Grade 12
Applied Mathematics
Achievement Test

Marking Guide

June 2018

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Order online at www.manitobalrc.ca.
This resource will also be available on the Manitoba Education and Training website at www.edu.gov.mb.ca/k12/assess/archives/index.html.
Websites are subject to change without notice.

## Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.
Available in alternate formats upon request.

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## General Marking Instructions

Please ensure that

- the student booklet number matches the number on the Scoring Sheet
- only a pencil is used to complete the Scoring Sheet
- the final test mark is recorded on the Scoring Sheet
- the Scoring Sheet is complete and a copy has been made for school records

Do not make any marks in the student booklets. Booklets may be selected by Manitoba Education and Training for sample marking.

Once marking is completed, please forward the Scoring Sheets to Manitoba Education and Training using the envelope provided (for more information, see the administration manual).

## Marking

Explanations for student errors for selected-response questions have been provided, if applicable.
To receive full marks for a question, a student's response must be complete and correct. Partial marks may be awarded for an "appropriate strategy" with execution errors. An appropriate strategy is defined as one that is consistent with the learning outcomes and mathematical processes associated with the question and, if properly executed, would lead to the correct answer.

Some questions require a form of explanation or justification from students. Explanation or justification can be given through a labelled diagram, in words, by showing mathematical operations for answer verification, or by providing output from a technological tool. For this reason, appropriate flexibility is required when marking student responses.

## Errors

Marks are deducted if conceptual or communication errors are committed.

## Conceptual Errors

As a guiding principle, students should only be penalized once for each error committed in the context of a test question. For example, students may choose an inappropriate strategy for a question, but carry it through correctly and arrive at an incorrect answer. In such cases, students should be penalized for having selected an inappropriate strategy for the task at hand, but should be given credit for having arrived at an answer consistent with their choice of strategy.

## Communication Errors

Communication errors are errors that are not related to the concepts and are tracked on the Scoring Sheet in a separate section. There will be a 0.5 mark deduction for each type of communication error committed, regardless of the number of errors committed for that type (see example on next page).

## (1) Final Answer

- does not include a percent sign
- does not identify the answer (e.g., TVM solver, Venn diagram)
- does not use the given contextual variables
- incorrectly states the final answer


## (22) Notation

- does not include braces when using set notation
- does not include a box when using a Venn diagram
- does not include one of the following in the equation: " $y=$ ", "sin", " $\ln$ ", or " $x$ ", or writes parameters separately from the equation


## (B3) Transcription/Transposition

- makes a transcription error (inaccurate transferring of information)
- makes a transposition error (changing order of digits)


## (ㄹ) Whole Units

- does not use whole units for materials purchased in design and measurement questions
- does not use whole units in contextual questions involving discrete data (e.g., people)


## ( ${ }^{5}$ ) Units

- does not include the dollar sign for monetary values
- uses incorrect units of measure
- does not include the units in the final answer
- confuses square and cubic units (e.g., $\mathrm{cm}^{2}$ instead of $\mathrm{cm}^{3}$, or vice versa)
- does not include units with labels on a graph


## (E6) Rounding

- rounds incorrectly
- rounds too soon
- does not express the answer to the appropriate number of decimal places, including monetary values to two decimal places

When a given response includes multiple types of communication errors, deductions are indicated in the order in which the errors occur in the response. No communication errors are recorded for work that has not been awarded marks. The total deduction may not exceed the marks awarded.

## Scoring

The marks allocated to questions are based on the concepts associated with the learning outcomes in the curriculum. For each question, shade in the circle on the Scoring Sheet that represents the mark awarded based on the concepts. A total of these marks will provide the preliminary mark.

The student's final mark is determined by subtracting the communication errors from the preliminary mark.

## Example:

A student has a preliminary mark of 46 . The student committed one E1 error ( 0.5 mark deduction) and three E6 errors ( 0.5 mark deduction).


## Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If a Scoring Sheet is marked with " 0 " and/or "NR" only (e.g., student was present but did not attempt any questions) please document this on the Irregular Test Booklet Report.

## Assistance

If any issue arises that cannot be resolved locally during marking, please call Manitoba Education and Training at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the marking keys.

Allison Potter<br>Assessment Consultant<br>Grade 12 Applied Mathematics<br>Telephone: 204-945-3411<br>Toll-Free: 1-800-282-8069, ext. 3411<br>Email: allison.potter@gov.mb.ca

## Marking Keys

Please note that this Marking Guide contains screen captures taken from a TI-84 Plus graphing calculator.

## Relations and Functions

## Question 1

Total: 1 mark
Learning Outcome: 12A.R. 1
Question Type: Selected Response

Select the answer that best completes the statement.
The end behaviour of a cubic function with a negative leading coefficient extends from:

## (A)) quadrant II to quadrant IV

B) quadrant III to quadrant I
C) quadrant I to quadrant III
D) quadrant IV to quadrant II

Select the best answer.
A cubic function has two $x$-intercepts and a positive $y$-intercept.
The graph that has these characteristics is:
A)

C)

(B)

D)


The heart pumps blood throughout the body. As the blood leaves the heart, it is replaced with new blood.

Muna's heart contains 70 mL of blood. With each heartbeat, the volume of original blood in her heart is reduced by $53 \%$ and replaced with new blood.
a) Determine the exponential regression equation that models the volume of original blood remaining in Muna's heart as a function of the number of heartbeats. Show your work.
(2 marks)

| Heartbeats | Volume of Original <br> Blood Remaining $(\mathbf{m L})$ |
| :---: | :---: |
| 0 | 70 |
| 1 | 32.9 |
| 2 | 15.463 |
| 3 | 7.26761 |
| 4 | 3.4157767 |

$$
y=70(0.47)^{x}
$$

b) Using your equation in (a), determine the volume of original blood remaining in Muna's heart after 6 heartbeats.
(1 mark)

|  | OR |
| ---: | :--- |
| $y$ | $=70(0.47)^{6}$ |
|  | $=0.754545$ |$\quad$| Heartbeats | Volume of Original <br> Blood Remaining (mL) |
| :---: | :---: |
| 5 | 1.605415 |
| 6 | 0.754545 |

The volume of original blood remaining is 0.75 mL .

| Marking Key |  |
| :--- | :--- |
| $\mathbf{1}$ | 1 mark for correct initial value in equation in (a) |
| $\mathbf{2}$ | 1 mark for correct rate of change in equation in (a) |
| $\mathbf{3}$ | 1 mark for consistent answer in (b) |

The following equation models the path of a basketball as it is shot by Sarah:

$$
h=-0.51 d^{2}+4.72 d+6.09
$$

where $h$ represents the height in feet
and $d$ represents the horizontal distance, in feet, the ball has travelled.
a) Create a clearly labelled graph of the equation.
(3 marks)

b) The standard height for a basketball net is 10 feet. Sarah shoots the ball and it goes directly through the net on its way down. Using the given equation, determine how far, horizontally, the ball was from the net when it was shot. Show your work.
(2 marks)

$\mathrm{Y}_{2}=10$
CALC 5: intersect (8.335..., 10)
$d=8.34$

The ball was shot 8.34 feet from the net.

## Marking Key

(1) 1 mark for communicating the context of the graph with appropriate title and/or labels in (a)
1 mark for using an appropriate domain and range (i.e., window settings/grid range) for the context of the question in (a)
(3) 1 mark for an appropriate shape that illustrates key characteristics of the function (e.g., maximum, minimum, asymptotes, intercepts) in (a)
(4) 1 mark for appropriate work in (b)
© 1 mark for consistent answer in (b)

## Question 5

Total: 5 marks
Learning Outcome: 12A.R. 3

## Question Type: Constructed Response

A satellite was launched from Cape Canaveral and set to orbit the Earth. Instruments measured its distance from the equator at certain time intervals, using positive numbers to indicate distances north of the equator and negative numbers to indicate distances south as shown below.

| Time (minutes) | 20 | 40 | 60 | 80 | 100 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Distance from equator (miles) | 3929 | 637 | -2468 | -254 | 3620 |

a) Determine the sinusoidal regression equation that models this data.
(1 mark)

$$
y=3249.86 \sin (0.07 x+0.38)+750.17
$$

b) After 180 minutes in orbit, the satellite passes directly over Cape Canaveral. How far is Cape Canaveral from the equator?
(1 mark)
CALC 1: value $x=180, y=1968.8583$
Cape Canaveral is 1968.86 miles from the equator.
c) Determine the northern and southern limits of the satellite's path relative to the equator. (2 marks)

CALC 3: minimum (61.999..., -2499.694)
$y=-2499.69$
CALC 4: maximum (17.002..., 4000.033)

$$
y=4000.03
$$

The northern limit is 4000.03 miles north and the southern limit is 2499.69 miles south.
OR

$$
\begin{aligned}
\operatorname{minimum} & =d-a & \text { maximum } & =d+a \\
& =750.17-3249.86 & & =750.17+3249.86 \\
& =-2499.69 & & =4000.03
\end{aligned}
$$

The northern limit is 4000.03 miles north and the southern limit is 2499.69 miles south.
d) Determine the period for this sinusoidal model.
(1 mark)

$$
\begin{aligned}
& \text { OR } \\
& \begin{aligned}
\text { Period } & =\frac{2 \pi}{b} \\
& =\frac{2 \pi}{0.0698 \ldots} \\
& =90
\end{aligned}
\end{aligned}
$$

$$
\begin{aligned}
\text { Period } & =2\left(\begin{array}{l}
x \text {-value at } \\
\text { minimum }
\end{array}-\begin{array}{c}
x \text {-value at } \\
\text { maximum }
\end{array}\right) \\
& =2(62-17) \\
& =90
\end{aligned}
$$

) Determine the period for this sinusoidal model

The period is 90 minutes.

## Marker Note(s):

$\rightarrow$ Regression equations may vary depending on the software used.
$\rightarrow$ Award mark $\left(\begin{array}{l}\text { for } 2056.40 \text { miles; answer reflects using rounded values from (a). }\end{array}\right.$
$\rightarrow$ Award mark $\mathbf{5}$ for 89.76 minutes; answer reflects using rounded values from (a).

| Marking Key |  |
| :--- | :--- |
| $\mathbf{1}$ | 1 mark for correct equation in (a) |
| $\mathbf{2}$ | 1 mark for consistent answer in (b) |
| $\mathbf{3}$ | 1 mark for consistent northern limit in (c) |
| $\mathbf{4}$ | 1 mark for consistent southern limit in (c) |
| $\mathbf{5}$ | 1 mark for consistent answer in (d) |

## Financial Mathematics

## Question 6

Total: 1 mark
Learning Outcome: 12A.FM. 2
Question Type: Selected Response
Select the best answer.

The table below shows the value of a luxury vehicle over a two-year period.

| Year | Value |
| :---: | :---: |
| 0 | $\$ 58500$ |
| 1 | $\$ 42100$ |
| 2 | $\$ 30300$ |

The value of this vehicle depreciates at an annual rate of:
A) $72 \%$
B) $52 \%$
C) $39 \%$
D) $\mathbf{2 8 \%}$

## Question 7

Total: 3 marks
Learning Outcome: 12A.FM. 2

## Question Type: Constructed Response

Johannes wants to apply for a bank loan. Information regarding his financial situation is given below.

- He has a house valued at $\$ 225000.00$ with a mortgage of $\$ 175000.00$.
- He has a cottage valued at $\$ 115000.00$ with a mortgage of $\$ 75000.00$.
- He has $\$ 9000.00$ in his savings account.
- He owes a total of $\$ 25000.00$ on his credit cards.
a) Calculate his net worth.
(1 mark)

| Assets | Liabilities | Net Worth |
| ---: | ---: | ---: |
| $\$ 225000.00$ | $\$ 175000.00$ | $\$ 349000.00$ |
| $\$ 115000.00$ | $\$ 75000.00$ | $-\$ 275000.00$ |
| $+\quad \$ 9000.00$ | $+\quad \$ 25000.00$ | $\$ 74000.00$ |
| $\$ 349000.00$ | $\$ 275000.00$ |  |

His net worth is $\$ 74000.00$.
b) Calculate his debt-to-equity ratio.
(1 mark)

$$
\begin{aligned}
\text { Debt-to-equity ratio } & =\frac{(\text { Total liabilities }- \text { Mortgage })}{\text { Net worth }} \times 100 \\
& =\frac{\$ 25000.00}{\$ 74000.00} \times 100 \\
& =33.78 \%
\end{aligned}
$$

His debt-to-equity ratio is $33.78 \%$.
c) Based on his debt-to-equity ratio, would the bank lend him money? Explain.
(1 mark)
The bank would lend him money since
his debt-to-equity ratio is less than $50 \%$.

| Marking Key |  |
| :--- | :--- |
| $\mathbf{1}$ | 1 mark for correct net worth value in (a) |
| $\mathbf{2}$ | 1 mark for consistent debt-to-equity ratio in (b) |
| $\mathbf{3}$ | 1 mark for appropriate explanation with reference to $50 \%$ in (c) |

Rémi deposited a sum of money into an account 36 years ago that earned an annual interest rate of $8.00 \%$. Today, there is $\$ 12800.00$ in his account.

Use the Rule of 72 to estimate the initial amount that Rémi deposited.

$$
t=\frac{72}{i}=\frac{72}{8}=9
$$

It takes 9 years for the money to double in value.

$$
\begin{aligned}
& \frac{36}{9}=4 \text { doubling periods } \\
& \begin{aligned}
\text { Initial amount } & =\frac{\$ 12800.00}{(2)^{4}} \\
& =\$ 800.00
\end{aligned}
\end{aligned}
$$

The initial amount Rémi deposited is approximately $\$ 800.00$.

## OR

$$
t=\frac{72}{i}=\frac{72}{8}=9
$$

It takes 9 years for the money to double in value.

| Time (years) | Amount (\$) |
| :---: | :---: |
| 36 | 12800.00 |
| 27 | 6400.00 |
| 18 | 3200.00 |
| 9 | 1600.00 |
| 0 | 800.00 |

The initial amount Rémi deposited is approximately $\$ 800.00$.

| Marking Key |  |
| :--- | :--- |
| $\mathbf{1}$ | 1 mark for correct doubling time (9 years) |
| $\mathbf{2}$ | 1 mark for consistent answer |

Mr. Smythe makes a one-time donation to a university. The university decides to invest this money and use only the amount earned in simple interest from the investment to finance a scholarship.

- The initial amount of the donation was $\$ 650000.00$.
- The amount earned in simple interest annually is $\$ 40000.00$.
- The university awards the scholarship to one student each year.

At what interest rate must the donation be invested to obtain the $\$ 40000.00$ needed to award the scholarship each year? Show your work.

$$
\begin{aligned}
I & =P r t \\
r & =\frac{I}{P t} \\
& =\frac{\$ 40000.00}{\$ 650000.00 \times 1} \\
& =0.0615
\end{aligned}
$$

The donation must be invested at an interest rate of $6.15 \%$.

## OR



The donation must be invested at an interest rate of $6.15 \%$.

|  |  | Marking Key |
| :--- | :--- | :--- |
| $\mathbf{1}$ | 1 mark for appropriate work |  |
| (2) | 1 mark for consistent answer |  |

Bonnie and Claude want to buy a house. They can afford monthly payments of $\$ 1125.00$. The bank offers them a mortgage at an interest rate of $3.10 \%$, compounded semi-annually, with an amortization period of 25 years.
a) What is the maximum amount of money the bank will lend them for their mortgage?

Show your work.
(2 marks)


The maximum amount of money the bank will lend them for their mortgage is $\$ 235163.06$.
b) If they have $\$ 30000.00$ saved for a down payment, what is the maximum house price they can afford?
(1 mark)

$$
\begin{aligned}
\text { Maximum house price } & =\$ 235163.06+\$ 30000.00 \\
& =\$ 265163.06
\end{aligned}
$$

The maximum house price they can afford is $\$ 265163.06$.

## Marker Note(s):

$\rightarrow$ Award a maximum of 1 mark in (a) for one incorrect input value; award no marks for two or more incorrect input values.

| Marking Key |  |
| :--- | :--- |
| $\mathbf{1}$ | 1 mark for appropriate work in (a) |
| $\mathbf{2}$ | 1 mark for consistent answer in (a) |
| $\mathbf{3}$ | 1 mark for consistent answer in (b) |

Bernard is exploring financing options for a new house. The bank offers him a mortgage of $\$ 245827.00$ at an interest rate of $3.75 \%$, compounded semi-annually. He has the following payment options:

Option 1: monthly payments of $\$ 1260.00$
Option 2: biweekly payments of $\$ 630.00$
a) How many years will it take Bernard to pay off the mortgage with each option?

Show your work.
(3 marks)

Option 1

$\frac{300}{12}=25$ years

Option 2

$\frac{571.3247}{26}=21.97$ years

It will take 25 years to pay off the mortgage with Option 1 and 21.97 years with Option 2.
b) If Bernard makes biweekly payments instead of monthly payments, how much money will he save? Show your work.
(2 marks)

$$
\begin{aligned}
\text { Total cost of monthly payments } & =\$ 1260.00 \times 300.0003 \ldots \\
& =\$ 378000.50
\end{aligned}
$$

Total cost of biweekly payments $=\$ 630.00 \times 571.3247 \ldots$

$$
=\$ 359934.59
$$

$$
\$ 378000.50-\$ 359934.59=\$ 18065.91
$$

He will save $\$ 18065.91$.

## Marker Note(s):

$\rightarrow$ Award marks $\mathbf{2}$ and $\mathbf{3}$ with an © communication error for answers of 300 payments for Option 1 and 571.32 payments for Option 2.
$\rightarrow$ Award mark $\mathbf{\Xi}$ for consistent answer reflecting the use of rounded values from (a).

| Marking Key |  |
| :--- | :--- |
| $\mathbf{1}$ | 1 mark for appropriate work in (a) |
| $\mathbf{2}$ | 1 mark for consistent answer for Option 1 in (a) |
| $\mathbf{3}$ | 1 mark for consistent answer for Option 2 in (a) |
| $\mathbf{4}$ | 1 mark for appropriate work in (b) |
| $\boldsymbol{\Theta}$ | 1 mark for consistent answer in (b) |

## Design and Measurement

## Question 12

Total: 1 mark
Learning Outcome: 12A.D. 1
Question Type: Selected Response

Select the best answer.

Melia baked the following cake and will ice the top and the sides.


45 cm
The surface area that needs to be iced is:
A) $1290 \mathrm{~cm}^{2}$
(B) $\mathbf{1 6 8 0} \mathrm{cm}^{2}$
C) $2580 \mathrm{~cm}^{2}$
D) $5400 \mathrm{~cm}^{2}$

A Canadian two-dollar coin consists of a gold-coloured centre and a silver-coloured outer ring. The coin has a diameter of 2.8 cm and is 0.2 cm thick while its centre has a diameter of 1.6 cm .

Diagram is not drawn to scale.


Calculate the volume of the silver-coloured outer ring of the coin.

$$
\begin{array}{rlrl}
V_{\text {coin }} & =\pi r^{2} h & V_{\text {centre }} & =\pi r^{2} h \\
& =\pi(1.4 \mathrm{~cm})^{2}(0.2 \mathrm{~cm}) & =\pi(0.8 \mathrm{~cm})^{2}(0.2 \mathrm{~cm}) \\
& =1.2315 \ldots \mathrm{~cm}^{3} & =0.4021 \ldots \mathrm{~cm}^{3} \\
V_{\text {ring }} & =V_{\text {coin }}-V_{\text {centre }} \\
& =1.2315 \mathrm{~cm}^{3}-0.4021 \mathrm{~cm}^{3} \\
& =0.83 \mathrm{~cm}^{3} &
\end{array}
$$

The volume of the outer ring is $0.83 \mathrm{~cm}^{3}$.

## Marking Key

| (1) | 1 mark for appropriate work |
| :--- | :--- | :--- |
| (2) | 1 mark for consistent answer |

## Question 14

Total: 4 marks
Learning Outcome: 12A.D. 1
Question Type: Constructed Response
A farmer is selling a cone-shaped pile of grain that has a diameter of 30 feet and a height of 20 feet. The grain needs to be transported to the market by truck.
a) The grain box of the farmer's truck has a volume of 850 cubic feet. What is the minimum number of times the farmer must go to the market to transport all of the grain? Show your work.
(2 marks)

$$
\begin{aligned}
V_{\text {pile }} & =\frac{\pi r^{2} h}{3} \\
& =\frac{\pi(15 \mathrm{ft} .)^{2}(20 \mathrm{ft} .)}{3} \\
& =4712.39 \mathrm{ft}^{3} \\
\frac{V_{\text {pile }}}{V_{\text {box }}} & =\frac{4712.39 \mathrm{ft}^{3}}{850 \mathrm{ft}^{3}}=5.54
\end{aligned}
$$

The minimum number of times is 6 .
b) Grain is sold by the whole bushel. This grain has a current value of $\$ 8.50 /$ bushel. If one bushel is equal to 1.24 cubic feet, calculate the value of the pile of grain.
Show your work.
(2 marks)

$$
\begin{aligned}
& \frac{4712.39 \mathrm{ft}^{3}}{1.24 \mathrm{ft}^{3}}=3800.31 \text { bushels } \\
&=3800 \text { whole bushels } \\
& 3800 \times \$ 8.50=\$ 32300.00
\end{aligned}
$$

The value of the pile of grain is $\$ 32300.00$.

Marker Note(s):
$\rightarrow$ Award mark $(4$ with an (⑨) communication error for an answer of $\$ 32302.67$.

| $\quad$ Marking Key |  |
| :--- | :--- |
| $\mathbf{1}$ | 1 mark for appropriate work in (a) |
| $\mathbf{2}$ | 1 mark for consistent answer in (a) |
| $\mathbf{3}$ | 1 mark for appropriate work in (b) |
| $\mathbf{4}$ | 1 mark for consistent answer in (b) |

## Probability

## Question 15

Total: 1 mark
Learning Outcome: 12A.P. 6
Question Type: Selected Response

Select the best answer.
There are 16 girls and 11 boys enrolled in a physical education class. A volleyball team of 4 girls and 2 boys will be formed from this class.

Which of the following expressions could be used to determine the number of teams possible?
A) ${ }_{27} C_{6}$
B) ${ }_{27} P_{6}$
(C) ${ }_{16} C_{4} \times{ }_{11} C_{2}$
D) ${ }_{16} P_{4} \times{ }_{11} P_{2}$

## Question 16

Total: 2 marks
Learning Outcome: 12A.P. 2
Question Type: Constructed Response

Rylan rolls two 4-sided dice with sides numbered 1 through 4. What is the probability that the sum of the rolled numbers is greater than or equal to 6 ? Show your work.

\[

\]

The probability is $\frac{3}{8}, 0.38$, or $37.5 \%$.

|  |  | Marking Key |
| :--- | :--- | :--- |
| $\mathbf{1}$ | I mark for appropriate work |  |
| $\mathbf{2}$ | I mark for consistent answer |  |

How many different routes can Martin take to the mall if he can only travel north and east and wants to meet Amy on the way? Show your work.


He can take 18 different routes.
OR

$$
\begin{array}{ll}
\text { arrange } & \text { arrange } \\
\text { NNEE } & \text { NNE } \\
\frac{4!}{2!2!} \times \frac{3!}{2!1!} \\
=6 \times 3 & \\
=18 &
\end{array}
$$

He can take 18 different routes.

Other methods are possible.

|  |  | Marking Key |
| :--- | :--- | :--- |
| $\mathbf{1}$ | 1 mark for appropriate work |  |
| $\mathbf{2}$ | 1 mark for consistent answer |  |

Arif, Simba, and Maritza ran for student council treasurer. Of the 650 students who voted:

- $44 \%$ voted for Arif
- $36 \%$ voted for Simba
- the remaining students voted for Maritza
a) Determine the number of students that voted for Maritza.
(1 mark)

$$
\begin{gathered}
100 \%-44 \%-36 \%=20 \% \\
0.20 \times 650=130
\end{gathered}
$$

There are 130 students who voted for Maritza.
b) One of the students is selected at random. Determine the odds against this student having voted for Arif.
(1 mark)

$$
\begin{gathered}
44 \% \text { of students voted for Arif } \\
100 \%-44 \%=56 \% \text { of students did not vote for Arif }
\end{gathered}
$$

$$
56: 44
$$

The odds against this student having voted for Arif are 56:44.
OR

$$
\begin{gathered}
0.44 \times 650=286 \text { students voted for Arif } \\
650-286=364 \text { students did not vote for Arif }
\end{gathered}
$$

$$
364: 286
$$

The odds against this student having voted for Arif are $364: 286$.

## Marker Note(s):

$\rightarrow$ Accept equivalent representations in (b).

|  |  |
| :--- | :--- |
| $\mathbf{1}$ | Marking Key |
| 2 | 1 mark for correct answer in (a) |

Jack is late for the bus $15 \%$ of the time. When he is late for the bus, the probability that he will see Jill at the bus stop is $8 \%$. When he is not late, the probability that he will see Jill at the bus stop is $82 \%$.
a) What is the probability that Jack did not see Jill today? Show your work.
(2 marks)


$$
\begin{aligned}
& P(\text { did not see Jill })= \\
& P(\text { late, did not see Jill })+ \\
& P(\text { not late, did not see Jill }) \\
&= 0.15(0.92)+0.85(0.18) \\
&= 0.138+0.153 \\
&= 0.291
\end{aligned}
$$

The probability is $\frac{291}{1000}, 0.29$, or $29.1 \%$.
b) Jack did not see Jill today. Using your answer in (a), what is the probability that Jack was late for the bus?
(1 mark)

$$
\begin{aligned}
P(\text { late } \mid \text { did not see Jill }) & =\frac{P(\text { late } \cap \operatorname{did} \text { not see Jill })}{P(\text { did not see Jill })} \\
& =\frac{0.138}{0.291} \\
& =0.4742
\end{aligned}
$$

The probability is $\frac{46}{97}, 0.47$, or $47.42 \%$.

| Marking Key |  |
| :--- | :--- |
| $\mathbf{1}$ | 1 mark for appropriate work in (a) |
| $\mathbf{2}$ | 1 mark for consistent answer in $(a)$ |
| $\mathbf{3}$ | 1 mark for consistent answer in $(b)$ |

Guy's baseball team is playing in a tournament. There are six teams entered in the tournament. All teams play each other once and each game is played on the same baseball field.
a) Determine the total number of games played in the tournament.
(1 mark)

$$
{ }_{6} C_{2}=15
$$

The total number of games played is 15 .
$\qquad$

$$
\begin{array}{lll} 
& 5+4+3+2+1=15 \\
& \uparrow & \uparrow \\
\text { \# of games played } & \text { remaining \# of games } \\
\text { by Team 1 } & \text { played by Team 2 }
\end{array}
$$

The total number of games played is 15 .
b) Determine the probability that Guy's team plays the first game of the tournament.
(1 mark)

$$
\begin{aligned}
P & =\frac{\text { number of games Guy's team plays }}{\text { total number of games played }} \\
& =\frac{{ }_{5} C_{1}}{{ }_{6} C_{2}} \\
& =\frac{5}{15}=\frac{1}{3}
\end{aligned}
$$

The probability is $\frac{1}{3}, 0.33$, or $33.33 \%$.
OR

$$
\begin{aligned}
P & =\frac{\text { number of teams required for the first game }}{\text { total number of teams }} \\
& =\frac{2}{6}=\frac{1}{3}
\end{aligned}
$$

The probability is $\frac{1}{3}, 0.33$, or $33.33 \%$.

| $\quad$ Marking Key |  |
| :--- | :--- |
| $\mathbf{1}$ | 1 mark for correct answer in (a) |
| $\mathbf{2}$ | 1 mark for consistent answer in $(b)$ |

## Question 21

Total: 3 marks
Learning Outcomes: 12A.P.4, 12A.P. 5
Question Type: Constructed Response

Shivani needs to create a new password for her computer. The password must begin with three upper case letters followed by five digits.
a) How many passwords are possible if repetition is not allowed? Show your work.
(2 marks)

$$
\underbrace{26 \times \underline{25} \times \underline{24}}_{\text {letters }} \times \underbrace{10 \times \underline{9} \times \underline{8} \times \underline{7} \times \underline{6}}_{\text {digits }}=471744000
$$

There are 471744000 passwords possible.
$\qquad$

$$
{ }_{26} P_{3} \times{ }_{10} P_{5}=471744000
$$

There are 471744000 passwords possible.
b) How many passwords are possible if repetition is not allowed and the password must begin with the letter M?
(1 mark)

$$
\underset{\mathrm{M}}{\frac{1}{\mathrm{M}}} \times \underbrace{\underline{25} \times \underline{24}}_{\text {remaining }} \times \underbrace{10 \times \underline{9} \times \underline{8} \times \underline{7} \times \underline{6}}_{\text {digits }}=18144000
$$

There are 18144000 passwords possible.
$\qquad$

$$
\frac{471744000}{26}=18144000
$$

There are 18144000 passwords possible.

| Marking Key |  |  |  |
| :--- | :--- | :--- | :---: |
| $\mathbf{1}$ | 1 mark for appropriate work in (a) |  |  |
| $\mathbf{2}$ | 1 mark for consistent answer in (a) |  |  |
| $\mathbf{3}$ | 1 mark for consistent answer in (b) |  |  |
|  |  |  |  |
| 28 | Applied Mathematics: Marking Guide (June 2018) |  |  |

Joe is getting dressed in the dark. The only socks in his drawer are 12 white socks and 10 green socks. He randomly picks two socks from the drawer, one after the other.
a) What is the probability that both socks are the same colour? Show your work.
(2 marks)

$$
\begin{aligned}
P(\text { white, white })+P(\text { green, green }) & =\left(\frac{12}{22} \times \frac{11}{21}\right)+\left(\frac{10}{22} \times \frac{9}{21}\right) \\
& =\frac{222}{462}
\end{aligned}
$$

The probability is $\frac{37}{77}, 0.48$, or $48.05 \%$.
OR

$$
\begin{aligned}
P(\text { white, white })+P(\text { green, green }) & =\frac{{ }_{12} C_{2}+{ }_{10} C_{2}}{{ }_{22} C_{2}} \\
& =\frac{111}{231}
\end{aligned}
$$

The probability is $\frac{37}{77}, 0.48$, or $48.05 \%$.

## Logical Reasoning

b) Using logical reasoning, what is the minimum number of socks Joe would have to pick to guarantee a pair of the same-coloured socks?
(1 mark)
Joe would have to pick a minimum of 3 socks.

| Marking Key |  |
| :--- | :--- |
| $\mathbf{1}$ | 1 mark for appropriate work in (a) |
| $\mathbf{2}$ | 1 mark for consistent answer in (a) |
| $\mathbf{3}$ | 1 mark for correct answer in (b) |

Select the best answer.
Given the following sets:

$$
\begin{aligned}
& A=\{3,4,5,6,7,8,9,10\} \\
& B=\{8,9,10,11,12\} \\
& C=\{8,9,10\}
\end{aligned}
$$

Which of the following statements represents set $C$ ?
A) $A^{\prime} \cap B^{\prime}$
B) $B \subset A$
C) $A \cup B$
D) $A \cap B$

Given hypothesis, $p$, and conclusion, $q$, complete the truth table below.

| $\boldsymbol{p}$ | $\boldsymbol{q}$ | $\sim \boldsymbol{p}$ | $\sim \boldsymbol{p} \leftrightarrow \boldsymbol{q}$ |
| :---: | :---: | :---: | :---: |
| True | True | False | False |
| True | False | False | True |
| False | True | True | True |
| False | False | True | False |

## Marking Key

(1) 1 mark for correct $\sim p$ column
(2) 1 mark for consistent $\sim p \leftrightarrow q$ column

Mrs. Dela Cruz teaches German and Spanish. She has 31 students of which 21 study German and 17 study Spanish.

How many of Mrs. Dela Cruz's students study German only? Show your work.

$$
\begin{aligned}
& 21+17=38 \\
& 38-31=7 \text { students study both }
\end{aligned}
$$

German: $21-7=14$
There are 14 students who study German only.
OR


There are 14 students who study German only.

|  |  | Marking Key |
| :--- | :--- | :--- |
| $\mathbf{1}$ | 1 mark for appropriate work |  |
| $\mathbf{2}$ | 1 mark for consistent answer |  |

## Exemplars

Exemplars may contain screen captures taken from software or Internet pages.

## Exemplar 1

## Question 3

Total: 3 marks

The heart pumps blood throughout the body. As the blood leaves the heart, it is replaced with new blood.

Muna's heart contains 70 mL of blood. With each heartbeat, the volume of original blood in her heart is reduced by $53 \%$ and replaced with new blood.
a) Determine the exponential regression equation that models the volume of original blood remaining in Buna's heart as a function of the number of heartbeats. Show your work.
(2 marks)

$$
\begin{aligned}
& F(x)=d(b)^{x} \\
& F(x)=70(0.53)^{x}
\end{aligned}
$$

b) Using your equation in (a), determine the volume of original blood remaining in Buna's heart after 6 heartbeats.
(1 mark)

$$
\begin{aligned}
& F(x)=70(0.53)^{6} \\
& F(x)=1.55 \mathrm{~mL}
\end{aligned}
$$

## 2 marks:

(1) $\rightarrow 1$ mark for correct initial value in equation in (a)
(3) 1 mark for consistent answer in (b)

## Exemplar 2

## Question 3

Total: 3 marks

The heart pumps blood throughout the body. As the blood leaves the heart, it is replaced with new blood.

Muna's heart contains 70 mL of blood. With each heartbeat, the volume of original blood in her heart is reduced by $53 \%$ and replaced with new blood.
exp Res
$y=M L$ of blood $x=$ of hart heats
a) Determine the exponential regression equation that models the volume of original blood remaining in Mona's heart as a function of the number of heartbeats. Show your work.

b) Using your equation in (a), determine the volume of original blood remaining in Buna's heart after 6 heartbeats.
(1 mark)


3 marks:
(1) $\rightarrow 1$ mark for correct initial value in equation in (a)
(2) $\rightarrow 1$ mark for correct rate of change in equation in (a)
(3 $\rightarrow 1$ mark for consistent answer in (b)
(巨6) $\rightarrow$ rounds too soon

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## Exemplar 1

## Question 4

## Total: 5 marks

The following equation models the path of a basketball as it is shot by Sarah:

$$
h=-0.51 d^{2}+4.72 d+6.09
$$

where $h$ represents the height in feet and $d$ represents the horizontal distance, in feet, the ball has travelled.
a) Create a clearly labelled graph of the equation.


Distance (ft)

## Exemplar 1 (continued)

b) The standard height for a basketball net is 10 feet. Sarah shoots the ball and it goes directly through the net on its way down. Using the given equation, determine how far, horizontally, the ball was from the net when it was shot. Show your work.
put a slider of $y=10 f+$
The ball was shot 0.92ft awau from the net.


## 3 marks:

(1) $\rightarrow 1$ mark for communicating the context of the graph with appropriate title and/or labels in (a)
(3) $\rightarrow 1$ mark for an appropriate shape that illustrates key characteristics of the function (e.g., maximum, minimum, asymptotes, intercepts) in (a) (4) $\rightarrow 1$ mark for appropriate work in (b)
(B3) $\rightarrow$ makes a transcription error (inaccurate transferring of information)

## Exemplar 2

## Question 4

## Total: 5 marks

The following equation models the path of a basketball as it is shot by Sarah:

$$
h=-0.51 d^{2}+4.72 d+6.09
$$

where $h$ represents the height in feet and $d$ represents the horizontal distance, in feet, the ball has travelled.
a) Create a clearly labelled graph of the equation.
(3 marks)


$$
\begin{aligned}
& h=-0.51 d^{2}+4.72 d+6.09 \\
& y=10
\end{aligned}
$$

## Exemplar 2 (continued)

b) The standard height for a basketball net is 10 feet. Sarah shoots the ball and it goes directly through the net on its way down. Using the given equation, determine how far, horizontally, the ball was from the net when it was shot. Show your work.


## The distance at which the ball was shot is 8.335 from the net

## 4 marks:

(1) $\rightarrow 1$ mark for communicating the context of the graph with appropriate title and/or labels in (a)
(3) $\rightarrow 1$ mark for an appropriate shape that illustrates key characteristics of the function (e.g., maximum, minimum, asymptotes, intercepts) in (a)
(4) $\rightarrow 1$ mark for appropriate work in (b)
$\boldsymbol{\Theta} \rightarrow 1$ mark for consistent answer in (b)
(55) $\rightarrow$ does not include the units in the final answer

## Exemplar 1

## Question 5

Total: 5 marks

A satellite was launched from Cape Canaveral and set to orbit the Earth. Instruments measured its distance from the equator at certain time intervals, using positive numbers to indicate distances north of the equator and negative numbers to indicate distances south as shown below.

a) Determine the sinusoidal regression equation that models this data.
(1 mark)

$$
y=3249.86 \sin (.07 x+.38)+750.17
$$

b) After 180 minutes in orbit, the satellite passes directly over Cape Canaveral. How far is Cape Canaveral from the equator?
(1 mark)

$$
\text { and calc value } x=180
$$

$$
V=1968.86 \text { miles }
$$

## Exemplar 1 (continued)

c) Determine the northern and southern limits of the satellite's path relative to the equator.
(2 marks)

d) Determine the period for this sinusoidal model.
(1 mark)

## 4 marks:

(1) $\rightarrow 1$ mark for correct equation in (a)
(2) $\rightarrow 1$ mark for consistent answer in (b)
(3) $\rightarrow 1$ mark for consistent northern limit in (c)
(4) $\rightarrow 1$ mark for consistent southern limit in (c)
(ㄹ) $\rightarrow$ does not identify the answer
(Es) $\rightarrow$ does not include the units in the final answer
(EG) $\rightarrow$ does not express the answer to the appropriate number of decimal places

## Exemplar 2

## Question 5

Total: 5 marks

A satellite was launched from Cape Canaveral and set to orbit the Earth. Instruments measured its distance from the equator at certain time intervals, using positive numbers to indicate distances north of the equator and negative numbers to indicate distances south as shown below.

| Time (minutes) | 20 | 40 | 60 | 80 | 100 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Distance from equator (miles) | 3929 | 637 | -2468 | -254 | 3620 |

a) Determine the sinusoidal regression equation that models this data.
(1 mark)
After much experimentation I got it to work.

$$
\begin{aligned}
& y_{1} \sim a \sin (b x,+c)+d\{-1<b c .7\} \\
& y=-3249.9 \sin (-0.07 x+408.02)+750.17\{-1<b<.7\}
\end{aligned}
$$

b) After 180 minutes in orbit, the satellite passes directly over Cape Canaveral. How far is Cape Canaveral from the equator?
(1 mark)

$$
x=180 \text { minutes } \quad y=1968.86 \text { miles north of equator }
$$

Desmos showed me where $x=180$ and the original equation (crossed.

## Exemplar 2

c) Determine the northern and southern limits of the satellite's path relative to the equator.

Northern: I tapped on a high point on the Desmos equation and found that the highest northern point is 4000.03 miles above equator

Southern: I tapped on a low point on the Desmos equation and found that the lowest southern point is -2499.70 miles below the equator.

d) Determine the period for this sinusoidal model.
(1 mark)
The period is the amount of space for graph to repeat itself. The 2 points I use are:

| $132.833-42.837=$ | 90 minutes |
| ---: | :--- |
|  | or |
| 1.5 hours |  |

## 5 marks:

(1) $\rightarrow 1$ mark for correct equation in (a)
(2) $\rightarrow 1$ mark for consistent answer in (b)
(3) $\rightarrow 1$ mark for consistent northern limit in (c)
(4) $\rightarrow 1$ mark for consistent southern limit in (c)
© $\rightarrow 1$ mark for consistent answer in (d)
(巨6) $\rightarrow$ does not express the answer to the appropriate number of decimal places
(巨6) $\rightarrow$ rounds incorrectly

## Exemplar 1

## Question 7

Total: 3 marks

Johannes wants to apply for a bank loan. Information regarding his financial situation is given below.

- He has a house valued at $\$ 225000.00$ with a mortgage of $\$ 175000.00$.
- He has a cottage valued at $\$ 115000.00$ with a mortgage or $\$ 75000.00$.
- He has $\$ 9000.00$ in his savings account.
- He owes a total of $\$ 25000.00$ on his credit cards.
a) Calculate his net worth.
(1 mark)

> 25000
> (all things owing)
> net worth $=$ assets - liabilities
> net worth $=225000$
b) Calculate his debt-to-equity ratio.
(1 mark)

$$
\frac{\text { Total liabilities - Mortgage }}{\text { Net worth }} \times 100
$$

$$
\text { DER= } \frac{275000-250000}{225000} \times 100=11 .
$$

(1 mark)

$$
\begin{aligned}
& \therefore \text { Yes the bank would lend him money } \\
& \text { because his DER is under } 32 \%
\end{aligned}
$$

## 1 mark:

(2) $\rightarrow 1$ mark for consistent debt-to-equity ratio in (b)

## Exemplar 2

## Question 7

Total: 3 marks

Johannes wants to apply for a bank loan. Information regarding his financial situation is given below.

- He has a house valued at $\$ 225^{\prime} 000.00$ with a mortgage of $\$ 175^{/} 000.00$.
- He has a cottage valued at $\$ 115000.00$ with a mortgage of $\$ 75000.00$.
- He has $\$ 9000.00$ in his savings account.
- He owes a total of $\$ 25000.00$ on his credit cards.
a) Calculate his net worth.
(1 mark)
Net worth

$$
\begin{gathered}
349000-275000 \\
=74000 \\
=
\end{gathered}
$$

b) Calculate his debt-to-equity ratio.
(1 mark)

$$
\begin{aligned}
\begin{array}{l}
\text { ark) } \begin{array}{c}
\text { ebt-to-equlfy }= \\
\text { ratio } \%
\end{array}
\end{array} \begin{aligned}
&(275000-250000) \\
&= 33.78 \\
& \therefore \text { the debt-to-equity } \\
& \text { ratio is } 33.78 \\
& \text { c) Based on his debt-to-equity ratio, would the bank lend him money? Explain. }
\end{aligned}
\end{aligned}
$$

(1 mark)
yes
because
it's below 50\%

## 3 marks:

(1) $\rightarrow 1$ mark for correct net worth value in (a)
(2) $\rightarrow 1$ mark for consistent debt-to-equity ratio in (b)
(3 $\rightarrow 1$ mark for appropriate explanation with reference to $50 \%$ in (c)
(ㅌ) $\rightarrow$ does not include a percent sign
(Es) $\rightarrow$ does not include the dollar sign for monetary values

Exemplar 1
Question 8
Rémi deposited a sum of money into an account 36 years ago that earned an annual interest rate of $8.00 \%$. Today, there is $\$ 12800.00$ in his account.

Use the Rule of 72 to estimate the initial amount that Rémi deposited.

$$
\begin{aligned}
& \text { Initial } \\
& \text { Time }=\frac{72}{8} \\
& =9 \text { years } \therefore \frac{36}{9}=4 \text { doubles in } 36 \text { years } \\
& \frac{\$ 12800}{4}=\$ 3200 \\
& \text { Initial }=\$ 3200
\end{aligned}
$$

## Exemplar 2

## Question 8

Total: 2 marks

Rémi deposited a sum of money into an account 36 years ago that earned an annual interest rate of $8.00 \%$. Today, there is $\$ 12800.00$ in his account.

Use the Rule of 72 to estimate the initial amount that Rémi deposited.

$$
t=\frac{72}{i} \quad=\frac{72}{8}=9 \quad \frac{12800,00}{9}=1422.22
$$

(1) $\rightarrow 1$ mark for correct doubling time (9 years)

## Exemplar 1

## Question 9

Total: 2 marks

Mr. Smythe makes a one-time donation to a university. The university decides to invest this money and use only the amount earned in simple interest from the investment to finance a scholarship.

- The initial amount of the donation was $\$ 650000.00$.
- The amount earned in simple interest annually is \$40 000.00.
- The university awards the scholarship to one student each year.

At what interest rate must the donation be invested to obtain the $\$ 40000.00$ needed to award the scholarship each year? Show your work.

$$
\begin{aligned}
I & =\operatorname{Rrt} \\
& =\frac{(650,000.00)}{40000.00}=16.25 \%
\end{aligned}
$$

## 1 mark:

(2) $\rightarrow 1$ mark for consistent answer
(1) $\rightarrow$ incorrectly states the final answer

## Exemplar 2

## Question 9

Total: 2 marks

Mr. Smythe makes a one-time donation to a university. The university decides to invest this money and use only the amount earned in simple interest from the investment to finance a scholarship.

- The initial amount of the donation was $\$ 650000.00$.
- The amount earned in simple interest annually is $\$ 40000.00$.
- The university awards the scholarship to one student each year.

At what interest rate must the donation be invested to obtain the $\$ 40000.00$ needed to award the scholarship each year? Show your work.


## 2 marks:

(1) $\rightarrow 1$ mark for appropriate work
(2) $\rightarrow 1$ mark for consistent answer
(巨6) $\rightarrow$ does not express the answer to the appropriate number of decimal places

## Exemplar 1

## Question 10

Bonnie and Claude want to buy a house. They can afford monthly payments of $\$ 1125.00$. The bank offers them a mortgage at an interest rate of $3.10 \%$, compounded semi-annually, with an amortization period of 25 years.
a) What is the maximum amount of money the bank will lend them for their mortgage? Show your work.

```
Initial: \(\stackrel{1}{1} 234654.21\)
Final: 0.00
monthly: 1125.00
Interest rate : 3.10
    \# years: 25
    \(P / Y: 12\)
\(C / Y: 12\)
```

b) If they have $\$ 30000.00$ saved for a down payment, what is the maximum house price they can afford?
(1 mark)

$$
\begin{aligned}
& 234.654 .21+30,000=264,654.21 \\
& \text { the maximum price they can } \\
& \text { afford is } \$ 264,654.21 \text {. }
\end{aligned}
$$

## 2 marks:

(2) $\rightarrow 1$ mark for consistent answer in (a)
(3 $\rightarrow 1$ mark for consistent answer in (b)
(Es) $\rightarrow$ does not include the dollar sign for monetary values

## Exemplar 2

## Question 10

Total: 3 marks

Bonnie and Claude want to buy a house. They can afford monthly payments of $\$ 1125.00$. The bank offers them a mortgage at an interest rate of $3.10 \%$, compounded semi-annually, with an amortization period of 25 years.
a) What is the maximum amount of money the bank will lend them for their mortgage?

Show your work.

$$
\begin{aligned}
& T M V \text { Solver: } \\
& N=300 \\
& I=3.1 \\
& P V=0 \\
& P m t=1125 \\
& F V=?=-\$ 1507417.57 \\
& P / y=12 \\
& C / y=2
\end{aligned}
$$

b) If they have $\$ 30000.00$ saved for a down payment, what is the maximum house price they can afford?
(1 mark)

$$
\begin{aligned}
& \qquad 507417.57+30000=\$ 537417.57 \\
& \text { The maximum house price that they can } \\
& \text { afford in } \$ 537417.57
\end{aligned}
$$

(2) $\rightarrow 1$ mark for consistent answer in (a)
(3) $\rightarrow 1$ mark for consistent answer in (b)

## Exemplar 1

## Question 11

## Total: 5 marks

Bernard is exploring financing options for a new house. The bank offers him a mortgage of $\$ 245827.00$ at an interest rate of $3.75 \%$, compounded semi-annually. He has the following payment options:

Option 1: monthly payments of $\$ 1260.00$
Option 2: biweekly payments of $\$ 630.00$
a) How many years will it take Bernard to pay off the mortgage with each option?

Show your work.
(3 marks)

| Option 1 | Option 2 |
| :--- | :--- |
| $\$ N=300$ | $\$ N=228.91$ |
| $I=3.75$ | $I=3.75$ |
| $P V=245827$ | $P V=245827$ |
| $P M T=-1260$ | $P M T=-1260$ |
| $F V=0$ | $F V=0$ |
| $P Y=12$ | $P y=26$ |
| $C y=2$ | $C y=2$ |

$$
\frac{229}{12}=19 \mathrm{yss} \quad \frac{300}{12}=25 \mathrm{yss}
$$

(1) $\rightarrow 1$ mark for appropriate work in (a)
(2) $\rightarrow 1$ mark for consistent answer for Option 1 in (a)
(4) $\rightarrow 1$ mark for appropriate work in (b)
$\boldsymbol{5} \rightarrow 1$ mark for consistent answer in (b)
(B3) $\rightarrow$ makes a transcription error (inaccurate transferring of information)
b) If Bernard makes biweekly payments instead of monthly payments, how much money will he save? Show your work.
(2 marks)

| Option 1 | Option 2 |
| :--- | :--- |
| $N=300$ | $N=228.91$ |
| $I=3.75$ | $I=3.75$ |
| $P V=245827$ | $P V=245827$ |
| $P M T=-1260$ | $P M T=-1260$ |
| $F V=$ | $F V=$ |
| $P Y=12$ | $P Y=26$ |
| $C Y=2$ | $C Y=2$ |

$$
\begin{aligned}
& 300 \times 1260 \\
& 229 \times 1260=\$ 288000
\end{aligned}
$$

$$
\frac{-42713}{\$ 80460} \cdot \$ 80460 \text { in interest }
$$

## Exemplar 2

## Question 11

Total: 5 marks

Bernard is exploring financing options for a new house. The bank offers him a mortgage of $\$ 245827.00$ at an interest rate of $3.75 \%$, compounded semi-annually. He has the following payment options:

Option 1: monthly payments of $\$ 1260.00$
Option 2: biweekly payments of $\$ 630.00$
a) How many years will it take Bernard to pay off the mortgage with each option?

Show your work.
(3 marks)

b) If Bernard makes biweekly payments instead of monthly payments, how much money will he save? Show your work.
(2 marks)

$\quad 5$ marks:
$\boldsymbol{1} \rightarrow 1$ mark for appropriate work in (a)
$\boldsymbol{2} \rightarrow 1$ mark for consistent answer for Option 1 in (a)
$\boldsymbol{3} \rightarrow 1$ mark for consistent answer for Option 2 in (a)
$\boldsymbol{4} \rightarrow 1$ mark for appropriate work in (b)
$\boldsymbol{5} \rightarrow 1$ mark for consistent answer in (b)

Exemplar 1
Question 13

A Canadian two-dollar coin consists of a gold-coloured centre and a silver-coloured outer ring. The coin has a diameter of 2.8 cm and is 0.2 cm thick while its centre has a diameter of 1.6 cm .

Diagram is not drawn to scale.


$$
\begin{aligned}
\text { center } d & =1.6 \\
r & =0.8 \\
\hline \text { coin } d & =2.8 \\
r & =1.4
\end{aligned}
$$

Calculate the volume of the silver-coloured outer ring of the coin.

$$
\begin{aligned}
\text { Volume of centre } & =\frac{4}{3} \pi 0.8^{3} \\
& =2.1446 \ldots{ }^{3}
\end{aligned}
$$

$$
\begin{aligned}
\text { Volume of coin } & =\frac{4}{3} \pi 1.4^{3} \\
& =11.494 \ldots{ }^{3}
\end{aligned}
$$

$$
\begin{aligned}
\text { Volume of ring } & =\text { volume of coin }- \text { volume of centre } \\
& =11.494 \ldots \\
& =9.349 \ldots
\end{aligned}
$$

Volume of the ring is $9.35 \mathrm{~cm}^{3}$

1 mark:
(2) $\rightarrow 1$ mark for consistent answer

## Exemplar 2

## Question 13

Total: 2 marks

A Canadian two-dollar coin consists of a gold-coloured centre and a silver-coloured outer ring. The coin has a diameter of 2.8 cm and is 0.2 cm thick while its centre has a diameter of 1.6 cm .

Diagram is not drawn to scale.


Calculate the volume of the silver-coloured outer ring of the coin.

$$
\begin{aligned}
V & =M \times r^{2} \times h \\
V & =\pi \times 1.4^{2} * 0.2 \\
& =1.23 \mathrm{~cm}^{3}
\end{aligned}
$$

## Exemplar 1

## Question 14

Total: 4 marks

A farmer is selling a cone-shaped pile of grain that has a diameter of 30 feet and a height of 20 feet. The grain needs to be transported to the market by truck.
a) The grain box of the farmer's truck has a volume of 850 cubic feet. What is the minimum number of times the farmer must go to the market to transport all of the grain? Show your work.
(2 marks)

$$
\begin{aligned}
& V=\pi r^{2} h \\
& U=\pi(15)^{2} 20 \\
& V=14137.17 \mathrm{ft}^{3} \\
& \frac{14137.17}{850}=16.63 \\
& \\
& =17 \text { times }
\end{aligned}
$$

b) Grain is sold by the whole bushel. This grain has a current value of $\$ 8.50 /$ bushel. If one bushel is equal to 1.24 cubic feet, calculate the value of the pile of grain. Show your work.
(2 marks)

$$
\begin{aligned}
& 1.24^{3} \cdot 8.50 \\
& =\$ 10.54
\end{aligned}
$$

## Exemplar 2

## Question 14

Total: 4 marks

A farmer is selling a cone-shaped pile of grain that has a diameter of 30 feet and a height of 20 feet. The grain needs to be transported to the market by truck.
a) The grain box of the farmer's truck has a volume of 850 cubic feet. What is the minimum number of times the farmer must go to the market to transport all of the grain? Show your work.
(2 marks)

b) Grain is sold by the whole bushel. This grain has a current value of $\$ 8.50 /$ bushel. If one bushel is equal to 1.24 cubic feet, calculate the value of the pile of grain.
Show your work.
(2 marks)

$$
\begin{array}{r}
\frac{4712.39}{1.24}=\frac{3800.31}{\$ 8.50} \\
\$ 32302.67
\end{array}
$$

## 3 marks:

(1) $\rightarrow 1$ mark for appropriate work in (a)
(3) $\rightarrow 1$ mark for appropriate work in (b)
(4) $\rightarrow 1$ mark for consistent answer in (b)
(④) $\rightarrow$ does not use whole units in contextual questions involving discrete data

## Exemplar 1

## Question 16

Total: 2 marks

Rylan rolls two 4 -sided dice with sides numbered 1 through 4. What is the probability that the sum of the rolled numbers is greater than or equal to 6 ? Show your work.


## Exemplar 2

## Question 16

Total: 2 marks

Rylan rolls two 4 -sided dice with sides numbered 1 through 4. What is the probability that the sum of the rolled numbers is greater than or equal to 6 ? Show your work.
$\frac{16}{6}$
$=2.67$

(1) $\rightarrow 1$ mark for appropriate work

## Exemplar 1

## Question 17

Total: 2 marks

How many different routes can Martin take to the mall if he can only travel north and east and wants to meet Amy on the way? Show your work.


1 mark:
(2) $\rightarrow 1$ mark for consistent answer

Exemplar 2
Question 17

How many different routes can Martin take to the mall if he can only travel north and east and wants to meet Amy on the way? Show your work.


There are 9 walls
Martin can set to Mall and meet Amy on the way.

1 mark:

## Exemplar 1

## Question 18

Total: 2 marks

Arif, Simba, and Maritza ran for student council treasurer. Of the 650 students who voted:

- $44 \%$ voted for Arif
- $36 \%$ voted for Simba
- the remaining students voted for Maritza
a) Determine the number of students that voted for Maritza.
(1 mark)

$$
\begin{array}{ll}
44+36=80 & 20 \% \text { of students voted } \\
20+80=100 \% & \text { for Maritza }
\end{array}
$$

b) One of the students is selected at random. Determine the odds against this student having voted for Arif.
(1 mark)

$$
\begin{array}{r}
36+20=56 \\
56: 44
\end{array}
$$

## Exemplar 2

## Question 18

Total: 2 marks
Arif, Simba, and Maritza ran for student council treasurer. Of the 650 students who voted:

- $44 \%$ voted for Arif 286
- $36 \%$ voted for Simba 234
- the remaining students voted for Maritza
a) Determine the number of students that voted for Maritza.

> (l mark)


100
$\frac{80}{20}$

$$
\begin{aligned}
& 650 \times 0.44=286 \text { votes - Arif } \\
& 650 \times 0.36=234 \text { votes - Simba } \\
& 650 \times 0.20 \frac{130 \text { votes }}{650 \text { votes }} \text { - Maritza }
\end{aligned}
$$

$\overline{20} \quad 130$ votes were lasted for Maritza
b) One of the students is selected at random. Determine the odds against this student having voted for Arif.
(1 mark)

$$
650: 286
$$

Exemplar 1
Question 19

Jack is late for the bus $15 \%$ of the time. When he is late for the bus, the probability that he will see Jill at the bus stop is $8 \%$. When he is not late, the probability that he will see Jill at the bus stop is $82 \%$.
a) What is the probability that Jack did not see Jill today? Show your work.
(2 marks)


$$
\begin{aligned}
p(\text { not see Jill }) & =(0.15)(0.92)+(0.85)(0.18) \\
& =0.138+0.153 \\
& =.291 \rightarrow 29.1 \%
\end{aligned}
$$

b) Jack did not see Jill today. Using your answer in (a), what is the probability that Jack was late for the bus?
(1 mark)

$$
\begin{aligned}
& \text { ablifythay } \\
& \text { late is } 13.8 \%
\end{aligned}
$$

2 marks:
(1) $\rightarrow 1$ mark for appropriate work in (a)
(2) $\rightarrow 1$ mark for consistent answer in (a)

## Exemplar 2

## Question 19

Total: 3 marks

Jack is late for the bus $15 \%$ of the time. When he is late for the bus, the probability that he will see Jill at the bus stop is $8 \%$. When he is not late, the probability that he will see Jill at the bus stop is $82 \%$.
a) What is the probability that Jack did not see Jill today? Show your work.
(2 marks)

b) Jack did not see Jill today. Using your answer in (a), what is the probability that Jack was late for the bus?
(1 mark)

$$
\begin{aligned}
P(\text { late })= & P(\text { late } \$ \text { not see })+P(\text { not late } B \text { not see }) \\
= & P((.15)(.92))+((.85)(.18) \\
= & (P(138)+P(.153)) \\
= & .291 \times 100 \\
= & 29.10 \% \text { that Jack was late for the bus given } \\
& \text { that he did not see Jul today. }
\end{aligned}
$$

## 1 mark:

(1) $\rightarrow 1$ mark for appropriate work in (a)

## Exemplar 1

## Question 20

Total: 2 marks

Guy's baseball team is playing in a tournament. There are six teams entered in the tournament. All teams play each other once and each game is played on the same baseball field.
a) Determine the total number of games played in the tournament.
(l mark)
$A: B, C, D, E, F$
$B: A, C, D, E, F \quad 6 \times 5=30$ games
$D$
$E$
$F$

b) Determine the probability that Guy's team plays the first game of the tournament.
(1 mark)

$$
\frac{5}{30} \times 100=16.67 \%
$$

## Exemplar 2

## Question 20

Total: 2 marks

Guy's baseball team is playing in a tournament. There are six teams entered in the tournament. All teams play each other once and each game is played on the same baseball field.
a) Determine the total number of games played in the tournament.
(1 mark)

$$
\begin{aligned}
\underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times 2 \times \underline{1} & =6! \\
& =720 \text { games }
\end{aligned}
$$

b) Determine the probability that Guy's team plays the first game of the tournament.
(1 mark)

$$
\begin{aligned}
\frac{1}{\text { G's }} \times 5 \times 4 \times 3 \times 2 \times 1 & =1 \times 51 \\
& =120
\end{aligned}
$$

$\frac{120}{720}=16.67 \%$
$\therefore$ there is a $16.67 \%$ chance guys team
will play first. will play first.
(2) $\rightarrow 1$ mark for consistent answer in (b)

## Exemplar 1

## Question 21

Total: 3 marks

Shivani needs to create a new password for her computer. The password must begin with three upper case letters followed by five digits.
a) How many passwords are possible if repetition is not allowed? Show your work.
(2 marks)

b) How many passwords are possible if repetition is not allowed and the password must begin with the letter M?
(1 mark)

$$
\begin{aligned}
& 1222498,650106 \\
& 1 \times 25 \times 24+9 \times 8 \times 7 \times 615=9072000
\end{aligned}
$$

## 2 marks:

(2) $\rightarrow 1$ mark for consistent answer in (a)
(3 $\rightarrow 1$ mark for consistent answer in (b)

## Exemplar 2

## Question 21

Total: 3 marks

Shivani needs to create a new password for her computer. The password must begin with three upper case letters followed by five digits.
a) How many passwords are possible if repetition is not allowed? Show your work.
(2 marks)

b) How many passwords are possible if repetition is not allowed and the password must begin with the letter M?
(1 mark)


| 2 marks: <br> (2) $\rightarrow 1$ mark for consistent answer in (a) <br> (3) $\rightarrow 1$ mark for consistent answer in (b) |  |
| :---: | :---: |
|  |  |
|  |  |

## Exemplar 1

## Question 22

Total: 3 marks

Joe is getting dressed in the dark. The only socks in his drawer are 12 white socks and 10 green socks. He randomly picks two socks from the drawer, one after the other.
a) What is the probability that both socks are the same colour? Show your work.
(2 marks)

$$
\begin{aligned}
& \text { white } \\
& \frac{12}{22} \times \frac{11}{22}=\frac{132}{484} \\
& \text { green } \\
& \frac{10}{22} \times \frac{9}{22}=\frac{90}{484} \\
& \frac{132+90}{484}=\frac{222}{484} \times 100=45.87 \%
\end{aligned}
$$

b) Using logical reasoning, what is the minimum number of socks Joe would have to pick to guarantee a pair of the same-coloured socks?
(1 mark)

$$
\left.\begin{array}{ccccccccc}
G & G & G & G & G & G & G & G & G \\
\hline & W & W & W & W & W & W & W & W
\end{array}\right] w W
$$

1 mark:
(2) $\rightarrow 1$ mark for consistent answer in (a)

## Exemplar 2

## Question 22

Joe is getting dressed in the dark. The only socks in his drawer are 12 white socks and 10 green socks. He randomly picks two socks from the drawer, one after the other.
a) What is the probability that both socks are the same colour? Show your work.
(2 marks)

b) Using logical reasoning, what is the minimum number of socks Joe would have to pick to guarantee a pair of the same-coloured socks?
(1 mark)

$$
12 \text { socks }
$$

(1) $\rightarrow 1$ mark for appropriate work in (a)

## Exemplar 1

Given hypothesis, $p$, and conclusion, $q$, complete the truth table below.

| $\boldsymbol{p}$ | $\boldsymbol{q}$ | $\sim \boldsymbol{p}$ | $\sim \boldsymbol{p} \leftrightarrow \boldsymbol{q}$ |
| :---: | :---: | :---: | :---: |
| True | True | $\boldsymbol{F}$ | $\boldsymbol{F}$ |
| True | False | $\boldsymbol{T}$ | $\boldsymbol{F}$ |
| False | True | $\boldsymbol{T}$ | $\mathbf{T}$ |
| False | False | $\boldsymbol{T}$ | $\boldsymbol{F}$ |

## 1 mark: <br> (2) $\rightarrow 1$ mark for consistent $\sim p \leftrightarrow q$ column

## Exemplar 2

Given hypothesis, $p$, and conclusion, $q$, complete the truth table below.

| $\boldsymbol{p}$ | $\boldsymbol{q}$ | $\sim \boldsymbol{p}$ | $\sim \boldsymbol{p} \leftrightarrow \boldsymbol{q}$ |
| :---: | :---: | :---: | :---: |
| True | True | false | true |
| True | False | false | false |
| False | True | True | True |
| False | False | True | false |

## Exemplar 1

## Question 25

Total: 2 marks

Mrs. Dela Cruz teaches German and Spanish. She has 31 students of which 21 study German and 17 study Spanish.

How many of Mrs. Dela Cruz's students study German only? Show your work.


## 2 marks:

(1) $\rightarrow 1$ mark for appropriate work
(2) $\rightarrow 1$ mark for consistent answer
(①) $\rightarrow$ does not identify the answer
(22) $\rightarrow$ does not include a box when using a Venn diagram

## Exemplar 2

## Question 25

Total: 2 marks

Mrs. Dela Cruz teaches German and Spanish. She has 31 students of which 21 study German and 17 study Spanish.

How many of Mrs. Bela Cruz's students study German only? Show your work.

$$
\begin{aligned}
& 31=21+17-x \\
& x=21+17-31 \\
& x=7 \text { doboth } \\
& 21-7=14 \text { only do German }
\end{aligned}
$$

(1) $\rightarrow 1$ mark for appropriate work
(2) $\rightarrow 1$ mark for consistent answer

## Appendices

## Appendix A: Table of Questions by Unit and Learning Outcome

| Relations and Functions |  |  |
| :---: | :---: | :---: |
| Question | Learning Outcome | Mark |
| 1 | 12A.R. 1 | 1 |
| 2 | 12A.R. 1 | 1 |
| 3 | 12A.R. 2 | 3 |
| 4 | 12A.R. 1 | 5 |
| 5 | 12A.R. 3 | 5 |
|  |  | Total $=15$ |
| Financial Mathematics |  |  |
| Question | Learning Outcome | Mark |
| 6 | 12A.FM. 2 | 1 |
| 7 | 12A.FM. 2 | 3 |
| 8 | 12A.FM. 3 | 2 |
| 9 | 12A.FM. 3 | 2 |
| 10 | 12A.FM. 1 | 3 |
| 11 | 12A.FM. 2 | 5 |
|  |  | Total $=16$ |
| Design and Measurement |  |  |
| Question | Learning Outcome | Mark |
| 12 | 12A.D. 1 | 1 |
| 13 | 12A.D. 1 | 2 |
| 14 | 12A.D. 1 | 4 |
|  |  | Total $=7$ |
| Probability |  |  |
| Question | Learning Outcome | Mark |
| 15 | 12A.P. 6 | 1 |
| 16 | 12A.P. 2 | 2 |
| 17 | 12A.P. 5 | 2 |
| 18 | 12A.P. 1 | 2 |
| 19 | 12A.P. 3 | 3 |
| 20 | 12A.P. 6 | 2 |
| 21 | 12A.P.4, 12A.P. 5 | 3 |
| 22 a) | 12A.P. 3 | 2 |
| Total $=17$ |  |  |
| Logical Reasoning |  |  |
| Question | Learning Outcome | Mark |
| $22 \mathrm{~b})$ | 12A.L. 1 | 1 |
| 23 | 12A.L. 2 | 1 |
| 24 | 12A.L. 3 | 2 |
| 25 | 12A.L.2, 12A.P. 2 | 2 |
|  |  | Total $=6$ |

# Appendix B: Irregularities in Provincial Tests 

## A Guide for Local Marking

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an Irregular Test Booklet Report should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all "NR") or only incorrect responses ("0")

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an Irregular Test Booklet Report.

Except in the case of cheating or plagiarism where the result is a provincial test mark of $0 \%$, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an Irregular Test Booklet Report documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the department along with the test materials.

## Irregular Test Booklet Report

Test: $\qquad$
Date marked:

Booklet No.: $\qquad$

Problem(s) noted: $\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Question(s) affected: $\qquad$
$\qquad$
$\qquad$

Action taken or rationale for assigning marks: $\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Follow-up: $\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Decision: $\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Marker's Signature: $\qquad$

Principal's Signature: $\qquad$

For Department Use Only—After Marking Complete
Consultant:
$\longrightarrow$
Date: $\qquad$

