Module 5

Supporting Students with *Mathematics Disabilities*

This module provides information about the characteristics of students with mathematics disabilities (MD) and the interventions and adaptations needed to support their learning.

Key Ideas in this Module

1. The term 'mathematics disability', as used in this module, encompasses a range of difficulties that include number sense, memorization of arithmetic facts, accurate or fluent calculation, and/or accurate mathematics reasoning.
2. A combination of direct instruction, strategy instruction, and time to practice is effective instructional practice for all students, particularly for students with a learning disability.
3. There are various instructional strategies that could be considered differentiated instruction or adaptations, depending upon how they are used.
4. If a particular strategy is essential to the success of a student with MD, the adaptation needs to be documented in the student's IEP.

What is a Mathematics Disability (MD)?

"MDs are learning disabilities that affect the development of skills in mathematics, such as understanding quantitative concepts, translating language-based problems into mathematical symbols, and following sequences of steps." Students diagnosed with a disability in mathematics may also have difficulty recalling and understanding basic mathematics facts and often cannot remember the multiplication tables despite spending hours trying to memorize them. Students with MDs may have difficulty reading mathematical signs and copying numbers or figures correctly, as well as difficulty with direction and orientation, working memory, long-term memory, processing speed, and visual-spatial ability. (American Psychiatric Association, DSM-IV-TR, 2000; Payne and Turner, 1999)

In the Diagnostic and Statistical Manual of Mental Disorders, Fifth Edition (2013) (DSM-5), the diagnostic term "Specific Learning Disorder with Impairment in Mathematics" is used as outlined in Module 1, specifically for individuals who experience difficulty with number sense, memorization of arithmetic facts, accurate or fluent calculation, and/or accurate math reasoning. (p. 67, DSM-5)

Dyscalculia is an alternative term used to refer to a pattern of difficulties characterized by problems processing numerical information, learning arithmetic facts, and performing accurate or fluent calculations. (p. 67, DSM-5) The term 'mathematics disability', used in this module, encompasses a range of difficulties that include number sense, memorization of arithmetic facts, accurate or fluent calculation, and/or accurate math reasoning.

> "There is no need to change the content of the curriculum for students with learning disabilities. What must be done is what all good constructivist teachers do, and that is pay careful attention to the child and how he or she learns and design instruction (not content) that maximizes the strengths of the child while minimizing the impact of weaknesses." Van de Walle. 2001.

Characteristics of Students with Mathematics Disabilities

Students with MDs may have difficulties ranging from mild to severe that affect many areas of their lives. Students with MDs may have difficulty with the following:

- Telling and keeping track of time;
- Sequencing past and future events;
- Understanding abstract concepts of time and direction;
- Developing spatial orientation and space organization;
- Reading maps;
- Understanding mechanical processes;
- Following directions in sports that involve sequencing and rules;
- Keeping track of scores and players during games such as card or board games.

In the classroom, students with MDs often have difficulty with the following:

- Retrieving facts;
- Making estimates;
- Retaining information;
- Understanding aspects of counting (often students with MDs count on their fingers, commit counting errors, and use counting-all rather than counting-on);
- Remembering facts and formulas for completing calculations;
- Following sequential directions;
- Sequencing, which includes reading numbers out of sequence, making substitutions, reversals, omissions, and doing operations backwards;
- Problem solving (can become lost in the problem-solving process).

Supporting Students with Mathematics Disabilities through Direct Instruction, Strategy Instruction, and Rehearsal and Practice

Students with MDs, like students with other learning disabilities, respond best to a combination of direct instruction, strategy instruction, and rehearsal and practice. The strategies outlined in this module will require direct instruction in order for students to reach mastery in their use.

The focus of mathematics instruction for all students should be to make mathematics experiences meaningful. Students make sense of mathematics when they are actively involved in doing mathematics and actively constructing meaning out of what they are doing. Students need opportunities to explore, develop, discuss, apply, and test ideas.

Mathematical experiences that develop from concrete experiences, followed by pictorial, and then abstract or symbolic representations allow students to attach meaning to what they do. This is important for all students, and critical for students with MDs.

In addition, "there are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics." Many of the strategies discussed in this module are related to the seven mathematical processes (communication, connections, reasoning, mental mathematics and estimation, problem solving, visualization, and technology) which are "intended to permeate teaching and learning" (Manitoba Education. *Kindergarten to Grade 8 Mathematics Manitoba Curriculum Framework of Outcomes*. Winnipeg, Manitoba: Manitoba Education, 2013. 11. www.edu.gov.mb.ca/k12/cur/math/frameworks.html.)

Van de Walle (2001) states the following:

- Every day, students must experience that mathematics makes sense.
- Students must come to believe that they are capable of making sense of mathematics.
- Teachers must stop teaching by telling and start letting students make sense of the mathematics they are learning.

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary. (Manitoba Education and Advanced Learning. Kindergarten to Grade 8 Mathematics Support Documents for Teachers. Winnipeg, Manitoba: Manitoba Education and Advanced Learning, 2012 to 2015. 2.) (Mathematics Curriculum Supports, Manitoba Education and Advanced Learning. www.edu.gov.mb.ca/k12/cur/math/supports.html.)

Supportive Strategies for Teaching Mathematical Concepts

During their years at school, students are expected to learn many complex mathematical concepts, especially in algebra, geometry, and advanced mathematics courses. The following strategies do not address specific mathematics concepts to be taught. Instead, these strategies can be generalized for teaching mathematics concepts to all students and may particularly benefit students who have MDs.

Any of these strategies can be considered differentiated instruction or adaptations, depending on how they are used. If a particular strategy is essential to supporting the needs of a specific student, it should be documented as an adaptation in the student's individual education plan (IEP) so that this information can follow the student to other classes and schools (refer to the Student-Specific Assessment section of Module 2).

- **List steps for completion of mathematics problems on the board:** Use direct instruction to teach mathematics concepts by writing examples of multi-step mathematics problems on the board and making note of what is required to complete the problems. Always put the problem in the same place on the board and leave it there throughout the class. Number (don't use letters) the steps in the order they are to be completed.
- **Keep sample math problems visible:** Keep a step-by-step model of a problem on the board for students to refer to. If students have difficulty with working memory and can't hold the problem in mind while looking back and forth from the board, have them copy the problem onto a coloured card and keep it next to their work. If they can't remember to do this, ask another student to copy the problem for them.
- **Use a "paired-learning" teaching strategy:** Begin by demonstrating a problem, then put students into pairs and ask each student in the pair to make up his or her own problem, solve it, and write down the answer. The student then gives the blank problem to his or her partner to solve. If their answers are not the same, the students discuss the differences and make corrections. This strategy, also known as class-wide peer tutoring, provides the practise and rehearsal that all students, particularly students with MDs, require to learn new concepts.
- **Pair with another student:** If the teacher cannot use class-wide peer tutoring, the student in need of support can help identify another student who is willing and capable of answering the student's questions and can double-check to make sure homework assignments are written down.
- **Use group response:** One way to increase student involvement in class is to ask all students to solve a math problem and write the answer on a notebook-sized dry-erase board they can hold up for the teacher to see. (Go to www.kleenslate.com for an example.) Another suggestion is to have students whisper the answer as the teacher circulates around the room (which gives other students time to formulate their answers). This strategy allows the teacher to quickly assess which students don't yet understand the concept or outcome. Some teachers use wireless audience response systems (also known as "clickers", "keypads", or "clicker" apps for a similar purpose. (See http://replysystems.com for an example.)
- **Use songs or chants:** Examples of helpful resources can be found at Remedia Publications: www.rempub.com/math.
- **Use association:** When setting up a problem, create an association with something familiar to the student. Try using the student's name in a sample word problem, or using metaphors, analogies, or examples from the students' daily lives.
- **Use Mathematics manipulatives:**

  One way educators can begin teaching a concept in a concrete way that allows students to explore and experience the particular concept is through the use of manipulatives (objects which facilitate hands-on, tactile learning). When introducing manipulatives to a group of students, it is important to give the students some time to 'play with' and explore the items before the teacher begins using the items as a teaching tool. This is essential at all grade levels, from Kindergarten to Grade 12. It is worthwhile to plan ahead regarding how to manage this essential 'playtime', as well as how to manage the process of distributing and collecting the manipulatives. Once students have had a sanctioned opportunity to explore the materials, they will likely be more ready to use the manipulatives for learning mathematical concepts.

> For more classroom organization strategies, see:
>
> Manitoba Education. "Key Element Two: Classroom Organization." *Towards Inclusion: Supporting Positive Behaviour in Manitoba Classrooms.* Manitoba Education: Winnipeg, Manitoba. 2011. www.edu.gov.mb.ca/k12/specedu/behaviour/index.html.

Manipulatives are important in providing **concrete, visual representation** for students and enhancing a student's learning of mathematics in the following ways (Small, 2009).

- Students can refer to the visual model even when they are not using the manipulative anymore.
- Students can work together cooperatively to solve mathematical problems.
- Students can discuss and verbalize their thinking about mathematical ideas.
- Students independence increases once the modelling has been provided by the teacher. (Refer to the Gradual Release of Responsibility section for further information.)

Once the students and teacher are ready to use the manipulatives to teach/learn mathematics concepts, teachers need to model the use of the manipulatives and then guide students in their use, providing commentary that links the concrete model with the concept being taught. In other words, the teacher needs to directly teach the concept and the use of the manipulatives. Marian Small's *Making Math Meaningful to Canadian Students, 2nd Edition,* provides many examples of concepts and how manipulatives can be used to represent the concept.

It is recommended that Mathematics teachers begin with **Base ten blocks** and **Attribute blocks** when building their classroom collection of manipulatives, as they are both extremely versatile in their use and can be used to support many concept development activities.

- **Base ten blocks** are used to teach basic mathematical concepts like place value, addition, subtraction, number sense, and counting.

  For further information, go to the following URLs:
  www.bing.com/images/search?q=base+10+blocks&qpvt=base+10+blocks&qpvt=base+10+blocks&FORM=IGREwww.mathatube.com/base-ten-block-understanding.html

- **Attribute blocks** are usually used to describe a geometric pattern.

  For further information, go to the following URL:
  www.bing.com/images/search?q=attribute+blocks&qpvt=attribute+blocks&qpvt=attribute+blocks&FORM=IGRE

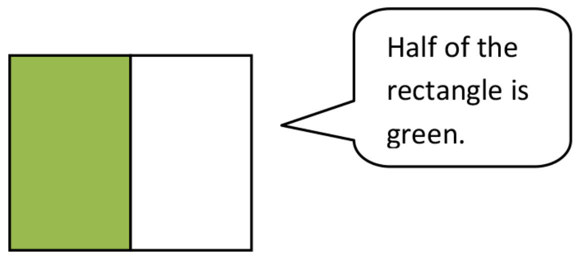
The following is a list of other useful manipulatives and examples of how they may be used:

- **Cuisenaire rods** are used to represent why 20-8=12 and can show why the least common multiple of 4 and 6 is 12 and also why 2/3 of 3/8 is 2/8. (www.cuisenaire.co.uk/)
- **Fraction strips** are useful in showing less than and greater than when it comes to comparing one fraction (e.g., when you are comparing 3/8 to another fraction such as 2/5). (http://lrt.ednet.ns.ca/PD/BLM/pdf_files/fraction_strips/fs_to_twelfths_labelled.pdf)
- **Coloured counters** can be used when discussing positive and negative integers [e.g., 5-(-2)]. (www.bing.com/images/search?q=coloured+counters&qpvt=coloured+counters&qpvt=coloured+counters&FORM=IGRE)
- **Geoboards** can be used to represent possible distances and slopes when you connect one pin to another on the board. (www.bing.com/images/search?q=geoboards&qs=n&form=QBIR&pq=geoboards&sc=2-9&sp=-1&sk=)
- **Algebraic tiles** can be used to represent variables and constants, and to represent and solve algebra problems. (www.bing.com/images/search?q=algebra%20tiles&qs=n&form=QBIR&pq=algebra%20tiles&sc=0-0&sp=-1&sk=)

- **Use pictorial representations** (see Appendix 5-A).

  Once students have had the opportunity to explore and experience a mathematical concept concretely, the teacher can support them in moving on to **pictorial representations**. Students can use technology or hand-drawn pictures to represent numbers and fractions, and to communicate their understanding of what they have learned.

Figure 3 **Pictorial Representation**



At this stage, students are able to create abstract or symbolic representations of concepts, numbers, fractions, and so forth. Through the use and creation of symbolic representations, students are demonstrating a deeper understanding of the concept. Their skills and understanding solidify further when they have frequent opportunities to describe, identify, and create representations of the mathematical concept. If students are unable to represent a concept in this more abstract way, the teacher will know that the student requires more practice at the concrete and/or pictorial stage.

The following is a simplistic example that illustrates the development of a concept through increasing levels of abstraction.

- Have students engage in everyday situations that use numbers and fractions (e.g., cutting a granola bar in half and sharing it with a friend).
- Have students demonstrate the concept of "half" using manipulatives such as attribute blocks or algebraic tiles (in high school) to build the foundation of their knowledge.
- Have students illustrate the concept of 'half' in various means (e.g., graph paper, art-mirror images, etc.).
- Have students move to the symbolic level by writing various fractions to represent 'half' (e.g., ½, 2/4, 36/72, etc.).

- **Develop mnemonics:** (see Appendix 5-B)
- **Use Mathematics games:**

Many early, middle, and senior years teachers occasionally incorporate mathematics games into their teaching to practise concepts in a fun, interactive, and novel way. Van de Walle, Karp, Lovin, Bay-William (2014) indicate that games provide low-stress approaches to practicing basic facts while helping students to be more fact fluent. Also, when a student can choose from a collection of reasoning strategies, it allows the student to become more adept at selecting strategies. There are different mathematics games for every possible level of mathematics, from simple addition to complex algebra. Games also help build critical thinking skills which will help in all aspects of learning.

Examples of mathematics games are listed below. For additional ideas, see the Manitoba Education and Advanced Learning supports at: www.edu.gov.mb.ca/k12/cur/math/supports.html.

- Middle Years Mathematics Activities and Games: www.edu.gov.mb.ca/k12/cur/math/my_games/index.html.
- Early Years Mathematics Activities and Games: www.edu.gov.mb.ca/k12/cur/math/games/index.html.
- The Center for Education in Mathematics and Computing, University of Waterloo, is another good source: www.cemc.uwaterloo.ca/.

In his book, *Styles and Strategies for Teaching Middle Grades Mathematics*, Dr. Thomas provides samples of Bingo questions and explains how teachers can make their own cards.

Many popular board games and game shows can be used to create Mathematics games.

There are also many free apps for tablets. Examples include the following:

- www.kidsmathgamesonline.com;
- www.sheppardsoftware.com/math.htm;
- www.dyscalculia.org/math-tools;

- www.brighthubeducation.com;
- www.softschools.com/math/games/;
- mrnussbaum.com/mathgames/;
- www.funbrain.com/brain/Adventure/SlingShot/index.html.

- **Reduce note taking and writing:**

  Photocopy pages for students so they do not have to take notes or rewrite math problems. Enlargements of original copies give students extra room to show work and write answers. This enables them to use their cognitive energy for completing the problems instead of copying them.

- **Work toward a deeper understanding of concepts allowing students to rework problems/questions where errors were made:**

  Recommended resource:

  Marzano, Robert J. *Marzano's (Nine) High-Yield Instructional Strategies*. www.palmbeachschools.org.

  (Adapted from the book: Marzano, Robert J., Debra J. Pickering, and Jane E. Pollock. *Classroom Instruction that Works: Research-based Strategies for Increasing Student Achievement*. Alexandria, VA: Association for Supervision and Curriculum Development (ASCD), 2001.

Supporting Students with Mathematics Disabilities through Adaptations

As in the previous section, the following strategies do not address specific mathematics concepts to be taught. Instead, these strategies can be generalized for teaching mathematics concepts to all students and may particularly benefit students who have MDs.

These strategies can also be considered as differentiating instruction or adaptations, depending on how they are used. If a particular strategy is essential to supporting the needs of a specific student, it should be documented as an adaptation in the student's IEP so that this information can follow the student to other classes and schools. (Refer to the Student-Specific Assessment section for further information.)

- **Teach students how to use a calculator for classwork and homework:** For example, when doing a calculation such as 145 x 32, a student can estimate 100 x 30 as a low answer and 200 x 30 as a high answer. Then, the student can check the calculator and should expect an answer greater than 3000 but less than 6000. Students with mathematics disabilities, should still be encouraged to use "easier facts" like multiples of 10 to estimate answers. They shouldn't just "trust" the calculator. Students must understand basic math skills when solving problems and when learning new concepts. As such, students using a calculator in class may need a periodic review of how to perform basic math skills to continue to develop their number sense.

- **Use graph paper for place value instruction:** Students can use graph paper to separate the places and decimal (ones, tens, hundreds, thousands; one-tenth, one-hundredth, one-thousandth); this helps students keep columns straight and reduce the likelihood of errors.
- **Use notebook paper to model appropriate spacing of math problems:** Turn notebook paper 90 degrees to help students organize math problems and line up numbers; when the paper is turned sideways, the student can write numbers between vertical lines. Show the student how to write a few problems correctly.
- **Use graphic organizers:** Graphic organizers can help students demonstrate, draw, or explain their thinking in math. Students can explore different ways of thinking about concepts by using strategies such as comparing and contrasting, a 3-point approach to vocabulary. (See Appendix 5-C.)

  For examples of graphic organizers that can be used in mathematics courses, see the following documents:

  Manitoba Education and Training. *Success for All Learners: A Handbook on Differentiating Instruction*. Winnipeg, Manitoba: Manitoba Education and Training, 1996. p. 6.36. This document is available for purchase from the Manitoba Learning Resource Centre (formerly the Manitoba Text Book Bureau).

  Zike, Dinah. Foldables. www.dinah.com/.

- **Use informative posters in the classroom:** Posters can provide visual reminders of important mathematical facts or concepts. The more interesting and novel the illustrations, the more likely students are to remember the information.
- **Use colour to highlight key facts:** Ask students to highlight key mathematics operations or issues *before* beginning work on mathematics problems. For instance, highlight each time the signs change. For geometry problems, ask them to highlight perimeter, area, or volume, as well as the name of the shape (e.g., triangle, square, trapezoid). Use different coloured highlighters to differentiate information.
- **Provide an extra textbook at home:** Students with LD and ADD/ADHD may tend to forget their books. Since mathematics can be one of their most challenging subjects, the possibility of successful learning depends on frequent opportunities to practise curricular outcomes. Missing opportunities for practise because of forgotten books can make a difference in a student's success.
- **List strategy or action words and the associated mathematics procedures:**
  - Encourage students to develop their own list and create a graphic organizer or foldable.
  - Create a wall poster wall for students—much like a word wall.

- Make it clear that memorization is not required.

  Example

  What operation would you use?

  Sometimes, the terms in word problems such as "combined", "what amount," or "how much less" can confuse students with regard to deciding which mathematics procedures to use. (See the mathematical language section in the Mathematics Curriculum at www.edu.gov.mb.ca/k12/cur/math/index.html.)

Supporting Students with Mathematics Disabilities through Assistive Technology (AT)

In Marian Small's book, "Making Math Meaningful to Canadian Students, K-8", educators are provided with abundant examples of how to infuse technology into mathematics learning for students. Assistive technology can be considered as differentiating instruction or adaptations, depending upon how the AT is used. If a particular type of AT is essential to supporting the needs of a specific student, it should be documented as an adaptation in the student's IEP so that this information can follow the student to other classes and schools. (Refer to the Student-Specific Assessment section for further information.)

- Calculators: Calculators can be used for exploring patterns which involve addition, subtraction, multiplication, division, and algebraic functions.

  The National Council of Teachers of Mathematics (NCTM) has made specific recommendations on calculators used in school Mathematics courses including the following:

  - All students should have access to calculators to explore mathematical ideas and experiences, to develop and reinforce skills, to support problem-solving activities, and to perform calculations and manipulations.
  - Mathematics teachers at all levels should promote the appropriate use of calculators to enhance instruction by modeling calculator applications, by using calculators in instructional settings, by integrating calculator use in assessment and evaluation, by remaining current with state-of-the-art calculator technology, and by considering new applications of calculators to enhance the study and the learning of mathematics.

  Further information is available at Government of Newfoundland and Labrador Department of Education: Division of Program Development: Calculators in Mathematics Instruction and Assessment: A Position Statement. 2004. 3. www.ed.gov.nl.ca/edu/k12/curriculum/documents/mathematics/CalculatorsinInstruction.PDF.

- **Computers:** There are many software programs available that increase conceptual learning of specific mathematics concepts (see *10 Tips for Software Selection for Math Instruction* at www.ldonline.org/article/6243/).

- **Interactive Whiteboard:** This is an instructional tool that allows computer images to be displayed onto a board using a digital projector. The teachers can then manipulate the elements on the board by using their fingers as a mouse, directly on the screen. Items can be dragged, clicked, and copied and the teacher can handwrite notes, which can be transformed into text and saved (see *Interactive Math Websites for Interactive Whiteboards* at www.theteachersguide.com/InteractiveSitesMathSmartBoard.htm).

There are countless websites which provide support to teachers and students in the area of Mathematics. The following is a list of some of the resources available:

- Math/LD Online at www.ldonline.org;
- Infusing Assistive Technology for Learning into the IPP Process at https://archive.education.alberta.ca/;
- Khan Academy at https://www.khanacademy.org/;
- DragonBox—Discover the Game of Math at www.dragonboxapp.com/;
- Meaningful Algebra with CAS—Compass Learning Technologies at www.compasstech.com.au/ARNOLD/MAC/index.html.

In addition, you may find the following **'search terms'** helpful in locating further websites that provide support to teachers and students in Mathematics:

addition, algebra, algorithm, angles, area, assistive technology, base ten, capacity, data collection, decimals, division, fraction, geometry, graphical representation, length, learn money, multiplication, number relationship, percentage, problem solving, shape and property, subtraction, telling time

Supporting Students with Mathematics Disabilities through Adaptations to Formative and Summative Assessment

As in the previous sections, some of these strategies can also be considered as differentiated instruction or as adaptations, depending on how they are used. As in the use of differentiated instruction, adaptations should be made available to all the students in the classroom. Certain adaptations will be essential for certain students. These must be documented in the students' IEPs. Adaptations offered during formative or summative assessments should be the same adaptations that the students use throughout the semester or school year. It is important to keep in mind that the adaptation(s) should not affect the validity of the assessment.

If a particular strategy is essential to supporting the needs of a specific student, it should be documented as an adaptation in the student's IEP so that this information can follow the student to other classes and schools (refer to the Student-Specific Assessment section for further information).

- **Share outcomes, criteria, and learning goals with students.** Identify the outcomes related to each assessment so that students understand expectations, as well as their strengths and learning needs.
- **Allow students to refer to formulas, definitions, and key mathematics facts during tests.** Students could also be allowed the use of a mathematics dictionary that they create and use when doing work. Examples of summary sheets, which allow students to create one-page review summaries of notes, formulas, or definitions, are located in the Foldables book by Dinah Zike at www.dinah.com/. Similarly, students can use "word banks" or laminated cards with formulas, definitions, and/or key mathematics facts. This method is already used in provincial tests to assess the application of concepts rather than the recall of facts. Students must understand concepts in order to know which formulas to use; therefore, the validity of the assessment is not impacted.
- **Laminate a copy of important mathematics facts.** Make wallet-sized copies of multiplication tables and divisibility rules and allow students to keep them and use them when a calculator is not permitted or available. Also allow students to keep small cards with formulas or acronyms. Pictorial representation of the facts could be laminated and displayed as well.
- **Provide options for students to demonstrate learning.** Instead of written tests, students can demonstrate learning through interviews, models, discussions, presentations, graphic organizers, and so forth.
- **Read the questions to the student out loud.**
- **Allow extended time for tests.**
- **Allow students opportunities to rewrite tests.**
- **Shorten homework assignments.** Assign every second, third, or fourth mathematics problem that still covers the skills required for mastering major mathematics concepts.

For more information, see Manitoba Education and Advanced Learning. *Policies and Procedures for Provincial Tests*. Winnipeg, MB: Manitoba Education and Advanced Learning. Available online at www.edu.gov.mb.ca/k12/assess/docs/pol_proc/.

> Adaptations to Provincial Tests
>
> **3.3.1 Procedures for Requesting Adaptations**
>
> Requests for adaptations are made at the time of registration when entering student data using the web application (*Provincial Test Student Registration*). Adaptations must be requested separately for each student for each test. (Manitoba Education and Advanced Learning) www.edu.gov.mb.ca/k12/asses/docs/pol_proc/.

Module Summary

A student with a mathematics disability can achieve a high level of success at school. Teachers must understand that a mathematics disability is a neurological disorder and, as such, provide students with direct/explicit instruction, along with strategy instruction, and time to practice skills to mastery. Students with mathematics disabilities face many challenges; however, with appropriate interventions and adaptations, they can become independent with their mathematics skills and have the same opportunities as their peers.

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Van de Walle, John A., Karen S. Karp, Jennifer M. Bay-Williams, Lynn M. McGarvey, and Sandra Folk. *Elementary and Middle School Mathematics, Teaching Developmentally*, 4th edition. Reading, MA: Addison Wesley Longman, Inc., 2001.

Van de Walle, John A., Karen S. Karp, LouAnn H. Lovin, and Jennifer M. Bay-Williams. *Teaching Student-Centered Mathematics, Developmentally Appropriate instruction for grades 3-5*, Second Edition, Boston, MA: Pearson, 2014.

Resources

Dacey, Linda, and Rebeca Eston Salemi. *Math for All: Differentiating Instruction, Grades 6-8*. Math Solutions Publications, 2007. www.pearsoncanadaschool.com/index.cfm?locator=PS1zOt&PMDBSOLUTIONID=25862&PMDBSITEID=2621&PMDBCATEGORYID=25876&PMDBSUBSOLUTIONID=&PMDBSUBJECTAREAID=&PMDBSUBCATEGORYID=26215&PMDbProgramID=53801&elementType=programComponents.

This book offers practical, research-based guidance to differentiating instruction in the mathematics classroom.

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This book offers practical, research-based guidance to differentiating instruction in the mathematics classroom.

Murray, Miki, and Jennifer Jorgensen. *Differentiated Math Classroom, A Guide for Teachers, K-8*. Heinemann, 2007. www.pearsoncanadaschool.com/index.cfm?locator=PS1zOt&PMDBSOLUTIONID=25862&PMDBSITEID=2621&PMDBCATEGORYID=25876&PMDBSUBSOLUTIONID=&PMDBSUBJECTAREAID=&PMDBSUBCATEGORYID=26215&PMDbProgramID=52822&elementType=programComponents.

This resource describes both the big ideas of differentiation and the day-to-day teaching that makes it work.

Small, Marian. *Eyes on Math: A Visual Approach to Teaching Math Concepts.*, NewYork, NY: Teachers College Press, 2013. www.nelsonschoolcentral.com/CGI-BIN/lansaweb?webapp=WKWSEARCH+webrtn=search+ml=LANSA:XHTML+partition=CEP+language=ENG.

This resource shows how to use images to stimulate mathematical teaching conversations around K-8 Mathematics concepts.

——. *Good Questions: Great Ways to Differentiate Mathematics Instruction*, 2nd edition. New York, NY: Teachers College Press, 2012. www.nelsonschoolcentral.com/cgi-bin/lansaweb?webapp=WBOOKCMP+webrtn=bookcomp+F(LW3ITEMCD)=9780807753132.

This resource provides clear, ready-to-use ideas for differentiating instruction in mathematics.

——. *Leaps and Bounds Toward Math Understanding: Teacher's Resource, 3-4*. Toronto, Canada: Nelson Education Ltd., 2011.

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This resource help teachers support students who are struggling in mathematics.

——. *Leaps and Bounds Toward Math Understanding: Teacher's Resource*, 7-8. Toronto, Canada: Nelson Education Ltd., 2011. www.nelsonschoolcentral.com/cgi-bin/lansaweb?webapp=WBOOKCMP+webrtn=bookcomp+F(LW3ITEMCD)=9780176351526.

——. *More Good Questions*. Toronto, Canada: Nelson Education Ltd., 2010. www.nelsonschoolcentral.com/CGI-BIN/lansaweb?webapp=WKWSEARCH+webrtn=search+ml=LANSA:XHTML+partition=CEP+language=ENG.

Specifically for secondary mathematics teachers, presents two powerful and universal strategies that teachers can use to differentiate instruction across all mathematics content.

——. *Uncomplicating Algebra*. Toronto, Canada: Nelson Education Ltd., 2014. www.nelsonschoolcentral.com/cgi-bin/lansaweb?webapp=WBOOKCMP+webrtn=bookcomp+F(LW3ITEMCD)=9780807755174.

This ressource helps teachers understand how to teach and assess student learning of algebra.

Thomas, Ed, Ph.D. *Dr. Thomas' P3CR Paired Learner Math Activities for General Math, Pre-Algebra, Algebra, and Geometry*. Woodbridge, NJ: Dimension 2000, 1999.

Appendix 5-A

Pictorial examples

Array or area model

a) Multiplication

When a student makes an error in performing a multiplication task, it most likely stems from a lack of understanding of the underlying principal, such as numeration and multiplication. When a student continues to make these kinds of errors and doesn't seem to understand what they are doing wrong they should be using concrete examples, such as the one below, to record the written algorithms. (Small, 2009)

Here is an example of increasing levels of abstraction using the concept of multiplication. It shows the progression from concrete to pictorial to symbolic.

Figure 4 **Pictorial Example of Multiplication**

1) $56 \times 38=$

Begin by building this concretely with base ten blocks, then move to pictorial representation as illustrated below.

	50	6
30	1500	180
8	400	48

$56 \times 38= 1500+180+400+48=2128$

Now move to symbolic representation as shown below.

$$\begin{array}{r} 56 \\ \underline{\times 38} \\ 1500 \\ 400 \\ 180 \\ \underline{+48} \\ 2128 \end{array}$$

If students are ready, they can progress to the standard algorithm, which is just a shortcut of the above.

$$\begin{array}{r} 56 \\ \underline{\times 38} \\ 1900 \\ \underline{+228} \\ 2128 \end{array}$$

1) $(2x+3)\ (x+2)=$

Using algebra tiles...

	x	x	3
x	x^2	x^2	3x
2	2x	2x	6

$2x^2+7x+6$

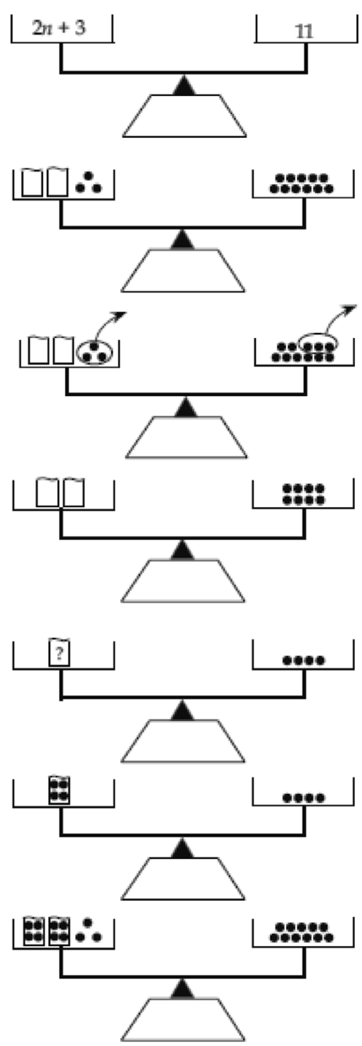
b) Algebraic Equations

Explain to students that an equation is like a balance scale. If the same number is subtracted from each side, for example, the equation remains balanced. Whatever students do to one side of an equation (e.g., add, subtract, multiply, divide) they must do to the other side of the equation.

An actual balance scale can be used to demonstrate this in a concrete way. The following illustration shows a pictorial representation of the concept.

Figure 5 ***Algebraic Equations***

Example:



Represent $2n + 3 = 11$ as a balance.

● represents a chip.

☐ represents a tag containing an unknown number of chips.

$2n + 3 = 11$

Show this concretely (or pictorially).

$$\begin{aligned} 2n + 3 &= 11 \\ -3 \quad & \quad -3 \end{aligned}$$

Maintaining balance, remove 3 chips from each side.

$2n = 8$

Simplify.

$$\frac{2n}{2} = \frac{8}{2}$$

Determine the number of chips that would be in each bag.

$n = 4$

Simplify.

$2n + 3 = 11$ (?)

$8 + 3 = 11$ (?)

$11 = 11$ (✔)

Check.

Source: Manitoba Education. *Grade 7 Mathematics: Support Document for Teachers*. Winnipeg, Manitoba: Manitoba Education. 2012. 19. www.edu.gov.mb.ca/k12/cur/math/support_gr7/patterns.pdf.

Appendix 5-B

Mnenomics

a) Order of Operations

VandeWalle, 2001, states that ***"Operations"*** refers to adding, subtracting, multiplying, dividing, squaring, etc.

But, when you see something like ...

7 + (6 × 52 + 3)

... a student asks themselves, "what part should I calculate first?"

Start at the left and go to the right?

Or go from right to left?

VandeWalle warns that *"if you calculate them in the wrong order, you will get a wrong answer!"*

For this very reason, people agreed to follow rules when doing calculations.

These rules are

Do things in brackets first. For example,

✔	**6 x (5 + 3)**	**=**	**6 x 8**	**=**	**48**
✘	**6 x (5 + 3)**	**=**	**30 + 3**	**=**	**33 (wrong)**

Calculate **Exponents (Powers, Roots) before multiplying, dividing, adding, or subtracting.** For example,

✔	**5 x 2^2**	**=**	**5 x 4**	**=**	**20**
✘	**5 x 2^2**	**=**	**10^2**	**=**	**100 (wrong)**

Multiply or divide before adding or subtracting. For example,

✔	**2 + 5 x 3**	**=**	**2 + 15**	**=**	**17**
✘	**2 + 5 x 3**	**=**	**7 X 3**	**=**	**21 (wrong)**

Otherwise just go left to right. For example,

✔	**30 ÷ 5 x 3**	**=**	**6 x 3**	**=**	**18**
✘	**30 ÷ 5 x 3**	**=**	**30 ÷ 15**	**=**	**2 (wrong)**

B	**B**rackets first
E	**E**xponents (i.e., powers, square roots, etc.)
MD	Multiplication and **D**ivision (left-to-right)
AS	**A**ddition and **S**ubtraction (left-to-right)

b) Properties of Whole Numbers

There are five common properties of whole numbers, two of which—the Communicative Property (CO) and the Associative Property (AP)— are often confusing for students. Mnemonics can help students differentiate between the two.

Communicative Property (CO) applies

- To addition and multiplication
- Even though the order changes
- Even though the sum or product is still the same

CO allows a change in order in numbers.

- 5 + 7 = 7 + 5
- A + b = b + a
- 3 x 7 = 7 x 3

Associative Property (AP) of addition applies

- Even though the parentheses move
- Even though (the grouping of numbers changes) the sum is still the same

AP arranges parentheses

- (12 + 8) + 5 = 12 + (8 + 5)
- (3X + 2) + 2X = 3X + (2 + 2X)

c) Linear Equations

Linear functions are described by both a formula and a line drawing on a graph. Typically students have problems with two special cases: a case where the line has no slope (vertical line) and a case where the line has a 0 slope (horizontal line). Students can make up their own mnemonics to help them understand their difficulties in linear equations.

An example of a mnemonic created by a high school senior is the acronym **HOY**.

- Slope of Horizontal Line: **HOY**—**H**orizontal, **0** slope, **Y**=?

  Example: A **H**orizontal line has a slope of 0 and is represented by an equation, Y = a constant.

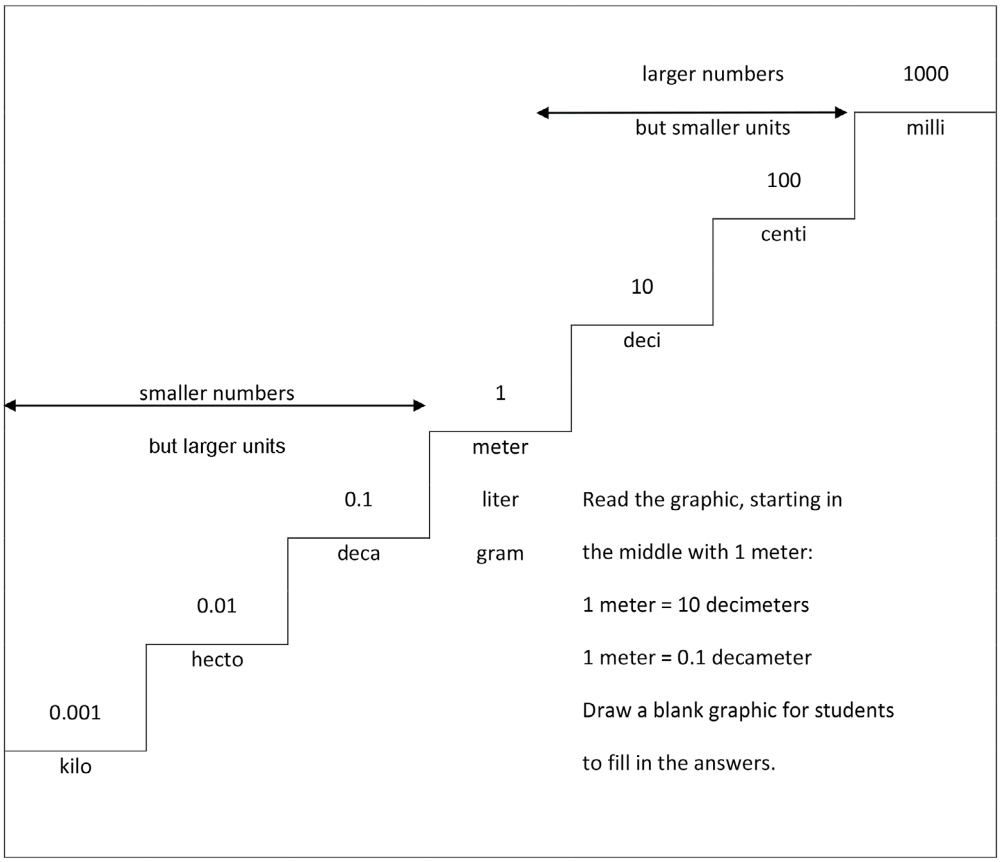
  If you know the mnemonic HOY, it is easier to remember that a vertical line has no slope and is represented by an equation x = a constant.

Appendix 5-C

Metric System Graphic Organizer

Students can use the following “staircase” graphic organizer to help them see and remember the relationship of prefixes in the metric system.

Figure 6 **Metric System Graphic Organizer**



Appendix 5-D

Strategies in Supporting Students with Mathematical Disabilities

The following are two useful strategies in supporting students with mathematical disabilities, as well as all other students. These examples happen to use early years scenarios; however, the strategies can be used in middle and senior years as well.

What Questions Can You Answer?

This is an example where numerical data are provided. Students generate a list of questions that can be answered from the data and then answer at least one of their questions.

Figure 7 **Maya's Toys**

Maya has a box of toys. Each toy is different. Maya sorts them into 3 groups.

Write three questions you can answer about the groups.

1. ______________________________

2. ______________________________

3. ______________________________

4. Find the answer to one of your questions. Show your work.

What Number Makes Sense?

This is an example of a problem situation from which numerical data are missing. Students choose from a list of numbers to fill in the blanks so that the problem and solution make sense.

Figure 8 **Football Points**

Yan played 2 football games last week. In the first game Yan's team scored _____ points. In the second game Yan's team scored _____ more points than in the first game. Altogether Yan's team scored _____ points in 2 games.

6	48	21

1. Read the problem.
2. Look at the numbers in the box.
3. Put the numbers where you think they fit best.
4. Read the problem again. Do the numbers make sense?
5. Explain how you know you have the numbers in the correct blanks.