Grade 12
Applied Mathematics
Achievement Test

## Student Booklet

June 2017

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Websites are subject to change without notice.

## Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

## Grade 12 Applied Mathematics Achievement Test

## DESCRIPTION

Total Possible Marks: 61
Time: 3 hours

| Unit | Description | Marks |
| :---: | :--- | :---: |
| A | Relations and Functions | 16 |
| B | Probability | 15 |
| C | Financial Mathematics | 15 |
| D | Design and Measurement | 6 |
| E | Logical Reasoning | 9 |

## Resources

You may use the following resources:

- Formula Sheet (tear-out page at the back of this booklet)
- one $8^{1 / 2 \prime} \times 11^{\prime \prime}$ study sheet
- ruler
- graphing calculator, computer software, and/or app
- Internet access for tools such as applets or mortgage payment calculators

Use of the Internet to communicate or access other content, including but not limited to course notes, definitions, or conceptual information is not permitted during the test.

## Electronic communication between students through phones, email, or file sharing during the test is strictly prohibited.

## Directions

Read all instructions on the test carefully.
If you need extra paper or you print out an answer to a question, let the teacher know. Indicate in the response space of the question that your answer is on a separate sheet.

Remember to

- indicate your input values by writing them in your booklet or printing a copy if using a technology tool
- include your booklet number and question number on additional pages (e.g., printouts) and attach them to the corresponding page in the booklet
- express your answers in decimal and percentage form to the nearest hundredth (two decimal places) when rounding, unless otherwise indicated
Example: $\frac{15}{29}=0.52$ or $51.72 \%$
- state any assumptions you make

A "graphic organizer" is a visual representation of information. Examples include a tree diagram, a chart, a list, a Venn diagram, a truth table, Pascal's triangle, etc.

## A clearly communicated answer

- is easily identified in the response space
— includes the parameters in the equation, and " $y=$ ", "sin", " $\ln$ ", or " $x$ ", as applicable
- includes the units of measure, where applicable
- includes labels, units, and scales for the axes on graphs
— is expressed as an exact value or is appropriately rounded
Marks may be deducted for errors relating to any of the above.



## PLEASE WAIT UNTIL INSTRUCTED TO TURN THE PAGE.

## RELATIONS AND Functions

## Question 1

Select the answer that best completes the statement.
The end behaviour of a cubic function can extend from:
A. quadrant I to quadrant II
B. quadrant II to quadrant I
C. quadrant II to quadrant IV
D. quadrant III to quadrant IV

Use the graph below to answer the following question and select the best answer.


Which equation does the graph represent?
A. $y=6 \sin (4 x)+5$
B. $y=12 \sin (4 x)+5$
C. $y=-6 \sin (4 x)+5$
D. $y=-12 \sin (4 x)+5$

Jordan is making an open-top box using an 8 in. by 10 in. sheet of cardboard. He plans to cut squares of length $x$ from the corners and fold up the sides. The function that represents the volume of the box, $V$, can be modelled by the equation:

$$
V=(x)(10-2 x)(8-2 x)
$$



Determine the maximum volume of the box. Show your work.

When jumping rope, the centre of the rope reaches a maximum height of 1.90 m at 0.38 s and a minimum height of 0.08 m at 0.88 s .


For the sinusoidal function that models the height at the centre of the rope as a function of time,
a) determine the period.
(1 mark)
b) determine the median height.
(1 mark)

A microbiologist is studying bacterial growth over 2 full days. She notices that the bacterial count doubles every 3 hours. The initial bacterial count is 6 .
a) Determine an exponential equation that represents this situation. Show your work.
(2 marks)
b) How much time does it take until the bacterial count is 1000 ? Show your work. (2 marks)
c) State the domain for this situation.
(1 mark)

A small town gets cellphone service. The number of users increases as a function of time as shown in the table below.

| Time (weeks) | 1 | 2 | 4 | 8 | 12 | 20 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of users | 202 | 319 | 480 | 656 | 743 | 821 |

a) Create a clearly labelled graph by plotting the given data. Draw the curve of best fit. (3 marks)

b) Determine the regression equation that best models the data in this situation.
c) According to your equation in (b), how many users did the town have at 15 weeks?
(1 mark)

## Probability

## Question 7

## Total: 1 mark

Select the best answer.
The probability of rain tomorrow is $\frac{5}{8}$. What are the odds against rain tomorrow?
A. $3: 8$
B. $5: 8$
C. $3: 5$
D. $5: 3$

## Question 8

Total: 2 marks

A survey of 25 students found that the probability of a student having a cat or a dog is $72 \%$. Of the 25 students, $44 \%$ said they have a cat and $64 \%$ said they have a dog.

How many students have both a cat and a dog? Show your work.

How many different routes are there from A to B if you only go east and south? Show your work.


Andrei and Sergei are playing soccer. They each take a shot on goal. Andrei shoots first and Sergei shoots second. The probability of Andrei scoring is 0.70 and the probability of Sergei scoring is 0.60 .
a) Use a graphic organizer to show all the possible outcomes for this situation. (1 mark)
b) What is the probability that at least one of them will score? Show your work. (2 marks)

An ultimate frisbee team consists of 7 players. A team is formed by randomly choosing players from a group of 8 men and 7 women.
a) Determine the number of ways 7 players can be chosen to form a team.
(1 mark)
b) Determine the probability that the team has exactly 3 women. Show your work.
(2 marks)

## Question 12

While playing a game, Cally pulls out the following letters from a bag containing some vowels (A, E, I, O, U) and some consonants:

a) How many different arrangements can be made using all of the letters above? Show your work.
(2 marks)
b) If Cally places all the vowels together and all the consonants together, how many different arrangements can be made using all of the letters above? Show your work.

## Financial Mathematics

## Question 13

Total: 1 mark

Select the best answer.
Xavier needs a loan to purchase a car. Which of the following options would result in the least interest paid?

A loan with an interest rate of:
A. $5.50 \%$, compounded daily
B. $5.50 \%$, compounded weekly
C. $5.50 \%$, compounded monthly
D. $5.50 \%$, compounded annually

## Question 14

Total: 1 mark
Select the best answer.
Shalini invests $\$ 25000.00$ in a Canada Savings Bond that earns a simple interest rate of $1.90 \%$. Determine the value of the investment at the end of 5 years.
A. $\$ 2375.00$
B. $\$ 27375.00$
C. $\$ 27489.41$
D. $\$ 237500.00$

The Bashir family wants to buy a house. They can afford monthly payments of $\$ 1325.00$. The bank offers them an interest rate of $3.25 \%$, compounded semi-annually over 15 years or 25 years.
a) Determine the maximum amount they can borrow if the mortgage is amortized over 15 years. Show your work.
(2 marks)
b) Determine the maximum amount they can borrow if the mortgage is amortized over 25 years.
(1 mark)
c) Give one reason why the Bashir family would choose to buy the less expensive house. (1 mark)

## Question 16

You are presented with two different investment plans:
Plan A: Invest $\$ 1000.00$ every year for 10 years at an interest rate of $6.00 \%$, compounded monthly.

Plan B: Invest a lump sum of $\$ 10000.00$ at an interest rate of $6.00 \%$, compounded monthly for 10 years.
a) Determine the value of each investment plan after 10 years. Show your work. (3 marks)
b) Calculate the total amount of interest earned for each plan. (1 mark)
c) Which plan would you choose? Explain. (1 mark)

The Connors are purchasing a condominium that costs $\$ 190000.00$. They have $\$ 25000.00$ saved for a down payment and will finance the balance at an interest rate of $5.50 \%$ for 20 years, compounded semi-annually.
a) Determine their monthly mortgage payment. Show your work.
(2 marks)
b) There are condominium fees of $\$ 300.00$ per month in addition to the down payment and mortgage payments. Calculate the total amount they will have paid after 5 years. Show your work.
(2 marks)

## Design and Measurement

## Question 18 <br> Total: 3 marks

A spherical balloon has a diameter of 30 cm . Air is added to the balloon until its surface area increases by $500 \mathrm{~cm}^{2}$. What is the new diameter?

## Question 19

Total: 3 marks

Students are building birdhouses as a class project. The birdhouses are built to the specifications shown in the diagram below. (Diagram is not drawn to scale.)


Identify all of the faces of one birdhouse. Use the graphing paper on the next page to illustrate the number of birdhouses the students can build from a $4 \mathrm{ft} . \times 3 \mathrm{ft}$. sheet of plywood.

Face
Dimensions

## Sheet of Plywood



3 ft .

The students can build a maximum of $\square$ birdhouse(s).

## LOGICAL REASONING

## Question 20

Select the best answer.
Which of the following statements is not biconditional?
A. If you invest at $6 \%$, then your investment will double in about 12 years.
B. If a quadrilateral is a square, then it is a rectangle.
C. If $x+y=3$, then $y=3-x$.
D. If a glass is half empty, then it is half full.

## Question 21

Total: 1 mark

Complete the pattern in the lower-right quadrant.


A student is shown 5 sprockets and a chain. The student predicts that when sprocket A is turned clockwise the chain will go downwards. Is he correct? Explain.


Consider the conditional statement below:
"If the probability of snow tomorrow is $\frac{4}{5}$, then the odds in favour of snow tomorrow are $4: 1 . "$
a) Write the converse of the conditional statement.
(1 mark)
b) Determine if a biconditional statement is possible. If yes, write the biconditional statement. If not, provide a counterexample.
(1 mark)

A survey of 95 families found that:

- 54 families have a car
- 46 families have a truck
- 28 families have a motorcycle
- 24 families have only a truck
- 10 families have a car and a truck
- 8 families have only a car and a motorcycle
- 5 families have all three
a) How many families do not have any vehicle? Show your work. (3 marks)
b) How many families have at least two vehicles?
(1 mark)


## NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

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## Formula Sheet: Applied Mathematics

| Relations and Functions $\begin{gathered} y=a x+b \\ y=a x^{2}+b x+c \\ y=a x^{3}+b x^{2}+c x+d \\ y=a b^{x} \\ y=a+b \ln (x) \\ y=a \log _{b} x \\ y=a \sin (b x+c)+d \\ y=a \cos (b x+c)+d \end{gathered}$ | Financial Mathematics $\begin{gathered} t=\frac{72}{i} \\ I=P r t \\ A=P\left(1+\frac{r}{n}\right)^{n t} \end{gathered}$ $\begin{aligned} & \text { Net worth }=\text { Total assets }- \text { Total liabilities } \\ & \begin{array}{l} \text { Debt-to-equity } \\ \text { ratio (\%) } \end{array}=\frac{(\text { Total liabilities }- \text { Mortgage) })}{\text { Net worth }} \times 100 \\ & \begin{array}{l} \text { Gross debt } \\ \text { service } \\ \text { ratio (\%) } \end{array}=\frac{\left(\begin{array}{c} \text { Monthly Monthly Monthly } \\ \text { mortgage } \left.+\begin{array}{c} \text { property } \\ \text { payment heating } \\ \text { taxes } \end{array}\right) \\ \text { Gross monthly income } \end{array}\right)}{} \times 100 \\ & \begin{array}{l} \text { Rate of } \\ \text { return (\%) } \end{array}=\frac{\left(\begin{array}{c} \text { Current value Previous value } \\ \text { of portfolio } \left.-\begin{array}{c} \text { of portfolio } \end{array}\right) \end{array}\right.}{\text { Previous value of portfolio }} \times 100 \end{aligned}$ |
| :---: | :---: |
| Probability $\begin{gathered} P(A \text { or } B)=P(A)+P(B)-P(A \text { and } B) \\ P(A \text { and } B)=P(A) \times P(B) \\ P(A \text { and } B)=P(A) \times P(B \mid A) \\ { }_{n} P_{r}=\frac{n!}{(n-r)!} \\ { }_{n} C_{r}=\frac{n!}{r!(n-r)!} \end{gathered}$ | Design and Measurement <br> Prism: Surface area $=P h+2 B$ $\text { Volume }=B h$ $\begin{gathered} \text { Pyramid: Surface area }=B+\frac{P s}{2} \quad(s=\text { slant height }) \\ \text { Volume }=\frac{B h}{3} \end{gathered}$ <br> Sphere: Surface area $=4 \pi r^{2}$ $\text { Volume }=\frac{4}{3} \pi r^{3}$ $\begin{gathered} \text { Cylinder: Surface area }=2 \pi r h+2 \pi r^{2} \\ \text { Volume }=\pi r^{2} h \end{gathered}$ <br> Cone: Surface area $=\pi r^{2}+\pi r s$ $\text { Volume }=\frac{\pi r^{2} h}{3}$ |

