Grade 12
Applied Mathematics
Achievement Test

Marking Guide

June 2013

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## Disponible en français.

Available in alternate formats upon request.

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## General Marking Instructions

Please do not make any marks in the test booklets. Any marks in a test booklet will have to be erased by departmental staff before the sample marking if the booklet is selected.

Please ensure that

- the student booklet number and the number on the Scoring Sheet are identical
- only a pencil is used to complete the Scoring Sheet
- each student's final result is recorded, by booklet number, on the corresponding Scoring Sheet
- the Scoring Sheet is complete and a copy has been made for school records

Once marking is completed, please forward the Scoring Sheets to Manitoba Education in the envelope provided (for more information, see the administration manual).

## Marking the Questions

Explanations for common errors for multiple-choice questions have been provided if applicable.
To receive full marks for a question, a student's response must be complete and correct. Partial marks may be awarded for an "appropriate strategy" with execution errors. An appropriate strategy is defined as one that is consistent with the learning outcomes and mathematical processes associated with the question and, if properly executed, would lead to the correct answer.

Some questions require a form of explanation or justification from students. Depending on the student's learning style, the explanation or justification can be given through a labelled diagram, in words, by showing mathematical operations for answer verification, or by referring to a software or calculator program. For this reason, appropriate flexibility is required when marking student responses.

## Student Errors

As a guiding principle, students should only be penalized once for each error committed in the context of a test question. For example, students may choose an inappropriate strategy for a question, but carry it through correctly and arrive at an incorrect answer. In such cases, students should be penalized for having selected an inappropriate strategy for the task at hand, but should be given credit for having arrived at an answer consistent with their choice of strategy.

Errors which are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

## Communication Errors

The marks allocated to questions are primarily based on the concepts associated with the learning outcomes in the curriculum. For each question, shade in the circle on the Scoring Sheet that represents the mark awarded based on the concepts. A total of these marks will provide the preliminary mark.

Errors that are not related to the concepts are called "Communication Errors" and these will be indicated on the Scoring Sheet in a separate section (see example below). There will be a 0.5 mark deduction for each type of communication error committed, regardless of the number of errors committed for a certain type (i.e., committing a second error for any type will not further affect a student's mark).

The total mark deduction for communication errors for any student response is not to exceed the marks given for that response. When multiple communication errors are made in a given response, any deductions are to be indicated in the order in which the errors occur in the response, without exceeding the given marks.

There is a maximum deduction of 3 marks (approximately $5 \%$ of the total test mark) for communication errors.

The student's final mark is determined by subtracting the communication errors from the preliminary mark.

## Example:

A student has a preliminary mark of 46 . The student committed two E1 errors ( 0.5 mark deduction) and three E4 errors ( 0.5 mark deduction).


## Communication Errors

Preliminary Mark $-\binom{0.5 \times \#$ of error types for a }{ maximum deduction of 3 marks }$=$ Final Mark
$46-\quad(0.5 \times 2) \quad=\quad 45$

## Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix C provides examples of such irregularities as well as procedures to follow to report irregularities.

If a Scoring Sheet is marked with " 0 " and/or "NR" only (e.g., student was present but did not attempt any questions) please document this on the Irregular Test Booklet Report.

## Assistance

If, during marking, any issue arises that cannot be resolved locally, please call Manitoba Education at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the marking keys.

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## Marking Keys

Please note that this Marking Guide contains screen captures taken from a TI-83 Plus graphing calculator.

## Relations and Functions

## Learning Outcome: 12A.R. 1

## Question Type: Multiple Choice

## Question No. 1 and Answer

Total: 1 mark
Circle the graph below which best represents a cubic function.
A)

B)

C)

$\checkmark$ D)


## Common Errors

A: exponential function
B: logarithmic function
C: quadratic function

| Learning Outcome: 12A.R. 2 | Question Type: Short Answer |
| :--- | ---: |
| Question No. 2 and Answer | Total: 2 marks |

Given the following function which represents the change in a town's population with respect to time:

$$
y=1000(1.05)^{x}
$$

Referring to the town, explain the meaning of:
a) " $1000 "$
(1 mark)
" 1000 " is the initial population.
b) " 1.05 "
(1 mark)
" 1.05 " is the growth rate of $5 \%$.

| Marking Key |  |
| :--- | :--- |
| $\mathbf{1}$ | 1 mark for correct explanation in (a) |
| $\mathbf{2}$ | 1 mark for correct explanation in (b) |

## Exemplar 1

## Question No. 2

Total: 2 marks

Given the following function which represents the change in a town's population with respect to time:

$$
y=1000(1.05)^{x}
$$

Referring to the town, explain the meaning of:
a) " 1000 "
(1 mark)
"1000" is the population
b) " 1.05 "
(1 mark)
"1.05" is the time

Exemplar 2
Question No. 2
Given the following function which represents the change in a town's population with respect to time:

$$
y=100(1.05)^{x}
$$

Referring to the town, explain the meaning of:
a) " $1000 "$
(1 mark)

$$
\begin{aligned}
& 1000 \text { = The point where the tx intercepts } \\
& \text { the yaxis. }
\end{aligned}
$$

b) " 1.05 "
(1 mark)

$$
\begin{aligned}
& 105=\text { wether the } f x \text { is Increasing or } \\
& \text { decreasing and in this graph it is Increasing. }
\end{aligned}
$$

0 marks:

A football player wants to kick a football so it will go over a crossbar that is $\mathbf{3 5}$ yards away and 3.33 yards high. (Diagram is not drawn to scale.)

The horizontal distance ( $d$, in yards) and the height ( $h$, in yards) that the football travels are represented by the following equation:

$$
h=-0.04 d^{2}+1.51 d
$$



How far above or below the crossbar will the football travel? Show your work.


The football will travel 0.52 yards above the crossbar.

|  |  | Marking Key |
| :--- | :--- | :--- |
| $\mathbf{1}$ | 1 mark for appropriate work |  |
| $\mathbf{2}$ | 1 mark for correct answer |  |

## Exemplar 1

## Question No. 3

A football player wants to kick a football so it will go over a crossbar that is 35 yards away and 3.33 yards high. (Diagram is not drawn to scale.)

The horizontal distance ( $d$, in yards) and the height ( $h$, in yards) that the football travels are represented by the following equation:

$$
h=-0.04 d^{2}+1.51 d
$$



How far above or below the crossbar will the football travel? Show your work.

```
on cale
y=-0.04 \mp@subsup{\partial}{}{2}+1.51d
    graph
2nd trace
maximum
    14.25m-3.33m
    =10.92m
```



```
    of }10.92\textrm{m}
```


## Exemplar 2

## Question No. 3

Total: 2 marks

A football player wants to kick a football so it will go over a crossbar that is 35 yards away and 3.33 yards high. (Diagram is not drawn to scale.)

The horizontal distance ( $d$, in yards) and the height ( $h$, in yards) that the football travels are represented by the following equation:

$$
h=-0.04 d^{2}+1.51 d
$$



How far above or below the crossbar will the football travel? Show your work.

$$
\begin{aligned}
& h=-0.04 d^{2}+1.51 d \\
& \begin{array}{l}
\text { (height) } \\
h=-0.04(35)^{2}+1.51(35) \\
(\text { height }) \\
\left.h=-0.04(35)^{2}\right) \\
(\text { height }) \\
h=39.00+52.85 \\
\\
3.85=h
\end{array}
\end{aligned}
$$

The football will travel 3.85 above the crossbar.

1 mark:
(1) $\rightarrow 1$ mark for appropriate work

The mass of a steel ball varies with respect to its diameter.

| diameter (mm) | 0 | 5 | 10 | 15 | 20 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| mass (g) | 0 | 2 | 10 | 32 | 80 |

Determine the cubic equation that best represents the data. Sketch a clearly labelled graph of the equation.

$$
\text { cubic equation: } \quad y=0.01 x^{3}-0.10 x^{2}+0.61 x-0.06
$$



## Question No. 4 continued

## Marker Note(s):

$\rightarrow$ No mark deduction for rounding errors, unless this results in a non-cubic equation.
$\rightarrow$ Regression equations may vary depending on the software used.

## Marking Key

(1) 1 mark for correct cubic equation
(2) 1 mark for correct graph with appropriate shape

31 mark for including: labels for the axes, units for the axes, and scales for the axes

## Exemplar 1

## Question No. 4

The mass of a steel ball varies with respect to its diameter.

| diameter (mm) | 0 | 5 | 10 | 15 | 20 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| mass (g) | 0 | 2 | 10 | 32 | 80 |

Determine the cubic equation that best represents the data. Sketch a clearly labelled graph of the equation.
cubic equation: $\qquad$


## Exemplar 1 (continued)

## 2 marks:

(1) $\rightarrow 1$ mark for correct cubic equation
(2) $\rightarrow 1$ mark for correct graph with appropriate shape
(11) $\rightarrow 0.5$ mark deduction (if applicable) for not including one of the following in the equation: " $y=$ ", "sin", " $\ln$ ", or " $x$ ", or for writing parameters separately from the equation

## Exemplar 2

Question No. 4
The mass of a steel ball varies with respect to its diameter. stat plot

| diameter (mm) | 0 | 5 | 10 | 15 | 20 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| mass (g) | 0 | 2 | 10 | 32 | 80 |

Determine the cubic equation that best represents the data. Sketch a clearly labelled graph of the equation. cubic equation: $a x^{3}+b x^{2}+c x+d=0.013 x^{3}+-0.097 x^{2}+0.61 x+-0.057$


## Exemplar 2 (continued)

## 2 marks:

(1) $\rightarrow 1$ mark for correct cubic equation
(2) $\rightarrow 1$ mark for correct graph with appropriate shape
(11) $\rightarrow 0.5$ mark deduction (if applicable) for not including one of the following in the equation: " $y=$ ", "sin", " $\ln$ ", or " $x$ ", or for writing parameters separately from the equation

A mass is suspended by a spring and is in a resting position 0.50 metres above a table.


The mass is pulled down 0.40 metres and is then released. The following information is obtained:

- It takes $\mathbf{1 . 2 0}$ seconds for the mass to return to its lowest position.
- The mass reaches a maximum height of 0.90 metres.
a) Determine the sinusoidal equation that best represents the distance of the mass with respect to the table as a function of time since it was released. Show your work.
(2 marks)

| time $(\mathrm{s})$ | 0.00 | 0.30 | 0.60 | 0.90 | 1.20 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| distance $(\mathrm{m})$ | 0.10 | 0.50 | 0.90 | 0.50 | 0.10 |

$$
\text { Using SinReg: } y=0.40 \sin (5.24 x-1.57)+0.50
$$

b) When will the mass be 0.75 metres above the table for the first time?
(1 mark)

$$
\begin{aligned}
& \text { intersect with } y=0.75 \\
& \text { time }=0.43 \text { seconds }
\end{aligned}
$$

Marker Note(s):
$\rightarrow$ Regression equations may vary depending on the software used.

| Marking Key |  |
| :--- | :--- |
| $\mathbf{1}$ | 1 mark for appropriate work in (a) |
| $\mathbf{2}$ | 1 mark for correct sinusoidal equation in (a) |
| $\mathbf{3}$ | 1 mark for correct answer in (b) consistent with sinusoidal equation in (a) |

## Exemplar 1

## Question No. 5

Total: 3 marks

A mass is suspended by a spring and is in a resting position 0.50 metres above a table.


The mass is pulled down 0.40 metres and is then released. The following information is obtained:

- It takes 1.20 seconds for the mass to return to its lowest position.
- The mass reaches a maximum height of 0.90 metres.
a) Determine the sinusoidal equation that best represents the distance of the mass with respect to the table as a function of time since it was released. Show your work.
(2 marks)


$$
y=104.642 \sin (0.2526 x-0.0828)+100.63
$$

## 1 mark:

(1) $\rightarrow 1$ mark for appropriate work in (a)
b) When will the mass be $\underline{0.75}$ metres above the table for the first time?
(1 mark)

$$
\begin{aligned}
& \begin{array}{l}
y_{1}=104.642 \sin (0.2526 x-0.0828)+100.63 \\
\begin{array}{ll}
y_{2}=0.75 & \text { t will be above the } \\
\text { intersection } & \text { table at o.75 metres } \\
x=1.69, y=0.75 & \text { for the first time after } \\
& \text { 1. }
\end{array}
\end{array} .
\end{aligned}
$$

## Exemplar 2

## Question No. 5

Total: 3 marks

A mass is suspended by a spring and is in a resting position 0.50 metres above a table.


The mass is pulled down 0.40 metres and is then released. The following information is obtained:

- It takes 1.20 seconds for the mass to return to its lowest position.
- The mass reaches a maximum height of 0.90 metres.
a) Determine the sinusoidal equation that best represents the distance of the mass with respect to the table as a function of time since it was released. Show your work.
(2 marks)

a $\sin (6 x+c)+d$



## 1 mark:

(2) $\rightarrow 1$ mark for correct sinusoidal equation in (a)
(1) $\rightarrow 0.5$ mark deduction (if applicable) for not including one of the following in the equation: " $y=$ ", "sin", "ln", or " $x$ ", or for writing parameters separately from the equation
b) When will the mass be 0.75 metres above the table for the first time?
(1 mark)

$$
\begin{aligned}
h(0.75) & =0.40 \sin (5.24(0.75))+0.50 \\
& =0.527 \\
& =0.53 \text { seconds ofter the mass is released. }
\end{aligned}
$$

## THIS PAGE WAS INTENTIONALLY LEFT BLANK.

It becomes easier and easier to see the headlights of an oncoming car the closer that it gets. The distance ( $d$, in metres) between the car and an observer can be described as a function of the intensity ( $I$, in lumens) of the headlight brightness:

$$
d=350-72 \ln (I)
$$

a) Sketch a clearly labelled graph of the equation.
(2 marks)


## Question No. 6 continued

b) Determine the distance to an oncoming car if the intensity of its headlights is 75 lumens.
(1 mark)

2nd TRACE 1: value $x=75, y=39.14$
The car is 39.14 metres away.

$$
\begin{aligned}
& \boldsymbol{O R} \\
& \begin{aligned}
d & =350-72 \ln (75) \\
& =39.14 \mathrm{~m}
\end{aligned}
\end{aligned}
$$

c) What is the maximum intensity of the headlights? Justify your answer.
(2 marks)
The intensity of the headlights will be greatest when the distance is 0 metres.
The maximum intensity of the headlights is 129.17 lumens.
OR

$$
\begin{aligned}
& \mathrm{Y}_{2}=0 \\
& \text { 2nd TRACE 5: intersect } x=129.17, y=0
\end{aligned}
$$

The maximum intensity of the headlights is 129.17 lumens.

## Marker Note(s):

$\rightarrow$ For the appropriate shape mark to be awarded, the graph must curve, must not cross the $y$-axis and must show an appropriate x-intercept.

## Marking Key

(1) 1 mark for correct graph with appropriate shape in (a)
(2) 1 mark for including: labels for the axes, units for the axes, and scales for the axes in (a)
(3) 1 mark for correct answer in (b)
(4) 1 mark for appropriate justification in (c)
© 1 mark for correct maximum intensity in (c)

## Exemplar 1

## Question No. 6

Total: 5 marks

It becomes easier and easier to see the headlights of an oncoming car the closer that it gets. The distance ( $d$, in metres) between the car and an observer can be described as a function of the intensity ( $I$, in lumens) of the headlight brightness:

$$
d=350-72 \ln (I)
$$

a) Sketch a clearly labelled graph of the equation.


Exemplar 1 (continued)
b) Determine the distance to an oncoming car if the intensity of its headlights is 75 lumens. (1 mark)

$$
\begin{aligned}
& d=-72 \ln (25)+350 \\
&=39 \mathrm{~m}
\end{aligned}
$$

4 marks:
(1) $\rightarrow 1$ mark for correct graph with appropriate shape in (a)
(2) $\rightarrow 1$ mark for including: labels for the axes, units for the axes, and scales for the axes in (a)
3 $\rightarrow 1$ mark for correct answer in (b)
$\boldsymbol{5} \rightarrow 1$ mark for correct maximum intensity in (c)
(Es) $\rightarrow 0.5$ mark deduction (if applicable) for rounding too soon or rounding incorrectly
c) What is the maximum intensity of the headlights? Justify your answer.
(2 marks)
It should be a hundred lumens as the max intensity because you can only get a hunted percent. In this case the max intensity is 129 lumens. This is because it must be meanses out of a different pecent or they can have that marbecause its inlumens.

## Exemplar 2

Question No. 6
Total: 5 marks

It becomes easier and easier to see the headlights of an oncoming car the closer that it gets. The distance ( $d$, in metres) between the car and an observer can be described as a function of the intensity ( $I$, in lumens) of the headlight brightness:

$$
d=350-72 \ln (I)
$$

a) Sketch a clearly labelled graph of the equation.


## Exemplar 2 (continued)

b) Determine the distance to an oncoming car if the intensity of its headlights is 75 lumens.
(1 mark)

$$
y=75 \quad x=45,58
$$

## 45,58 meters

c) What is the maximum intensity of the headlights? Justify your answer.
(2 marks)

$$
\text { infinite intesensity because their is no } y \text { intercept }
$$

4 marks:
(1) $\rightarrow 1$ mark for correct graph with appropriate shape in (a)
(3) 1 mark for correct answer in (b)
(4) $\rightarrow 1$ mark for appropriate justification in (c)
$\boldsymbol{5} \rightarrow 1$ mark for correct maximum intensity in (c)
Note: No mark awarded for labels, units, and scale in (a) since axes are reversed.
Marks (3, 4, and © were awarded based on the student's answer in (a).

## Probability

Learning Outcome: 12A.P. 4

Question Type: Multiple Choice
Question No. 7 and Answer
Total: 1 mark

Licence plates in Ontario contain 4 upper case letters followed by 3 digits, with repetition allowed. Circle the maximum possible number of licence plates that begin with the letters: MMBA, MANI, or BNTP.
A) 2160
B) 2880
$\checkmark$ C) $\mathbf{3 0 0 0}$
D) 4000

## Common Errors

A: $3 \times 10 \times 9 \times 8$
B: $4 \times 10 \times 9 \times 8$
D: $4 \times 10 \times 10 \times 10$

| Learning Outcome: 12A.P. 1 | Question Type: Multiple Choice |
| :--- | ---: |
| Question No. 8 | Total: 1 mark |

A bag contains 6 white marbles, 8 blue marbles, 2 yellow marbles, and $\mathbf{4}$ green marbles. What are the odds in favour of selecting a white marble?
$\checkmark$ A) $6: 14$
B) $6: 20$
C) $14: 6$
D) $20: 6$

> | Common Errors |
| :--- |
| B: probability |
| C: odds against selection |
| D: probability reversed |

A cookie jar contains 10 chocolate chip cookies, 12 double chocolate cookies, and 15 oatmeal cookies. Allison says that the odds against selecting a cookie with chocolate are 15 to 37. Ryan says that the odds against are 15 to 22 . Who is correct? Explain your answer.

Ryan is correct. Odds are expressed as part:part while probabilities are expressed as part: whole.

## OR

Ryan is correct.

| Oatmeal |  | Chocolate |  |
| :---: | :---: | :---: | :---: |
| Double Chocolate |  |  |  |
| 15 | $:$ | 12 |  |
| $=15: 22$ |  |  | 10 |
|  |  |  |  |


|  |  | Marking Key |
| :--- | :--- | :--- |
| (1) | 1 mark for correct explanation |  |

Exemplar 1
Question No. 9
A cookie jar contains 10 chocolate chip cookies, 12 double chocolate cookies, and 15 oatmeal cookies. Allison says that the odds against selecting a cookie with chocolate are 15 to 37 . Ryan says that the odds against are 15 to 22 . Who is correct? Explain your answer.

Ryan is correct as his answer is in the proper format.

Allison just kept hers as a fraction and didn't
change to odds.

Exemplar 2
Question No. 9

A cookie jar contains 10 chocolate chip cookies, 12 double chocolate cookies, and 15 oatmeal cookies. Allison says that the odds against selecting a cookie with chocolate are 15 to 37 . Ryan says that the odds against are 15 to 22 . Who is correct? Explain your answer.

the sum of the
needs

up


$$
37
$$



Describe a situation containing mutually exclusive events. Explain why the events are mutually exclusive.

- Selecting a 5 and an even number from a set of cards numbered from 1 to 10 . These are mutually exclusive events since 5 is not an even number.

Other answers are possible.

| Marking Key |  |
| :--- | :--- |
| $\mathbf{1}$ | 1 mark for appropriate example |
| $\mathbf{2}$ | 1 mark for appropriate explanation |

Exemplar 1

Describe a situation containing mutually exclusive events. Explain why the events are mutually exclusive.

In school, taking either math or history which are in the same time block.
This is mutually exclusive because you can only takeone. They are in the same time block so you cannot take both, making it mutually exclusive.

Exemplar 2
Question No. 10

Describe a situation containing mutually exclusive events. Explain why the events are mutually exclusive.


An organization consisting of 15 women and 19 men must create a 10 -person committee.
a) How many committees can be created that will include 4 women and 6 men? Show your work.
(2 marks)

$$
\begin{aligned}
\text { women: }{ }_{15} C_{4} & =1365 \\
\text { men: }{ }_{19} C_{6} & =27132
\end{aligned}
$$

4 women and 6 men: $1365 \times 27132=37035180$ committees
b) If a 10-person committee is randomly selected, what is the probability that the committee will include 4 women and 6 men? Show your work.
(2 marks)

$$
\begin{aligned}
\frac{37035180}{{ }_{34} C_{10}} & =\frac{37035180}{131128140} \\
& =0.28=28.24 \%
\end{aligned}
$$

| $\quad$ Marking Key |  |
| :--- | :--- |
| $\mathbf{1}$ | 1 mark for appropriate work in (a) |
| $\mathbf{2}$ | 1 mark for correct answer in $(a)$ |
| $\mathbf{3}$ | 1 mark for appropriate work in $(b)$ |
| $\mathbf{4}$ | 1 mark for correct answer in $(b)$ |

## Exemplar 1

An organization consisting of 15 women and 19 men must create a 10 -person committee.
a) How many committees can be created that will include 4 women and 6 men? Show your work.
(2 marks)

$$
\begin{aligned}
& 15 C_{4}+19 C_{6} \\
& 1365+27132 \\
& =28497 \text { ways }
\end{aligned}
$$

b) If a 10-person committee is randomly selected, what is the probability that the committee will include 4 women and 6 men? Show your work.
(2 marks)


28497:/31128140

3 marks:
(1) $\rightarrow 1$ mark for appropriate work in (a)

3 $\rightarrow 1$ mark for appropriate work in (b)
(4) $\rightarrow 1$ mark for correct answer in (b)

Exemplar 2
Question No. 11

An organization consisting of 15 women and 19 men must create a 10-person committee.
a) How many committees can be created that will include 4 women and 6 men? Show your work.
(2 marks)


You Con have 3 committer of 4 women and 6 men.
b) If a 10-person committee is randomly selected, what is the probability that the committee will include 4 women and 6 men? Show your work.
(2 marks)

2 marks:
(3 $\rightarrow 1$ mark for appropriate work in (b) (4) $\rightarrow 1$ mark for correct answer in (b)

You have been asked to create a four-character password for your computer using:

- the 26 upper case letters of the alphabet (A, B, C, ...)
- the 26 lower case letters of the alphabet ( $\mathbf{a}, \mathrm{b}, \mathrm{c}, \ldots$ )
- the digits from 0 to 9
- the symbols: $\sim$ ! @ \# \$ \% ^ \& *
a) How many different four-character passwords are possible if any of the letters, digits, or symbols can be used for each character if repetition is allowed?
(1 mark)

$$
\begin{aligned}
& 26+26+10+9=71 \text { characters } \\
& 71 \times 71 \times 71 \times 71=71^{4}=25411681 \text { passwords }
\end{aligned}
$$

b) How many different four-character passwords are possible if repetition is not allowed? (1 mark)

$$
{ }_{71} \mathrm{P}_{4}=71 \times 70 \times 69 \times 68=23319240 \text { passwords }
$$

c) How many four-character passwords begin with a letter and end with a digit if repetition is allowed?
(1 mark)

$$
52 \times 71 \times 71 \times 10=2621320 \text { passwords }
$$

## Question No. 12 continued

d) How many different four-character passwords containing at least one symbol are possible if repetition is allowed? Show your work.
(2 marks)

$$
\begin{aligned}
\text { Total } & =\text { all passwords }- \text { passwords without symbols } \\
& =71^{4}-62^{4} \\
& =25411681-14776336 \\
& =10635345
\end{aligned}
$$

OR—— 1 symbol: $(9 \times 62 \times 62 \times 62) \times 4$

$$
=8579808
$$

2 symbols: $(9 \times 9 \times 62 \times 62) \times \frac{4!}{(2!2!)}$

$$
=1868184
$$

3 symbols: $(9 \times 9 \times 9 \times 62) \times 4$

$$
=180792
$$

4 symbols: $(9 \times 9 \times 9 \times 9)$

$$
=6561
$$

$$
8579808+1868184+180792+6561=10635345 \text { passwords }
$$

| $\quad$ Marking Key |  |  |
| :--- | :--- | :---: |
| $\mathbf{1}$ | 1 mark for correct answer in (a) |  |
| $\mathbf{2}$ | 1 mark for correct answer in (b) |  |
| $\mathbf{3}$ | 1 mark for correct answer in $(c)$ |  |
| $\mathbf{4}$ | 1 mark for appropriate work in $(d)$ |  |
| $\mathbf{5}$ | 1 mark for correct answer in $(d)$ |  |

## Exemplar 1

## Question No. 12

Total: 5 marks

You have been asked to create a four-character password for your computer using:

- the 26 upper case letters of the alphabet (A, B, C, ...) 26
- the 26 lower case letters of the alphabet $(a, b, c, \ldots) \quad 26$
- the digits from 0 to 9
- the symbols:


$$
+\quad 9
$$

a) How many different four-character passwords are possible if any of the letters, digits, or symbols can be used for each character if repetition is allowed?
(1 mark)

$$
\text { 71. } 71 \cdot \frac{71}{\substack{\text { everything allowed }}}=25411681 \text { passwords possible }
$$

b) How many different four-character passwords are possible if repetition is not allowed?
(1 mark)

$$
7 . \underline{70} \cdot \underline{\substack{\text { no repetition }}}=23319240 \text { passwords }
$$

c) How many four-character passwords begin with a letter and end with a digit if repetition is allowed?
(1 mark)

$$
\frac{52}{[\text { letter }]} \cdot 71 \cdot 71 \cdot \frac{10}{[\#]}=2621320 \text { passwords }
$$

Exemplar 1 (continued)
d) How many different four-character passwords containing at least one symbol are possible if repetition is allowed? Show your work.
(2 marks)

$$
\begin{aligned}
& \text { Case \#1, symbol } \frac{9}{5} \cdot 62 \cdot 62 \cdot 62=2144952 \\
& \text { case \#2, } 2 \text { symbols } \frac{9}{5} \cdot \frac{9}{5} \cdot \frac{62 \cdot 62}{}=311364 \\
& \text { Case \#3 3 symbols } \frac{9}{5} \cdot \frac{9}{5} \cdot \frac{9}{5} \cdot \frac{62}{}=45198 \\
& \text { Case \#4 4 symbols } \frac{9}{5} \cdot \frac{9}{5} \cdot \frac{9}{5} \cdot \frac{9}{5}=\frac{+6561}{2,508,075}
\end{aligned}
$$

$$
2508075 \text { passurds possible. }
$$

Exemplar 2
Question No. 12
You have been asked to create a four-character password for your computer using:

- the 26 upper case letters of the alphabet (A, B, C, ...)
- the 26 lower case letters of the alphabet ( $\mathrm{a}, \mathrm{b}, \mathrm{c}, \ldots$ )
- the digits from 0 to 9
- the symbols: ~! @ \# \$ \% ^ \& *
a) How many different four-character passwords are possible if any of the letters, digits, or symbols can be used for each character if repetition is allowed?
(1 mark)
b) How many different four-character passwords are possible if repetition is not allowed?
(1 mark)

c) How many four-character passwords begin with a letter and end with a digit if repetition is allowed?
(1 mark)

$$
\begin{aligned}
& \frac{52}{10} \times \frac{52}{10 \times 10}=1002=14068 \text { letters } \\
& 9 \times 9=81 \\
& 14068+1000+81=15149
\end{aligned}
$$

Exemplar 2 (continued)
d) How many different four-character passwords containing at least one symbol are possible if repetition is allowed? Show your work.
(2 marks)

$$
\begin{aligned}
& 26 \times 26 \times 26=7034^{+} \\
& 26 \times 26 \times 26=7034^{+}=21629 \\
& 10 \times 10 \times 10=1000^{+}=2 \times 9=6561^{+} \\
& 9 \times 9 \times 9 \times 9=
\end{aligned}
$$

## Financial Mathematics

Brigitte invests $\$ 5000.00$ at an interest rate of $\mathbf{6 \%}$ for 5 years. Circle the compounding period below that would maximize the rate of return on the investment.
$\checkmark$ A) daily
B) monthly
C) quarterly
D) semi-annually

Learning Outcome: 12A.FM. 2
Question Type: Multiple Choice
Question No. 14 and Answer

Circle the asset below which is most likely to depreciate in value.
A) rare coin collection
B) classic car
C) house
$\checkmark$ D) computer

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Mr. Chang is 64 and plans to retire next year. His portfolio includes the following investments:

- $\$ 50000.00$ in mutual funds
- $\$ 100000.00$ in stocks
- $\$ 20000.00$ in guaranteed investment certificates (GICs)
a) Does this portfolio include an appropriate level of risk for Mr. Chang at this stage in his life? Explain your answer.
(1 mark)
No. Given that Mr. Chang is so close to retirement, this portfolio involves too much risk because of the high proportion of stock.

Other answers are possible.

## Question No. 15 continued

b) Mr. Chang's investments had the following returns last year: mutual funds increased by $12.00 \%$, stocks decreased by $\mathbf{4 . 0 0 \%}$, and GICs had an annual interest rate of $\mathbf{3 . 0 0 \%}$. Calculate the average rate of return for the year for this portfolio. Show your work.
(2 marks)

| Type of investment | Principal (\$) | Return (\$) | End of the year (\$) |
| :---: | :---: | :---: | :---: |
| mutual funds | $\mathbf{5 0 0 0 0 . 0 0}$ | 6000.00 | 56000.00 |
| stocks | $\mathbf{1 0 0 0 0 0 . 0 0}$ | -4000.00 | 96000.00 |
| GICs | $\mathbf{2 0 0 0 0 . 0 0}$ | 600.00 | 20600.00 |
| Total: | $=\mathbf{\$ 1 7 0 0 0 0 . 0 0}$ | $=\$ \mathbf{2 6 0 0 . 0 0}$ | $=\mathbf{\$ 1 7 2 \mathbf { 6 0 0 . 0 0 }}$ |


| Average |
| :--- |
| rate of |
| return |$=\frac{(\$ 172600.00-\$ 170000.00)}{\$ 170000.00} \times 100$


$=1.53 \%$$|$| OR |
| :--- |$\quad$| Average |
| :--- |
| rate of |
| return |$=\frac{\$ 2600.00}{\$ 170000.00} \times 100$


| Marking Key |  |  |
| :--- | :--- | :---: |
| (1) | 1 mark for appropriate explanation in (a) |  |
| (2) | 1 mark for correct total return amount or correct total end of the year amount in (b) |  |
| $\mathbf{3}$ | 1 mark for correct answer in (b) consistent with work |  |

Mr. Chang is 64 and plans to retire next year. His portfolio includes the following investments:

- $\$ 50000.00$ in mutual funds
- \$100 000.00 in stocks
- $\$ 20000.00$ in guaranteed investment certificates (GICs)
a) Does this portfolio include an appropriate level of risk for Mr. Chang at this stage in his life? Explain your answer.
(1 mark)
NO, assuming Mr. Chang lives in Canada a wise option would be to invest money in a savings account. If he had entered money into an account that he couldn't spend, the money would accumulate and increase in value, he would have more money than he initially entered. Mutual funds you are able to spend money you putin, "Daily Access".
stocks are a risky investment because you risk the chance of losing money so 1 don't think this was wise especially since he was retiring
Even though the Canadian Pension Plan directly takes money off your cheques (if you choose), assuming Mr. Chang, doesn't have CPP listed in his portfolio I think that wald have been an appropriate investment of money as he grew older.

Exemplar 1 (continued)
b) Mr. Chang's investments had the following returns last year: mutual funds increased by $12.00 \%$, stocks decreased by $4.00 \%$, and GICs had an annual interest rate of $3.00 \%$. Calculate the average rate of return for the year for this portfolio. Show your work.
(2 marks)

| Type of investment | Principal (\$) | Return (\$) | End of the year (\$) |
| :---: | :---: | :---: | :---: |
| mutual funds | 50000.00 | 6000 | $\$ 56000$ |
| stocks | 100000.00 | -4000 | $\$ 96000$ |
| GICs | 20000.00 | 600 | $\$ 20600$ |
| Total: |  | 2600 | $\$ 172600$ |

$$
\begin{aligned}
& \text { MF= } 50000 \times 1.12=56000 \\
& 56000-50000=6000 \\
& =6000 \\
& \begin{aligned}
& S=100000 \times 0.04=4000 \\
& 100000-4000 \\
&= 96000 \\
& * \text { lost money } \\
& \text { GIC'S }= 20000 \times 1.03=20600 \\
& 20600-20000 \\
&= 600
\end{aligned}
\end{aligned}
$$

2 marks:
(1) $\rightarrow 1$ mark for appropriate explanation in (a) (2) $\rightarrow 1$ mark for correct total return amount or correct total end of the year amount in (b)

## Exemplar 2

## Question No. 15

Total: 3 marks

Mr. Chang is 64 and plans to retire next year. His portfolio includes the following investments:

- $\$ 50000.00$ in mutual funds
- \$100 000.00 in stocks
- $\$ 20000.00$ in guaranteed investment certificates (TICs)
a) Does this portfolio include an appropriate level of risk for Mr. Chang at this stage in his life? Explain your answer.
(1 mark)
No, te should have more like maybe, a mortage, a car. He should own mare things of value He is 64, he should have more then 170,000.

Exemplar 2 (continued)
b) Mr. Chang's investments had the following returns last year: mutual funds increased by $12.00 \%$, stocks decreased by $4.00 \%$, and GICs had an annual interest rate of $3.00 \%$. Calculate the average rate of return for the year for this portfolio. Show your work.
(2 marks)

| Type of investment | Principal (\$) | Return (\$) | End of the year (\$) |
| :---: | :---: | :---: | :---: |
| mutual funds | 50000.00 | $+12 \%$ | 56000 |
| stocks | 100000.00 | $-4 \%$ | 96000 |
| GICs | 20000.00 | $+3 \%$ | 20600 |
| Total: |  |  | 172600 |

$$
\begin{aligned}
& \text { mutual funds } \\
& \qquad 5000 \times 1.12 \\
& \text { Average rate of return }=\frac{\text { current }}{P} \\
& =\frac{56000-50000}{50000} \times 100=12 \%
\end{aligned}
$$

$$
\begin{aligned}
& \text { Average rate of return }=\frac{\text { current value -Previous }}{\text { Previous }} \times 100 \\
& =\frac{56000-50000}{50000} \times 100=12 \%
\end{aligned}
$$

Stocks
$100000 \times 0.04$

$$
\begin{gathered}
100000=96000 \\
10000-4000=
\end{gathered}
$$

$$
\frac{96000-100000}{100000} \times 100=-4 \%
$$

GITs $20000 \times 1.03=20600$

$$
20000
$$

1 mark:
(2) $\rightarrow 1$ mark for correct total return amount or correct total end of the year amount in (b)

Amar dreams of retiring at the age of 55. He had planned on starting to save for his retirement at the age of 50 , but his financial advisor does not agree. He recommends that Amar starts to save sooner.
a) If Amar invests $\mathbf{\$ 1 0 0 0 . 0 0}$ on his 25th birthday and contributes $\mathbf{\$ 2 0 0 . 0 0}$ every month to an account that earns $8.00 \%$ compounded monthly, what will be the value of the investment on his 55th birthday? Show your work.
(2 marks)


The value of the investment will be $\$ 309007.62$.
b) If Amar invests $\mathbf{\$ 1 0 0 0 . 0 0}$ on his 50th birthday, how much will he have to contribute every month to match the final value of the investment in (a)? Assume that the interest rate and the compounding periods are the same.
(1 mark)


Amar will have to contribute $\$ 4185.23$ every month.

## Question No. 16 continued

c) Calculate the difference between Amar's total contribution in (a) and in (b)? Show your work.
(2 marks)
Total contribution in (a): $(360)(\$ 200.00)+\$ 1000.00=\$ 73000.00$
Total contribution in $(\mathrm{b}):(60)(\$ 4185.23)+\$ 1000.00=\$ 252113.80$

$$
\$ 252113.80-\$ 73000.00=\$ 179113.80
$$

## Marker Note(s):

$\rightarrow$ A maximum of 1 error is allowed in the input values of a financial template in (a) and (b) (award the mark for appropriate work, but not the mark for correct answer).

| Marking Key |  |
| :--- | :--- |
| $\mathbf{1}$ | 1 mark for appropriate work in (a) |
| $\mathbf{2}$ | 1 mark for correct answer in (a) |
| $\mathbf{3}$ | 1 mark for correct answer in (b) consistent with work in (a) |
| $\mathbf{4}$ | 1 mark for appropriate work in (c) |
| $\boldsymbol{5}$ | 1 mark for correct answer in (c) consistent with work in (a) and (b) |

## Exemplar 1

Question No. 16
Total: 5 marks

Amar dreams of retiring at the age of 55 . He had planned on starting to save for his retirement at the age of 50, but his financial advisor does not agree. He recommends that Amar starts to save sooner.
a) If Amar invests $\$ 1000.00$ on his 25 th birthday and contributes $\$ 200.00$ every month to an account that earns $8.00 \%$ compounded monthly, what will be the value of the investment on his 55th birthday? Show your work.
(2 marks)

$$
\begin{aligned}
& N=360 \\
& I=8 \\
& P=1000 \\
& P M t=-200 \\
& P V=?=\$ 287136.16 \\
& P / y=12 \\
& C / y=12
\end{aligned}
$$

b) If Amer invests $\$ 1000.00$ on his 50th birthday, how much will he have to contribute every month to match the final value of the investment in (a)? Assume that the interest rate and the compounding periods are the same.
(1 mark)

$$
\begin{aligned}
& N=60 \\
& I=8 \\
& P=1000 \\
& P n t=?=-\$ 3928.12 \\
& F V=287136.16 \\
& P / y=12 \\
& C / y=12
\end{aligned}
$$

Exemplar 1 (continued)
c) Calculate the difference between Amar's total contribution in (a) and in (b)? Show your work.
(2 marks)

$$
\text { a) } 360 x-200 \equiv \$ 72000
$$

$$
\text { b) }-3928.12 \times 60=\$-235687.2
$$



$$
235687.2
$$

$$
-72000
$$



Option b earns less in interest and has to pay the difference of $-\$ 163687.2$ in monthly payments.

4 marks:
(1) $\rightarrow 1$ mark for appropriate work in (a)
(3) $\rightarrow 1$ mark for correct answer in (b) consistent with work in (a)

4 $\rightarrow 1$ mark for appropriate work in (c)
$\boldsymbol{\Theta} \rightarrow 1$ mark for correct answer in (c) consistent with work in (a) and (b)
(EA) $\rightarrow 0.5$ mark deduction (if applicable) for not stating or incorrectly stating the final answer

## Exemplar 2

## Question No. 16

Total: 5 marks

Amar dreams of retiring at the age of 55. He had planned on starting to save for his retirement at the age of 50, but his financial advisor does not agree. He recommends that Amar starts to save sooner.
a) If Amar invests $\$ 1000.00$ on his 25 th birthday and contributes $\$ 200.00$ every month to an account that earns $8.00 \%$ compounded monthly, what will be the value of the investment on his 55th birthday? Show your work.
(2 marks)

$$
\begin{aligned}
& N=360 \\
& I \%=8 \\
& P v=-1000 \\
& P_{m t}=-200 \\
& P / y=12 \\
& C / y=12
\end{aligned}
$$


b) If Amer invests $\$ 1000.00$ on his 50th birthday, how much will he have to contribute every month to match the final value of the investment in (a)? Assume that the interest rate and the compounding periods are the same.
(1 mark)

$$
\begin{aligned}
& N=300 \\
& I \%=8 \\
& P=-1000 \\
& F v=709007.62 \quad P_{m}+=.317 .20 \\
& P / Y=12 \\
& C H=12
\end{aligned}
$$

## Exemplar 2 (continued)

c) Calculate the difference between Amar's total contribution in (a) and in (b)? Show your work.
(2 marks)

$$
\begin{array}{ll}
a)=73000 & 96160 \\
6)=96160 & -73000 \\
& 23160
\end{array}
$$

4 marks:
(1) $\rightarrow 1$ mark for appropriate work in (a)
(2) $\rightarrow 1$ mark for correct answer in (a)
(4) $\rightarrow 1$ mark for appropriate work in (c)
$\boldsymbol{5} \rightarrow 1$ mark for correct answer in (c) consistent with work in (a) and (b)
(EA) $\rightarrow 0.5$ mark deduction (if applicable) for not stating or incorrectly stating the final answer

The Reimers have purchased a house valued at $\$ 250000.00$ and have made a down payment of $\$ 25 \mathbf{0 0 0 . 0 0}$.
a) Calculate their monthly mortgage payment if they obtain a mortgage amortized over 15 years at an interest rate of $5.50 \%$ compounded semi-annually. Show your work.
(2 marks)


Their monthly mortgage payment is $\$ 1831.05$.

## Question No. 17 continued

b) How much equity will the Reimers have in their house after 5 years if the value of the house appreciates at a rate of $\mathbf{2 . 0 0 \%}$ per year? Show your work.
(3 marks)

$$
\$ 250000.00 \times 1.02^{5}=\$ 276020.20
$$

```
    bal(5*12)
```

    169197.5283
    equity $=$ appreciated value of the house - balance owing
$=\$ 276020.20-\$ 169197.53$
$=\$ 106822.67$
OR

$$
\begin{aligned}
\text { appreciation } & =\left(\$ 250000.00 \times 1.02^{5}\right)-\$ 250000.00 \\
& =\$ 26020.20
\end{aligned}
$$

equity $=$ down payment $+\sum \operatorname{Prn}(1,60)+$ appreciation
$=\$ 25000.00+\$ 55802.47+\$ 26020.20$
$=\mathbf{\$ 1 0 6 ~} 822.67$

## Marker Note(s):

$\rightarrow$ A maximum of 1 error is allowed in the input values of a financial template in (a) and (b) (award the mark for appropriate work, but not the mark for correct answer).

| Marking Key |  |
| :--- | :--- |
| $\mathbf{1}$ | 1 mark for appropriate work in (a) |
| $\mathbf{2}$ | 1 mark for correct answer in (a) |
| $\mathbf{3}$ | 1 mark for correct calculation of appreciation in (b) |
| $\mathbf{4}$ | 1 mark for correct calculation of balance owing or sum of the principal paid in (b) |
| $\mathbf{3}$ | consistent with answer in (a) |
| 1 mark for correct equity in (b) consistent with work |  |

## Exemplar 1

## Question No. 17

The Reimers have purchased a house valued at $\$ 250000.00$ and have made a down payment of \$25000.00.
a) Calculate their monthly mortgage payment if they obtain a mortgage amortized over 15 years at an interest rate of $5.50 \%$ compounded semi-annually. Show your work.
(2 marks)

$$
\begin{aligned}
N & =180 \\
I \% & =5.9 \\
P V & =-250000 \\
A P M T & =2034.50 \\
F V & =0 \\
P / y & =12 \\
C / y & =2
\end{aligned}
$$

Exemplar 1 (continued)
b) How much equity will the Reimers have in their house after 5 years if the value of the house appreciates at a rate of $2.00 \%$ per year? Show your work.
(3 marks)

$$
\begin{aligned}
& 25000 \times 1.02=255000 \times 1.02=260100 \times 1.02=2650100 \times 1.02 \\
& =27.0608 .04 \times 1.02=276020.20 \\
& 180 \times 2034.50=366390 \\
& 276020.2-366390 \\
& \quad-90369.8
\end{aligned}
$$

(®2) $\rightarrow 0.5$ mark deduction (if applicable) for not including the units in the final answer
(Es) $\rightarrow 0.5$ mark deduction (if applicable) for rounding too soon or rounding incorrectly

Exemplar 2
Question No. 17

The Reimers have purchased a house valued at $\$ 250000.00$ and have made a down payment of $\$ 25$ 000.00.

$$
250,000-25000=225,000
$$

a) Calculate their monthly mortgage payment if they obtain a mortgage amortized over 15 years at an interest rate of $5.50 \%$ compounded semi-annually. Show your work.
(2 marks)

$$
\begin{aligned}
n & =180 \\
I & =5.5 \\
P V & =225,000 \\
P M T & =? \text { Alp silver } \\
F V & =0 \\
P / Y & =12 \\
C / Y & =2
\end{aligned}
$$

Exemplar 2 (continued)
b) How much equity will the Reimers have in their house after 5 years if the value of the house appreciates at a rate of $2.00 \%$ per year? Show your work.

$$
\begin{aligned}
& (3 \text { marks } \\
& n=60 \\
& I=5 \\
& P V=225,000 \\
& P M T=-1831.05 \\
& F V=? \text { Alpha Solve }-169197.52 \\
& P / y=12 \\
& C l y=2
\end{aligned}
$$

(1) $\rightarrow 1$ mark for appropriate work in (a)
(2) $\rightarrow 1$ mark for correct answer in (a)
(4) $\rightarrow 1$ mark for correct calculation of balance owing or sum of the principal paid in (b) consistent with answer in (a) $\boldsymbol{\Theta} \rightarrow 1$ mark for correct equity in (b) consistent with work
(55) $\rightarrow 0.5$ mark deduction (if applicable) for rounding too soon or rounding incorrectly

## Design and Measurement

A cake mix will produce 230 cubic inches of batter. You are using cylinder-shaped baking cups that have a diameter of 3 inches and a depth of 2 inches for the batter. How many cupcakes will you be able to make? Show your work.

$$
\begin{aligned}
& \text { radius }=1.5 \mathrm{in} \text {. } \\
& V=\pi r^{2} h \\
& =\pi(1.5 \mathrm{in} .)^{2}(2 \mathrm{in} .) \\
& =14.14 \mathrm{in}^{3} \text { for each cupcake } \\
& \frac{230 \mathrm{in}^{3}}{14.14 \mathrm{in}^{3} / \text { cupcake }}=16.27 \text { cupcakes } \\
& \text { I will be able to make } 16 \text { cupcakes. }
\end{aligned}
$$

Accept the following answers: $16,16.27$, or 17 .

|  |  | Marking Key |
| :--- | :--- | :--- |
| $\mathbf{1}$ | 1 mark for appropriate work |  |
| $\mathbf{2}$ | 1 mark for correct answer |  |

Question No. 18

A cake mix will produce 230 cubic inches of batter. You are using cylinder-shaped baking cups that have a diameter of 3 inches and a depth of 2 inches for the batter. How many cupcakes will you be able to make? Show your work.

$$
r=3 / 2=1.5
$$

$$
\begin{aligned}
& V_{\text {сир саке }}=2 \pi r h+2 \pi r^{2} \\
&=(2 \pi(1.5)(2))+\left(2 n(1.5)^{2}\right) \\
&=32.99 \mathrm{in}^{3} \\
& 230 / 32.99=6.97
\end{aligned}
$$

You would be able to make approximately 7 cupcakes.

Exemplar 2
Question No. 18
A cake mix will produce 230 cubic inches of batter. You are using cylinder-shaped baking cups that have a diameter of 3 inches and a depth of 2 inches for the batter. How many cupcakes will you be able to make? Show your work.
 $230 \mathrm{in}^{3}$ of batter
$\pi r^{2} \times h=\pi 1.5^{2} \times 2=14.14^{3} \downarrow$ to $14^{3}$ to mate full cupcakes.

$$
\frac{230^{3}}{14^{x}}=16.43 \rightarrow 16 \text { full cop cafes }
$$

you will be able to make 16 cupcakes.
(1) $\rightarrow 1$ mark for appropriate work
(2) $\rightarrow 1$ mark for correct answer (Es) $\rightarrow 0.5$ mark deduction (if applicable) for rounding too soon or rounding incorrectly

A goat is tied to the corner of a barn with a 50 -foot rope. The barn measures
60 feet by 40 feet. Calculate the total area outside of the barn that is available to the goat. Show your work.


$$
\begin{aligned}
\text { Total area } & =\frac{3 \pi(50 \mathrm{ft} .)^{2}}{4}+\frac{\pi(10 \mathrm{ft} .)^{2}}{4} \\
& =5890.49+78.54 \\
& =5969.03 \mathrm{ft}^{2}
\end{aligned}
$$

Marker Note(s):
$\rightarrow$ Deduct a maximum of 1 mark if student calculates only one of the two areas correctly.

|  |  | Marking Key |
| :--- | :--- | :--- |
| $\mathbf{1}$ | 1 mark for appropriate work |  |
| $\mathbf{2}$ | 1 mark for correct answer |  |

## Exemplar 1

A goat is tied to the corner of barm with a 50-footrepe. The barn measures 60 feet by 40 feet. Calculate the total aresoutside of the barn that is available the goat. Show your work.


$$
\text { 1/4 of circle }=\frac{1963.495404}{\times 3}
$$

$$
3 / 4 \text { of circle } 5890.49 \text { square } f t \text {. }
$$

1 mark:
(1) $\rightarrow 1$ mark for appropriate work

## Exemplar 2

A goat is tied to the corner of a barn with a 50 -foot rope. The barn measures 60 feet by 40 feet. Calculate the total area outside of the barn that is available to the goat. Show your work.

$7853.98-1000-50=6803.98 \mathrm{ft}^{2}=$ Total area available

The Manitoba Beach Volleyball Association has asked you to design a souvenir beach ball according to the following information:

- The beach ball must have a volume between 1 and 3 cubic feet.
- The plastic material costs $\$ 0.15$ per $\mathrm{ft}^{2}$.
- Labour and other materials cost $\$ 1.25$ per beach ball.
- The Association wants to make a profit of $80 \%$ of the cost of making each beach ball. Based on your design, what is the minimum selling price for each souvenir beach ball? Show your work.

$$
\begin{aligned}
V & =\mathbf{1} \mathbf{f t}^{3}=\frac{4}{3} \pi r^{3} \\
r & =0.62 \mathrm{ft} .
\end{aligned}
$$

Surface area $=4 \pi r^{2}$

$$
\begin{aligned}
& =4 \pi(0.62 \mathrm{ft} .)^{2} \\
& =4.84 \mathrm{ft}^{2}
\end{aligned}
$$

OR

| $V=\mathbf{2} \mathbf{f t}^{\mathbf{3}}=\frac{4}{3} \pi r^{3}$ <br> $r=0.78 \mathrm{ft}$. | Cost of <br> production |
| :--- | :--- |
| $=7.65(\$ 0.15)+\$ 1.25$ |  |
| Surface area $=4 \pi(0.78 \mathrm{ft} .)^{2}$ |  |
|  | $=7.65 \mathrm{ft}^{2}$ | | Profit | $=\$ 2.40 \times 0.80$ |
| ---: | :--- |
|  | $=\$ 1.92$ |

OR
$V=\mathbf{3} \mathbf{f t}^{\mathbf{3}}=\frac{4}{3} \pi r^{3}$
$r=0.89 \mathrm{ft}$.
$\begin{aligned} \text { Surface area } & =4 \pi(0.89 \mathrm{ft} .)^{2} \\ & =10.06 \mathrm{ft}^{2}\end{aligned} \begin{aligned} \text { Profit } & =\$ 2.76 \times 0.80 \\ & =\$ 2.21\end{aligned}$

Other answers are possible.

## Marker Note(s):

$\rightarrow$ The answer may vary as a result of rounding.

| Marking Key |  |  |
| :--- | :--- | :---: |
| $\mathbf{1}$ | 1 mark for correct radius |  |
| $\mathbf{2}$ | 1 mark for correct surface area consistent with radius |  |
| $\mathbf{3}$ | 1 mark for correct cost of production consistent with surface area |  |
| $\mathbf{4}$ | 1 mark for correct selling price consistent with surface area |  |

Exemplar 1
Question No. 20
The Manitoba Beach Volleyball Association has asked you to design a souvenir beach ball according to the following information:

- The beach ball must have a volume between $(1)$ and 3 cubic feet.
- The plastic material costs $\$ 0.15$ per $\mathrm{ft}^{2}$.
- Labour and other materials cost $\$ 1.25$ per beach ball.
- The Association wants to make a profit of $80 \%$ of the cost of making each beach ball.

Based on your design, what is the minimum selling price for each souvenir beach ball? Show your work.


4 marks:
(1) $\rightarrow 1$ mark for correct radius
(2) $\rightarrow 1$ mark for correct surface area consistent with radius
(3) $\rightarrow 1$ mark for correct cost of production consistent with surface area (4) $\rightarrow 1$ mark for correct selling price consistent with surface area (E9) $\rightarrow 0.5$ mark deduction (if applicable) for not stating or incorrectly stating the final answer

Exemplar 2
Question No. 20
The Manitoba Beach Volleyball Association has asked you to design a souvenir beach ball according to the following information:

- The beach ball must have a volume between 1 and 3 cubic feet.
- The plastic material costs $\$ 0.15$ per $\mathrm{ft}^{2}$.
- Labour and other materials cost $\$ 1.25$ per beach ball.
- The Association wants to make a profit of $80 \%$ of the cost of making each beach ball.

Based on your design, what is the minimum selling price for each souvenir beach ball? Show your work.


$$
\begin{aligned}
& r=0.75 \\
& \text { volume }=\frac{4}{3} \pi 0.75^{2}=2.36 \mathrm{ft}
\end{aligned}
$$

$$
S . A=4 \pi 0.75^{2}=7.06 \ldots f t^{2}
$$

cost for plastic $=0.15 \times 7.06=\$ 1.06$ Total cost for manufacturing ball $=\$ 1.06+\$ 1.25=\$ 2.31$
$2.31=20 \%$ of the total cost
represents $\quad 2.31 \times 5=\$ 11.55$
$20 \%$ ob
final cost $\$ 11.55 \times 0.8=\$ 9.24$

$$
\$ 2.31+\$ 9.24=11.55
$$

The organization will have to charge
$\$ 11.55$ for each ball.
3 marks:
(1) $\rightarrow 1$ mark for correct radius
(2) $\rightarrow 1$ mark for correct surface area consistent with radius
(3) $\rightarrow 1$ mark for correct cost of production consistent with surface area

## LOGICAL REASONING

Learning Outcome: 12A.L. 3

Question Type: Multiple Choice
Question No. 21 and Answer
Total: 1 mark

Given the statement: "If the temperature outside is below $-40^{\circ} \mathrm{C}$, then schools will be closed." Circle the contrapositive below.
A) "If schools are closed, then the temperature outside is below $-40^{\circ} \mathrm{C}$."
$\checkmark$ B) "If schools are not closed, then the temperature outside is not below $-40^{\circ} \mathrm{C}$."
C) "If the temperature outside is not below $-40^{\circ} \mathrm{C}$, then schools will not be closed."
D) "Schools will be closed if and only if the temperature outside is below $-40^{\circ} \mathrm{C}$."

Common Errors
A: converse
C: inverse
D: biconditional

A sample of 100 families was surveyed regarding the electronic devices they have in their homes. The Venn diagram below shows the number of families that have a computer, a DVD player, or a Blu-Ray player.

a) How many families have all three electronic devices in their homes? (1 mark)
b) How many families do not have any of these electronic devices in their homes?
(1 mark)

$$
100-(14+65+5+3+5+4+2)=2
$$

|  |  |
| :--- | :--- |
| Marking Key |  |
| $\mathbf{1}$ | 1 mark for correct answer in (a) |
| $\mathbf{2}$ | 1 mark for correct answer in (b) |

Marc wrote the statement: "An isosceles triangle is equilateral."
a) Rewrite the statement in "if-then" form.
"If a triangle is isosceles, then it is equilateral."
b) Provide a counter-example to show that the "if-then" statement in (a) is false. (1 mark)

See the diagram below for a counter-example.


Other answers are possible.

## Marking Key

| 1 |
| :--- |
| $(2)$ |

1 mark for correct "if-then" statement in (a)
1 mark for appropriate counter-example in (b)

## Exemplar 1

## Question No. 23

Marc wrote the statement: "An isosceles triangle is equilateral."
a) Rewrite the statement in "if-then" form.
(1 mark)
If it is an isosceles triangle, then it is equilateral
b) Provide a counter-example to show that the "if-then" statement in (a) is false.
(1 mark)
If it is equilateral it doesn't always have to be an isosceles triangle

Exemplar 2
Question No. 23

Marc wrote the statement: "An isosceles triangle is equilateral."
a) Rewrite the statement in "if-then" form.
(1 mark)
If a triangle is equilateral, then it must be $a_{n}$ isoseceles triangle
b) Provide a counter-example to show that the "if-then" statement in (a) is false.
(1 mark)
If a triangle is not equilateral, then it is not an isosceles triangle.

0 marks:
$\rightarrow$ no criteria met

The following students attend the same school and participate in the extracurricular activities as indicated below.

The basketball team consists of:

$$
B=\{\text { Jacquie, Lisa, Mangu, Maya, Nora, Sabrina }\}
$$

The student tutoring group consists of:

$$
T=\{\text { Jacquie, Mangu, Paul, Sabrina, Sam, Simon }\}
$$

The volleyball team consists of:

$$
V=\{\text { Nick, Paul, Pieter, Quinton, Sam, Simon }\}
$$

a) Identify the two sets from above that are disjoint.

The basketball and volleyball teams are disjoint.
b) Determine $B \cap T$.
(1 mark)

$$
\{\text { Jacquie, Mangu, Sabrina }\}
$$

|  |  |
| :--- | :--- |
| $\mathbf{1}$ | Marking Key |
| $\mathbf{2}$ | 1 mark for correct answer in (a) |

## Exemplar 1

## Question No. 24

Total: 2 marks
The following students attend the same school and participate in the extracurricular activities as indicated below.

The basketball team consists of:

$$
B=\{\text { Lace, Lisa, Manet, Maya, Nora,-Sabrinat }\}
$$

The student tutoring group consists of:

$$
T=\{\text { Laurie, -Mangur, Paul, Sabrina; } \text {, Sam, Simon }\}
$$

The volleyball team consists of:

$$
V=\{\text { Nick, Paul, Dieter, Quinton, Sam, Simon }\}
$$

a) Identify the two sets from above that are disjoint.
(1 mark)

## The basket ball team and basket ball team are disjointed.

b) Determine $B \cap T$.
(1 mark)

$$
\begin{aligned}
& \text { Jacque, Mango, and Sabrina are in } \\
& \text { both basket ball and tutoring. }
\end{aligned}
$$



## Exemplar 2

## Question No. 24

Total: 2 marks

The following students attend the same school and participate in the extracurricular activities as indicated below.

The basketball team consists of:
H $B=\{$ Jacque, Lisa, Mangu, Maya, Nora, Sabrina $\}$
The student tutoring group consists of:
$B T=\{$ Jacque, Mangu, Paul, Sabrina, Sam, Simon\} ~
The volleyball team consists of:
C $V=\{$ Nick, Paul, Peter, Quinton, Sam, Simon $\}$
a) Identify the two sets from above that are disjoint.
(1 mark)

$$
\Lambda(A \cap C)=0
$$

Basketball and volleyball are disjoint.
b) Determine $B \cap T$.
(1 mark)


Bare in both.
(1) $\rightarrow 1$ mark for correct answer in (a)

## Appendices

## Appendix A:

Table of Questions by Unit and Learning Outcome

| Unit | Question | Type | Learning Outcome | Mark |
| :---: | :---: | :---: | :---: | :---: |
| A | 1 | MC | 12A.R. 1 | 1 |
| A | 2 | SA | 12A.R. 2 | 2 |
| A | 3 | SA | 12A.R. 1 | 2 |
| A | 4 | LA | 12A.R. 1 | 3 |
| A | 5 | LA | 12A.R. 3 | 3 |
| A | 6 | LA | 12A.R. 2 | 5 |
| Total $=16$ |  |  |  |  |
| B | 7 | MC | 12A.P. 4 | 1 |
| B | 8 | MC | 12A.P. 1 | 1 |
| B | 9 | SA | 12A.P. 1 | 1 |
| B | 10 | SA | 12A.P. 2 | 2 |
| B | 11 | LA | 12A.P. 6 | 4 |
| B | 12 | LA | 12A.P.4, 12A.P. 5 | 5 |
| Total $=14$ |  |  |  |  |
| C | 13 | MC | 12A.FM. 1 | 1 |
| C | 14 | MC | 12A.FM. 2 | 1 |
| C | 15 | LA | 12A.FM. 3 | 3 |
| C | 16 | LA | 12A.FM.1, 12A.FM. 3 | 5 |
| C | 17 | LA | 12A.FM.1, 12A.FM. 2 | 5 |
| Total $=15$ |  |  |  |  |
| D | 18 | SA | 12A.D. 1 | 2 |
| D | 19 | SA | 12A.D. 1 | 2 |
| D | 20 | LA | 12A.D. 1 | 4 |
| Total $=8$ |  |  |  |  |
| E | 21 | MC | 12A.L. 3 | 1 |
| E | 22 | SA | 12A.L. 2 | 2 |
| E | 23 | SA | 12A.L. 3 | 2 |
| E | 24 | SA | 12A.L. 2 | 2 |
| Total $=7$ |  |  |  |  |

## Legend for Units:

A: Relations and Functions
B: Probability
C: Financial Mathematics
D: Design and Measurement
E: Logical Reasoning

## Legend for Question Types:

MC: Multiple Choice
SA: Short Answer
LA: Long Answer

## Appendix B: <br> Table of Questions by Type and Learning Outcome

| Type | Question | Unit | Learning Outcome | Mark |
| :---: | :---: | :---: | :---: | :---: |
| MC | 1 | A | 12A.R. 1 | 1 |
| MC | 7 | B | 12A.P. 4 | 1 |
| MC | 8 | B | 12A.P. 1 | 1 |
| MC | 13 | C | 12A.FM. 1 | 1 |
| MC | 14 | C | 12A.FM. 2 | 1 |
| MC | 21 | E | 12A.L. 3 | 1 |
| Total $=6$ |  |  |  |  |
| SA | 2 | A | 12A.R. 2 | 2 |
| SA | 3 | A | 12A.R. 1 | 2 |
| SA | 9 | B | 12A.P. 1 | 1 |
| SA | 10 | B | 12A.P. 2 | 2 |
| SA | 18 | D | 12A.D. 1 | 2 |
| SA | 19 | D | 12A.D. 1 | 2 |
| SA | 22 | E | 12A.L. 2 | 2 |
| SA | 23 | E | 12A.L. 3 | 2 |
| SA | 24 | E | 12A.L. 2 | 2 |
| Total $=17$ |  |  |  |  |
| LA | 4 | A | 12A.R. 1 | 3 |
| LA | 5 | A | 12A.R. 3 | 3 |
| LA | 6 | A | 12A.R. 2 | 5 |
| LA | 11 | B | 12A.P. 6 | 4 |
| LA | 12 | B | 12A.P.4, 12A.P. 5 | 5 |
| LA | 15 | C | 12A.FM. 3 | 3 |
| LA | 16 | C | 12A.FM.1, 12A.FM. 3 | 5 |
| LA | 17 | C | 12A.FM.1, 12A.FM. 2 | 5 |
| LA | 20 | D | 12A.D. 1 | 4 |
| Total $=37$ |  |  |  |  |

```
Legend for Question Types:
MC: Multiple Choice
SA: Short Answer
LA: Long Answer
```


## Legend for Units:

```
A: Relations and Functions
B: Probability
C: Financial Mathematics
D: Design and Measurement
E: Logical Reasoning
```


## Appendix C: <br> Irregularities in Provincial Tests

## A Guide for Local Marking

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an Irregular Test Booklet Report should be completed and sent to the Department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all "NR") or only incorrect responses ("0")

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the Department is made aware that follow-up has taken place by completing an Irregular Test Booklet Report.

Except in the case of cheating or plagiarism where the result is a provincial test mark of $0 \%$, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an Irregular Test Booklet Report documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the Department along with the test materials.

## Irregular Test Booklet Report

Test: $\qquad$
Date marked:

Booklet No.: $\qquad$

Problem(s) noted: $\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Question(s) affected: $\qquad$
$\qquad$
$\qquad$

Action taken or rationale for assigning marks: $\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Follow-up: $\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Decision: $\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Marker's Signature: $\qquad$

Principal's Signature: $\qquad$

For Department Use Only—After Marking Complete
Consultant:
$\longrightarrow$
Date: $\qquad$

