Grade 12
Applied Mathematics
Achievement Test

# Student Booklet

January 2024

Manitoba

# GRADE 12 APPLIED MATHEMATICS
## ACHIEVEMENT TEST

## DESCRIPTION

**Time Required to Complete the Test: 3 hours**          **Total Possible Marks: 66**
**Additional Time Allowed: 30 minutes**

| Unit | Marks |
|---|---|
| Relations and Functions | 15 |
| Financial Mathematics | 18 |
| Probability | 15 |
| Design and Measurement | 10 |
| Logical Reasoning | 8 |

## DIRECTIONS

- Indicate your input values by writing them in your booklet or printing a copy if using a technology tool.

- State any assumptions you make.

- When rounding, express your answers in decimal or percentage form to at least the nearest hundredth (two decimal places), except for monetary values or when otherwise indicated.

  Example: $\dfrac{15}{29} = 0.52$ or $51.72\%$

- When no tax calculation is necessary, the wording "taxes included" will be used. When you are required to add taxes, the wording "plus GST and/or PST" will be used and current tax rates will be given (e.g., GST = 5%, PST = 7%).

**Note:** Rounding too soon in your solution may result in an inaccurate final answer for which full marks will not be awarded.

**A clearly communicated answer**

— is easily identified in the response space

— includes the parameters in the equation, and "$y =$", "sin", "ln", or "$x$", as applicable

— includes the units of measure, where applicable

— includes labels, units, scales for the axes on graphs, and key characteristics of functions (e.g., maximum, minimum, intercepts, and appropriate shape)

— is expressed as an exact value or is appropriately rounded

Marks may be deducted for errors relating to any of the above.

> **Electronic communication between students through phones,
> email, or file sharing during the test is strictly prohibited.**

# Terminology Sheet

Some questions may include directing words such as *calculate* and *determine*. These directing words are explained below.

| Directing words | The question is asking for… |
| --- | --- |
| **Calculate/Determine** | a mathematical formula, an algebraic equation, or a numerical calculation to solve a problem |
| **Complete** | a table, diagram, or graph to be filled in |
| **Create/Draw/Use a graphic organizer** | a visual representation of information such as a graph, tree diagram, chart, list, Venn diagram, truth table, or Pascal's triangle |
| **Describe/Explain** | words or symbols, diagrams, charts or graphs, or other methods that clearly show what you are thinking |
| **Indicate/Select** | a stated or circled answer |
| **Show your work/Justify** | reasons or facts that support a position by using mathematical computations, words, or diagrams |
| **State/Write** | a word, sentence, or number, without an explanation |

# Formula Sheet: Applied Mathematics

| RELATIONS AND FUNCTIONS | FINANCIAL MATHEMATICS |
|---|---|

### RELATIONS AND FUNCTIONS

$$y = ax + b$$

$$y = ax^2 + bx + c$$

$$y = ax^3 + bx^2 + cx + d$$

$$y = ab^x$$

$$y = a + b\ln(x)$$

$$y = a\sin(bx + c) + d$$

### PROBABILITY

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A \cap B) = P(A) \times P(B|A)$$

$$_nP_r = \frac{n!}{(n - r)!}$$

$$_nC_r = \frac{n!}{r!(n - r)!}$$

### FINANCIAL MATHEMATICS

$$t = \frac{72}{i}$$

$$I = Prt$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Net worth = Total assets − Total liabilities

$$\text{Debt-to-equity ratio (\%)} = \frac{(\text{Total liabilities} - \text{Mortgage})}{\text{Net worth}} \times 100$$

$$\text{Gross debt service ratio (\%)} = \frac{\left(\begin{array}{ccc}\text{Monthly} & \text{Monthly} & \text{Monthly} \\ \text{mortgage} + \text{property} + \text{heating} \\ \text{payment} & \text{taxes} & \text{costs}\end{array}\right)}{\text{Gross monthly income}} \times 100$$

$$\text{Rate of return (\%)} = \frac{\left(\begin{array}{cc}\text{Current value} & \text{Previous value} \\ \text{of portfolio} & - & \text{of portfolio}\end{array}\right)}{\text{Previous value of portfolio}} \times 100$$

### DESIGN AND MEASUREMENT

Prism: Surface area $= Ph + 2B$

Volume $= Bh$

Pyramid: Surface area $= B + \frac{1}{2}Ps$

Volume $= \frac{1}{3}Bh$

Cube: Surface area $= 6l^2$

Volume $= l^3$

Rectangular prism: Surface area $= 2lw + 2lh + 2wh$

Volume $= lwh$

Triangular prism: Surface area $= bh + l(a + b + c)$

Volume $= \frac{1}{2}bhl$

Square-based pyramid: Surface area $= b^2 + 2bs$

Volume $= \frac{1}{3}b^2h$

Sphere: Surface area $= 4\pi r^2$

Volume $= \frac{4}{3}\pi r^3$

Cylinder: Surface area $= 2\pi r^2 + 2\pi rh$

Volume $= \pi r^2 h$

Cone: Surface area $= \pi r^2 + \pi rs$

Volume $= \frac{1}{3}\pi r^2 h$

**NO MARKS WILL BE AWARDED
FOR WORK DONE ON THIS PAGE.**

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FOR WORK DONE ON THIS PAGE.**

# RELATIONS AND FUNCTIONS

## Question 1                                          Total: 1 mark

Select the pair of functions that might not have *x*-intercepts.

A)  exponential and quadratic

B)  quadratic and logarithmic

C)  logarithmic and sinusoidal

D)  sinusoidal and cubic

A pilot determined a function that shows the relation between height and atmospheric pressure:

$$H = 45.786 - 6.902 \ln p$$

where $H$ represents the height of the plane above the ground (in km)
and $p$ represents the atmospheric pressure (in mm of mercury).

102

a) Determine the atmospheric pressure at ground level in mm of mercury.

*(1 mark)*

103

b) A plane is flying at a height of 11 km and the air pressure inside is 561 mm of mercury. Determine the difference between the air pressure inside the plane and the atmospheric pressure outside the plane in mm of mercury. Show your work.

*(2 marks)*

# Question 3                                                           **Total: 4 marks**

For a math project, a student visits an amusement park. While riding the roller coaster, they use their smartphone to record their height above the ground as a function of time for a portion of the ride.

They collect the following data:

| Time (s)   | 1  | 3  | 15 | 20 | 25 | 30 |
|------------|----|----|----|----|----|----|
| Height (m) | 10 | 25 | 28 | 20 | 13 | 20 |

a) Determine the cubic regression equation that models this data.

*(1 mark)*

b) Determine the *y*-intercept using your equation in (a).

*(1 mark)*

c) Explain what the *y*-intercept represents in this situation.

*(1 mark)*

d) Using your equation in (a), determine the maximum height of the roller coaster in the first 20 seconds.

*(1 mark)*

## Question 4                                    Total: 3 marks

Hummingbirds beat their wings with a period of approximately 0.006 seconds. A transmitter is placed at the tip of a hummingbird's wing to measure the height above the ground.

a) Select the sinusoidal function that could model the relationship between the height, $h$ (in feet) and the time, $t$ (in seconds).

*(1 mark)*

      A)  $h = 0.15\sin(t) + 6$

      B)  $h = 0.15\sin(10t) + 6$

      C)  $h = 0.15\sin(100t) + 6$

      D)  $h = 0.15\sin(1000t) + 6$

b) Using your chosen function in (a), determine the height of the wing at the 17th second.

*(1 mark)*

c) Determine the range of the function you chose in (a).

*(1 mark)*

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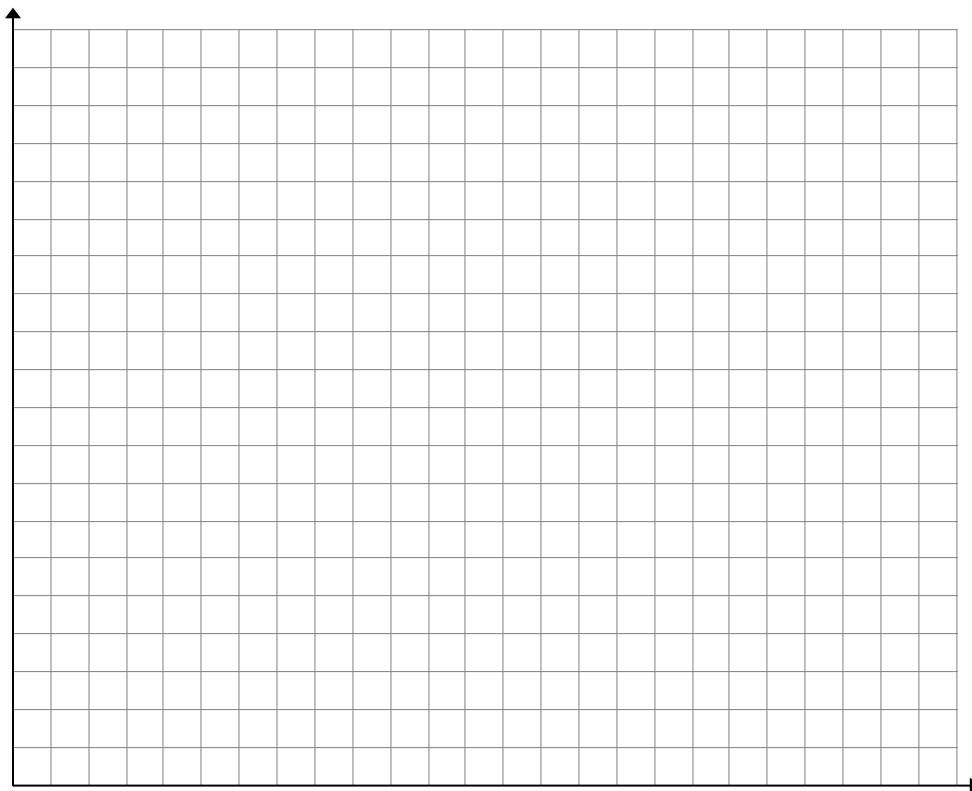## Question 5                                                 Total: 6 marks

When Jennika was 22 years old, she received $7500.00 from her grandmother. She invested the money and the following data was collected throughout the term (rounded to the nearest dollar).

| Time (years) | 0 | 1 | 3 | 5 | 10 | 15 |
|---|---|---|---|---|---|---|
| Account Value ($) | 7500 | 7827 | 8523 | 9281 | 11 486 | 14 215 |

a)   Create a clearly labelled graph by plotting the given data. Draw the curve of best fit.

*(3 marks)*



b)   Determine the exponential equation that best models the data in this situation.

*(1 mark)*

c) Determine the rate of return of Jennika's investment when she is 36 years old.
Show your work.

*(2 marks)*

## Question 6                                                            **Total: 5 marks**

Georgio has two options when investing $15 000.00.

    **Option 1:** He can invest the money at an interest rate of 6.50%, compounded monthly for 5 years.

    **Option 2:** He can invest the money in an account that earns simple interest for 5 years.

114

a) Determine the value of the investment if he chooses Option 1. Show your work.

*(2 marks)*

115

b) Georgio wants to earn the same amount of interest as he did in (a). Determine the simple interest rate, as a percent, that he would need if he chooses Option 2. Show your work.

*(2 marks)*

116

c) Explain why the simple interest rate in (b) is higher than 6.50%.

*(1 mark)*

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**Question 7**                                                    **Total: 5 marks**

Luke and Autumn want to buy a house. The bank offers them a mortgage with the following terms:

- an interest rate of 2.85%, compounded semi-annually
- an amortization period of 20 years

117

a) Luke and Autumn want to make

- a down payment of $18 000.00
- monthly payments of $1450.00

Given the terms above, determine the maximum house price they can afford.
Show your work.

*(2.5 marks)*

118

b) Luke and Autumn find a house they want to buy that is valued at $343 000.00. The bank offers the same mortgage terms. They have $18 000.00 saved for a down payment. Determine the monthly mortgage payment.

*(1 mark)*

c)    The house in (b) is in a neighbourhood where

      •   the average monthly property taxes are $280.00
      •   the monthly heating costs are $345.00

Luke and Autumn have an annual gross income of $83 000.00. Based on their gross debt service ratio (GDSR), would the bank lend them money? Explain.

*(1.5 marks)*

**Question 8**                                                                    **Total: 6 marks**

It is Joelyn's 18th birthday and she is planning for retirement.

- Her grandparents gave her $10 000.00.
- She will retire when her investment reaches $500 000.00.

**Option 1:** She invests $10 000.00 initially and will make regular monthly deposits at 5% interest, compounded monthly.

**Option 2:** She invests $1500.00 initially and makes regular monthly deposits of $200.00. She receives a 5% interest rate, compounded monthly.

a) If she selects Option 1, determine how much she will have to invest monthly to retire at age 60. Show your work.

*(2 marks)*

b) If she selects Option 2, determine how old she will be when she retires. Show your work.

*(3 marks)*

                                          

c)    Explain which option you would recommend.

*(1 mark)*

**Question 9**                                               **Total: 1 mark**

123

Describe a situation with two events where the probability of the second event is dependent on the first event.

124

There were 12 students at a camp. Over the course of the weekend

- 4 students went swimming
- 9 students went biking

Is participation in these events over the course of the weekend mutually exclusive?
Justify your answer.

A food inspector has 5 cafeterias and 4 restaurants to inspect.

Determine how many ways he can choose 3 places to inspect today if he must go to at least one restaurant. Show your work.

## Question 12 Total: 2 marks

Serena has a collection of 17 superhero books. Superman defeats Batman in 11 of these books.

a) If Serena randomly chooses a book from these 17 books, determine the odds in favour of choosing a book in which Superman defeats Batman.

*(1 mark)*

b) Serena buys 4 more books to add to her collection. Among these 4 books, Superman defeats Batman in 3 of them. Determine the odds against randomly choosing a book from her collection in which Superman defeats Batman.

*(1 mark)*

A teacher asks her students: "How many ways can the 11 letters of the word PROBABILITY be arranged?"

A student provides the following incorrect solution:

$$\frac{11!}{4!} = 1\ 663\ 200$$

There are 1 663 200 ways.

Correct the student's work.

## Question 14                                    **Total: 3 marks**

Arjun has 5 extra concert tickets to give away. He has 9 friends who would like to go to the concert.

a)   Determine how many ways he can choose to give away the tickets to his friends.

*(1 mark)*

b)   Paul is one of the 9 friends. If Arjun gives one of the tickets to Paul, determine how many ways Arjun can choose to give away the remaining tickets to his other friends.

*(1 mark)*

c)   Determine the probability that Arjun chooses to give Paul a ticket.

*(1 mark)*

**Question 15**                                                      **Total: 3 marks**

The weather report calls for a 72% probability of snow tomorrow. If it snows, the probability that Juan will go skiing tomorrow is 63%. If it does not snow, the probability that Juan will go skiing tomorrow is 46%.

132

a)   Use a graphic organizer to show all possible outcomes for this situation.

*(1 mark)*

133

b)   Determine the probability that Juan goes skiing tomorrow. Show your work.

*(2 marks)*

134

Using the digits 0 through 9, Haaziq needs to create a 4-digit or 5-digit code for his new bank card. Determine the total number of codes possible if repetition is allowed. Show your work.

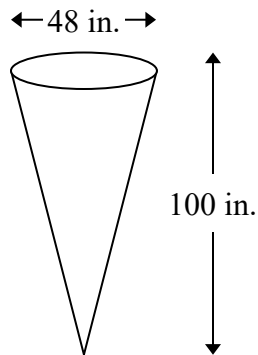**Question 17**                                                    **Total: 1 mark**

135

Select the volume of the following cone, in cubic feet, given the measurements shown below in inches.



A)  34.91 ft$^3$

B)  418.88 ft$^3$

C)  5026.55 ft$^3$

D)  60 318.58 ft$^3$

## Question 18           Total: 3 marks

A food company sells soup in a cylindrical container with a radius of 3.3 cm and a height of 9.8 cm.

<span style="float:right">136</span>

a)    Calculate the surface area of the soup container.

*(1 mark)*

<span style="float:right">137</span>

b)    The aluminum used to make the containers costs \$0.10 per 1000 cm$^2$, taxes included. The company wants to make 4500 soup containers. Determine the total cost (ignore waste). Show your work.

*(2 marks)*

Madelaine and Ryan both want to change the flooring in their bedrooms. They both have bedrooms that are 14 feet long and 8 feet wide.

138

a) Madelaine is using vinyl planks. The planks are sold in cases. Each case can cover 24 ft$^2$ and costs $47.50, taxes included. Calculate the cost of Madelaine's flooring. Show your work.

*(2 marks)*

139

b) Ryan is using sheet vinyl. Rolls are 12 feet wide and can be cut to any length. Ryan wants to lay the vinyl as one single rectangular sheet. The vinyl costs $23.88 per linear foot, taxes included. Calculate the cost of Ryan's flooring. Show your work.
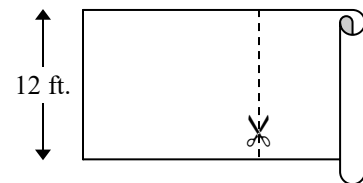
*(1 mark)*

12 ft.

Diagram is not drawn to scale.

c) Calculate the amount of waste (unused material) in each bedroom, in square feet.
Show your work.

*(2 marks)*

d) Madelaine's flooring costs $1.98 per square foot and Ryan's flooring costs $1.99 per square foot. Explain why these unit costs are so close but the total flooring costs are so different.

*(1 mark)*

## LOGICAL REASONING

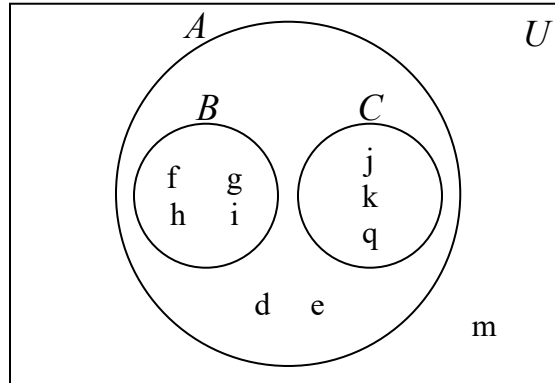**Question 20**                                          **Total: 1 mark**

Given the following sets:

$U = \{d, e, f, g, h, i, j, k, m, q\}$

$A = \{d, e, f, g, h, i, j, k, q\}$

$B = \{f, g, h, i\}$

$C = \{j, k, q\}$



Select the correct statement from below.

A)   $n(C' \cap B') = 5$

B)   $n(C \cap B) = 0$

C)   $n(C \cup B) = 10$

D)   $n(C' \cup B') = 2$

## Question 21           Total: 2 marks

Given a group of 25 high school students, 13 students attend a country festival and 8 students attend a folk festival.

$$C = \{\text{students who attend the country festival}\}$$
$$F = \{\text{students who attend the folk festival}\}$$

143

a) Given $n(C \cup F) = 19$, draw a Venn diagram to represent this situation.

*(1.5 marks)*

144

b) Determine $n(C \cap F)'$.

*(0.5 mark)*

Let *p* represent "a rock is wet" and *q* represent "it is raining outside".

145

a)  Write a conditional statement based on the following symbolic form:

$$\neg p \Rightarrow \neg q$$

*(1 mark)*

146

b)  Provide a counterexample to the statement in (a).

*(1 mark)*

## Question 23                                                     Total: 2 marks

Given the following statement:

"If it takes about 8 years to double your investment,
then you invest at an annual interest rate of 9%."

147

a)   Write the converse of this statement.

*(1 mark)*

148

b)   Determine if the original conditional statement is true using the Rule of 72.

*(1 mark)*

149

Complete the following square using the numbers 1 to 9 only once. Each row, column, and diagonal must add up to 15.

| 4 |   |   |
|---|---|---|
|   |   | 1 |
|   | 7 |   |

**END OF TEST**