INTRODUCTION

This document will provide teachers with practical strategies for classroom-based assessment.

Today, teachers face many demands in the classroom that include implementing curriculum and ensuring that students successfully master prescribed learning outcomes. Having a plan for instruction and assessment is essential. Planning needs to take place both in and out of the classroom. The school—both students and staff—can work together as a community of learners to accomplish this. Communication among teachers of various grades is also important to facilitate discussion about grade-level outcomes, student progress related to those outcomes, and strategies for the continued success of all students. Without this communication, teachers are repeatedly starting over with each new class of students. School-based planning helps teachers to see the big picture: the role of mathematics in the whole school and in the community. Using classroom-based assessments and other teaching/assessment tools that are available, teachers are able to see general strengths and weaknesses of both individual students and the program.

Curriculum changes, increased or changing student needs, research related to how students learn, and a rapidly changing society are just some of the challenges teachers face today. Teachers cannot do everything themselves. There has never been a greater need for cooperation and collaboration among teachers of all grade levels than there is now in the 21st century.

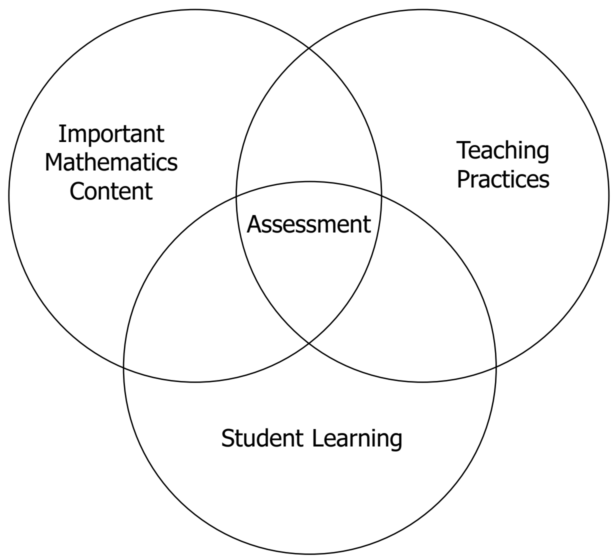
Cooperative planning allows teachers to share the work. The sharing of ideas and assessment tasks can be very beneficial. It is important that students have the opportunity to experience assessment tasks from a variety of sources. Although students become comfortable with the presentation format used by their classroom teachers, they need to be exposed to presentation formats used by other teachers. All students do not respond in the same way to the same lesson; exposing them to various presentation formats takes this into consideration.

ASSESSMENT TO ASSIST LEARNING AND INFORM INSTRUCTION

Research on Assessment

Mathematics Classroom Assessment

Assessment is a "systematic process of gathering information about what a student knows, is able to do, and is learning to do" (Manitoba Education and Training, *Reporting on Student Progress and Achievement* 5). Assessment is an integral part of instruction that enhances, empowers, and celebrates student learning. "Assessment occurs at the intersection of important mathematics content, teaching practice, and student learning" (National Council of Teachers of Mathematics, *Assessment Standards for School Mathematics* 6).



Meaningful Assessment

One purpose of meaningful assessment is to inform instruction by providing information about student learning. This information can then be used to provide direction for planning further instruction. Assessment should occur in authentic contexts that allow students to demonstrate learning by performing meaningful tasks. Decisions about a student's ability in mathematics should be made after evidence is gathered from a wide variety of sources.

Meaningful content and contexts for assessment help students by engaging their attention and encouraging them to share their work and talk about their progress. Students need to take an active part in assessment. When students understand assessment criteria and procedures, and take ownership for assessing the quality, quantity, and processes of their own work, they develop self-assessment skills. The ultimate goal of assessment is to help students develop into independent, lifelong learners who regularly monitor and assess their own progress.

The Teacher's Role in Assessment

In the classroom, teachers are the primary assessors of students. Teachers design assessment tools with two broad purposes: to collect information that will inform classroom instruction and to monitor students' progress toward achieving year-end mathematics learning outcomes. Teachers also assist students in developing self-monitoring and self-assessment skills and strategies. To do this effectively, teachers must ensure that students are involved in setting learning goals, developing action plans, and using assessment processes to monitor their achievement of goals. Teachers also create opportunities to celebrate students' progress and successes.

Teachers learn about student learning and progress by regularly and systematically observing students in action, and by interacting with students during instruction. Because students' knowledge and many of their skills, strategies, and attitudes are internal processes, teachers gather data and make judgments based on observing and assessing students' interactions, performances, and products or work samples. Teachers demonstrate that assessment is an essential part of learning. They model effective assessment strategies and include students in the development of assessment procedures, such as creating rubrics or checklists.

Assessment Purposes and Audiences

The quality of assessment largely determines the quality of evaluation. Evaluation is "the process of making judgments and decisions based on the interpretation of evidence gathered through assessment" (Manitoba Education and Training, *Reporting on Student Progress and Achievement* 39). Valid judgments can be made only if accurate and complete assessment data are collected in a variety of contexts over time. Managing assessment that serves a multitude of purposes and audiences is a challenging task. Teachers must continually balance the assessment of their students' progress in the development of skills, strategies, and attitudes, with the purposes and audiences for the information collected.

Important Mathematics Content

Learning outcomes and standards assist classroom teachers and other educators to

- plan learning activities that support student achievement
- establish goals for learning, instruction, and assessment
- monitor student progress in achieving learning outcomes and standards
- communicate with students, parents, and guardians about student progress
- develop a mathematics plan for a school

Teaching Practices

Developing an effective mathematics program is a dynamic process. The program is shaped by the teaching style and resources of each teacher, by the interests and abilities of the students, and by the needs of the community. An effective mathematics program contains a balance between many factors.

Planning a balanced mathematics program should take into account that

- specific learning outcomes stated are year-end learning outcomes—students may achieve the learning outcomes at any time during the year
- students need to practice in many meaningful contexts to consolidate new knowledge, skills, and strategies because learning is recursive and cumulative
- planning is continual, informed by ongoing classroom assessment
- a variety of instructional approaches, classroom management techniques, assessment practices, tools and strategies, and problem-solving activities are essential

Areas to consider when planning a balanced program may include

- learning outcomes for each course
- processes of mathematics: communicating, estimating and using mental mathematics, making connections, organizing and structure, problem solving, reasoning, using technology, and visualizing
- standards and student performance for Senior 1 and Senior 4
- student grouping patterns: individual, pairs, small groups, large groups, whole class, heterogeneous, homogeneous, student-directed, teacher-directed
- various learning styles and multiple intelligences
- various rates of student learning, addressed by providing pre-teaching, review, additional practice for some students, and challenging extension activities for others
- student diversity: students representing certain cultures, languages, and faiths may view, organize, or classify the world differently from those who do not share their background.

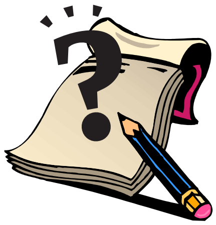
Planning an effective mathematics program is a challenging task. Looking at how the outcomes are related across grade levels is helpful. This information can be used to create instructional activities and to help define areas in which initial teaching and practice are necessary and those in which practice and maintenance through problem-solving activities are the main emphases.

In every planning decision, reflective teachers need to ask:

- What is an appropriate balance for my students?
- Am I achieving the balance in my classroom, both in the short term and the long term?
- Is my instruction helping students achieve the appropriate learning outcomes of my mathematics program?

When planning to achieve learning outcomes, it is important to read each specific learning outcome carefully and clarify the types of learning that are expected of the student. This will provide guidance for developing appropriate instructional activities to help students meet the expectations of the mathematics curriculum.

Before teachers choose assessment tasks, tools, and strategies, it is critical to define the reasons for assessing, the audience who will use the information, and the ways the results will be used. Appropriate assessment activities must focus on tasks that allow students to demonstrate their competence in applying learning in authentic ways.



Questions to Guide Planning

1. What do we want students to know and be able to do?

   This question can be answered by reading the learning outcome to determine knowledge, skills, and strategies that it includes.

2. What do students already know?

   This question can be answered by having students work through activities that demonstrate their prior knowledge related to particular outcomes. When planning a particular unit, teachers may want to consider the following questions: “How do I access prior knowledge? How do I build on that knowledge to teach the outcomes of the unit? What other units or outcomes in this course are related or connected? How do I build on those connections?”

3. How can activities be connected to students’ real-life experience?

   This question can be answered by drawing from teachers’ experiences, professional resources, and classroom discussion.

4. What instructional methods, materials, and strategies will help students develop these competencies?

   These will be drawn from teachers’ experiences, professional resources, or instructional strategies provided in the Foundation for Implementation documents for each course in Senior 1 to Senior 4 Mathematics. When planning a particular unit, teachers may want to consider the following questions: “What activities will the students engage in? Which of the mathematical processes will the activities highlight? Are there appropriate mental math questions that students should be able to do? Are there appropriate journal questions for students to answer? What technology or other materials are required for this unit?”

5. What is the purpose of assessment? How will the assessment be used?

   The purpose of assessment could be to monitor students’ progress, to make instructional decisions, to evaluate student achievement, or to evaluate programs. Each of these purposes has a distinct audience. A single assessment activity may have more than one purpose. When planning a particular unit, teachers may want to consider the following question: “What items will I assess?”

6. What assessment tasks will allow students to demonstrate their understanding in authentic ways?

   Assessment tasks will be drawn from teachers' experiences, professional resources, research on multiple intelligences, educational research, or suggestions provided in the Foundation for Implementation documents for each course in Senior 1 to Senior 4 Mathematics. When planning a particular unit, teachers may want to consider the following specific questions: "What could be placed in a student's portfolio to demonstrate that the learning outcome has been achieved? Will a paper-and-pencil test be a reasonable final assessment of this unit or should I use a performance assessment or a project for the final assessment of this unit?"

Planning with the End in Mind

To plan effectively, teachers need to have a clear sense of the purpose of instruction. The primary intent of mathematics education is that students attain the mathematics learning outcomes identified and mandated for each course. The learning outcomes are not a checklist of what a teacher will teach. They are a description of the knowledge, skills, strategies, and attitudes that students are expected to demonstrate.

Placing the focus on student learning in this way means that all instructional planning must begin with students' present levels of performance. Teachers cannot decide on the first priorities for instruction until they know what knowledge, skills, strategies, and attitudes students bring into class. Teachers are better able to make appropriate choices of topics, learning resources, groupings, and instructional strategies when they know students as individuals. For this reason, teachers need to gather information about students from a variety of sources.

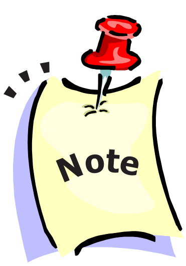
Formative Assessment

Formative assessment is data collected about a whole group and/or individual students during classroom instruction.

Formative assessment is designed to guide instruction and improve student learning. This is done by

- identifying specific learning needs
- providing feedback describing students' performance

The instruments used in formative assessment provide information or data that teachers, parents, and students may use to identify factors that facilitate student learning. The immediacy of feedback and the opportunity to practice are critical for student learning.



"The thrust of formative assessment is toward improving learning and instruction. Therefore, the information should not be used for assigning marks, as the assessment often occurs before students have had full opportunities to learn content or develop skills" (Manitoba Education and Training, *Reporting on Student Progress and Achievement* 9–10).

Summative Assessment

Summative assessment occurs after students have had an opportunity to practice their learning independently. It often occurs during reporting times and is used to describe individual progress related to meeting the learning outcomes.

Tools and Strategies

Possible assessment tools and strategies that may be used for assessment include

- observations recorded on checklists or in teacher notes
- diagnostic interviews
- group/peer assessments
- self-assessment
- paper-and-pencil tasks
- student journal/learning log entries

Each of these strategies is suitable for either formative or summative assessment. The purpose of the assessment will determine whether the tool is a formative tool or a summative tool.

Self-Assessment

In order to help students become lifelong learners who are independent, teachers need to allow students to regularly monitor and self-assess their work. Helping students understand assessment criteria and procedures will allow students to take ownership for assessing the quality, quantity, and processes of their own work. Taking an active part in assessment will help develop self-assessment skills.

A SENIOR 3 TEACHER'S PLANNING AND ASSESSMENT PROCESS

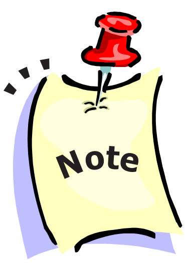
To provide specific examples of how to implement a planning and assessment process, Senior 3 options will be used. This allows us a chance to look back at Senior 1 and Senior 2 and ahead to Senior 4 as we plan the mathematics programming.

Choice Model

One way for students to take responsibility for their learning is to have them choose how they will be evaluated. At the beginning of a course (or unit), students could be presented with two evaluation models.

For example:

Option A		Option B	
Mental Math	5%	Mental Math	5%
Journals	5%	Journals	5%
Assignments	5%	Assignments	15%
Portfolio	15%	Portfolio	25%
Tests	40%	Tests	20%
Exam	30%	Exam	30%



Certain aspects of the options are the same for both. For example, the weighting of the final examination may not be a decision the teacher has.

A student who chooses Option A is most likely a student who does well on tests but does not always complete assignments. A student who chooses Option B is most likely a student who works hard on assignments but has trouble writing tests. There may be other reasons why students make the choices they do. To make the selection, students complete a form requesting their option, provide reasons for their choice, and have a parent or guardian sign the form. In this way, parents or guardians are informed of the possible marking schemes used by the teacher. Once a student makes the choice, he or she must live with it until the teacher offers the choice again (at the beginning of a new unit or a new marking period or whenever the teacher chooses). The criteria for the choices should be clearly spelled out to students and parents before the decision date is reached.

Note that all students are expected to try all assignments and activities in the classroom. The value of each of the activities will be different for each student.

The options for the students could involve different categories:

Option A		Option B	
Mental Math	5%	Mental Math	5%
Journals	5%	Journals	5%
Assignments	10%	Assignments	10%
Portfolio	10%	Portfolio	10%
Tests	40%	Project	20%
Exam	30%	Tests	20%
		Exam	30%

In this scenario, the student choosing Option B would be required to complete a project on his or her own time. The student and the teacher would agree upon the topic of the project. The criteria for the project would be presented with the outline of the options for marking.

For example:

Senior 3 Pre-Calculus Mathematics

Teacher: Mrs. B
School: Any School Manitoba
In order to calculate the grade for each student, the following options are available:

Option A:		**Option B:**	
Mental Math	5%	Mental Math	5%
Journal	5%	Journal	5%
Assignments	10%	Assignments	5%
Tests	40%	Tests	25%
Portfolio	10%	Portfolio	10%
Exam	30%	Project	20%
		Exam	30%

Students are to select one of the above choices and fill in the form below. This form must be returned to the teacher by Sept. 10, 2002. The student's mark on the first report card will be calculated using the method that the student chooses. After the first report card, students will have a chance to choose again.

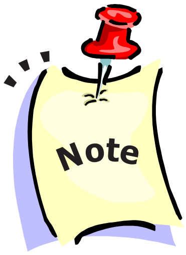
See attached sheet for a description of each of the categories above.
Student Name:
I choose Option A B (Circle one)
My reasons for choosing this option are:

Student Signature: Date:
Parent Signature:

Senior Years Curricula

The following chart outlines the Manitoba curricula for Senior Years Mathematics.

Senior 1	Senior 2	Senior 3	Senior 4
Senior 1 Mathematics (10F)	Applied Mathematics (20S)	Applied Mathematics (30S)	Applied Mathematics (40S)
Senior 1 Transitional Mathematics (10F) (optional course)	Consumer Mathematics (20S)	Consumer Mathematics (30S)	Consumer Mathematics (40S)
	Pre-Calculus Mathematics (20S)	Pre-Calculus Mathematics (30S)	Pre-Calculus Mathematics (40S)



Accounting 30S and Accounting 40S can be used to fulfill the Mathematics requirement at the Senior 3 and Senior 4 level.

The outcomes for Senior 1 Mathematics (10F), Applied Mathematics (20S, 30S, 40S) and Pre-Calculus Mathematics (20S, 30S, 40S) are adapted from the *Common Curriculum Framework for K–12 Mathematics, Grades 10 to 12.*

When planning for instruction and assessment, Senior Years mathematics teachers have the Foundation for Implementation documents to guide their planning process. Teachers should investigate how the unit they are planning for fits in with prior knowledge and with future expectations. A unit has been selected from each of the Senior 3 Mathematics courses to illustrate this planning process.

Senior 3 Applied Mathematics

Unit: Precision Measurement

General Learning Outcome: Use measuring devices to make estimates and to perform calculations in solving problems

Specific Learning Outcomes:

G-1: Enlarge or reduce the diagram of a dimensioned object

G-2: Calculate maximum and minimum values using tolerances for lengths, areas, and volumes

G-3: Solve problems involving percentage error.

Previous Knowledge: In Senior 2 Applied Mathematics, students studied how to use measurement instruments such as vernier calipers and micrometers. As a part of this study, students were to select the appropriate measurement device and solve problems involving measurement. As well, students needed to consider the concepts of precision and accuracy.

Future Expectations: In Senior 4 Applied Mathematics, students will be expected to solve problems involving the design and measurement of various objects, shapes, or processes. Budgeting constraints will be important in most of the design schemes. Students will also be expected to create scale diagrams in the vectors unit. Outside of school, students are likely to be faced with situations where they must make measurements as part of a decorating or construction project. They need to understand the importance of accuracy in these projects.

Planning for Assessment and Instruction:

How do I access prior knowledge? How do I build on that knowledge to teach the outcomes of the unit?

Present students with the following problem:

You have been asked to determine the amount of money in loonies that a coffee cup can hold. Work with a partner to answer this problem.

Students can work in pairs. The teacher can observe students working and ask questions like the following:

- *What information do you need to answer the question?*
- *What materials would be useful for this problem?*

By recording answers on an observation chart, teachers can assess how much the students know about measurement. This problem can lead to a discussion about the need for accuracy and precision. A discussion of the usefulness of scale diagrams could also take place.

> What other units or outcomes in this course are related or connected? How do I build on those connections?

This unit is unrelated to the other units in the course.

> What activities will students engage in? Which of the mathematical processes will the activities highlight? Are there appropriate mental math questions that students should be able to do? Are there appropriate journal questions for students to answer?

One of the activities students will engage in is the drawing of objects to scale. This activity will highlight the process of visualization.

Students should be able to answer mental math questions like the following:

- *If a drawing has a scale of 1 cm : 2 m and one part of the drawing is 3 cm long, how long is the actual piece?*
- *If the dimension of an object is 3 cm ± 1 mm, what is the maximum size of the object?*

Students should be able to respond to journal questions like the following:

- *If the scale on a map is 1 cm : 300 km, explain to a friend how to find the actual distance between two places.*
- *You have a drawing that you would like to reduce. What kind of scale should you use? Explain why.*
- *Explain to a friend who missed class how to find the maximum area of a rectangular object if you are given dimensions with tolerances.*

What items will I assess?

- mental math questions
- journal entries
- scale drawings
- problems related to percentage error
- the final project for the unit

What could be placed in a student’s portfolio to show learning has occurred?

- an example of a diagram that has been reduced or enlarged
- sample mental math questions that the student has created related to the unit
- a problem that was solved
- the final project for the unit

Will a paper-and-pencil test be a reasonable final assessment for this unit or should I use a performance assessment or a project for the final assessment of this unit?

Since this unit stresses performance rather than a final product, a performance assessment or a project would be more suitable than a paper-and-pencil test.

What technology or other materials are required for this unit?

- measuring devices (calipers, micrometers, rulers, protractors)
- grid paper
- computer drawing program

Senior 3 Consumer Mathematics

Unit: Data Analysis and Interpretation

General Learning Outcome: Analyze data with a focus on the validity of the presentation and the inferences made.

Specific Learning Outcomes:

E-1: Display and analyze data on a line plot

E-2: Use measures of central tendency to support decisions

E-3: Manipulate the presentation of data to represent a point of view

Prior Knowledge: In Senior 2 Consumer Mathematics, students looked at various sampling techniques to generate data.

Future Expectations: In Senior 4 Consumer Mathematics, students will examine measures of variability like the standard deviation to help analyze data. After graduation, students will be faced with many sources of data and asked to form opinions and make decisions based on the data presented.

Planning for Assessment and Instruction:

How do I access prior knowledge? How do I build on that knowledge to teach the outcomes of the unit?

Present the class with the following problem:

A school principal is concerned about the types of activities students engage in during their spare time and the effect the activity has on school performance. Your group has been asked to make a presentation to the parent council about one of the following problems:

- *part-time jobs*
- *the Internet*
- *video games*
- *hunting*
- *sports activities*
- *other ______________________ (please specify).*

Divide the class into groups of three or four. Ask the groups to brainstorm for ideas to start this project. Have students think about what data need to be collected, how the data will be collected, and how the data will be analyzed. Have each group report their thoughts to the class. Use this activity as a unit project to return to as you learn new ways to analyze data.

What other units or outcomes in this course are related or connected? How do I build on those connections?

The analysis of Games and Numbers unit is connected to this unit. Students should be aware of the use of statistics in the media. One way to increase their awareness might be to collect examples of statistics used in the media and create a worksheet based on the information collected. Students could be asked to bring in examples from newspapers and magazines. A class discussion around the samples would help to make the connections.

What activities will students engage in? Which of the mathematical processes will the activities highlight? Are there appropriate mental math questions that students should be able to do? Are there appropriate journal questions for students to answer?

Students could answer mental math questions like the following:

- *Find the mode of the following set of data: 2, 2, 3, 4, 5, 5, 6, 6, 7, 7, 7, 8*
- *Find the mean of the following set of data: 2, 3, 4, 5, 6, 7, 8*
- *Scott arranged the data in numerical order and then used the middle number as a measure of central tendency. Which measure of central tendency is he using?*

Students could respond to journal questions such as:

- *Compare and Contrast the three measures of central tendency. (State both differences and similarities.)*
- *How can statistics be used to misrepresent the truth?*

What items will I assess?

- mental math questions
- journal entries
- group work while working on the project
- the project and the oral presentations

What could be placed in a student's portfolio to show learning has occurred?

- a copy of a newspaper or magazine article that uses statistics to present a case, along with a critique by the student of the possible problems with the article
- data found on the internet or in some other source displayed on a line plot
- a compare and contrast frame with the three measures of central tendency compared
- the final project from the group on the presentation to the school board
- math dictionary entries

Will a paper-and-pencil test be a reasonable final assessment for this unit or should I use a performance assessment or a project for the final assessment of this unit?

The project introduced at the beginning of the unit with the presentation to the school board could be used as the final assessment. Different groups could be asked to take different viewpoints and present the findings of the group to the class. Self, peer, and teacher assessment could be components of the mark for the presentation.

What technology or other materials are required for this unit?

- Internet access for statistics
- spreadsheets
- magazines
- newspapers

Senior 3 Pre-Calculus Mathematics

Unit: Functions

General Learning Outcomes: Examine the nature of relations with an emphasis on functions.

Represent and analyze situations that involve expressions, equations, and inequalities.

Represent and analyze quadratic, polynomial, and rational functions using technology as appropriate.

Specific Learning Outcomes:

H-1: perform operations on functions and compositions of functions

H-2: determine the inverse of a function

H-3: use the remainder theorem to evaluate polynomial expressions and the factor theorem to determine factors of polynomials.

H-4: describe, graph, and analyze polynomials and rational functions, using technology

Prior Knowledge: In Senior 2 Pre-Calculus Mathematics, students were introduced to functions and function notation. They graphed linear functions and discussed the properties of linear functions.

Future Expectations: In Senior 4 Pre-Calculus, students will be expected to analyze and perform transformations on functions and relations including inverse transformations and rational transformations. They will study trigonometric functions, exponential functions, logarithmic functions, and conic sections. In Calculus, students will study functions in greater depth using the first and second derivatives to help analyze and graph functions.

Planning for Assessment and Instruction:

How do I access prior knowledge? How do I build on that knowledge to teach the outcomes of the unit?

Present students with a word cycle with the following words:

- function
- relation
- domain
- range
- zeros
- *x*-intercept
- *y*-intercept
- linear function

Have students complete the word cycle individually and then share the results with a partner. After the teacher assesses the word cycles, various cycles can be shared with the class.

What other units or outcomes in this course are related or connected? How do I build on those connections?

The outcomes on quadratic functions, absolute functions, radical functions, and inequalities relate to this unit. By selecting examples for this unit from those specific functions, connections can be made. Compare and Contrast frames can be used as well. Students could create a word web based on the word “function”.

What activities will the students engage in? Which of the mathematical processes will the activities highlight? Are there appropriate mental math questions that students should be able to do? Are there appropriate journal questions for students to answer?

Students should be able to answer the following mental math questions:

- State the inverse of $f(x) = \{(2,3), (-4,5)\}$.
- Is $(x-1)$ a factor of $f(x) = 5x^3 + x^2 - 8x - 2$?
- What is the remainder when $f(x) = 27x^4 - 27x^3 + x^2 - x + 8$ is divided by $(x-1)$?
- What are the zeros of $f(x) = (x-1)(x-4)(x-5)(x+3)$?

Students should be able to answer the following journal questions:

- *Explain to a friend who missed class how to find the inverse of a function.*
- *Write an explanation to solve the following problem. (You do not have to solve the problem—only explain how to solve it.)*
  Graph $f(x) = x^4 - 5x^3 - 13x^2 + 53x + 60$. State all intercepts.
- *What are the advantages and disadvantages of synthetic division?*
- *How are the remainder theorem and the factor theorem similar? How are they different?*
- *Create a question that could be used on a test for this unit. Provide the solution and a possible marking scheme.*

What items will I assess?

- mental math questions
- journal entries
- word web
- word cycle
- compare and contrast frame
- quiz
- final unit test

What could be placed in a student's portfolio to show learning has occurred?

- journal entries
- quizzes
- mental math questions that the student has created
- original questions created by the student
- final unit test

Will a paper-and-pencil test be a reasonable final assessment for this unit or should I use a performance assessment or a project for the final assessment of this unit?

A paper-and-pencil test would be a reasonable final assessment of this unit. If cumulative testing is done, several tests may have the same concepts.

What technology or other materials are required for this unit?

While not necessary, a graphing calculator could be useful to check graphs of functions. Other graphing software could also be used.

SUMMATIVE ASSESSMENT

Summative assessment is usually conducted at the end of a block of instruction and is designed to determine to what extent students have attained student learning outcomes. It is primarily used for assigning marks (Manitoba Education and Training, *Reporting on Student Progress and Achievement* 10).

Summative assessment may take many forms (for example, paper-and-pencil tasks, interviews, portfolios, journal entries, open-ended questions, performance tasks, projects, and so on).

Portfolios

Portfolio assessment allows a teacher to gather evidence from a wide variety of sources to get a more complete picture of a student's progress in mathematics and a more complete assessment of the extent to which a student has achieved the outcomes of the course.

A portfolio is a purposeful collection of work. Assessing students using a portfolio can allow a teacher the opportunity to assess a wide variety of mathematical skills and attitudes. By collecting, reflecting, and selecting work, students share in the assessment process.

The purpose of the portfolio could be to show growth over a period of time or to provide evidence that a student has met the outcomes of a unit or course. Before beginning work with portfolios, teachers need to decide what the purpose of the portfolio will be. Specific parameters for the portfolio should be established and communicated to students before the collection of materials for the portfolio begins.

Once students have collected sufficient pieces of evidence, they need to sort through the collection and select the most appropriate pieces of work. As they select pieces, they should reflect on the piece and why it makes sense to include it in the portfolio.

Teachers who have not used portfolios in their mathematics classroom should begin with a smaller project and build on the successes and learn from the failures.

For example, a Senior 1 Mathematics teacher may choose the Statistics unit in *Senior 1 Mathematics (10F)* to start a portfolio. At the beginning of the unit, the teacher would present the student with the purpose of the portfolio, the criteria for success, and some indication of the process to be used in gathering evidence.

Purpose: To demonstrate that you have achieved the outcomes of this unit, you will

- assess the strengths, weaknesses, and biases of samples and data collection methods
- critique ways in which statistical information and conclusions are presented by the media and other sources
- create scatterplots for discrete and continuous variables
- interpret a scatterplot to determine if there is an apparent relationship

- determine the line of best fit from a scatterplot for an apparent linear relationship, by
  — inspection
  — using technology (equations are not expected)
- draw and justify conclusions from the line of best fit
- design, conduct, and report on an experiment to investigate a relationship between two variables

Criteria: Your completed portfolio will have between 5 and 10 pieces of work. There should be no more than two of the same kind of work included (for example, no more than 2 quizzes or 2 journal entries). The entire portfolio should provide evidence that you have achieved the outcomes listed above. You should include work that would show growth over the course of the unit.

Examples of work that could be included:

- journal entries
- mental math questions
- reviews of newspaper articles
- critiques of television broadcasts
- scatterplots
- assignments
- quizzes
- projects
- reports
- any other item that illustrates knowledge and understanding

For every piece of work in the portfolio, you should attach a brief note that tells why the piece was included, what outcome(s) it provides evidence of, and which of the mathematical processes was used for the item (see attached chart). When you have finished selecting the items to be included, you should organize the portfolio and complete a table of contents.You should include a statement of introduction for your portfolio that explains how you organized your portfolio and why you organized it that way, what your best piece of work is and why you feel it is your best work, and the personal value of completing a portfolio.

The portfolio will be due one week after the project for Statistics is handed in. The project should be included as one of the pieces of work.

Suggested Marking Rubric for Portfolio Assessment

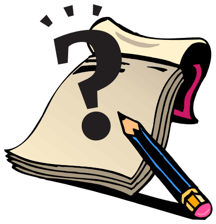
Senior 1 Mathematics (10F) Statistics Portfolio

	Low			High
Statement of Introduction (3 marks)				
1. Did student answer all three questions? (organization, best work, value of portfolio)	❑	❑	❑	❑
2. Reponse to questions (content)	❑	❑	❑	❑
3. Overall quality of letter (presentation and form)	❑	❑	❑	❑
Table of Contents (2 marks)				
1. Identification of processes: Communication, Reasoning, Technology, Problem Solving, Organization & Structure, Connections, Estimation/Mental Math, Visualization	❑	❑	❑	❑
2. Coverage of Outcomes	❑	❑	❑	❑
Notes on Items (5 marks)				
1. Reason Piece was Included (x3)	❑	❑	❑	❑
2. Identification of Outcomes	❑	❑	❑	❑
3. Identification of Processes	❑	❑	❑	❑
Overall Impression (5 marks)				
1. Selection of Items (5–10 items, variety of items)	❑	❑	❑	❑
2. Format and Presentation (x3) (organization, neatness, clarity, questions with answers)	❑	❑	❑	❑
3. Mathematical Content (skills, concepts, problem solving, corrections)	❑	❑	❑	❑
Other: ______________________ ______________________ (bonus for above and beyond requirements)	❑	❑	❑	❑

Journals

A writing journal allows a student an opportunity to communicate about mathematics. Regular journal entries will allow the teacher to find out what the student knows, what misconceptions a student may have, and what the student does not know. “Enhanced student comprehension is one definite advantage of journal writing” (Holens 20).

One way to incorporate journals into the mathematics classroom is to have a notebook that remains in the classroom for each student. If journals remain in the classroom, the teacher can read them at any time.



Sample types of journal questions that involve content might include the following:

Describe two different ways that you could find 35% of 120.

Explain to your friend who missed class why the following law is true: $(x^a)^b = x^{ab}$. Try to explain it in two different ways.

Describe the real number system to a younger student.

For each of your games of NAB, record who went first and who won. Were there any strategies that you used to win the game? If so, explain any strategies that you found.

Explain how factoring is like dividing. How is it different?

Why is budgeting important?

Why do some people say that credit cards are dangerous?

Discuss the similarities and differences among zeros of a function, x-intercepts, and roots of an equation.

State two other ways to represent the direction N30°W. Illustrate your solutions with a diagram.

How are percent and percentile rank similar? How are they different?

Describe the process used to change a degree measure to a radian measure.

After a test, students could answer the following type of question:

Out of 25, I think I got ___________, because ...

This type of reflective question could help students become aware of their strengths and weaknesses in a particular unit. The answers to this question may also reveal students' attitudes toward a particular unit.

Before a test, students could answer a question such as the following:

In this unit, I found ______________ to be the easiest topic and __________ to be the hardest topic. Before the test, I need to…

A journal can also be a place for students to make connections between the classroom and the world outside the classroom:

What did you do this weekend that involved using math?

When using journals, it is important for students to feel safe to write what they know and don't know. It is in the attempts and trials that students grow in their mathematical understanding. When assessing journals, the prime factor for determining a student's mark should be **effort**.

Open-Ended Questions

Open-ended questions can be categorized into three groups:

- Open-process
- Open-ended product
- Open-problem formulation

Open-process questions are questions that can be solved in more than one way. This type of open-ended question is appropriate for most topics in mathematics. Having students share their solutions on the board so other students can see alternative solution strategies is useful in showing students that mathematics is much more than one correct answer to every question and one correct way to get to the answer.

When students are working in groups, they could be asked to come up with two different ways to solve a question. In groups, there are more ideas to share and it may be easier to come up with more than one solution. After students have had practice in thinking of different ways to solve problems, students could come up with different solutions on their own.

Example:

Solve: $3^{2x} = 2^{x+1}$

Solution 1: (Algebraically)

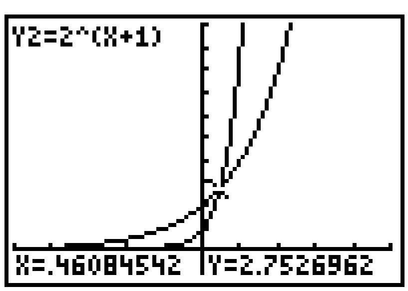
$$\log 3^{2x} = \log 2^{x+1}$$
$$(2x)\log 3 = (x+1)\log 2$$
$$2x\log 3 = x\log 2 + \log 2$$
$$2x\log 3 - x\log 2 = \log 2$$
$$x = \frac{\log 2}{2\log 3 - \log 2}$$

$x = 0.4608$

Solution 2: (Graphically, using the TI-83 graphing calculator)

Let $y_1 = 3^{2x}$ and $y_2 = 2^{x+1}$

Graph and find the point of intersection using CALC, intersect



Solution 3: (Changing to the same base)

Since $3 \approx 2^{1.585}$, change the equation to:

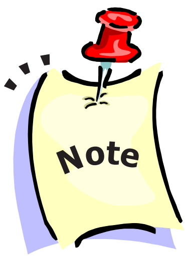
$$\left(2^{1.585}\right)^{2x} = 2^{x+1}$$

$$2^{3.17x} = 2^{x+1}$$

$$3.17x = x + 1$$

$$2.17x = 1$$

$$x = 0.4608$$



Note: Variations on each of the three solutions listed above are possible.

Open-ended product questions are questions that have multiple correct answers. This is the type of question most people think of when they hear “open-ended.” In mathematics, there are a few places where questions may have more than one correct answer.

Example:

1. *Give an example of a number that is an integer but is not a whole number.* (Any number that is negative would be a correct answer to this question.)

2.

Senior 4 Consumer Mathematics Assigned Project

The purpose of the assigned project is to give you an opportunity to collect and analyze data. Frequently, analysis of data requires a situation in which to frame conclusions. The project is not meant to have one correct answer/conclusion or one approach to its solution. It is intended that you **set up situations, collect data,** and **draw reasonable conclusions** based on your data and circumstances in the real world.

The format of your work is critical. You should provide

- An **introduction** that presents the problem, pertinent definitions, and any special circumstances that will have an impact on the decision(s) reached
- A **body** of the project that presents the data collected and the arguments used to support all claims
- A **conclusion** that follows logically from the data
- A **bibliography** that identifies all sources

By collecting and anayzing data, answer the following question:

What is the best non-alcoholic beverage?

Define the terms. Use multiple situations. Use data to justify your answer. Use mathematics to support your conclusion.

Each project will be scored on the following:

- Introduction
- Data Presentation
- Data Analysis
- Presentation and Organization

You may work on this project in a group of no more than 3 students.

3. *A sinusoidal function has a maximum at (5, 2) and the next minimum at (7, –4). Find an equation for this function. State the amplitude, period, phase shift, and vertical shift. Graph the function.*

Solution:

Amplitude: $\dfrac{2-(-4)}{2}=3$

Vertical Shift: $2-3=-1$ or $-4+3=-1$

Period: $2(7-5)=4$

$\therefore b=\dfrac{2\pi}{4}$ or $\dfrac{\pi}{2}$

Horizontal Shift:

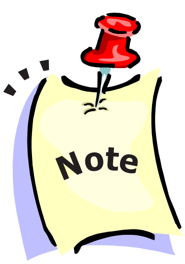
For cosine and a = 3:	c = 5 or 1 or …
For cosine and a = –3:	c = 7 or 3 or …
For sine and a = 3:	c = 8 or 4 or …
For sine and a = –3:	c = 6 or 2 or …

$\therefore$ equation is $y=3\cos\dfrac{\pi}{2}(x-5)-1$ **or**

$$y=-3\cos\frac{\pi}{2}(x-7)-1$$ **or**

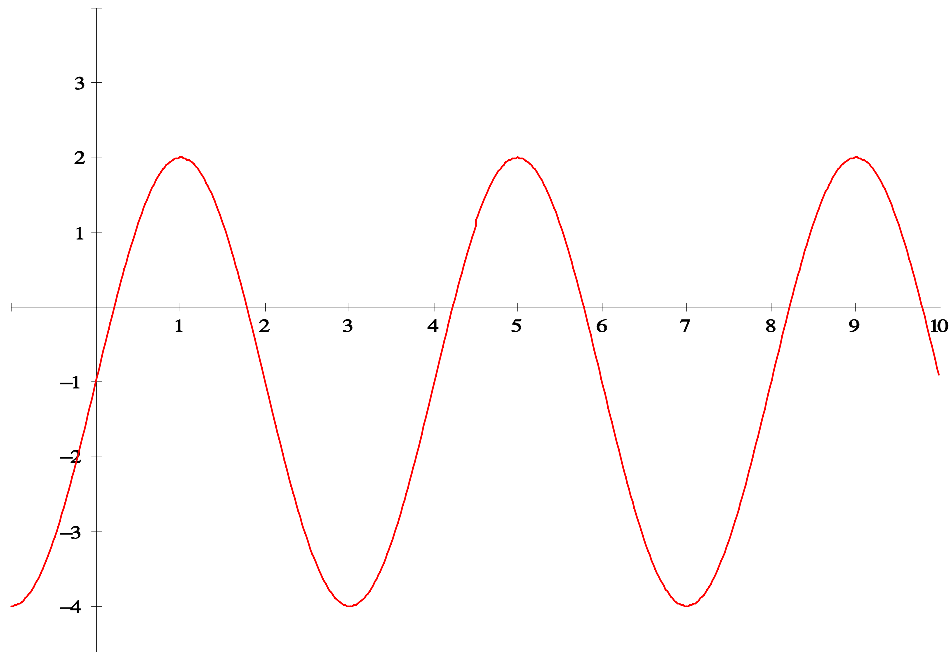
$$y=3\sin\left(\frac{\pi}{2}x\right)-1$$ **or**

$$y=-3\sin\left(\frac{\pi}{2}(x+2)\right)-1$$



Note: There are many other possible equations.

Graph:



Performance Tasks

Performance tasks are tasks designed to provide students with an opportunity to apply mathematical concepts, skills, and processes. While students are engaged in a task, the teacher observes students individually, gathering information about the ways in which they assess personal performance, apply mathematical concepts to the task, solve problems, and make connections with other knowledge and skills.

Structured interviews used along with performance task(s) provide teachers with a great deal of information about what a student is able to do. A structured interview usually begins and ends with easy questions. At the beginning, an easy question will put the student at ease, and at the end it allows the student to finish with a sense of accomplishment.

Interviews require specific classroom management. While working with one student, the rest of the class should be working independently. Consider having students work on learning activities or assignments in small groups so that group members can field some of the questions normally dealt with by the teacher. Activities chosen should be ones in which students have had previous experience—not ones that require new teaching.

In some situations, it may be possible to interview two or three students at a time. Two students could sit back-to-back and the teacher could observe both students as they performed the tasks. The questions could be directed to each student in turn. Three students could be observed if the interview mainly required a hands-on demonstration of skills or concepts.

Example: *Senior 2 Consumer Mathematics (20S)*—Spatial Geometry

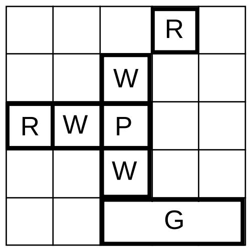
While students are working with the cuisenaire rods, observe how they manipulate the rods and ask questions about the work they are doing.

Example:

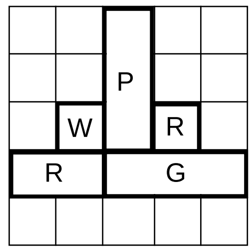
Using the top, front, and side views, construct the 3-D model of cuisenaire rods and sketch your model on the isometric paper provided.

The following model uses 1 purple, 2 green, 1 red, and 2 white rods.

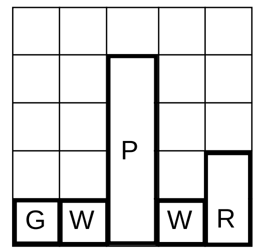
Top View



Front View



Side View



Structured Interview Questions:

1. What is the problem asking you to do? What materials do you require to solve the problem?
2. What have you tried? What steps did you take? What did not work?
3. What is the relationship between the top and front views? What is the relationship between the front and side views? What is the relationship between the top and side views? What do the views tell you?
4. Have you tried making a guess? What did that tell you?
5. How would you explain what you know right now?
6. Is that the only possible answer? How did you know when you were finished?

When asking questions of students, it is important to have some questions prepared ahead of time. Allow the students plenty of time to respond thoughtfully. Make a written record of your observations. A checklist may or may not be relevant to the situation. When using the above in your classroom, you may want to have several similar tasks for students to work on so that the first person interviewed and observed does not have the same model for the task.

Projects

The term *project* can refer to many kinds of assignments. For this document, the term will be used to describe a piece of work that a student creates over an extended period of time. Projects may be assigned in different ways; students may research a project topic of their choosing, or they research one topic that has been assigned to the entire class.

Rubrics

When creating rubrics or checklists, the following table of common expressions used to describe levels of performance may prove useful.

Common Expressions Used to Describe Levels of Performance*

Dimension	Level 1	Level 2	Level 3
Assistance	considerable assistance	moderate assistance	independent
Breadth	narrow	moderate	broad
Clarity	confusing	some confusion	clear
Completeness	incomplete	some omissions	thorough
Complexity	simple	moderately complex	complex
Consistency	inconsistent	some inconsistencies	consistent
Dependence	dependent on others	partial dependence	independent
Depth	superficial	moderate depth	substantial depth
Development	undeveloped	developing	developed
Difficulty	easy	moderately difficult	difficult
Effectiveness	ineffective	moderately effective	effective
Error	major flaws	some errors	minor errors
Experience	inexperienced	limited experience	experienced
Focus	unfocused	divided attention	focused
Frequency	rarely	sometimes	frequently
Innovation	copy	some difference	creative
Organization	disorganized	partially organized	organized
Skill	limited skill	some skill	skilled
Success	unsuccessful	limited success	successful
Systemization	random	some evidence of a plan	systematic
Understanding	no/limited understanding	some understanding	understanding

*Adapted from Gary Flewelling and William Higginson, *A Handbook on Rich Learning Tasks: Realizing a Vision of Tomorrow's Mathematics Classroom* (Kingston, ON: Centre for Mathematics, Science and Technology Education, Queen's University, 2000) 50. Adapted by permission.

ASSESSING MATHEMATICAL PROCESSES

Communicating

Students should be able to communicate, both orally and in written form, their mathematical understanding of a problem. Students should be able to use their own language to explain and clarify in such a way so that others can understand.

Students should

- use mathematical language and concepts
- explain reasoning
- report evidence
- state a conclusion
- draw and label
- reflect on what they are learning

Journals can be used to help students communicate. Opportunities for classroom discourse should be encouraged. When students work in groups, they can be working on communication skills.

Estimating and Using Mental Mathematics

Mental math consists of a collection of strategies that enable a person to estimate, visualize, and manipulate numbers in their heads. Mental math strategies allow students to apply their knowledge of basic facts to compute problems that involve larger numbers.

There are specific methods and procedures that can be taught and practised the same way pencil-and-paper algorithms are taught and practised. Mental math should begin when students enter school and continue through to the end of high school.

In assessing mental math strategies, teachers should look for both oral and written evidence. For example, when asked how to find 25% of 40, a student could answer "I divided by 4 since 25% is one-quarter of the whole." The student could also have answered, "I found 10% of 40, then half of that to find 5%, and added 20% (2 x 10%) plus the 5% to get 10." Another student may say "I found half and then half again." Students should be able to describe more than one way to solve a question.

Once mental math strategies have been taught, they must be practised. This can be done through daily mental math time that is time restricted to ensure mental math strategies are being used.

Making Connections

Students should move beyond understanding a mathematical concept to identifying how mathematical concepts are related to one another, to other subject areas, and to everyday life. A teacher- or student-led discussion may explore concepts relative to mathematics such as

- measurement in industrial arts
- ratio in social studies
- integers in banking

- transformations in art
- collecting and interpreting data and estimating and recognizing patterns in science

Students also need to make connections between the concrete, pictorial, symbolic, oral, and written representations of a concept.

Example:

Represent the quantity in the centre box
in as many ways as possible.

$$\boxed{\frac{1}{2}}$$

Students should be able to transfer and apply their mathematical skills in different situations and contexts. By representing a concept in many different ways, students demonstrate understanding of that concept.

Concept Frame

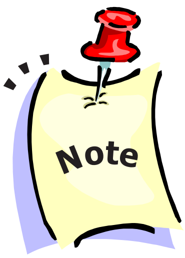
<table>
<tr><td>Definition</td><td>Key Facts</td></tr>
<tr><td colspan="2">FUNCTION</td></tr>
<tr><td>Make a drawing or a diagram</td><td>Non-examples</td></tr>
<tr><td>Examples</td><td>How is __________ like __________?</td></tr>
</table>

Organizing and Structure

It is important that students are able to organize and structure mathematical information. Order and structure can give meaning to information that appears chaotic or random. Students may organize and apply structure based on their background knowledge or they may develop structure to extend beyond their own experiences in order to provide meaning to a context. Organization and structure allow students to develop connections and patterns in mathematics. Conversely, the perception of connections and patterns in mathematics may allow students to develop skills in organizing and applying structure.

Problem Solving

Non-routine problem solving is the process in which students apply their understanding of mathematical concepts and skills. This process involves both mathematical investigations and open problems.



Routine problems are generally problems in which the way to a solution is immediately evident. These problems are often "operation questions" presented in word form.

Teachers need to look at four main areas in problem solving and to assess students' progress within each area.

1. **Understanding a problem:** Before students can begin to solve a problem they must be able to interpret its meaning. Teachers can assess students' understanding of a problem by having them do one of the following:
   - rephrase the question in their own words
   - identify the question and draw a diagram
   - highlight the relevant information in one colour and the question in another
   - sort sentence strips from a problem into three groups: needed information, extra information, and the question
2. **Using appropriate strategies:** Students need opportunities to evaluate the effectiveness of using different strategies to address the same problem. In this manner, they are able to move beyond one or two favoured strategies. Teachers can assess the use of strategies by having students explain how they solved the problem. A checklist can be used to assess the explanation and the answer. It may include the following criteria:
   - the chosen strategy is appropriate for the question
   - the strategy is applied correctly
   - the answer is correct
   - the explanation is clear
   - the student states strategy used
3. **Verifying solutions:** Students should self-assess by checking their work or by using another strategy.
4. **Formulating their own problems:** After students have had practice interpreting problems and working with various strategies, they should be able to develop problems of their own. Teachers can assess the problems by using a rubric that is developed together with the students.

Students often have difficulty explaining the strategy they used when solving problems. One way to introduce problem-solving strategies is to adapt word problems so that students will focus on the data and the question being asked, rather than jumping into trying to find an answer. For example, provide the equation for a problem and ask the students to solve it. Then, ask the students to use the equation to fill in the missing information. Finally, ask for the solution to the problem.

Example:

For the following problem:

i. *Solve the equation.*

ii. *Fill in the blanks in the problem statement. State the final answer.*

> Curtis has ________ more quarters in his collection than Sharon has in hers. Together, they have ________ quarters. How many quarters does Sharon have?
>
> Equation: $q + q + 50 = 190$.

A second approach is to provide the information in a word problem but leave out the question. The students can be asked to write at least two questions that could be answered using the information.

Example:

For the following problem, state two questions that could be asked using the information provided. Answer your two questions, showing all work.

> Mavis jogs west at 12 km/h for 40 minutes, and then turns south and jogs at 14 km/h for 30 minutes.

Another area of difficulty for students is the verification of solutions. One way to get students to verify their solution to a problem is to ask *"Do you think your answer is correct? Explain."* By asking this, students are told explicitly to verify their solution and their strategy. Some students may be able to tell you that their answer is incorrect but they do not know how to fix it.

On the following page is an example of an assignment that has students consider each of the aspects of problem solving:

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*Adapted from Gerald Kulm, *Mathematics Assessment: What Works in the Classroom* (San Francisco, CA: Jossey-Bass Inc., Publishers, 1994) 145. This material is used by permission of John Wiley & Sons, Inc.

Purpose of this activity: To have you solve a problem and then write your solution in such a way that you reflect on the problem-solving methods used.

Problem: ____

____

____

Write-Up:

1. Problem Statement:

   In your own words, state the problem clearly enough so that someone unfamiliar with the problem could understand what it is that you are asked to do. *(2 marks)*

2. Process:

   Based on your notes, describe what you did in attempting to solve this problem.

   - How did you get started?
   - What approaches did you try?
   - Where did you get stuck?
   - What drawings did you use?

   Include things that didn't work or that seemed like a waste of time. Do this part of the write-up even if you didn't solve the problem. *(5 marks)*

3. Solution:

   a. State your solution(s) as clearly as you can. (If you only obtained a partial solution, give that.)

   b. Explain how you know that your solution is the best one possible. Your explanation should be written in a way that will be convincing to someone else—even someone who initially disagrees with your answer. Remember that merely stating your answer will count for nothing! *(4 marks)*

4. Extensions:

   Invent some extensions or variations to this problem. That is, write down some related problems. They can be easier, harder, or about the same level of difficulty as the original problem. (You are not expected to solve or answer these additional problems.) *(2 marks)*

5. Evaluation:

   Was this problem too easy, too hard, or about right? Explain why. *(2 marks)*

Reasoning

Students need to have a real understanding of mathematics. They need to move beyond memorizing sets of rules and procedures into investigations that answer the "why" questions. In order to do this, students must be provided with many opportunities to explain, justify, and refine their thinking. Listening to the explanations of their peers and being able to share their own thinking in a safe environment that fosters risk-taking will help them solidify their understandings. Sharing in this way will also help them identify and eliminate some of their misconceptions.

By having students construct, illustrate, write, and present their ideas, conceptualizations, and conclusions, progress related to reasoning can be assessed.

Using Technology

The teacher who values and encourages the use of today's learning technologies will find that most students are keen to participate in a higher level of mathematical thinking and problem solving than is possible when technologies are not used.

All students need access to calculators that can help them perform the task of solving real-world problems involving arithmetic operations. In addition, every mathematics classroom needs access to at least one computer for teacher presentation and for student use. Additional computers, perhaps in a computer lab, but ideally in each classroom, should be available as small groups and whole classes investigate, apply, and practise mathematical concepts.

Software programs that provide databases, spreadsheets, and geometric drawing programs are critical to students' achievement of learning outcomes for Senior Years Mathematics. Computer programs that provide computer-assisted instruction (CAI), or that motivate students to practice basic facts, estimation, mental calculation, and spatial problem solving also help support a modern mathematics curriculum.

To summarize, teachers and students need access to appropriate technology. Using a variety of tools enhances the opportunities for—and the likelihood of—students developing a deeper understanding of mathematical ideas and procedures.

Visualizing

Visualization is the construction of mental models and/or images of mathematical concepts and processes.Visualization of mathematical concepts can be demonstrated by building, drawing, and describing. Students should also be able to identify mathematical concepts in the models and images around them.