Grade 12
Applied Mathematics
Achievement Test

## Student Booklet

June 2018

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While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

## Grade 12 Applied Mathematics Achievement Test

## DESCRIPTION

## Total Possible Marks: 61

Time: 3 hours

| Unit | Marks |
| :--- | :---: |
| Relations and Functions | 15 |
| Financial Mathematics | 16 |
| Design and Measurement | 7 |
| Probability | 17 |
| Logical Reasoning | 6 |

## Directions

Remember to

- indicate your input values by writing them in your booklet or printing a copy if using a technology tool
- express your answers in decimal and percentage form to the nearest hundredth (two decimal places) when rounding, unless otherwise indicated
Example: $\frac{15}{29}=0.52$ or $51.72 \%$
- state any assumptions you make

A "graphic organizer" is a visual representation of information. Examples include a tree diagram, a chart, a list, a Venn diagram, a truth table, Pascal's triangle, etc.

## A clearly communicated answer

- is easily identified in the response space
- includes the parameters in the equation, and " $y=$ ", "sin", "ln", or " $x$ ", as applicable
- includes the units of measure, where applicable
- includes labels, units, and scales for the axes on graphs
- is expressed as an exact value or is appropriately rounded

Marks may be deducted for errors relating to any of the above.

## Electronic communication between students through phones, email, or file sharing during the test is strictly prohibited.

## RELATIONS AND Functions

## Question 1

Select the answer that best completes the statement.
The end behaviour of a cubic function with a negative leading coefficient extends from:
A) quadrant II to quadrant IV
B) quadrant III to quadrant I
C) quadrant I to quadrant III
D) quadrant IV to quadrant II

Select the best answer.
A cubic function has two $x$-intercepts and a positive $y$-intercept.
The graph that has these characteristics is:
A)

C)

B)

D)


The heart pumps blood throughout the body. As the blood leaves the heart, it is replaced with new blood.

Muna's heart contains 70 mL of blood. With each heartbeat, the volume of original blood in her heart is reduced by $53 \%$ and replaced with new blood.
a) Determine the exponential regression equation that models the volume of original blood remaining in Muna's heart as a function of the number of heartbeats. Show your work.
(2 marks)
b) Using your equation in (a), determine the volume of original blood remaining in Muna's heart after 6 heartbeats.
(1 mark)

The following equation models the path of a basketball as it is shot by Sarah:

$$
h=-0.51 d^{2}+4.72 d+6.09
$$

where $h$ represents the height in feet and $d$ represents the horizontal distance, in feet, the ball has travelled.
a) Create a clearly labelled graph of the equation.
(3 marks)

b) The standard height for a basketball net is 10 feet. Sarah shoots the ball and it goes directly through the net on its way down. Using the given equation, determine how far, horizontally, the ball was from the net when it was shot. Show your work.
(2 marks)


A satellite was launched from Cape Canaveral and set to orbit the Earth. Instruments measured its distance from the equator at certain time intervals, using positive numbers to indicate distances north of the equator and negative numbers to indicate distances south as shown below.

| Time (minutes) | 20 | 40 | 60 | 80 | 100 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Distance from equator (miles) | 3929 | 637 | -2468 | -254 | 3620 |

a) Determine the sinusoidal regression equation that models this data.
(1 mark)
b) After 180 minutes in orbit, the satellite passes directly over Cape Canaveral. How far is Cape Canaveral from the equator?
c) Determine the northern and southern limits of the satellite's path relative to the equator. (2 marks)
d) Determine the period for this sinusoidal model.
(1 mark)

## Financial Mathematics

## Question 6

Select the best answer.
The table below shows the value of a luxury vehicle over a two-year period.

| Year | Value |
| :---: | :---: |
| 0 | $\$ 58500$ |
| 1 | $\$ 42100$ |
| 2 | $\$ 30300$ |

The value of this vehicle depreciates at an annual rate of:
A) $72 \%$
B) $52 \%$
C) $39 \%$
D) $28 \%$

Johannes wants to apply for a bank loan. Information regarding his financial situation is given below.

- He has a house valued at $\$ 225000.00$ with a mortgage of $\$ 175000.00$.
- He has a cottage valued at $\$ 115000.00$ with a mortgage of $\$ 75000.00$.
- He has $\$ 9000.00$ in his savings account.
- He owes a total of $\$ 25000.00$ on his credit cards.
a) Calculate his net worth.
(1 mark)
b) Calculate his debt-to-equity ratio.
(1 mark)
c) Based on his debt-to-equity ratio, would the bank lend him money? Explain.
(1 mark)

Rémi deposited a sum of money into an account 36 years ago that earned an annual interest rate of $8.00 \%$. Today, there is $\$ 12800.00$ in his account.

Use the Rule of 72 to estimate the initial amount that Rémi deposited.

Mr. Smythe makes a one-time donation to a university. The university decides to invest this money and use only the amount earned in simple interest from the investment to finance a scholarship.

- The initial amount of the donation was $\$ 650000.00$.
- The amount earned in simple interest annually is $\$ 40000.00$.
- The university awards the scholarship to one student each year.

At what interest rate must the donation be invested to obtain the $\$ 40000.00$ needed to award the scholarship each year? Show your work.

Bonnie and Claude want to buy a house. They can afford monthly payments of $\$ 1125.00$. The bank offers them a mortgage at an interest rate of $3.10 \%$, compounded semi-annually, with an amortization period of 25 years.
a) What is the maximum amount of money the bank will lend them for their mortgage? Show your work.
(2 marks)
b) If they have $\$ 30000.00$ saved for a down payment, what is the maximum house price they can afford?
(1 mark)

Bernard is exploring financing options for a new house. The bank offers him a mortgage of $\$ 245827.00$ at an interest rate of $3.75 \%$, compounded semi-annually. He has the following payment options:

Option 1: monthly payments of $\$ 1260.00$
Option 2: biweekly payments of $\$ 630.00$
a) How many years will it take Bernard to pay off the mortgage with each option? Show your work.
(3 marks)
b) If Bernard makes biweekly payments instead of monthly payments, how much money will he save? Show your work.
(2 marks)

## Design and Measurement

## Question 12

Select the best answer.
Melia baked the following cake and will ice the top and the sides.


The surface area that needs to be iced is:
A) $1290 \mathrm{~cm}^{2}$
B) $1680 \mathrm{~cm}^{2}$
C) $2580 \mathrm{~cm}^{2}$
D) $5400 \mathrm{~cm}^{2}$

A Canadian two-dollar coin consists of a gold-coloured centre and a silver-coloured outer ring. The coin has a diameter of 2.8 cm and is 0.2 cm thick while its centre has a diameter of 1.6 cm .

Diagram is not drawn to scale.


Calculate the volume of the silver-coloured outer ring of the coin.

A farmer is selling a cone-shaped pile of grain that has a diameter of 30 feet and a height of 20 feet. The grain needs to be transported to the market by truck.
a) The grain box of the farmer's truck has a volume of 850 cubic feet. What is the minimum number of times the farmer must go to the market to transport all of the grain? Show your work.
(2 marks)
b) Grain is sold by the whole bushel. This grain has a current value of $\$ 8.50 /$ bushel. If one bushel is equal to 1.24 cubic feet, calculate the value of the pile of grain. Show your work.

## Probability

## Question 15

Total: 1 mark

Select the best answer.
There are 16 girls and 11 boys enrolled in a physical education class. A volleyball team of 4 girls and 2 boys will be formed from this class.

Which of the following expressions could be used to determine the number of teams possible?
A) ${ }_{27} C_{6}$
B) ${ }_{27} P_{6}$
C) ${ }_{16} C_{4} \times{ }_{11} C_{2}$
D) ${ }_{16} P_{4} \times{ }_{11} P_{2}$

Question 16
Total: 2 marks

Rylan rolls two 4-sided dice with sides numbered 1 through 4. What is the probability that the sum of the rolled numbers is greater than or equal to 6 ? Show your work.


## Question 17

How many different routes can Martin take to the mall if he can only travel north and east and wants to meet Amy on the way? Show your work.


## Question 18

Arif, Simba, and Maritza ran for student council treasurer. Of the 650 students who voted:

- $44 \%$ voted for Arif
- $36 \%$ voted for Simba
- the remaining students voted for Maritza
a) Determine the number of students that voted for Maritza.
(1 mark)
b) One of the students is selected at random. Determine the odds against this student having voted for Arif.

Jack is late for the bus $15 \%$ of the time. When he is late for the bus, the probability that he will see Jill at the bus stop is $8 \%$. When he is not late, the probability that he will see Jill at the bus stop is $82 \%$.
a) What is the probability that Jack did not see Jill today? Show your work.
b) Jack did not see Jill today. Using your answer in (a), what is the probability that Jack was late for the bus?
(1 mark)

Guy's baseball team is playing in a tournament. There are six teams entered in the tournament. All teams play each other once and each game is played on the same baseball field.
a) Determine the total number of games played in the tournament. (1 mark)
b) Determine the probability that Guy's team plays the first game of the tournament. (1 mark)

Shivani needs to create a new password for her computer. The password must begin with three upper case letters followed by five digits.
a) How many passwords are possible if repetition is not allowed? Show your work. (2 marks)
b) How many passwords are possible if repetition is not allowed and the password must begin with the letter M?
(1 mark)

Joe is getting dressed in the dark. The only socks in his drawer are 12 white socks and 10 green socks. He randomly picks two socks from the drawer, one after the other.
a) What is the probability that both socks are the same colour? Show your work.
(2 marks)

## LOGICAL REASONING

b) Using logical reasoning, what is the minimum number of socks Joe would have to pick to guarantee a pair of the same-coloured socks?
(1 mark)

Select the best answer.
Given the following sets:

$$
\begin{aligned}
A & =\{3,4,5,6,7,8,9,10\} \\
B & =\{8,9,10,11,12\} \\
C & =\{8,9,10\}
\end{aligned}
$$

Which of the following statements represents set $C$ ?
A) $A^{\prime} \cap B^{\prime}$
B) $B \subset A$
C) $A \cup B$
D) $A \cap B$

## Question 24

Total: 2 marks

Given hypothesis, $p$, and conclusion, $q$, complete the truth table below.

| $\boldsymbol{p}$ | $\boldsymbol{q}$ | $\sim \boldsymbol{p}$ | $\sim \boldsymbol{p} \leftrightarrow \boldsymbol{q}$ |
| :---: | :---: | :---: | :---: |
| True | True |  |  |
| True | False |  |  |
| False | True |  |  |
| False | False |  |  |

Mrs. Dela Cruz teaches German and Spanish. She has 31 students of which 21 study German and 17 study Spanish.

How many of Mrs. Dela Cruz's students study German only? Show your work.

## NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

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## Formula Sheet: Applied Mathematics

| Relations and Functions $\begin{gathered} y=a x+b \\ y=a x^{2}+b x+c \\ y=a x^{3}+b x^{2}+c x+d \\ y=a b^{x} \\ y=a+b \ln (x) \\ y=a \log _{b} x \\ y=a \sin (b x+c)+d \\ y=a \cos (b x+c)+d \end{gathered}$ | Financial Mathematics $\begin{gathered} t=\frac{72}{i} \\ I=P r t \\ A=P\left(1+\frac{r}{n}\right)^{n t} \end{gathered}$ $\begin{aligned} & \text { Net worth }=\text { Total assets }- \text { Total liabilities } \\ & \begin{array}{l} \text { Debt-to-equity } \\ \text { ratio (\%) } \end{array}=\frac{(\text { Total liabilities }- \text { Mortgage })}{\text { Net worth }} \times 100 \\ & \begin{array}{l} \text { Gross debt } \\ \text { service } \\ \text { ratio (\%) } \end{array}=\frac{\left(\begin{array}{c} \text { Monthly Monthly Monthly } \\ \text { mortgage } \left.+\begin{array}{c} \text { property }+\begin{array}{c} \text { heating } \\ \text { payment } \\ \text { taxes } \\ \text { costs } \end{array} \end{array}\right) \\ \text { Gross monthly income } \end{array} 100\right.}{\begin{array}{l} \text { Rate of } \\ \text { return (\%) } \end{array}=\frac{\left(\begin{array}{c} \text { Current value } \left.\begin{array}{c} \text { Previous value } \\ \text { of portfolio } \\ \text { of portfolio } \end{array}\right) \end{array}\right.}{\text { Previous value of portfolio }} \times 100} \end{aligned}$ |
| :---: | :---: |
| Probability $\begin{gathered} P(A \text { or } B)=P(A)+P(B)-P(A \text { and } B) \\ P(A \text { and } B)=P(A) \times P(B) \\ P(A \text { and } B)=P(A) \times P(B \mid A) \\ { }_{n} P_{r}=\frac{n!}{(n-r)!} \\ { }_{n} C_{r}=\frac{n!}{r!(n-r)!} \end{gathered}$ | Design and Measurement <br> Prism: Surface area $=P h+2 B$ $\text { Volume }=B h$ $\begin{gathered} \text { Pyramid: Surface area }=B+\frac{P s}{2} \quad(s=\text { slant height }) \\ \text { Volume }=\frac{B h}{3} \end{gathered}$ <br> Sphere: Surface area $=4 \pi r^{2}$ $\text { Volume }=\frac{4}{3} \pi r^{3}$ $\begin{gathered} \text { Cylinder: Surface area }=2 \pi r h+2 \pi r^{2} \\ \text { Volume }=\pi r^{2} h \end{gathered}$ <br> Cone: Surface area $=\pi r^{2}+\pi r s$ $\text { Volume }=\frac{\pi r^{2} h}{3}$ |

