Grade 12 Applied Mathematics Achievement Test

Student Booklet

June 2015



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Disponible en français.

Available in alternate formats upon request.

GRADE 12 APPLIED MATHEMATICS ACHIEVEMENT TEST

DESCRIPTION

Total Possible Marks: 60

Time: 3 hours

Unit	Description	Marks
А	Relations and Functions	16
В	Probability	15
С	Financial Mathematics	17
D	Design and Measurement	7
E	Logical Reasoning	5

RESOURCES

You may use the following resources:

- Formula Sheet (tear-out page at the back of this booklet)
- one $8\frac{1}{2}'' \times 11''$ study sheet
- ruler
- graphing calculator, computer software, and/or app
- Internet access for tools such as applets or mortgage payment calculators

Use of the Internet to communicate or access other content, including but not limited to course notes, definitions, or conceptual information is not permitted during the test.

Electronic communication between students through phones, email, or file sharing during the test is strictly prohibited.

DIRECTIONS

Read all instructions on the test carefully.

If you need extra paper or you print out an answer to a question, let the teacher know. Indicate in the response space of the question that your answer is on a separate sheet.

Remember to

- indicate your input values by writing them in your booklet or printing a copy if using a technology tool
- include your booklet number and question number on additional pages (e.g., printouts) and attach them to the corresponding page in the booklet
- express answers in decimal and percentage form to **two decimal places** when rounding, unless otherwise indicated

Example:
$$\frac{15}{29} = 0.52$$
 or 51.72%

• state any assumptions you make

A "graphic organizer" is a visual representation of information. Examples include a tree diagram, a chart, a list, a Venn diagram, a truth table, Pascal's triangle, etc.

A clearly communicated answer

- is easily identified in the response space
- includes the parameters in the equation, and "y =", "sin", "ln", or "x", as applicable
- includes the units of measure, where applicable
- includes labels, units, and scales for the axes on graphs
- is expressed as an exact value or is appropriately rounded

Marks may be deducted for errors relating to any of the above.



PLEASE WAIT UNTIL INSTRUCTED TO TURN THE PAGE.

Applied Mathematics: Student Booklet (June 2015)

RELATIONS AND FUNCTIONS

Question 1

Select the best answer.

Which of the following functions has an unrestricted domain and an unrestricted range?

- A. cubic
- **B.** exponential
- C. logarithmic
- **D.** quadratic

Total: 1 mark

102

Select the best answer.

Which graph below represents a logarithmic function?





Sam is studying the characteristics of quadratic functions.

She states the following:

Statement 1: A quadratic function always has a degree of 2.

Statement 2: The graph of a quadratic function always extends from quadrant II to quadrant I.

Statement 3: The graph of a quadratic function always has one turning point.

Statement 4: A quadratic function always has two *x*-intercepts.

Two of these statements are incorrect. Identify which statements are incorrect and provide a counterexample for each.

The table below shows the electricity consumption in gigawatt-hours (GWh) of a large city since 1960.

Years (since 1960)	Consumption (GWh)
0	11 967
10	21 139
20	19 877
30	19 173
40	23 333
50	24 087

a) Determine the equation for the cubic regression that models the data.

(1 mark)

b) Using your equation in (a), estimate the electricity consumption in 2015. Show your work and round your answer to the nearest whole value.

(2 marks)

105

George is diabetic and the amount of sugar in his blood (concentration) gradually decreases over time according to the function:

 $c = 9(0.995)^t$

where *c* represents the concentration and *t* represents the time (in minutes).

His initial blood sugar concentration is 9.

a) State the range in this situation.

(1 mark)

b) George forgets to eat his morning snack. He experiences dizziness once his blood sugar concentration drops below 3. After how many minutes will this happen? Show your work.

(2 marks)

108

The top of a flagpole moves back and forth on a windy day. It sways 15 cm to the left (-15 cm) and 15 cm to the right (+15 cm) from its resting position (0 cm). It moves back and forth every two (2) seconds. At t = 0, the pole was at its resting position before swaying to the left.

a) Determine a sinusoidal equation that models this situation. Show your work.

(2 marks)

b) Graph the equation for two (2) full cycles.

(3 marks)

c) On an even windier day, the flagpole moves back and forth every second. Explain how your equation will change.

(1 mark)

PROBABILITY

Question 7

Select the best answer.

A game involves drawing one card from a set of cards numbered from 1 to 20. The desired outcomes in the game are to draw an even number or a multiple of 5.

The outcomes in this game can best be described as:

- A. non-mutually exclusive
- **B.** mutually exclusive
- C. independent
- **D.** dependent

Question 8

Select the best answer.

A student is given two spinners: one divided into three equal sections and the other divided into four equal sections, as shown below.





D. $\frac{2}{7}$

12



111

Total: 1 mark

Total: 1 mark



The probability of rain is 60%. What are the odds against rain?

Question 10

A pizza place offers the following choices:

- 3 types of crust
- 2 types of sauce
- 5 types of cheese
- 6 meat toppings
- 8 vegetable toppings

Charles would like to create a pizza by choosing one item from each category. Determine how many different pizzas can be made.

113

115

Create a scenario where the calculation $_{8}C_{5} \times _{9}C_{2}$ would be appropriate.

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116

A contractor has five workers he can assign to a task. How many ways can the contractor assign at least one worker to the task?

On January 21, the probability of the overnight low temperature being below -27° C is 11%. If the temperature is below -27° C, the probability of Cara's car starting is 55%. If it is not below -27° C, the probability of Cara's car starting is 91%.

a) Create a graphic organizer to represent all possibilities for this situation.

(1 mark)

b) What is the probability that Cara's car will not start on January 21? Show your work. *(2 marks)*

A teacher surveys her class of 29 students and finds out that in the past week, 15 students worked on an assignment, 12 students studied for a test, and 7 students did both.

a) How many students did not work on an assignment nor study for a test?

(1 mark)

120

119

b) The teacher decides to randomly put students in groups of three for a class project. What is the probability that all three members of a group will not have worked on an assignment nor studied for a test?

(2 marks)

There are 50 high school students in a small town. The school's cross-country coach wants to randomly select 9 students to form a team.

a) How many different teams can be created?

(1 mark)

b) The coach rents a van and drives his team to the race. Calculate the number of ways the students can be seated, if there are 9 passenger seats.

(1 mark)

122

FINANCIAL MATHEMATICS

Question 16

Select the best answer.

The value of a house appreciates 4.10% per year. If the house was purchased for \$180 000 in June 2010, what is the approximate value of the house in June 2015?

- **A.** \$184 000
- **B.** \$220 000
- **C.** \$937 000
- **D.** \$1 003 000

Total: 1 mark

Deema is saving for a home renovation. She deposits \$50.00 every month into a new high-interest savings account that earns 4.60%, compounded monthly.

a) What will be the value in Deema's account after 4 years? Show your work.

(2 marks)

b) How much interest will she have earned after 4 years? Show your work.

(2 marks)

126

Lisette is 50 years old and would like to retire at age 55. Her assets include \$60 000.00 in a guaranteed investment certificate (GIC), \$78 000.00 in a chequing account, and \$192 000.00 in stocks.

Make two recommendations to help Lisette strengthen her investment portfolio. Justify your recommendations.

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127

Serge invests \$12 000.00 on his 30th birthday. If his investment earns 6.00% interest, compounded annually, apply the Rule of 72 to estimate how much money he will have when he retires on his 66th birthday.

Harsimran invested \$20 000.00 in a simple interest savings bond. After 12 years, the future value of the bond was \$26 768.00.

a) Determine the annual interest rate.

(2 marks)

(1 mark)

Maurice needs a truck for his new job that will last four years. He has the following two options:

- **Option 1:** He can purchase the truck for \$45 194.35 (taxes included) with no down payment, at an interest rate of 4.00%, compounded monthly, for four years.
- **Option 2:** He can lease the same truck for four years with a down payment of \$5000.00 and monthly payments of \$850.00 (taxes included). He would be allowed 20 000 km per year and there is a \$0.18 per km (taxes included) charge above this limit.
- a) What would be Maurice's total cost in Option 1 if he makes monthly payments? Show your work.

(2 marks)

b) If Maurice drives 25 000 km annually, what would be his total cost in Option 2? Show your work.

(2 marks)

c) Which option would you suggest to Maurice and why?

(1 mark)

131

132

DESIGN AND MEASUREMENT

Question 22

Select the best answer.

How many cubic yards are in 54 cubic feet?

- **A.** 2
- **B.** 3
- **C.** 6
- **D.** 18

Question 23	Total: 1 mark

One can of paint can cover an area of 200 ft^2 .

How many cans need to be purchased to paint a 60 ft. by 8 ft. wall?

Total: 1 mark

134

James is landscaping his 50 ft. by 40 ft. yard. He will construct a concrete walkway with a uniform width of x around the centre of the yard which is to be covered in sod, as illustrated below. (Diagram is not drawn to scale.)



Consider the following:

- The walkway must be at least 3.5 feet wide.
- The concrete must be poured 6 inches deep.
- The concrete costs \$3.00 per cubic foot, plus GST and PST.
- The sod costs \$0.40 per square foot, plus GST and PST.
- The budget for this project is \$2150.00.
- a) Design a walkway that fits within the budget. Indicate the width of the walkway and the dimensions of the sod below.

(1 mark)

Width of the walkway (*x*): ______ ft.

Dimensions of the sod: _____ ft. by _____ ft.

b) Calculate the total cost of your design. (Note: GST = 5%, PST = 8%)

(4 marks)

LOGICAL REASONING

Question 25

137

Select the best answer.

What values of *m* and *n* correctly complete the following truth table?

р	q	$p \rightarrow q$
True	True	True
True	False	т
False	True	п
False	False	True

- A. m = True n = True
- **B.** m = True n = False
- **C.** m = False n = True
- **D.** m =False n =False

Consider this conditional statement:

"If *x* has a positive value, then $x^2 \ge x$."

Provide a counterexample to the statement above.

A universal set is defined as:

 $S = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

Two subsets are given below:

 $O = \{ \text{odd whole numbers of } S \}$ $P = \{ \text{prime numbers of } S \}$

(1 mark)

b) Determine $O \cup P$.

(1 mark)

c) Determine $n(O \cap P')$.

(1 mark)

END OF TEST

Formula Sheet: Applied Mathematics

Relations and Functions	Financial Mathematics
y = ax + b	$t = \frac{72}{i}$
$y = ax^2 + bx + c$	I = Prt
$y = ax^3 + bx^2 + cx + d$	$(r)^{nt}$
$y = ab^x$	$A = P\left(1 + \frac{r}{n}\right)$
$y = a + b \ln(x)$	Net worth = Total assets – Total liabilities
$y = a \log_b x$	Debt to equity ratio (%) = $\frac{\text{(Total liabilities - Mortgage)}}{\text{Net worth}} \times 100$
$y = a\sin(bx + c) + d$	(Monthly Monthly Monthly)
$y = a\cos(bx + c) + d$	$\frac{\text{Gross debt}}{\text{service}}_{\text{ratio (%)}} = \frac{\begin{pmatrix} \text{mortgage + property + heating} \\ \text{payment taxes costs} \end{pmatrix}}{\text{Gross monthly income}} \times 100$
	(Current value Previous value)
	Rate of return (%) = $\frac{(\text{ of portfolio } - \text{ of portfolio })}{\text{Previous value of portfolio}} \times 100$
Probability	Design and Measurement
	Design and wreasurement
P(A or B) = P(A) + P(B) - P(A and B)	Prism: Surface area = $Ph + 2B$
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $P(A \text{ and } B) = P(A) \times P(B)$	Prism: Surface area = $Ph + 2B$ Volume = Bh
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $P(A \text{ and } B) = P(A) \times P(B)$ $P(A \text{ and } B) = P(A) \times P(B A)$	Prism: Surface area = $Ph + 2B$ Volume = Bh Pyramid: Surface area = $B + \frac{Ps}{2}$ (s = slant height)
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $P(A \text{ and } B) = P(A) \times P(B)$ $P(A \text{ and } B) = P(A) \times P(B A)$ $P = -\frac{n!}{n!}$	Prism: Surface area = $Ph + 2B$ Volume = Bh Pyramid: Surface area = $B + \frac{Ps}{2}$ (s = slant height) Volume = $\frac{Bh}{3}$
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