Grade 12
Applied Mathematics
Achievement Test

## Student Booklet

January 2013

## Manitoba Education Cataloguing in Publication Data

Grade 12 applied mathematics achievement test. Student booklet. January 2013 [electronic resource]

## ISBN: 978-0-7711-5295-5

1. Educational tests and measurements-Manitoba.
2. Mathematical ability-Testing.
3. Mathematics-Examinations, questions, etc.
4. Mathematics-Study and teaching (Secondary)-Manitoba.
I. Manitoba. Manitoba Education.
510.76

## Manitoba Education

School Programs Division
Winnipeg, Manitoba, Canada
All images found in this document are copyright protected and should not be extracted, accessed, or reproduced for any purpose other than for their intended educational use in this document.

Permission is hereby given to reproduce this document for non-profit educational purposes provided the source is cited.

After the administration of this test, print copies of this resource will be available for purchase from the Manitoba Text Book Bureau. Order online at <www.mtbb.mb.ca>.
This resource will also be available on the Manitoba Education website at <www.edu.gov.mb.ca/k12/assess/archives/index.html>.
Websites are subject to change without notice.

## Disponible en français.

Available in alternate formats upon request.

## Grade 12 Appled Mathematics Achievement Test

## DESCRIPTION:

Total Possible Marks: 60
Time: 3 hours

| Unit | Description | Marks |
| :---: | :--- | :---: |
| A | Relations and Functions | 14 |
| B | Probability | 18 |
| C | Financial Mathematics | 16 |
| D | Design and Measurement | 6 |
| E | Logical Reasoning | 6 |

## Formula Sheet: Applied Mathematics

| Relations and Functions $\begin{gathered} y=a x^{2}+b x+c \\ y=a x^{3}+b x^{2}+c x+d \\ y=a b^{x} \\ y=a+b \ln (x) \\ y=a \sin (b x+c)+d \\ y=a \cos (b x+c)+d \\ \text { Period }=\frac{2 \pi}{\text { Frequency }} \text { or } \frac{2 \pi}{b} \end{gathered}$ | Financial Mathematics $\begin{gathered} I=P r t \\ A=P\left(1+\frac{r}{n}\right)^{n t} \\ \begin{array}{l} \text { Debt to equity } \\ \text { ratio } \end{array}=\frac{(\text { Total liabilities }- \text { Mortgage })}{\text { Net worth }} \times 100 \\ \begin{array}{l} \text { Gross debt } \\ \text { service } \\ \text { ratio } \end{array}=\frac{\left(\begin{array}{c} \text { Monthly } \left.\begin{array}{c} \text { Mortgage }+ \text { property }+ \text { heating } \\ \text { payment } \\ \text { taxes } \end{array}\right) \\ \text { Gross monthly income } \end{array}\right.}{} \times 100 \\ \begin{array}{l} \text { Average } \\ \text { rate of } \\ \text { return } \end{array}=\frac{\left(\begin{array}{c} \text { Current value } \\ \text { of portfolio } \end{array}-\begin{array}{c} \text { Previous value } \\ \text { of portfolio } \end{array}\right)}{\text { Previous value of portfolio }} \times 100 \end{gathered}$ |
| :---: | :---: |
| Probability $\mathrm{P}(\mathrm{~A} \text { or } \mathrm{B})=\mathrm{P}(\mathrm{~A})+\mathrm{P}(\mathrm{~B})-\mathrm{P}(\mathrm{~A} \text { and } \mathrm{B})$ $\mathrm{P}(\mathrm{~A} \text { and } \mathrm{B})=\mathrm{P}(\mathrm{~A}) \times \mathrm{P}(\mathrm{~B})$ ${ }_{n} P_{r}=\frac{n!}{(n-r)!}$ ${ }_{n} C_{r}=\frac{n!}{r!(n-r)!}$ | Design and Measurement <br> Prism: Surface area $=P h+2 B$ $\text { Volume }=B h$ $\begin{gathered} \text { Pyramid: Surface area }=B+\frac{P l}{2} \quad(l=\text { slant height }) \\ \text { Volume }=\frac{B h}{3} \end{gathered}$ <br> Sphere: Surface area $=4 \pi r^{2}$ $\text { Volume }=\frac{4}{3} \pi r^{3}$ $\begin{gathered} \text { Cylinder: Surface area }=2 \pi r h+2 \pi r^{2} \\ \text { Volume }=\pi r^{2} h \end{gathered}$ |

## Test Resources and Directions:

- You may consult your $8^{1 / 2} 2^{\prime \prime} \times 11^{\prime \prime}$ study sheet during the test.
- You may use a ruler, a graphing calculator, and computer software. You may also have access to the Internet for tools such as applets or a mortgage payment calculator. Use of the Internet to access course notes, find definitions, or search for conceptual information about the course is prohibited during the test.
- For short-answer and long-answer questions, you may print out diagrams from the computer or your calculator where applicable. Indicate your booklet number and the question number on the printouts. Remain seated and your teacher will distribute these printouts to you.
Indicate in the response space of the question that the answer is on a printed sheet and staple it to the page.
- If you need more space to answer a question, extra paper may be provided by your teacher. Write your booklet number and the question number on any extra paper used and staple it into the booklet where your answer begins. Indicate in the response space of the question that the answer is on a separate sheet.
- Provide clear explanations or justifications where applicable. This can be done through labelled diagrams, in words, by showing mathematical operations to verify your answer, or by referring to a calculator or software program.
- If you refer to a calculator program, indicate all your input values.
- If you refer to a software program or a website, indicate all your input values and print or copy the screen showing the answers.
- If you refer to a spreadsheet, print a copy of the answers.
- A graphic organizer is a visual representation of information. Examples include a tree diagram, a chart, a list, a Venn diagram, a truth table, Pascal's triangle, etc.
- Round your final answers to the nearest two decimal places unless otherwise indicated.
- An answer without any work shown will be considered incomplete.
- Always state your assumptions.

The following errors may result in a 0.5 mark deduction:

- not including one of the following in the equation: " $y=$ ", "sin", "ln", or " $x$ ", or writing parameters separately from the equation
- not including the units in the final answer
- not including one of the following on the graph: labels for the axes, units for the axes, or scales for the axes
- not stating or incorrectly stating the final answer
- rounding too soon or rounding incorrectly
- not using whole units appropriately


## RELATIONS AND Functions

## Question No. 1

Circle the sinusoidal equation below which is best represented by the following graph.

A) $y=2.5 \sin (x)+5.5$
B) $y=2.5 \sin (x)-5.5$
C) $y=5 \sin (x)+5.5$
D) $y=5 \sin (x)-5.5$

A swing is located directly in front of a tree. As Danielle swings, she is 7.3 metres away from the tree at the furthest point of her swing and 3.5 metres away from the tree at the closest point. (Diagram is not drawn to scale.)


If a sinusoidal equation is used to represent Danielle's position with respect to the tree, determine:
a) the median
(1 mark)
b) the amplitude
(1 mark)

The future deer population in a provincial park is described by the function:

$$
P=365(0.98)^{t}
$$

where $t$ is the number of years from now and $P$ is the population.
a) What is the present size of the deer population?
(1 mark)
b) How can you tell that the deer population is decreasing?
(1 mark)
c) The park ranger will implement a conservation plan if the deer population decreases below 100. Could this happen in the next 20 years? Use the function to support the reasoning in your answer.
(1 mark)

Felix is examining the growth of bean plants under different growing conditions. The results of one trial are as follows:

| Day | Average height of bean plants (cm) |
| :---: | :---: |
| 1 | 5.7 |
| 3 | 12.8 |
| 5 | 16.5 |
| 9 | 19.3 |
| 11 | 19.8 |
| 15 | 20.1 |

a) Determine a logarithmic equation that best represents the data.
(1 mark)
b) Using your equation in (a), predict the average height of the plants on the 30th day. Indicate your answer to 1 decimal place.
(1 mark)
c) A logarithmic function can represent the average height of the plants, but it has limitations. Explain why the domain or the range is limited in this situation.
(1 mark)

During a science experiment, Roger, who is on a platform, throws a ball toward the ground. He collects the following data:

| time (s) | 0.0 | 0.4 | 0.8 | 1.2 |
| :--- | :---: | :---: | :---: | :---: |
| height of the ball <br> above the ground (m) | 4.50 | 4.72 | 3.36 | 0.44 |

a) Determine the quadratic equation that best represents the data. Sketch a clearly labelled graph of the equation.
quadratic equation: $\qquad$
b) How much time (to 2 decimal places) will it take for the ball to hit the ground? Show your work.

## (2 marks)

## Probability

A multiple-choice test has 3 questions. Each question has 4 possible answers. A student randomly selects an answer for each of the 3 questions. What is the probability that the student will select all the correct answers?

## Question No. 7

Mr. Ramesh asked his 25 students how they got to school that day.

- 12 students said they rode the bus.
- 11 students said they walked.

Are these events mutually exclusive? Explain your reasoning.

Describe a scenario that involves dependent events. Explain how you know that the events are dependent.

Twenty-five cards numbered from 1 to 25 are placed in a bag. What is the probability of selecting a card that is a multiple of 7 or an odd number? Show your work.

In October, the Leopards have a football game. If it snows, the probability that they will win is 0.8 . If it does not snow, the probability that they will win is 0.5 . The probability of snow is 0.3 . Calculate the probability that the Leopards will win. Show your work.

A couple plans to have four children. The probability of a child being a girl is $50 \%$. Determine the probability of the couple having at least 2 girls. Show your work using a graphic organizer. (A graphic organizer is a visual representation of information. Examples include a tree diagram, a chart, a list, a Venn diagram, a truth table, Pascal's triangle, etc.)

Four students were asked to determine how many four-digit numbers could be created using the digits $0,1,2$, and 3 , based on their assumptions. They gave the following answers:

- Aaron: 256
- Beth: 192
- Carol: 24
- David: 18

Choose one answer and show how it was calculated. Identify two assumptions made by the student.

A volleyball team consists of 8 players, a coach, and a manager. If the members of the team are asked to line up for a photograph, determine the number of ways that the members can line up
a) if there are no restrictions
(1 mark)
b) if the coach and the manager must stand beside each other (1 mark)

A group of 3 objects is selected from 8 objects.
a) Describe a scenario in which the order of selection of the 3 objects is not important. Calculate the total possible outcomes for your scenario.
(2 marks)
b) Describe a scenario in which the order of selection of the 3 objects is important. Calculate the total possible outcomes for your scenario.
(2 marks)

## Financial Mathematics

## Question No. 15

Circle the graph below that best represents an investment earning compound interest over a period of years.
A)

B)

C)

D)


Diane invests $\$ 100$ and doubles her money in 8 years. Circle the operation below which illustrates the correct use of the Rule of 72 to estimate the annual interest rate.
A) $72 \div 8$
B) $8 \div 72 \times 100$
C) $100 \div 8$
D) $72 \div 100$

## Question No. 17

Identify an asset which is likely to appreciate in value. Justify your answer.

Amanda wants to invest $\$ 15000$. She wonders which would be the better investment in terms of the amount of interest earned.
a) Option 1: Buy a $\$ 15000$ Canada Savings Bond that earns simple interest at an annual rate of $4.5 \%$. Calculate the amount of interest earned after 5 years.
(l mark)
b) Option 2: Buy a $\$ 15000$ Guaranteed Investment Certificate that earns $4.5 \%$ compounded annually. Calculate the amount of interest earned after 5 years. Show your work. (2 marks)
c) Amanda's friend suggests that she should invest the $\$ 15000$ in the stock market. Do you agree or disagree with her friend? Explain your answer.
(1 mark)

A dealership advertises a sale price of $\$ 45899$ (including taxes) for a new truck. The salesman offers Darrel two options:

## Option 1: Lease

- down payment of $\$ 5000$
- monthly payments of $\$ 577.50$ (including taxes) for 4 years
- residual value of $\$ 15000$ after 4 years
- allowable limit of $20000 \mathrm{~km} /$ year and $\$ 0.15$ for each additional kilometre


## Option 2: Purchase

- down payment of $\$ 5000$
- loan with monthly payments, compounded monthly for 4 years at an annual rate of $4.9 \%$
a) Darrel estimates that he will drive $30000 \mathrm{~km} /$ year. If he chooses Option 1, how much will he pay in total if he returns the truck at the end of the lease? Show your work.
b) What will be the total price of the truck, including the down payment, if Darrel decides to purchase it according to Option 2? Show your work.


## (3 marks)

The Richards have a mortgage of $\$ 200000$ at an interest rate of $6 \%$ compounded semi-annually and amortized over 25 years. The monthly mortgage payment is $\$ 1279.61$.
a) If the Richards divide their monthly payment in half and make their payment every two weeks instead, how many payments will be required to pay off the mortgage? Show your work.
(2 marks)
b) If the Richards make their payment every two weeks, how much interest will they have saved by the end of the mortgage? Show your work.

## Design and Measurement

## Question No. 21

## Total: 2 marks

The Bertrands want to empty their circular swimming pool. There is 3 feet of water left in the pool which has a diameter of 16 feet. Using a pump which can remove $400 \mathrm{ft}^{3}$ of water per hour, how many hours will it take to remove all the water?

You have been asked to install floor tiles and paint your aunt's bathroom based on the following information:

- The floor measures $5 \mathrm{ft} . \times 7 \mathrm{ft}$.
- The walls are 8 ft . high.
- The door measures $80 \mathrm{in} . \times 30 \mathrm{in}$.
- The window measures $24 \mathrm{in} . \times 30 \mathrm{in}$.
a) You must cover the entire bathroom floor with tiles. Each tile measures $1 \mathrm{ft} . \times 1 \mathrm{ft}$. You will need an extra $5 \%$ of tiles to account for waste. How many tiles will you need to purchase for the project?
(1 mark)
b) You must apply two coats of paint to the walls of the bathroom. The door and the window will not be painted. Determine the total area to be painted. How many cans of paint will you need to purchase if one can covers $100 \mathrm{ft}^{2}$ ? Show your work.


## LOGICAL REASONING

Circle the statement below which has the same meaning as: "If a quadrilateral is a square, then it is a rectangle."
A) If a quadrilateral is not a square, then it is not a rectangle.
B) If a quadrilateral is a rectangle, then it is a square.
C) If a quadrilateral is not a rectangle, then it is not a square.
D) If it is not a square or a rectangle, then it is not a quadrilateral.

Given the statement: "Multiples of 6 are always multiples of 3."
a) Write the converse statement.
(1 mark)
b) Provide a counter-example that shows the converse statement is false.
(1 mark)

A sports club noted that its members participated in at least one of the following sports: football, tennis, or badminton.

- 36 played football
- 42 played tennis
- 51 played badminton
- 14 played football and tennis
- 16 played football and badminton
- 15 played tennis and badminton
- 11 participated in all three sports
a) Use a graphic organizer to illustrate this situation. (A graphic organizer is a visual representation of information. Examples include a tree diagram, a chart, a list, a Venn diagram, a truth table, Pascal's triangle, etc.)
(1 mark)
b) How many members only played badminton?
(1 mark)
c) How many members played football or tennis?
(1 mark)


## NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

## NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

## NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

