

Grade 11 Pre-Calculus Mathematics (30S)

A Course for
Independent Study

***GRADE 11 PRE-CALCULUS
MATHEMATICS (30S)***

*A Course for
Independent Study*

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$$0^\circ \leq \theta \leq 360^\circ \quad 5$$

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Lesson 1

Relations and Functions

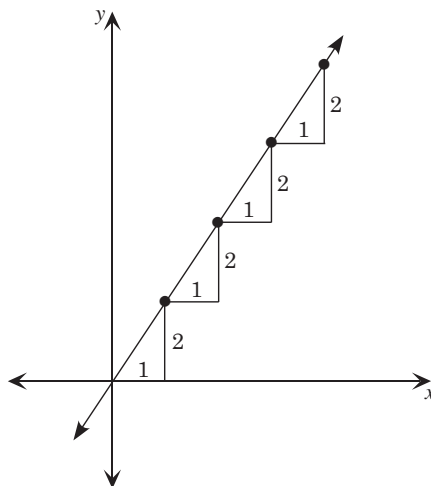
Outcomes

When you complete this lesson, you will be able to

- define a relation and a function
- state the domain and range of a given relation or function using set notation
- represent a relation or function in
 - a) graph mode
 - b) rule mode
 - c) ordered pair mode
- determine whether a relation is or is not a function
- use functional notation to evaluate a function

Overview

In *Grade 10 Pre-Calculus Mathematics* you studied the linear function $y = mx + b$ or $f(x) = mx + b$ where m is the slope of the line and b is the y -intercept. The slope (m) of its graph shows you that equal changes in the x -value gives equal changes in the y -value.



The rate of change of y with respect to x is always constant. In the diagram, for every 1 unit of change in x , y changes 2 units.

In mathematics if two quantities are related in such a way that the value of the one quantity determines the value of the second quantity, you have a mathematical model of a relation.

Definition

A **relation** is a set of ordered pairs (x, y) .

The first components of the ordered pairs form a set called the **domain** of the relation. The second components form a set called the **range** of the relation.

For example:

Given the relation $A = \{(2, 1), (3, 2), (4, 3)\}$

Domain of A : $\{2, 3, 4\}$

Range of A : $\{1, 2, 3\}$

Notice how each set is enclosed in braces, $\{\}$.

A special type of relation is the function.

Definition

A **function** is a set of ordered pairs (x, y) such that for each value of x there is exactly one value of y .

Example

1. Is the relation $B = \{(1, 4), (6, 2), (5, 3), (4, -4)\}$ a function?
2. Is the relation $C = \{(1, 6), (4, 10), (1, 4), (2, -2)\}$ a function?
3. Is the relation given by $y^2 = x$ a function?
4. Is the relation given by $y = x^2$ a function?

Solution

1. Yes, the relation is a function because each of the first components is different.
2. No, the first component 1 is associated with the second components of 6 and 4 rather than only one second component.
3. No, because $x = 4$ produces $y = 2$ or $y = -2$, so $(4, 2)$ and $(4, -2)$ are ordered pairs of the relation and the first component 4 has two second components associated with it.
4. Yes, if you square the value of x , you get just one value of y .



Relations and functions can be represented in five main ways:

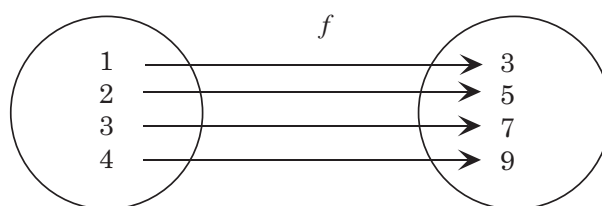
1. Ordered pairs in a set

$$\{(1, 3), (2, 5), (3, 7), (4, 9)\}$$

2. Ordered pairs in a table of values

| | | | | |
|-----|---|---|---|---|
| x | 1 | 2 | 3 | 4 |
| y | 3 | 5 | 7 | 9 |

3. Mapping diagram

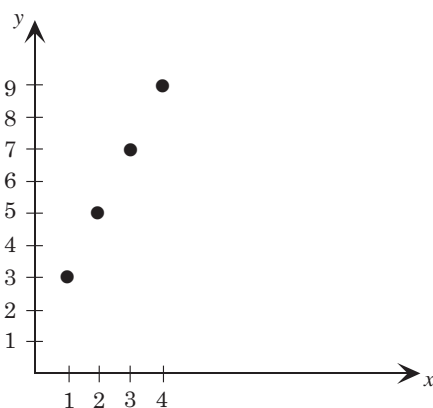


input x (domain)

output y (range)

The arrowhead points to the range element associated with the domain element of a function f .

4. Graph



5. Equation or rule

$$y = 2x + 1 \text{ where } x \in \{1, 2, 3, 4\}$$

In functional notation, this rule would be written as $f(x) = 2x + 1$, where $x \in \{1, 2, 3, 4\}$.

You can write $f(1) = 3$ which means if you substitute 1 into the rule f , the answer is 3.

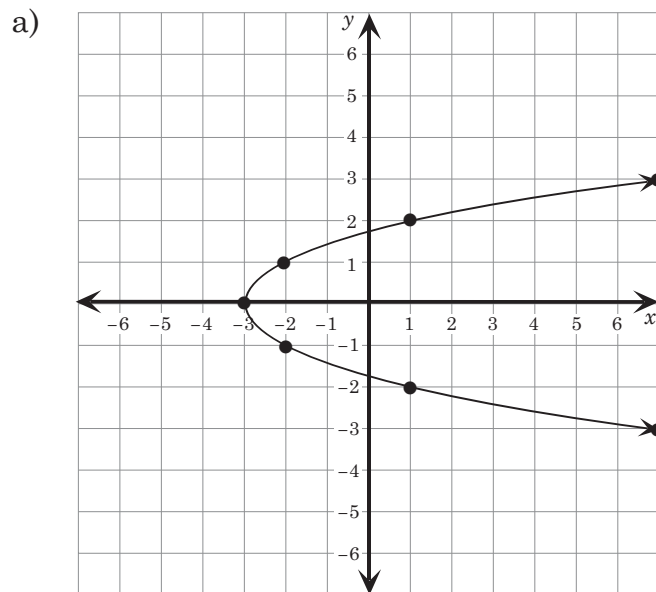
Functional notation is used only for functions, and not for relations.

If a relation is presented in the graphical mode, you can determine whether it is, or is not, a function by using the **vertical line test**.

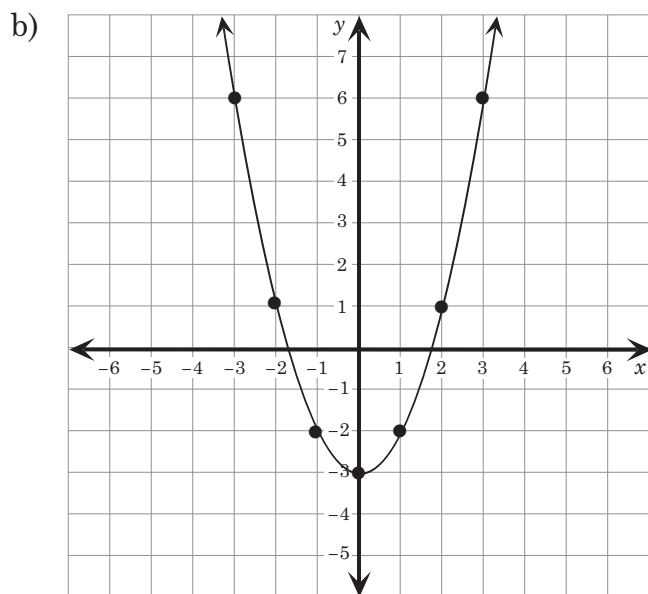
Vertical Line Test

If any vertical line intersects the graph of a relation in more than one point then the graph does not represent a function.

For example:



$x = y^2 - 3$ is not a function since any vertical line $x = h$ with $h > -3$ will intersect the graph in two points.



$y = x^2 - 3$ is a function since no vertical line will intersect the graph in more than one point.

System of Numbers

A review of the types of systems of numbers may be helpful in writing domain and range.

Natural Numbers = $\mathbb{N} = \{1, 2, 3, \dots\}$

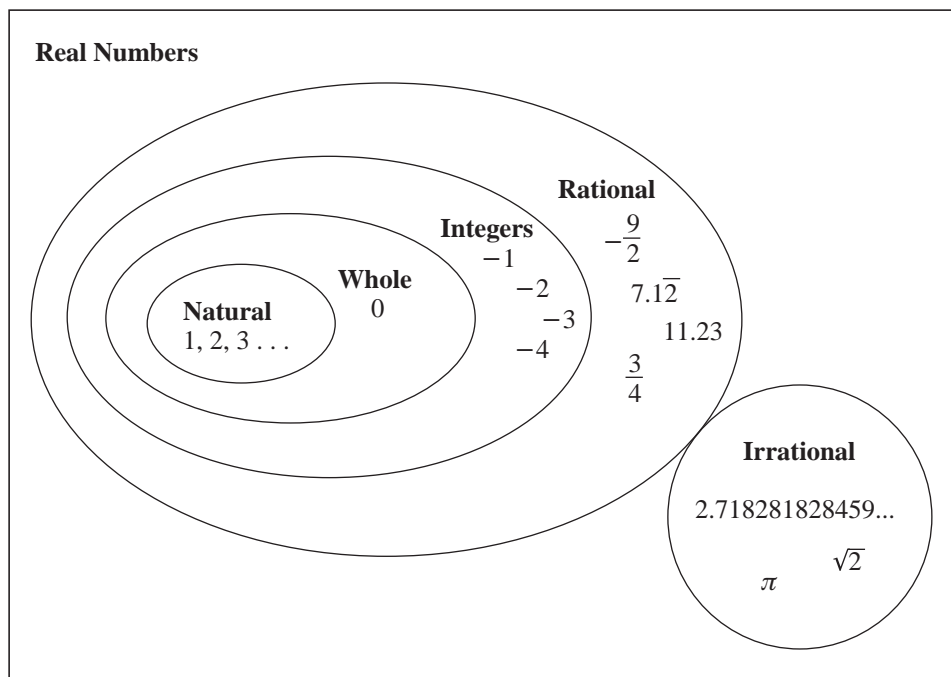
Whole Numbers = $\mathbb{W} = \{0, 1, 2, 3, \dots\}$

Integers = $\mathbb{I} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

Rational Numbers = $\left\{ \frac{a}{b} \mid a \text{ and } b \text{ are integers, } b \neq 0 \right\}$

Irrational Numbers = $\{x \mid x \text{ is not rational}\}$

Real Numbers = $\mathbb{R} = \{x \mid x \text{ is rational or } x \text{ is irrational}\}$



Note that the natural numbers are embedded in the whole numbers, the whole numbers are embedded in the integers, and the integers are embedded in the rationals. The rationals and irrationals are two completely separate sets and do not overlap. The reals are the union of the two distinct sets of the rationals and the irrationals.



Assignment

1. Determine which of the following relations are functions:

a) $A = \{(2, 1), (2, 3), (2, 5), (2, 6)\}$

b) $B = \{(2, 1), (3, 1), (4, 1), (5, 1)\}$

c) $y = 2x + 3$

d) $y = x^2 + 2$

2. In each case, the following ordered pairs are part of a relation. Write an equation representing the relation.

a)

| | | | | |
|-----|---|---|---|----|
| x | 1 | 2 | 3 | 4 |
| y | 3 | 6 | 9 | 12 |

b)

| | | | | |
|-----|---|---|---|----|
| x | 1 | 2 | 3 | 4 |
| y | 1 | 4 | 9 | 16 |

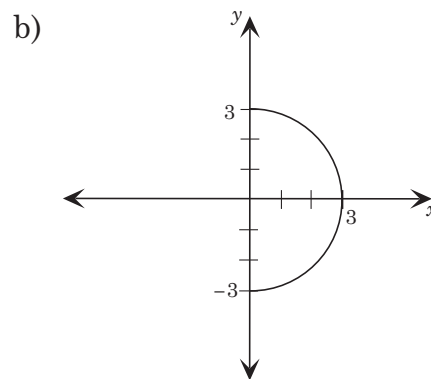
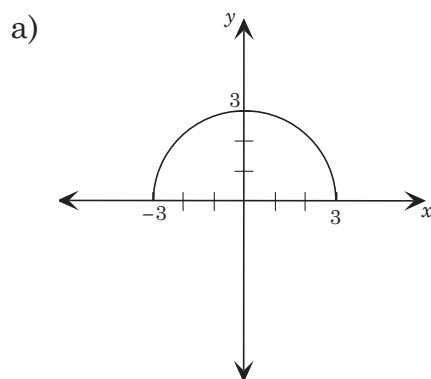
c)

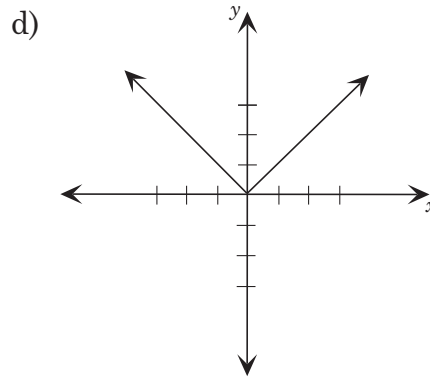
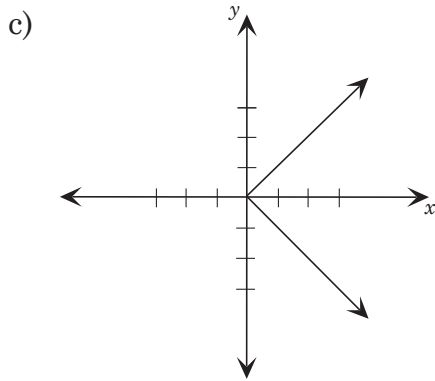
| | | | | |
|-----|---|---|---|---|
| x | 2 | 4 | 6 | 8 |
| y | 1 | 2 | 3 | 4 |

d)

| | | | | |
|-----|---|---|---|---|
| x | 1 | 2 | 3 | 4 |
| y | 1 | 3 | 5 | 7 |

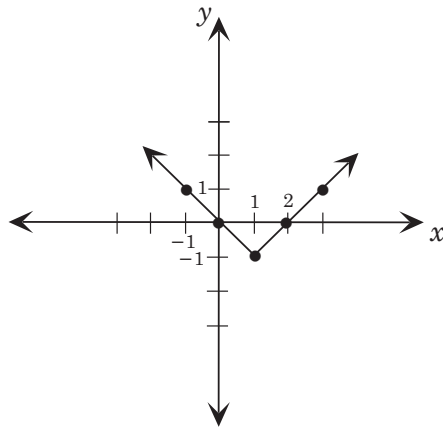
3. Use the vertical line test to determine which relations are functions.





4. Given the function $f = \{(-1, 3), (2, 6), (5, -3), (7, 7)\}$, find the value of
- $f(2)$
 - $f(5)$
 - $f(-1)$
 - $f(4)$

5. Given the graph of the function h , find the value of
- $h(0)$
 - $h(-1)$
 - $h(2)$



6. Given the function $f(x) = 3 - x$, find the value of
- $f(0)$
 - $f(-1)$
 - $f(3)$
 - $f(t)$

7. If $f(x) = 2x^2 - 3x - 1$, find
- $f(-1)$
 - $f(\sqrt{2})$
 - $f(a)$
 - $f(2x)$
 - $f(x - 1)$
 - $f(f(1))$
8. If $f(x) = x^2 - 2$ and $g(x) = -2x$, find the value of
- $f(-3)$
 - $g(-2)$
 - $f(g(3))$
 - $g(f(3))$
9. Find the domain and range of the relations in Question 1, using correct set notation.
10. Find the domain and range of the relations in Question 3, using correct set notation.



Check your answers in the Module 1 Answer Key.

Notes
