Grade 12
Applied Mathematics
Achievement Test

## Student Booklet

January 2015

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## Disponible en français.

Available in alternate formats upon request.

## Grade 12 Applied Mathematics Achievement Test

## DESCRIPTION

| Unit | Description | Marks |
| :---: | :--- | :---: |
| A | Relations and Functions | 15 |
| B | Probability | 15 |
| C | Financial Mathematics | 17 |
| D | Design and Measurement | 7 |
| E | Logical Reasoning | 6 |

## Resources

You may use the following resources:

- Formula Sheet (tear-out page at the back of this booklet)
- one $81 / 2^{\prime \prime} \times 11^{\prime \prime}$ study sheet
- ruler
- graphing calculator, computer software, and/or app
- Internet access for tools such as applets or mortgage payment calculators

Use of the Internet to communicate or access other content, including but not limited to course notes, definitions, or conceptual information is not permitted during the test.

> Electronic communication between students through phones, email, or file sharing during the test is strictly prohibited.

## Directions

Read all instructions on the test carefully.
If you need extra paper or you print out an answer to a question, let the teacher know. Indicate in the response space of the question that your answer is on a separate sheet.

Remember to

- indicate your input values by writing them in your booklet or printing a copy if using a technology tool
- include your booklet number and question number on additional pages (e.g., printouts) and attach them to the corresponding page in the booklet
- express answers in decimal and percentage form to two decimal places when rounding, unless otherwise indicated
Example: $\frac{15}{29}=0.52$ or $51.72 \%$
- state any assumptions you make

A "graphic organizer" is a visual representation of information. Examples include a tree diagram, a chart, a list, a Venn diagram, a truth table, Pascal's triangle, etc.

## A clearly communicated answer

- is easily identified in the response space
- includes the parameters in the equation, and " $y=$ ", "sin", "ln", or " $x$ ", as applicable
- includes the units of measure, where applicable
- includes labels, units, and scales for the axes on graphs
- is expressed as an exact value or is appropriately rounded

Marks may be deducted for errors relating to any of the above.


PLEASE WAIT UNTIL INSTRUCTED TO TURN THE PAGE.

## RELATIONS AND FUNCTIONS

## Question 1 <br> Total: 1 mark

Select the best answer.
If the end behaviour of the graph of a function extends from Quadrant III to Quadrant I, then the function is a
A. constant
B. quadratic
C. cubic
D. sinusoidal

## Question 2

Total: 1 mark

Select the best answer.
Which of the following statements is false?
A. A linear function has only one $y$-intercept.
B. A quadratic function must have two $x$-intercepts.
C. An exponential function has exactly one asymptote.
D. A logarithmic function has a range that extends from negative infinity to positive infinity.

Joshua builds canoes. He has discovered that he can sell 120 canoes per year if the price is $\$ 450.00$ per canoe. For every $\$ 100.00$ increase in price, he sells 20 fewer canoes per year.
a) Complete the table below.
(1 mark)

| Canoes | Selling <br> Price (\$) | Revenue from <br> Canoe Sales (\$) |
| :---: | :---: | :---: |
| 120 |  |  |
| 100 |  |  |
| 80 |  |  |
| 60 |  |  |
| 40 |  |  |

b) Using the information in (a), determine the quadratic regression equation that models the relationship between the selling price and the revenue from canoe sales.
(1 mark)
c) According to your equation in (b), what is Joshua's maximum revenue?
(l mark)
d) According to your equation in (b), what is the highest selling price Joshua can charge for a canoe that results in an annual revenue of $\$ 30000.00$ ? Show your work.
(2 marks)

Tania has invested $\$ 10000.00$ in a savings account. The approximate growth of her investment is modelled by the equation:

$$
t=-288.00735+31.27 \ln A
$$

where $A$ represents the future value of the investment (in dollars) and $t$ represents the time (in years).
a) State the domain and the range in this situation.
(2 marks)
Domain: $\qquad$

Range: $\qquad$
b) How long will it take Tania's investment to triple?
(1 mark)
c) How much will her investment be worth after 10 years?
(1 mark)

A car is driving down the street and a pebble gets caught in one of its tire treads.
The tire rotates and the height of the pebble varies sinusoidally with the horizontal distance. This situation is modelled by the equation:

$$
h=30 \sin (0.0334 d-1.57)+30
$$

where $d$ represents the distance the tire travels (in centimetres) and $h$ represents the height of the pebble (in centimetres).
a) Create a clearly labelled graph of the equation for two revolutions of the tire starting from the time the pebble is caught in the tire tread.
(3 marks)
b) Determine the circumference of the tire.
(1 mark)

## Probability

## Question 6

## Total: 1 mark

Select the best answer.
A card is drawn from a set of cards numbered from 1 to 10 . Which situation below shows an event and its complement?
A. Drawing a 3 and drawing a 6 .
B. Drawing an even number and drawing a 2.
C. Drawing an odd number and drawing an even number.
D. Drawing an odd number and drawing a 6 .

## Question 7

Total: 1 mark

Select the best answer.

Ashton has some coins in his pocket. He reaches into his pocket and pulls out a coin at random. If the odds in favour of the chosen coin being a quarter are $4: 7$, what is the probability of the coin not being a quarter?
A. $\frac{7}{4}$
B. $\frac{3}{7}$
C. $\frac{4}{11}$
D. $\frac{7}{11}$

How many different routes are there from point A to point B, if you only go east and south? Show your work.


## Question 9

Evaluate:
100 !
$98!$

## Question 10

Total: 2 marks

How many different arrangements can be made using all the letters of the word "WINNIPEG", if the first letter must be P and the last letter W? Show your work.

The social justice committee at a high school is made up of 8 boys and 7 girls. From the committee, 4 students will be randomly chosen to attend a conference. What is the probability that all 4 students will be girls? Show your work.

A hockey team has practice jerseys in three colours. The team bag has 5 black, 4 white, and 6 red jerseys. The coach reaches into the bag and randomly selects a jersey for Peter and a jersey for Paul. What is the probability that both jerseys are the same colour? Show your work.

Cindy has an MP3 player that can play songs in a random order.
a) How many different ways can a 12 -song playlist be arranged, if each song is played only once?
(1 mark)
b) What is the probability that Cindy's 3 favourite songs will be played together when she plays the 12 -song playlist? Show your work.
(2 marks)

## Financial Mathematics

## Question 14

Select the best answer.
Kayla has an investment of $\$ 2000.00$ at $3.00 \%$ simple interest for 4 years. Which of the following statements is false?
A. The interest earned doubles if the time period doubles.
B. The interest earned halves if the interest rate halves.
C. The interest earned doubles if the interest rate doubles and the time period doubles.
D. The interest earned remains the same if the investment halves and the interest rate doubles.

## Question 15

Mr. and Mrs. Belair want to purchase a house.
Mr. Belair is a biologist and his annual salary is $\$ 81000.00$. Mrs. Belair is a pharmacist and her annual salary is $\$ 85250.00$.

The monthly mortgage payment for the house they want to purchase is $\$ 2750.00$, annual property taxes are $\$ 3600.00$, and the monthly heating costs are $\$ 240.00$.
a) Find the gross debt service ratio (GDSR) for the Belairs.
(2 marks)
b) Using your answer in (a), explain if they can afford to buy the house.
(1 mark)

Sara used her credit card to pay for a group hot air balloon ride. The cost was $\$ 997.50$, taxes included. Her credit card has a promotional offer of $0 \%$ interest for 2 months. After this period, the annual interest rate is $19.90 \%$ on any outstanding balance, compounded daily.

Sara decides to make payments of $\$ 110.00$ at the end of every month, even during the promotional period. How long will it take Sara to pay off the balance? Show your work.

## Question 17

Sheena receives $\$ 20000.00$ from an insurance settlement. She wants to invest her money for three years in either a guaranteed investment certificate (GIC) or in real estate.

Option 1: The GIC has an annual interest rate of $2.75 \%$, compounded semi-annually.
Option 2: The real estate investment generates annual returns of $5.90 \%$, compounded annually.
a) Determine the value of the GIC after 3 years. Show your work.
(2 marks)
b) Determine the value of the real estate investment after 3 years.
(1 mark)
c) Which would be the better investment for Sheena? Justify your answer.
(1 mark)

## Question 18

At the age of 18 , Justine invests $\$ 1000.00$ at an interest rate of $7.20 \%$, compounded annually.
a) Using the Rule of 72, estimate how old Justine will be when her investment equals $\$ 8000.00$. Show your work.
(2 marks)
b) Using a technology tool, determine the number of years it will take to reach $\$ 8000.00$. Show your work and indicate your answer to two decimal places.
(2 marks)

Use the information below to answer the questions on the next page.
Mario has decided to make an investment for a period of 40 years. He has two options:
Option 1: a fund that earns simple interest at $5.00 \%$ annually
Option 2: a savings account that earns $5.00 \%$ interest, compounded annually

a) Given the graphs of Option 1 and Option 2, estimate the value of the initial investment for each option.
(1 mark)
b) Which graph represents Option 1? Explain your answer.
(1 mark)

## Design and Measurement

## Question 20

The zoo has asked you to design a structure for its monkeys and owls using the following guidelines:

- The structure will back against the wall of a building and will be fenced at the top, front, and sides. (No fence is needed on the ground or at the back.)
- The structure will be divided into two enclosures by a separation fence and have a height of 15 ft .
- The monkeys require an enclosure with a ground area between $600 \mathrm{ft}^{2}$ and $1000 \mathrm{ft}^{2}$.
- The owls require an enclosure with a ground area between $400 \mathrm{ft}^{2}$ and $800 \mathrm{ft}^{2}$.
- The entire structure will be created using chain-linked fence, which is sold in $50 \mathrm{ft} . \times 5 \mathrm{ft}$. $\left(250 \mathrm{ft}^{2}\right)$ segments. Each segment costs $\$ 160.00$, plus GST and PST.

a) Determine a possible set of dimensions for your design.
(1 mark)
Ground dimensions of monkey enclosure: $\qquad$ $\mathrm{ft} . \times$ $\qquad$ ft .

Ground dimensions of owl enclosure: $\qquad$ $\mathrm{ft} . \times$ $\qquad$ ft .
b) Determine the minimum number of fence segments needed for your design. Show your work.

## (3 marks)

c) Calculate the total cost of the structure. (Note: GST $=5 \%, \mathrm{PST}=8 \%$ ) (1 mark)

The coffee mug shaded in the diagram below is based on a cone with the bottom portion removed. (Diagram is not drawn to scale.)


Determine the volume of the mug. Show your work.

## LOGICAL REASONING

Use the Venn diagram below to answer the following question and select the best answer.


What is $n(M \cup N)$ ?
A. 5
B. 10
C. 50
D. 60

## Question 23

Given the following universal set:

$$
U=\{\text { Alain, Betty, Candace }\}
$$

Write all the subsets of $U$ that have exactly 2 elements.

## Question 24

A survey of 100 students was conducted to find the most popular ice cream flavour. The findings are displayed below.

- 60 students like vanilla
- 77 students like chocolate
- 42 students like both vanilla and chocolate

Use a Venn diagram to represent this situation.

Consider the original statement:
"If a polygon is a triangle, then this polygon has exactly three sides."
a) Write the converse of the statement.
(1 mark)
b) Determine if a biconditional statement can be made using the original statement. If it is possible, write the biconditional statement. If not, provide a counterexample.
(1 mark)

## NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

## Formula Sheet: Applied Mathematics

| Relations and Functions $\begin{gathered} y=a x+b \\ y=a x^{2}+b x+c \\ y=a x^{3}+b x^{2}+c x+d \\ y=a b^{x} \\ y=a+b \ln (x) \\ y=a \log _{b} x \\ y=a \sin (b x+c)+d \\ y=a \cos (b x+c)+d \end{gathered}$ | Financial Mathematics $\begin{gathered} t=\frac{72}{i} \\ I=P r t \\ A=P\left(1+\frac{r}{n}\right)^{n t} \end{gathered}$ $\begin{aligned} & \text { Net worth }=\text { Total assets }- \text { Total liabilities } \\ & \begin{array}{l} \text { Debt to equity } \\ \text { ratio }(\%) \end{array}=\frac{(\text { Total liabilities }- \text { Mortgage })}{\text { Net worth }} \times 100 \\ & \begin{array}{l} \text { Gross debt } \\ \text { service } \\ \text { ratio (\%) } \end{array} \end{aligned}=\frac{\left(\begin{array}{c} \text { Monthly Monthly Monthly } \\ \text { mortgage } \left.+ \text { property }+\begin{array}{c} \text { heating } \\ \text { payment } \\ \text { taxes costs } \end{array}\right) \end{array}\right. \text { Gross monthly income }}{} \times 100 .$ |
| :---: | :---: |
| Probability $\begin{gathered} P(A \text { or } B)=P(A)+P(B)-P(A \text { and } B) \\ P(A \text { and } B)=P(A) \times P(B) \\ P(A \text { and } B)=P(A) \times P(B \mid A) \\ { }_{n} P_{r}=\frac{n!}{(n-r)!} \\ { }_{n} C_{r}=\frac{n!}{r!(n-r)!} \end{gathered}$ | Design and Measurement <br> Prism: Surface area $=P h+2 B$ $\text { Volume }=B h$ $\begin{gathered} \text { Pyramid: Surface area }=B+\frac{P s}{2} \quad(s=\text { slant height }) \\ \text { Volume }=\frac{B h}{3} \end{gathered}$ <br> Sphere: Surface area $=4 \pi r^{2}$ $\text { Volume }=\frac{4}{3} \pi r^{3}$ <br> Cylinder: Surface area $=2 \pi r h+2 \pi r^{2}$ $\text { Volume }=\pi r^{2} h$ <br> Cone: Surface area $=\pi r^{2}+\pi r s$ $\text { Volume }=\frac{\pi r^{2} h}{3}$ |

