GRADE 12 APPLIED MATHEMATICS (40S)

Midterm Practice Examination
Answer Key

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Midterm Practice Examination
Answer Key

Name: _____

Student Number: _____

Attending ❑   Non-Attending ❑

Phone Number: _____

Address: _____

_____

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For Marker's Use Only

Date: _____

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Answer Key

Instructions

The midterm examination is based on Modules 1 to 4 of the Grade 12 Applied Mathematics course. It is worth 20% of your final mark in this course.

Time

You will have a maximum of **3.0 hours** to complete the midterm examination.

Notes

You are allowed to bring the following to the examination: pens/pencils (2 or 3 of each), metric and imperial rulers, a graphing and/or scientific calculator, and your Midterm Exam Resource Sheet. Your Midterm Exam Resource Sheet must be handed in with the examination. Graphing technology (either computer software or a graphing calculator) **is required** to complete this examination.

Show all calculations and formulas used. Use all decimal places in your calculations and round the final answers to the correct number of decimal places. Include units where appropriate. Clearly state your final answer. Final answers without supporting calculations or explanations will **not** be awarded full marks. Indicate equations and/or keystrokes used in calculations.

When using graphing technology, include a screenshot or printout of graphs **or** sketch the image and indicate the window settings (maximum and minimum *x*- and *y*-values), increments, and axis labels, including units.

Name: _____

Answer all questions to the best of your ability. Show all your work.

1. Given the function graphed below, complete the table with the required information. *(8 marks)*
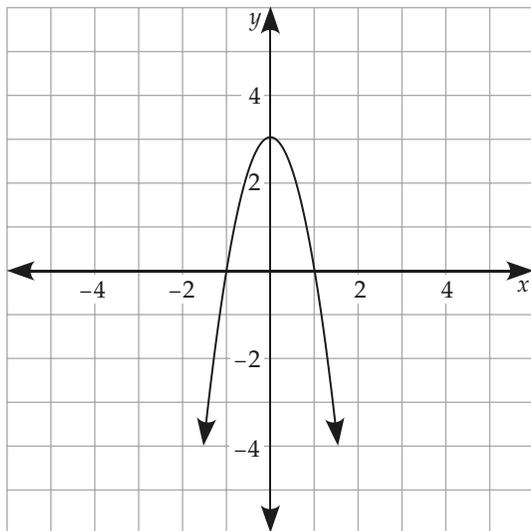
Answer: (Module 1, Lesson 1)

Type of Function	Quadratic
Degree	2
Coordinates of x-intercept(s)	$(-1, 0)$, $(1, 0)$
Coordinates of y-intercept(s)	$(0, 3)$
End behaviour	Quadrant III to Quadrant IV
Absolute or relative maximum and/or minimum—state y-value	Absolute maximum at $y = 3$
Domain	$\{x \mid x \in \Re\}$
Range	$\{y \mid y \leq 3, y \in \Re\}$

2. The loudness of a sound, L, measured in decibels (dB), is defined by the formula $L = 10 \log \left(\dfrac{I}{I_o} \right)$, where $\left(\dfrac{I}{I_o} \right)$ is the intensity of a sound, I, in W/cm², compared to the minimum sound intensity, I_o, your ear can detect. *(5 marks)*

a) The volume at a concert is measured to be 115 dB. Calculate the ratio, $\left(\dfrac{I}{I_o} \right)$, for this sound in W/cm². *(3 marks)*

Answer: (Module 1, Lesson 6)

$$L = 10 \log \left(\frac{I}{I_o} \right)$$

$$115 = 10 \log \left(\frac{I}{I_o} \right)$$

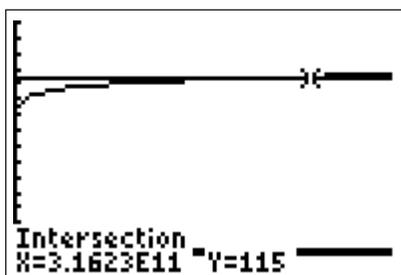$$\frac{115}{10} = \frac{10}{10} \log \left(\frac{I}{I_o} \right)$$

$$11.5 = \log \left(\frac{I}{I_o} \right)$$

$$10^{11.5} = \left(\frac{I}{I_o} \right)$$

$$\left(\frac{I}{I_o} \right) = 3.16227766 \times 10^{11}$$

Or graphical solution:

The value of the ratio, $\left(\dfrac{I}{I_o} \right)$, of this sound is approximately 3.16×10^{11} W/cm².

b) The value of $\left(\dfrac{I}{I_o}\right)$ for a jet engine is approximately $3.16227766 \times 10^{12}$ W/cm^2.

How many decibels is the loudness of a jet engine? *(2 marks)*

Answer: (Module 1, Lesson 6)

$$L = 10 \log\left(\dfrac{I}{I_o}\right)$$

$$L = 10 \log\left(3.16227766 \times 10^{12}\right)$$

$$L = 125$$

The loudness of a jet engine is 125 dB.

3. When an object is projected vertically upward, its height, h, in metres, after t seconds, is given by the equation $h = -4.9t^2 + vt + s$, where v is the initial velocity of the object and s is the initial height (if any) above the ground. *(9 marks)*

a) From your location at the top of a cliff, 55 m above a lake, you throw a rock straight up into the air with a velocity of 15 m/sec. It lands in the water directly below you. Write an equation that models this situation. *(1 mark)*
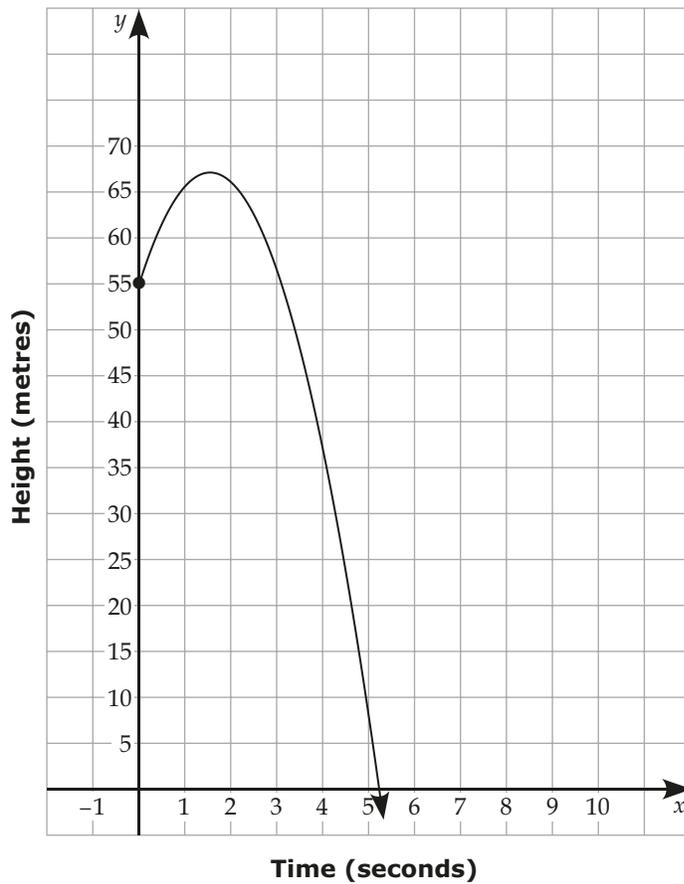
Answer: (Module 1, Lesson 2)

$h = -4.9t^2 + vt + s$

$h = -4.9t^2 + (15)t + (55)$

b) Sketch a graph of this equation, showing the height of the rock as a function of time. Label the axes of the graph. *(3 marks)*

Answer: (Module 1, Lesson 2)

Note: Students may use technology and either sketch or print the screenshot. Alternatively, students may create a table of values, plot the points, and sketch the line. The graph must have scales and labels on axes, have a parabola shape, and have the correct y-intercept.

c) Determine the maximum height the rock attains and the number of seconds it takes to attain the maximum height. Show your work or explain your strategy. Round your final answers to the nearest tenth. *(3 marks)*

Answer: (Module 1, Lesson 2)

This solution shows a TI-84 graphing calculator screen finding the maximum point using 2nd– Calc, Maximum. The coordinates of the vertex are (1.53, 66.48).
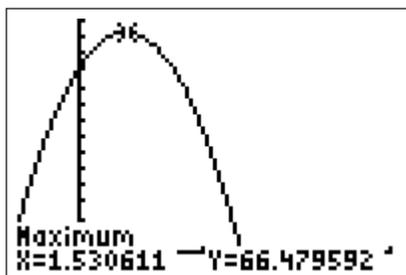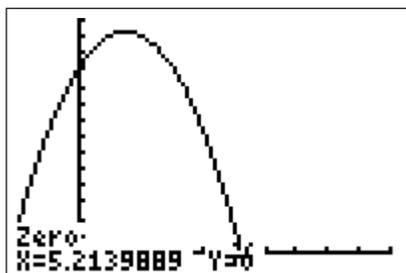

The rock reaches a height of 66.5 m, 1.5 seconds after it is released.

d) How long is the rock in the air? Show your work or explain your strategy. Round your final answer to the nearest tenth. *(2 marks)*

Answer: (Module 1, Lesson 2)

This solution shows a TI-84 graphing calculator screen finding the *x*-intercept using 2nd– Calc, Zero.

The rock hits the water after 5.2 seconds.

4. The village of Winkler was first established in 1906 and was recognized as a city in 2002. The city population recorded during various years between 1911 and 2011 is reported below. *(11 marks)*
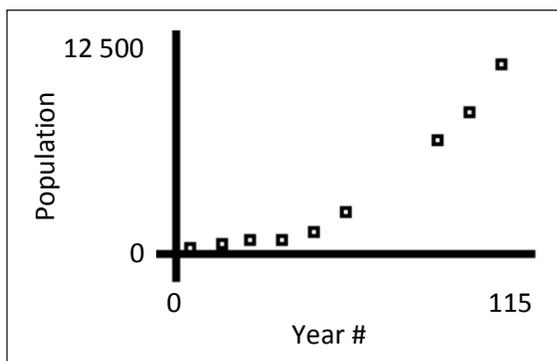
Year	Year #	Population
1911	5	458
1921	15	812
1931	25	1,005
1941	35	957
1951	45	1,331
1961	55	2,529
1991	85	6,400
2001	95	7,999
2011	105	10,670

Source: http://en.wikipedia.org/wiki/Winkler,_Manitoba

a) Plot the population data on a graph using the Year # as the independent variable. Sketch and label your graph below. *(4 marks)*

Answer: (Module 1, Lesson 4)

b) Using technology, find the exponential regression equation that best fits this data. *(2 marks)*

Answer: (Module 1, Lesson 4)

The following information is from the TI-84 Stat, Calc, ExpReg function.

```
ExpReg
 y=a*b^x
 a=410.992473
 b=1.031654821
 r²=.9812288321
 r=.9905699531
```

$y = 410.99(1.0317)^x$

c) Winkler was officially incorporated as a town when the population was approximately 1835 inhabitants. According to this function model, during what year did this happen? *(3 marks)*

Answer: (Module 1, Lesson 4)

The following information is from the TI-84. The line, "$y = 1835$" was graphed. Then the 2nd–Calc, Intersect function was used to find the x-value of the regression function when $y = 1835$.

Intersection
X=48.011114 Y=1835
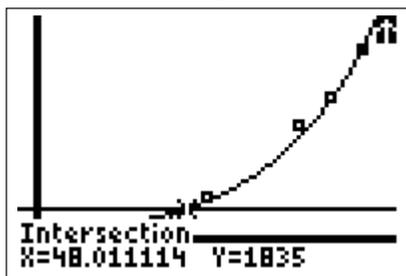
The population of Winkler was approximately 1835 during its 48th year, or in 1954.

d) Using the exponential regression equation or technology, determine the approximate population of the city of Winkler during its centennial anniversary, in 2006. *(2 marks)*
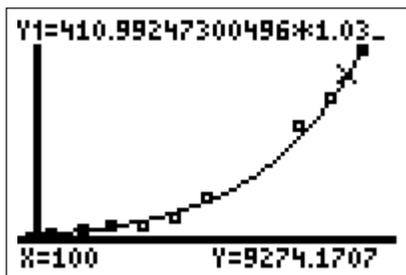
Answer: (Module 1, Lesson 4)

$$y = 410.99 \times 1.0317^x$$

$$y = 410.99 \times 1.0317^{100}$$

$$y = 9314.8$$

Or

Y1=410.99247300496*1.03_

X=100   Y=9274.1707

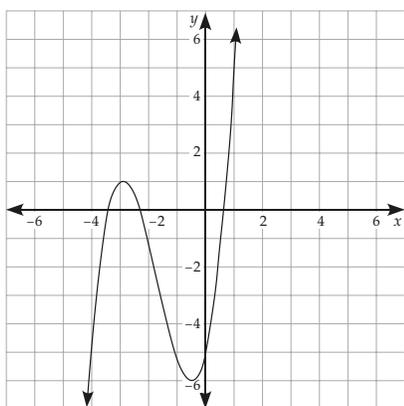The population was approximately 9300 inhabitants.

5. Match each the following equations with its corresponding graph. Write the letter of the equation below the correct graph. *(6 marks)*

   a) $y = \log(x) + 5$

   b) $y = 5^x$

   c) $y = \ln x$

   d) $y = e^x$

   e) $y = 0.5(x^2 + x - 6)$

   f) $y = x^3 + 5x^2 + 4x - 5.1$

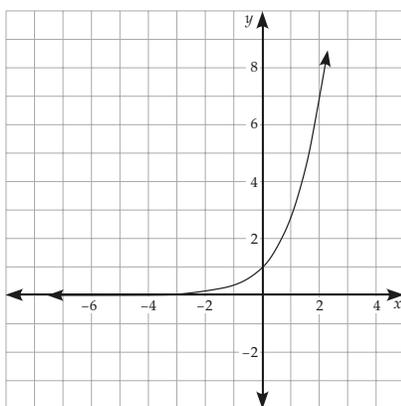   *Answers:*   (Module 1, Lessons 1, 3, and 5)
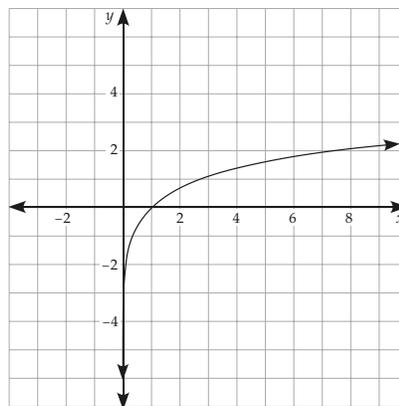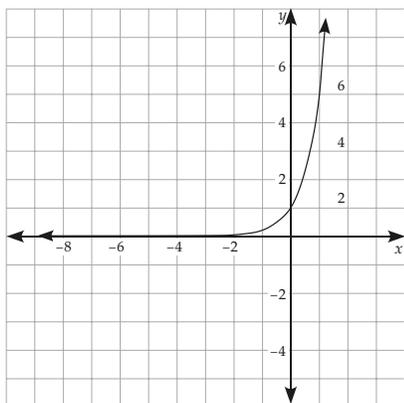
A

f) $y = x^3 + 5x^2 + 4x - 5.1$

B

d) $y = e^x$

C
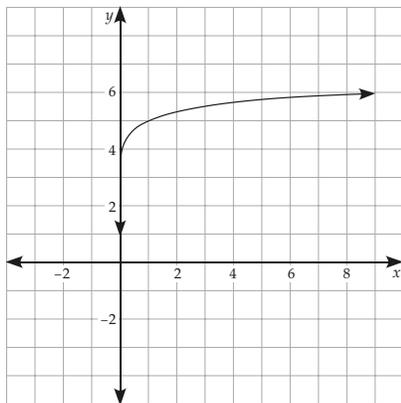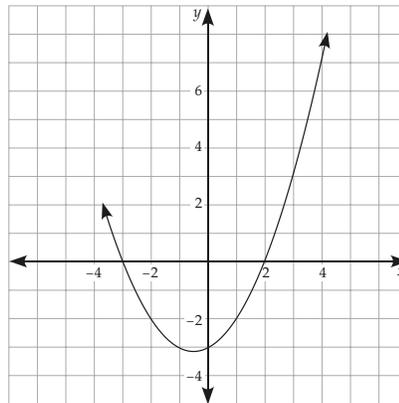
c) $y = \ln x$

D

b) $y = 5^x$

E

a) $y = \log(x) + 5$

F

e) $y = 0.5(x^2 + x - 6)$

6. List **five** things to consider when assessing the accuracy, reliability, and relevance of data and information. *(5 marks)*

    Answer: (Module 2, Lesson 2)

    Answers will vary. Possible answers include:

    Is the data consistent with multiple reputable sources? Do qualified experts agree with the data and information? Is the data primary or secondary? Has the data been taken from a sample that was an appropriately selected random and representative cross-section of the population? Is the data free from errors, carefully collected, and recorded?

    Are the sources used by the author trustworthy and do they adhere to high standards, or are they opinionated and far-fetched? When was the page published or last updated? Why was the information published? Is the site trying to sell something, to inform others, to teach, to persuade? Who is the intended audience?

    Find out who actually authored the information you are considering and check their credentials. Do they speak with authority? Are they working for a reputable organization? Is the author's name and contact information given? Who holds the copyright to the information published?

    Advertising or sponsorship on the site may indicate that you need to be suspicious of the content of that page. Look for biased wording or examples of opinion rather than the use of facts. Determine if underlying assumptions are affecting the reliability of the data.

7. Create a Venn diagram to represent the following information and answer the question below. *(5 marks)*

Students at a private dance studio may take ballet, hip hop, or tap dance classes.

> 3 students at the studio take all three.
>
> 7 students take tap and hip hop.
>
> 5 are in ballet and hip hop classes.
>
> All students who take tap also take either ballet or hip hop.
>
> 21 of the students take hip hop or tap.
>
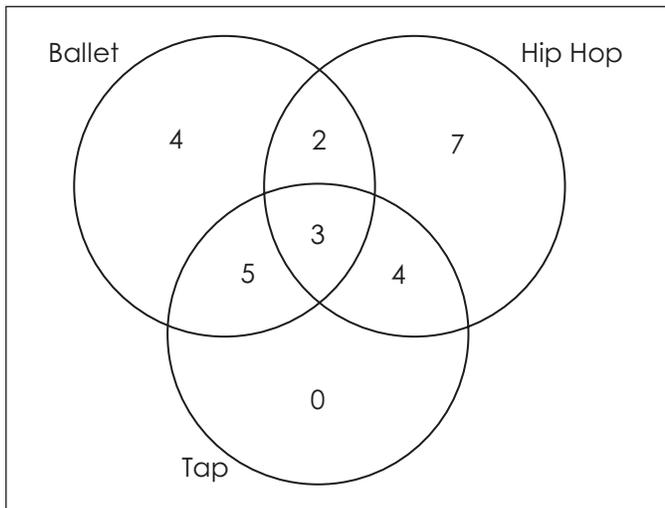> 14 are in ballet classes.
>
> 12 students take tap.

How many students attend the studio? _____

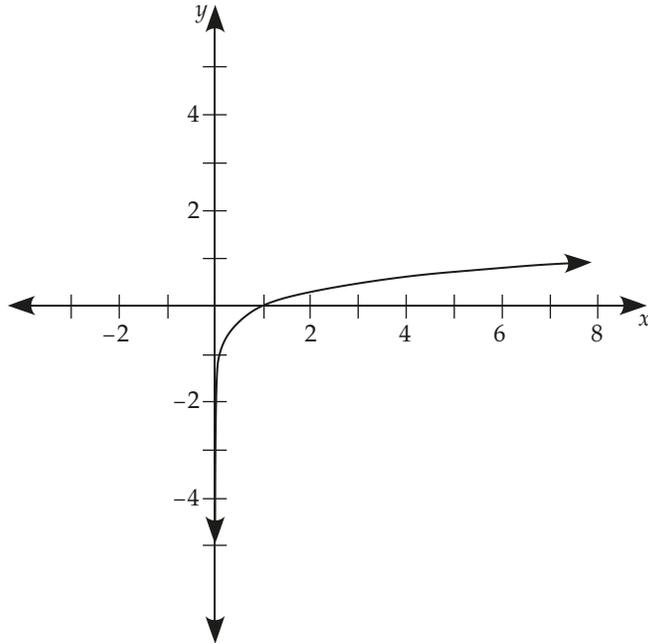*Answer:*   (Module 3, Lesson 3)

How many students attend the studio? _____25_____

Name: _____

8. Use graphing technology to sketch the graph of $y = \log(x)$ and complete the table of information. *(5 marks)*

   Answer: (Module 1, Lesson 5)

Domain	$\{x > 0\}$
Range	$\{y \in \Re\}$
x-intercept	1
Equation of asymptote	$x = 0$

9. In a class of 25 students, 8 are on student council, 5 are on the volleyball team, and 2 students are on both the council and the team. How many students are neither on the council nor on the team? Justify your answer. *(3 marks)*

   Answer: (Module 3, Lesson 3)

   There are 2 students on both the student council and the volleyball team. This means that there are 6 students who are only on the student council and 3 students who are only on the volleyball team. That leaves 14 students who are on neither $(25 - 2 - 6 - 3)$.

10. Sets A, B, and C are defined as:

$A = \{x \mid x < 5, x \in \Re\}$

$B = \{x \mid x \geq -2, x \in \Re\}$

$C = \{x \mid -3 < x \leq 2, x \in \Re\}$

a) Graph sets A, B, and C using the number line below. *(3 marks)*

Answer: (Module 3, Lesson 1)
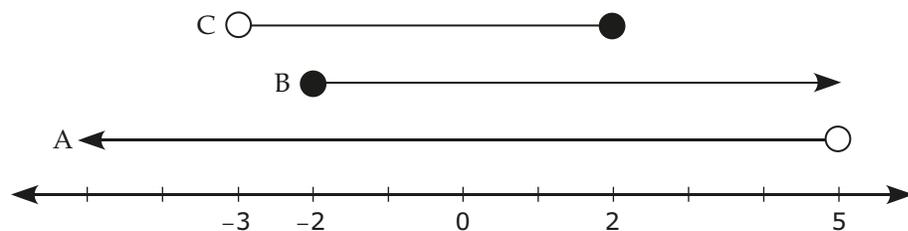
b) Using set notation, define the following: *(3 marks)*

i) A ∩ B =

ii) B ∪ C =

iii) A ∩ B′ =

Answers: (Module 3, Lesson 1)

i) $A \cap B = \{x \mid -2 \leq x < 5, x \in \Re\}$

ii) $B \cup C = \{x \mid x > -3, x \in \Re\}$

iii) $A \cap B′ = \{x \mid x < -2, x \in \Re\}$

11. Answer the following questions based on the conditional statement, "If a polygon is an octagon, then it has 8 sides." *(8 marks)*

a) State the hypothesis and the conclusion of this statement. *(1 mark)*

Answer: (Module 3, Lesson 2)

Hypothesis: The polygon is an octagon.

Conclusion: The polygon has 8 sides.

b) Complete the following truth table for the conditional statement. Justify your results by describing the possible states of the hypothesis and conclusion for each of the four possible cases. *(5 marks)*

Answer: (Module 3, Lesson 2)

Hypothesis	Conclusion	Conditional Statement
T	T	T
T	F	F
F	T	F
F	F	T

Case 1: The polygon is an octagon. The polygon has 8 sides. True

Case 2: The polygon is an octagon. The polygon does not have 8 sides. False

Case 3: The polygon is not an octagon. The polygon has 8 sides. False

Case 4: The polygon is not an octagon. The polygon does not have 8 sides. True

c) Write the converse of the given statement. *(1 mark)*

Answer: (Module 3, Lesson 2)

"If the polygon has 8 sides, then it is an octagon."

d) Is the given statement biconditional? Explain. *(1 mark)*

Answer: (Module 3, Lesson 2)

The statement is biconditional because both the statement and the converse are true.

12. An event consists of tossing three coins and noting if they land Heads, H, or Tails, T. Write the sample space, S, of this event. *(1 mark)*

Answer: (Module 4, Lesson 1)

S = {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}

13. Each of the letters in the words PRACTICE EXAM are written on individual cards and placed in a hat. *(5 marks)*

a) One card is randomly drawn from the hat. Determine the probability of drawing a C. Write the final answer as a reduced fraction. *(1 mark)*

Answer: (Module 4, Lesson 1)

$$P(C) = \frac{2}{12} = \frac{1}{6}$$

b) One card is randomly drawn from the hat. Determine the probability of drawing a vowel. Write the final answer as a percentage. *(1 mark)*

Answer: (Module 4, Lesson 1)

$$P(\text{vowel}) = \frac{5}{12} = 41.7\%$$

c) One card is drawn, the letter is noted, and the card is replaced. A second card is drawn. Determine the probability of drawing the P and then the M. Write the final answer as a fraction. *(1 mark)*

Answer: (Module 4, Lesson 1)

$$P(P, M) = \frac{1}{12} \times \frac{1}{12} = \frac{1}{144}$$

d) One card is drawn, the letter is noted, and then a second card is drawn without replacing the first card. What is the probability of drawing two consonants? Write the final answer as a decimal to the nearest thousandth. *(1 mark)*

Answer: (Module 4, Lesson 1)

$$P(\text{consonant, consonant}) = \frac{7}{12} \times \frac{6}{11}$$

$$P(\text{consonant, consonant}) = \frac{42}{132} = \frac{7}{22} = 0.3\overline{18}$$

e) One card is drawn at random from the hat. Determine the probability that it is not the X. *(1 mark)*

Answer: (Module 4, Lesson 1)

$$P(X') = 1 - P(X) = 1 - \frac{1}{12}$$

$$P(X') = \frac{11}{12} \text{ or } 0.917$$

14. A dart is thrown randomly and lands on a square dartboard with three areas, as pictured below. Note that the sides of square A are 5 times as long as the sides of square C, and the sides of square B are 3 times as long as the sides of square C.
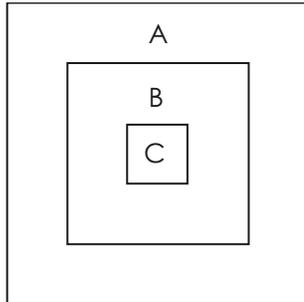
Calculate: *(4 marks)*

a) Probability that a dart lands in area A

Answer: (Module 4, Lesson 1)

$$A = \frac{16}{25}$$

b) Probability that a dart lands in area B

Answer: (Module 4, Lesson 1)

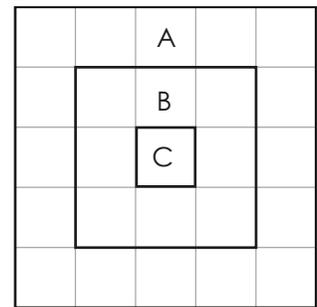$$B = \frac{8}{25}$$

c) Odds in favour of a dart landing in area B

Answer: (Module 4, Lesson 2)

$B = 8:17$

d) Odds against a dart landing in area C

Answer: (Module 4, Lesson 2)

$C = 24:1$

15. An experiment consists of randomly drawing a card from a deck of 50 cards numbered from 1 to 50. What is the probability of drawing a card that is an even number or a multiple of 5? *(4 marks)*

Answer: (Module 4, Lesson 3)

$P(\text{even or multiple of five}) = P(\text{even}) + P(\text{multiple of five}) - P(\text{even and multiple of five})$

$P(\text{even or multiple of five}) = \dfrac{25}{50} + \dfrac{10}{50} - \dfrac{5}{50}$

$P(\text{even or multiple of five}) = \dfrac{30}{50} = \dfrac{3}{5}$

16. Three bags contain marbles:

Bag 1	Bag 2	Bag 3
3 green	2 green	4 green
2 red	1 red	3 white
1 white	5 black	2 black

Answer the following questions regarding one or more of the bags. *(11 marks)*

a) Using Bag 1, determine the probability of drawing a green marble. *(1 mark)*

Answer: (Module 4, Lesson 2)

$P(\text{green, Bag 1}) = \dfrac{3}{6} = \dfrac{1}{2}$

b) Using Bag 2, determine the odds in favour of drawing a green marble. *(1 mark)*

Answer: (Module 4, Lesson 2)

$2:6$ or $1:3$

c) Using Bag 3, determine the odds against drawing a black marble. *(1 mark)*

Answer: (Module 4, Lesson 2)

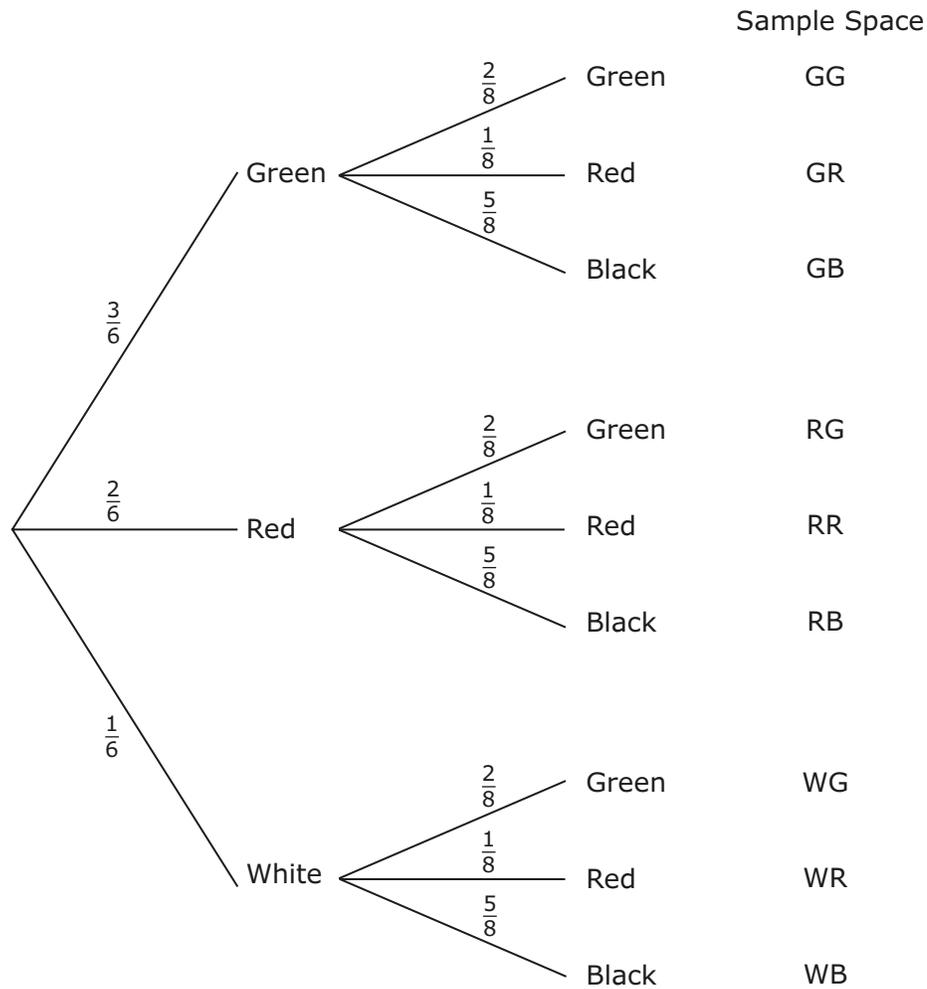$7:2$

d) Create a tree diagram to represent the sample space for the event of drawing one marble from Bag 1 and then one marble from Bag 2. Include the probability of drawing each colour along each branch. *(5 marks)*

Answer: (Module 4, Lesson 1)

Sample Space

Bag 1		Bag 2	Sample Space
Green $\frac{3}{6}$	$\frac{2}{8}$	Green	GG
	$\frac{1}{8}$	Red	GR
	$\frac{5}{8}$	Black	GB
Red $\frac{2}{6}$	$\frac{2}{8}$	Green	RG
	$\frac{1}{8}$	Red	RR
	$\frac{5}{8}$	Black	RB
White $\frac{1}{6}$	$\frac{2}{8}$	Green	WG
	$\frac{1}{8}$	Red	WR
	$\frac{5}{8}$	Black	WB

e) When choosing one marble from each of Bag 1 and Bag 2, what is the probability of choosing two marbles of the same colour? *(2 marks)*

Answer:

$$P(\text{GG or RR}) = \frac{3}{6} \cdot \frac{2}{8} + \frac{2}{6} \cdot \frac{1}{8} = \frac{6}{48} + \frac{2}{48} = \frac{8}{48} = \frac{1}{6}$$

f) When choosing one marble from each of Bag 1 and Bag 2, what is the probability of choosing two marbles of different colours? *(1 mark)*

Answer:

$$1 - P(\text{GG or RR}) = 1 - \frac{1}{6} = \frac{5}{6}$$

17. A customer enters a restaurant. The probability that the customer orders at least one of either a steak or a salad is $\frac{8}{11}$. The probability that the customer orders steak is $\frac{2}{11}$, while the probability of ordering salad is $\frac{7}{11}$. What is the probability that the customer orders both a steak and a salad? *(4 marks)*

Answer:

$P(\text{Steak or Salad}) = P(\text{Steak}) + P(\text{Salad}) - P(\text{Steak and Salad})$

$$\frac{8}{11} = \frac{2}{11} + \frac{7}{11} - P(\text{Steak and Salad})$$

$$\frac{8}{11} - \frac{2}{11} - \frac{7}{11} = -P(\text{Steak and Salad})$$

$$\frac{-1}{11} = -P(\text{Steak and Salad})$$

$$\frac{1}{11} = P(\text{Steak and Salad})$$

The probability the customer orders both a steak and a salad is $\frac{1}{11}$.