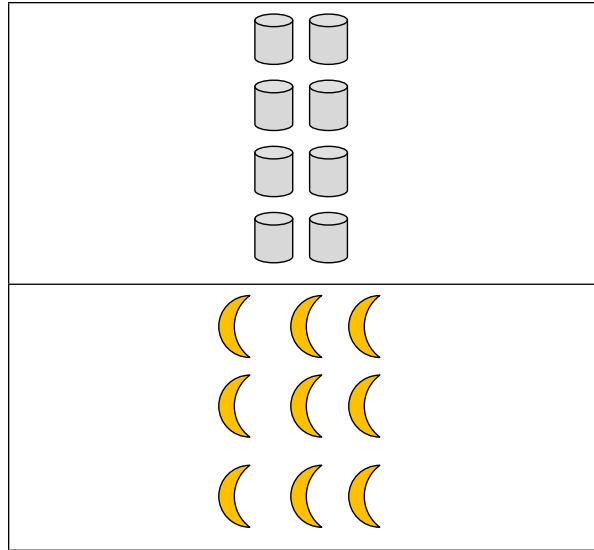
❏ write a division number sentence for each problem

- **Relate division to multiplication by using arrays and writing related number sentences.**
- **Solve a problem involving division.**

Suggestions for Instruction

- **Array cards:** Use the array cards (BLM 3.N.12.4).
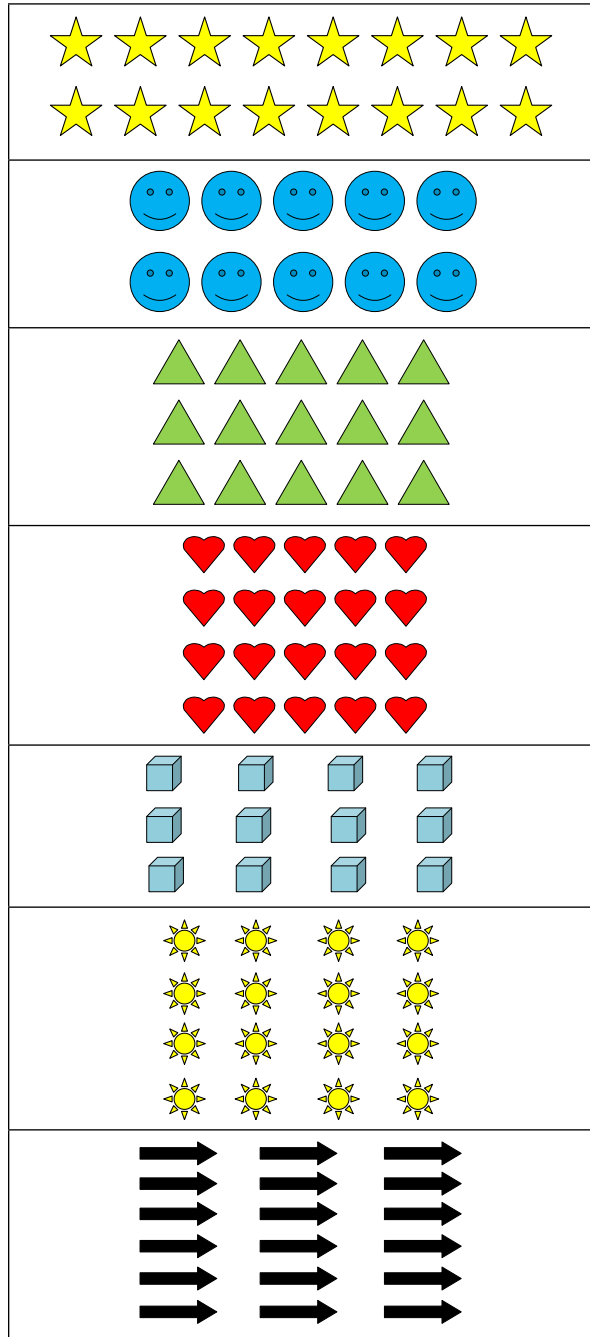
Student Directions:

1. Pick an array card.

2. Write 2 multiplication number sentences for your array.

3. Write 2 division number sentences for your array.

4. Write one multiplication story problem to match your array.

5. Write one division story problem to match your array.

■ **Triangular Flashcards and Fact Families:** Talk to students about the relationship between multiplication and division. Triangular flashcards help students see this relationship. Use the cards to make the fact family for a given set of numbers.

Example:
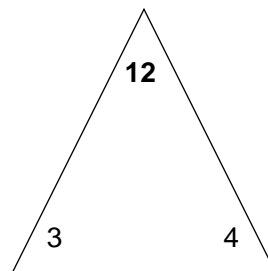
If the 12 is covered students are looking for the product of 3 and 4.

If either the 3 or 4 are covered students are looking for the quotient of 12 ÷ 3 or 4.

The numbers 12, 3, and 4 are a fact family. Students can write
2 multiplication and 2 division number sentences for the card.

$3 \times 4 = 12$

$4 \times 3 = 12$

$12 \div 4 = 3$

$12 \div 3 = 4$

If you think about the part-part-whole, then the 12 represents the whole and the 3 and 4 represent the parts. If the whole is unknown, multiplication is the operation needed. If one of the parts is unknown, then division is needed. Students can use this understanding to help them interpret multiplication and division problems.

■ **Using Part-Part-Whole for Problem Solving:** Give students a story problem.

Sean bought a bag of 12 candies.
He wants to share his candies with his 4 friends.
How many candies will each person get?
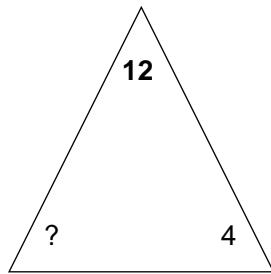
Ask the following:

- Do you know the whole? (12 candies)
- Do you know the parts? (We know one part, one part is missing.)
- If we know the whole and one of the parts, what operation will you use to solve the problem? (division)

Use the triangle to record the problem information.

Mason has 3 pages of stickers.
Each page has 5 stickers.
How many stickers does he have altogether?
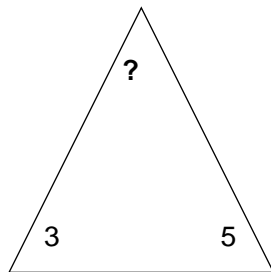
Ask the following:

- Do you know the whole? (no)

- Do you know the parts? (Yes: 3 and 5.)

- If we know both of the parts but not the whole, what operation will you use to solve the problem? (multiplication)

Note: Students can make their own triangular cards. The cards can be used in multiplication and division stations/centres.

- **Open-Ended Problems:** Have students solve the problems giving multiple solutions.

  1. The Grade 3 class is playing a game in gym.
     The teacher wants them to be in equal groups with no remainders.
     If there are 20 students in the class, how many different sizes of groups can you make?

  2. The answer to a multiplication question is 12.
     What might the question be?

Assessing Understanding: Journal Entry

Explain how multiplication and division are related.

Use words, pictures, arrays, number lines, and/or numbers and symbols in your explanation.

Look for evidence of understanding that

- ❑ multiplication and division are inverse operations
- ❑ multiplication is repeated addition
- ❑ division is repeated subtraction
- ❑ an array can represent both operations

Grade 3: Number (3.N.13)

Enduring Understandings:

A fraction represents a part of a whole.

Fractions can be compared using a variety of models.

The size of the fractional part depends on the size of the whole.

Equal parts do not have to look the same but they must be the same size or have the same amount of the whole.

Essential Questions:

What is a fraction?

Where do you use fractions in everyday life?

What is the numerator?

What is the denominator?

Specific Learning Outcome(s):	Achievement Indicators:
3.N.13 Demonstrate an understanding of fractions by ■ explaining that a fraction represents a portion of a whole divided into equal parts ■ describing situations in which fractions are used ■ comparing fractions of the same whole with like denominators [C, CN, ME, R, V]	➤ Identify common characteristics of a set of fractions. ➤ Describe everyday situations where fractions are used. ➤ Cut or fold a whole into equal parts, or draw a whole in equal parts; demonstrate that the parts are equal, and name the parts. ➤ Sort a set of diagrams of regions into those that represent equal parts and those that do not, and explain the sorting. ➤ Represent a fraction concretely or pictorially. ➤ Name and record the fraction represented by the shaded and non-shaded parts of a region. ➤ Compare fractions with the same denominator using models. ➤ Identify the numerator and denominator for a fraction. ➤ Model and explain the meaning of *numerator* and *denominator*.

PRIOR KNOWLEDGE

Students may have had no formal experience with fractions. Most children will have had experience dividing something in half in a sharing situation. However, many children use the word *half* to mean a part of something. They do not, however, differentiate between *half* and *part*.

BACKGROUND INFORMATION

Terminology

Fraction: A number that represents part of a whole, part of a set, or a quotient in the form $\frac{a}{b}$, which can be read as *a* divided by *b*.

Numerator: The number above the line in a fraction that can state one of the following:

- the number of equal parts in a set to be considered
- the number of equal parts of a whole to be considered

Denominator: The number below the line in a fraction that can state one of the following:

- the number of elements in a set
- the number of equal parts into which the whole is divided

In Grade 3, the focus is on developing a beginning understanding of fractions less than one, relating fractions to real-life situations, and comparing fractions with the same denominator. The focus is on having students explore parts of a whole that has been divided into "fair shares" or "fractional parts" (equal-sized pieces). Finding a fraction of a set is introduced in Grade 4.

It is vital that students represent fractions with concrete models. A variety of materials must be used so that students understand what fractions mean. It is also important that students develop visual images for fractions and be able to tell "about how much" a particular fraction represents.

Note: It is important to use a wide variety of models so that students do not come to believe that fractions only relate to parts of a circle (pizza, pie).

MATHEMATICAL LANGUAGE

fraction	third
numerator	quarter
denominator	fifth
fair share	sixths
whole	eighths
one whole	tenths
half	one of _____ equal parts

LEARNING EXPERIENCES

Assessing Prior Knowledge:

Performance Task to assess student understanding of *half*:

Give students a strip of paper, piece of string, or a piece of licorice. (Play dough or plasticine "worms" would work as well.) Ask them to cut the strip/piece in half.

Ask, "How do you know that it is half?"

Observe students to determine if they

❑ know that there should be two pieces

❑ fold the object in half before cutting or cut without folding

❑ match the two pieces to see if they are the same length or say that it is half because there are two pieces even if they are not equal

Class Discussion: Web or Concept Map

Ask: "What do you know about fractions?"

Create a web or concept map to record their responses.

■ **Describe everyday situations where fractions are used.**

Suggestions for Instruction

■ Have students identify everyday situations that use fractions (e.g., sharing a chocolate bar with a friend, cutting an apple or sandwich in half, telling time—half past/quarter to, distance—halfway).

After symbols are introduced, have students find examples of fractions in magazines/newspapers/signs. Use the examples to create a fraction poster.

■ **Cut or fold a whole into equal parts, or draw a whole in equal parts; demonstrate that the parts are equal and name the parts.**

■ **Sort a set of diagrams of regions into those that represent equal parts and those that do not, and explain the sorting.**

Suggestions for Instruction

■ Begin with concept of one half. Give students a letter-sized piece of paper. Ask them to fold it in half. Do not give further directions. Have students compare their halves. Some students will fold the paper on short side (hotdog) and others on the long side (hamburger).
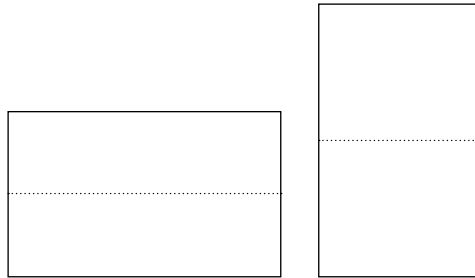
Ask the following:

- How many parts do you have?

- How do you know that each part is one half?

Expected answer: "There are two parts and both of them are equal."

- **Exploring Quarters/Fourths and Eighths**

  Have students fold a strip into half and then into half again.

  Ask them the following:

  - How many parts they have altogether? (4)

  - Are they equal?

  - Does anyone know what we call one part out of the 4 parts?

  Introduce both terms—*one-fourth* and *one-quarter*. Connect to money (4 quarters in a loonie) to help students understand the terminology.

  Refold and have the strip folded in half again. Before opening it up, ask them to predict how many parts they think they have now. After opening it, ask how many parts altogether? Does anyone know what we call one part out of 8 parts? Introduce the term *one-eighth*.

  Extend the thinking by asking what they think you would call

  - one part out of 3 parts?

  - one part out of 6 parts?

  - one part out of 10 parts?

BLM
3.N.13.1

- **Sorting Diagrams into Equal Parts and Unequal Parts:** Give students pictures of shapes divided into equal and unequal parts (BLM 3.N.13.1). Have students sort them and give the sorting rule.

Assessing Understanding: Journal Entry

Sadie and Ben were sharing a piece of cake. Ben cut the cake in two pieces and then said to Sadie, "You can have the bigger half."

Does this make sense? Why or why not?

In response, the student should indicate that the word *half* means that the whole/object has been divided into two equal parts so there is no bigger half.

Suggestions for Instruction

- **Exploring Fractions with Manipulatives**

  1. Pattern Blocks: Use the hexagon as the whole. Have students use the other pattern blocks to show half, thirds, and sixths.

  2. Cuisenaire Rods: Use differently sized rods to explore a variety of fractions (e.g., using the ten rod as the whole, students can explore tenths, fifths, and halves).

  These concrete representations should help students make the connection to the symbolic representations easier.

■ **Read the Book:** *Whole-y Cow! Fractions Are Fun* by Taryn Souders, illustrated by Tatjiana Mai-Wyss. In this book students are asked a fraction question about each illustration. It is a good book to use before introducing the symbols.

Note: You might want to have a discussion about some of the illustrations (e.g., in one picture the cow is painted blue [rear part] and white [front part], and the words on the page say, "Moo while her friends paint one half blue!"—students may question whether the two parts are actually equal and therefore should not be referred to as "half").

- ■ **Name and record the fraction represented by the shaded and non-shaded parts of a region.**
- ■ **Identify the numerator and denominator for a fraction.**
- ■ **Model and explain the meaning of *numerator* and *denominator*.**

## Suggestions for Instruction

Note

Before the teacher introduces the symbols, students should have multiple opportunities to explore and talk about fractions. Modelling the language of "one of _____ equal parts" will help students make the connection between language and the symbol.

■ **Introducing the Symbols:** Reading a book such as *Apple Fractions* by Jerry Pallotta, illustrated by Rob Bolster, *Full House: An Invitation to Fractions* by Dayle Ann Dodds, illustrated by Abby Carter, or *If You Were a Fraction* by Trisha Speed Shaskan, illustrated by Francesca Carabelli, will help make connections among the pictorial representation, the words, and the symbol.

Show students pictures with fractional parts shaded along with the fraction symbol. Ask students if they can tell what each number/digit in the fraction represents. For example:

$$\frac{1}{2}$$

$$\frac{1}{3}$$

$$\frac{2}{5}$$

$$\frac{3}{4}$$
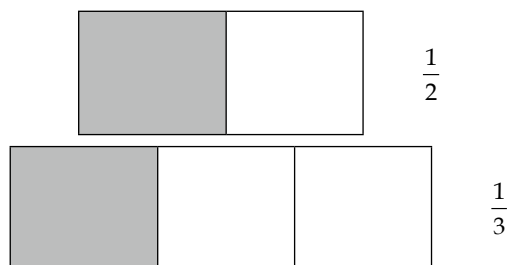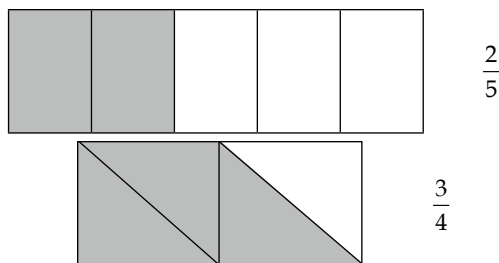
Ask questions such as the following:

- Look at the shaded part of the rectangle. Now look at the top number/digit in the fraction. Does the top number in the fraction tell you something about the shaded part of the rectangle? *(The top number in the fraction tells how many parts of the rectangle are shaded.)*

- Look at the bottom number/digit in the fraction. Does this number tell you anything about the rectangle? *(The bottom number in the fraction tells how many parts there are in the rectangle.)*

Show pictures of other shapes (circles, triangles, etc.) to see if students have generalized the relationship between the fraction symbol and the picture beyond the rectangle.

Once students have made this connection introduce the terms *numerator* and *denominator.*

- **From Picture to Fraction:** Provide students with pictures and have them write the matching fraction. Have students explain how they determined each digit in the fraction.

- **Fraction Concentration:** Prepare a set of fraction picture cards and a set with the matching fraction symbols. Have students use them to play a game of concentration.
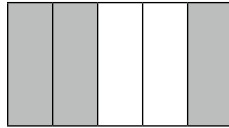
Assessing Understanding: Paper-and-Pencil Task

1. Draw a picture for these fractions:

   $\dfrac{2}{4}$

   $\dfrac{4}{6}$

2. What fraction of each shape is shaded? What fraction is not shaded?

   shaded _____
   unshaded _____

   shaded _____
   unshaded _____

3. Julie says that her fraction has a denominator of 8 and a numerator of 3. Draw a picture to match Julie's fraction.

   Look for:

   The student understands that

   ❏ the denominator represents the total number of parts the whole has been divided into

   ❏ the numerator represents the number of shaded parts or the parts being focused on

Suggestions for Instruction

- **Comparing Fractions Using Pattern Blocks:** Use hexagons as wholes and green triangles as the parts. Have students show the fraction $\frac{1}{6}$ and the fraction $\frac{3}{6}$.

  Ask: Which fraction is the largest? How do you know?

  ($\frac{3}{6}$ is larger because there are 3 pieces out of the six. In the fraction $\frac{1}{6}$, you only have 1 piece out of the six.)

  Have students compare $\frac{3}{6}$ and $\frac{5}{6}$ as well as $\frac{4}{6}$ and $\frac{2}{6}$.

  Do the same with Cuisenaire rods. For example: Use an 8 rod and the ones cubes. Have students show $\frac{3}{8}$ and $\frac{5}{8}$. Ask them to identify the largest (smallest) fraction and to explain their choice.

  Once students have had practice comparing fractions in this way ask, "If you have two fractions with the same denominators, how can you tell which fraction is the largest?" Students should indicate that when two fractions have the same denominator the fraction with the largest numerator represents the larger part of the whole.

Assessing Understanding: Journal Entry

- Hugh said that his fraction is greater than $\frac{2}{10}$ but less than $\frac{6}{10}$. What might his fraction be?

- You are going to share a piece of cake with your friend. Would you rather have $\frac{4}{6}$ of the cake or $\frac{3}{6}$ of the cake? Explain your thinking. Use pictures, numbers, and words in your explanation.

- Order these fractions from the smallest to the largest:
  $$\frac{7}{8} \qquad \frac{2}{8} \qquad \frac{3}{8} \qquad \frac{5}{8} \qquad \frac{4}{8} \qquad \frac{6}{8}$$

Introduction: Read the following fraction story (patterned after *The Doorbell Rang*).

> Mrs. Hurd is famous for her homemade pizza.
>
> On Saturday she baked a large pepperoni pizza for her two children, Mary and Tom. The children were excited about sharing the pizza.
> *How much of the pizza will each child get? How should Mrs. Hurd cut the pizza?*
>
> As they sat down to eat, the doorbell rang. It was Paul and Jim from next door. Mrs. Hurd invited them in to share the pizza.
> *Now, how much of the pizza will each person get? How should the pizza be cut?*
>
> Mrs. Hurd was just getting ready to cut the pizza when the doorbell rang again. It was Elly and Kelly, the twins from across the street. Mary and Tom invited them in.
> *How much of the pizza will each person get now? How should Mrs. Hurd cut the pizza?*
>
> The children began to worry. Their share of the pizza was getting smaller with each ring of the doorbell. They finally decided to hurry up and eat, but before they could cut the pizza the doorbell rang. This time it was their cousins, Hans and Heidi.
> *How much of the pizza will each person get? How should Mrs. Hurd cut the pizza?*
>
> Mrs. Hurd cut the pizza and everyone started eating. After the pizza was finished all of the children were still hungry. Luckily Mrs. Hurd saved the day by popping another pizza into the oven.
> *If the children share the second pizza, how many pieces will each child eat altogether?*
>
> As the story is read, have students visually represent how the pizza should be cut each time and identify the fraction both in words and with a symbol.

Student Directions:

Write a fraction story. You can pattern it after a story you know or you can make up your own story.

Note: The story should illustrate the sharing of a whole not a set.

Criteria:

- The object (whole) in your story should be shared at least 3 times.
- Illustrations should show how the object is shared each time.
- Fraction symbols should be included.

Notes