

# Grade 3 Mathematics

Support Document for Teachers





**GRADE 3 MATHEMATICS**

Support Document for Teachers

Manitoba Education and Training Cataloguing in Publication Data

Grade 3 mathematics : support document for teachers

Includes bibliographical references.

ISBN: 978-0-7711-7469-8 (pdf)

ISBN: 978-0-7711-8043-9 (print)

1. Mathematics—Study and teaching (Primary).  
2. Mathematics—Study and teaching (Primary)—Manitoba.  
I. Manitoba. Manitoba Education and Training.  
372.7044

Copyright © 2017, the Government of Manitoba, represented by the Minister of Education and Training.

Manitoba Education and Training  
School Programs Division  
Winnipeg, Manitoba, Canada

Every effort has been made to acknowledge original sources and to comply with copyright law. If cases are identified where this has not been done, please notify Manitoba Education and Training. Errors or omissions will be corrected in a future edition. Sincere thanks to the authors, artists, and publishers who allowed their original material to be used.

All images found in this resource are copyright protected and should not be extracted, accessed, or reproduced for any purpose other than for their intended educational use in this resource.

Any websites referenced in this resource are subject to change without notice. Educators are advised to preview and evaluate websites and online resources before recommending them for student use.

Print copies of this resource (stock number 80646) can be purchased from the Manitoba Learning Resource Centre. Order online at [www.manitobalrc.ca](http://www.manitobalrc.ca).

This resource is available on the Manitoba Education and Training website at [www.edu.gov.mb.ca/k12/cur/math/supports.html](http://www.edu.gov.mb.ca/k12/cur/math/supports.html).

Websites are subject to change without notice.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

# C O N T E N T S

---

<b>List of Blackline Masters (BLMs)</b>	v
Grade 3 Mathematics Blackline Masters	v

---

<b>Acknowledgements</b>	vii
-------------------------	-----

---

<b>Introduction</b>	1
Overview	2
Conceptual Framework for Kindergarten to Grade 9 Mathematics	5
Assessment	9
Instructional Focus	11
Document Organization and Format	12

---

<b>Number</b>	1
Number (3.N.1)	3
Number (3.N.2)	11
Number (3.N.3, 3.N.4)	20
Number (3.N.5)	28
Number (3.N.6, 3.N.7, 3.N.8, 3.N.9, 3.N.10)	39
Number (3.N.11, 3.N.12)	63
Number (3.N.13)	92

---

<b>Patterns and Relations</b>	1
Patterns and Relations (3.PR.1, 3.PR.2)	3
Patterns and Relations (3.PR.3)	17

---

<b>Shape and Space</b>	1
Shape and Space (Measurement) (3.SS.1, 3.SS.2)	3
Shape and Space (Measurement) (3.SS.3)	12
Shape and Space (Measurement) (3.SS.4)	22
Shape and Space (Measurement) (3.SS.5)	29
Shape and Space (3-D Objects and 2-D Shapes) (3.SS.6, 3.SS.7)	36

---

<b>Statistics and Probability (Data Analysis)</b>	1
Statistics and Probability (Data Analysis) (3.SP.1, 3.SP.2)	3

---

<b>Bibliography</b>	1
---------------------	---

## List of Blackline Masters (BLMs)

### Grade 3 Mathematics Blackline Masters

#### Number (N)

- BLM 3.N.1.1: Number Cards 100 to 1000
- BLM 3.N.1.2: Hundred Chart
- BLM 3.N.1.3: Correct or Incorrect Patterns
- BLM 3.N.2.1: Number of the Day
- BLM 3.N.2.2: I Can—Number of the Day
- BLM 3.N.2.3: Small Ten Frames
- BLM 3.N.3.1: Number Cards 100 to 1000
- BLM 3.N.3.2: Empty Hundred Chart
- BLM 3.N.3.3: Roll and Order
- BLM 3.N.3.4: Largest or Smallest? Game Board
- BLM 3.N.3.5: Assessment of Ordering Numbers
- BLM 3.N.4.1: Estimation Station Recording Sheet
- BLM 3.N.4.2: Estimation Assessment
- BLM 3.N.5.1: Place Value Cards—Units and Tens, Hundreds
- BLM 3.N.6.1: Race to 100
- BLM 3.N.6.2: Mental Math Strategies—Two-Digit Addition
- BLM 3.N.7.1: Race to Zero
- BLM 3.N.7.2: Mental Math Strategies—Two-Digit Subtraction
- BLM 3.N.10.1: Assessing Understanding of Mental Math Strategies
- BLM 3.N.10.2: Doubles
- BLM 3.N.10.3: Let's Make 5
- BLM 3.N.10.4: Let's Make 10
- BLM 3.N.11.1 and 3.N.12.1: Multiplication and Division Templates
- BLM 3.N.11.2: Multiplication Performance Task
- BLM 3.N.11.3: Multiplication Sentence and Draw Array Assessment
- BLM 3.N.12.2: Sharing Cookies—*The Doorbell Rang*
- BLM 3.N.12.3: Division Story Problems
- BLM 3.N.12.4: Array Cards
- BLM 3.N.13.1: Sorting Shapes

#### Patterns and Relations (PR)

- BLM 3.PR.1.1 and 3.PR.2.1: Hundred Chart

**Shape and Space (SS)**

BLM 3.SS.1.1: Months and Days Cards

BLM 3.SS.1.2: Duration Cards—Longer or Shorter

BLM 3.SS.1.3: Calendar Template

BLM 3.SS.3.1: Wiggler Derby Recording Sheet

BLM 3.SS.4.1: Mystery Egg/Container Recording Sheet

BLM 3.SS.5.1: Find the Perimeter Recording Sheet

BLM 3.SS.5.2: Measuring Cylinders Recording Sheet

BLM 3.SS.5.3: Perimeter

BLM 3.SS.6.1: 3-D Objects

BLM 3.SS.6.2: Shapes

**Statistics and Probability (Data Analysis) (SP)**

BLM 3.SP.2.1: Data about Me



## ACKNOWLEDGEMENTS

Manitoba Education and Training wishes to thank the members of the Kindergarten to Grade 4 Mathematics Support Document Development Team for their contribution to this document. Their dedication and hard work have made this document possible.

Writer		
<b>Kindergarten to Grade 4 Mathematics Support Document Development Team (2006–2008)</b>	Dianne Soltess	St. James-Assiniboia School Division
	Rosanne Ashley	Winnipeg School Division
	Joanne Barre	Louis Riel School Division
	Debbie Furdyk	Lord Selkirk School Division
	Trish Goosen	Lord Selkirk School Division
	Brad Hayward	Fort la Bosse School Division
	Catherine Ko-Heinrichs	Winnipeg Mennonite Elementary School
	Carol Matsumoto	Seven Oaks School Division
	Shannon Oikawa	Pembina Trails School Division
	Louisa Petznick	Frontier School Division
Donna Slobodzian	Red River Valley School Division	
<b>Manitoba Education and Training School Programs Division Staff</b>	Pat Steuart	St. James-Assiniboia School Division
	Carole Bilyk Project Manager	Development Unit Instruction, Curriculum and Assessment Unit
	Louise Boissonneault Coordinator	Document Production Services Unit Educational Resources Branch
	Sherry Perih Project Leader	Development Unit Instruction, Curriculum and Assessment Unit
	Marjorie Poor Publications Editor	Document Production Services Unit Educational Resources Branch
	Lindsay Walker Desktop Publisher	Document Production Services Unit Educational Resources Branch



# INTRODUCTION

## Purpose of This Document

*Grade 3 Mathematics: Support Document for Teachers* provides various suggestions for instruction, assessment strategies, and learning resources that promote the meaningful engagement of mathematics learners in Grade 3. The document is intended to be used by teachers as they work with students in achieving the learning outcomes and achievement indicators identified in *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)* (Manitoba Education).

## Background

*Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes* is based on *The Common Curriculum Framework for K–9 Mathematics*, which resulted from ongoing collaboration with the Western and Northern Canadian Protocol (WNCP). In its work, WNCP emphasizes

- common educational goals
- the ability to collaborate and achieve common goals
- high standards in education
- planning an array of educational activities
- removing obstacles to accessibility for individual learners
- optimum use of limited educational resources

The growing effects of technology and the need for technology-related skills have become more apparent in the last half century. Mathematics and problem-solving skills are becoming more valued as we move from an industrial to an informational society. As a result of this trend, mathematics literacy has become increasingly important. Making connections between mathematical study and daily life, business, industry, government, and environmental thinking is imperative. The Kindergarten to Grade 12 mathematics curriculum is designed to support and promote the understanding that mathematics is

- a way of learning about our world
- part of our daily lives
- both quantitative and geometric in nature

### Beliefs about Students and Mathematics Learning

The Kindergarten to Grade 8 mathematics curriculum is designed with the understanding that students have unique interests, abilities, and needs. As a result, it is imperative to make connections to all students' prior knowledge, experiences, and backgrounds.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with unique knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of manipulatives and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

Students need frequent opportunities to develop and reinforce their conceptual understanding, procedural thinking, and problem-solving abilities. By addressing these three interrelated components, students will strengthen their ability to apply mathematical learning to their daily lives.

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

Conceptual understanding: comprehending mathematical concepts, relations, and operations to build new knowledge. (Kilpatrick, Swafford, and Findell 5)

Procedural thinking: carrying out procedures flexibly, accurately, efficiently, and appropriately.

Problem solving: engaging in understanding and resolving problem situations where a method or solution is not immediately obvious. (OECD 12)

## First Nations, Métis, and Inuit Perspectives

First Nations, Métis, and Inuit students in Manitoba come from diverse geographic areas with varied cultural and linguistic backgrounds. Students attend schools in a variety of settings, including urban, rural, and isolated communities. Teachers need to recognize and understand the diversity of cultures within schools and the diverse experiences of students.

First Nations, Métis, and Inuit students often have a whole-world view of the environment; as a result, many of these students live and learn best in a holistic way. This means that students look for connections in learning, and learn mathematics best when it is contextualized and not taught as discrete content.

Many First Nations, Métis, and Inuit students come from cultural environments where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding.

A variety of teaching and assessment strategies are required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences, and learning styles of students. The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education (Banks and Banks).

### Affective Domain

A positive attitude is an important aspect of the affective domain that has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help students develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom learning activities, persist in challenging situations, and engage in reflective practices.

Teachers, students, and parents\* need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward reaching these goals.

Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessment of personal goals.

---

\* In this document, the term *parents* refers to both parents and guardians and is used with the recognition that in some cases only one parent may be involved in a child's education.

## Early Childhood

Young children are naturally curious and develop a variety of mathematical ideas before they enter Kindergarten. Children make sense of their environment through observations and interactions at home, in daycares, preschools, and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home.

Activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities.

Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

## Mathematics Education Goals for Students

The main goals of mathematics education are to prepare students to

- communicate and reason mathematically
- use mathematics confidently, accurately, and efficiently to solve problems
- appreciate and value mathematics
- make connections between mathematical knowledge and skills and their applications
- commit themselves to lifelong learning
- become mathematically literate citizens, using mathematics to contribute to society and to think critically about the world

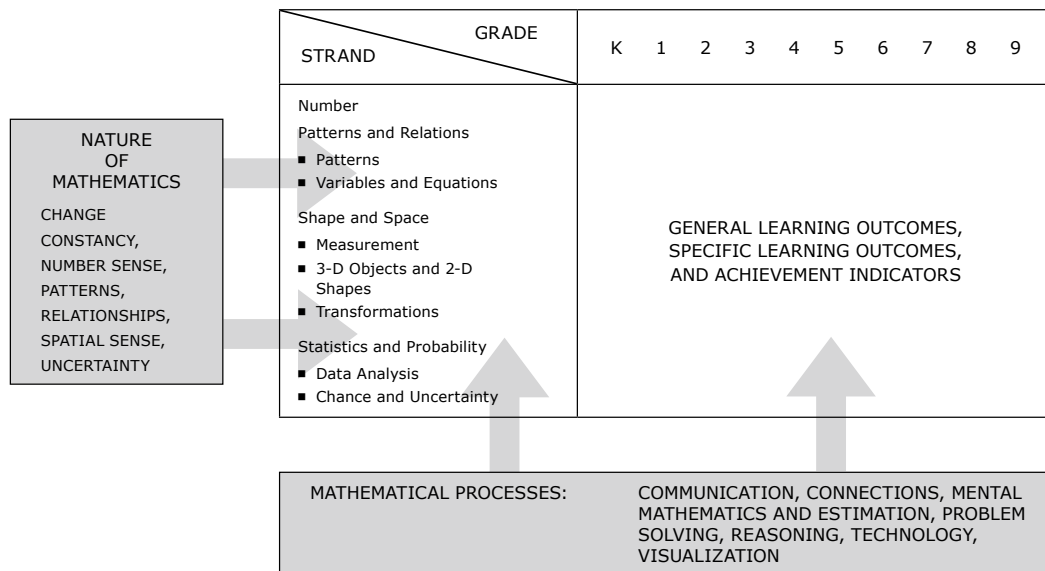
Mathematics education must prepare students to use mathematics to think critically about the world.

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, a philosophy, and an art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity

# CONCEPTUAL FRAMEWORK FOR KINDERGARTEN TO GRADE 9 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



## Mathematical Processes

There are critical components that students must encounter in mathematics to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and solving problems
- develop visualization skills to assist in processing information, making connections, and solving problems

The common curriculum framework incorporates these seven interrelated mathematical processes, which are intended to permeate teaching and learning:

- **Communication [C]:** Students communicate daily (orally, through diagrams and pictures, and by writing) about their mathematics learning. They need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. This enables them to reflect, to validate, and to clarify their thinking. Journals and learning logs can be used as a record of student interpretations of mathematical meanings and ideas.
- **Connections [CN]:** Mathematics should be viewed as an integrated whole, rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—concrete, pictorial, and symbolic (the symbolic mode consists of oral and written word symbols as well as mathematical symbols). The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas.
- **Mental Mathematics and Estimation [ME]:** The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences, and students should be provided with frequent opportunities to practise these skills. Mental mathematics and estimation is a combination of cognitive strategies that enhances flexible thinking and number sense.
- **Problem Solving [PS]:** Students are exposed to a wide variety of problems in all areas of mathematics. They explore a variety of methods for solving and verifying problems. In addition, they are challenged to find multiple solutions for problems and to create their own problems.
- **Reasoning [R]:** Mathematics reasoning involves informal thinking, conjecturing, and validating—these help students understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers.
- **Technology [T]:** The use of calculators is recommended to enhance problem solving, to encourage discovery of number patterns, and to reinforce conceptual development and numerical relationships. They do not, however, replace the development of number concepts and skills. Carefully chosen computer software can provide interesting problem-solving situations and applications.
- **Visualization [V]:** Mental images help students to develop concepts and to understand procedures. Students clarify their understanding of mathematical ideas through images and explanations.

These processes are outlined in detail in *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.



## Strands

The learning outcomes in the Manitoba curriculum framework are organized into four strands across Kindergarten to Grade 9. Some strands are further subdivided into substrands. There is one general learning outcome per substrand across Kindergarten to Grade 9.

The strands and substrands, including the general learning outcome for each, follow.

### Number

- Develop number sense.

### Patterns and Relations

- Patterns
  - Use patterns to describe the world and solve problems.
- Variables and Equations
  - Represent algebraic expressions in multiple ways.

### Shape and Space

- Measurement
  - Use direct and indirect measure to solve problems.
- 3-D Objects and 2-D Shapes
  - Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.
- Transformations
  - Describe and analyze position and motion of objects and shapes.

### Statistics and Probability

- Data Analysis
  - Collect, display, and analyze data to solve problems.
- Chance and Uncertainty
  - Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

## Learning Outcomes and Achievement Indicators

The Manitoba curriculum framework is stated in terms of general learning outcomes, specific learning outcomes, and achievement indicators:

- **General learning outcomes** are overarching statements about what students are expected to learn in each strand/substrand. The general learning outcome for each strand/substrand is the same throughout the grades from Kindergarten to Grade 9.
- **Specific learning outcomes** are statements that identify the specific skills, understanding, and knowledge students are required to attain by the end of a given grade.
- **Achievement indicators** are samples of how students may demonstrate their achievement of the goals of a specific learning outcome. The range of samples provided is meant to reflect the depth, breadth, and expectations of the specific learning outcome. While they provide some examples of student achievement, they are not meant to reflect the sole indicators of success.

In this document, the word *including* indicates that any ensuing items **must be addressed** to meet the learning outcome fully. The phrase *such as* indicates that the ensuing items are provided for illustrative purposes or clarification, and are **not requirements that must be addressed** to meet the learning outcome fully.

## Summary

The conceptual framework for Kindergarten to Grade 9 mathematics describes the nature of mathematics, the mathematical processes, and the mathematical concepts to be addressed in Kindergarten to Grade 9 mathematics. The components are not meant to stand alone. Learning activities that take place in the mathematics classroom should stem from a problem-solving approach, be based on mathematical processes, and lead students to an understanding of the nature of mathematics through specific knowledge, skills, and attitudes among and between strands. *Grade 3 Mathematics: Support Document for Teachers* is meant to support teachers to create meaningful learning activities that focus on formative assessment and student engagement.

Authentic assessment and feedback are a driving force for the suggestions for assessment in this document. The purposes of the suggested assessment activities and strategies are to parallel those found in *Rethinking Classroom Assessment with Purpose in Mind: Assessment for Learning, Assessment as Learning, Assessment of Learning* (Manitoba Education, Citizenship and Youth). These include the following:

- assessing *for, as, and of* learning
- enhancing student learning
- assessing students effectively, efficiently, and fairly
- providing educators with a starting point for reflection, deliberation, discussion, and learning

Assessment *for* learning is designed to give teachers information to modify and differentiate teaching and learning activities. It acknowledges that individual students learn in idiosyncratic ways, but it also recognizes that there are predictable patterns and pathways that many students follow. It requires careful design on the part of teachers so that they use the resulting information to determine not only what students know, but also to gain insights into how, when, and whether students apply what they know. Teachers can also use this information to streamline and target instruction and resources, and to provide feedback to students to help them advance their learning.

Assessment *as* learning is a process of developing and supporting metacognition for students. It focuses on the role of the student as the critical connector between assessment and learning. When students are active, engaged, and critical assessors, they make sense of information, relate it to prior knowledge, and use it for new learning. This is the regulatory process in metacognition. It occurs when students monitor their own learning and use the feedback from this monitoring to make adjustments, adaptations, and even major changes in what they understand. It requires that teachers help students develop, practise, and become comfortable with reflection, and with a critical analysis of their own learning.

Assessment *of* learning is summative in nature and is used to confirm what students know and can do, to demonstrate whether they have achieved the curriculum learning outcomes, and, occasionally, to show how they are placed in relation to others. Teachers concentrate on ensuring that they have used assessment to provide accurate and sound statements of students' proficiency so that the recipients of the information can use the information to make reasonable and defensible decisions.

<b>Overview of Planning Assessment</b>			
	<b>Assessment for Learning</b>	<b>Assessment as Learning</b>	<b>Assessment of Learning</b>
Why Assess?	<ul style="list-style-type: none"> <li>to enable teachers to determine next steps in advancing student learning</li> </ul>	<ul style="list-style-type: none"> <li>to guide and provide opportunities for each student to monitor and critically reflect on his or her learning and identify next steps</li> </ul>	<ul style="list-style-type: none"> <li>to certify or inform parents or others of student's proficiency in relation to curriculum learning outcomes</li> </ul>
Assess What?	<ul style="list-style-type: none"> <li>each student's progress and learning needs in relation to the curriculum outcomes</li> </ul>	<ul style="list-style-type: none"> <li>each student's thinking about his or her learning, what strategies he or she uses to support or challenge that learning, and the mechanisms he or she uses to adjust and advance his or her learning</li> </ul>	<ul style="list-style-type: none"> <li>the extent to which each student can apply the key concepts, knowledge, skills, and attitudes related to the curriculum outcomes</li> </ul>
What Methods?	<ul style="list-style-type: none"> <li>a range of methods in different modes that make a student's skills and understanding visible</li> </ul>	<ul style="list-style-type: none"> <li>a range of methods in different modes that elicit the student's learning and metacognitive processes</li> </ul>	<ul style="list-style-type: none"> <li>a range of methods in different modes that assess both product and process</li> </ul>
Ensuring Quality	<ul style="list-style-type: none"> <li>accuracy and consistency of observations and interpretations of student learning</li> <li>clear, detailed learning expectations</li> <li>accurate, detailed notes for descriptive feedback to each student</li> </ul>	<ul style="list-style-type: none"> <li>accuracy and consistency of a student's self-reflection, self-monitoring, and self-adjustment</li> <li>engagement of the student in considering and challenging his or her thinking</li> <li>the student records his or her own learning</li> </ul>	<ul style="list-style-type: none"> <li>accuracy, consistency, and fairness of judgments based on high-quality information</li> <li>clear, detailed learning expectations</li> <li>fair and accurate summative reporting</li> </ul>
Using the Information	<ul style="list-style-type: none"> <li>provide each students with accurate descriptive feedback to further his or her learning</li> <li>differentiate instruction by continually checking where each student is in relation to the curriculum outcomes</li> <li>provide parents or guardians with descriptive feedback about student learning and ideas for support</li> </ul>	<ul style="list-style-type: none"> <li>provide each student with accurate, descriptive feedback that will help him or her develop independent learning habits</li> <li>have each student focus on the task and his or her learning (not on getting the right answer)</li> <li>provide each student with ideas for adjusting, rethinking, and articulating his or her learning</li> <li>provide the conditions for the teacher and student to discuss alternatives</li> <li>the student reports his or her learning</li> </ul>	<ul style="list-style-type: none"> <li>indicate each student's level of learning</li> <li>provide the foundation for discussions on placement or promotion</li> <li>report fair, accurate, and detailed information that can be used to decide the next steps in a student's learning</li> </ul>

Source: Manitoba Education, Citizenship and Youth. *Rethinking Classroom Assessment with Purpose in Mind: Assessment for Learning, Assessment as Learning, Assessment of Learning*. Winnipeg, MB: Manitoba Education, Citizenship and Youth, 2006, 85.

## INSTRUCTIONAL FOCUS

---

The Manitoba curriculum framework is arranged into four strands. These strands are not intended to be discrete units of instruction. The integration of learning outcomes across strands makes mathematical experiences meaningful. Students should make the connection between concepts both within and across strands.

Consider the following when planning for instruction:

- Routinely incorporating conceptual understanding, procedural thinking, and problem solving within instructional design will enable students to master the mathematical skills and concepts of the curriculum.
- Integration of the mathematical processes within each strand is expected.
- Problem solving, conceptual understanding, reasoning, making connections, and procedural thinking are vital to increasing mathematical fluency, and must be integrated throughout the program.
- Concepts should be introduced using manipulatives and gradually developed from the concrete to the pictorial to the symbolic.
- Students in Manitoba bring a diversity of learning styles and cultural backgrounds to the classroom and they may be at varying developmental stages. Methods of instruction should be based on the learning styles and abilities of the students.
- Use educational resources by adapting to the context, experiences, and interests of students.
- Collaborate with teachers at other grade levels to ensure the continuity of learning of all students.
- Familiarize yourself with exemplary practices supported by pedagogical research in continuous professional learning.
- Provide students with several opportunities to communicate mathematical concepts and to discuss them in their own words.

“Students in a mathematics class typically demonstrate diversity in the ways they learn best. It is important, therefore, that students have opportunities to learn in a variety of ways—individually, cooperatively, independently, with teacher direction, through hands-on experience, through examples followed by practice. In addition, mathematics requires students to learn concepts and procedures, acquire skills, and learn and apply mathematical processes. These different areas of learning may involve different teaching and learning strategies. It is assumed, therefore, that the strategies teachers employ will vary according to both the object of the learning and the needs of the students” (Ontario 24).

## DOCUMENT ORGANIZATION AND FORMAT

---

This document consists of the following sections:

- **Introduction:** The Introduction provides information on the purpose and development of this document, discusses characteristics of and goals for Early Years learners, and addresses Aboriginal perspectives. It also gives an overview of the following:
  - **Conceptual Framework for Kindergarten to Grade 9 Mathematics:** This framework provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.
  - **Assessment:** This section provides an overview of planning for assessment in mathematics, including assessment *for, as, and of* learning.
  - **Instructional Focus:** This discussion focuses on the need to integrate mathematics learning outcomes and processes across the four strands to make learning experiences meaningful for students.
  - **Document Organization and Format:** This overview outlines the main sections of the document and explains the various components that comprise the various sections.
- **Number:** This section corresponds to and supports the Number strand for Grade 3 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.
- **Patterns and Relations:** This section corresponds to and supports the Patterns and Variables and Equations substrands of the Patterns and Relations strand for Grade 3 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.
- **Shape and Space:** This section corresponds to and supports the Measurement and 3-D Objects and 2-D Shapes substrands of the Shape and Space strand for Grade 3 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.
- **Statistics and Probability:** This section corresponds to and supports the Data Analysis substrand of the Statistics and Probability strand for Grade 3 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.
- **Blackline Masters (BLMs):** Blackline masters are provided to support student learning. They are available in *Microsoft Word* format so that teachers can alter them to meet students' needs, as well as in *Adobe PDF* format.
- **Bibliography:** The bibliography lists the sources consulted and cited in the development of this document.

## Guide to Components and Icons

Each of the sections supporting the strands of the Grade 3 Mathematics curriculum includes the components and icons described below.

### **Enduring Understanding(s):**

These statements summarize the core idea of the particular learning outcome(s). Each statement provides a conceptual foundation for the learning outcome. It can be used as a pivotal starting point in integrating other mathematics learning outcomes or other subject concepts. The integration of concepts, skills, and strands remains of utmost importance.

### **General Learning Outcome(s):**

General learning outcomes (GLOs) are overarching statements about what students are expected to learn in each strand/substrand. The GLO for each strand/substrand is the same throughout Kindergarten to Grade 8.

<b>SPECIFIC LEARNING OUTCOME(S):</b>	<b>ACHIEVEMENT INDICATORS:</b>
<p>Specific learning outcome (SLO) statements define what students are expected to achieve by the end of the grade.</p> <p>A code is used to identify each SLO by grade and strand, as shown in the following example:</p> <p><b>3.N.1</b> The first number refers to the grade (Grade 3). The letter(s) refer to the strand (Number). The last number indicates the SLO number. [C, CN, ME, PS, R, T, V]</p> <p>Each SLO is followed by a list indicating the applicable mathematical processes.</p>	<p>Achievement indicators are examples of a representative list of the depth, breadth, and expectations for the learning outcome. The indicators may be used to determine whether students understand the particular learning outcome. These achievement indicators will be addressed through the learning activities that follow.</p>

## PRIOR KNOWLEDGE

---

Prior knowledge is identified to give teachers a reference to what students may have experienced previously. Teachers should assess students' prior knowledge before planning instruction.

## BACKGROUND INFORMATION

---

Background information is identified to give teachers knowledge about specific concepts and skills related to the particular learning outcome(s).

## MATHEMATICAL LANGUAGE

---

Lists of terms students will encounter while achieving particular learning outcomes are provided. These terms can be placed on mathematics word walls or used in a classroom mathematics dictionary. *Kindergarten to Grade 8 Mathematics Glossary: Support Document for Teachers* (Manitoba Education, Citizenship and Youth) provides teachers with an understanding of key terms found in Kindergarten to Grade 7 mathematics. The glossary is available on the Manitoba Education and Training website at [www.edu.gov.mb.ca/k12/cur/math/supports.html](http://www.edu.gov.mb.ca/k12/cur/math/supports.html).

## LEARNING EXPERIENCES

---

Suggested instructional strategies and assessment ideas are provided for the specific learning outcomes and achievement indicators. In general, learning activities and teaching strategies related to specific learning outcomes are developed individually, except in cases where it seems more logical to develop two or more learning outcomes together. Suggestions for assessment include information that can be used to assess students' progress in their understanding of a particular learning outcome or learning experience.



### **Assessing Prior Knowledge:**

#### **Observation Checklist:**

#### **Assessing Understanding:**

Suggestions are provided for assessing prior to and after lessons, and checklists are provided for observing during lessons to direct instruction.



## Suggestions for Instruction

- **Achievement indicators appropriate to particular learning experiences are listed.**

The instructional suggestions include the following:

- **Materials/Resources:** Outlines the resources required for a learning activity.
- **Organization:** Suggests groupings (individual, pairs, small group, and/or whole class).
- **Procedure:** Outlines detailed steps for implementing suggestions for instruction.

Some learning activities make use of BLMs, which are found in the Blackline Masters section in *Microsoft Word* and *Adobe PDF* formats.

## PUTTING THE PIECES TOGETHER



Putting the Pieces Together tasks, found at the end of the learning outcomes, consist of a variety of assessment strategies. They may assess one or more learning outcomes across one or more strands and may make cross-curricular connections.

---

## Notes



# GRADE 3 MATHEMATICS

Number



## Grade 3: Number (3.N.1)

### Enduring Understanding:

Counting is a strategy for finding the answer to how many.

### Essential Question:

Is there a quicker way to find the answer than counting by ones from one?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
<p><b>3.N.1</b> Say the number sequence between any two given numbers forward and backward</p> <ul style="list-style-type: none"><li>■ from 0 to 1000 by<ul style="list-style-type: none"><li>■ 10s or 100s, using any starting point</li><li>■ 5s, using starting points that are multiples of 5</li><li>■ 25s, using starting points that are multiples of 25</li></ul></li><li>■ from 0 to 100 by<ul style="list-style-type: none"><li>■ 3s, using starting points that are multiples of 3</li><li>■ 4s, using starting points that are multiples of 4</li></ul></li></ul> <p>[C, CN, ME]</p>	<ul style="list-style-type: none"><li>→ Extend a skip-counting sequence by 10s or 100s, forward and backward, using a given starting point.</li><li>→ Extend a skip-counting sequence by 5s, forward and backward, starting at a given multiple of 5.</li><li>→ Extend a skip-counting sequence by 25s, forward and backward, starting at a given multiple of 25.</li><li>→ Extend a given skip-counting sequence by 3s, forward, starting at a given multiple of 3.</li><li>→ Extend a given skip-counting sequence by 4s, starting at a given multiple of 4.</li><li>→ Identify and correct errors and omissions in a skip-counting sequence.</li><li>→ Determine the value of a set of coins (nickels, dimes, quarters, loonies) by using skip counting.</li><li>→ Identify and explain the skip-counting pattern for a number sequence.</li></ul>

## PRIOR KNOWLEDGE

Students may be able to say the number sequence from 0 to 100 by

- 2s, 5s, and 10s, forward and backward, using starting points that are multiples of 2, 5, and 10 respectively
- 10s, using starting points from 1 to 9
- 2s, starting from 1

## BACKGROUND INFORMATION

---

Students in Grade 3 are expanding their experiences with numbers to 1000 and may struggle with the increase in numbers. It is important to provide many opportunities for students to bridge the decades through the hundreds (e.g., 98, **99, 100, 101**, 102, 103, 104, 105, 106, 107, 108, **109, 110, 111**, 112...). When students are working with larger numbers the goal is to have students understand that there is a pattern within our number system that enables us to predict numbers. Have students recognize and explain errors and omissions in a given skip-counting sequence to help to reinforce the development of counting, number relationships, and place value.

When skip-counting with students, the focus should be on looking for patterns. Understanding patterns can support children's use of invented strategies and prepare students for working with money. Exploring the patterns should strengthen children's understanding of number relationships and properties. Asking children, "What did you observe about the pattern?" can help facilitate children's sense making about number relationships with the patterns and other math concepts.

Counting on and counting back by 5s, 10s, and 100s are important mental math strategies for addition and subtraction. Skip-counting by 2s, 3s, and 4s is a foundation for multiplicative understanding.

Counting a mixed collection of coins can be difficult for students because they are expected to shift how they are skip counting several times (e.g., counting by loonies [1s], then by quarters [25s], and then by dimes [10s]). They need practice switching the count using a set of like coins before counting mixed collections of money.

## MATHEMATICAL LANGUAGE

---

### Counting numbers:

one to one thousand	count back
count on	penny
skip count	nickel
set	dime
number	quarter
numeral	loonie
multiple	money



### Assessing Prior Knowledge: Interview

Ask the student to

- start at 42 and count by 2s (stop at 60)
- start at 13 and count by 2s (stop at 31)
- start at 78 and count backward by 2s (stop at 64)
- start at 30 and count by 10s (stop at 100)
- start at 7 and count by 10s (stop at 57)
- start at 100 and count backward by 10s (stop at 40)
- start at 15 and count by 5s (stop at 60)
- start at 85 and count backward by 5s (stop at 55)
- start at 3 and count by 3s (stop at 24)
- start at 4 and count by 4s (stop at 28)
- count a set of counters by 2s, 5s, or 10s, and count on
- determine the value of 5 quarters

The student is able to

- count by 2s
  - forward on the multiple
  - forward off the multiple
  - backward on the multiple
- count by 10s
  - forward on the multiple
  - forward off the multiple
  - backward on the multiple
- count by 5s
  - forward on the multiple
  - backward on the multiple
- count by 3s
  - forward on the multiple
- count by 4s
  - forward on the multiple

- count a set in groups and count on
    - by 2s
    - by 5s
    - by 10s
  - determine the value of set of coins
    - nickels
    - dimes
    - quarters
- 

- **Extend a skip-counting sequence by 10s, or 100s, forward and backward, using a given starting point.**
- **Extend a skip-counting sequence by 5s, forward and backward, starting at a given multiple of 5.**
- **Extend a skip-counting sequence by 25s, forward and backward, starting at a given multiple of 25.**

### Suggestions for Instruction

- **Count around the Circle:** Have the class sit in a circle. Give the start and finish for the count. Before counting ask students questions such as the following:
  - We're going to start at 0 and stop at 110. Who do you think will say 110?
  - We're going to count by 25s to \$10.00. How many times will we have to go around the circle?Have students (each student saying one number in turn) count to confirm their predictions.
- **Problem Solving:** Have students think about counting by asking questions such as the following:
  - If you start with 126, how many 10s do you need to add to get to more than 200?
  - If you start with 345, how many 10s do you need to add to get to 415?
  - If you count on in 100s from 350 to 850, how many 100s do you count?
  - If you count back in 10s from 934 to 854, how many 10s do you count?
  - If you count back in 100s from 978 to 178, how many 100s do you count?
  - If you start at 175 and count on eight 25s, what number will you say last?

BLM  
3.N.1.1

For students in group situations, use a set of number cards to select the starting point.



- **Change the Count:** Start counting together from a given starting point. After a short time clap your hands, ring a bell, or use another signal to change the skip-counting sequence. Change the sequence several times.

Example: Start counting together in 5s from 50. After reaching 150, clap your hands and announce that they will now be counting by 10s. When 290 is reached clap again and announce that the counting will be by 100s. When the class reaches 990, clap once more and announce that they will be counting backwards by 5s, and so on.

- **Extend a skip-counting sequence by 3s, forward, starting at a given multiple of 3.**
- **Extend a skip-counting sequence by 4s, starting at a given multiple of 4.**

### Suggestions for Instruction

BLM  
3.N.1.2

- **Patterns in Hundred Chart:** Have students place counters on a hundred chart on multiples of 3 and 4. Have students describe the pattern made by the counters.
- **Calculator Exploration:** Students use different starting points (multiples of 3 or 4) and the constant feature on a calculator to check counting by 3s and 4s.

Constant feature on the calculator (Not all calculators have the feature.):

Example:

Counting by 5s:

Press the following keys:

$0 + 5 = = = 15$

Counting by 3s starting at 9:

Press the following keys:

$9 + 3 = = = 18$

Challenge the students by having them predict the next number before they press the equal button.

- Present problems that have students skip-counting and comparing them.

Example:

Jan counted the blocks 3 at a time.

Gill counted the blocks 4 at a time.

Write the first 10 numbers in Jan's and Gill's pattern.

How are the patterns the same and how are they different?

**Note:** Counting can be built in to classroom routines. Students can use the "Number of the Day" and count by 3s, 4s, 5s, 10s, 100s, and 25s (if applicable). It can also be done when there are a few minutes of class time available (e.g., just before dismissal, waiting for class changes).



---

### Assessing Understanding: Interview

Have the student

- count on by 10s from 460 to 510
- count on by 5s from 185 to 230
- count on by 100s from 138 to 838
- count on by 25s from 125 to 250
- count back by 10s from 773 to 683
- count back by 5s from 530 to 485
- count back by 100s from 996 to 106
- count back by 25s from 625 to 500
- count on by 3s starting at 18 to 36
- count on by 4s starting at 24 to 48

The student is able to

- count on by 10s
  - count on by 5s
  - count on by 100s
  - count on by 25s
  - count back by 10s
  - count back by 5s
  - count back by 100s
  - count back by 25s
  - count by 3s on multiples of 3
  - count by 4s on multiples of 4
-

- **Identify and correct errors and omissions in a skip-counting sequence.**
- **Identify and explain the skip-counting pattern for a number sequence.**

## Suggestions for Instruction

BLM  
3.N.1.3

- Prepare a set of cards containing both correct and incorrect skip-counting sequences. Students sort them into two groups (Correct/Incorrect) and then correct the incorrect sequences.
- **What's the Pattern?** Present students with number sequences such as
  - 298, 398, 498, 598, 698
  - 175, 200, 225, 250, 275
  - 327, 337, 347, 357, 367

Have students identify the skip-counting pattern (e.g., counting by 10s) and then give the next three numbers in the sequence.



### Assessing Understanding: Paper-and-Pencil Task 1

Give students the set of number sequences. Have them identify the skip counting pattern and then give the next four numbers in the sequence.

1. 275, 300, 325, 350, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_  
Counting forward/backward by \_\_\_\_\_.
2. 456, 446, 436, 426, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_  
Counting forward/backward by \_\_\_\_\_.
3. 660, 665, 670, 675, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_  
Counting forward/backward by \_\_\_\_\_.
4. 123, 223, 323, 423, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_  
Counting forward/backward by \_\_\_\_\_.
5. 708, 718, 728, 738, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_  
Counting forward/backward by \_\_\_\_\_.

### Paper-and-Pencil Task 2

If you start at 465 and count by 5s to 550, and your friend starts at 410 and counts by 10s to 550, what numbers would you both say? Explain how you know.

- **Determine the value of a set of coins (nickels, dimes, quarters, loonies) by using skip counting.**

### Suggestions for Instruction

- Initially have students count sets of like coins (e.g., all dimes or all nickels). Then have them count sets with two different coins. Make sure that students start with the coins that have the greatest value first. Keep adding coins until they are able to count a mixed collection.
- **Money Counting Centre:** Prepare bags/containers with coins of varying amounts. Have students select a bag of coins, count the money in the bag, and record the answer. Vary the number and types of coins in each bag/container in order to meet the needs of your students.

**Note:** When possible involve students in counting money for classroom events such as special lunches, field trips, and so on.

The following site contains BLMs of Canadian coins:

[http://lrt.ednet.ns.ca/PD/BLM/table\\_of\\_contents.htm](http://lrt.ednet.ns.ca/PD/BLM/table_of_contents.htm)



---

### Assessing Understanding: Observation Checklist

Give the student a collection of coins. Observe them as they count the collection.

The student

- sorts the coins
  - starts with the coins that have the greatest value
  - is able to continue the count as coin values change (e.g., quarters to dimes)
  - counts the collection accurately
-

## Grade 3: Number (3.N.2)

### Enduring Understanding:

Quantities can be represented in a variety of ways with objects, pictures, and numerals.

### Essential Questions:

How can quantities be shown?

How many different ways can you represent a number?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
3.N.2 Represent and describe numbers to 1000, concretely, pictorially, and symbolically. [C, CN, V]	<ul style="list-style-type: none"><li>→ Read a 3-digit numeral without using the word “and” (e.g., 321 is three hundred twenty-one, NOT three hundred AND twenty-one).</li><li>→ Read a number word (0 to 1000).</li><li>→ Represent a number as an expression (e.g., <math>300 - 44</math> for 256 or <math>20 + 236</math>).</li><li>→ Represent a number using manipulatives, such as base-10 materials.</li><li>→ Represent a number pictorially.</li><li>→ Write number words for multiples of ten to 90.</li><li>→ Write number words for multiples of a hundred to 900.</li><li>→ Determine compatible number pairs for 100.</li></ul>

## PRIOR KNOWLEDGE

---

Students may have had experience representing and describing numbers to 100, concretely, pictorially, and symbolically. They may have represented numbers using

- concrete materials (e.g., ten frames, base-10 materials)
- coins
- tallies
- pictures
- expressions
- words
- symbols
- place value

## BACKGROUND INFORMATION

---

To develop a good sense of number, students have to develop an intuition about numbers and their relationships. Flexible intuitive thinking about numbers develops gradually as a result of exploring numbers and visualizing in a variety of contexts. Provide the use of concrete materials and models such as base-10 materials, hundred charts, number lines, place value charts, and money to help students make connections between the concrete and pictorial to the symbolic representations of the numbers.

The reading of number words such 625 should be read as “six hundred twenty-five.” When reading numbers the word *and* denotes the decimal. When writing four-digit numbers symbolically, there is usually no space or comma between the thousands and hundreds place. Writing numbers that are five or more digits requires a space between the thousands and hundreds place.

When students are representing numbers in a variety of ways, they demonstrate their understanding of the use of a number (e.g., my house number is 34), how a number compares to another number (e.g., 34 is 1 less than 35), how a number can be broken into parts (e.g., 34 is  $32 + 2$ ), and place value (e.g., 34 is  $30 + 4$  or  $20 + 14$  or  $10 + 24$ ).

The ability to represent numbers in a variety of ways will benefit students when doing operations and mental mathematics problems. Present number sentences horizontally as well as vertically, to encourage students to use different representations of numbers and part-part-whole thinking.

Example:

$$\begin{array}{r} 25 \\ \downarrow \\ 25 \end{array} + \begin{array}{r} 26 \\ \swarrow \downarrow \\ 25 + 1 \end{array} \quad \text{or} \quad \begin{array}{r} 25 \\ \swarrow \downarrow \\ 20 + 5 \end{array} + \begin{array}{r} 26 \\ \swarrow \downarrow \\ 20 + 6 \end{array}$$

Developing part-part-whole relationships allows students to think of a number as a composition of other numbers. This includes knowing the parts and being able to find the whole, knowing the whole and finding the parts, knowing a part and a whole, and finding the missing part. Students who develop a deep understanding of numbers will be able to partition numbers in flexible ways. This learning outcome can be connected to Specific Learning Outcome 3.N.5, which involves the development of place value.

Encourage students to represent numbers in a variety of ways (e.g., using manipulatives, words and pictures, number sentences, place value, money, ten frames, horizontal and vertical number lines, connections to other strands, and real-life situations).

Students need something to remind them of the different ways in which they can represent numbers. Use a chart or table/desk file folders. Brainstorm with the class different representations in order to build the chart/file folder. As new representations are introduced add them to the chart/file folder.

Example:

<b>Ways to Represent Numbers</b>	
words	pictures
materials—base-10 blocks, etc.	story problems
place value	expanded notation/form
comparisons—greater/less than	1, 10, 100 more or less than
half of/double/twice as many as	number line
number sentences/expressions	money
odd/even	real-world connections (house number, date, etc.)

It would be beneficial to set criteria related to the number of ways expected when students are asked to represent a number. Criteria might also specify specific ways required as well (e.g., place value).

An important facet to building representations of numbers is to provide opportunities for students to discuss and reflect upon their thinking and the connections they are making through the representations. Allow time to discuss these connections during daily routines.

## MATHEMATICAL LANGUAGE

---

represent	loonie
place value	number words to 1000
base-10 blocks	number sentence
coins	expression
penny	greatest
nickel	least
dime	money
quarter	

## LEARNING EXPERIENCES

---



### Assessing Prior Knowledge: Performance Task

Student Directions:

38      65      99

1. Choose one of the numbers above.
2. Represent the number in at least 6 different ways.

The student is able to represent a 2-digit number using

- concrete materials (ten frames, base-10 materials)
  - tallies
  - pictures
  - words
  - expressions/number sentences
  - number line
  - comparisons (greater/less than, 1 more/less, 10 more/less, etc.)
  - other
- 

- **Read a 3-digit numeral without using the word “and” (e.g., 321 is three hundred twenty-one, NOT three hundred AND twenty-one).**
- **Read a number word (0 to 1000).**
- **Write number words for multiples of ten to 90.**
- **Write number words for multiples of a hundred to 900.**

### Suggestions for Instruction

- **Gotcha!** Place cards with a set of numbers between 101 and 1000 in a container. Include ten (or more) cards with the word “Gotcha!” written on them. Have students sit in a circle. Pass the container around the circle. Each student draws a card and reads the number. If read correctly the student keeps the card. If a “Gotcha!” card is selected the student must return all of his/her cards to the container. The game continues until all cards are played. The student with the most (or least) cards is the winner.

**Note:** The game can also be played with number words.



- **Concentration**

**Materials:** a set of 12 number cards along with a matching set of number word cards

**Players:** 2

**Directions:** Shuffle the cards together and lay them face down in four rows of six. Players take turns drawing 2 cards. The player reads both cards. If the cards match, the player keeps the pair and plays again. If the cards do not match, the cards are returned to the grid. Play continues until all cards have been matched. The winner is the player with the most cards.

- Over a period of time, have students collect number words from newspapers and/or magazines and write the corresponding numerals beside them. Students can sort and graph the numbers they have found.
- Have students practise writing the number words for the multiples of ten to 90 and multiples of a hundred to 900. This will enable them to write any number from 1 to 1000 in words.
- **“Multiple” Stories:** Have students write a story that uses as many of the number words for the multiples of 10 (to 90) or the multiples of 100 (to 900) as possible. Share the stories with the class and/or compile them into a class book.

- **Daily Routine—Number of the Day**

Organize students into teams of two. Assign the routine to a different team each day. This can be used as assessment for learning. It also helps to keep the concepts fresh in the minds of the students over the course of the year.

**Note:** The Number of the Day can be done on a laminated chart (although after a while it becomes difficult to erase). Some teachers have put words/phrases from the chart on individual strips of paper with a magnetic strip on the back and for use on a white board. This enables the teacher to differentiate for groups of students by adding or deleting representations.

**Number of the Day**

--

Write the number in words:

Show the number using:

Pictures/Models	Base-10 Materials
Expanded Form	Comparisons (more than/less than, etc.)
Money (in two different ways)	
Number Sentences/Expressions	



**Assessing Understanding: Interview**

Show the following numbers. Have the student read them.

671    904    297    880    536    355

Show the following number words. Have the student read them.

- eight hundred six
- two hundred thirty-seven
- nine hundred forty-one
- six hundred fifty
- four hundred ninety-eight
- one hundred sixty-three

The student is able to

- read three-digit numerals
  - confidently
  - with hesitation
- read number words to 999
  - confidently
  - with hesitation

### Student Self-Assessment

BLM  
3.N.2.2

Have students add to the chart several times during the year (perhaps at reporting times).

Name: \_\_\_\_\_

<b>Represent and describe numbers to 1000, concretely, pictorially, and symbolically.</b>				
	<b>September</b>	<b>November</b>	<b>March</b>	<b>June</b>
I can				
I can				
I can				
My goal(s) for next term is/are				

Sample:

Name: \_\_\_\_\_

<b>Represent and describe numbers to 1000, concretely, pictorially, and symbolically.</b>				
	<b>September</b>	<b>November</b>	<b>March</b>	<b>June</b>
I can	represent numbers to 100	represent numbers to 500	represent numbers to 1000	work with numbers larger than 1000
I can	represent numbers using pictures, tallies, ten frames, cubes, counters, money, and other math materials	represent numbers using base-10 materials, a number line, and comparisons	represent numbers using regular and irregular place value	represent numbers in multiple ways
I can	write addition and subtraction number sentences for numbers to 100	write addition and subtraction number sentences for numbers to 500	write addition and subtraction number sentences for numbers to 1000	write addition and subtraction number sentences and some multiplication and division number sentences for numbers to 1000 (e.g., $\times 1$ , $\times 0$ , $\div 1$ )
My goal(s) for next term is/are	<ul style="list-style-type: none"> <li>■ to be able to represent larger numbers</li> <li>■ to find new ways to represent these numbers</li> </ul>			

- **Determine compatible number pairs for 100.**

### Suggestions for Instruction

- **Compatible Number Pairs to 100:** A pair of numbers that is easy to work with mentally (also known as friendly or nice numbers) are said to be **compatible or complementary**. Compatible numbers can help students understand how numbers work. If you know parts of the number, you can put the number together to find the whole. If you know the whole and one of the parts, you can take away the part you know to find the other part.

Part-whole relationships refers to the idea that numbers can be broken down into parts, and that these parts can be compared to the whole. According to John Van de Walle, to conceptualize a number as being made up of two or more parts is the most important understanding that can be developed about number relationships.

Have the students brainstorm the list of compatible numbers to 100 and find the patterns.

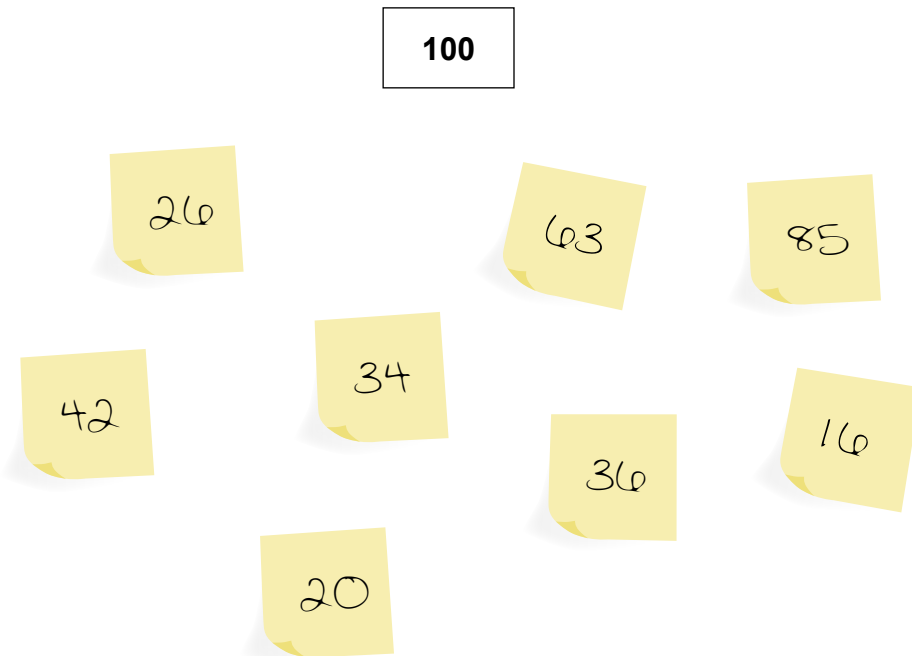
Example:

$0 + 100$	$99 + 1$	$89 + 11$
$10 + 90$	$98 + 2$	$88 + 12$
$20 + 80$	$97 + 3$	$87 + 13$
$30 + 70$	$96 + 4$	$86 + 14$
$40 + 60$	$95 + 5$	$85 + 15$
$50 + 50$	$94 + 6$	$84 + 16$
	$93 + 7$	$83 + 17$
	$92 + 8$	$82 + 18$
	$91 + 9$	$81 + 19 \dots$

Students can make connections with the patterns and build a better sense of number by listing the compatible number.

BLM  
3.N.2.3

- Make a two-digit number with ten frames. Students work together to determine what goes with the ten frame to make 100.
- Post the numeral 100 on a chart. Place numbers less than 100 on sticky notes under the number. Students must choose a number from the wall and give its compatible number to 100.



## Grade 3: Number (3.N.3, 3.N.4)

### Enduring Understandings:

Place value patterns are repeated in large numbers and these patterns can be used to compare and order numbers.

The position of a digit in a number determines the quantity it represents.

Estimation is a way to get an approximate answer.

### Essential Questions:

How does changing the order of the digits in a number affect its placement on a number line?

How are place value patterns repeated in numbers?

How does the position of a digit in a number affect its value?

What are strategies to make reasonable estimates?

Does the use of referents help us make a more reasonable estimate?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
3.N.3 Compare and order numbers to 1000. [CN, R, V]	<ul style="list-style-type: none"><li>→ Place a set of numbers in ascending or descending order, and verify the result by using a hundred chart (e.g., a one hundred chart, a two hundred chart, a three hundred chart), a number line, or by making references to place value.</li><li>→ Create as many different 3-digit numerals as possible, given three different digits. Place the numbers in ascending or descending order.</li><li>→ Identify errors in an ordered sequence.</li><li>→ Identify missing numbers in parts of a hundred chart.</li><li>→ Identify errors in a hundred chart.</li></ul>

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
<p>3.N.4. Estimate quantities less than 1000 using referents. [ME, PS, R, V]</p>	<ul style="list-style-type: none"> <li>→ Estimate the number of groups of ten in a quantity using 10 as a referent (known quantity).</li> <li>→ Estimate the number of groups of a hundred in a quantity using 100 as a referent.</li> <li>→ Estimate a quantity by comparing it to a referent.</li> <li>→ Select an estimate for a quantity by choosing among three possible choices.</li> <li>→ Select and justify a referent for determining an estimate for a quantity.</li> </ul>

## PRIOR KNOWLEDGE

---

Students may have had experience comparing and ordering numbers to 100 and estimating quantities to 100 using referents.

## BACKGROUND INFORMATION

---

Students need experiences naming numbers that are greater than, less than, or between numbers. As students place numbers in ascending or descending order, have them explain and justify the result using a number line or number chart. As students become confident with this concept, they should progress to using place value positional names to determine size. An understanding of place value (SLO 3.N.5) is important for students to compare and order larger numbers.

It is not necessary to introduce the symbols “<” or “>” in Grade 3 but more specifically students should not be assessed using the symbols. The focus should be on using and understanding what *greater than* and *less than* mean.

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Estimation is also used to make mathematical judgments and to develop useful, efficient strategies for dealing with situations in daily life. When estimating, students need to know which strategy to use and how to use it.

Students use reasoning skills to help estimate. It is important to provide referents for estimation. Through the process of choosing and using referents, students will be able to justify a referent for determining an estimate.

**Referent:** A known quantity used to estimate or compare.

Examples:

- Use one layer of marbles from a jar to estimate the number of marbles in the entire jar.
- The height of the room is about twice as tall as a student.
- Know the width of your pinky finger is approximately 1 cm.
- The length of a piece of paper is approximately 30 cm.

## MATHEMATICAL LANGUAGE

---

order

compare

greater than

less than

greatest

least

three-digit number

estimate

referent

approximate

## LEARNING EXPERIENCES

---



### Assessing Prior Knowledge

1. Give each student a hundred chart. Ask them to put a marker on
  - a number greater than 45
  - a number less than 93
  - a number between 28 and 55
2. Have each student select eight number cards (from a random collection 1 to 100) and order them from least to greatest.
3. Ask students, “Which is greater: 87 or 78? Explain/show how you know.”
4. Have students take a handful of centimetre cubes, bingo chips, or other small objects such as buttons or beans, estimate the quantity, and then explain how they decided on the answer.

The student is able to

- identify and compare numbers to 100 correctly
  - order numbers to 100
  - use a referent to make a reasonable estimate
-



- Place a set of numbers in ascending or descending order, and verify the result by using a hundred chart (e.g., a one hundred chart, a two hundred chart, a three hundred chart), a number line, or by making references to place value.
- Create as many different 3-digit numerals as possible, given three different digits. Place the numbers in ascending or descending order.
- Identify errors in an ordered sequence.
- Identify missing numbers in parts of a hundred chart.
- Identify errors in a hundred chart.

### Suggestions for Instruction

- Grade 3 students do not have a good understanding of what a 1000 of something looks like. Use a book such as *How Much, How Many, How Far, How Heavy, How Long, How Tall Is 1000?* by Helen Nolan, illustrated by Tracy Walker, to help give students a sense of the magnitude of 1000.

BLM  
3.N.3.1

- Use a collection of number cards with values shown to 1000. Have students choose 10 cards and order them from least to greatest. You could also have 10 different students each select a card and then order themselves.

BLM  
3.N.3.2

- **Missing Pieces:** Use parts of hundred charts and have students fill in the missing numbers. Ask students to explain how they know what number to write.

Examples:

These were taken from larger charts. Fill in the missing numerals.

	826			
835				
			848	
		857		
				869

436			
	457		

Have students make their own charts with an empty chart.

BLM  
3.N.3.3

■ **Roll and Order**

**Objective:** to use number sense to order three-digit numbers

**Materials:** 3 (0 to 9) dice, a game board for each player

**Directions:**

1. Determine the Start and End numbers (e.g., start on 1 and end on 1000).
2. Player 1 rolls the three dice and arranges them into a three-digit number. He or she writes the number where he or she feels it belongs on his or her game board.
3. Player 2 takes a turn.
4. Play continues until one player has filled his or her board (numbers are in sequence).

**Note:** Differentiate the game by

- changing the Start and End numbers (e.g., narrow the range [200 to 300])
- having students play in partners
- reducing or increasing the number of dice
- increasing or decreasing the number of spaces on the game board

BLM  
3.N.3.4

■ **Largest or Smallest?**

**Objective:** to make the largest/smallest number

**Materials:** one die (0 to 9), game board for each player

**Directions:**

1. Decide whether players are making the largest or smallest number.
2. Player 1 rolls the die and calls out the number.
3. Both players choose to place the number (digit) in the hundreds, tens, or ones place on their game boards.
4. The die is rolled two more times with players filling in their game boards each time until the number is complete.
5. Players compare their numbers. The player with the largest/smallest number scores 1 point. In the event of a tie both players score a point.
6. Player 2 then rolls the die. Play continues until one player scores 10 points.

Hundreds	Tens	Ones

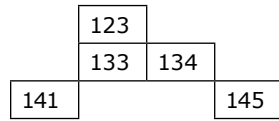
Observe students as they play.

Ask

- How do you decide where to place each number?
- Does your strategy help you score points?

- **Finding Errors:** Provide students with portions of hundred charts and number sequences that contain errors. Have students identify and correct the errors.

Example:



- **Ordering on a Number Line:** Use a clothesline/piece of string and a set of cards with numbers between 1 and 1000. (Tent cards—cards that are folded over so that they sit on the line without flipping or moving—work best.) Label the start and finish of the line. Pass out number cards to several students. Have them place their number on the line. Ask students to explain their placement choice.

**Note:** This activity could be used as a classroom routine. Have one or two students each day place two or three numbers on the line explaining their placement choice.



BLM  
3.N.3.5

### Assessing Understanding: Checklist

Use a checklist to record observations from the learning activities above.

Students	Order numbers from 1 to 200	Order numbers from 300 to 500	Order numbers from 600 to 1000

- **Estimate the number of groups of ten in a quantity using 10 as a referent (known quantity).**
- **Estimate the number of groups of a hundred in a quantity using 100 as a referent.**
- **Estimate a quantity by comparing it to a referent.**
- **Select an estimate for a quantity by choosing among three possible choices.**
- **Select and justify a referent for determining an estimate for a quantity.**

### Suggestions for Instruction

- Use a book such as *Great Estimations* by Bruce Goldstone to help students understand how referents are used for estimation.
- Fill a container with cereal, popcorn, or manipulative materials (up to 1000). Ask students to estimate the number of individual pieces in the container, using a referent. The referent can be a set of 10, 25, or 50—this will assist them in making reasonable estimates. Refill the container with other materials to improve the students’ estimation skills.

- **Investigation:** Pairs of students open a small box of raisins and count the visible (top layer) of raisins. They use this referent to estimate the number of raisins in a box. Graph the estimates on the chalkboard. Have students spill out and count the actual number of raisins. Graph the actual number counts on the chalkboard. Discuss the differences.

Repeat the investigation using the same empty raisin boxes filled with puffed rice or using different size boxes of raisins.

- **Estimation Station:** Set up a weekly estimation station with a quantity to be estimated and an appropriate referent (e.g., 2 containers—one filled with centimetre cubes and the other with ten/one hundred cubes [depending on the number of cubes]). Have students record their estimate and then count to find the actual.

Objects	Estimate	Actual	My estimate is		
			too low	just right	too high

- Show students a quantity of an object (e.g., beans, bingo chips, craft sticks). Provide students with three possible choices and have them choose the best estimate and record it. Show them a referent and allow students to change their estimate.

BLM  
3.N.4.1

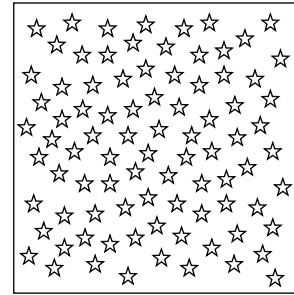
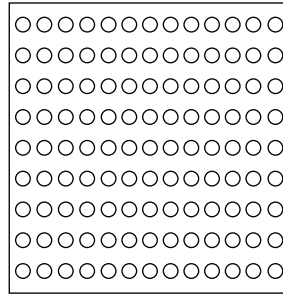
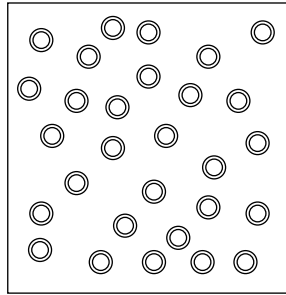


BLM  
3.N.4.2

---

## Assessing Understanding: Paper-and-Pencil Tasks

1. Estimate the number of objects in each box. Explain how you decided on your estimate.



2. **Journal/Learning Log Entry**

Display a transparent container with small objects such as pennies in it and a matching container with 10 (or 100) of the same objects to use as a referent (depending on the quantity in the container). Ask students to record their estimate in their journal and then write about how they arrived at their estimate.

For both tasks, the student is able to

- select and use an appropriate referent (10, 25, 100)
- make a reasonable estimate
- explain his or her thinking

## Grade 3: Number (3.N.5)

### Enduring Understandings:

The position of a digit in a number determines the quantity it represents.

The groupings of 1s, 10s, and 100s can be taken apart in different ways.

### Essential Question:

How many ways can a number be taken apart?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
<p>3.N.5 Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000. [C, CN, R, V]</p>	<ul style="list-style-type: none"><li>→ Record in more than one way the number represented by proportional and non-proportional concrete materials.</li><li>→ Represent a number in different ways using proportional and non-proportional concrete materials, and explain how they are equivalent (e.g., 351 can be represented as three 100s, five 10s, and one 1, or two 100s, fifteen 10s, and one 1, or three 100s, four 10s, and eleven 1s).</li><li>→ Explain, and show with counters, the meaning of each digit for a 3-digit numeral with all digits the same (e.g., for the numeral 222, the first digit represents two hundreds [two hundred counters], the second digit represents two tens [twenty counters], and the third digit represents two ones [two counters]).</li></ul>

### PRIOR KNOWLEDGE

---

Students may have illustrated, concretely and pictorially, the meaning of place value for numbers to 100 in Grade 2 using proportional base-10 materials.

### BACKGROUND INFORMATION

---

In order to understand and use place value students need to be able to “think in groups” or to unitize. They need to see ten as a unit and not as a collection of 10 individual parts.

In 1989, Sharon Ross (cited in Van de Walle, Karp, Lovin, and Bay-Williams 157) identified five distinct levels of understanding of place value based on responses to the following task:

1. Place 36 blocks on the table and have the students count them.
2. Have them write the number that tells how many there are.
3. Circle the 6. Ask, "Does this part of your 36 have anything to do with how many blocks there are?"
4. Circle the 3. Ask, "Does this part of your 36 have anything to do with how many blocks there are?"

Do not give clues.

### Levels of Understanding of Place Value

1. **Single numeral:** The student writes 36 but views it as a single numeral. The individual digits 3 and 6 have no meaning by themselves.
2. **Position names:** The student identifies correctly the tens and ones/units position but still makes no connections between individual digits and the counters.
3. **Face value:** The student matches 6 counters with the 6 and 3 counters with the 3.
4. **Transition to place value:** The 6 is matched with the 6 counters and the 3 with the remaining 30 counters but they are not seen as 3 groups of 10.
5. **Full understanding:** The 3 is correlated with 3 groups of 10 counters and the 6 with 6 single counters.

In proportional models for base 10, such as craft (popsicle) sticks, Digi-Blocks, and base-10 blocks, the representation for 10 is ten times the size of the representation for 1, 100 is ten times the size of 10, and so on.

Non-proportional models, such as money, do not have any size relationship.

Students need opportunities to construct their own understanding of our place value system. In order to do this, they should work with materials such as craft sticks that they have to group themselves. Base-10 blocks and Digi-Blocks are useful materials once their understanding is in place, but they are adult- rather than student- or child-developed constructs.

**Preventing Misconceptions:** The way we talk about concepts/ideas can create misconceptions for students. For example: Students are shown the number 68 and asked, "How many ones are in this number?" Generally the expected response is "8" but in fact, there are sixty-eight ones in 68. Rephrasing the question to ask how many extra ones/remaining ones (the number left over after grouping in tens) or how many ones are in the ones/unit place in this number may help prevent misconceptions.

## MATHEMATICAL LANGUAGE

---

place value  
expanded form/notation  
hundreds  
tens  
ones  
units

## LEARNING EXPERIENCES

---



### Assessing Prior Knowledge: Interview

Use the Sharon Ross assessment (see background information) to identify the student's understanding.

**Note:** At the end of Grade 2 students should be at Stage 4: Transition to place value.

---

- **Record in more than one way the number represented by proportional and non-proportional concrete materials.**
- **Represent a number in different ways using proportional and non-proportional concrete materials, and explain how they are equivalent (e.g., 351 can be represented as three 100s, five 10s and one 1, or two 100s, fifteen 10s, and one 1, or three 100s, four 10s, and eleven 1s).**
- **Explain, and show with counters, the meaning of each digit for a 3-digit numeral with all digits the same (e.g., for the numeral 222, the first digit represents two hundreds [two hundred counters], the second digit represents two tens [twenty counters], and the third digit represents two ones [two counters]).**

### Suggestions for Instruction

- **Toothpick 1000:** Use toothpicks, glue and cardstock/manila tag to make a visual representation of 1000.

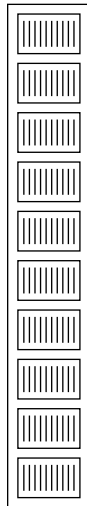


**Directions:**

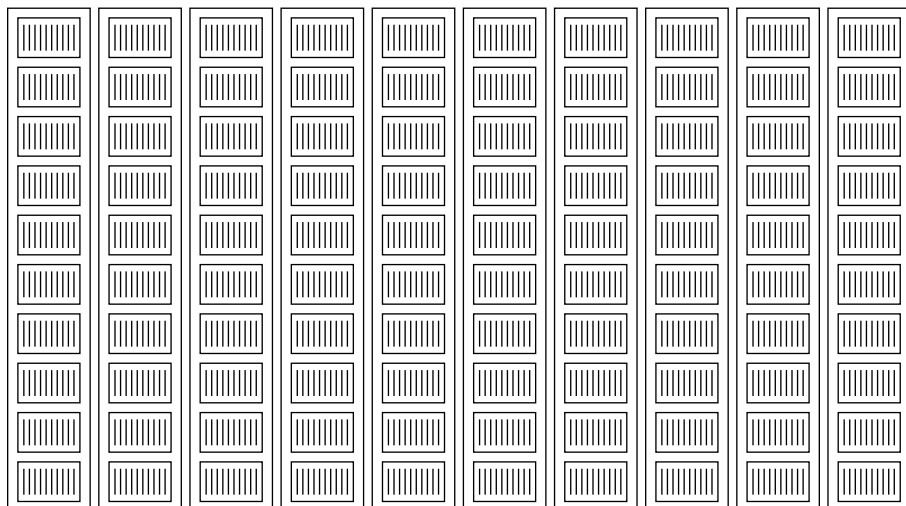
1. Cut 100 cards (don't use white paper) large enough to hold 10 toothpicks with small spaces between them. Have students glue 10 toothpicks on each card. It is important the toothpicks are oriented the same way on the cards (e.g., the "hotdog" way).



2. Cut 10 strips of cardstock/manilla tag (in a contrasting colour) wide enough to fit the toothpick cards leaving a small space on either side and long enough to hold 10 toothpick cards with a small space between each one. Staple the toothpick cards on each strip to represent a hundred.



3. Put the 10 hundred strips together (with a small space between each one) to represent a thousand.



Students are now able to identify a particular toothpick by counting the hundred strips, the ten cards, and the individual toothpicks on the cards.

Students can play “Guess My Number” or “Find the Number” using this representation.

Examples:

■ **Guess My Number:**

1. “I am thinking of a number between 325 and 650. Guess my number.”
2. Students ask questions that can be answered with a “yes” or a “no”:
  - Does the number have 5 hundreds?
  - Is the number greater than 500?
  - Does the number have 6 tens?

And so on.

3. Count the number of questions it takes to find the number.
4. Each time they play, students try to decrease the number of questions needed.

■ **Find the Number:**

- Find the number that has 3 hundreds, 6 tens, and 2 ones.
- Find the number that is 100 less than 976.
- Find the number that has 4 hundreds, 15 tens, and 9 ones.

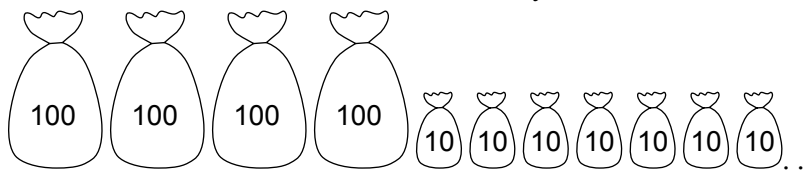
And so on.

- Use bagged or bundled manipulatives (e.g., beans, straws) to represent numbers concretely. Have students record the number in more than one way.

Example:

What number is shown in the picture below?

Record the number in three different ways.



Possible answers:

- four hundred seventy-three
- 4 hundreds, 7 tens, 3 ones
- $100 + 100 + 100 + 100 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 1 + 1 + 1$
- $400 + 70 + 3$

- It is very important that students have experience using zero as a place holder.

Have students trade hundreds, tens, and ones in order to represent a number in a variety of ways.

Examples:

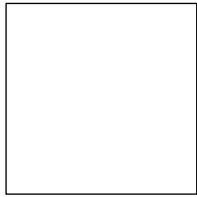


$$473 = 3 \text{ hundreds} + 17 \text{ tens} + 3 \text{ ones}$$

$$473 = 4 \text{ hundreds} + 6 \text{ tens} + 13 \text{ ones}$$

$$209 = 2 \text{ hundreds} + 0 \text{ tens} + 9 \text{ ones}$$

$$209 = 1 \text{ hundred} + 10 \text{ tens} + 9 \text{ ones}$$

Students might find that using a place-value grid helps them to organize their thinking and to begin to see patterns in their answers. For example:

Hundreds	Tens	Ones
		
4	7	3
3	17	3
2	27	3
1	37	3
4	6	13
4	5	23
4	4	33

**BLM**  
**3.N.5.1**

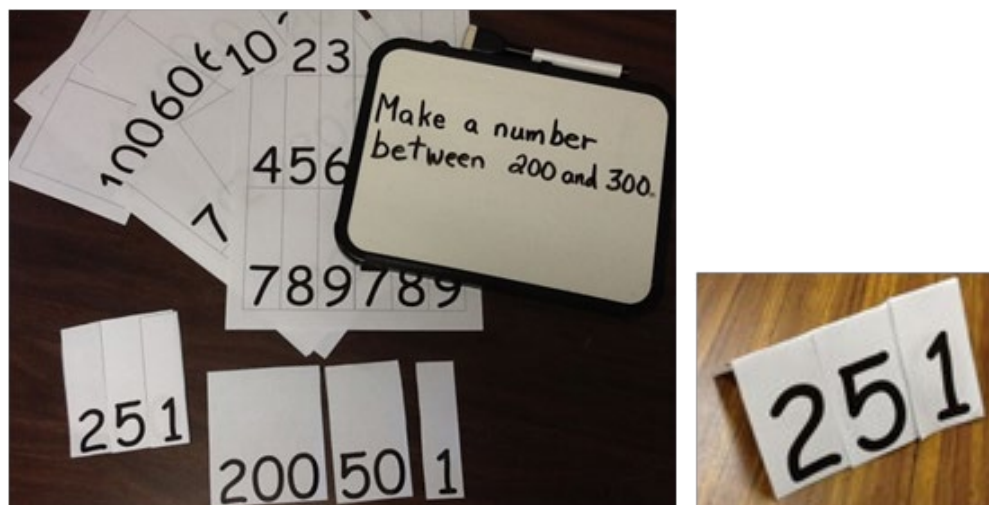
- Place Value Tents or Arrows Cards:** Place value tent/arrow cards help students to see the relationship between a digit and its value based on its position in the number. Using the cards, illustrate how numbers are built. The cards are made so that the tens card is twice as long as the ones cards, and the hundreds card is three times as long as the ones cards.

Have students use place value tent/arrow cards to make numerals.

Examples:

- Make a three-digit number between 350 and 500.
- Make an even three-digit number that is greater than 400 but less than 475.
- Make a three-digit number that is 100 more than 652.
- Make a three-digit number that has is 10 more than 492.
- Make a three-digit number in which the ones digit is greater than the tens digit.

Starting with the hundreds card, layer the others on top, right aligned. This approach will show how the number is built, while allowing students to see the individual components of the number.



- Create and share number riddles using the place value tent or arrow cards.

Example:

I am thinking of a number less than 300. The ones digit is greater than the tens digit. The sum of the digits is 12. What could the number be?

- **What's the Value?** Give students a three-digit number with all three digits the same. Ask them to explain what each of the digits represents using both manipulatives and words.

- **Calculator Challenge:** Have students enter a three-digit number with three identical digits (e.g., 555).

Ask the following:

- Can you change the number so that it has a zero in the ten's place? What did you enter? (- 50)
- Can you change the number so that it has a zero in the hundred's place? What did you enter? (- 500)
- Can you change the number so that it has a zero in the one's place? What did you enter? (- 5)

This activity can be used for numbers without identical digits.

- **Classroom Routine: “Stretch It!–Place Value”** Have students work in partners. Give them a two- or three-digit number or have them select one. Using a strip of chart paper, students represent the number (using place value) in as many different ways as they can.

Example:

<b>58</b>
5 tens, 8 ones
4 tens, 18 ones
3 tens, 28 ones
2 tens, 38 ones
1 ten, 48 ones
58 ones



---

### Assessing Understanding: Interview

Give the student a three-digit number such as 264. Have them explain the meaning of each digit using materials to support their explanation.

The student is able to

- use materials to represent a three-digit number
- explain that the first digit represents 2 hundreds (two hundred counters)
- explain that the second digit represents 6 tens (sixty counters)
- explain that the third digit represents 4 ones (four counters)

### Paper-and-Pencil Task

1. Roll a die (0 to 9) three times. Record the numbers. (If any of the numbers are the same roll the dice again.) Make as many three-digit numbers as you can. Order the numbers from greatest to least.
  2. Choose one of the numbers you made. Explain the value of each digit. Use pictures and words.
  3. Choose another number. Represent it in at least six different ways, using what you know about place value.
-

## PUTTING THE PIECES TOGETHER: PERFORMANCE TASK

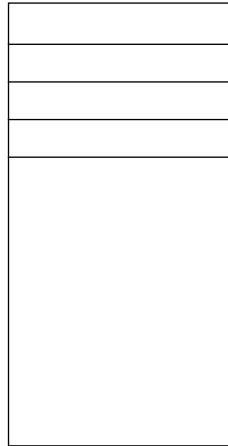


### Making a Step Book

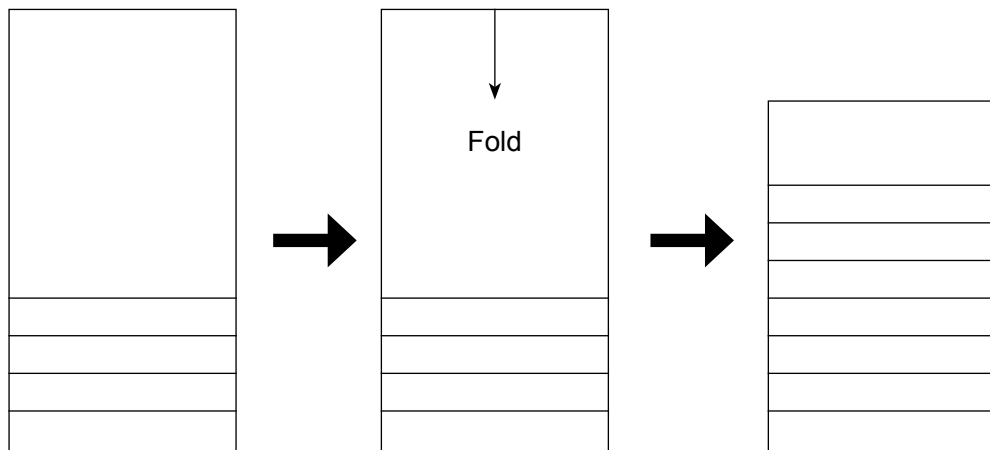
Tell students that they are going to make their own Step Book for representing a number.

Directions:

1. Take four pieces of letter-sized paper. Stagger the sheets by placing each sheet of paper about 2.5 cm (1 inch) lower than the one before. You will have four steps.

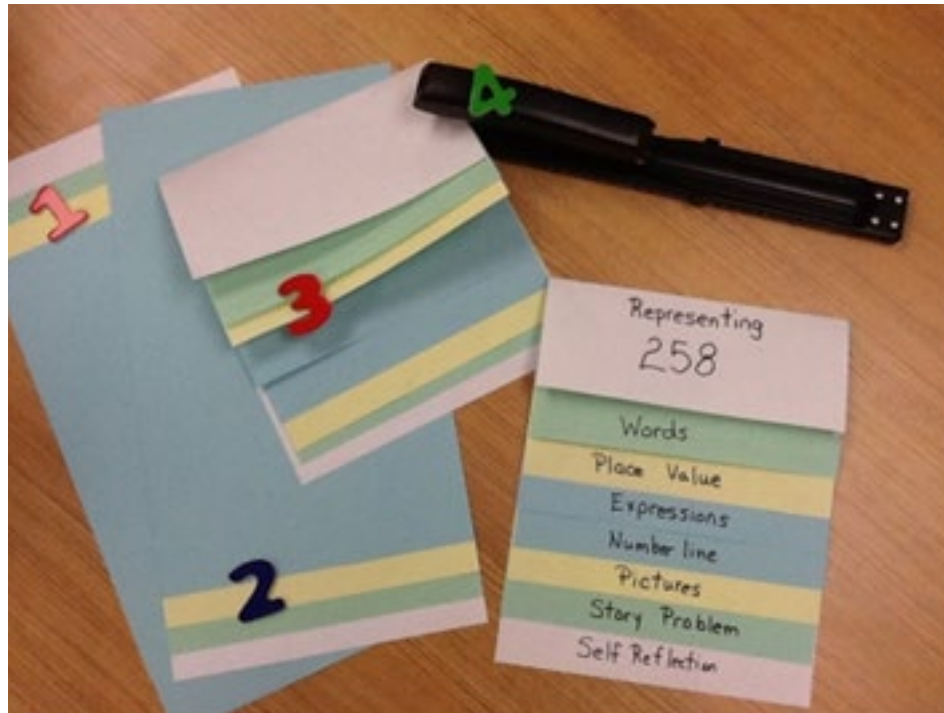


2. Hold the papers together. Turn them over so that the overlapping sections are on the bottom. Fold the top over so that you now have eight steps. Check to see if the steps are even and adjust as necessary. Open the top four steps and put two staples along the crease.

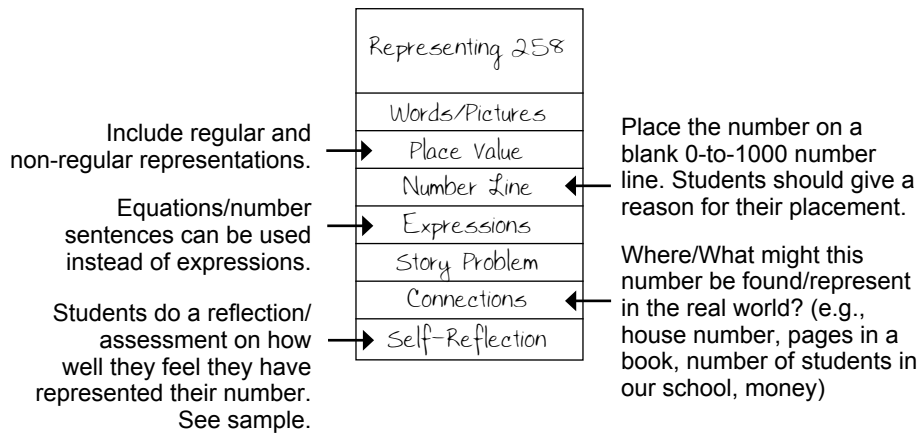


3. Brainstorm the different ways you/the students want to represent their number. Write one method on each of the second through seventh flaps. The last flap can be used for self-reflection/assessment.

- Assign or have students select a three-digit number to represent.



Example:



Sample Reflection/Assessment (done on a separate sheet and glued/taped in):

I represented my number using

- words
- pictures
- place value-regular and non-regular
- number line
- expressions
  - addition
  - subtraction
- a story problem
- connections

Next time I will

---

---

**Note:** Differentiate the assignment by

- having students select their own number
- assigning numbers, varying the number size and range based on student needs
- having students pick the ways to represent their number
- having students work with a partner



## Grade 3: Number (3.N.6, 3.N.7, 3.N.8, 3.N.9, 3.N.10)

### Enduring Understandings:

Quantities can be taken apart and put together.

Addition and subtraction are inverse operations.

### Essential Question:

How can symbols be used to represent quantities, operations, or relationships?

How can strategies be used to compare and combine numbers?

What questions can be answered using subtraction and/or addition?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
<p><b>3.N.6</b> Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as</p> <ul style="list-style-type: none"><li>■ adding from left to right</li><li>■ taking one addend to the nearest multiple of ten and then compensating</li><li>■ using doubles</li></ul> <p>[C, ME, PS, R, V]</p>	<ul style="list-style-type: none"><li>→ Add two 2-digit numerals using a mental mathematics strategy, and explain or model the strategy.</li><li>→ Explain how to use the “adding from left to right” strategy (e.g., to determine the sum of <math>23 + 46</math>, think <math>20 + 40</math> and <math>3 + 6</math>).</li><li>→ Explain how to use the “taking one addend to the nearest multiple of ten” strategy (e.g., to determine the sum of <math>28 + 47</math>, think <math>30 + 47 - 2</math> or <math>50 + 28 - 3</math>).</li><li>→ Explain how to use the “using doubles” strategy (e.g., to determine the sum of <math>24 + 26</math>, think <math>25 + 25</math>; to determine the sum of <math>25 + 26</math>, think <math>25 + 25 + 1</math> or doubles plus 1).</li><li>→ Apply a mental mathematics strategy for adding two 2-digit numerals.</li></ul>

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
<p><b>3.N.7</b> Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as</p> <ul style="list-style-type: none"> <li>■ taking the subtrahend to the nearest multiple of ten and then compensating</li> <li>■ thinking of addition</li> <li>■ using doubles</li> </ul> <p>[C, ME, PS, R, V]</p>	<ul style="list-style-type: none"> <li>→ Subtract two 2-digit numerals using a mental mathematics strategy, and explain or model the strategy.</li> <li>→ Explain how to use the “taking the subtrahend to the nearest multiple of ten and then compensating” strategy (e.g., to determine the difference of <math>48 - 19</math>, think <math>48 - 20 + 1</math>).</li> <li>→ Explain how to use the “thinking of addition” strategy (e.g., to determine the difference of <math>62 - 45</math>, think <math>45 + 5</math>, then <math>50 + 12</math> and then <math>5 + 12</math>).</li> <li>→ Explain how to use the “using doubles” strategy (e.g., to determine the difference of <math>24 - 12</math>, think <math>12 + 12</math>).</li> <li>→ Apply a mental mathematics strategy for subtracting two 2-digit numerals.</li> </ul>
<p><b>3.N.8</b> Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem-solving context.</p> <p>[C, ME, PS, R]</p>	<ul style="list-style-type: none"> <li>→ Estimate the solution for a story problem involving the sum of two 2-digit numerals (e.g., to estimate the sum of <math>43 + 56</math>, use <math>40 + 50</math>; the sum is close to 90).</li> <li>→ Estimate the solution for a story problem involving the difference of two 2-digit numerals (e.g., to estimate the difference of <math>56 - 23</math>, use <math>50 - 20</math>; the difference is close to 30).</li> </ul>
<p><b>3.N.9</b> Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1-, 2-, and 3-digit numerals) by</p> <ul style="list-style-type: none"> <li>■ using personal strategies for adding and subtracting with and without the support of manipulatives</li> <li>■ creating and solving problems in contexts that involve addition and subtraction of numbers concretely, pictorially, and symbolically.</li> </ul> <p>[C, CN, ME, PS, R]</p>	<ul style="list-style-type: none"> <li>→ Model the addition of two or more numbers using concrete or visual representations, and record the process symbolically.</li> <li>→ Model the subtraction of two numbers using concrete or visual representations, and record the process symbolically.</li> <li>→ Create an addition or subtraction story problem for a solution.</li> <li>→ Determine the sum of two numbers using a personal strategy (e.g., for <math>326 + 48</math>, record <math>300 + 60 + 14</math>).</li> <li>→ Determine the difference of two numbers using a personal strategy (e.g., for <math>127 - 38</math>, record <math>38 + 2 + 80 + 7</math> or <math>127 - 20 - 10 - 8</math>).</li> <li>→ Solve a problem involving the sum or difference of two numbers.</li> </ul>

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
<p>3.N.10 Apply mental math strategies to determine addition facts and related subtraction facts to 18 (9 + 9). [C, CN, ME, R, V]</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Recall of addition and related subtraction facts to 18 is expected by the end of Grade 3.</p> </div>	<ul style="list-style-type: none"> <li>→ Describe a mental mathematics strategy that could be used to determine a given basic fact, such as <ul style="list-style-type: none"> <li>■ doubles (e.g., for 6 + 8, think 7 + 7)</li> <li>■ doubles plus one (e.g., for 6 + 7, think 6 + 6 + 1)</li> <li>■ doubles take away one (e.g., for 6 + 7, think 7 + 7 - 1)</li> <li>■ doubles plus two (e.g., for 6 + 8, think 6 + 6 + 2)</li> <li>■ doubles take away two (e.g., for 6 + 8, think 8 + 8 - 2)</li> <li>■ making 10 (e.g., for 6 + 8, think 6 + 4 + 4 or 8 + 2 + 4)</li> <li>■ commutative property (e.g., for 3 + 9, think 9 + 3)</li> <li>■ addition to subtraction (e.g., for 13 - 7, think 7 + □ = 13).</li> </ul> </li> <li>→ Provide a rule for determining answers for adding and subtracting zero.</li> </ul>

## PRIOR KNOWLEDGE

Students may have an understanding of addition and subtraction (limited to one- and two-digit numerals) to 100 using personal strategies.

They may be able to apply the following mental math strategies:

- making ten
- using doubles
- using one more, one less
- using two more, two less
- building on a known double
- using addition for subtraction

## BACKGROUND INFORMATION

---

One goal of mathematics education is to have students calculate using number sense, which in part means having students learn flexible and accurate methods of computation. We do not want students to rely on one single method of computing but rather to use methods that are efficient depending on the context and the numbers used. Mental math and estimation is one of the math processes that needs to be incorporated throughout the year.

As students progress in their understanding of operations, they continue to build their understanding of place value concepts. Place value understanding is a vital concept that students must possess as they learn multi-digit computations.

Students will develop, apply, and describe strategies to add and subtract. The strategies listed in the learning outcomes should be explored with students. The strategies listed in the learning outcomes are not an exclusive list, however. Students may have their own strategies they use on a regular basis. Students need to be given regular opportunities to ensure that they have ample practice and application with mental math strategies. Instruction needs to focus on a balanced approach that provides students with opportunities to develop their conceptual understanding, procedural thinking, and problem solving that mutually reinforce student learning.

In order for students to be computationally fluent with basic facts, they must exhibit efficiency, accuracy, and flexibility with numbers. Students must be able to develop quick and accurate recall of the basic facts, but this skill is most effective only after achieving conceptual understanding. Recall is a developmental process that improves computational fluency by developing efficiency, accuracy, and flexibility with numbers. It is more than just producing a quick answer to a computation.

Educational math resources define quick recall of math facts as the ability to solve a basic number computation in 3 seconds or less (Van de Walle, Karp, Lovin, and Bay-Williams). However, math educators have found that a premature demand for a quick performance could cause anxiety and undermine understanding for some students. The 3-second recall is an assessment guideline for teachers and does not need to be shared with students because it may cause undue anxiety or may also interfere with students' mathematical thinking.

The recall of basic math facts develops through procedural fluency and proficiency in the other strands of mathematics and leads to higher-order math skills. Recall consists of and is taught through

- number sense
- strategy development
- reasoning skills
- effective practice (practice with meaning)

When teaching for understanding, we require students to learn certain prerequisite skills and knowledge to achieve recall. Students develop their number sense over a period of time. Final recall of math facts is a consolidation of understanding and purposeful practice developed starting in Kindergarten.

**Addend:** One of the numbers in a designated sum of two or more numbers (e.g., in  $3 + 5 + 1 = 9$ ; 3, 5, and 1 are addends).

**Minuend:** In a subtraction problem, the number from which another number is to be subtracted (e.g., in  $5 - 3 = 2$ , the 5 is the minuend).

**Subtrahend:** In subtraction, the number being subtracted from a given number (e.g., in  $5 - 2 = 3$ , 2 is the subtrahend).

**Note:** Addition and subtraction should be taught together. This will enable students to see the relationships between the two operations.

There are many different types of addition and subtraction problems. Students should have experience with all types.

Addition				Both + and -
Result Unknown ( $a + b = ?$ )	Change Unknown ( $a + ? = c$ )	Start Unknown ( $? + b = c$ )	Combine ( $a + b = ?$ )	Compare
Pat has 8 marbles. Her brother gives her 4. How many does she have now? ( $8 + 4 = ?$ )	Pat has 8 marbles but she would like to have 12. How many more does she need to get? ( $8 + ? = 12$ )	Pat has some marbles. Her brother gave her 4 and now she has 12. How many did she have to start with? ( $? + 4 = 12$ )	Pat has 8 blue marbles and 4 green marbles. How many does she have in all? ( $8 + 4 = ?$ )	Pat has 8 blue marbles and 4 green marbles. How many more blue marbles does she have? ( $8 - 4 = ?$ or $4 + ? = 8$ )
Subtraction				Pat has 8 blue marbles and some green marbles. She has 4 more blue marbles than green ones. How many green marbles does she have? ( $8 - 4 = ?$ or $4 + ? = 8$ )
Result Unknown ( $a - b = ?$ )	Change Unknown ( $a - ? = c$ )	Start Unknown ( $? - b = c$ )	Combine	
Pat has 12 marbles. She gives her brother 4 of them. How many does she have left? ( $12 - 4 = ?$ )	Pat has 12 marbles. She gives her brother some. Now she has 8. How many marbles did she give to her brother? ( $12 - ? = 8$ )	Pat has some marbles. She gives her brother 4 of them. Now she has 8. How many marbles did she have to start with? ( $? - 4 = 8$ )	Pat has 12 marbles. 8 are blue and the rest are green. How many are green? ( $12 - 8 = ?$ )	

## MATHEMATICAL LANGUAGE

---

Operations:

addition

take away

add

compatible number pairs

sum

story problem

total

number sentence

more

multiple

subtraction

estimate

subtract

addition fact

difference

subtraction fact

less

strategy

## LEARNING EXPERIENCES

---



### Assessing Prior Knowledge: Paper-and-Pencil Task or Interview

Give students the following questions:

$34 + 42$

$28 + 57$

$89 - 65$

$73 - 18$

Solve the problems. Be sure to show your thinking.

The student is able to use

- expanded form or adding from left to right [ $34 + 42 \longrightarrow (30 + 40) + (4 + 2)$ ]
  - compensating [ $28 + 57 \longrightarrow (28 + 2) + 55$ ]
  - number line
  - other \_\_\_\_\_
-

- **Add two 2-digit numerals using a mental mathematics strategy, and explain or model the strategy.**
- **Explain how to use the “adding from left to right” strategy (e.g., to determine the sum of  $23 + 46$ , think  $20 + 40$  and  $3 + 6$ ).**

In the “adding from left to right” strategy, each number is mentally broken into its place value components (expanded notation) and then they are added, starting with the largest place value.

Examples:

$$43 + 26 =$$

$$40 + 20 = 60$$

$$3 + 6 = 9$$

$$60 + 9 = 69$$

$$38 + 15 =$$

$$30 + 10 = 40$$

$$8 + 5 = 13$$

$$40 + 13 = 53$$

- **Explain how to use the “taking one addend to the nearest multiple of ten” strategy (e.g., to determine the sum of  $28 + 47$ , think  $30 + 47 - 2$  or  $50 + 28 - 3$ ).**

This strategy uses the idea of making “friendly” or “nice” numbers. Friendly numbers are numbers that are easier to work with (e.g., multiples of ten).

Example:

$$58 + 17$$

58 is close to 60

$$60 + 17 = 77$$

58 + 17 is 2 less

$$77 - 2 = 75$$

- **Explain how to use the “using doubles” strategy (e.g., to determine the sum of  $24 + 26$ , think  $25 + 25$ ; to determine the sum of  $25 + 26$ , think  $25 + 25 + 1$  or doubles plus 1).**

In this strategy students use a double fact they know or can determine easily to help them find the total.

Example:

$$43 + 40$$

$$40 + 40 = 80$$

43 is 3 more so  $43 + 40 = 83$

- **Apply a mental mathematics strategy for adding two 2-digit numerals.**

### Suggestions for Instruction

- **Classroom Routine—“What’s My Strategy?”**

Prepare cards with expressions adding two 2-digit numbers. Have students work in partners. Each student draws a card, determines the strategy he/she will use to solve it, and then finds the answer. When students have a solution, they take turns explaining to their partner the strategy used.

Both students could work with the same expression and solve it separately. Once the problem is solved, students can compare the strategies used.

In both cases, providing an opportunity for students to share their strategies and solutions with the whole class will help all students further develop their “strategies vocabulary.”

BLM  
3.N.6.1

- **Race to 100**

**Materials:**

2 (1 to 6) dice, recording sheet, and pencil

**Directions:**

1. Each player, in turn, rolls the two dice, makes a two-digit number, and records it on their recording sheet.
2. On the second roll, the new two-digit number is added to the first and the total is recorded. The player explains the strategy used.
3. Play continues in this manner until one player gets to 100.

**Note:** Before playing the game, students need to decide if the winner is the first person to go over 100 or if it is the person closest to 100 without going over.

Example:

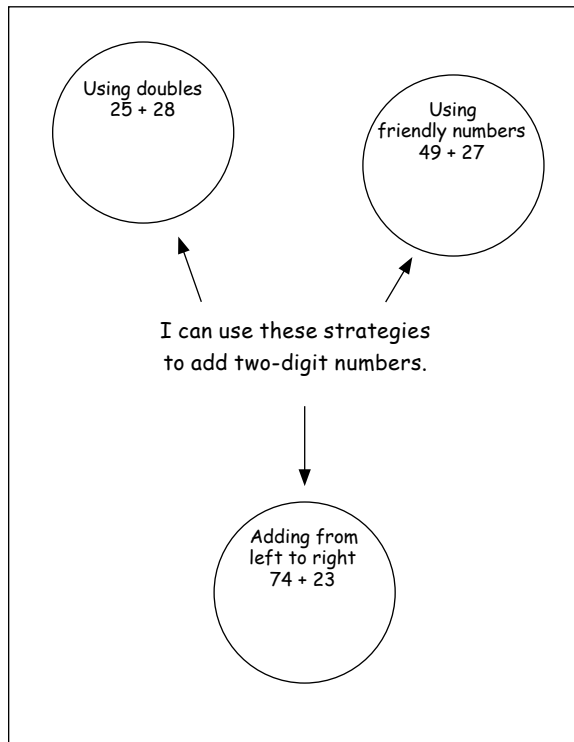
Race to 100	
Roll	Total
0	
25	25
16	41
32	73 and so on





BLM  
3.N.6.2

## Assessing Understanding: Paper-and-Pencil Task/Portfolio Entry



- Subtract two 2-digit numerals using a mental mathematics strategy, and explain or model the strategy used.
- Explain how to use the “taking the subtrahend to the nearest multiple of ten and then compensating” strategy (e.g., to determine the difference of  $48 - 19$ , think  $48 - 20 + 1$ ).

This strategy uses the idea of making “friendly” or “nice” numbers. Friendly numbers are numbers that are easier to work with (in this case, multiples of ten).

Example:

$$74 - 28$$

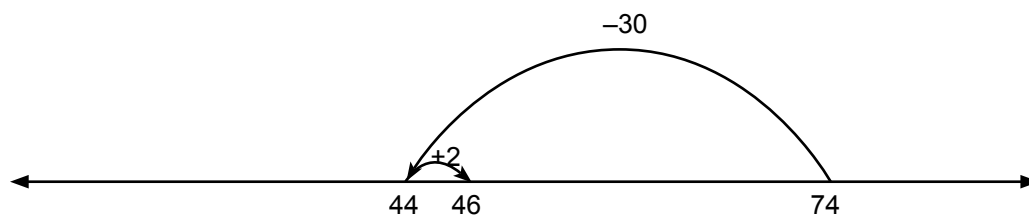
28 is close to/two away from 30

$$\text{Think } 74 - 30 + 2$$

$$74 - 30 = 44$$

$$44 + 2 = 46$$

Using a number line:



- Explain how to use the “thinking of addition” strategy (e.g., to determine the difference of  $62 - 45$ , think  $45 + 5$ , then  $50 + 12$  and then  $5 + 12$ ).

“Think addition” is an efficient strategy for subtraction.

Example:

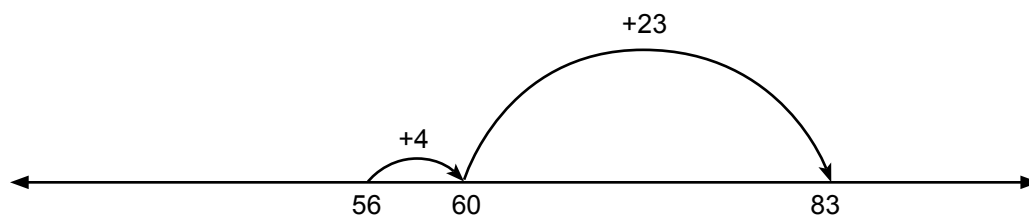
$83 - 56$  (think “56 and how many more to make 83”)

$56 + 4 = 60$  (think “60 and 23 more to make 83”)

$60 + 23 = 83$

Therefore, the difference between 83 and 56 is  $4 + 23$  or 27.

Using a number line:



- Explain how to use the “using doubles” strategy (e.g., to determine the difference of  $24 - 12$ , think  $12 + 12$ ).

In this strategy students use a double fact they know or can determine easily to help them find the difference.

Example:

$28 - 14$ , think “ $14 + 14$  is 28, so 28 minus 14 is 14”

- **Apply a mental mathematics strategy for subtracting two 2-digit numerals.**

### Suggestions for Instruction

- **Classroom Routine—“What’s My Strategy?”**

Prepare cards with expressions subtracting two 2-digit numbers. Have students work in partners. Each student draws a card, determines the strategy he/she will use to solve it, and then finds the answer. When students have a solution they take turns explaining the strategy used to their partner.

Both students could work with the same expression and solve it separately. Once solved, students can compare the strategies used.

In both cases, providing an opportunity for students to share their strategies and solutions with the whole class will help all students further develop their “strategies vocabulary.”

- **Race to Zero**

**Materials:**

2 (1 to 6) dice, recording sheet, and pencil

**Directions:**

1. Each player, in turn, rolls the two dice, makes a two-digit number, and records it on their recording sheet. This number is subtracted from 100.
2. On the second roll the new two-digit number is subtracted from the difference and is recorded. Each time the player should explain the strategy used.
3. Play continues in this manner for four rolls. The player with a difference closest to zero is the winner.

Example:

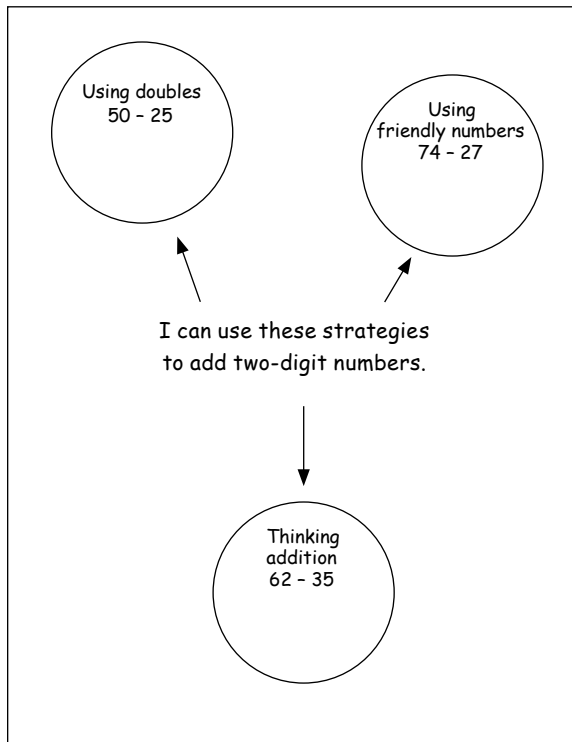
Race to Zero	
Roll	Difference
Start 100	
25	75
16	59
32	27

BLM  
3.N.7.1



BLM  
3.N.7.2

## Assessing Understanding: Paper-and-Pencil Task/Portfolio Entry



- **Estimate the solution for a story problem involving the sum of two 2-digit numerals (e.g., to estimate the sum of  $43 + 56$ , use  $40 + 50$ ; the sum is close to 90).**
- **Estimate the solution for a story problem involving the difference of two 2-digit numerals (e.g., to estimate the difference of  $56 - 23$ , use  $50 - 20$ ; the difference is close to 30).**

### Suggestions for Instruction

**Note:** The rules for rounding numbers to the nearest ten are not intended to be taught. Students should be given the freedom to determine the multiples for the estimation.

- Provide multiple opportunities for students to estimate the solutions to a variety of addition and subtraction story problems. Have them explain the strategy used.



## Assessing Understanding: Paper-and-Pencil Task/Journal Entry

- Meg estimated that  $86 - 38$  would be about 50. What strategy might she have used for her estimate?
- Sam had \$1.10. He wants to buy a pencil for 17¢, a notebook for 43¢, and a candy bar for 49¢. He estimates that he has enough money to buy all three items. Is he correct? Explain your thinking.

- **Model the addition/subtraction of two or more numbers using concrete or visual representations, and record the process symbolically.**
- **Determine the sum of two numbers using a personal strategy (e.g., for  $326 + 48$ , record  $300 + 60 + 14$ ).**
- **Determine the difference of two numbers using a personal strategy (e.g., for  $127 - 38$ , record  $38 + 2 + 80 + 7$  or  $127 - 20 - 10 - 8$ ).**

### Suggestions for Instruction

#### Strategies for Addition

$$382 + 26$$

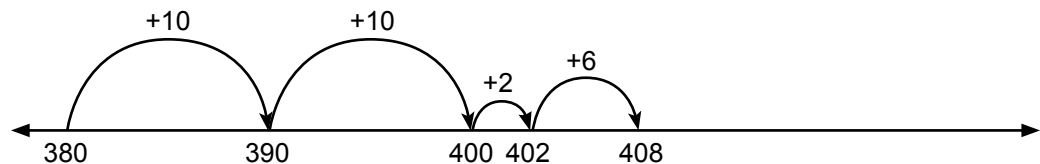
#### Breaking up Numbers (Split Strategy)

This method requires place value understanding.

$$\begin{array}{r} 382 + 26 \\ \swarrow \downarrow \searrow \swarrow \downarrow \searrow \\ 300 \ 80 \ 2 \ 20 \ 6 \\ \swarrow \downarrow \searrow \swarrow \downarrow \searrow \\ 300 + 100 + 8 = 408 \\ (80+20)(2+6) \end{array}$$

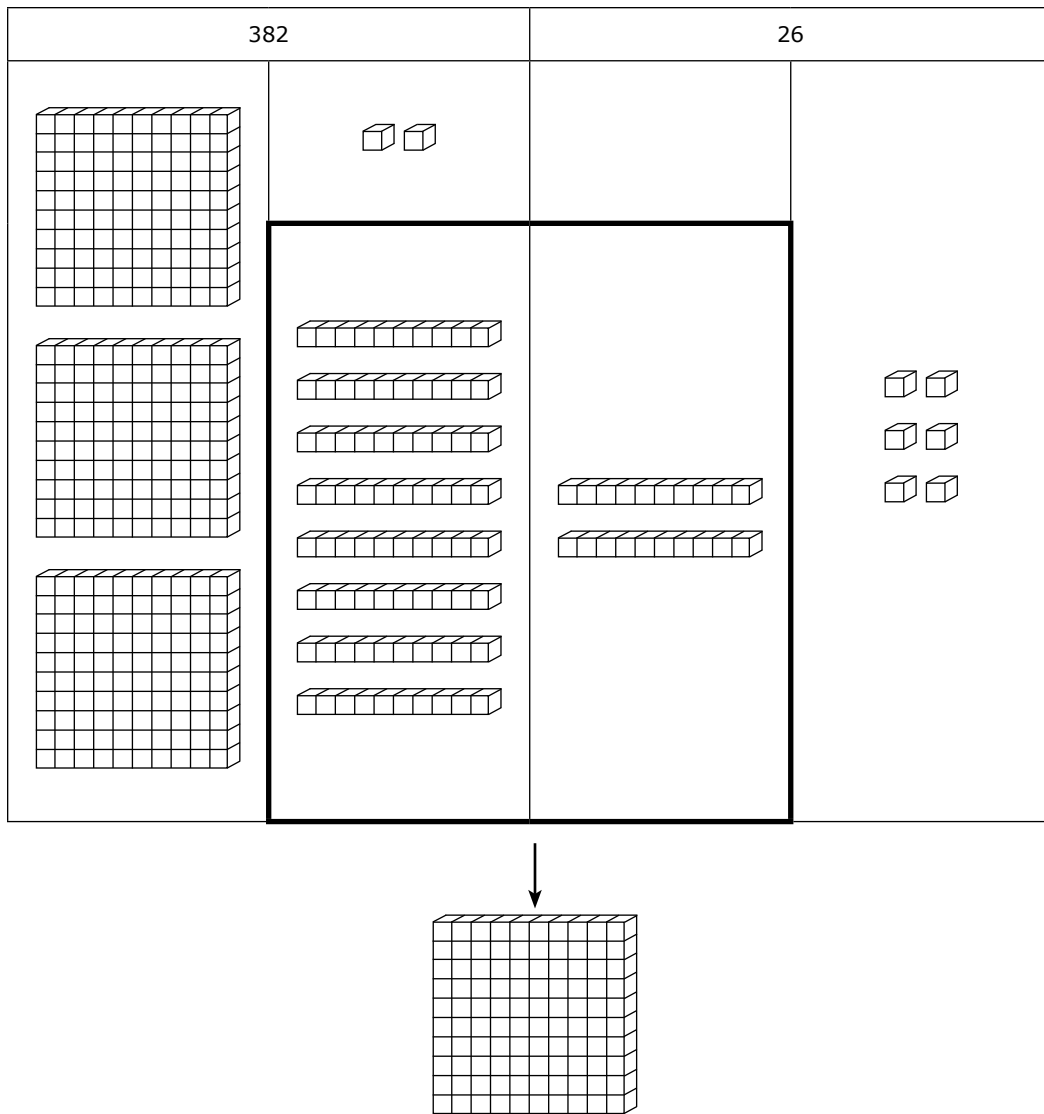
**Note:** As the size of the numbers increases it is more difficult for students to use this method mentally.

#### Empty Number Line (Jump Strategy)



There are many other possibilities.

Use representations of materials such as base-10 blocks.



**Compensating (making “nice” or “friendly” numbers)**

$$\begin{array}{c}
 390 \\
 382 + 26 \longrightarrow (382 + 8) + 18 \longrightarrow 408 \\
 \swarrow \searrow \\
 8 + 18
 \end{array}$$

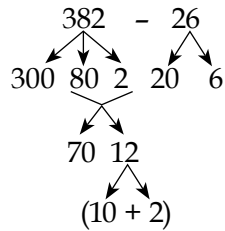
**Note:** Students need to use their knowledge of compatible number pairs for 10 to use this strategy.

## Strategies for Subtraction

$$382 - 26$$

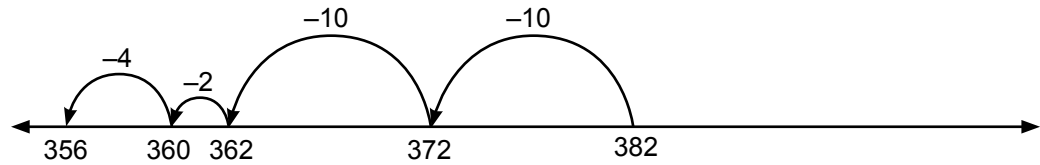
### Breaking up Numbers (Split Strategy)

This method requires place value understanding.



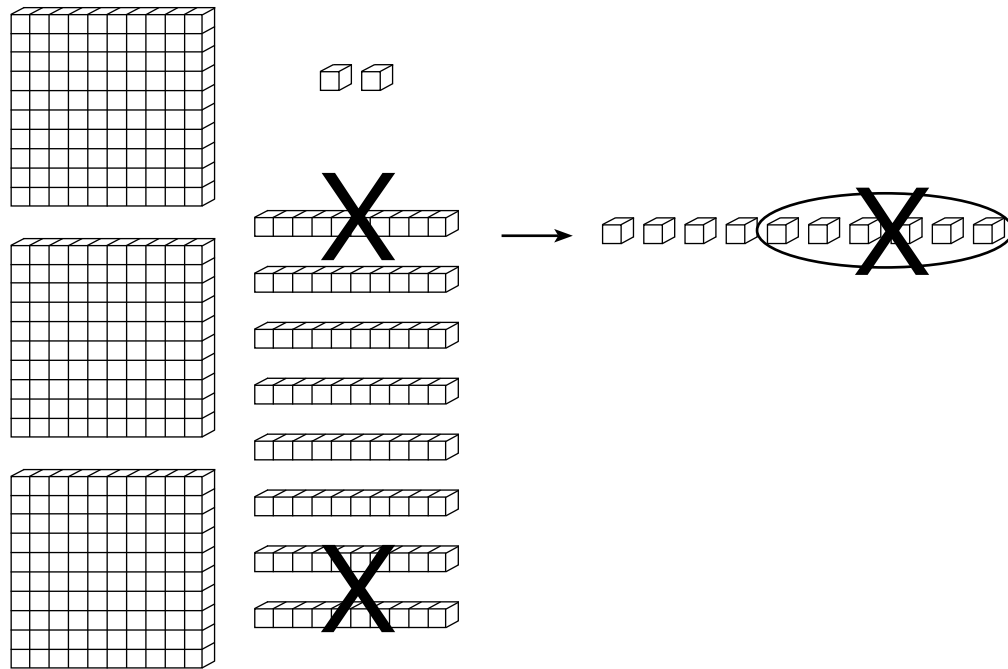
$$300 + (70 - 20) + (12 - 6) = 356$$

### Empty Number Line (Jump Strategy)



There are many other possibilities.

Use representations of materials such as base-10 blocks.



**Compensating (making "nice" or "friendly" numbers)**

Add 4 to both numbers:

$$(382 + 4) - (26 + 4) \longrightarrow 386 - 30 = 356$$

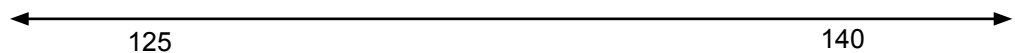


- **Create an addition or subtraction story problem for a solution.**
- **Solve a problem involving the sum or difference of two numbers.**

### Suggestions for Instruction

- **“The Answer Is...” Books:** Make a booklet with 6 to 8 pages. On the front cover write “The answer is 376 [or any other number]. What is the question?” Have students create their own addition and subtraction word problems that would result in an answer of 376 and write them on a page in the booklet.
- Build problem writing into class routines by having two students each day responsible for writing an addition and a subtraction problem that would result an answer equal to a given number. Students could also roll three dice to make their number.
- **What Lies Between?** Present students with a section of a number line.

Example:



Have students find different addition and subtraction number sentences that will result in a solution that falls between the two given numbers.

Example:  $100 + 35 = 135$  (135 is between 125 and 140.)

$$175 - 42 = 133$$

And so on.

Extend the activity by increasing the number size and/or the range.

Modify the activity by decreasing the size of the numbers.

- Present students with a set of data.

Example:

<b>Davidson Elementary School</b>		
<b>Attendance for the Week of January 11 to 15</b>		
Date	Number of Boys (147)	Number of Girls (138)
January 11	145	136
January 12	139	135
January 13	142	134
January 14	137	130
January 15	141	133

Have students use the data to create story problems.

**Extension:** Give students an answer and ask them to decide the question using the data.

Examples:

- The answer is 9. What is the question? (How many students were absent on January 13?)
- The answer is 274. What is the question? (How many students were present on January 15?)



---

### Assessing Understanding: Paper-and-Pencil Task

Solve the problems.

Be sure to show your work.

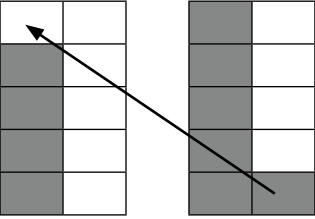
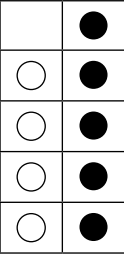
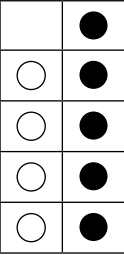
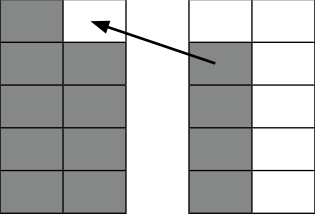
1. The students in Mrs. Johnson's class collected aluminum cans for recycling. Jana collected 124 cans, Mason collected 204 cans, and Marilyn collected 235 cans. How many cans did they collect altogether?
2. The elementary school has 457 students. If 232 of the students are boys, how many girls are in the school?
3. Manjeet has two jars of pennies. One jar has 326 pennies, and the other jar has 387 pennies. How many pennies does Manjeet have altogether?
4. The answer is 245. What is the question? Write an addition problem that has an answer of 245.
5. The answer is 136. What is the question? Write a subtraction problem that has an answer of 136.

- 
- **Describe a mental mathematics strategy that could be used to determine a given basic fact, such as**
    - **doubles (e.g., for  $6 + 8$ , think  $7 + 7$ )**
    - **doubles plus one (e.g., for  $6 + 7$ , think  $6 + 6 + 1$ )**
    - **doubles take away one (e.g., for  $6 + 7$ , think  $7 + 7 - 1$ )**
    - **doubles plus two (e.g., for  $6 + 8$ , think  $6 + 6 + 2$ )**
    - **doubles take away two (e.g., for  $6 + 8$ , think  $8 + 8 - 2$ )**
    - **making 10 (e.g., for  $6 + 8$ , think  $6 + 4 + 4$  or  $8 + 2 + 4$ )**
    - **commutative property (e.g., for  $3 + 9$ , think  $9 + 3$ )**
    - **addition to subtraction (e.g., for  $13 - 7$ , think  $7 + ? = 13$ ).**

## Suggestions for Instruction

### Mental Math

**Note:** The development of mental math strategies is greatly enhanced by sharing and discussion. Students should be given the freedom to adapt, combine, and invent their own strategies. Students should be able to apply these strategies to larger numbers.

Strategy	Teaching Strategies
Using doubles: ■ for $4 + 6$ , think $5 + 5$	Use ten frames to help students visualize the strategies. Example: $4 + 6$  Students can see that moving the one square (counter) to the other ten frame will make the addition easier by adding $5 + 5$ .
Using doubles plus one or two: ■ for $4 + 5$ , think $4 + 4 + 1$ ■ for $4 + 6$ , think $4 + 4 + 2$	Use two-colour counters (beans). Example: $4 + 5$ 
Using doubles take away one or two: ■ for $4 + 5$ , think $5 + 5 - 1$ ■ for $4 + 6$ , think $6 + 6 - 2$	 Students can see that they can either add $4 + 4 + 1$ or $5 + 5 - 1$ .
Making 10: ■ for $7 + 5$ , think $7 + 3 + 2$	Use a double ten frame to help students visualize the strategy. When adding $9 + 4$ students can see that moving one from the 4 to make 10 makes adding easier. This is a practical application of part-part-whole understanding. Eventually students will be able to show the steps without the ten frames. Example:  $\begin{array}{r} 9 + 4 \\ \downarrow \quad \swarrow \downarrow \\ 9 + 1 + 3 \\ \downarrow \quad \downarrow \quad \downarrow \\ 10 + 3 = 13 \end{array}$

Strategy	Teaching Strategies
Using the commutative property: ■ for $3 + 9$ think $9 + 3$	These are sometimes referred to as “turn-around facts.” Reversing the order of the addends can make it easier for students to apply one of the other mental math strategies (count on, make 10, etc.).
Using addition for subtraction: ■ for $7 - 3$ , think $3 + ? = 7$	Note: Thinking addition is an efficient strategy for subtraction. Teaching addition and subtraction at the same time helps students to see this relationship between the operations. Example: for $9 - 5$ think “5 and how many more to make 9?” ( $5 + \square = 9$ )

- Use a set of double nine dominoes. Have students sort them into five groups:
  - Doubles
  - Doubles +/- 1
  - Doubles +/- 2
  - Make 10
  - Other

**■ Provide a rule for determining answers for adding and subtracting zero.**

### Suggestions for Instruction

- Act out problems involving the addition and subtraction of zero.  
Examples:
  - There are 12 candies in the box. Mark looks at them but doesn’t eat any. How many candies are left in the box?
  - Samantha is playing a game of Yahtzee. She rolls the dice and gets three sixes. She rolls again but does not get another six. How many sixes does she have altogether?
 Have students write their own problems to act out.
- Present the class with the following equations (one at a time):
  - $14 - 0 =$
  - $19 + 0 =$
  - $62 + 0 =$
  - $43 - 0 =$
  - $18 + 0 =$
  - $75 - 0 =$

After the equations have been solved, ask students if they notice anything about the start number and the sum/difference in each equation. Is there a pattern?

From student observations, develop a class “rule” for adding and subtracting zero (property of zero).



---

## Assessing Understanding: Checklist

Show students a number sentence. Have individual students explain the strategy used to solve the problem. Record the strategy used on the chart.

**Note:** Some facts will be known by students. Asking them to have to explain a strategy will provide more information about their reasoning strategies and about whether the student understands the fact.

BLM  
3.N.10.1

Student	Doubles	Doubles $\pm$ 1	Doubles $\pm$ 2	Make 10	Commutative Property	Addition for Subtraction	Adding and Subtracting 0	Known Fact	Other

---

- **Recall doubles to 18.**



### Assessing Understanding

Observe students as they play a game involving doubles.

#### Doubles

**Players:** 2

**Materials:**

- one game board for each player
- four dice (or spinners) labelled with the digits 5, 6, 7, 8, 9, 10
- counters for each player

BLM  
3.N.10.2

Doubles			
12	20	18	16
14	18	12	10
10	20	16	14

#### Directions:

Players take turns to

- roll all four dice
- look for doubles on the dice (e.g., 5, 9, 5, 6 gives double 5)
- cover the sum on their board

If a player throws 2 doubles at once, they can use both of them.

If a player rolls 3 of one number, they make only one double.

The first player to fill in a row is the winner.

#### Look for:

The student

- calculates the double mentally
- calculates the double using manipulatives (number line, counters, fingers, etc.)

- Recall compatible number pairs for 5 and 10.



### Assessing Understanding

Use five and ten frames. Flash each frame and have students give the compatible number to make 5 or 10.



### Assessing Understanding

Observe students as they play a game involving compatible numbers.

**Example: Let's Make 5/10**

#### Players:

- 2 to 4 for observation purposes
- whole class for practice

#### Materials:

- one game board for each player (either the "Let's Make 5" board or the "Let's Make 10" board)
- a spinner with the numbers 0 to 5 (0 to 10), or a die labelled 0 to 5 (10-sided die)
- counters for each player

BLM  
3.N.10.3

Let's Make 5				
4	2	0	1	5
3	1	5	2	4
0	3	4	1	2
5	4	2	3	1
1	5	3	4	0

BLM  
3.N.10.4

Let's Make 10					
10	5	7	2	9	4
1	8	3	0	6	5
4	3	6	10	0	7
9	2	1	8	5	3
7	0	5	6	1	9
3	8	2	4	10	6

**Directions:**

1. The teacher spins the spinner or rolls the die and calls out the number shown.
2. Students cover the compatible number to make 5 or 10.
3. The first player to fill in a row, column, or diagonal is the winner.

**Look for:**

The student

- calculates the compatible number mentally within a reasonable length of time
  - calculates the compatible number using manipulatives (number line, counters, fingers, etc.)
-



## Grade 3: Number (3.N.11, 3.N.12)

### Enduring Understandings:

Multiplication and division are inverse operations.

Multiplication is repeated addition.

Division is repeated subtraction.

### Essential Questions:

How can skip counting and arrays be used to demonstrate multiplication and division?

How are addition and multiplication related?

How are subtraction and division related?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
<p><b>3.N.11</b> Demonstrate an understanding of multiplication to <math>5 \times 5</math> by</p> <ul style="list-style-type: none"> <li>■ representing and explaining multiplication using equal grouping and arrays</li> <li>■ creating and solving problems in context that involve multiplication</li> <li>■ modelling multiplication using concrete and visual representations, and recording the process symbolically</li> <li>■ relating multiplication to repeated addition</li> <li>■ relating multiplication to division</li> </ul> <p>[C, CN, PS, R]</p>	<p>(It is intended that students show their understanding of strategies using manipulatives, pictorial representations, and/or patterns when determining products.)</p> <ul style="list-style-type: none"> <li>→ Identify events from experience that can be described as multiplication.</li> <li>→ Represent a story problem (orally, shared reading, written) using manipulatives or diagrams, and record in a number sentence.</li> <li>→ Skip-count by 2s, 3s, 4s, and 5s to determine the answer to a multiplication problem represented as equal groups.</li> <li>→ Represent a multiplication expression as repeated addition.</li> <li>→ Represent a repeated addition as multiplication.</li> <li>→ Create and illustrate a story problem for a number sentence.</li> <li>→ Represent, concretely or pictorially, equal groups for a number sentence.</li> <li>→ Represent a multiplication expression using an array.</li> <li>→ Create an array to model the commutative property of multiplication.</li> <li>→ Relate multiplication to division by using arrays and by writing related number sentences.</li> <li>→ Solve a problem in context involving multiplication.</li> </ul>

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
<p><b>3.N.12</b> Demonstrate an understanding of division by</p> <ul style="list-style-type: none"> <li>■ representing and explaining division using equal sharing and equal grouping</li> <li>■ creating and solving problems in context that involve equal sharing and equal grouping</li> <li>■ modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically</li> <li>■ relating division to repeated subtraction</li> <li>■ relating division to multiplication (limited to division related to multiplication facts up to <math>5 \times 5</math>).</li> </ul> <p>[C, CN, PS, R]</p>	<p>(It is intended that students show their understanding of strategies using manipulatives, pictorial representations, and/or patterns when determining quotients.)</p> <ul style="list-style-type: none"> <li>→ Identify events from experience that can be described as equal sharing.</li> <li>→ Identify events from experience that can be described as equal grouping.</li> <li>→ Illustrate, with counters or a diagram, a story problem involving equal sharing, presented orally or through shared reading, and solve the problem.</li> <li>→ Illustrate, with counters or a diagram, a story problem involving equal grouping, presented orally or through shared reading, and solve the problem.</li> <li>→ Listen to a story problem, represent the numbers using manipulatives or a sketch, and record the problem with a number sentence.</li> <li>→ Create, and illustrate with counters, a story problem for a number sentence.</li> <li>→ Represent a division expression as repeated subtraction.</li> <li>→ Represent a repeated subtraction as a division expression.</li> <li>→ Relate division to multiplication by using arrays and writing related number sentences.</li> <li>→ Solve a problem involving division.</li> </ul>

## PRIOR KNOWLEDGE

---

Students may have

- skip-counted by 2s, 5s, and 10s, forward and backward, using starting points that are multiples of 2, 5, and 10 respectively
- represented a number in a variety of equal groups with and without singles
- grouped a set of counters into equal groups with and without singles in more than one way and explained which grouping makes counting easier
- worked with doubles to 20

## BACKGROUND INFORMATION

---

### Terminology

**Multiplication:** A mathematical operation of combining groups of equal amounts; repeated addition; the inverse of division.

**Product:** The number obtained when two or more factors are multiplied (e.g., in  $6 \times 3 = 18$ , 18 is the product).

**Division:** A mathematical operation involving two numbers that tells how many groups there are or how many are in each group.

**Quotient:** The answer to the division of two numbers (in  $12 \div 3 = 4$ , the quotient is 4).

**Array:** A set of objects or numbers arranged in an order, usually in rows and/or columns.

The Grade 3 focus for multiplication and division is on having students understand the meaning of multiplication and division as well as on understanding the inverse relationship between the two operations.

### Meanings of Multiplication

1. Repeated addition

For example:  $3 + 3 + 3 = 9$

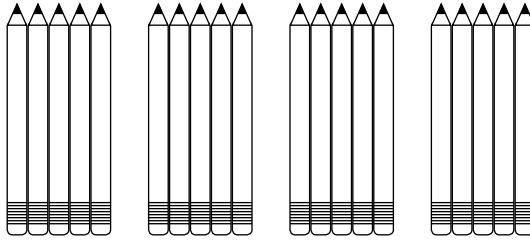


**Note:** Repeated addition is helpful to understand multiplication, but students need to move beyond this strategy as their knowledge develops and other strategies become more efficient.

2. Equal groups or sets

For example:

Pencils come in packages of 5.

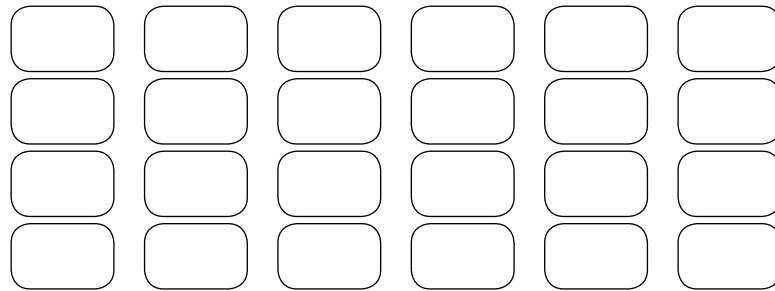


How many pencils are in 4 packages?

3. An array

For example:

A classroom has 4 rows with 6 desks in each row.



How many desks are in the classroom?

### Multiplication Problems

In a multiplication problem, both the number of objects in each group and the number of groups are given. The total number of objects is the unknown.

### Types of Division Problems

1. Partitive or equal sharing division:

In this type of problem, the total number of groups is known. The unknown is the number of items in each group. The quotient tells the amount for each share.

For example:

- Jonas has 24 pieces of candy.

He wants to share them equally among his 4 friends.

How many candies will each friend get?

2. Quotative, equal grouping, or measurement division:

In this type of problem, the number of items in each group is known. The unknown is the number of groups that can be made from the given quantity. The quotient tells how many groups can be made from the given quantity.

For example:

- Martha bought cookies for 10¢ each.  
She spent 90¢.  
How many cookies did she buy?

### Preventing Misconceptions

Common misconceptions students may develop include “multiplication makes bigger” and “division makes smaller.” This is not true when working with fractions or decimals less than one, as students will learn in later years.

For example:

$$8 \times 0.25 = 2 \text{ or } 8 \div 0.25 = 32$$

**Note:** Division should be taught together with multiplication so that students can see the inverse relationship between the two operations.

## MATHEMATICAL LANGUAGE

---

sets of	divide
groups of	division
multiply	equal groups
multiplication	sharing
product	array
quotient	times



### Assessing Prior Knowledge: Individual Interview

#### Skip Counting

Ask the student to

- start at 2 and skip-count by 2s to 24
- start at 5 and skip-count by 5s to 100
- start at 10 and skip-count by 10s to 100
- start at 20 and skip-count backwards by 2s to 8

#### Thinking in Groups

Give the student a collection of 30 objects/counters. Ask them to count the collection by

- 2s
- 5s
- 10s

Does the student arrange the counters into groups as they are counting?

The student is able to

- orally skip-count forward by 2s
  - orally skip-count forward by 5s
  - orally skip-count forward by 10s
  - orally skip-count backward by 2s
  - count a collection by 2s using groups of 2
  - count a collection by 5s using groups of 5
  - count a collection by 10s using groups of 10
-

- **Identify events from experience that can be described as multiplication.**

### Suggestions for Instruction

- **Thinking about Groups:** Have students brainstorm real-life objects that are sold in groups (juice boxes, eggs, crayons, tires on cars, boxes of granola bars, etc.). If possible bring in some actual packages of objects. These objects can be used for problem solving later on.

Create a class poster using pictures of objects that are sold in groups.

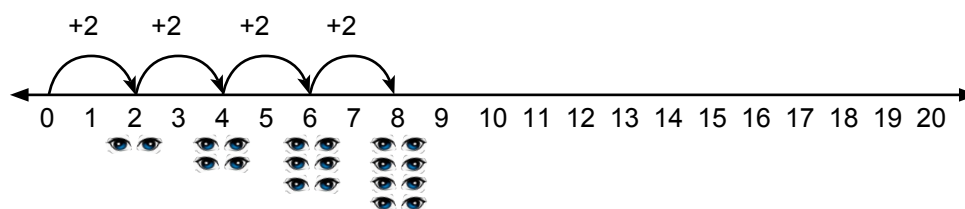
- ***What Comes in 2s, 3s, and 4s?*** by Suzanne Aker, illustrated by Bernie Karlin. This book shows everyday objects that come in groups of 2, 3, or 4. Read the book and make a list/chart of the things that come in 2s, 3s, or 4s. Use these examples to ask questions such as

- If there are 5 children, how many eyes are there?
- There are 4 cars. How many tires are there?

Initially, have students write repeated addition number sentences to solve these problems. After the multiplication symbol has been introduced, go back and rewrite the number sentences as multiplication.

Extend the activity by having students make a poster or write a page for a class book about things that come in different sized groups (e.g., What Comes in 5s or 6s?)

- **Skip Counting/Repeated Addition on a Number Line:** Reinforce the idea of thinking in groups using a number line. For example:





---

## Assessing Understanding

Give students the following problems:

Solve the problems. Be sure to show your work.

- There are four tricycles at the playground. How many wheels are there?
- Nine children are lined up for recess. How many legs are there?
- Marie bakes cookies. Each cookie has 5 chocolate chips on top. How many chocolate chips are on five cookies?

### Look for:

The student is able to solve the problems using

- skip counting
  - repeated addition
  - other (multiplication)
- 

- **Represent a story problem (orally, shared reading, written) using manipulatives or diagrams, and record in a number sentence.**
- **Skip count by 2s, 3s, 4s, and 5s to determine the answer to a multiplication problem represented as equal groups.**
- **Represent a multiplication expression as repeated addition.**
- **Represent a repeated addition as multiplication.**
- **Create and illustrate a story problem for a number sentence.**
- **Represent, concretely or pictorially, equal groups for a number sentence.**

## Suggestions for Instruction

- **Introducing the Concept of Multiplication:** Read the book *Amanda Bean's Amazing Dream: A Mathematical Story* by Cindy Neuschwander. This is the story of a girl who counts “everything and anything” by 1s, 2s, 5s, and 10s. Her teacher tries to convince her that multiplication is a faster way of counting, but it is not until Amanda has a dream that overwhelms her counting skills that she decides multiplication is something she wants to learn.

Have students compare skip counting, adding, and multiplication. Do they all arrive at the same answer? Which one do you think is faster (more efficient)?

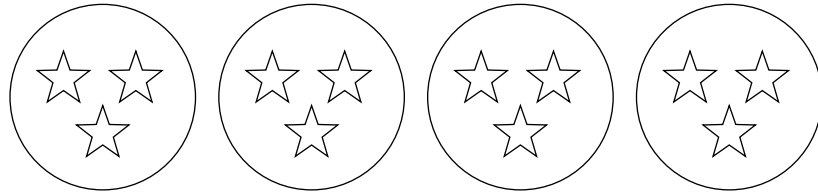


**Note:** Before introducing the symbol for multiplication, it is important that students understand its meaning. Many students will be able to tell you that the “x” is “times” but have no idea what that actually means.

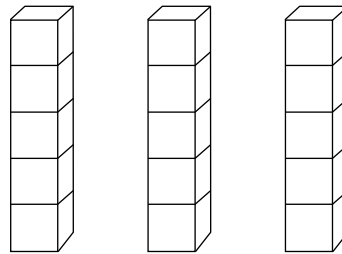
■ **Interpreting a Diagram/Picture or Sets of Manipulatives as Multiplication:**

Show students pictures of groups of equal sets and/or groups of manipulative materials and have them describe what they see. If they do not use the language of groups and sets, model it for them.

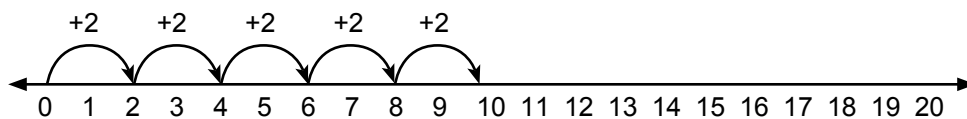
Examples:



- 4 circles with 3 stars in each
- 4 circles with 3 stars per circle
- 4 sets of 3 stars
- 4 groups of 3 stars
- $3 + 3 + 3 + 3$
- 3, 6, 9, 12 stars



- 3 towers with 5 cubes in each
- 3 towers with 5 cubes per tower
- 3 groups of 5 cubes
- 3 sets of 5 cubes
- $5 + 5 + 5$
- 5, 10, 15 cubes



- 5 jumps of 2
- 5 groups of 2
- 5 sets of 2
- $2 + 2 + 2 + 2 + 2 = 10$
- 2, 4, 6, 8, 10



### Assessing Understanding

Give students a story problem, and have them represent it using materials and pictures. Have them describe their representations using groups/sets language.

Example:

Rudy puts his stickers into a sticker book.  
 So far he has filled 3 pages.  
 Each page has 6 stickers.  
 How many stickers does Rudy have in his book?

**Look for:**

- a correct representation using materials
- a correct representation using a picture
- group/set language
  - 3 pages with 6 stickers on each page
  - 3 groups of 6 stickers
  - 3 sets of 6 stickers
  - $6 + 6 + 6$
  - 6, 12, 18 stickers

### Suggestions for Instruction

- **Introducing the Multiplication Symbol:** Explain to students that we have a symbol we use in place of writing out the words “groups of” or “sets of.” For some students it might help if this is represented pictorially. For example:

$$\begin{array}{c} \times \\ 3 \text{ groups of } 6 = 18 \end{array}$$

$$\begin{array}{c} \times \\ 4 \text{ sets of } 2 = 8 \end{array}$$

**Note:** It is important that students read/interpret the multiplication number sentence in the same way. The first number (factor) gives the number of equal groups and the second number (factor) tells the number of items within each group. For example the number sentence  $5 \times 4 = 20$  should be read/interpreted as 5 groups/sets with 4 objects in each group/set.

Have students practise reading and representing multiplication number sentences.

Example:

$$5 \times 2 = 10$$



5 groups of 2 equal 10

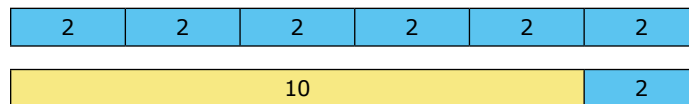
$$2 + 2 + 2 + 2 + 2 = 10$$

### ■ Manipulatives/Materials to Support Students' Understanding

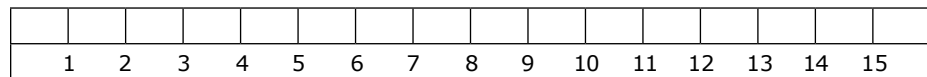
- **Cuisenaire Rods:** For example:  $6 \times 2 = \square$

Students get six 2-cm rods and place them in a row end to end.

Have them find the rod(s) that is (are) the same length as the row containing the six twos.



**Note:** A centimetre ruler/number line could also be used.



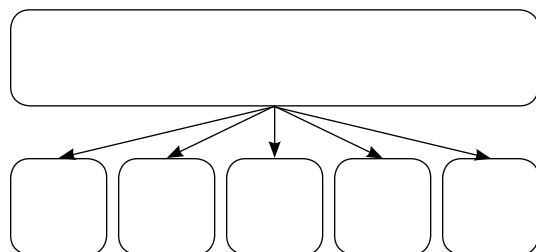
Therefore,  $6 \times 2$  is the same as 12.

- **Egg Cartons:** Egg cartons (or ice cube trays) can be used to make/hold groups of objects.
- **Multiplication/Division Templates:** The templates can be used for both multiplication and division. For multiplication, students use the template that has the number of groups asked for in the question/problem. They make the groups with objects or a dry erase marker and then find the product using repeated addition or skip counting.

For division, students use the template that has the number of groups asked for in the question (divisor). They begin with the whole and then divide it among the groups to determine the quotient.

BLM  
3.N.11.1/12.1

For example: Template for  $5 \times ?$  or  $? \div 5$



- Problem Writing:** Prepare a set of multiplication number sentences using index cards. Have students choose one of the number sentences, and write a story problem to match (on an index card).

The number sentence cards and problem cards can be used as a **Concentration/Matching** game. Put an *N* on the back of the number sentence cards and a *P* on the back of the problem cards.

Turn cards up-side-down in an array. Students pick up a number card and a problem card. If they match, they keep the pair. If they don't match, the cards are returned to the array. Play continues until all cards have been matched. The player with the greatest number of cards is the winner.



BLM  
3.N.11.2

### Assessing Understanding: Performance Task

<b>Multiplication Performance Task</b>
Use the number sentence $4 \times 3 =$ Show different ways to find the answer (product).
<b>Picture</b>
<b>Repeated addition</b>
<b>Number line</b>
<b>Skip counting</b>
<b>Story problem</b>

The student is able to solve/represent the number sentence using

- a picture
- repeated addition
- number line
- skip counting

The student is able to

- create a story problem to match the number sentence

- **Represent a multiplication expression using an array.**
- **Create an array to model the commutative property of multiplication.**
- **Solve a problem in context involving multiplication.**

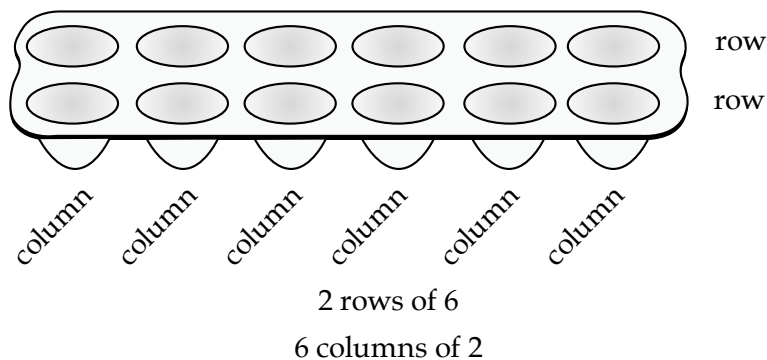
## Suggestions for Instruction

- **Introducing Arrays:** Show students a variety of arrays (pictures or actual objects).

Ask:

- What do you notice about how the arrays are arranged?
- How can we find out how many are in each array (repeated addition, skip counting, and multiplication)?

Explain to students that an array is a way of organizing a set of objects into rows and columns. Use an egg carton (or another array) to help students understand the difference between rows and columns.



Turn the carton on end and show them that now we have 6 rows of 2 and 2 columns of 6.

Have students brainstorm examples of arrays in the environment.

Examples:

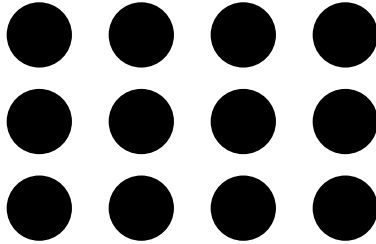
- desks/tables in a classroom
- eggs in an egg carton
- muffins in a muffin tin
- cookies on a cookie sheet
- pictures in a photo album
- cereal boxes in a grocery store
- dots on a ten frame
- windows/window panes
- calendar

If possible have students find actual arrays or pictures of arrays that they can bring in to share with the class.

**Note:** Students could use a digital camera to take pictures of arrays they find in the school or neighbourhood.

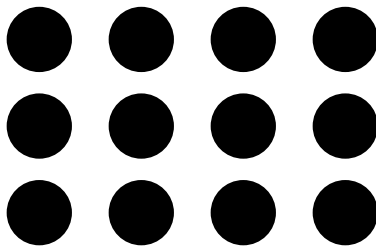
- **Number Sentences and Arrays:** Show students an array and ask them to describe what they see. For example:

There are 3 rows with 4 dots in each row.



Ask: "How can we write what we see as a multiplication number sentence?"

Guide students to see that the multiplication sign can be read as "rows of."



can be written as 3 rows of 4 or  $3 \times 4 = 12$ .

Show additional arrays and have students give a multiplication number sentence for each. If students brought in pictures of arrays, have them write matching multiplication number sentences for them.

Show the class multiplication number sentences, and have groups of students line up to show the matching array.

- **Revisiting *Amanda Bean's Amazing Dream*:** Pages 1 to 9 have examples of arrays in the illustrations. Have students find the arrays. Use sticky notes to add multiplication number sentences to the pages. For example: On the top left-hand side of page 1 there are windows arranged in 2 rows with 3 windows in each ( $2 \times 3 = 6$ ). Each window is divided into 6 rows with 3 panes in each ( $6 \times 3 = 18$ ).

Students can do their own pictures to show arrays.

## ■ Array Game

**Materials:** centimetre grid paper, 2 (0 to 5 or 1 to 6) dice preferably different colours (spinners could be used), markers (a different colour for each player)

**Players:** 2

Establish game **rules:**

- If different coloured dice (spinners) are used, decide which colour will represent the number of rows and which colour will represent the number in each row.
- If the dice are the same colour (or one spinner is used), roll them separately. The first die represents the number of rows, and the second die represents the number in each row.
- Arrays cannot overlap.

**Directions:**

1. The first player rolls the dice, determines the number of rows and the number in each row, and then outlines the matching array on the grid paper. The player writes the corresponding number sentence inside the array.
2. Player 2 takes his/her turn.
3. If the array will not fit on the grid paper, the player loses a turn.
4. Play continues until each player has had 5 turns.

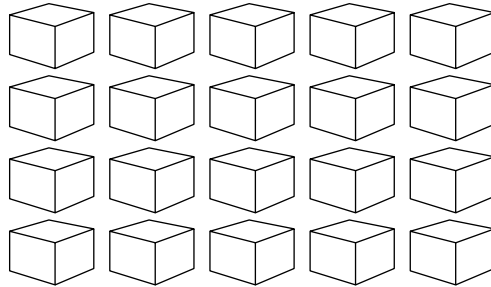
The player with the most arrays on the board is the winner. In the event of a tie, total up the number of squares outlined by each player. The player with the largest number is the winner.



---

## Assessing Understanding: Paper-and-Pencil Task

1. Write a multiplication number sentence for each array.



2. Draw an array for each multiplication number sentence.

$$2 \times 5 = 10$$

$$6 \times 1 = 6$$

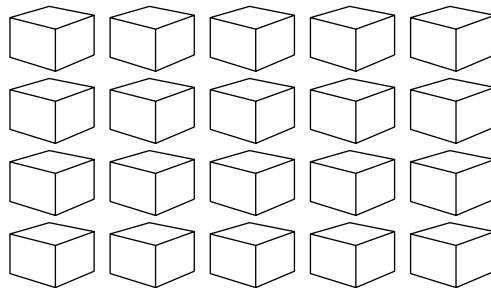
3. Give two examples of arrays in the real world.
- 

### Suggestions for Instruction

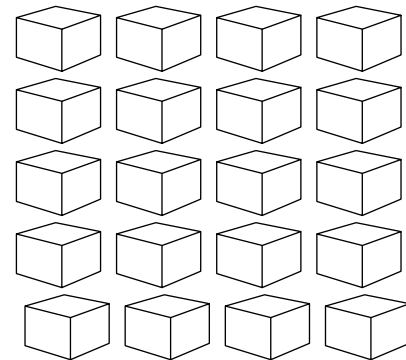
- **Commutative Property of Multiplication:** Show students an array. Have them give the matching multiplication number sentence. Now, turn the array and have students give the matching multiplication number sentence.

Ask:

- Did the number of objects change? Why or why not?



$$4 \times 5 = 20$$



$$5 \times 4 = 20$$

- Do you think that this will be true for all arrays?

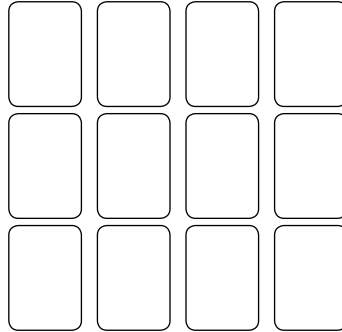
Have students make their own arrays from grid paper to investigate. Have them explain their findings in their math journals or with the class.



- **Relate multiplication to division by using arrays and writing related number sentences.**

### Suggestions for Instruction

- **Arrays and Division:** Show students an array, and explain that this is a page of baseball cards.



Ask:

- How many cards are there altogether?
- What multiplication sentence matches the array?
- Can you use the array to answer these questions?
  - If the cards are shared with 3 people, how many cards does each person get?  
Students should make a connection to the 3 rows with 4 cards in each row.
  - If the cards are shared with 4 people, how many cards does each person get?  
Students should make a connection to the 4 columns (4 rows if the array is rotated) with 3 cards in each.
  - Do you think that this will work for all arrays?

Students work with a partner or small group. Give each group an array. Ask them to write two “sharing” problems that can be answered using their array.

Have groups share their problems with the class. Problems and arrays could be assembled into a class book.



**BLM**  
**3.N.11.3**

---

### Assessing Understanding: Paper-and-Pencil Task

Solve the following problems.

Write a multiplication number sentence and draw an array for each problem.

1. Margo planted 3 rows of flower seeds. She put 5 seeds in each row. How many seeds did Margo plant altogether? Number sentence:	Array
2. There are 4 bags of apples. There are 4 apples in each bag. How many apples are there altogether? Number sentence:	Array
3. Juan invited 7 friends for ice cream cones. Juan and his friends each had 2 scoops of ice cream. How many scoops of ice cream did they have altogether? Number sentence:	Array
4. Pencils come in packages of 6. Mrs. Fast buys 3 packages for her class. How many pencils did Mrs. Fast get? Number sentence:	Array
5. Hans buys a sheet of stickers. On the sheet there are 4 rows with 3 stickers in each row. How many stickers did Hans buy? Number sentence:	Array

---

## PUTTING THE PIECES TOGETHER: PERFORMANCE TASK



Use the book *Each Orange Had 8 Slices: A Counting Book* by Paul Giganti, Jr., illustrated by Donald Crews. Each page or double-page spread has a picture of a group of objects (flowers, tricycles, horses, etc.). The pictures are described and then multiplication questions are asked.

For example: On page 1 the illustration shows 3 flowers with the following description and set of questions:

On my way to the playground  
I saw 3 red flowers  
Each flower had  
pretty petals.  
Each petal had 2 tiny black bugs.

How many red flowers were there?  
How many pretty petals were there?  
How many tiny black bugs were there in all?

Read the book to the students. Go back and explore each page. (This could be done over a period of several days.) Discuss the format of each page.

As a class, write multiplication number sentences and an answer for each question.

### **Task Directions:**

We are going to make a class book like *Each Orange Had 8 Slices: A Counting Book*.

1. Choose an object that can be described using groups.
2. Draw a picture using the object you have chosen.
3. Write a description of your picture. Include 3 numbers in your description.
4. Write three questions that could be solved using multiplication.
5. On the back of your picture, write a multiplication number sentence and an answer for each of your questions.



---

### Assessment Criteria: Student Self-Assessment

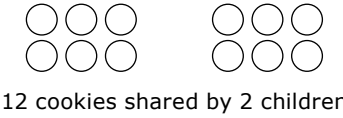
- My picture has objects that can be described using groups.
  - I have included a description of my picture.
  - My description includes 3 numbers.
  - I have included 3 questions for my picture.
  - I have written a multiplication number sentence and an answer for each of my questions on the back of my picture.
- 

- **Identify events from experience that can be described as equal sharing.**
- **Identify events from experience that can be described as equal grouping.**
- **Illustrate, with counters or a diagram, a story problem involving equal sharing, presented orally or through shared reading, and solve the problem.**
- **Illustrate, with counters or a diagram, a story problem involving equal grouping, presented orally or through shared reading, and solve the problem.**
- **Listen to a story problem, represent the numbers using manipulatives or a sketch, and record the problem with a number sentence.**

### Suggestions for Instruction

- **Equal Sharing Division:** Have students share examples of times when they had to share something so that everyone got the same amount/number.  
Tell students that you are going to read them a story all about sharing equally. Read the book *The Doorbell Rang* by Pat Hutchins. Ma bakes a dozen cookies and her two children are excited about sharing them until the doorbell rings. Each time the doorbell rings the children's share of the cookies becomes smaller until they are down to one each. In the end, Grandma comes with more cookies to save the day.  
Reread the book, and have students act out the story. Use paper cookies and paper plates to show the sharing. Each time, record the sharing using words instead of the division symbol. For example: 12 shared by 2 = 6 cookies each.

BLM  
3.N.12.2

Number of cookies	Number of children	Sharing (picture)	Number of cookies per child
12	2	 12 cookies shared by 2 children	6
12			
12			
12			

**Extension:** Students work with a partner to write a story patterned after *The Doorbell Rang*. The story should include at least 3 division situations. Have them read their story to the class while acting out the division problems. Record the division number sentences.

- **Introducing the Division Symbol:** Before introducing the symbol for division, it is important that students understand its meaning. Have students use the following vocabulary to describe division situations:

- shared by
- is how many groups of

Explain to students that we have a symbol to use for division in place of writing out the words *shared by* or *is how many groups of*. For some students it might help if this is represented pictorially. For example:

$$\begin{array}{c} \div \\ 12 \text{ shared by } 2 = 6 \end{array}$$

$$\begin{array}{c} \div \\ 12 \text{ is how many groups of } 4 = 3 \end{array}$$

- **Equal Grouping Division:** Use the book *A Remainder of One* by Elinor J. Pinczes, illustrated by Bonnie MacKain. In the story, Joe is part of a group of 25 bugs that is marching in a parade before the queen. When the bugs divide themselves up, Joe becomes the odd bug out.

Before reading the book, ask the students what they know about the word *remainder*. For the first reading, read to enjoy the language and the overall concept of the book.

As you reread the story, stop each time the insects decide on a different formation (“The troop had divided by two for the show.”) and ask students to predict how many insects will be in each line and how many (if any) insects will be left over. Act out the lineup using students (if numbers permit) or manipulatives.

The insects are organized in arrays in the illustrations. Use the arrays to support the predictions. Have division number sentences written for each formation either in words (25 divided into 2 lines = 12 in each line with one leftover) or in symbols ( $25 \div 2 = 12 \text{ r}1$ ).

**Note:** The National Council of Teachers of Mathematics (NCTM) Illuminations website (<https://illuminations.nctm.org/Lesson.aspx?id=3751>) provides a detailed lesson plan for the story as well as a way to create a similar story about lining up that you can use if you do not have the book.

- **Equal Grouping Problems:** Give students equal grouping division problems to solve. Provide objects and bags so that students can act them out. Have a number sentence written for each question.
  1. Marc has 20 trinkets for his birthday bags. How many children can he invite if he
    - puts 5 trinkets in each bag?
    - puts 2 trinkets in each bag?
    - puts 4 trinkets in each bag?
  2. Hannah has 12 apples. How many bags can she make if she
    - puts 6 apples in each bag?
    - puts 3 apples in each bag?
    - puts 2 apples in each bag?
    - puts 4 apples in each bag?
  3. Groups of Grade 3 students are packaging cookies for a bake sale. Each group has 18 cookies to package. How many packages can they make if they
    - put 9 cookies in each package?
    - put 6 cookies in each package?
    - put 3 cookies in each package?

Ask students to give examples of situations where they have had to use equal grouping. Have students use one of the examples to write their own equal grouping problems for the class to solve.

- **Create, and illustrate with counters, a story problem for a number sentence.**

### Suggestions for Instruction

BLM  
3.N.12.3

- **Writing Problems:** Prepare a set of cards (at least one per student) with a division number sentence written on each one. Have students select a card, write a story problem to match, and then act out the problem using counters/objects.

$10 \div 5 = 2$	$7 \div 1 = 7$
$6 \div 2 = 3$	$20 \div 4 = 5$
$15 \div 3 = 5$	$12 \div 4 = 3$
$10 \div 2 = 5$	$16 \div 4 = 4$
$8 \div 4 = 2$	$9 \div 3 = 3$
$12 \div 6 = 2$	$20 \div 5 = 4$
$8 \div 2 = 4$	$12 \div 3 = 4$
$14 \div 7 = 2$	$5 \div 5 = 1$
$15 \div 5 = 3$	$6 \div 3 = 2$
$4 \div 2 = 2$	$14 \div 2 = 7$



---

### Assessing Understanding: Journal Entry

1. Use the numbers 12, 3, and 4.
2. Write a division number sentence.
3. Make up a story problem to match your number sentence.
4. Use counters or draw a diagram to act out your story problem.

#### Extension

Is there another division number sentence you could write using the same numbers?

Can you change your story problem to match the new number sentence?

**Look for** evidence that the student

- sees the relationship between the three numbers
- is able to write a division number sentence
- can create a story problem to match the number sentence
- is able to act out the problem (with counters or a diagram)

**Extension** (if done)

- is able to write a second division number sentence
  - is able to change the story problem to reflect the new number sentence
-

- **Represent a division expression as repeated subtraction.**
- **Represent a repeated subtraction as a division expression.**

## Suggestions for Instruction

- **Investigating Division as Repeated Subtraction:** Present students with this problem.

Anna is thinking about multiplication and division.

She knows that multiplication can be shown as repeated addition. She wonders if division can be shown as repeated subtraction.

Can you help answer her question?

Use materials (counters, hundred chart, etc.) and number lines to help with your investigation.

Have students work with a partner to investigate this question.

Support students as needed by asking questions such as the following:

- What number sentence are you using for your investigation?
- What number will you have to start with on your number line?
- How do you show subtraction on a number line?
- If you are using cubes, how many do you need to start with?
- How can you show subtraction using the cubes?
- What number in the number sentence tells you how many cubes to take away?
- How can you record what you are doing each time?

Have students present their findings to the class.

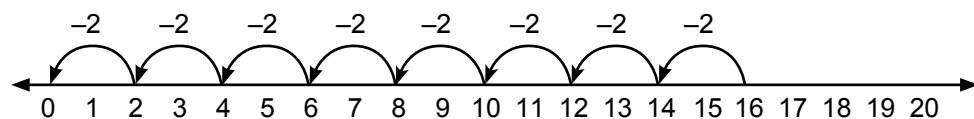
- **Division and the Number Line:** Tell students that Greg is walking on a number line. He starts at 16 and jumps back two numbers each time. How many jumps will do until he gets to zero?

Show the jumps on a number line (or get students to act it out using a large floor number line).

How can we write this as a repeated subtraction number sentence?

$$(16 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 = 0)$$

What division number sentence will match what is shown on the number line? ( $16 \div 2 = 8$ )





Have students make up their own division problems to represent (act out) on a number line. Use both a repeated subtraction and a division number sentence to show the solution for each problem.

- **Repeated Subtraction and Division:** Give students a set of repeated subtraction number sentences and have them write the matching division number sentence.

$12 - 3 - 3 - 3 - 3 = 0$	$15 - 5 - 5 - 5 = 0$
$20 - 4 - 4 - 4 - 4 - 4 = 0$	$10 - 2 - 2 - 2 - 2 - 2 = 0$
$18 - 6 - 6 - 6 = 0$	$14 - 7 - 7 = 0$



---

### Assessing Understanding: Paper-and-Pencil Task

#### Fun on the Farm

Use repeated subtraction and a division number sentence to show how to solve the following problems:

1. There are 20 legs in the pigpen.  
How many pigs are there?
2. There are 16 eyes in the chicken coop.  
How many chickens are in the chicken coop?
3. There are 10 hands in the farmhouse.  
How many people are in the house?

The student is able to

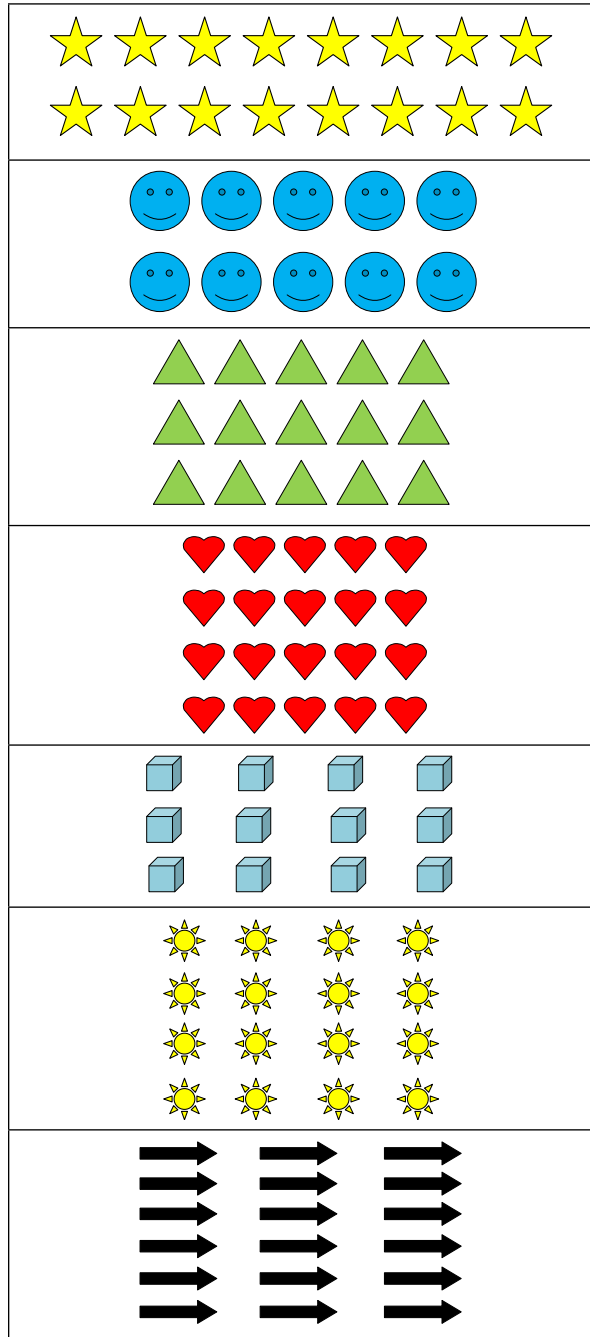
- read and interpret the problem
  - write a repeated subtraction number sentence for each problem
  - write a division number sentence for each problem
-

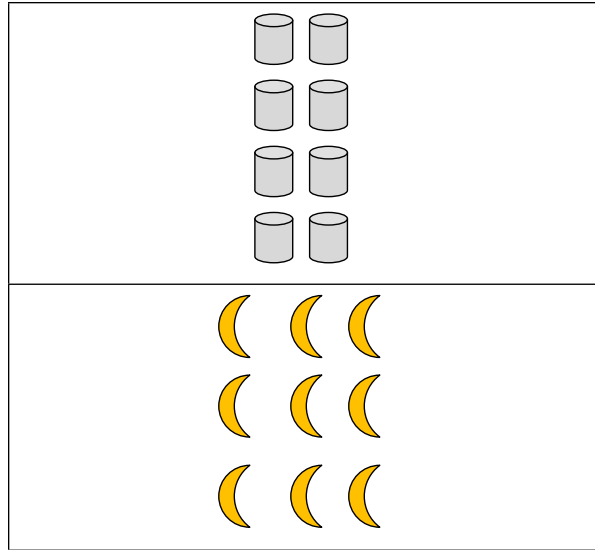
- **Relate division to multiplication by using arrays and writing related number sentences.**
- **Solve a problem involving division.**

### Suggestions for Instruction

BLM  
3.N.12.4

- **Array cards:** Use the array cards (BLM 3.N.12.4).



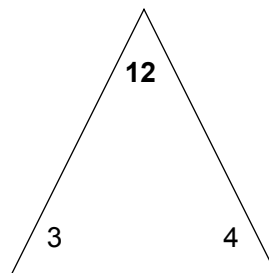


**Student Directions:**

1. Pick an array card.
2. Write 2 multiplication number sentences for your array.
3. Write 2 division number sentences for your array.
4. Write one multiplication story problem to match your array.
5. Write one division story problem to match your array.

- **Triangular Flashcards and Fact Families:** Talk to students about the relationship between multiplication and division. Triangular flashcards help students see this relationship. Use the cards to make the fact family for a given set of numbers.

Example:



If the 12 is covered students are looking for the product of 3 and 4.

If either the 3 or 4 are covered students are looking for the quotient of  $12 \div 3$  or 4.

The numbers 12, 3, and 4 are a fact family. Students can write 2 multiplication and 2 division number sentences for the card.

$$3 \times 4 = 12$$

$$4 \times 3 = 12$$

$$12 \div 4 = 3$$

$$12 \div 3 = 4$$

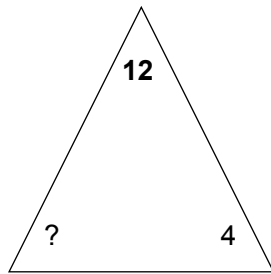
If you think about the part-part-whole, then the 12 represents the whole and the 3 and 4 represent the parts. If the whole is unknown, multiplication is the operation needed. If one of the parts is unknown, then division is needed. Students can use this understanding to help them interpret multiplication and division problems.

- **Using Part-Part-Whole for Problem Solving:** Give students a story problem.

Sean bought a bag of 12 candies.

He wants to share his candies with his 4 friends.

How many candies will each person get?



Ask the following:

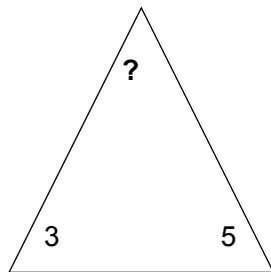
- Do you know the whole? (12 candies)
- Do you know the parts? (We know one part, one part is missing.)
- If we know the whole and one of the parts, what operation will you use to solve the problem? (division)

Use the triangle to record the problem information.

Mason has 3 pages of stickers.

Each page has 5 stickers.

How many stickers does he have altogether?



Ask the following:

- Do you know the whole? (no)
- Do you know the parts? (Yes: 3 and 5.)
- If we know both of the parts but not the whole, what operation will you use to solve the problem? (multiplication)

**Note:** Students can make their own triangular cards. The cards can be used in multiplication and division stations/centres.

- **Open-Ended Problems:** Have students solve the problems giving multiple solutions.
  1. The Grade 3 class is playing a game in gym. The teacher wants them to be in equal groups with no remainders. If there are 20 students in the class, how many different sizes of groups can you make?
  2. The answer to a multiplication question is 12. What might the question be?



---

### Assessing Understanding: Journal Entry

Explain how multiplication and division are related.

Use words, pictures, arrays, number lines, and/or numbers and symbols in your explanation.

**Look for** evidence of understanding that

- multiplication and division are inverse operations
  - multiplication is repeated addition
  - division is repeated subtraction
  - an array can represent both operations
-

## Grade 3: Number (3.N.13)

### Enduring Understandings:

A fraction represents a part of a whole.

Fractions can be compared using a variety of models.

The size of the fractional part depends on the size of the whole.

Equal parts do not have to look the same but they must be the same size or have the same amount of the whole.

### Essential Questions:

What is a fraction?

Where do you use fractions in everyday life?

What is the numerator?

What is the denominator?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
<p><b>3.N.13</b> Demonstrate an understanding of fractions by</p> <ul style="list-style-type: none"><li>■ explaining that a fraction represents a portion of a whole divided into equal parts</li><li>■ describing situations in which fractions are used</li><li>■ comparing fractions of the same whole with like denominators</li></ul> <p>[C, CN, ME, R, V]</p>	<ul style="list-style-type: none"><li>→ Identify common characteristics of a set of fractions.</li><li>→ Describe everyday situations where fractions are used.</li><li>→ Cut or fold a whole into equal parts, or draw a whole in equal parts; demonstrate that the parts are equal, and name the parts.</li><li>→ Sort a set of diagrams of regions into those that represent equal parts and those that do not, and explain the sorting.</li><li>→ Represent a fraction concretely or pictorially.</li><li>→ Name and record the fraction represented by the shaded and non-shaded parts of a region.</li><li>→ Compare fractions with the same denominator using models.</li><li>→ Identify the numerator and denominator for a fraction.</li><li>→ Model and explain the meaning of <i>numerator</i> and <i>denominator</i>.</li></ul>

## PRIOR KNOWLEDGE

---

Students may have had no formal experience with fractions. Most children will have had experience dividing something in half in a sharing situation. However, many children use the word *half* to mean a part of something. They do not, however, differentiate between *half* and *part*.

## BACKGROUND INFORMATION

---

### Terminology

**Fraction:** A number that represents part of a whole, part of a set, or a quotient in the form  $\frac{a}{b}$ , which can be read as  $a$  divided by  $b$ .

**Numerator:** The number above the line in a fraction that can state one of the following:

- the number of equal parts in a set to be considered
- the number of equal parts of a whole to be considered

**Denominator:** The number below the line in a fraction that can state one of the following:

- the number of elements in a set
- the number of equal parts into which the whole is divided

In Grade 3, the focus is on developing a beginning understanding of fractions less than one, relating fractions to real-life situations, and comparing fractions with the same denominator. The focus is on having students explore parts of a whole that has been divided into “fair shares” or “fractional parts” (equal-sized pieces). Finding a fraction of a set is introduced in Grade 4.

It is vital that students represent fractions with concrete models. A variety of materials must be used so that students understand what fractions mean. It is also important that students develop visual images for fractions and be able to tell “about how much” a particular fraction represents.

**Note:** It is important to use a wide variety of models so that students do not come to believe that fractions only relate to parts of a circle (pizza, pie).

## MATHEMATICAL LANGUAGE

---

fraction	third
numerator	quarter
denominator	fifth
fair share	sixths
whole	eighths
one whole	tenths
half	one of ____ equal parts

## LEARNING EXPERIENCES

---



### Assessing Prior Knowledge:

**Performance Task** to assess student understanding of *half*:

Give students a strip of paper, piece of string, or a piece of licorice. (Play dough or plasticine “worms” would work as well.) Ask them to cut the strip/piece in half.

Ask, “How do you know that it is half?”

Observe students to determine if they

- know that there should be two pieces
- fold the object in half before cutting or cut without folding
- match the two pieces to see if they are the same length or say that it is half because there are two pieces even if they are not equal

### Class Discussion: Web or Concept Map

Ask: “What do you know about fractions?”

Create a web or concept map to record their responses.

---



- **Describe everyday situations where fractions are used.**

### Suggestions for Instruction

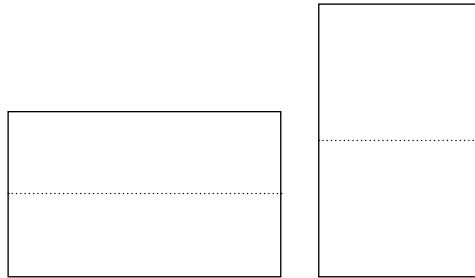
- Have students identify everyday situations that use fractions (e.g., sharing a chocolate bar with a friend, cutting an apple or sandwich in half, telling time—half past/quarter to, distance—halfway).

After symbols are introduced, have students find examples of fractions in magazines/newspapers/signs. Use the examples to create a fraction poster.

- **Cut or fold a whole into equal parts, or draw a whole in equal parts; demonstrate that the parts are equal and name the parts.**
- **Sort a set of diagrams of regions into those that represent equal parts and those that do not, and explain the sorting.**

### Suggestions for Instruction

- Begin with concept of one half. Give students a letter-sized piece of paper. Ask them to fold it in half. Do not give further directions. Have students compare their halves. Some students will fold the paper on short side (hotdog) and others on the long side (hamburger).



Ask the following:

- How many parts do you have?
- How do you know that each part is one half?

Expected answer: “There are two parts and both of them are equal.”

■ **Exploring Quarters/Fourths and Eighths**

Have students fold a strip into half and then into half again.

Ask them the following:

- How many parts they have altogether? (4)
- Are they equal?
- Does anyone know what we call one part out of the 4 parts?

Introduce both terms—*one-fourth* and *one-quarter*. Connect to money (4 quarters in a loonie) to help students understand the terminology.

Refold and have the strip folded in half again. Before opening it up, ask them to predict how many parts they think they have now. After opening it, ask how many parts altogether? Does anyone know what we call one part out of 8 parts? Introduce the term *one-eighth*.

Extend the thinking by asking what they think you would call

- one part out of 3 parts?
- one part out of 6 parts?
- one part out of 10 parts?

BLM  
3.N.13.1

- **Sorting Diagrams into Equal Parts and Unequal Parts:** Give students pictures of shapes divided into equal and unequal parts (BLM 3.N.13.1). Have students sort them and give the sorting rule.



---

**Assessing Understanding: Journal Entry**

Sadie and Ben were sharing a piece of cake. Ben cut the cake in two pieces and then said to Sadie, “You can have the bigger half.”

Does this make sense? Why or why not?

In response, the student should indicate that the word *half* means that the whole/object has been divided into two equal parts so there is no bigger half.

---

**Suggestions for Instruction**

■ **Exploring Fractions with Manipulatives**

1. Pattern Blocks: Use the hexagon as the whole. Have students use the other pattern blocks to show half, thirds, and sixths.
2. Cuisenaire Rods: Use differently sized rods to explore a variety of fractions (e.g., using the ten rod as the whole, students can explore tenths, fifths, and halves).

These concrete representations should help students make the connection to the symbolic representations easier.

- **Read the Book:** *Whole-y Cow! Fractions Are Fun* by Taryn Souders, illustrated by Tatjana Mai-Wyss. In this book students are asked a fraction question about each illustration. It is a good book to use before introducing the symbols.

**Note:** You might want to have a discussion about some of the illustrations (e.g., in one picture the cow is painted blue [rear part] and white [front part], and the words on the page say, “Moo while her friends paint one half blue!”—students may question whether the two parts are actually equal and therefore should not be referred to as “half”).

- **Name and record the fraction represented by the shaded and non-shaded parts of a region.**
- **Identify the numerator and denominator for a fraction.**
- **Model and explain the meaning of *numerator* and *denominator*.**

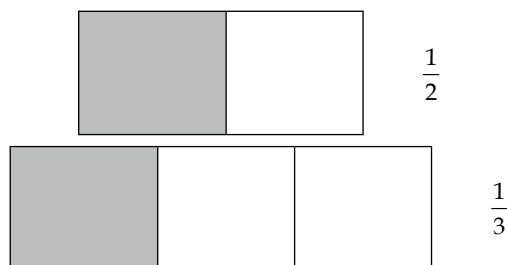
## Suggestions for Instruction

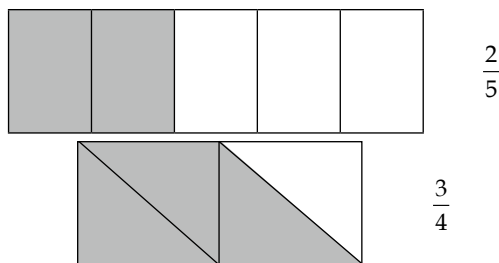
### Note

Before the teacher introduces the symbols, students should have multiple opportunities to explore and talk about fractions. Modelling the language of “one of \_\_\_\_ equal parts” will help students make the connection between language and the symbol.

- **Introducing the Symbols:** Reading a book such as *Apple Fractions* by Jerry Pallotta, illustrated by Rob Bolster, *Full House: An Invitation to Fractions* by Dayle Ann Dodds, illustrated by Abby Carter, or *If You Were a Fraction* by Trisha Speed Shaskan, illustrated by Francesca Carabelli, will help make connections among the pictorial representation, the words, and the symbol.

Show students pictures with fractional parts shaded along with the fraction symbol. Ask students if they can tell what each number/digit in the fraction represents. For example:





Ask questions such as the following:

- Look at the shaded part of the rectangle. Now look at the top number/digit in the fraction. Does the top number in the fraction tell you something about the shaded part of the rectangle? (*The top number in the fraction tells how many parts of the rectangle are shaded.*)
- Look at the bottom number/digit in the fraction. Does this number tell you anything about the rectangle? (*The bottom number in the fraction tells how many parts there are in the rectangle.*)

Show pictures of other shapes (circles, triangles, etc.) to see if students have generalized the relationship between the fraction symbol and the picture beyond the rectangle.

Once students have made this connection introduce the terms *numerator* and *denominator*.

- **From Picture to Fraction:** Provide students with pictures and have them write the matching fraction. Have students explain how they determined each digit in the fraction.
- **Fraction Concentration:** Prepare a set of fraction picture cards and a set with the matching fraction symbols. Have students use them to play a game of concentration.



---

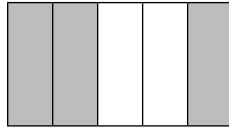
## Assessing Understanding: Paper-and-Pencil Task

1. Draw a picture for these fractions:

$$\frac{2}{4}$$

$$\frac{4}{6}$$

2. What fraction of each shape is shaded? What fraction is not shaded?



shaded \_\_\_\_\_

unshaded \_\_\_\_\_



shaded \_\_\_\_\_

unshaded \_\_\_\_\_

3. Julie says that her fraction has a denominator of 8 and a numerator of 3. Draw a picture to match Julie's fraction.

**Look for:**

The student understands that

- the denominator represents the total number of parts the whole has been divided into
  - the numerator represents the number of shaded parts or the parts being focused on
-

- **Compare fractions with the same denominator using models.**

### Suggestions for Instruction

- **Comparing Fractions Using Pattern Blocks:** Use hexagons as wholes and green triangles as the parts. Have students show the fraction  $\frac{1}{6}$  and the fraction  $\frac{3}{6}$ .

Ask: Which fraction is the largest? How do you know?

*( $\frac{3}{6}$  is larger because there are 3 pieces out of the six. In the fraction  $\frac{1}{6}$ , you only have 1 piece out of the six.)*

Have students compare  $\frac{3}{6}$  and  $\frac{5}{6}$  as well as  $\frac{4}{6}$  and  $\frac{2}{6}$ .

Do the same with Cuisenaire rods. For example: Use an 8 rod and the ones cubes. Have students show  $\frac{3}{8}$  and  $\frac{5}{8}$ . Ask them to identify the largest (smallest) fraction and to explain their choice.

Once students have had practice comparing fractions in this way ask, "If you have two fractions with the same denominators, how can you tell which fraction is the largest?" Students should indicate that when two fractions have the same denominator the fraction with the largest numerator represents the larger part of the whole.




---

### Assessing Understanding: Journal Entry

- Hugh said that his fraction is greater than  $\frac{2}{10}$  but less than  $\frac{6}{10}$ . What might his fraction be?
- You are going to share a piece of cake with your friend. Would you rather have  $\frac{4}{6}$  of the cake or  $\frac{3}{6}$  of the cake? Explain your thinking. Use pictures, numbers, and words in your explanation.
- Order these fractions from the smallest to the largest:

$\frac{7}{8}$      $\frac{2}{8}$      $\frac{3}{8}$      $\frac{5}{8}$      $\frac{4}{8}$      $\frac{6}{8}$

---

## PUTTING THE PIECES TOGETHER: PERFORMANCE TASK



**Introduction:** Read the following fraction story (patterned after *The Doorbell Rang*).

Mrs. Hurd is famous for her homemade pizza.

On Saturday she baked a large pepperoni pizza for her two children, Mary and Tom. The children were excited about sharing the pizza.

*How much of the pizza will each child get? How should Mrs. Hurd cut the pizza?*

As they sat down to eat, the doorbell rang. It was Paul and Jim from next door. Mrs. Hurd invited them in to share the pizza.

*Now, how much of the pizza will each person get? How should the pizza be cut?*

Mrs. Hurd was just getting ready to cut the pizza when the doorbell rang again. It was Elly and Kelly, the twins from across the street. Mary and Tom invited them in.

*How much of the pizza will each person get now? How should Mrs. Hurd cut the pizza?*

The children began to worry. Their share of the pizza was getting smaller with each ring of the doorbell. They finally decided to hurry up and eat, but before they could cut the pizza the doorbell rang. This time it was their cousins, Hans and Heidi.

*How much of the pizza will each person get? How should Mrs. Hurd cut the pizza?*

Mrs. Hurd cut the pizza and everyone started eating. After the pizza was finished all of the children were still hungry. Luckily Mrs. Hurd saved the day by popping another pizza into the oven.

*If the children share the second pizza, how many pieces will each child eat altogether?*

As the story is read, have students visually represent how the pizza should be cut each time and identify the fraction both in words and with a symbol.

### Student Directions:

Write a fraction story. You can pattern it after a story you know or you can make up your own story.

**Note:** The story should illustrate the sharing of a whole not a set.

### Criteria:

- The object (whole) in your story should be shared at least 3 times.
- Illustrations should show how the object is shared each time.
- Fraction symbols should be included.

---

## Notes





# GRADE 3 MATHEMATICS

## Patterns and Relations



## Grade 3: Patterns and Relations (3.PR.1, 3.PR.2)

### Enduring Understandings:

Patterns can grow and repeat.

Patterns can be found in many different forms.

### Essential Questions:

What is the increasing or decreasing unit in the pattern?

What strategies can be used to continue an increasing or decreasing pattern?

What strategies can be used to continue a numerical sequence?

How is the pattern increasing or decreasing?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
<p><b>3.PR.1</b> Demonstrate an understanding of increasing patterns by</p> <ul style="list-style-type: none"><li>■ describing</li><li>■ extending</li><li>■ comparing</li><li>■ creating</li></ul> <p>patterns using manipulatives, diagrams, and numbers (to 1000). [C, CN, PS, R, V]</p>	<ul style="list-style-type: none"><li>→ Describe an increasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues.</li><li>→ Identify the pattern rule of an increasing pattern, and extend the pattern for the next three terms.</li><li>→ Identify and explain errors in an increasing pattern.</li><li>→ Identify and describe various increasing patterns found on a hundred chart, such as horizontal, vertical, and diagonal patterns.</li><li>→ Compare numeric patterns of counting by 2s, 3s, 4s, 5s, 10s, 25s, and 100s.</li><li>→ Create a concrete, pictorial, or symbolic representation of an increasing pattern for a pattern rule.</li><li>→ Create a concrete, pictorial, or symbolic increasing pattern, and describe the pattern rule.</li><li>→ Solve a problem using increasing patterns.</li><li>→ Identify and describe increasing patterns in the environment.</li><li>→ Identify and apply a pattern rule to determine missing elements for a pattern.</li><li>→ Describe the strategy used to determine missing elements in an increasing pattern.</li></ul>

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
<p><b>3.PR.2</b> Demonstrate an understanding of decreasing patterns by</p> <ul style="list-style-type: none"> <li>■ describing</li> <li>■ extending</li> <li>■ comparing</li> <li>■ creating</li> </ul> <p>patterns using manipulatives, diagrams, and numbers (starting from 1000 or less). [C, CN, PS, R, V]</p>	<ul style="list-style-type: none"> <li>→ Describe a decreasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues.</li> <li>→ Identify the pattern rule of a decreasing pattern, and extend the pattern for the next three terms.</li> <li>→ Identify and explain errors in a decreasing pattern.</li> <li>→ Identify and describe various decreasing patterns found on a hundred chart, such as horizontal, vertical, and diagonal patterns.</li> <li>→ Compare decreasing numeric patterns of counting backward by 2s, 3s, 4s, 5s, 10s, 25s, and 100s.</li> <li>→ Create a concrete, pictorial, or symbolic decreasing pattern for a pattern rule.</li> <li>→ Create a concrete, pictorial, or symbolic decreasing pattern, and describe the pattern rule.</li> <li>→ Solve a problem using decreasing patterns.</li> <li>→ Identify and describe decreasing patterns in the environment.</li> <li>→ Identify and apply a pattern rule to determine missing elements for a pattern.</li> <li>→ Describe the strategy used to determine missing elements in a decreasing pattern.</li> </ul>

## PRIOR KNOWLEDGE

Students may have worked with a wide variety of repeating patterns.

They may have reproduced, extended, and created increasing patterns using manipulatives, diagrams, sounds, and actions (numbers to 100).

## BACKGROUND INFORMATION

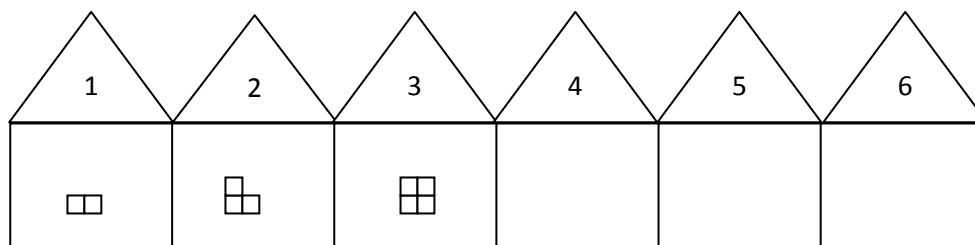
---

Patterns are found throughout mathematics and should be taught throughout the year through problem solving. Our number system is based on an increasing pattern.

Simple repeating and growing/increasing/decreasing patterns consist of a series of related elements—each new element is related to the previous in some manner. Students must be able to identify the relationship in order to understand the pattern.

Students who develop the ability to **identify, reproduce, extend, and create** patterns are able to make generalizations and see relationships with numbers. This is why identifying and extending patterns is an important process in algebraic reasoning. To work with increasing and decreasing patterns, it is important that Grade 3 students have a sound foundation with repeating patterns, as well as some abilities with increasing patterns.

In Grade 3, students are expanding their knowledge of increasing patterns. They are beginning to explore decreasing patterns and making more in-depth connections with number concepts. When presenting a pattern, encourage students to verbalize the rule for the patterns they are working with. To encourage students to make connections with numbers, present the pattern with numerical term positions. For example:



Increasing and decreasing patterns are patterns in which the basic core pattern grows/shrinks or changes in a predictable way.

## MATHEMATICAL LANGUAGE

---

pattern

decreasing pattern

increasing pattern

element

extend

reproduce

rule

hundred chart

## LEARNING EXPERIENCES




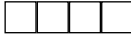
---



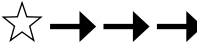


### Assessing Prior Knowledge

Present students with a variety of increasing patterns. Have them identify the missing elements and explain the rule used to create and extend the pattern.

Examples:

1.     \_\_\_\_\_   
Figure 1      Figure 2      Figure 3      Figure 4      Figure 5      Figure 6

   \_\_\_\_\_   
Figure 1      Figure 2      Figure 3      Figure 4      Figure 5

2. 2, 4, 6, 8, \_\_\_\_\_, 12, 14, \_\_\_\_\_, \_\_\_\_\_

3. Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23		25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63		65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The student is able to

- explain how the pattern increases/grows
  - extend the pattern
  - fill in the missing element
-

- Describe an increasing/decreasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues.
- Identify the pattern rule of an increasing/decreasing pattern, and extend the pattern for the next three terms.
- Create a concrete, pictorial, or symbolic increasing/decreasing pattern, and describe the pattern rule.
- Identify and explain errors in an increasing/decreasing pattern.

### Suggestions for Instruction

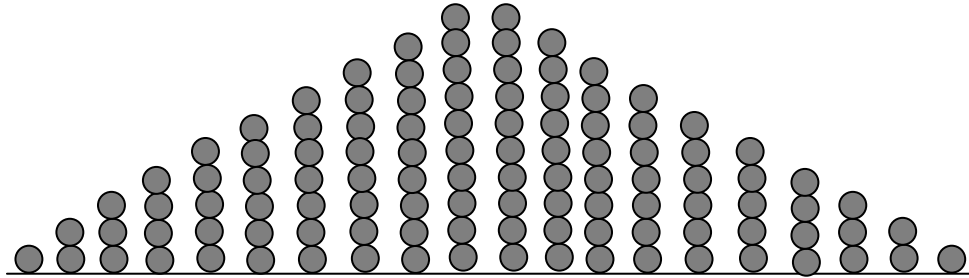
- Use the book *Rooster's Off to See the World* by Eric Carle to review increasing patterns and introduce decreasing patterns. The pattern is graphically displayed to follow the action in the text. Ask students to describe how the pattern changes from page to page.  
**Note:** The pattern images could be photocopied so students can build the pattern as they retell the story.
- Use a book such as *Mrs. McTats and Her Houseful of Cats* by Alyssa Satin Capucilli, illustrated by Joan Rankin. Read the book through first. Reread the book. Use pictures or counters to represent the changes in the number of cats. Have students identify and describe the increasing pattern.

**Note:** The pattern changes at the end of the story.

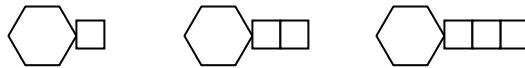


The pattern is +2, +3, +4, +5, and +6.

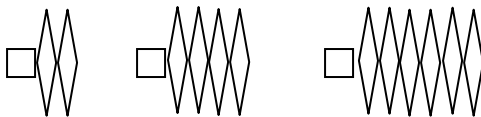
- Use a book such as *Counting Crocodiles* by Judy Sierra, illustrated by Will Hillenbrand, to explore both increasing and decreasing patterns. The first half of the book counts the number of crocodiles from 1 to 10. The second half counts the crocodiles from 10 to 1. Students can use pictures or counters to represent the actions in the story. For example,



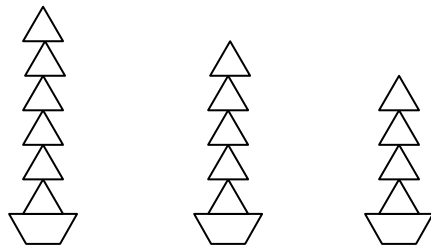
- Pattern Problem:** Use two different pattern block shapes. Make an increasing pattern. Use the same shapes to make a decreasing pattern. Record your patterns in your journal. Explain the pattern rules.
- Use pattern blocks. Have students identify the pattern rule and then extend the pattern for the next three terms/figures.



Term 1      Term 2      Term 3      Term 4      Term 5      Term 6



Term 1      Term 2      Term 3      Term 4      Term 5      Term 6



Term 1      Term 2      Term 3      Term 4      Term 5      Term 6

- Pattern Errors:** Students work with a partner. Each student creates a decreasing or increasing pattern that contains an error. Students exchange patterns and identify the errors in each others' pattern.

Example:





- **Counting Errors:** The teacher or student leader recites a counting sequence. When an error is detected students raise their hands (or use some other agreed upon signal). A student is selected to identify and correct the error.
- **Mystery Rule:** The teacher decides on a pattern rule. Students give a number (input), the teacher applies the rule and gives the result (output). This continues until someone can identify the pattern rule.  
Students can take turns being the leader. Have the rule written down and shown to the teacher first.



---

### Assessing Understanding: Performance Task

#### Student Directions:

Work with a partner.

1. Use colour tiles to make an increasing or decreasing pattern. Your pattern should have at least 4 terms.
2. Draw your pattern.
3. Explain your pattern rule.
4. Carefully describe your pattern so that someone else can make it increase or decrease three more times.
5. Look at your partner's pattern. Draw your partner's pattern.
6. Make it increase or decrease three more times.
7. What is your partner's pattern rule?
8. Compare your pattern with your partner's. How are they the same? How are they different?

**Paper-and-Pencil Task:** Answer the following questions in your journal/notebook:

1. What is an increasing pattern? Give an example.
  2. What is a decreasing pattern? Give an example.
-

- **Locate and describe various increasing/decreasing patterns found on a hundred chart, such as horizontal, vertical, and diagonal patterns.**

### Suggestions for Instruction

BLM  
3.PR.1.1 &  
3.PR.2.1

- **Patterns on the Hundred Chart:** Have students explore patterns on a hundred chart. Possible increasing or decreasing patterns include the following:
  - for each row, left to right, numbers increase by 1
  - for each row, right to left, numbers decrease by 1
  - for each column, top to bottom, numbers increase by 10
  - for each column, bottom to top, numbers decrease by 10
  - numerous skip counting patterns
  - on the diagonal (starting at the top) from left to right numbers increase by 11
  - on the diagonal (starting at the bottom) from right to left numbers decrease by 11
  - on the diagonal (starting at the top) from right to left numbers increase by 9
  - on the diagonal (starting at the bottom) from left to right numbers decrease by 9

Hundred Chart									
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

**Note:** Extend student thinking by

- changing the numbers on the chart to start at a larger number
- having the numbers of the chart increase by a multiple rather than by 1s (e.g., start at 0 and count up by 2s or start at 0 and count up by 10s)
- change the orientation of the chart by having the 1 to 10 row at the bottom



## Assessing Understanding: Missing Numbers

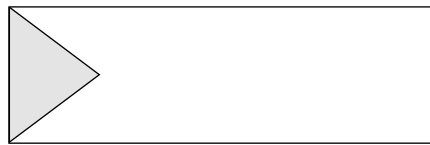
This is a piece from a hundred chart. Fill in the missing numbers. Explain how you figured out each answer.

	26			
	36		38	
45				

- Compare numeric patterns of counting by 2s, 3s, 4s, 5s, 10s, 25s, and 100s.
- Compare decreasing numeric patterns of counting backward by 2s, 3s, 4s, 5s, 10s, 25s, and 100s.

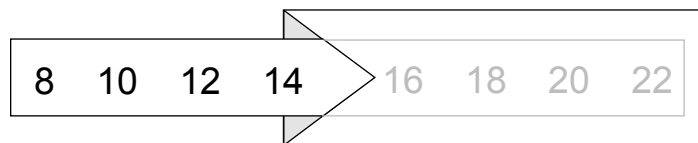
### Suggestions for Instruction

- Students compare counting patterns on the 100 chart.
  - Count by 2s. Colour the numbers on the hundred chart. Describe the patterns you see.
  - Count by 5s on the same 100 chart. Describe the patterns you see.
  - How are these patterns alike? How are they different?Use additional charts to compare counting by 3s, 4s, 10s, 25s, and 100s.
- Prepare a “pattern slider” to use with numeric patterns. Use a legal size sheet of paper and fold it to form a flat tube. Tape it together. Cut a V on one side.



Use pattern strips. Slide a strip through the slider. Gradually pull the pattern through until three or four numbers can be seen. Students predict the next number(s) in the pattern. Have students justify their prediction by identifying the pattern rule.

For example,





---

### Assessing Understanding: Paper-and-Pencil Task

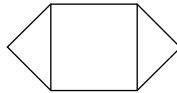
- Mark started at 60 and counted backward by 3s to 0.  
Pam started at 60 and counted backward by 4s to 0.  
Hans started at 60 and counted backward by 2s to 0.  
What numbers will each person say?  
How are the counting patterns the same?  
How are they different?
  - If you start at 465 and count by 5s to 550, and your friend starts at 410 and counts by 10s to 550, what numbers would you both say? Explain how you know.
- 

- **Create a concrete, pictorial, or symbolic representation of an increasing/decreasing pattern for a pattern rule.**
- **Identify and apply a pattern rule to determine missing elements for a pattern.**
- **Describe the strategy used to determine missing elements in an increasing/decreasing pattern.**
- **Solve a problem using increasing/decreasing patterns.**

### Suggestions for Instruction

- **The Rule Is \_\_\_\_:** Give the first term of an increasing or decreasing pattern along with the pattern rule. Have students create a representation of the pattern.  
**Note:** The representation should have at least 4 terms.

Examples:



Term 1

Rule: Add 2

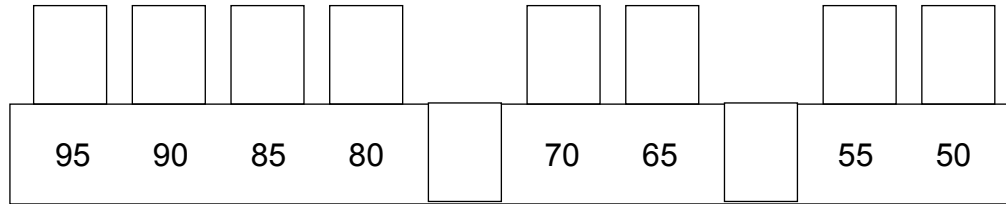
**24**

Term 1

Rule: Minus 2

- Use a Hidden Numeral Board for missing element problems.

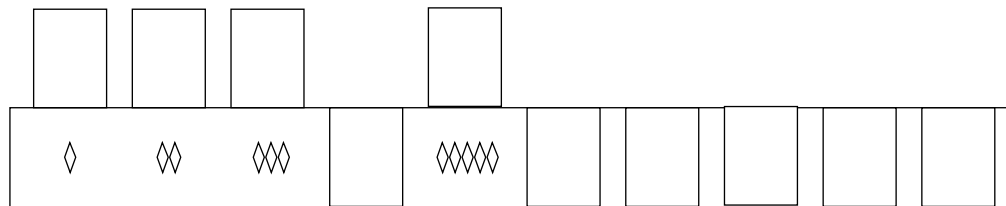
Examples:



How is the pattern changing?

What is the pattern rule? How did you figure it out?

Use the pattern rule to find the missing numbers.



How is the pattern changing?

What is the pattern rule? How did you figure it out?

Use the pattern rule to find the missing terms.

- Have students share the strategies they use to determine the missing elements in an increasing or decreasing pattern. Strategies can be recorded on a chart and put up for student reference.

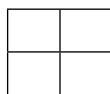
**Sample Problems:**

- How many stars are in Term 3?  
How many stars are in Term 5?  
What strategy did you use to solve the problem?

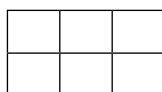
Term 1	Term 2	Term 3	Term 4	Term 5
☆☆☆☆☆☆	☆☆☆☆☆☆		☆☆☆☆☆☆	
☆☆☆☆☆☆	☆☆☆☆☆☆		☆☆☆☆☆☆	
☆☆☆☆☆☆	☆☆☆☆☆☆		☆☆☆☆☆☆	
☆☆☆☆☆☆	☆☆☆☆☆☆			
☆☆☆☆☆☆	☆☆☆☆☆☆			
☆☆☆☆☆☆	☆☆☆☆☆☆			

- Mara wants to find out the number of tires on 7 cars. Use a pattern to help Mara solve this problem.

- Sam has a problem. He is putting new tile in his house. The book he is using shows the first 3 terms of the pattern but he is not sure how to continue the pattern. Complete the next two terms for him. Write a note to Sam explaining how he should continue the pattern.



Term 1



Term 2



Term 3

- The Grade 3 class is having their picture taken. The photographer put them in rows. She put 1 person in the first row, 2 in the second, 3 in the third, and so on. If there were 7 rows of students in the picture, how many students were there in the class? Use a pattern to find your answer.
- Mrs. Allen has a box of chocolates. There are 24 chocolates in the box. She decides to eat 2 chocolates each day. How many days will the chocolates last? Use a pattern to explain your answer.
- Henri baked cookies over the weekend. Each day during the week he took three cookies to school for his lunch. On Saturday he had 18 cookies left. How many cookies had he baked? Use a pattern to solve the problem.
- Alex loves cookies. He ate 10 cookies on Monday, 9 on Tuesday, 8 on Wednesday, and so on through Sunday. How many cookies did he eat in all from Monday through Sunday? Use a pattern to help solve the problem.
- Write a problem that can be solved using an increasing or decreasing pattern. Share your problem with a partner.

**Note:** Problems could be compiled into a class book.



---

## Assessing Understanding

Observe students as they work on problems.

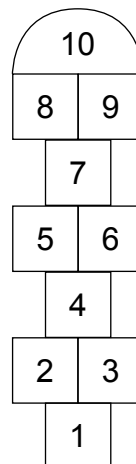
The student is able to

- identify the pattern rule and apply it to solve the problem
  - use mathematical language related to increasing and decreasing patterns
  - use an increasing pattern to solve a problem
  - use a decreasing pattern to solve a problem
  - discuss the strategies used to find missing elements
- 

- **Identify and describe increasing/decreasing patterns in the environment.**

## Suggestions for Instruction

- Have students use a digital camera to take pictures of increasing and decreasing patterns in the environment.
- **Home Pattern Hunt:** Students look for examples of increasing and decreasing patterns (concrete, pictorial, or symbolic) at home and bring them (or a replica) to school to share with the class.
- **Science Observation Centre:** Students bring in objects from nature that have increasing or decreasing patterns and place them at the science centre. Students identify and record the patterns observed.
- **Patterns in Hopscotch Exploration:** Draw a hopscotch grid on the board or outside on the tarmac. Ask students to describe the patterns they see.



Focus the exploration with questions such as the following:

- What is happening to the numbers in the single rectangles?
- What is happening to the numbers in the left-hand double rectangles?
- What is happening to the numbers in the right-hand double rectangles?
- What is the pattern rule?
- If the hopscotch grid was continued, what number would be in the seventh single rectangle? In the eighth left-hand double rectangle? In the tenth right-hand double rectangle?
- Would the number 14 be in a single rectangle? Explain your thinking.

**Extending student thinking:**

Design a hopscotch grid made up of single and double rectangles so that 6, 11, and 16 are in single rectangles.

## PUTTING THE PIECES TOGETHER: PERFORMANCE TASK



### Increasing and/or Decreasing Pattern Story

Introduce the task by reading an increasing/decreasing pattern book. Revisit a book used earlier or read a new one such as *Ten Little Ladybugs* by Melanie Gerth, illustrated by Laura Huliska-Beith.

Tell students that they are going to work with a partner to write and illustrate an increasing/decreasing pattern story to share with a younger class (reading buddy).

The activity can be differentiated by having students

- write an increasing story
- write a decreasing story
- write a story that has both an increasing and decreasing pattern
- write a story that increases or decreases by numbers greater than one



## Grade 3: Patterns and Relations (3.PR.3)

### Enduring Understandings:

"Equals" indicates equivalent sets.

Unknown quantities can be found by using the balance strategy.

### Essential Questions:

How is a number sentence like a balance scale?

What does the equal sign mean?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
<p><b>3.PR.3</b> Solve one-step addition and subtraction equations involving symbols representing an unknown number. [C, CN, PS, R, V]</p>	<ul style="list-style-type: none"><li>→ Explain the purpose of the symbol, such as a triangle or a circle, in an addition or a subtraction equation with one unknown.</li><li>→ Create an addition or a subtraction equation with one unknown to represent a combination or separation action.</li><li>→ Provide an alternative symbol for the unknown in an addition or a subtraction equation.</li><li>→ Solve an addition or a subtraction equation that represents combining or separating actions with one unknown, using manipulatives.</li><li>→ Solve an addition or a subtraction equation with one unknown using a variety of strategies including guess and test.</li><li>→ Explain why the unknown in an addition or a subtraction equation has only one value.</li></ul>

## PRIOR KNOWLEDGE

Students may be able to

- demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100)
- record equalities and inequalities symbolically using the equal symbol or the not-equal symbol

## BACKGROUND INFORMATION

---

The equal symbol represents a relation between two equal quantities. In other words, the expression on the left-hand side of the equal symbol represents the same quantity as the expression on the right-hand side of the equal symbol. The equal symbol means “is the same as.”

Many students have misconceptions about the equal symbol. Many think that the equal symbol means “give answer.” As a result they have difficulty with questions such as the following:

$4 + \underline{\quad} = 7$       Students will add across the equal sign and fill the blank with 11.

$\underline{\quad} = 2 + 5$       Students will say that the question itself is incorrect because the blank is on the wrong side.

$3 + 4 = 5 + \underline{\quad}$       Students will add all the numbers and put 12 in the blank.

### Exploring Relationship between Terms

Exploring the relationship between expressions/terms on either side of an equal symbol enables students to develop the understanding of the relationship that exists between the expressions/terms.

**Note:** It is important that students discover this relationship on their own.

Ask students what they notice about the expressions on both sides of the equal sign. If the students have difficulty seeing a relationship, provide additional examples using smaller numbers.

Examples:

$16 + 18 = 18 + 16$  (Think: “The order of the addends is reversed but the numbers are the same so they are equal.”)

$13 + 9 = 15 + 7$  (Think: “15 is 2 more than 13 and 7 is 2 less than 9 so they are equal.”)

$16 + 26 = 8 + 34$  (Think: “8 is 8 less than 16 and 34 is 8 more than 26 so they are equal.”)

$2 + 8 = 1 + 9$  (Think: “1 is 1 less than 2 and 9 is 1 more than 8 so they are equal.”)

Once students see the relationship between expressions you can introduce unknowns.

Examples:

$21 + 56 = \square + 50$  (Think: "50 is 6 less than 56. I need to add 21 + 6 to the right side in order to make them equal so  $21 + 56 = 27 + 50$ .")

$16 + 12 = \square + 17$  (Think: "17 is one more than 16. I need to take 1 away from the 12 to make them equal so  $16 + 12 = 11 + 17$ .")

$\square + 48 = 36 + 42$  (Think: "48 is 6 more than 42. I need to subtract 6 from the 36 in order to make them equal so  $30 + 48 = 36 + 42$ .")

## MATHEMATICAL LANGUAGE

---

same	equal sign
more	equal symbol
less	inequality
equal	equality
not equal	symbol
balance	unknown
match	equation

## LEARNING EXPERIENCES

---



### Assessing Prior Knowledge

Present the following equations. Have students fill in the missing numbers.

- $\_\_ + 5 = 11$
- $8 + 1 = 6 + \_\_$
- $6 = 10 - \_\_$
- $\_\_ + 5 = 2 + 4$
- $\_\_ = 10 + 5$
- $4 + 5 = \_\_ + 3$

Ask students to explain the strategies they used to solve the problems.

Students

- correctly fill in the missing numbers
  - demonstrate an understanding of the equal sign
  - explain their strategies using correct mathematical language
-

- Explain the purpose of the symbol, such as a triangle or a circle, in an addition or a subtraction equation with one unknown.

### Suggestions for Instruction

**Note:** Students have been solving equations with one unknown in previous grades. The unknown has been represented by a line or question mark. The use of a symbol should be an easy transition for them.

- Give students the following equations and ask them to solve them.

a.  $14 + 7 = \triangle$

b.  $20 - \bigcirc = 15$

c.  $\square + 2 = 18$

d.  $16 - 4 = \hexagon$

Ask the following:


- How did you know what the questions were asking?
- How did you know where to put the answers?
- What is the purpose of the symbols (triangle, circle, square, and hexagon) in each equation?

- Create an addition or a subtraction equation with one unknown to represent a combination or separation action.
- Provide an alternative symbol for the unknown in an addition or a subtraction equation.
- Solve an addition or a subtraction equation that represents combining or separating actions with one unknown using manipulatives.
- Solve an addition or a subtraction equation with one unknown using a variety of strategies including guess and test.
- Explain why the unknown in an addition or a subtraction equation has only one value.

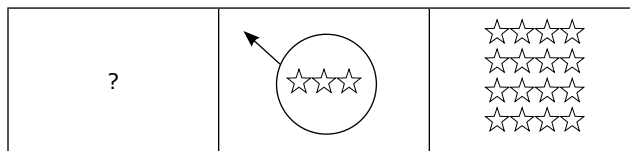
### Suggestions for Instruction

- Provide pictorial representations of addition and subtraction. Have students write an equation representing the unknown with a symbol of their choosing.

Examples:

1.   $18 + 10 = \underline{\quad}$

2.



$\underline{\quad} - 3 = 16$

- Have students create their own pictorial representations and the matching equations. Use the representations and equations to play a matching game such as concentration.
- **Equation Solving Stations:** Set up stations with different manipulative materials.

**Materials:**

- four equations for each station
- balance scale(s) and unifix cubes
- base-10 blocks
- counters
- ten frames

**Directions for Each Station:**

1. Solve the equations using the [balance scale and unifix cubes, base-10 blocks, counters, ten frames].
2. Record your solutions in your journal/notebook.

As students are working, rotate through the stations and ask questions such as the following:

- What is the equation asking you to find?
  - How do you know that the materials you are using match the equation?
  - What does the equal sign mean in the equation?
- Have students solve the following equations:
- a.  $16 + \square = 17 + 5$
  - b.  $\triangle - 6 = 17 - 5$
  - c.  $32 + 19 = \bigcirc + 20$
  - d.  $100 = 64 + \hexagon$

Ask students to explain the strategies they used to find the unknown.

- Mr. Nelson put this equation on the board.  
 $21 - 8 = \triangle$   
Tom said the answer was 12. Mark said the answer was 13.  
Is it possible that both boys have the correct answer? Why or why not?

■ **Classroom Routine—“Free Facts”**

Free facts are facts that are derived from a given equation. In order to identify these facts students must use their understanding of number relationships.

Example:

Free Facts for $16 + 18 = 34$		
$16 + 8 = 24$	$34 - 16 = 18$	$14 + 20 = 34$
$34 - 18 = 16$	$160 + 180 = 340$	$15 + 19 = 34$
Etc.		



---

### Assessing Understanding: Paper-and-Pencil Task

1. Solve these equations. Explain the strategies you used to figure them out in your journal.

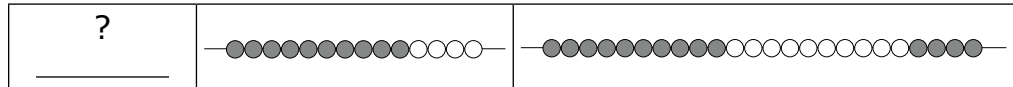
a.  $17 + 10 = \bigcirc + 4$

b.  $26 - \triangle = 12$

2. Write an equation for the picture below.



3. Write two possible equations for this picture.



$(\triangle + 14 = 24 \text{ or } \triangle - 14 = 24)$

4. Give at least eight "Free Facts" for the following equation:

$25 + 32 = 57$

---

---

## Notes





# GRADE 3 MATHEMATICS

## Shape and Space



# Grade 3: Shape and Space (Measurement)

## (3.SS.1, 3.SS.2)

### Enduring Understandings:

Elapsed time is the measure of the duration of an event.

Units within a system relate to each other (e.g., seconds, minutes, hours, day, week, month, year).

We can use measurement to sequence events.

### Essential Questions:

What activities take a minute, an hour, a day, a week, a month, or a year to complete?

How are seconds, minutes, and hours related?

How are days and months related?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
<p><b>3.SS.1</b> Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). [CN, ME, R]</p>	<ul style="list-style-type: none"> <li>→ Select and use a non-standard unit of measure, such as television shows or pendulum swings, to measure the passage of time, and explain the choice.</li> <li>→ Identify activities that can or cannot be accomplished in minutes, hours, days, months, and years.</li> <li>→ Provide personal referents for minutes and hours.</li> </ul>
<p><b>3.SS.2</b> Relate the number of seconds to a minute, the number of minutes to an hour, and the number of days to a month in a problem-solving context. [C, CN, PS, R, V]</p>	<ul style="list-style-type: none"> <li>→ Determine the number of days in any month using a calendar.</li> <li>→ Solve a problem involving the number of minutes in an hour or the number of days in a given month.</li> <li>→ Create a calendar that includes days of the week, dates, and personal events.</li> </ul>

## PRIOR KNOWLEDGE

---

Students may have had experience relating the number of days to a week and the number of months to a year. They may have had no formal experience with the passage of time related to minutes, hours, or seconds.

## BACKGROUND INFORMATION

---

Time is abstract and it cannot be seen, so it is a challenge to fully understand. Students need many experiences relating the passage of time to events in their life. They need to understand that the duration of an event/activity is determined by its start and end times and that there are standard units that we can use to describe the duration. Students should develop personal referents (length of a class or television show) to understand duration. This understanding is developed by having students do the following:

- Compare two events to determine which is longer or shorter.
- Measure the duration of events using non-standard units such as pendulums, sand timers, et cetera.
- Use standard units to measure the duration of events to ensure the measuring is uniform.

## MATHEMATICAL LANGUAGE

---

minutes	years
hours	seconds
days	calendar
weeks	passage of time
months	

## LEARNING EXPERIENCES

---



BLM  
3.SS.1.1

### Assessing Prior Knowledge: Interview

1. Use the cards included on BLM 3.SS.1.1. Have students order the months of the year.
2. Have students order the days of the week.
3. Ask the following:
  - How many months are there in one year?
  - How many days in one week?
  - How many days in two weeks?

January	February	March
April	May	June
July	August	September
October	November	December

Sunday	Monday	Tuesday
Wednesday	Thursday	Friday
Saturday		

#### Look for:

The student

- orders the months of the year
  - orders the days of the week
  - knows the number of months in a year
  - knows the number of days in a week
-

- **Select and use a non-standard unit of measure, such as television shows or pendulum swings, to measure the passage of time, and explain the choice.**

### Suggestions for Instruction

- Use non-standard units to measure the passage of time before introducing standard units. For example, have students make a pendulum using a weight suspended from a string. Students can count the number of swings it takes to complete tasks (count to 50, jump 10 times, walk to the classroom door, etc.). **Note:** Increasing or decreasing the length of the string will change the duration of time. This would be an interesting inquiry for students.
- Provide cards with pictures of familiar activities, some of which take a long time and some of which take a short time. Divide students into groups. In groups, have them sort the cards into short- and long-time activities, and discuss how they know. Share their reasons for their sorts with the class.
- **Longer or Shorter?** Have students work with a partner. Students select a duration card (BLM 3.SS.1.2). Each partner chooses one of the activities on the card. Both students start at the same time in order to determine which activity takes the longest to complete.

BLM  
3.SS.1.2

<p><b>Longer or Shorter?</b> counting backwards from 20 to 1 or counting to 100 by 10s</p>	<p><b>Longer or Shorter?</b> singing <i>Row, Row, Row Your Boat</i> or counting by 5s to 50</p>
<p><b>Longer or Shorter?</b> counting to 20 or clapping your hands 10 times</p>	<p><b>Longer or Shorter?</b> walking to the classroom door or stamping your foot 10 times</p>
<p><b>Longer or Shorter?</b> printing your first and last name or saying the alphabet</p>	<p><b>Longer or Shorter?</b> jumping 20 times or standing up and sitting down 10 times</p>
<p><b>Longer or Shorter?</b> bouncing a ball 10 times or touching your toes 8 times</p>	<p><b>Longer or Shorter?</b> saying your telephone number or tying your shoelace</p>

- **Identify activities that can or cannot be accomplished in minutes, hours, days, months, and years.**

### Suggestions for Instruction

- Reading a book such as *A Second Is a Hiccup: A Child's Book of Time* by Hazel Hutchins, illustrated by Kady MacDonald Denton, will provide real-life examples of the passage of time.
- **How long will it take?** Brainstorm a list of events/activities. Organize the list of events/activities into two groups: things that take a short time to complete (e.g., combing your hair, eating dinner, counting to 20) and things that take a long time to complete (e.g., reading a chapter book, building a house, growing a garden).  
Have students select an appropriate unit of measurement to describe the time it would take to complete each event/activity.

- **Provide personal referents for minutes and hours.**

### Suggestions for Instruction

- **How long is a minute?** Have students put their heads down. Ask them to raise their head when they think a minute is up. Ask questions such as the following:
  - What did you notice?
  - How did you decide when to put your head up?
  - If you had to do this activity again how would you decide now? What would you change?
- **Egg/Sand Timer Activity:** Set the egg timer for one minute or use a one-minute sand timer. Have students find out the following:
  - How many cubes can you join together in a minute? in two minutes?
  - How many times can you print your first name in a minute? in two minutes?
  - How far can you count in a minute? in two minutes?
  - How many times can you hop in a minute? in two minutes?

- **Second Hand:** Draw students' attention to the second hand on an analog clock. Use the egg or sand timer to show that one revolution (from the 12 to the 12) of the second hand is equal to one minute on the timer(s).
- **Just a Minute:** Provide pairs with a stopwatch or a clock with a second hand. One student acts as the timer, and the other as the estimator.

Student Instructions:

- a. Timer: Say "Go!"  
Estimator: Without looking at the time, signal when you think a minute has passed.  
Timer: Tell the Estimator whether the estimate was over or under a minute.
  - b. Try it three times, and see if the estimates improve. Record your results.
  - c. Switch roles.
- **How long does it take?** Work with a partner, list all the activities you can think of that take
    - about 5 minutes to do
    - about 10 minutes to do
    - about 30 minutes to do
    - more than 30 minutes to do
 Be prepared to share your list with the class. How are the lists the same?  
How are they different?
  - **What can you do in an hour?** Have students relate daily events/activities to one hour. Set a timer or an alarm clock and have students connect the hour to classroom activities (e.g., math class, lunch, TV shows, playing video games).
  - **A Day in the Life Of:** Have students keep a record of the activities they do over the course of a day. For each activity have them estimate the number of minutes or hours it takes to complete.

Examples:

Activity	Time
Brushing teeth	2 minutes
Getting dressed	10 minutes
Eating breakfast	20 minutes





---

### Assessing Understanding: Journal Entry

Would you use minutes or hours to measure

- the length of the school day?
- the length of recess?

Explain your choices.

---

- **Solve a problem involving the number of minutes in an hour.**

### Suggestions for Instruction

- **How long does it take?** Ask students what they notice about the face of an analog clock (with a second hand). Look for numbers 1 to 12, 3 hands, tick marks between the numbers, et cetera. Point out that the every time the second hand moves to one of the small tick marks a second has passed. Have students determine the number of seconds in a minute by counting as the second hand makes one revolution.
- **Minutes in an hour?** Review the observations made about the face of the analog clock. Introduce the long hand as the minute hand. Have students watch to see what happens to the minute hand as the second hand completes one revolution (It moves one tick mark.). Ask students how many tick marks there are altogether (60). Point out that when the minute hand completes one revolution an hour has passed. Have students use this information to determine the number of minutes in an hour.



---

### Assessing Understanding: Problems with Time

- Mike's video is 90 minutes long. Is that more or less than an hour? Explain how you know.
- Pat's mother said that she could play at her friend's for either 2 hours or for 150 minutes. Which one should she choose if she wants to play for as long as she can? Explain your choice.

#### Look for:

The student

- understands that there are 60 seconds in a minute
  - understands that there are 60 minutes in an hour
  - applies this information in a problem-solving situation
-

- **Determine the number of days in any month using a calendar.**
- **Solve a problem involving the number of days in a given month.**

## Suggestions for Instruction

- **How many days?** Have students work in groups. Give each group a calendar. Ask them to explore the calendar to determine the number of days in each month. What did they notice?
- **Thirty days . . .** Introduce the rhyme: *Thirty days hath September, April, June, and November, and all the rest have thirty-one, except, February having 28. But Leap Year coming once in four, February then hath one day more.*
- **Hands Can Help:** Show students how they can use their hands to determine the number of days in each month. Have students make a fist. Touch the first knuckle and say January. Touch the space between the knuckles and say February. The next knuckle is March, et cetera. After saying July, go back to the first knuckle for August and continue. Months landing on a knuckle have 31 days and those landing in the spaces have 30 except for February.
- **February Is Special!** Talk to students about the fact that February has only 28 days except in a leap year when it adds one more day. Explain that this is because a year is made up of a little bit more 365 days, so every four years we add an extra day (leap year).
- **Calendar Problems:**
  - Mira is 2 years old. How many months is that?
  - Mr. and Mrs. Jones hired Zack to walk their dog for the month of October.
    - If Zack walks the dog twice a day, how many times will he walk the dog during the month?
  - Today is March 1.
    - Jillian’s birthday is on May 2.
    - How many more days does Jillian have to wait until her birthday?
    - Explain how you know.

Have students create their own calendar problems to share with the class.

- **Create a calendar that includes days of the week, dates, and personal events.**

## Suggestions for Instruction

BLM  
3.SS.1.3

- **My Year:** Provide students with a calendar template (BLM 3.SS.1.3) for each month. Have them fill in the month, days of the week, and dates. With the help of their family, have students put important events on their calendar (birthdays, holidays, anniversaries, special events, etc.). Students can use the calendar to record special school events as well.  
**Note:** This activity could be done using technology.
- **100-Day Investigation:** Many classrooms celebrate the 100th day of school. Have pairs of students use their calendars to determine when the 100th day of school will occur. They will need to figure out how many school days there are each month beginning in September, but this information does not need to be given to students before they attempt the problem. Provide scaffolding to groups when appropriate. (Teachers will have to decide ahead of time whether in-service/administrative days will count as school days for this problem.)



---

### Assessing Understanding: Paper-and-Pencil Task or Interview

What unit of time would you use to measure the following?

- a ride to school by bike
- a picnic
- a journey to the moon
- a holiday
- a baby's age
- an adult's age
- recess
- a blink of an eye

Which month(s)

- is the shortest
  - have 30 days
  - have 31 days
-

## Grade 3: Shape and Space (Measurement) (3.SS.3)

### Enduring Understandings:

Objects have distinct attributes that can be measured with appropriate tools.  
Standard units provide a common language for communicating measurement.  
A measurement must contain a number and a unit.

### Essential Questions:

Why do we need standard measurement units?  
How are centimetres and metres related?  
What referent can you use to estimate length in centimetres?  
What referent can you use to estimate length in metres?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
<p><b>3.SS.3</b> Demonstrate an understanding of measuring length (cm, m) by</p> <ul style="list-style-type: none"><li>■ selecting and justifying referents for the units cm and m</li><li>■ modelling and describing the relationship between the units cm and m</li><li>■ estimating length using referents</li><li>■ measuring and recording length, width, and height [C, CN, ME, PS, R, V]</li></ul>	<ul style="list-style-type: none"><li>→ Provide a personal referent for one centimetre and explain the choice.</li><li>→ Provide a personal referent for one metre and explain the choice.</li><li>→ Match a standard unit to a referent.</li><li>→ Show that 100 centimetres is equivalent to 1 metre by using concrete materials.</li><li>→ Estimate the length of an object using personal referents.</li><li>→ Determine and record the length or width of a 2-D shape.</li><li>→ Determine and record the length, width, or height of a 3-D object.</li><li>→ Draw a line segment of a given length using a ruler.</li><li>→ Sketch a line segment of a given length without using a ruler.</li></ul>

## PRIOR KNOWLEDGE

---

Students may have had experience measuring length using non-standard units only.

## BACKGROUND INFORMATION

---

Students have had opportunities with measurement attributes using direct comparison and non-standard units. The use of non-standard units allows students to see the need for a standard unit. There should be ample discussion with the students to explore why standard units are necessary to ensure consistency when measuring. Centimetre and metre will be two standard units of measurement that will be introduced to students. It is important for students to have the time to discover personal referents for these standard units of measurement. Personal referents allow students to visualize measurement and make estimates more accurate.

**Iteration** means the act of repeating. In measurement, using a unit smaller than the object being measured and repeating it end-to-end is an example of iteration.

**Referent** is a point of reference used to compare in estimation (e.g., using the width of the baby finger as a referent for a centimetre).

**Length** is the distance from one end of an object to the other end, commonly measured in units of metres, centimetres, millimetres, and kilometres.

**Height** is the measurement from base to top.

**Width** is one dimension of a 2-D or 3-D figure.

**Standard unit of measure** is a unit commonly used to indicate the length, area, volume (capacity), mass, or temperature of an object; it is also used to indicate the passage of time.

**Note:** Students need to be taught how to use measuring tools (rulers, metre sticks). Some rulers leave a space before the zero and others use the end of the ruler as zero.

Students also need to learn the difference between the marks on the ruler (millimetres and centimetres). Using centimetre cubes and lining them up on the ruler will help students see which numbers represent centimetres. (Millimetres are introduced in Grade 5.)

## MATHEMATICAL LANGUAGE

---

measure	width
measurement	ruler
standard unit	metre stick
centimetre	referent
metre	estimate
length	line
height	

## LEARNING EXPERIENCES

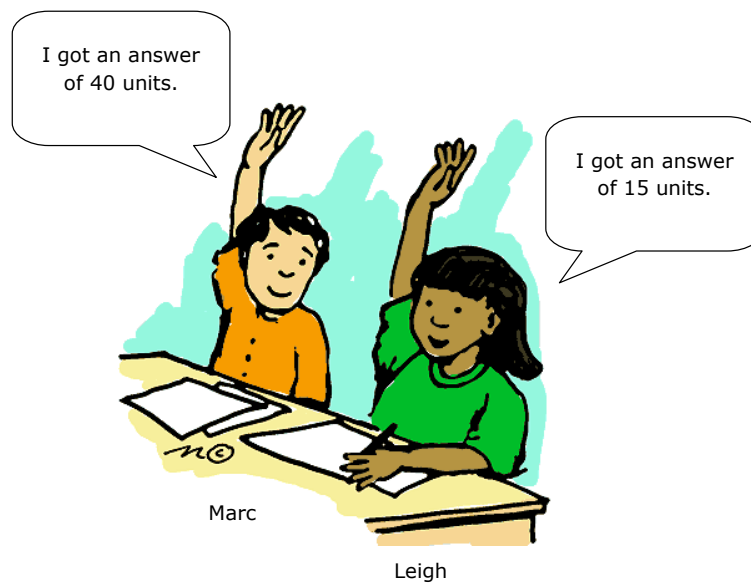
---



### Assessing Prior Knowledge: Measurement Stations

1. Station 1: Provide several objects along with a non-standard unit, such as paper clips, craft sticks, cubes, et cetera. Students measure the length and/or height of the objects and record their findings.
2. Station 2: Draw two different curved or zigzagged lines on sheets of paper. Provide string and a non-standard unit. Students measure the length of the lines.
3. Station 3: Present a problem such as the following one. Have students write an explanation in their journals.

Marc and Leigh measured the table.



Explain how this might have happened.

Observe students as they work.

**Look for:**

The student

- is able to use non-standard units to measure length and height
  - measures from one end to the other without gaps or overlaps
  - is able to record their measurements using a number and a unit
  - is able to measure a curved or zigzagged line
  - understands that the size of the unit determines the number of units needed (The larger the unit the fewer needed. The smaller the unit the more needed.)
- 

- **Determine and record the length or width of a 2-D shape.**
- **Determine and record the length, width, or height of a 3-D object.**
- **Provide a personal referent for one centimetre and explain the choice.**
- **Match a standard unit to a referent.**
- **Estimate the length of an object using personal referents.**

### Suggestions for Instruction

- **Creating the Need for a Standard Unit:** Read the book *How Big Is a Foot?* by Rolf Myller. This is the story about a young apprentice that is charged with making a bed for the queen. The king measures the distance around the queen using his foot as the unit. He sends those measurements to the apprentice who then uses his small feet as the unit to make the bed. Needless to say the bed is too small for the queen and the apprentice is sent to jail. While there, he comes to realize that the king's feet are bigger than his and that they need some sort of standard unit.  
Read the book but stop when the apprentice is thinking about the problem in jail.  
Have students write a letter to the apprentice explaining why the bed was too small for the queen.  
Discuss their explanations. Ask students how the marble copy of the king's foot helped the apprentice. Highlight the importance of the standard unit.
- **Centimetres:** Give students a centimetre cube (base-10 materials, centicubes, Cuisenaire rods). Explain that the length, width, and height of the cube are all equal to one centimetre. Have students use the cubes to measure various small objects recording the measurements with a number and centimetres as the unit.

- **Centimetre Walk:** Provide each student with a 1-cm cube or a 1-cm paper strip. Take students on a walk and have them measure and record items 1 cm long. Ask small groups to sort their findings and to display their results.
- **Measuring Me:** Have students work with a partner to measure various body parts using centimetres. Provide rulers and measuring tapes. Be sure that students know how to use the measuring tools.

Examples:

Body Part	Measurement
arm	
leg	
finger	
nose	
hand	
foot	

- **Measurement Detective:** Have students go on a hunt in the classroom for objects that are of a specified length.

Examples:

Length	Object
2 cm	
5 cm	
10 cm	
15 cm	
20 cm	
30 cm	

**Note:** This activity could be done at home instead of the classroom.

- **Determining a Referent for a Centimetre:** Have students take out their pencils. Ask them how they could measure the pencil without using a ruler. Brainstorm some solutions. If students do not come up with a referent, ask them to look at their hands. Ask, "Is there some part of your hand that might be about one centimetre in length or width?" Have students measure to check.

Once the referent has been determined have students use it to estimate the length or width of various objects and then measure to check their estimates.



BLM  
3.SS.3.1

■ **Wiggler Derby Game**

**Objective:** to accumulate a large number of centimetres, or to attain a large number of reasonable estimates

**Number of players:** 2

**Materials:** bags containing an even number of mixed up wigglers (e.g., ribbons, yarn, string, paper strips, pipe cleaners) rulers, tape measures, or metre sticks

**Directions:**

**Note:** Decide with the students the range for reasonable estimates (e.g., within 5 cm, 10 cm, and so on) based on the lengths of the wigglers provided and students' experiences before beginning the game.

In pairs, students take turns fishing out a wiggler, estimating the length, and then measuring the length. Record on the chart (BLM 3.SS.3.1). Continue taking turns until all the wigglers have been fished out of the bag.

<b>Wiggler Derby</b>			
<b>Wiggler</b>	<b>Estimate</b>	<b>Measurement</b>	<b>Reasonableness</b>
Red yarn	30 cm	24 cm	yes
Yellow yarn	22 cm	26 cm	yes

■ **Measuring Shapes/Objects**

- Give students a 2-D shape. Ask them to measure the length and width of the shape.
- Give students a 3-D object such as a cereal box, and have them measure the length, width, and height of the box.

■ **I Spy:** Play a game of "I Spy" using measurement. For example, "I spy with my little eye something in the classroom that is 10 cm long."

■ **Four (or more) Star Measurer:** Develop measurement criteria with students. Ask them to identify what makes a good measurer. Record their suggestions. Select the key elements for a "Four Star Measurer" chart.

Examples:

- I used a referent to estimate the length, width, or height.
- I started measuring at one end of the object.
- I started measuring at the zero mark on the ruler.
- I recorded using both a number and a unit.



---

## Assessing Understanding: Performance Tasks

1. Provide a collection of 2-D shapes and 3-D objects.

### Student Directions:

- Pick one 2-D shape. Measure its length and width. Record your results.
- Pick one 3-D object. Measure its length, width, and height. Record your results.

2. Provide a collection of different objects such as popsicle/craft sticks, straws, stir sticks, toothpicks, pipe cleaners, et cetera.

### Student Directions:

- Pick two of the objects.
- Estimate their length using your referent. Record.
- Use a ruler to find the actual measurement. Record.

**Self-Assessment:** Have students use the “Four Star Measurer” criteria to self-assess.

### Look for:

The student

- is able to measure the length, height, and width of a 3-D object accurately
- is able to measure the length and width of a 2-D shape accurately
- is able to use a referent to make a reasonable estimate

- 
- **Show that 100 centimetres is equivalent to 1 metre by using concrete materials.**
  - **Provide a personal referent for one metre and explain the choice.**

## Suggestions for Instruction

- **Introducing the Metre:** Present this problem. The classroom flooring is going to be replaced (or carpeted). The flooring company wants to know the length and width of the room so they can order enough flooring. How can we measure it?

Discuss possible solutions.

Have students measure the length of the room using a 30-cm ruler. Talk about any difficulties they might have had using a 30-cm ruler (small tool and unit, had to keep moving the ruler, had to keep track of the number of cm so they could add them up at the end, etc.).

Tell them that there is a larger unit that might be easier to use. Show students the metre stick. Ask them to estimate its length in centimetres. Place centicubes or base-10 materials along the top of the metre stick to show that it is equal to 100 cm.

Have them measure again using the metre stick.

Talk about why they might have found it easier to use (fewer iterations, count in metres rather than cm, faster, etc.). Students might have found it difficult to use the metre stick when there was less than a metre to measure. If so, discuss possible solutions.

- **Metre Search:** Have students find objects in the classroom that are about 1 metre, more than 1 metre, or less than 1 metre. Make a class list.
- **Referent for a Metre:** Let students experiment to find a personal referent for a metre. Provide an opportunity for sharing their referents and explaining their choices.
- **Measurement Game:** Students work in groups of 2 to 4.

**Note:** Before playing the game, have the class decide the degree of accuracy needed (e.g., within 5 cm [either higher or lower than the actual measure]).

**Materials:** 30-cm ruler, metre stick, tape measure for each group

**Directions:**

- Player 1 selects an object in the classroom and gives the dimension to be measured (length, width, height).
  - Each player writes down an estimate. (Encourage students to use their referent.)
  - Player 1 measures to check.
  - Those players that meet the degree of accuracy decided on previously score 1 point.
  - Player 2 takes a turn.
  - Play continues until one player has 10 points.
- **I Spy:** Extend the I Spy game to include measurements in metres.



---

## Assessing Understanding: Performance Assessment

1. Give the student adding machine tape or string. Ask them to cut a piece that is about one metre long (without the use of a measuring tool). Ask them to explain how they decided where to cut. Have them measure to check the accuracy of their estimate.
2. Ask the student to measure an object in the classroom using metres. Observe.

### Look for:

The student is able to

- use a referent to make a reasonable estimate of the size of a metre
  - explain how they used their referent to estimate
  - measure accurately in metres
  - record measurements using a number and a unit
- 

- **Draw a line segment of a given length using a ruler.**
- **Sketch a line segment of a given length without using a ruler.**

### Suggestions for Instruction

- **Line Segment Drawing:** Have students measure the length of specified parts of their body and then use a ruler to draw a line segment to equal that length.

Example:

Body Part	Length	Line Segment
Finger	6 cm	
Nose		

■ **Line Segment Estimation Game**

Work in pairs.

**Materials:** 10- or 20-sided die, rulers

**Directions:**

1. Player 1 rolls the die.
2. Both players draw a line equal to the number shown in centimetres without using a ruler.
3. Players then measure to check.
4. If the line drawn is close to the number, the player scores a chip or a point.
5. Player 2 rolls.
6. The game continues until one player has 5 chips/points (or more).

## Grade 3: Shape and Space (Measurement) (3.SS.4)

### Enduring Understandings:

Objects have distinct attributes that can be measured with appropriate tools.  
Specific tools measure specific attributes.

### Essential Questions:

How are grams and kilograms related?  
What referents can I use to estimate grams and kilograms?  
What tools can I use to measure mass?  
Will changing the shape of an object affect its mass?

### SPECIFIC LEARNING OUTCOME(S):

### ACHIEVEMENT INDICATORS:

<p><b>3.SS.4</b> Demonstrate an understanding of measuring mass (g, kg) by</p> <ul style="list-style-type: none"><li>■ selecting and justifying referents for the units g and kg</li><li>■ modelling and describing the relationship between the units g and kg</li><li>■ estimating mass using referents</li><li>■ measuring and recording mass</li></ul> <p>[C, CN, ME, PS, R, V]</p>	<ul style="list-style-type: none"><li>→ Provide a personal referent for one gram and explain the choice.</li><li>→ Provide a personal referent for one kilogram and explain the choice.</li><li>→ Match a standard unit to a referent.</li><li>→ Explain the relationship between 1000 grams and 1 kilogram using a model.</li><li>→ Estimate the mass of an object using personal referents.</li><li>→ Determine and record the mass of a 3-D object.</li><li>→ Measure, using a scale, and record the mass of everyday objects using the units g and kg.</li><li>→ Provide examples of 3-D objects that have a mass of approximately 1 g, 100 g, and 1 kg.</li><li>→ Determine the mass of two similar objects with different masses, and explain the results.</li><li>→ Determine the mass of an object, change its shape, re-measure its mass, and explain the results.</li></ul>
---	---

## PRIOR KNOWLEDGE

---

Students may have had experience

- determining which of two objects is heaviest/lightest by comparing
- selecting a non-standard unit for measuring the mass of an object and explaining why it was chosen
- estimating, measuring, and recording the mass (weight) of an object using non-standard units

## BACKGROUND INFORMATION

---

Students will be estimating and measuring mass using the gram and kilogram. Estimating mass can be a challenge because the object's size and shape are not directly related to its mass. Students need ample exploration with developing a sense of what a gram and kilogram feel like.

Ensure all scales (pan balance, kitchen scales, beam scales) are accurate prior to having students use them.

**Mass:** A measure of how much matter there is in an object. In the international system (SI) of units, the units of mass include grams (g) and kilograms (kg).

In daily life, the terms **mass** and **weight** are virtually interchangeable, but in reality they are not the same thing. An object on the moon weighs less than it does on Earth, and in space is weightless. The mass of the object does not change, regardless of where it is. Mass is measured with a balance.

**Weight:** A measure comprising a combination of the mass of an object and the pull of gravity on that mass. Weight is measured with a scale.

Students should be introduced to the correct terminology.

## MATHEMATICAL LANGUAGE

---

mass	kilogram
weight	compare
scale	estimate
balance	estimation
gram	referent



### Assessing Prior Knowledge: Performance Task

1. **Heavier or Lighter:** Have students compare (without a measuring tool) two objects to determine which is heavier/lighter.
2. **How Heavy?** Have students use non-standard units to estimate and then measure the mass of a given object.

Observe to determine if the student is able to

- determine the heaviest/lightest object using direct comparison
- use a non-standard unit to estimate the mass of an object
- use a non-standard unit to find the mass of an object

- **Provide a personal referent for one kilogram and explain the choice.**
- **Match a standard unit to a referent.**

### Suggestions for Instruction

- **Introducing the Kilogram:** Introduce the kilogram and make a list of items students know are purchased by the kilogram. Collect advertisements and package labels showing kilogram amounts.  
Use the terms *kilo* and *kilogram* so students become familiar with both. Introduce the format *kg* for recording.  
Have students make a class collection of kilogram items (i.e., items with a mass of 1 kilogram). Ask them to label each item and its mass.
- **Making a Kilogram:** In small groups have students make their own kilogram mass. Use materials such as a zippered bag and pennies or cubes, a container and sand, a zippered bag and marbles, et cetera. Use the balance scale and the standard kg mass to check their work.
- **Kilogram Referent:** Using their kilogram mass or a standard kg unit, have students see if they can find something in the classroom that can serve as a referent for the kg. Share findings with the class.
- **How much does it weigh?** Have students use a balance scale to find the mass of a set of objects. Make sure that some of the objects weigh either more or less than a kg. Have a class discussion around any difficulties they had completing the task. What happened when the object didn't weigh exactly one or more kilograms? Is there a smaller unit we can use to measure mass? Introduce the gram. (A centimetre cube weighs one gram.)



- **Explain the relationship between 1000 grams and 1 kilogram using a model.**
- **Provide a personal referent for one gram and explain the choice.**
- **Estimate the mass of an object using personal referents.**
- **Provide examples of 3-D objects that have a mass of approximately 1 g, 100 g, and 1kg.**
- **Determine and record the mass of a 3-D object.**
- **Measure, using a scale, and record the mass of everyday objects using the units g and kg.**

### Suggestions for Instruction

- **How are kilograms and grams related?** Have students test to find out how many centimetre cubes it takes to balance a kilogram mass (1000).  
The common unit for measuring mass is the kilogram; a gram is one-thousandth of a kilogram. Introduce the format g to record grams.
- **Referent for a Gram:** Have students explore (at school or at home) to determine a personal referent for a gram (a counter, paper clip, a small bean, a raisin, a jellybean, etc.). Share their findings with the class.
- **How much is 100 grams?** Ask students if they have ever been to a store that has a bulk food section. Have they ever bought anything from that section? Point out that bulk food prices usually tell you how much it costs to buy 100 grams. How much is 100 grams?  
Provide several different sets of objects (cubes, counters, craft sticks, pennies, or wrapped candies), zippered bags, a balance scale, and a 100-gram mass. Have students estimate (using their referent) how much of each object will make 100 grams. Put the objects in zippered bags. Measure to check.  
Ask why you can get more of one item than another if both bags weigh 100 grams (weight of each piece differs, the size of each piece differs, etc.).
- **Mass Ordering:** Have students bring in a collection of food boxes (cereal, crackers, pasta, etc.). Look at the boxes to find the mass. Order the boxes from lightest to heaviest. What boxes could be put together to make about a kilogram of food? 500 grams of food?

BLM  
3.SS.4.1

- **Mystery Eggs (Containers):** Fill plastic eggs or small sealable containers with a variety of items (different masses) such as cotton balls, flour, salt, pennies, pasta, rice, beans, chocolate chips, foam chips, et cetera. Seal the eggs/containers with tape and label each with a number or letter.

Have students use their referent to estimate the mass of each one. Record.

Now, have students use a balance scale and standard mass units to find the actual weight of the eggs/containers. Record on BLM 3.SS.4.1.

Students can calculate the difference between their estimate and the actual weight.

Example:

<b>Egg/Container</b>	<b>Estimate</b>	<b>Actual</b>	<b>Difference</b>
A	200 g	220 g	20 g
B			



---

### Assessing Understanding: Performance Task

1. **Estimating Mass:** Have students use their referents to estimate
  - the mass of an object in grams
  - the mass of an object in kilograms and grams
2. **Measuring Mass:** Have students measure the objects in question 1 to check their estimates.

#### Look for:

The student is able to

- use a referent to estimate the mass of an object in kilograms
  - use a referent to estimate the mass of an object in grams
  - measure accurately in kilograms
  - measure accurately in grams
  - measure accurately in kilograms and grams
-

- **Determine the mass of two similar objects with different masses, and explain the results.**
- **Determine the mass of an object, change its shape, re-measure its mass, and explain the results.**

## Suggestions for Instruction

- **Similar Objects, Different Mass?** Read the book *The Dragon's Scales* by Sarah Albee, illustrated by John Manders. This story addresses misconceptions related to measuring mass: the more objects the greater the mass and the larger the object the greater the mass.

Discuss the misconceptions with the students.

**Note:** Provide experiences with objects that are small but have a greater mass than large objects. Use a variety of shapes and sizes of objects, such that mass cannot be determined by appearance.

- **Same Size, Different Mass?** Provide similar objects. For example,
  - lunch bags containing different materials (e.g., one with feathers or marshmallows and one with cubes or craft sticks)
  - a golf ball and a table tennis ball
  - a baseball and a tennis ball

Have students find the mass of each.

Ask them to explain why the mass differs if the objects are the same size (e.g., made of/contain different materials, solid or hollow, different purposes).

- **Parcel Comparison:** Have students compare 3 parcels that are exactly the same size, but with different masses (under 1 kg). Discuss everyday applications.



- **Will it change?** Give students a piece of play dough or plasticine. Have them roll it into a ball and then find its mass.

Tell students that they are going to use all of the material in their ball to make something different. Do they think that the mass will change or will it stay the same? Record their thoughts.

Reshape the material and weigh it again.

What did they find out? Why did this happen? (The object will have the same mass unless additional material has been added or some has been taken away.)



---

### Assessing Understanding: Paper-and-Pencil Task

Answer the following questions in your journal/notebook:

1. Is it possible for a small object to weigh more than a large one? Explain your thinking.
  2. Two objects are the same size. Will they have the same mass? Explain your thinking.
  3. Why does the mass of an object stay the same when you change its shape?
-

## Grade 3: Shape and Space (Measurement) (3.SS.5)

### Enduring Understanding:

Perimeter is a measure of length.

### Essential Questions:

What is perimeter and how is it measured?

Where is perimeter used in the real world?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
<p><b>3.SS.5</b> Demonstrate an understanding of perimeter of regular and irregular shapes by</p> <ul style="list-style-type: none"><li>■ estimating perimeter using referents for centimetre or metre</li><li>■ measuring and recording perimeter (cm, m)</li><li>■ constructing different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter</li></ul> <p>[C, ME, PS, R, V]</p>	<ul style="list-style-type: none"><li>➔ Measure and record the perimeter of a regular shape, and explain the strategy used.</li><li>➔ Measure and record the perimeter of an irregular shape, and explain the strategy used.</li><li>➔ Construct a shape for a given perimeter (cm, m).</li><li>➔ Construct or draw more than one shape for the same perimeter.</li><li>➔ Estimate the perimeter of a shape (cm, m) using personal referents.</li></ul>

### PRIOR KNOWLEDGE

---

Students may have had no prior experience with perimeter.

### BACKGROUND INFORMATION

---

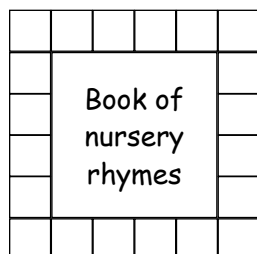
Students need to be immersed in investigations using a variety of concrete materials to help students develop strategies for finding the perimeter of an object. When students construct the meaning for perimeter they will begin to recognize the outside of any object as its perimeter. This idea will help students' later understanding of area. Estimating perimeter by using personal referents can help students verify if their measurements are reasonable.

**Perimeter** is the distance around a closed figure.

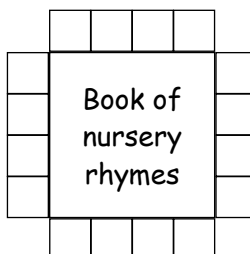
**Note:** There is no expectation that students are taught a formula for finding perimeter.

When measuring the perimeter using blocks or tiles, show students the correct method, as illustrated below:

Incorrect method:



Correct method:



## MATHEMATICAL LANGUAGE

---

perimeter

regular

irregular

construct

estimate

circumference

distance around

## LEARNING EXPERIENCES

---



### Assessing Prior Knowledge

Due to the fact that students have not had previous experience with perimeter, it is important to assess their prior knowledge.

Read a book such as *Racing Around* by Stuart J. Murphy, illustrated by Mike Reed.

This book tells the story of a young Mike's participation in a 15-kilometre race that takes place on "Perimeter Path." Before the race Mike rides around the athletic field and the zoo. His brother uses his odometer to figure out the perimeter for each location. The illustrations clearly demonstrate the meaning of perimeter.

---

- **Measure and record the perimeter of a regular shape, and explain the strategy used.**
- **Measure and record the perimeter of an irregular shape, and explain the strategy used.**

### Suggestions for Instruction

BLM  
3.SS.5.1

- **Finding Perimeter:** Provide students with a selection of items. Have them do the following:
  - Select an object (e.g., book) from the group. Using cubes, estimate and record the distance around (or the perimeter of) the object. Now measure and record the distance around (or perimeter).
  - Select a second object that has a greater perimeter than the first one measured. Estimate, measure, and record its perimeter.
  - Repeat step (b) with a third object, and then order these 3 objects based on their perimeters, from least to greatest or greatest to least.

Object	Unit Used for Measuring	Estimate	Actual
Object A			
Object B			
Object C			

- **Irregular Shapes and Perimeter:** Show students some irregular shapes. Ask how they might find the perimeter. (Use a ruler and measure each side then add the dimensions together.) Measure and record their findings.
- **“Really” Irregular:** Ask students how they think they would be able to find the perimeter of an outline of their hand or foot. Will any of the measurement tools we have been using to measure so far work? Why?

Let students investigate to see if they can find the perimeter.

Discuss their findings/difficulties.

If it does not come up in the discussion, suggest using a piece of string first and then measuring with a standard measuring tool.

Have students find the perimeter of their hand and foot.

BLM  
3.SS.5.2

- **Conflicting Cylinders:** Provide a set of differently sized cylinders. Label each with a number or letter.

Have students select a cylinder and then estimate whether the circumference (perimeter of a circle) of a cylinder is less than, equal to, or greater than its height.

Measure to check. Record their findings.

Example:

Cylinder	Circumference is <i>less than, equal to, or greater than the height.</i>	Circumference (Perimeter)	Height	Correct?
1	less than	15 cm	12 cm	no
2				

- **Larger Perimeters:** Have students measure the perimeter of the classroom using metres. If possible, have them measure the perimeter of the gymnasium or other larger space.

- **Construct a shape for a given perimeter (cm, m).**
- **Construct or draw more than one shape for the same perimeter.**

### Suggestions for Instruction

- **Same Perimeter, Different Shape:** Give students this problem:  
The perimeter of a shape is 12 cm. What might the shape look like? Find more than one possibility. Use tiles to make the shape. Record your shapes on centimetre-grid paper.  
**Note:** Use only horizontal or vertical lines. On centimetre-grid paper, the units formed by diagonal lines are not equal to the units formed by horizontal and vertical lines.  
(To prove this to students use two identical squares of paper. Fold one square on the diagonal. Match the diagonal to one of the sides of the second square. Students will notice that the diagonal is longer than the side.)
- **Tiles Problem:** Give students 8 tiles. Ask them to make a rectangle and find the perimeter.  
Now, have them use the 8 tiles to make other shapes.  
Record the shapes on centimetre-grid paper and record the perimeter of each.



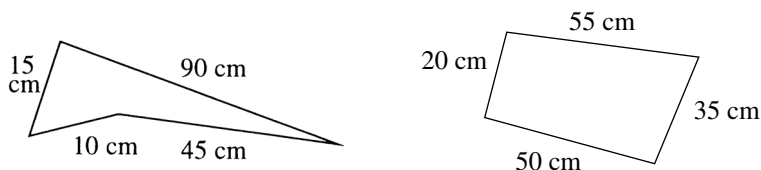
How many different shapes can you make?


- **Rope/Cord Investigation:** Give students a rope or strong cord 160 cm in length. Tie the rope together at the ends. Ask students, in groups of 4, to make as many different shapes as possible with the rope. Measure the sides of each shape and record the perimeter in a table.

Discuss the results.

**Note:** Tying a knot decreases the actual length, so you may want to provide extra length to allow for the knots.

Examples:



Shape (approximately 160 cm in length)	Perimeter

- **Estimate the perimeter of a shape (cm, m) using personal referents.**

### Suggestions for Instruction

- **Perimeter Search:** Have students select objects in the classroom, estimate their perimeter using their referents, and then measure to check.
- **Seating Problem:** Use the book *Spaghetti and Meatballs for All: A Mathematical Story* by Marilyn Burns, illustrated by Debbie Tilley, to explore perimeter in a problem-solving situation. In this book, the family plans a family dinner for 32 people. Throughout the story, they change the configuration of the tables as more guests arrive in order to have everyone sit together. Students can use tiles and chips to help solve the seating problems.



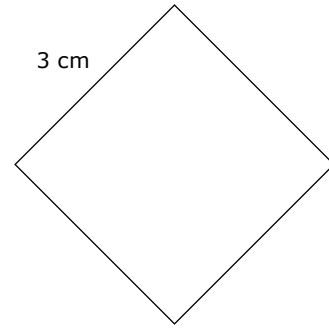
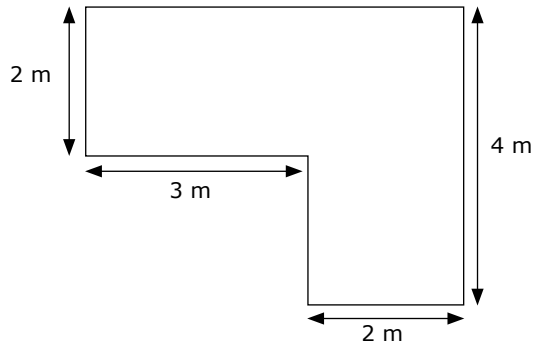
BLM  
3.SS.5.3

---

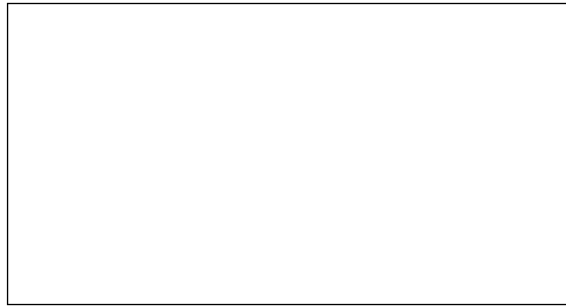
## Assessing Understanding: Paper-and-Pencil Task

### Perimeter

1. Mrs. Smith wants to make a rectangular garden with a perimeter of 24 metres. What might her garden look like? Give 3 possibilities. Record your shapes on centimetre-grid paper.
2. Find the perimeter of the following shapes:



3. Measure to find the perimeter.





**Measurement Activity Day:** Have students participate in a variety of measurement tasks.

Examples:

1. **Heavy Order to Fill:** (Provide 6 different items of varied weights.) Arrange the items from heaviest to lightest. Check by weighing or comparing by hand.
2. **Standing Long Step:** From the starting line take one giant step. Estimate the distance to the nearest centimetre from the starting line. Measure to check.
3. **Time on Your Hands:** Estimate how many times you can do certain activities (e.g., jump, hop on one foot, touch your toes) in 15 seconds, 30 seconds, and 1 minute.
4. **Around and About:** Estimate the distance around (perimeter) given objects and then measure to check.
5. **Suck It In:** Hold a 10-cm square of paper at the end of a straw by sucking in. Walk until the paper drops. Estimate and then measure the distance in metres.
6. **Backward Jump:** Stand with your feet together and take one jump backward. Estimate and then measure the distance.
7. **Paper Plate Toss:** Toss a paper plate like a discus. Estimate the distance in metres. Measure to check.
8. **Big Foot:** (Give students an extra large outline of a foot.) Estimate and then measure the perimeter.

Observe students as they work through the activities.

### Student Self-Assessment:

I can

- estimate centimetres and metres using my referents
- measure centimetres and metres accurately
- estimate the passage of time using referents
- estimate perimeter using referents
- measure perimeter accurately

My goal is \_\_\_\_\_.

## Grade 3: Shape and Space (3-D Objects and 2-D Shapes) (3.SS.6, 3.SS.7)

### Enduring Understandings:

Geometric shapes and objects can be classified by attributes.

Objects can be described and compared using geometric attributes.

A 3-D object can be analyzed in terms of its 2-D parts.

### Essential Questions:

How can 3-D shapes be described?

How can 2-D shapes be compared and sorted?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
<p><b>3.SS.6</b> Describe 3-D objects according to the shape of the faces, and the number of edges and vertices. [C, CN, PS, R, V]</p>	<ul style="list-style-type: none"> <li>→ Identify the faces, edges, and vertices of a 3-D objects, including cubes, spheres, cones, cylinders, pyramids, and prisms.</li> <li>→ Identify the shape of the faces of a 3-D object.</li> <li>→ Determine the number of faces, edges, and vertices of a 3-D object.</li> <li>→ Construct a skeleton of a 3-D object, and describe how the skeleton relates to the 3-D object.</li> <li>→ Sort a set of 3-D objects according to the number of faces, edges, or vertices.</li> </ul>
<p><b>3.SS.7</b> Sort regular and irregular polygons, including</p> <ul style="list-style-type: none"> <li>■ triangles</li> <li>■ quadrilaterals</li> <li>■ pentagons</li> <li>■ hexagons</li> <li>■ octagons</li> </ul> <p>according to the number of sides. [C, CN, R, V]</p>	<ul style="list-style-type: none"> <li>→ Classify a set of regular and irregular polygons according to the number of sides.</li> <li>→ Identify regular and irregular polygons having different dimensions.</li> <li>→ Identify regular and irregular polygons having different orientations.</li> </ul>

## PRIOR KNOWLEDGE

---

Students may have had experience

- sorting 3-D objects and 2-D shapes using two attributes
- describing, comparing, and constructing 3-D objects and 2-D shapes
- identifying 2-D shapes and 3-D objects in the environment

## BACKGROUND INFORMATION

---

Pierre van Hiele and Dina van Hiele-Geldof (cited in Van de Walle, Karp, Lovin, and Bay-Williams 346–351), mathematics teachers from the Netherlands in the 1950s, researched the development of geometry thinking. Through their research they identified five sequential levels of geometric thought. Two of the levels are listed below. Most Grade 3 students will be at Level 0 or 1.

There are four characteristics of these levels of thought:

- The levels of geometric reasoning/understanding are sequential. Students must pass through all prior levels to arrive at any specific level.
- These levels are not age-dependent.
- Geometric instructional experiences have the greatest influence on advancement through the levels.
- Instruction or language at a higher level than the level of the student may inhibit learning.

**Level 0** (sometimes labelled as Level 1): **Visual**

At this level students can name and recognize shapes by their appearance, but cannot specifically identify properties of shapes. Students may think that a rotated square is a “diamond” and not a “square” because it looks different from their visual image of square. Most students in Kindergarten to Grade 3 will be at Level 0 (visualization).

Suggestions for instruction at this level include

- sorting, identifying, and describing shapes
- working with physical models
- seeing different sizes and orientations of the same shape in order to distinguish the characteristics of the shape and to identify features that are not relevant
- building, drawing, making, putting together, and taking apart 2-D shapes and 3-D objects

**Level 1: Analysis** (Some students may be at this stage.)

At this level students begin to be able to identify the properties of shapes. They use appropriate geometric vocabulary related to properties. They are able to move beyond less important features such as size or orientation in order to sort and classify shapes. They start to describe the relationship between shapes and their properties.

Suggestions for instruction at this level include

- focusing on properties (defining, measuring, observing, or changing) by using concrete or virtual models
- problem solving involving shapes
- classifying shapes based on their properties

### Terminology

#### 3-D Objects:

**Vertex, Vertices:** Can be used in the following ways:

- The common endpoint of two sides of a polygon.
- The common endpoint of two rays that form an angle.
- The common point where three or more edges of a 3-D solid meet.

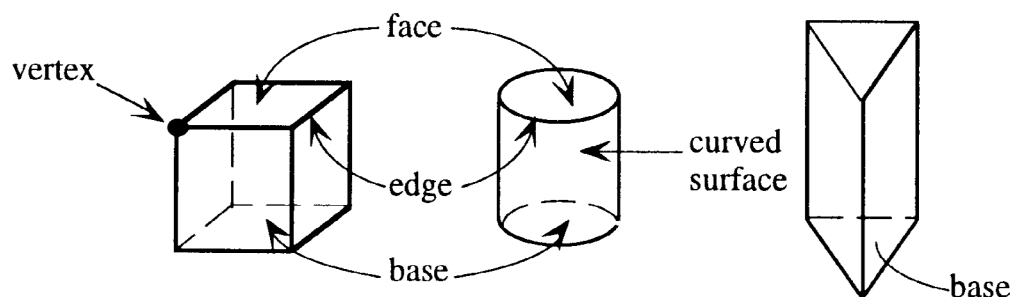
**Note:** A cone has an **apex**, but it is often referred to as a vertex.

**Edge:** The line segment where 2 plane faces of a solid figure meet.

**Face:** A flat surface of a solid.

**Surface:** Cones and spheres have curved surfaces (i.e., a cone has 1 face and 1 curved surface).

**Base:** A particular side or face of a geometric figure.



**Prism:** A 3-D figure (solid) that has two congruent and parallel faces that are polygons (the bases); the remaining faces are parallelograms.

**Pyramid:** A polyhedron whose base is a polygon and whose lateral faces are triangles that share a common vertex.

## 2-D Shapes:

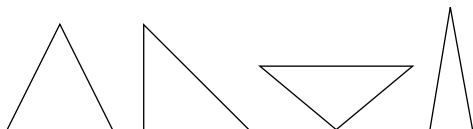
**Polygon:** A closed plane figure formed by three or more line segments.

**Regular polygon:** A polygon in which all sides and all angles are congruent.

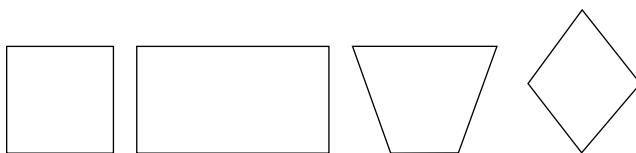
**Irregular polygon:** A polygon whose sides and angles are not all congruent.

**Polygon Names:** Note: Regular polygons are shown first.

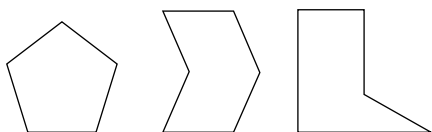
3 sides—triangle



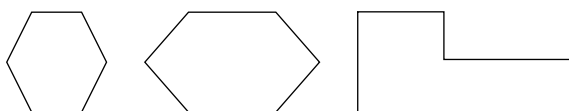
4 sides—quadrilateral



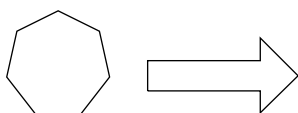
5 sides—pentagon



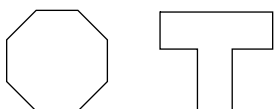
6 sides—hexagon



7 sides—heptagon



8 sides—octagon



## MATHEMATICAL LANGUAGE

---

cube	irregular polygon
sphere	triangle
cone	quadrilateral
cylinder	pentagon
prism	hexagon
pyramid	heptagon
face	octagon
edge	three-dimensional
vertex	two-dimensional
vertices	attribute
curved surface	property
skeleton	sides
polygon	shape
regular polygon	object

## LEARNING EXPERIENCES

---



### Assessing Prior Knowledge

Give students a small group of regular and irregular 3-D objects.

1. Have them sort them according to two attributes and then state their sorting rule.
2. Sort a set of objects into two groups. Have students identify the sorting rule.

Give students a small group of regular and irregular 2-D shapes.

1. Have them sort them according to two attributes and then state their sorting rule.
2. Sort a set of objects into two groups. Have students identify the sorting rule.

### Observation Checklist:

The students are able to

- sort a collection of 3-D objects according to two attributes
  - sort a collection of 2-D shapes according to two attributes
  - state the sorting rule     3-D     2-D
  - identify the sorting rule of a pre-sorted set     3-D     2-D
-



- Identify the faces, edges, and vertices of a 3-D object, including cubes, spheres, cones, cylinders, pyramids, and prisms.
- Identify the shape of the faces of a 3-D object.
- Determine the number of faces, edges, and vertices of a 3-D object.
- Construct a skeleton of a 3-D object, and describe how the skeleton relates to the 3-D object.
- Sort a set of 3-D objects according to the number of faces, edges, or vertices.

### Suggestions for Instruction

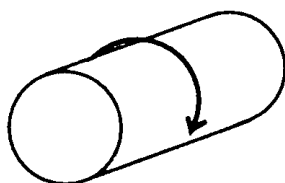
- **Prisms and Pyramids:** Present a pre-sorted set of prisms and pyramids (prisms in one group, pyramids in the other). Ask students why they think the objects are grouped in this way. Students should notice that
  - the triangular faces of pyramids meet at one vertex
  - the pyramid sits on one face (base)
  - the prisms have rectangular faces and 2 faces that are of a different shape (except in the case of a rectangular prism).

- **Exploration:** Allow students time to build and explore with 3-D objects (models, box collections, etc.). Direct discussions to enable students to determine attributes of 3-D objects. Model the terminology:

rolls	doesn't roll
flat faces	curved surfaces
vertices	no vertices
slides	doesn't slide
stacks	doesn't stack
type of edges (straight or curved)	

Record 3-D-object attributes on a reference chart. Provide illustrations and words to indicate attributes.

Example:



curved surface

- **How do they get their names?** Use a set of prisms (especially prisms with bases that are not rectangular) and pyramids. Beside each object place a tag with its name on it. Have students explore the objects to see if they can figure out how the object got its name. Discuss their findings. Guide students to see that pyramids get their names by the shape they sit on (base) and prisms get their names by the shape of the two congruent, non-rectangular faces (base) (except in the case of a rectangular prism or a cube).

Work with the students to arrive at a definition for both terms.

- **Vocabulary:** Using a prism and a pyramid, focus students' attention on the faces, edges, and vertices. Together as a class write a definition for each term.
- **Cones, Spheres, and Cylinders:** Read the book *Sir Cumference and the Sword in the Cone* by Cindy Neuschwander, illustrated by Wayne Geehan, to introduce students to cubes, cones, cylinders, and pyramids.

Have students describe cones, spheres, and cylinders in terms of faces, edges, and vertices. Introduce the term "curved surface."

BLM  
3.SS.6.1

- **Class Chart:** Have students work in pairs or small groups. Give each group a different 3-D object. Have them use their object to fill in the following information (BLM 3.SS.6.1):

Name of Object	Number of Flat Faces	Number of Curved Surfaces	Number of Edges	Number of Vertices	Shape of the Base

Have groups present their findings to the class. Record the information on a class chart.




---

### Assessing Understanding: Journal/Notebook Entry

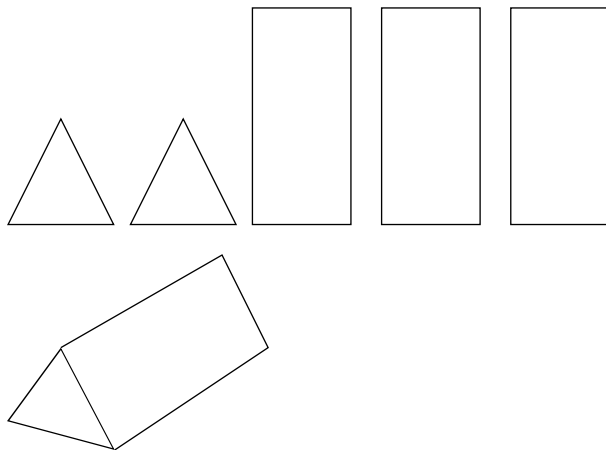
Explain how prisms and pyramids are similar. Explain how they are different.

---

## Suggestions for Instruction

- **Guess the Object:** Trace the faces of a variety of 3-D objects. Show students the faces and have them guess the object they came from.

Example:



triangular prism

Have students create their own “Guess the Object” to share with the class.

**Note:** This can be done with real-world materials such as food boxes, toothpaste boxes, chocolate boxes (e.g., Toblerone boxes). Students then have to identify the actual box the faces represent along with its geometric name.

- **Feely Can Puzzles:** Place a 3-D object in the can. Pass the can from one student to another. Each student reaches in, feels the object, and then gives one of its attributes. When it is felt that all possible attributes are exhausted, students can identify the object.
- **Riddle Me 3-D:** Have students write riddles for 3-D objects.

Example:

I am a 3-D object.  
I have 5 faces.  
I have 9 edges.  
I have 6 vertices.  
My base is triangular.  
What am I?

Create a class riddle book for 3-D objects.

- **Skeletons:** Have students construct skeletons for 3-D objects.

Use

- straight straws and twist ties



Insert twist tie into the ends of the straw.

- toothpicks and mini marshmallows/modelling clay

The straws/toothpicks are the edges and the twist ties/marshmallows are the vertices.

Have students describe their skeleton in relation to the object.

**Extension:**

1. Cut the straws into different lengths. Lengths can be given a monetary value and students can calculate the value of their completed skeleton.
2. Each student/pair makes a skeleton for a particular food stuff box. When finished, set up the skeletons in one location and the boxes in another. Have students match the skeleton to the correct box.

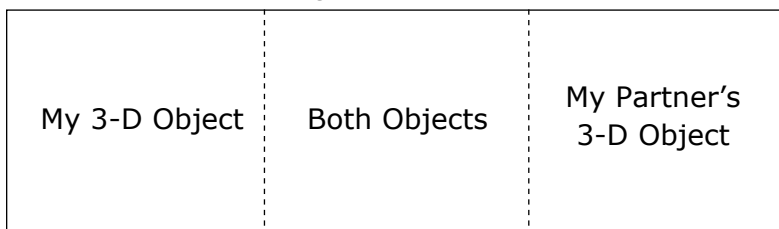
- **Object Comparison:** Fold a letter size paper the hot dog way.



Divide the front half in thirds and cut each line up to the fold.



Use this like a Venn diagram.



Each partner takes a different 3-D object.

Students compare and contrast their objects. In the “My 3-D Object” section (under the flap) the student describes attributes that are unique to their object. In the “My Partner’s 3-D Object” section the student lists attributes that are unique to the partner’s object. In the middle section attributes that are common to both are listed.

- **Object Sorting:** Use a Venn diagram. Have students sort the objects using their own sorting rules. Make a class list of the sorting rules used.

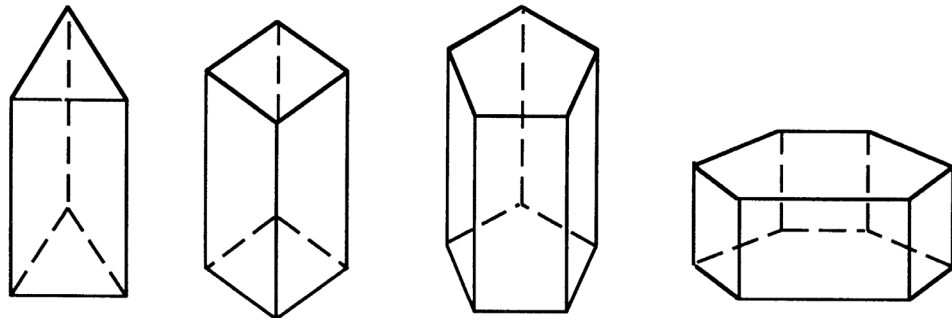
Examples:

- 6 faces, 12 edges
- more than 6 vertices, more than 9 edges



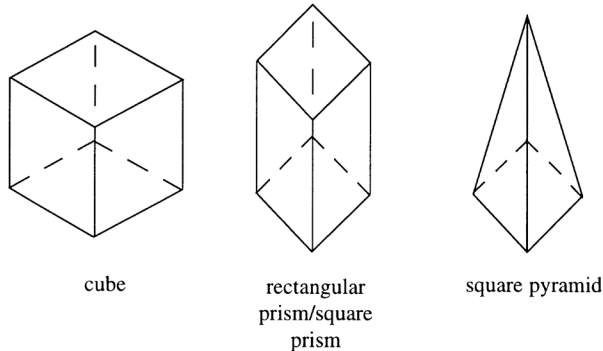
### Assessing Understanding

1. You have 12 straws and 8 marshmallows/twist ties. What skeleton of a 3-D object can you make using all of these straws and marshmallows?
2. Record the number of faces, vertices, and edges for each of these prisms.



3. The base of a solid is square. What is the name of the solid? Are there any other solids it might be? Use drawing and writing to explain your thinking.  
Sample rubric: Note: "Square prism" is an acceptable term.

Possibilities:



Low response: includes 2 names with attempted drawings and explanations

Medium response: includes 3 names with reasonable drawings and explanations

High response: includes 3 names with accurate drawings and complete explanations

- **Classify a set of regular and irregular polygons according to the number of sides.**
- **Identify regular and irregular polygons having different dimensions.**
- **Identify regular and irregular polygons having different orientations.**

### Suggestions for Instruction

- **Geoboard Sort:** Give each student a geoboard and one elastic. Have them make a shape that has 3 or more straight sides. Do a class sort of the shapes based on the number of sides.
- **Name the Shape:** Read the book such as *The Greedy Triangle* by Marilyn Burns, illustrated by Gordon Silveria. This is the story of a bored triangle that keeps going to the local shapeshifter to add another side/angle to his shape. Shape vocabulary from triangles to decagons is introduced. Real-world examples for the shapes are also given.

Have students use a geoboard to act out the story as it is being read.

**Note:** The book *Twizzlers Shapes and Patterns* by Jerry Pallotta, illustrated by Rob Bolster, also introduces the vocabulary for shapes up to ten sides.

- **2-D Sort:** Provide students with a collection of paper shapes of various sizes. Ask students to sort the collection into two groups and to name the groups. Have students regroup the collection and sort it in other ways.

With each sort, have students describe the attribute(s) common to the group (e.g., sides and vertices). Note: If students have not done a “circle, not circle” sort, it is important to direct their attention to this.

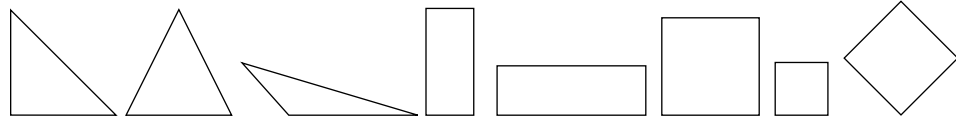
**Follow-Up:** Have the class develop a chart, organizing the attribute information of each 2-D shape.

BLM  
3.SS.6.2

Shape	Number of Sides	Number of Vertices
triangle	3	3
quadrilateral	4	4
pentagon	5	5

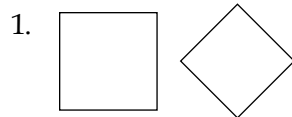
- **Regular or Irregular Polygons:** Read a book such as *If You Were a Polygon* by Marcie Aboff, illustrated by Sarah Dillard. This book introduces students to regular and irregular polygons. Discuss the difference between the two types of polygons. Have the class come up with a definition for both types.

- **Geoboard Shapes:** Ask students to make different sizes of 1 shape or different types of the same shape on their geoboard. Have them record their shapes on dot paper.

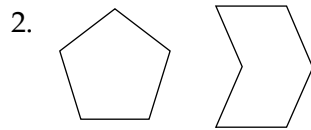



---

### Assessing Understanding: Paper-and- Pencil Task



How are the shapes the same? How are they different?



How are the shapes the same? How are they different?

#### Look for:

The student

- identifies all shapes as polygons
  - identifies specific shapes as quadrilaterals or pentagons
  - identifies specific shapes as regular and irregular polygons
  - gives specific names to square and rectangle
- 

### Suggestions for Instruction

- **Geoboard Challenges:** Have students work in pairs or small groups to solve the following challenges:
  - How many different pentagons can you make on your geoboard? Record your pentagons on dot paper.
  - How many different quadrilaterals can you make on your geoboard? Record your quadrilaterals on dot paper.
- **Feel a Shape:** Provide opportunities for students to identify 2-D objects by touch only. For example, place attribute blocks in a “feely can,” or pair students and have one student place a 2-D object in the hands of another student (with hands behind back).

- **Vocabulary Game:** Have students work in pairs. The first student draws a vocabulary card. They try to get their partner to identify the word by giving them clues (without saying the word). Students can keep track of the number of clues needed or a time limit can be set.

Vocabulary cards might include the following:

polygon	triangle	square
rectangle	quadrilateral	pentagon
hexagon	heptagon	octagon
side	vertex	regular polygon
irregular polygon	corner	2-D

- **Five Questions (similar to Twenty Questions):** Ask students to pose a maximum of five questions that require Yes/No answers in order to guess the name of a 2-D shape. Sample questions: Does it have less than 5 sides? Does it have square corners? Are its sides equal in length?
- **The Important Thing About \_\_\_\_\_:** Use the book *The Important Book* by Margaret Wise Brown, illustrated by Leonard Weisgard. The author uses a basic format for writing about the important aspects of everything from a spoon, an apple, and the wind to a shoe.

The following is a modified format.

The important thing about \_\_\_\_\_ is that [*the most important attribute*].

It \_\_\_\_\_.

It \_\_\_\_\_.

It/A \_\_\_\_\_,  
and it \_\_\_\_\_.

But the important thing about \_\_\_\_\_ is that [*repeat from the first line*].

Have students write their own page for a class book about 2-D (and 3-D) shapes (objects) using this format. (Students should be able to modify the format to make it work for their shape/object.)



Example:

The important thing about a square is that it has four equal sides.

It is a special rectangle.

It has four vertices.

It is a quadrilateral,

and it is also a polygon.

But the most important thing about a square is that it has four equal sides.

- **Fun with Tangrams:** Introduce the tangram puzzle to students. Have them identify the shapes of the seven tans (triangles, a parallelogram, and a trapezoid). Have students try to put the puzzle back into a square.

Read a book such as *Grandfather Tang's Story* by Ann Tompert, illustrated by Robert Andrew Parker, or *The Tangram Magician* by Lisa Campbell Ernst and Lee Ernst. Have students reproduce the animals/objects from the book(s).

**Problem Solving:** Have students explore to determine whether regular polygons can be made with a different number of tangram pieces. Make a large class chart and have students post their solutions. This could be a long term project.

Example:

Number of Pieces	Triangle	Square	Rectangle	Pentagon	Hexagon
1					
2					
3					
4					
5					
6					
7					

**Note:** Some of these are not possible.



---

### Assessing Understanding: Journal/Notebook Entry

1. How are 2-D shapes and 3-D objects related?  
(3-D objects are made up of 2-D shapes. 3-D objects get their names from their 2-D shaped bases.)
  2. Draw 5 different pentagons.
- 

## PUTTING THE PIECES TOGETHER: PERFORMANCE TASK



Present the following scenario:

The local art gallery wants to put up a geometry display, and they have asked our class to help. You are going to work in pairs. Each pair will need to

- pick either a 2-D shape or a 3-D object
- make a skeleton model of your shape/object and label its parts
- create a picture/collage using your shape or object (The picture could be done using the shape/object only and changing size and orientation. The collage could be real-world examples of the shape/object [digital pictures, pictures cut from magazines/newspapers, etc.]
- do research to see if you can find some artist(s) that has/have used your shape/object in their art work (painting, sculpture, quilting, crafts, construction, etc.)
- if possible, bring some real-world examples for the display
- present your work to the class and to any visitors

Have students set up their displays.

Have them present their work to the class. If possible, have them extend invitations to other classes or school personal. Parents might also be invited.

### Assessment:

Develop assessment criteria with the class. Students can use the criteria to do a self-assessment (group). It can also be used by the teacher as a summative piece.



# GRADE 3 MATHEMATICS

Statistics and Probability (Data Analysis)



# Grade 3: Statistics and Probability (Data Analysis) (3.SP.1, 3.SP.2)

## Enduring Understandings:

Data can be collected and organized in a variety of ways.

Data can be used to answer questions.

## Essential Questions:

Why do we collect data?

How can data be collected and recorded?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
<p><b>3.SP.1</b> Collect first-hand data and organize it using</p> <ul style="list-style-type: none"> <li>■ tally marks</li> <li>■ line plots</li> <li>■ charts</li> <li>■ lists</li> </ul> <p>to answer questions. [C, CN, V]</p>	<ul style="list-style-type: none"> <li>→ Record the number of objects in a set using tally marks.</li> <li>→ Determine the attributes of line plots.</li> <li>→ Organize a set of data using tally marks, line plots, charts, or lists.</li> <li>→ Collect and organize data using tally marks, line plots, charts, or lists.</li> <li>→ Answer questions arising from a line plot, chart, or list.</li> <li>→ Answer questions using collected data.</li> </ul>
<p><b>3.SP.2</b> Construct, label, and interpret bar graphs to solve problems. [PS, R, V]</p>	<ul style="list-style-type: none"> <li>→ Determine the attributes of bar graphs.</li> <li>→ Create bar graphs from a set of data including labelling the title and axes.</li> <li>→ Draw conclusions from a bar graph to solve problems.</li> <li>→ Solve problems by constructing and interpreting a bar graph.</li> </ul>

## PRIOR KNOWLEDGE

---

Students may have formulated questions and collected data using concrete objects, tallies, check marks, charts, or lists. They may have constructed and interpreted concrete graphs and pictographs to solve problems.

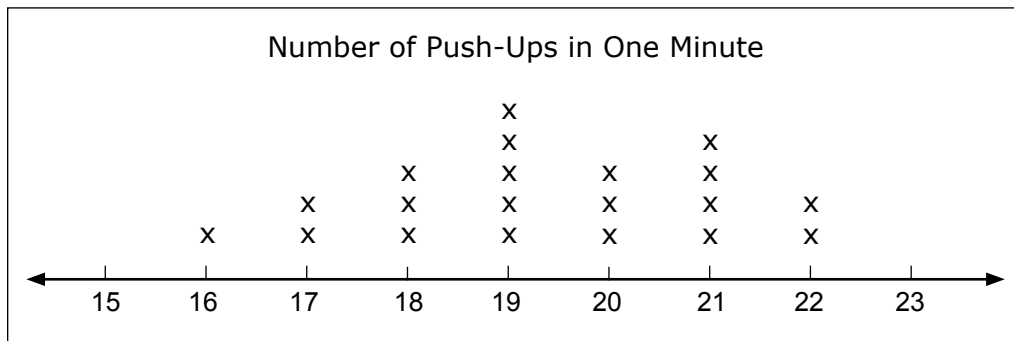
## BACKGROUND INFORMATION

---

Students need to develop strategies to collect and record information using first-hand data. First-hand data is information that people collect on their own by counting, conducting polls, conducting experiments, or using measuring devices. The first-hand data should relate to the students and the community they live in. The goal is to have students communicate their understanding by recording data in an organized manner and by answering, asking, and writing questions concerning the data.

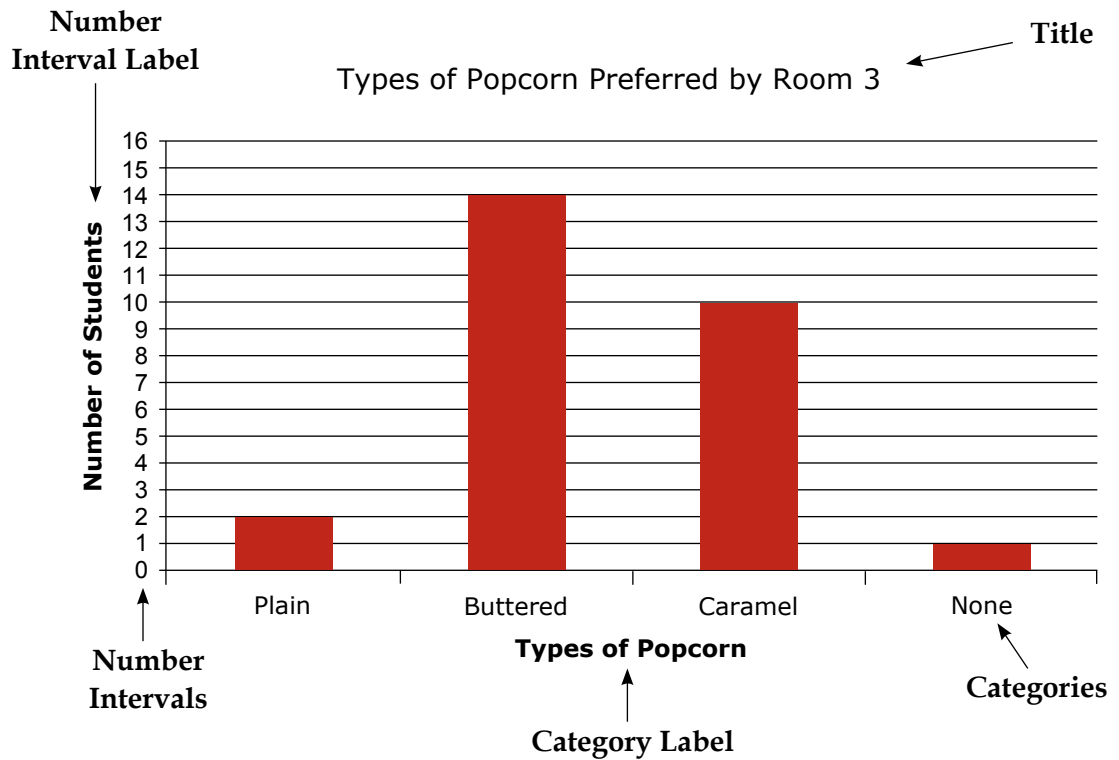
**Line Plot:** A number line on which each number in a set of data is plotted by making a mark (usually an X or a large dot) above that number on the number line.

Example:



**Bar Graph:** A bar graph is a graph that uses horizontal or vertical bars to display data.

Example:



A bar graph needs the following labels:

- title
- categories
- category label
- number intervals (Note: Numbers are labelled on the line not the space.)
- number interval label

Generally the data graphed at the elementary level is discrete data (data attained by counting in whole numbers). In this case, there are always spaces left between the bars.

**Chart:** A chart is a diagram that illustrates information in the form of a table, graph, or picture.

Example of a table:

<b>Name</b>	<b>Dog</b>	<b>Cat</b>
Mark	✓	
Allan	✓	
Dan		✓
Violet		✓
Sarah	✓	
Ryan	✓	
Jeanne	✓	✓

## MATHEMATICAL LANGUAGE

---

categories

label

title

data

tallies

match

more

less

same amount as

most

least

bar graph

line plot

compare

survey

number intervals

chart

list

axes

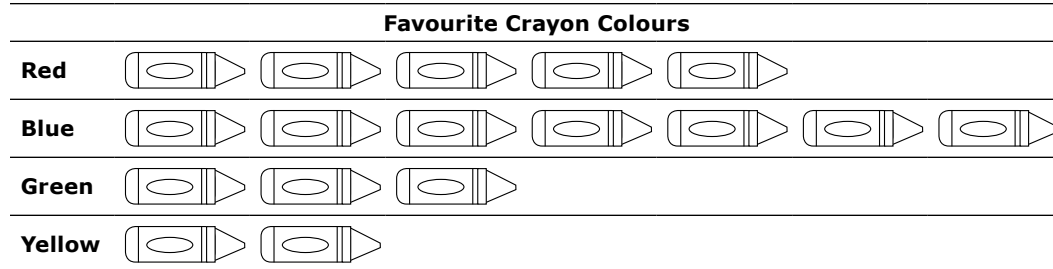


## LEARNING EXPERIENCES

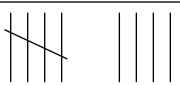



### Assessing Prior Knowledge

Present students with the following pictograph.



- Have students describe, orally or in writing, what the graph is showing (interpret the data). Use guiding questions if needed.
  - What does the pictograph show? How do you know?
  - What does this tell about the colours of crayons?
  - Which do we like most? Least?
  - How many more are there of our most favourite colour than our least favourite colour?
  - Which do we like more—red or green? How do you know?
  - How many people were surveyed? How do you know?
- Give students a set of data. Have them construct a concrete graph.

Do you like winter?	
Yes	No
	

#### Look for:

The student is able to

- describe the data represented on a pictograph
- answer questions pertaining to a pictograph
- create a concrete graph to display a given set of data
- answer questions pertaining to a concrete graph
- use mathematical language correctly

- Record the number of objects in a set using tally marks.
- Determine the attributes of line plots.
- Organize a set of data using tally marks, line plots, charts, or lists.
- Collect and organize data using tally marks, line plots, charts, or lists.
- Answer questions arising from a line plot, chart, or list.
- Answer questions using collected data.

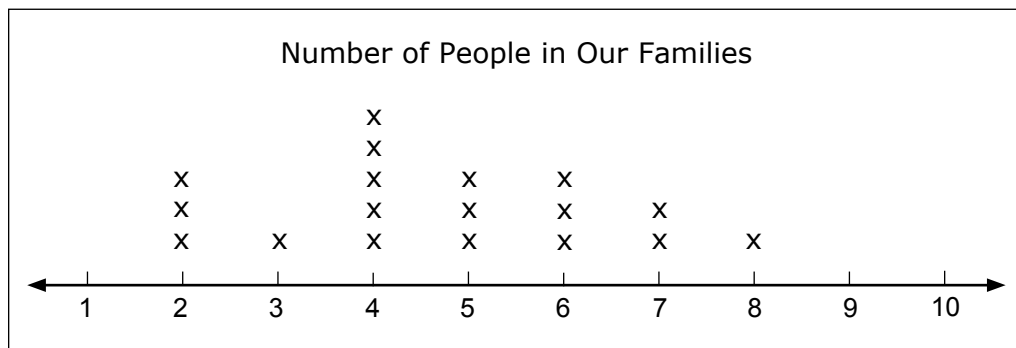
### Suggestions for Instruction

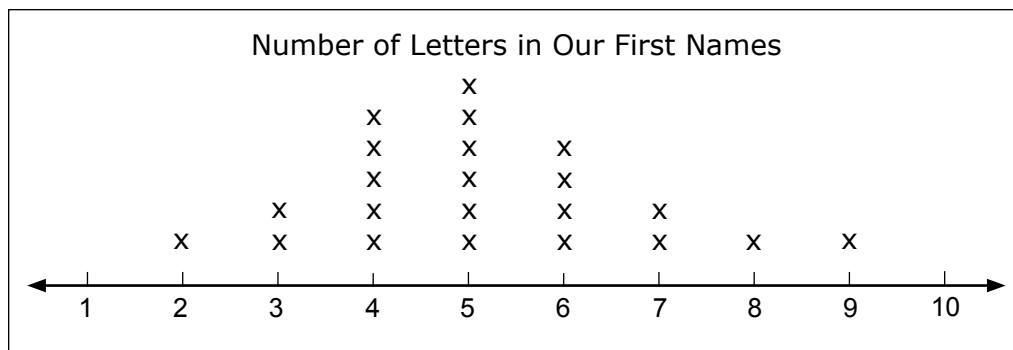
- Review the use of tally marks with students by
  - reading a book such as *Tally Charts* by Vijaya Khisty Bodach
  - having them take a handful of counters, cubes, et cetera, and record the number of objects in the set using tallies
  - having them do their own “Yes/No” survey and collect and record the data using tallies
- Create two or more line plots with the students. Use the class graphs or the pre-made graphs below. Have students look at the similarities between the line plots in order to identify their attributes.

Example:

Line plots have

- a number line
- Xs to represent the data
- a title





- **Investigation:** Each student in Room 10 was asked how many pockets they had.

This was the data collected.

Our Pockets—Room 10
3, 4, 7, 0, 1, 6
2, 4, 4, 0, 2, 4
2, 1, 0, 2, 0, 2
0, 4, 3, 2, 3, 5

- a. Show this information on a line plot.
- b. Collect the same information from the children in your class.
- c. Show this information on a line plot.
- d. Describe how your information is the same as or different from Room 10's.




---

### Assessing Understanding

Develop criteria with the students. Possible criteria might include the following:

- Room 10's information is correctly displayed on a line plot.
  - Our class information is collected in an organized way.
  - Our class information is correctly displayed on a line plot.
  - Both line plots have a title and a number line.
  - Information is compared using "math language."
-

- Ten Grade 3 students were asked how they came to school. Three different methods were used to gather the data.
  - What information does each method provide?
  - Which method do you prefer? Explain your thinking.

**Chart**

<b>Car</b>	<b>Bus</b>	<b>Walk</b>	<b>Bike</b>
Sean Pam	Sasha Chen Arthur	Henri	Sarah Otis George Raven

**List**

car
bus
walk
bus
bike
car
bike
bus
bike
bike

**Tally**

<b>Transport</b>	<b>Number</b>
Car	
Bus	
Walk	
Bike	

- Provide meaningful opportunities for students to collect, represent, and interpret data.

Examples:

- special lunches, events, et cetera, over the school year
- surveys (classroom/school) on current issues (e.g., sustainable development)
- student-created questions:
  - How many pets do you have at home?
  - How many windows/rooms/doors are in your house?
  - How many telephones/televisions are in your house?
  - How many letters are in your first name?
  - How many letters are in your last name?
  - How many vowels are in your name?
  - In which month were you born?
  - What is your favourite single-digit number?

**Note:** The science, social studies, and physical education/health education curricula provide meaningful contexts for working with data.

Literacy with ICT can be integrated into this strand (collecting and displaying data).

- **Determine the attributes of bar graphs.**
- **Create bar graphs from a set of data including labelling the title and axes.**
- **Draw conclusions from a bar graph to solve problems.**
- **Solve problems by constructing and interpreting a bar graph.**

## Suggestions for Instruction

**Note:** Students should be able to construct a bar graph, but more importantly they need to be able to interpret the graph and to use the information to solve problems. The majority of time spent working on bar graphs should be on the interpretation rather than on the construction.

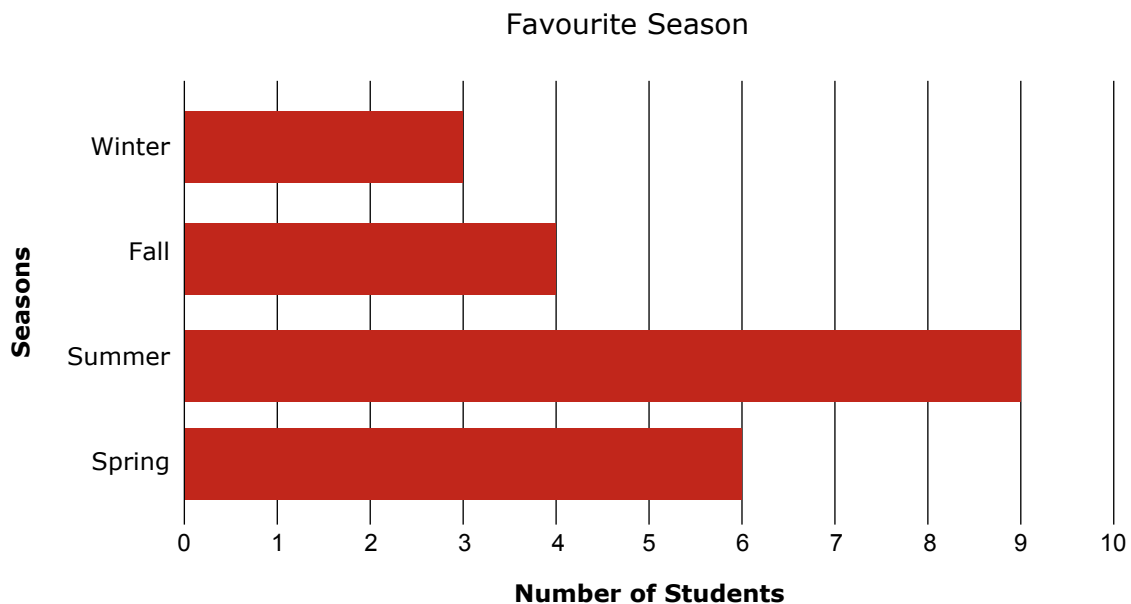
- Create two or more bar graphs with the students. Use the class graphs or the pre-made graphs below. Have students look at the similarities between the graphs in order to identify their attributes.

Example:

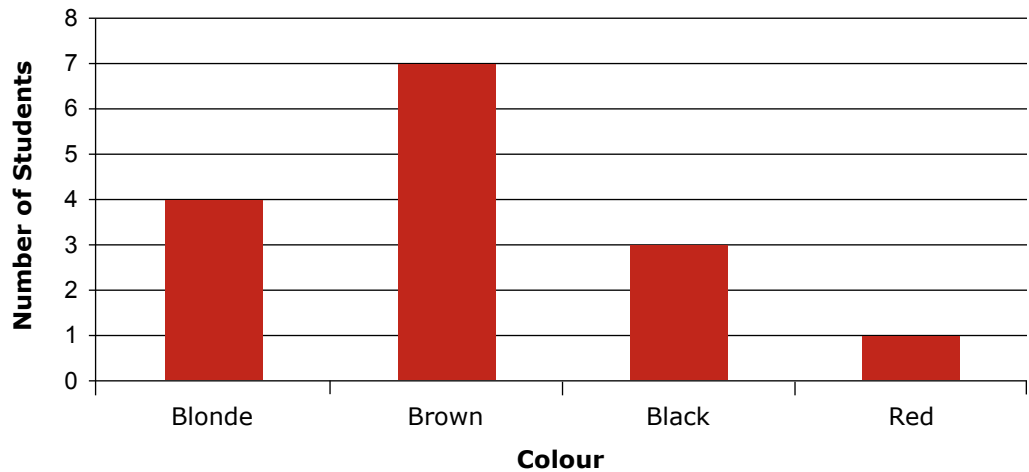
Bar graphs have

- title
- axes (numbers and categories)
- labels (number axis and categories axis)
- bars

**Note:** Spaces should be left between the bars because the data graphed at this level is generally discrete data.



Room 3 Hair Colour



- Use the identified attributes to develop criteria for students to use for self-assessment.

Example:

My Five Star Graph has
☆ a title
☆ both axes labelled
☆ the choices/categories labelled
☆ the numbers labelled on the lines
☆ the bars filled in correctly

- **Thinking Critically:** Once students are familiar with the attributes of bar graphs, read them a book such as *Lemonade for Sale* by Stuart J. Murphy, illustrated by Tricia Tusa (Math Start Level 3), or *Fair is Fair!* by Jennifer Dussling, illustrated by Diane Palmisciano (Math Matters). Both books have the characters use bar graphs to solve a problem, but there are problems in the way the graphs are presented (axes are not labelled in *Lemonade for Sale*, spaces are not left between the bars in *Fair is Fair!*).  
Read the book (showing the pictures) and see if the students notice the errors.

**BLM**  
**3.SP.2.1**

- Have students complete a “Data about Me” sheet after brainstorming a list of information the class wants to collect.

Post the data sheets (or have them in a binder/report cover).

Assign individuals or groups of students to one of the areas. Have them compile the data and then represent it on a bar graph.

When students have collected, organized, and displayed the data, they should share and discuss results with classmates.

Sample “Data about Me” sheet:

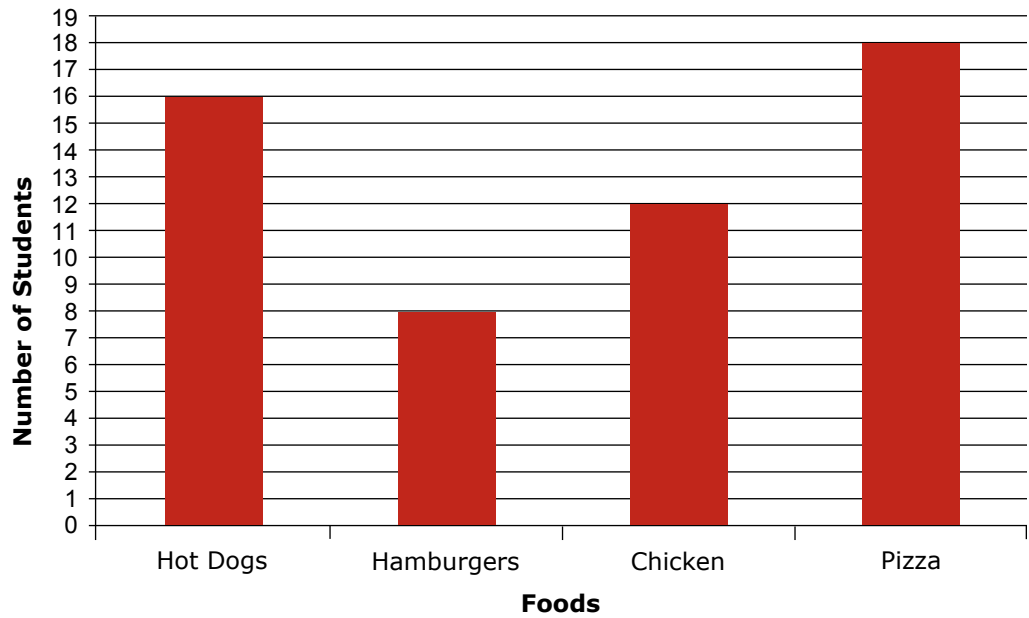
The image shows a sample "Data about Me" sheet template enclosed in a rectangular border. In the center is a starburst shape containing the text "Data about Me". Surrounding this central shape are six other shapes, each containing a question for data collection:

- A rectangle in the top-left corner with the text "Number of People in My Family".
- An octagon in the top-right corner with the text "Type of Pets".
- An oval on the left side with the text "Hair Colour".
- A pentagon on the right side with the text "Birthday Month".
- A triangle in the bottom-left corner with the text "Favourite Season".
- A rounded rectangle in the bottom-right corner with the text "Favourite Day of the Week".

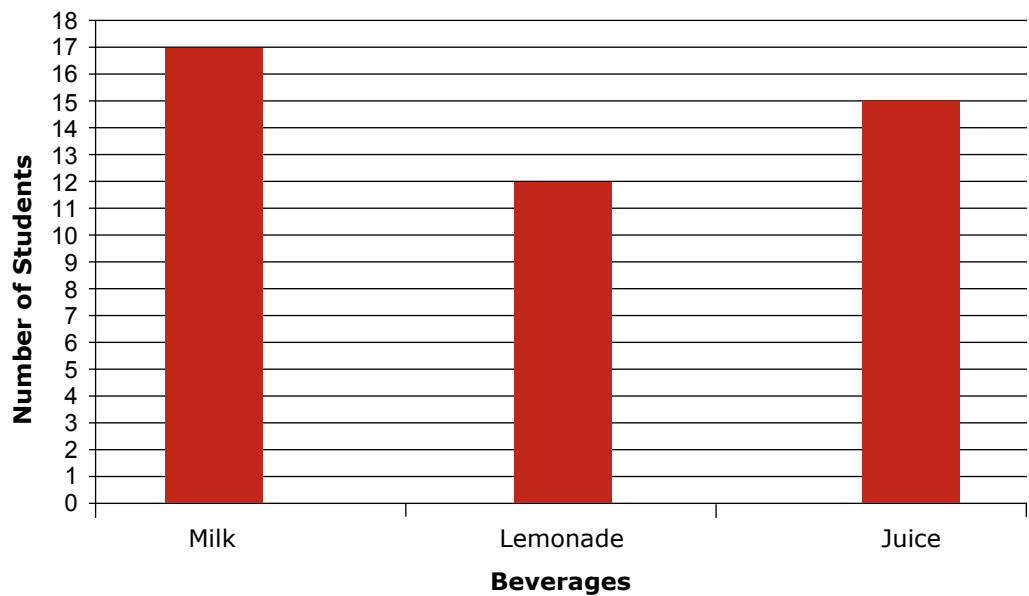


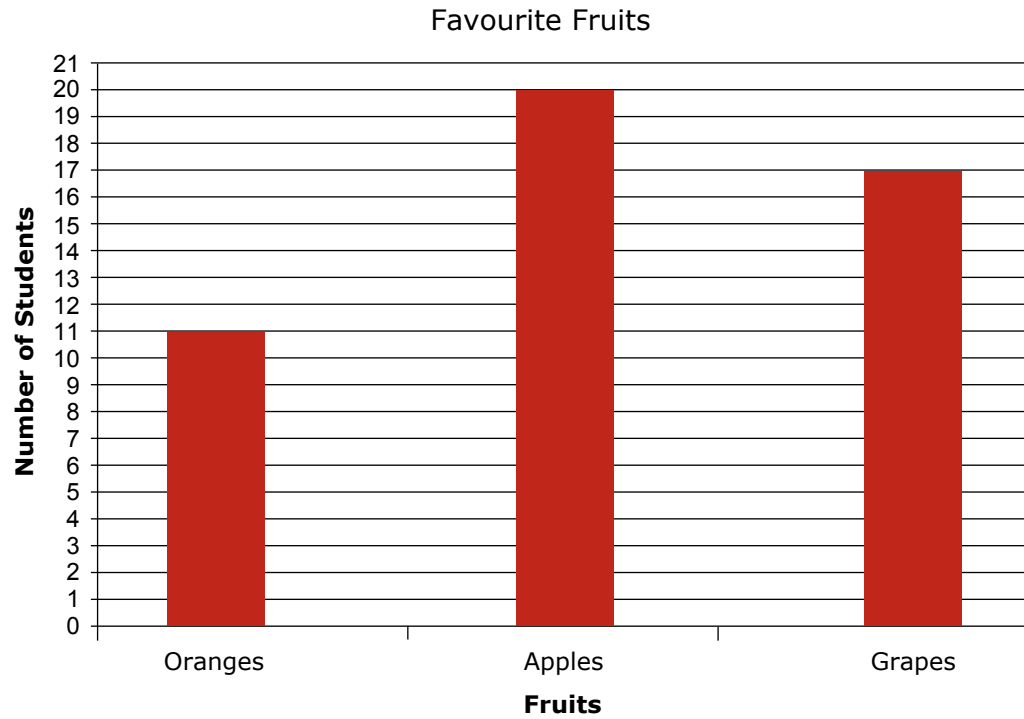
- **Problem Solving:** The parent group in the school is planning a special lunch. They surveyed some of the students to determine what they should serve. Here are the results:

Favourite Foods



Favourite Beverages





Use the data from the graphs. Write a note to the parent group explaining what food, beverage, and fruit they should serve. Be specific. Use both numbers and words to support your choices.

Expect students to use some comparison statements (e.g., Eleven students chose oranges. Six more students chose grapes than oranges. Apples had the most votes with 20).



### Planning a Math Night

**Context:** Explain to students that they are going to be planning a Math Night for their class/grade. Have them brainstorm things they need to consider (e.g., time, night of the week, refreshments, activities for each strand).

Have students work in partners or small groups. Assign each group (or have groups select) a topic or sub-topic (strand activities would have to be divided) from the brainstormed list. Have each group

- formulate a question
- determine the answer choices
- collect and organize the data
- represent the data using a line plot or bar graph
- summarize the data in written form
- present the results to the class

### Assessment Criteria:

Use the following assessment criteria:

- Question is clearly stated.
- Choices answer the question.
- Data is collected and organized.
- Representation of the data is accurate.
- Line plot/bar graph is correctly labelled.
- Data summary gives an accurate picture of the findings.
- Findings are presented in a clear, concise manner.

---

## Notes



# GRADE 3 MATHEMATICS

Bibliography



## BIBLIOGRAPHY

- Aboff, Marcie. *If You Were a Polygon*. Illus. Sarah Dillard. North Mankato, MN: Picture Window Books-Capstone, 2009.
- Aker, Suzanne. *What Comes in 2s, 3s, and 4s?* Illus. Bernie Karlin. New York, NY: Alladin, 1992.
- Albee, Sarah. *The Dragon's Scales*. Step into Reading series: A Step 2 Book. Illus. John Manders. Toronto, ON: Random House, 1998.
- Banks, J. A., and C. A. M. Banks. *Multicultural Education: Issues and Perspectives*. Boston: MA: Allyn and Bacon, 1993.
- Bodach, Vijaya Khisty. *Tally Charts*. North Mankato, MN: Capstone Books, 2008.
- Brown, Margaret Wise. *The Important Book*. Illus. Leonard Weisgard. New York, NY: HarperCollins, 1999.
- Burns, Marilyn. *The Greedy Triangle*. Illus. Gordon Silveria. New York, NY: Scholastic Press, 1994.
- . *Spaghetti and Meatballs for All: A Mathematical Story*. Illus. Debbie Tilley. New York, NY: Scholastic Press, 1997.
- Capucilli, Alyssa Satin. *Mrs. McTats and Her Houseful of Cats*. Illus. Joan Rankin. London, UK: Simon and Schuster UK, 2001.
- Carle, Eric. *Rooster's Off to See the World*. New York, NY: Aladdin Books, 1972.
- Dodds, Dayle Ann. *Full House: An Invitation to Fractions*. Illus. Abby Carter. Somerville, MA: Candlewick Press, 2009.
- Dussling, Jennifer. *Fair is Fair! Math Matters*. Illus. Diane Palmisciano. New York, NY: Kane Press, 2003.
- Ernst, Lisa Campbell, and Lee Ernst. *The Tangram Magician*. Maplewood, NJ: Blue Apple Books, 2005.
- Gerth, Melanie. *Ten Little Ladybugs*. Illus. Laura Huliska-Beith. London, UK: Gullane Childrewns Books, 2000.
- Giganti, Paul, Jr. *Each Orange Had 8 Slices: A Counting Book*. Illus. Donald Crews. New York, NY: Greenwillow Books, 1999.
- Goldstone, Bruce. *Great Estimations*. New York, NY: Square Fish, 2010.
- Hutchins, Hazel. *A Second Is a Hiccup: A Child's Book of Time*. Illus. Kady MacDonald Denton. New York, NY: Scholastic, 2004.
- Hutchins, Pat. *The Doorbell Rang*. New York, NY: Greenwillow Books, 1989.
- Kilpatrick, J., J. Swafford, and B. Findell, eds. *Adding it Up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press, 2001.

- Manitoba Education. *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes* (2013). Winnipeg, MB: Manitoba Education, 2013. Available online at <[www.edu.gov.mb.ca/k12/cur/math/framework\\_k-8/index.html](http://www.edu.gov.mb.ca/k12/cur/math/framework_k-8/index.html)>.
- Manitoba Education, Citizenship and Youth. *Kindergarten to Grade 8 Mathematics Glossary: Support Document for Teachers*. Winnipeg, MB: Manitoba Education, Citizenship and Youth, 2009.
- . *Rethinking Classroom Assessment with Purpose in Mind: Assessment for Learning, Assessment as Learning, Assessment of Learning*. Winnipeg, MB: Manitoba Education, Citizenship and Youth, 2006.
- Murphy, Stuart J. *Lemonade for Sale*. MathStart 3. Illus. Tricia Tusa. New York, NY: HarperCollins, 1997.
- . *Racing Around*. MathStart: Perimeter. Illus. Mike Reed. New York, NY: HarperCollins, 2002.
- Myller, Rolf. *How Big Is a Foot?* New York, NY: Yearling-Random House, 1990.
- Neuschwander, Cindy. *Amanda Bean's Amazing Dream: A Mathematical Story*. Illus. Liza Woodruff. Math activities by Marilyn Burns. New York, NY: Scholastic Press, 1998.
- . *Sir Cumference and the Sword in the Cone*. Illus. Wayne Geehan. Watertown, MA: Charlesbridge, 2003.
- Nolan, Helen. *How Much, How Many, How Far, How Heavy, How Long, How Tall Is 1000?* Illus. Tracy Walker. Toronto, ON: Kids Can Press, 2001.
- OECD (Organisation for Economic Co-operation and Development). *PISA 2012 Field Trial Problem Solving Framework*. September 30, 2012. <[www.oecd.org/pisa/pisaproducts/46962005.pdf](http://www.oecd.org/pisa/pisaproducts/46962005.pdf)>.
- Ontario. Ministry of Education. *The Ontario Curriculum Grades 1–8: Mathematics*. Ontario: Ministry of Education, 2005. Available online at <[www.edu.gov.on.ca/eng/curriculum/elementary/math18curr.pdf](http://www.edu.gov.on.ca/eng/curriculum/elementary/math18curr.pdf)>.
- Pallotta, Jerry. *Apple Fractions*. Illus. Rob Bolster. New York, NY: Scholastic, 2002.
- . *Twizzlers Shapes and Patterns*. Illus. Rob Bolster. New York, NY: Cartwheel-Scholastic, 2002.
- Pinczes, Elinor J. *A Remainder of One*. Illus. Bonnie MacKain. New York, NY: Houghton Mifflin Harcourt, 2003.
- Shaskan, Trisha Speed. *If You Were a Fraction*. Illus. Francesca Carabelli. North Mankato, MN: Picture Window Books-Capstone, 2008.
- Sierra, Judy. *Counting Crocodiles*. Illus. Will Hillenbrand. Orlando, FL: Harcourt, 1997.
- Souders, Taryn. *Whole-y Cow! Fractions Are Fun*. Illus. Tatjana Mai-Wyss. Ann Arbor, MI: Sleeping Bear Press, 2010.



Tompert, Ann. *Grandfather Tang's Story: A Tale Told with Tangrams*. Illus. Robert Andrew Parker. New York, NY: Random House, 1990.

Van de Walle, John A., Sandra Folk, Karen S. Karp, and Jennifer M. Bay-Williams. *Elementary and Middle School Mathematics: Teaching Developmentally*. 3rd Can. ed. Toronto, ON: Pearson Canada Inc., 2011.

Van de Walle, John A., Karen S. Karp, Lou Ann H. Lovin, and Jennifer M. Bay-Williams. *Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades 3–5*. 2nd ed. Vol. II The Van de Walle Professional Mathematics Series. Boston, MA: Pearson Education, 2014.

Western and Northern Protocol (WNCP). *The Common Curriculum Framework for K–9 Mathematics*. Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut Territory, Saskatchewan, and Yukon Territory: Ministries of Education of Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan, and the Yukon Territory, 2006.







Printed in Canada  
Imprimé au Canada