Grade 12
Applied Mathematics
Achievement Test

## Student Booklet

June 2014

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## Disponible en français.

Available in alternate formats upon request.

# Grade 12 Applied Mathematics Achievement Test 

## DESCRIPTION:

Total Possible Marks: 60
Time: 3 hours

| Unit | Description | Marks |
| :---: | :--- | :---: |
| A | Relations and Functions | 16 |
| B | Probability | 15 |
| C | Financial Mathematics | 18 |
| D | Design and Measurement | 6 |
| E | Logical Reasoning | 5 |

## Formula Sheet: Applied Mathematics

| Relations and Functions $\begin{gathered} y=a x+b \\ y=a x^{2}+b x+c \\ y=a x^{3}+b x^{2}+c x+d \\ y=a b^{x} \\ y=a+b \ln (x) \\ y=a \log _{b} x \\ y=a \sin (b x+c)+d \\ y=a \cos (b x+c)+d \end{gathered}$ | Financial Mathematics $\begin{gathered} t=\frac{72}{i} \\ I=P r t \\ A=P\left(1+\frac{r}{n}\right)^{n t} \end{gathered}$ |
| :---: | :---: |
| Probability $\begin{gathered} P(A \text { or } B)=P(A)+P(B)-P(A \text { and } B) \\ P(A \text { and } B)=P(A) \times P(B) \\ P(A \text { and } B)=P(A) \times P(B \mid A) \\ { }_{n} P_{r}=\frac{n!}{(n-r)!} \\ { }_{n} C_{r}=\frac{n!}{r!(n-r)!} \end{gathered}$ | Design and Measurement <br> Prism: Surface area $=P h+2 B$ $\text { Volume }=B h$ $\begin{gathered} \text { Pyramid: Surface area }=B+\frac{P s}{2} \quad(s=\text { slant height }) \\ \text { Volume }=\frac{B h}{3} \end{gathered}$ <br> Sphere: Surface area $=4 \pi r^{2}$ $\text { Volume }=\frac{4}{3} \pi r^{3}$ $\begin{gathered} \text { Cylinder: Surface area }=2 \pi r h+2 \pi r^{2} \\ \text { Volume }=\pi r^{2} h \end{gathered}$ <br> Cone: Surface area $=\pi r^{2}+\pi r s$ $\text { Volume }=\frac{\pi r^{2} h}{3}$ |

## Test Resources and Directions:

- You may consult your $8^{1 / 2} 2^{\prime \prime} \times 11^{\prime \prime}$ study sheet during the test.
- You may use a ruler, a graphing calculator, and computer software. You may also have access to the Internet for tools such as applets or a mortgage payment calculator. Use of the Internet to access course notes, find definitions, communicate, or search for conceptual information about the course is prohibited during the test.
- For short-answer and long-answer questions, you may print out diagrams from the computer or your calculator where applicable. Indicate your booklet number and the question number on the printouts. Remain seated and your teacher will distribute these printouts to you.
Indicate in the response space of the question that the answer is on a printed sheet and staple it to the page.
- If you need more space to answer a question, extra paper may be provided by your teacher. Write your booklet number and the question number on any extra paper used and staple it into the booklet where your answer begins. Indicate in the response space of the question that the answer is on a separate sheet.
- Provide clear explanations or justifications where applicable. This can be done through labelled diagrams, in words, by showing mathematical operations to verify your answer, or by referring to a calculator or software program.
- If you refer to a calculator program, indicate all your input values.
- If you refer to a software program or a website, indicate all your input values and print or copy the screen showing the answers.
- If you refer to a spreadsheet, print a copy of the answers.
- A graphic organizer is a visual representation of information. Examples include a tree diagram, a chart, a list, a Venn diagram, a truth table, Pascal's triangle, etc.
- Round your final answers to two decimal places unless otherwise indicated.
- Always state your assumptions.

The following errors may result in a 0.5 mark deduction:

- not including one of the following in the equation: " $y=$ ", "sin", "ln", or " $x$ ", or writing parameters separately from the equation
- not including the units in the final answer
- not including one of the following on the graph: labels for the axes, units for the axes, or scales for the axes
- not stating or incorrectly stating the final answer
- rounding too soon or rounding incorrectly
- not using whole units appropriately
- making a transcription or transposition error


## Relations and Functions

## Question 1

Total: 1 mark

Select the equation below that is best represented by the following graph.

A. $y=-4.70 \sin x$
B. $y=-1.00 x^{3}-4.70 x^{2}+5.00 x$
C. $y=5.00+4.70 \ln x$
D. $y=4.70 x^{2}+1.00 x+5.00$

## Question 2

Total: 2 marks

When inflating a balloon, the volume of air in the balloon can be modelled by the equation:

$$
V=0.02 c^{3}-0.73 c^{2}+11.30 c-12.79
$$

where $V$ represents the volume $\left(\mathrm{cm}^{3}\right)$ of air in the balloon and $c$ represents the circumference $(\mathrm{cm})$ of the balloon.

How much air would need to be blown into the balloon so that it has a circumference of 60 cm ? Show your work.

Cobalt-60 is an isotope used in medical imaging. It decays naturally over time according to the equation:

$$
t=35.01-7.60 \ln P
$$

where $t$ represents the time in years
and $P$ represents the percentage of the original material that is still radioactive.
State the domain and the range of the logarithmic function in the context of this situation.

Domain: $\qquad$

> Range:
$\qquad$

A store owner wants to increase his profits. Suppose that his operating costs and his earnings are modelled by the following equations:

$$
\begin{array}{ll}
\text { costs: } & y=0.04 x^{2}+44.00 x+1500.00 \\
\text { earnings: } & y=1.25^{x}
\end{array}
$$

where $x$ represents the time, in hours, that the store is open per week and $y$ represents the operating costs or earnings, in dollars.
a) Create a clearly labelled graph of both equations on the axes below.
b) Using a graphing calculator or graphing software, determine the minimum number of hours that the store should stay open in order to make a profit (earnings are greater than the cost). Explain how you arrived at your answer. State your answer to one decimal place.
(2 marks)

The average monthly temperatures for one year in Snow Lake are shown in the table below.

| Month | Average Monthly <br> Temperature $\left({ }^{\circ} \mathbf{C}\right)$ |
| :---: | :---: |
| January | -20.2 |
| February | -15.1 |
| March | -8.0 |
| April | 1.9 |
| May | 9.6 |
| June | 15.8 |
| July | 18.6 |
| August | 17.3 |
| September | 10.0 |
| October | 3.2 |
| November | -8.1 |
| December | -17.6 |

a) Determine the sinusoidal equation that best represents this data.
b) Using your sinusoidal equation in (a), calculate the length of time that the average monthly temperature was at or above $10^{\circ} \mathrm{C}$. Show your work.
(2 marks)

A water well has a pump that can initially extract 300 gallons of water per day. The water level in the well begins to drop according to the function:

$$
W=300\left(\frac{4}{5}\right)^{\frac{d}{10}}
$$

where $W$ represents the volume of water, in gallons, extracted daily and $d$ represents the number of days after the water level begins to drop.
a) Determine the volume of water extracted on the 100th day after the water level begins to drop. Show your work.
(2 marks)
b) On what day will the pump first extract less than 75 gallons of water per day? Show your work.
(2 marks)

## Probability

## Question 7

How many different ways can all 7 letters of the word "OAKBANK" be arranged?
A. 210
B. 1260
C. 2520
D. 5040

## Question 8

## Total: 1 mark

111
Brien states that taking a driver's education course and passing the road test on the first attempt are dependent events. Explain why Brien is correct.

Determine the number of paths you can use to go from point A to point B if you can only move south or east. Show your work.


John has 24 coins in his piggy bank and 6 of them are quarters. He reaches into his piggy bank and pulls out a coin at random.
a) Determine the probability that the coin will be a quarter.
(1 mark)
b) Determine the odds against the coin being a quarter.
(1 mark)

A group of 6 friends is going to a concert. How many different ways can they sit in a row if Jasmin and Leena must sit beside each other? Show your work.

Among a group of students, $65 \%$ will attend a hockey game, $55 \%$ will go out for supper, and $30 \%$ will attend a hockey game and go out for supper.

Determine the percentage of students who will neither attend a hockey game nor go out for supper. Show your work.

A fisherman knows that the probability of catching a fish depends on the weather. If it is raining, the probability of catching a fish is $30 \%$. If it is not raining, the probability of catching a fish is $10 \%$. During an average fishing season, it rains $20 \%$ of the time.

Determine the probability that the fisherman will catch a fish on any given day. Show your work.

A school's drama club includes 14 members: 8 boys and 6 girls. Four members are selected to attend a workshop.
a) How many possible groups of 4 members can be selected if there are no restrictions?
(1 mark)
b) How many possible groups of 4 members can be selected if at least one boy must be in the group? Show your work.
(2 marks)

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## Financial Mathematics

## Question 15

## Total: 1 mark

Zoe made an investment of $\$ 5000.00$. Ten (10) years later, the investment was worth $\$ 6917.11$. What was the annual interest rate, if the interest was compounded monthly?
A. $3.25 \%$
B. $3.30 \%$
C. $3.83 \%$
D. $39.59 \%$

## Question 16

Total: 1 mark

Which of the following assets is most likely to appreciate in value?
A. car
B. house
C. computer
D. television

Marko wants a new sports car. He wonders whether buying or leasing the car would be the better option.

## Option 1: Buying

- purchase price of $\$ 30000.00$ (taxes included)
- payments every two weeks
- down payment of $\$ 5000.00$
- interest rate of $3.00 \%$, compounded every two weeks


## Option 2: Leasing

- monthly payments of $\$ 300.00$ (taxes included) for 5 years
- residual value of $\$ 15000.00$ (taxes included)
a) If Marko chooses Option 1 and wants to pay off the car over a five-year period, how much would his payment be every two weeks? Show your work.
b) Calculate the total cost of Option 2 if Marko purchases the car for its residual value at the end of the lease.


## (l mark)

c) Which option should Marko choose? Explain your reasoning. (1 mark)

Mr. Van Wyck's assets are worth $\$ 650000.00$. The mortgage on his house is $\$ 250000.00$ and he owes $\$ 130000.00$ in total on his credit lines and credit cards.
a) Calculate Mr. Van Wyck's net worth.
(1 mark)
b) Calculate Mr. Van Wyck's debt to equity ratio. Based on your answer, do you think the bank will lend him money? Explain.
(2 marks)

Francis makes a one-time investment of \$12000.00 in a registered retirement savings plan at $5.00 \%$, compounded semi-annually. He plans to withdraw the money when he retires in 30 years.
a) Determine the value of the investment when Francis retires. Show your work.
(2 marks)
b) Calculate his rate of return over the 30 years. Show your work. (2 marks)

Therese and Alphonse purchased a house valued at $\$ 354000.00$. They made a $\$ 60000.00$ down payment and obtained a mortgage amortized over 25 years at an interest rate of $4.75 \%$, compounded semi-annually.
a) Determine Therese and Alphonse's monthly mortgage payment. Show your work. (2 marks)
b) What will be the balance owing on the mortgage after 5 years? (1 mark)
c) After the initial 5-year period, Therese and Alphonse renegotiate their mortgage. The bank offers them an interest rate of $2.25 \%$, compounded semi-annually. If their monthly payment remains the same, how much sooner will they be able to pay off their mortgage? Show your work.
(2 marks)

## Design and Measurement

## Question 21

Total: 2 marks

Philippa wants to cover her dining room floor with linoleum. The floor measures $14 \mathrm{ft} . \times 12 \mathrm{ft}$. The linoleum costs $\$ 13.99$ per square yard and must be purchased in whole units.

What will be the total cost for the flooring, including taxes? Show your work.
(Note: GST $=5 \%$, PST $=8 \%$ )

Mackenzie Construction was awarded the contract to build gravel shoulders along the highway between Wabowden and Thompson. (Diagram is not drawn to scale.)

The gravel shoulders will be

- along a 22 mile segment of the highway
- on both sides of the highway
- 10 feet wide
- 20 inches deep

Note: 1 mile $=5280$ feet


How many truckloads of gravel will be needed for the project if a truck holds 20 cubic yards of gravel? Show your work.

## LOGICAL REASONING

Select the statement below which best completes the following truth table.

| $\boldsymbol{p}$ | $\boldsymbol{q}$ |  |
| :---: | :---: | :---: |
| True | True | True |
| True | False | False |
| False | True | False |
| False | False | False |

A. $\quad p \cap q$
B. $p \cup q$
C. $p \Rightarrow q$
D. $p \Leftrightarrow q$

Given the following situation:

- the universal set $U=\{$ positive integers less than 10$\}$
- $A=\{2,3,4,5,6\}$
- $B=\{$ even numbers of $U\}$

Determine $A \cap B$.

Given the statement: "If I live in Winnipeg, then I live in Manitoba."
a) Write the inverse of the given statement.
(1 mark)
b) Is the given statement biconditional? Explain.
(1 mark)
c) Write the contrapositive of the given statement. (1 mark)

## NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

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