Grade 11 Applied Mathematics (30S)

Final Practice Examination
Answer Key

GRADE 11 APPLIED MATHEMATICS

Final Practice Examination
Answer Key

Name: ______________________________________

Student Number: ______________________________

Attending ❑ Non-Attending ❑

Phone Number: ________________________________

Address: _____________________________________

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For Marker's Use Onl

Date: _______________________________________

inal Mark: ________ /100 = ________ %

Comments:

Instructions

The final examination is based on Modules 5 to 8 of the Grade 11 Applied Mathematics course. It is worth 25% of your final mark in this course.

Time

You will have a maximum of **2.5 hours** to complete the final examination.

Notes

You are allowed to bring the following to the examination: pens/pencils (2 or 3 of each), metric and imperial rulers, a graphing and/or scientific calculator, and your Final Exam Resource Sheet. Your Final Exam Resource Sheet must be handed in with the examination. Graphing technology (either computer software or a graphing calculator) **is required** to complete this examination.

Show all calculations and formulas used. Use all decimal places in your calculations and round the final answers to the correct number of decimal places. Include units where appropriate. Clearly state your final answer. Final answers without supporting calculations or explanations will **not** be awarded full marks. Indicate equations and/or keystrokes used in calculations.

When using graphing technology, include a screenshot or printout of graphs **or** sketch the image and indicate the window settings (maximum and minimum *x*- and *y*-values), increments, and axis labels, including units.

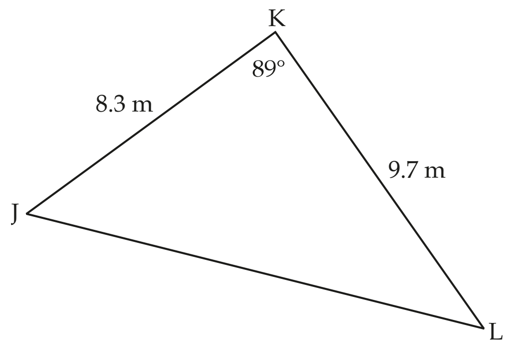
Name: ____________________

Answer all questions to the best of your ability. Show all your work.

Module 5: Trigonometry *(25 marks)*

1. Solve for all the missing angles and all the missing sides in the triangles below. Round your answers to one decimal place.

   a) *(6 marks)* (Module 5, Lesson 3)



Answer:

First, find the size of k, using the Cosine Law.

$$k^2 = j^2 + l^2 - 2(j)(l)\cos\angle K$$

$$k^2 = 9.7^2 + 8.3^2 - 2(9.7)(8.3)\cos 89°$$

$$k^2 = 162.98 - 161.02\cos 89°$$

$$k = \sqrt{162.98 - 161.02\cos 89°}$$

$$k = 12.7 \text{ m}$$

Now, find $\angle$L using the Sine Law.

$$\frac{\sin\angle L}{l} = \frac{\sin\angle K}{k}$$

$$\frac{\sin\angle L}{8.3} = \frac{\sin 89°}{12.7}$$

$$\angle L = \sin^{-1}\left(\frac{8.3\sin 89°}{12.7}\right)$$

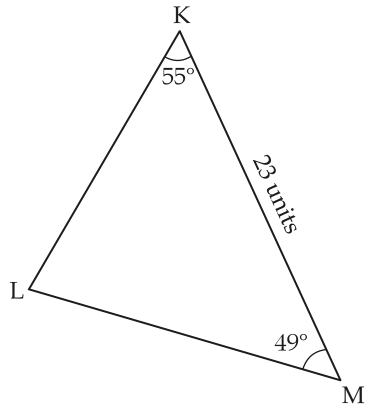
$$\angle L = 40.8°$$

Finally, find $\angle$J using the 180° rule.

$$\angle J = 180° - 89° - 40.8°$$

$$\angle J = 50.2°$$

b) *(5 marks)* (Module 5, Lesson 2)



Answer:

First, find ∠L using the 180° rule.

$180° - 55° - 49° = 76°$

Now use the Sine Law to find *k*.

$$\frac{k}{\sin 55°} = \frac{23}{\sin 76°}$$

$$k = 19.4 \text{ units}$$

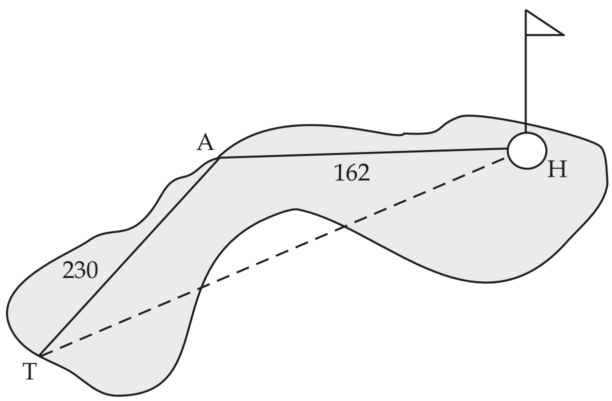
Now use the Sine Law again to find *m*.

$$\frac{m}{\sin 49°} = \frac{23}{\sin 76°}$$

$$m = 17.9 \text{ units}$$

Name: ____

2. A golf hole has a dogleg as shown in the diagram below. What is the angle at the dogleg (TAH), if the distance from the tee, T, to the hole, H, is 362 m? *(3 marks)*
(Module 5, Lesson 3)



Answer:

To find this angle, use the Cosine Law.

$$a^2 = t^2 + h^2 - 2(t)(h)\cos\angle A$$

$$362^2 = 162^2 + 230^2 - 2(162)(230)\cos\angle A$$

$$131044 = 79144 - 74520\cos\angle A$$

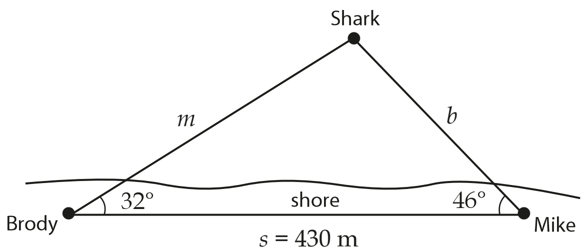
$$51900 = -74520\cos\angle A$$

$$\angle A = \cos^{-1}\left(-\frac{51900}{74520}\right)$$

$$\angle A = 134.1°$$

The angle at the dogleg is 134.1°.

3. Brody and Mike are two harbour masters who are tracking the position of a great white shark. The harbour masters are located in buildings on the ocean shore that are 430 m apart. Brody's line of sight to the shark makes an angle of 32° with his line of sight to Mike. Mike's line of sight to the shark makes an angle of 46° with his line of sight to Brody. Determine the distance of the shark from each harbour master. Round your answers to the nearest metre. *(5 marks)* (Module 5, Lesson 4)



Answer:

To solve this problem, first find the missing angle in the diagram of the triangle above, using the 180° rule.

$$\angle S = 180° - 32° - 46°$$

$$\angle S = 102°$$

Now, you can determine the distance between the shark and Mike using the Sine Law.

$$\frac{b}{\sin \angle B} = \frac{s}{\sin \angle S}$$

$$b = \frac{430 \sin 32°}{\sin 102°}$$

$$b = 233 \text{ m}$$

The shark and Mike are 233 m apart.

Finally, calculate the distance between the shark and Brody using the Sine Law.

$$\frac{m}{\sin \angle M} = \frac{s}{\sin \angle S}$$

$$m = \frac{430 \sin 46°}{\sin 102°}$$

$$m = 316 \text{ m}$$

The shark and Brody are 316 m apart.

Name: ______________________________________

4. Given points A, B, and C, determine measurements for angle A, side *a*, and side *b*, so that each of the following situations is created. *(3 × 2 marks each = 6 marks)*
(Module 5, Lesson 5)

a) no triangle is possible

Answer:

Note: Other solutions are possible.

In ΔABC, let ∠A = 32°, *a* = 1 m, and *b* = 3.5 m.

To show there is no triangle possible, you can use the Sine Law.

$$\frac{\sin 32^\circ}{1} = \frac{\sin B}{3.5}$$

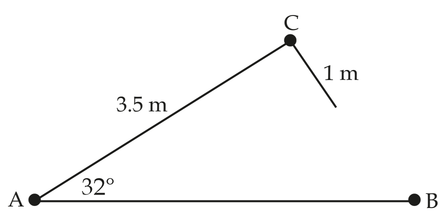
$$3.5\left(\frac{\sin 32^\circ}{1}\right) = \sin B$$

$$\sin B = 1.854$$

$$\angle B = \sin^{-1}(1.854)$$

$$\angle B = \text{ not possible}$$

As the value of sin B > 1, it is not possible to complete this triangle.



b) two triangles are possible

Answer:

In ΔABC, let $\angle A = 42°$, $a = 6$ m, and $b = 7$ m.

To show there are two solutions, you can use the Sine Law.

$$\frac{\sin 42°}{6} = \frac{\sin B}{7}$$

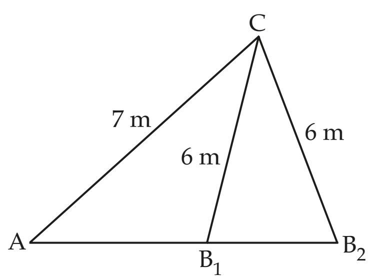
$$7\left(\frac{\sin 42°}{6}\right) = \sin B$$

$$\sin B = 0.7806$$

$$\angle B = \sin^{-1}(0.7806)$$

$$\angle B = 51.3°$$

As the value of $\sin B < 1$, and side a is shorter than side b, it is possible to create two triangles.



Name: ____

c) one right triangle is possible

Answer:

In ΔABC, let $\angle$A = 30°, a = 3.5 m, and b = 7 m.

To show that only one right triangle is possible, you can use the Sine Law.

$$\frac{\sin 30}{3.5} = \frac{\sin \text{B}}{7}$$

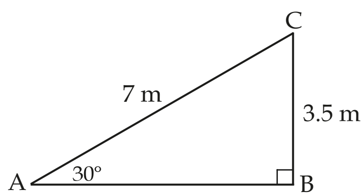
$$7\left(\frac{\sin 30°}{3.5}\right) = \sin \text{B}$$

$$\sin \text{B} = 1$$

$$\angle \text{B} = \sin^{-1}(1)$$

$$\angle \text{B} = 90°$$

As the value of sin B = 1, only one triangle is possible and it is a right triangle.



Module 6: Statistics *(23 marks)*

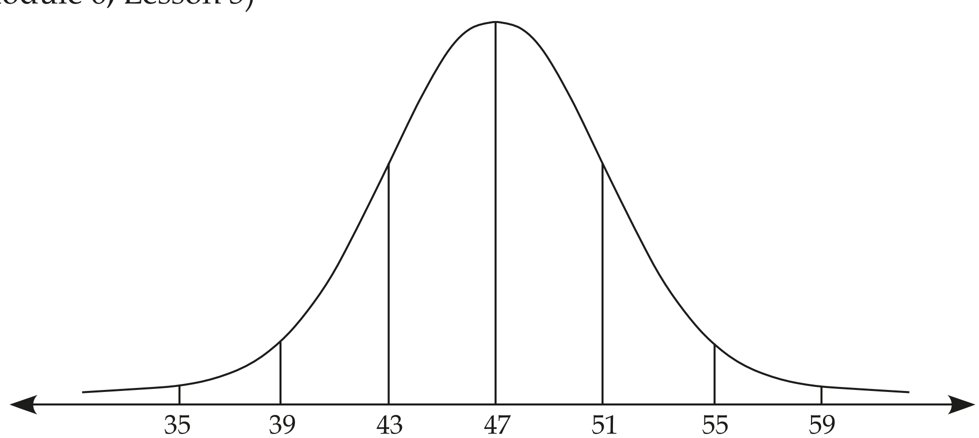
1. State two properties that apply to all normal distributions. *(2 marks)* (Module 6, Lesson 3)

   Answer:

   Any two of the following:

   - The curve is perfectly symmetrical about the mean. The two halves of the curve are a mirror image of each other around the mean.
   - The standard deviation determines whether the curve is tall and skinny or short and wide or any variation between.
     - If the standard deviation, σ, is large, the curve will be wider horizontally and shorter vertically.
     - If the standard deviation, σ, is small, the curve will be narrower horizontally and taller vertically.
   - The mean, the median and the mode are all the same value.
   - Almost all data, 99.7%, is within 3 standard deviations of the mean.
   - The probability that a score falls within one standard deviation of the mean on either side is approximately 68% or 0.68.
   - The probability that a score falls within two standard deviations of the mean is approximately 95% or 0.95.
   - The probability that a score falls within three standard deviations of the mean is approximately 99.7% or 0.997.
   - The total area under the curve is always 1, or 100%. This simply indicates that all of the data is represented by points under the curve.

2. Determine the mean and standard deviation of the following normal curve. *(2 marks)* (Module 6, Lesson 3)



   Answer:

   The mean is 47.

   The standard deviation is 4.

Name: ____

3. The following data represents a sample of the waiting times at a dentist's office. These times are expressed in minutes.

12	19	2	13	21	23	18	13	20
23	26	7	10	8	16	12	17	21

a) Determine the mean and standard deviation for the data. Round to one decimal place. *(2 marks)* (Module 6, Lesson 2)

Answer:

Mean: 15.6

Standard Deviation: 6.4

b) Calculate the percentage, to one decimal place, of the waiting times that fall within ±1 standard deviation of the mean. *(2 marks)* (Module 6, Lesson 3)

Answer:

One standard deviation less than the mean is 9.2. One standard deviation more than the mean is 22.0.

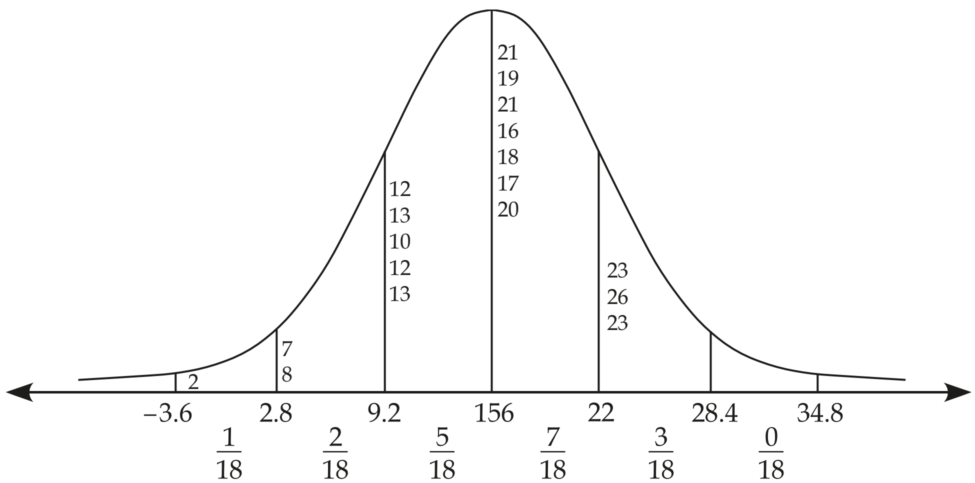
The data values that fall in this range are 12, 19, 13, 10, 21, 16, 18, 12, 13, 17, 20, and 21.

There are 12 out of 18 values in this range. This is equal to 67%.

c) Verify if the waiting times resemble a normal distribution. Justify your answer. *(3 marks)* (Module 6, Lesson 3)

Answer:

The percentage of data that falls within one standard deviation of the mean is close to what it should be. Therefore, we should sketch a normal curve to help determine if the data resembles a normal distribution.



$\pm$1 standard deviation: $\frac{12}{18} = 66.7\%$

$\pm$2 standard deviations: $\frac{17}{18} = 94.4\%$

$\pm$3 standard deviations: $\frac{18}{18} = 100\%$

This data is close to approximating a normal curve, but not exact. The percentages of data that should fall in each category in the normal distribution are similar to what they should be. The percentage of data within one standard deviation of the mean is close to 68%. the percentage of data that is within two standard deviations of the mean is close to 95%. All of the data is within three standard deviations of the mean, which is close to 99.7%. Therefore, this data does approximate a normal distribution.

Name: ____

4. The lifetime of a microwave is normally distributed with a mean of 4.7 years and a standard deviation of 0.4 years. (Module 6, Lesson 5)

   What percent of microwaves will last at least 5 years? *(2 marks)*

   Answer:

   The mean is 4.7.

   The standard deviation is 0.4.

   The lower bound is 5.

   Using a probability calculator, the percent of microwaves that will last at least 5 years is 22.66%.

5. For a university calculus course, the professor decided that only 5% of students would fail. Assume the marks are normally distributed. (Module 6, Lesson 5)

   a) What z-score represents this value? *(1 mark)*

      Answer:

      Using a probability calculator:

      The mean is 0.

      The standard deviation is 1.

      The percent is 0.05.

      The z-score at which 5% of data is below that value is -1.6449.

      5% of the data will have a z-score of -1.6449 or lower.

   b) If the mean mark was 71.4 and the standard deviation was 14.3, what is the lowest mark that would represent a passing mark? *(2 marks)*

      Answer:

      $$-1.6449 = \frac{X - 71.4}{14.3}$$

      The lowest passing mark is 48.

   c) If the mean was 60 and the standard deviation was 16.2, will you pass with a mark of 45? Justify your answer using mathematical calculations. *(2 marks)*

      Answer:

      $$z = \frac{45 - 60}{16.2} = -0.9259$$

      This is a z-score that is above the lowest z-score of -1.6449. Therefore, you will pass.

Name: ___________________________________

6. Based on survey results, 61 out of 100 people who go to a car dealership end up purchasing a vehicle. These results are accurate to within ±5%, 19 times out of 20. (Module 6, Lesson 6)

   a) Determine the confidence level. *(1 mark)*

      Answer:

      $\frac{19}{20} = 95\%$

   b) State the margin of error. *(1 mark)*

      Answer:

      ±5%

   c) What is the confidence interval? *(1 mark)*

      Answer:

      56% – 66%

7. Explain how the margin of error affects the confidence interval. *(1 mark)* (Module 6, Lesson 6)

   Answer:

   The smaller the margin of error, the smaller the confidence interval. The larger the margin of error, the larger the confidence interval.

8. In a study of the water quality of Manitoba lakes, pollutants are measured in parts per million (ppm). The average amount of pollutants found in the lakes for a particular region of Manitoba is 0.88 ppm, with a standard deviation of 0.27. The data follows a normal distribution. The measured pollutants at Green Lake are 1.21 ppm. Determine the equivalent *z*-score. *(1 mark)* (Module 6, Lesson 4)

   Answer:

   $$z = \frac{1.21 - 0.88}{0.27}$$

   $$z = 1.22$$

Module 7: Mathematical Models *(27 marks)*

1. A delivery truck is bringing goods to a store 180 km away. For the first half hour, the driver maintains a speed of 60 km/h. The driver then accelerates to 100 km/h for the remainder of the trip. Create a graph to represent this trip. *(3 marks)* (Module 7, Lesson 2)

   Answer:

Note:

1 mark for each correct slope

1 mark for correct axes

Name: ____________________________________

2. Determine a point that is in the solution region for the following system of linear inequalities. Prove that your point is in the solution region. *(3 marks)* (Module 7, Lesson 5)

$$y \le -\frac{1}{2}x + 2$$

$$y \ge -2x + 5$$

Answer:

A possible point is (3, 0).

For the first inequality:

$$0 \le -\frac{1}{2}(3) + 2$$

$$0 \le -1.5 + 2$$

$$0 \le 0.5$$

For the second inequality:

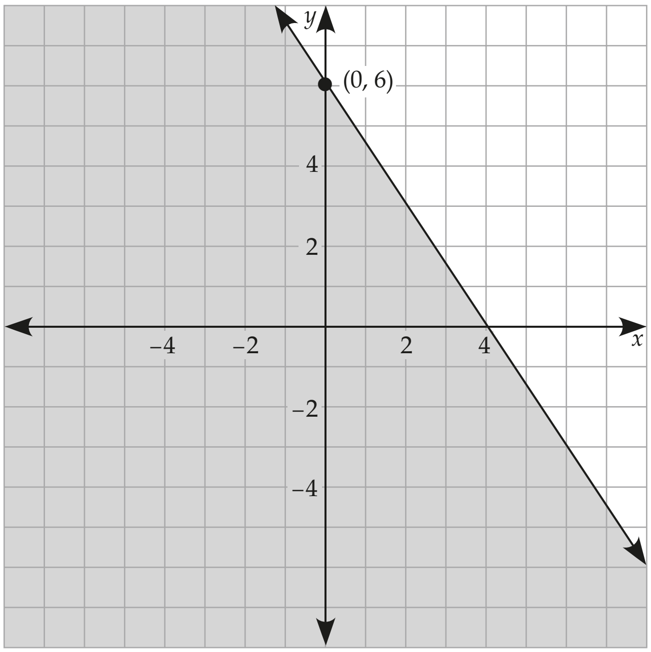
$$0 \ge -2(3) + 5$$

$$0 \ge -6 + 5$$

$$0 \ge -1$$

Since both statements are true, the point (3, 0) is in the solution region.

3. Write the linear inequality that is represented in the graph below. *(3 marks)*
(Module 7, Lesson 4)



Answer:

y-intercept: 6

Slope: $\frac{-3}{2}$

Therefore, the equation of the boundary line is $y = \frac{-3}{2}x + 6$.

Boundary line: solid

Therefore, the inequality is either $\leq$ or $\geq$.

Since the graph is shaded below the line, the inequality is $y \leq \frac{-3}{2}x + 6$.

Note:

1 mark for the correct slope

1 mark for the correct y-intercept

1 mark for the correct inequality sign

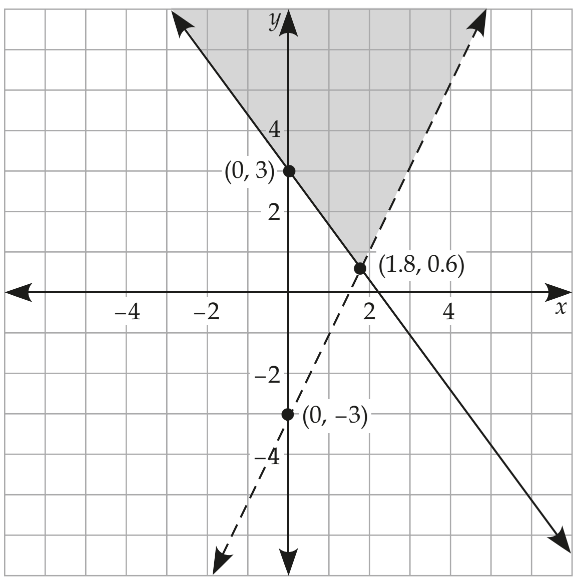
Name: ____

4. Sketch the following system of inequalities. Shade the region of overlap. *(5 marks)* (Module 7, Lesson 5)

$$4x + 3y \geq 9$$

$$-y < -2x + 3$$

Answer:



$4x + 3y \geq 9$	$-y < -2x + 3$
$3y \geq -4x + 9$	$y > 2x - 3$
$y \geq \frac{-4}{3}x + 3$	Slope: $\frac{2}{1}$
Slope: $\frac{-4}{3}$	y-intercept: -3
y-intercept: 3	
Boundary line: solid Shade above	Boundary line: dashed Shade above

Point of intersection: (1.8, 0.6)
(found using technology)

Note:

1 mark for each correct line (half-mark for slope and half-mark for y-intercept) × 2 lines

1 mark for each correct solid or dashed line × 2 lines

1 mark for the correctly shaded region

5. You have \$200 to spend on clothing. Shirts costs \$20 and pants cost \$35. Determine how many shirts and pants you can purchase. (Module 7, Lesson 4)

   a) Define the two variables used in this scenario. *(1 mark)*

      Answer:

      Let x represent the number of shirts purchased.

      Let y represent the number of pants purchased.

   b) What are the restrictions on these variables? Explain. *(2 marks)*

      Answer:

      $x \geq 0$

      $y \geq 0$

      You cannot buy a negative number of shirts and pants.

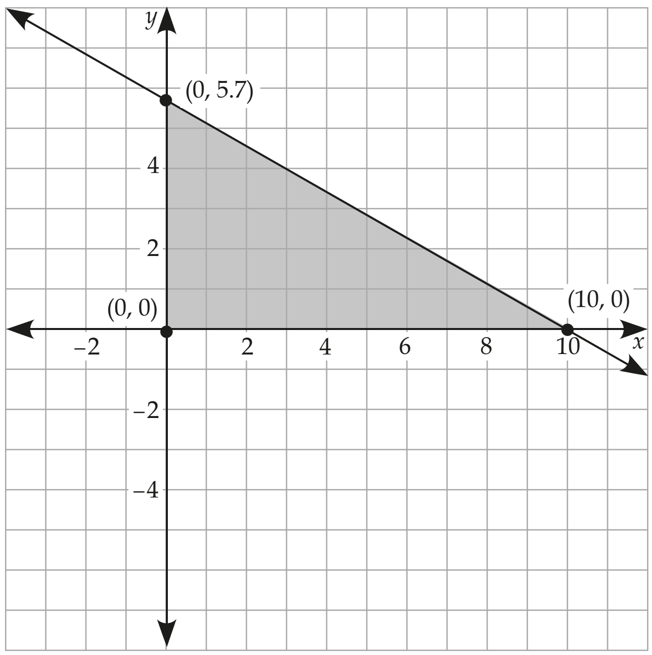
   c) Write an inequality to represent how many shirts and how many pants you can purchase. *(1 mark)*

      Answer:

      $20x + 35y \leq 200$

d) Graph the inequality you created in (b) and (c), and label the vertices of the solution region. *(3 marks)*

Answer:



$$20x + 35y \leq 200$$

$$35y \leq -20x + 200$$

$$y \leq \frac{-20}{35}x + \frac{200}{35}$$

$$y \leq \frac{-4}{7}x + \frac{40}{7}$$

y-intercept: $\frac{40}{7}$ or approximately 5.7

e) If you buy 4 shirts, how many pairs of pants can you possibly buy? *(1 mark)*

Answer:

You can buy 1, 2, or 3 pairs of pants.

f) If you buy 3 pairs of pants, how many shirts can you possibly buy? *(1 mark)*

Answer:

You can buy 1, 2, 3, or 4 shirts.

6. A food truck sells pizza and chicken wings. Each day, they sell at least three times as many orders of pizza as they do chicken wings. The food truck only has room to hold 42 orders of pizza and 30 orders of chicken wings. Pizzas are sold for $10 each and chicken wings are sold for $8 each. Determine the number of pizza and chicken wings that would have to be sold to maximize the amount of money the food truck receives in sales in one day. (Module 7, Lesson 6)

   a) Write the equation of the objective function. *(1 mark)*

      Answer:

      $P = 10x + 8y$

   b) The owner of the food truck graphs this inequality and finds the vertices of the solution region to be (0, 0), (42, 14), and (42, 0). How many of each type of food should be sold to maximize profits? Verify your solution by testing all vertices. *(3 marks)*

      Answer:

      Check the vertices:

      $P = 10x + 8y$

      (0, 0)

      $P = \$0$

      (42, 14)

      $P = \$532$

      (42, 0)

      $P = \$420$

      The greatest profit comes from selling 42 orders of pizza and 14 orders of chicken wings.

Name: ______________________________

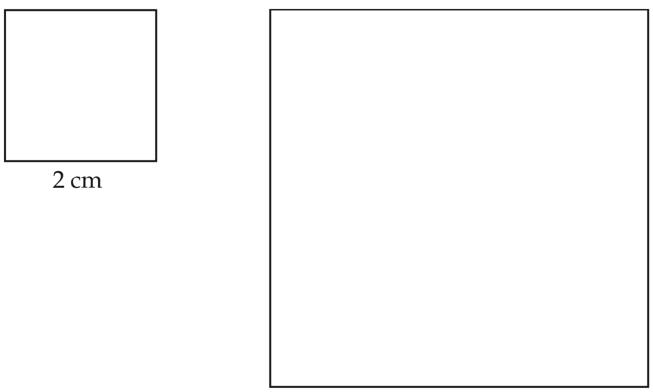
Module 8: Scale Factors for 2-D and 3-D Shapes *(25 marks)*

1. Provide an example in your life of when you would use a model of a 2-D shape. *(1 mark)* (Module 8, Lesson 1)

   Answer:

   Examples may vary. Possible answers could be drawing of a room floor plan or using a map.

2. The following squares are similar. Determine the scale factor if the larger square is the scale model and the area of the larger square is 25 cm^2. *(3 marks)* (Module 8, Lesson 2)

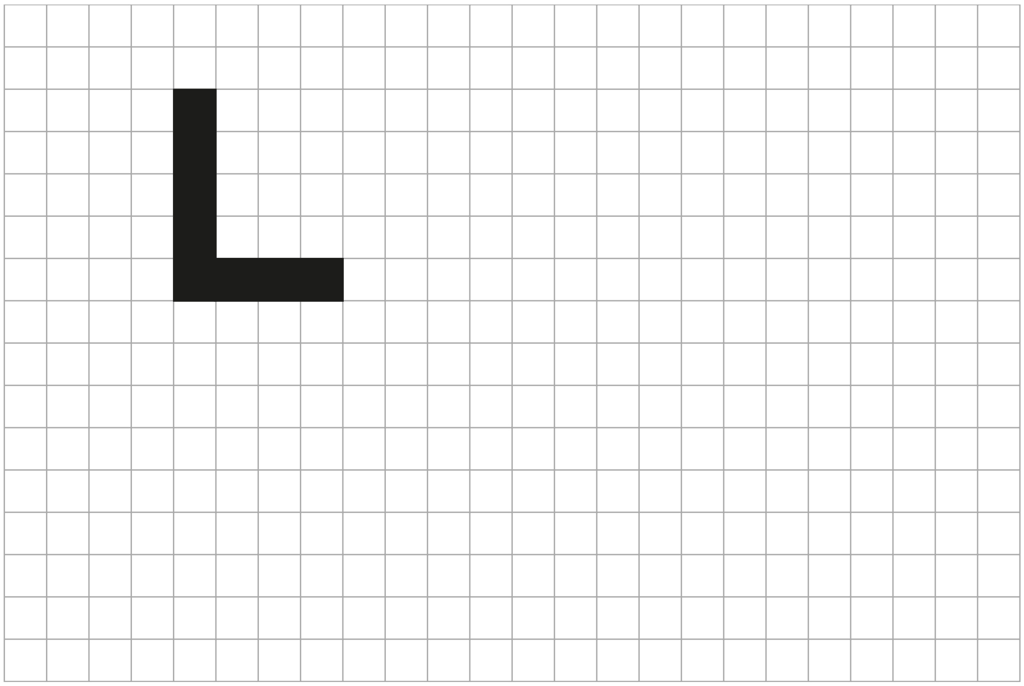


   Answer:

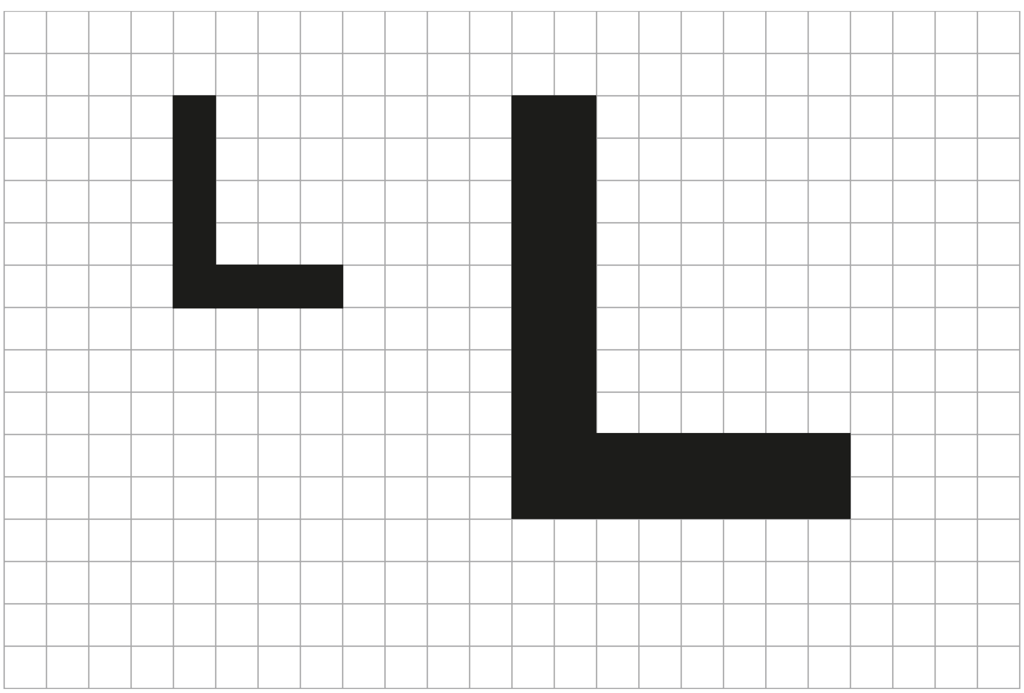
   The side length of the large square must be 5 cm. Then:

$$\text{scale factor} = \frac{\text{length of scale square}}{\text{length of actual square}} = \frac{5}{2} = 2.5$$

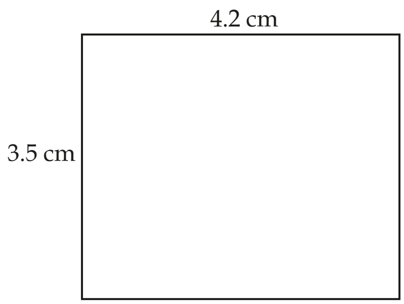
3. Draw a shape similar to the one below, using a scale of 2:1. *(2 marks)*
(Module 8, Lesson 1)



Answer:



4. a) The floor of a room is drawn to scale as shown in the diagram below, using a reduction factor of 100. Calculate the area of the actual room that the diagram represents. *(3 marks)* (Module 8, Lesson 3)



Answer:

$$\text{Area Scale} = 3.5 \text{ cm} \times 4.2 \text{ cm}$$
$$= 14.7 \text{ cm}^2$$

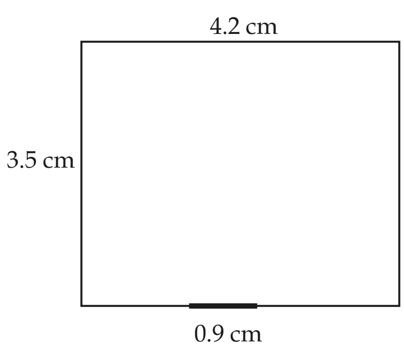
$$R^2 = \frac{\text{Area Scale}}{\text{Area Actual}}$$

$$\frac{1}{100^2} = \frac{14.7 \text{ cm}^2}{\text{Area Actual}}$$

$$\text{Actual} = 147\ 000 \text{ cm}^2$$

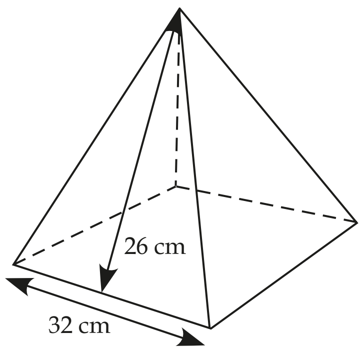
b) A doorway in the actual room is 0.9 m wide. Draw the doorway on the diagram above, using the reduction scale factor of 100. *(1 mark)* (Module 8, Lesson 1)

Answer:



Note: The door could be anywhere on the rectangle but needs to be exactly 0.9 cm.

5. The dimensions for a right square pyramid are shown below. A scale model of this pyramid is being created, with a surface area of 4023 cm^2. Calculate the dimensions of the model pyramid. *(5 marks)* (Module 8, Lesson 4)



Answer:

First, determine the surface area of the actual pyramid. Then, find the scale factor relating the two surface areas. Finally, calculate the dimensions of the scale pyramid.

To find the surface area of a square pyramid, first recognize that all four triangular faces are the same. Therefore, multiply the formula for the area of a triangle by 4. Then, determine the area of the square base.

$$SA_{\text{actual}} = 4\left(\frac{1}{2}\right)bh + b^2$$

$$= (4)\left(\frac{1}{2}\right)(32)(26) + (32)(32)$$

$$= 1664 + 1024$$

$$= 2688 \text{ cm}^2$$

Now find the scale factor.

$$k^2 = \frac{SA_{\text{scale}}}{SA_{\text{actual}}} = \frac{4023}{2688} = 1.5$$

$$k = 1.2$$

Now, multiply the dimensions of the actual pyramid by the scale factor to determine the dimensions of the scale pyramid.

Scale height: 26(1.2) = 31.2 cm

Scale base: 32(1.2) = 38.4 cm

Name: ____________________________________

6. Determine the scale factor if the volume of a scale diagram is 21 mm^3 and the volume of the actual shape is 68 cm^3. *(3 marks)* (Module 8, Lesson 5)

Answer:

You first need to convert both units to mm^3. Therefore, 68 cm^3 is 68 × 10 × 10 × 10 = 68 000 mm^3. You can then calculate the scale factor relating the two diagrams.

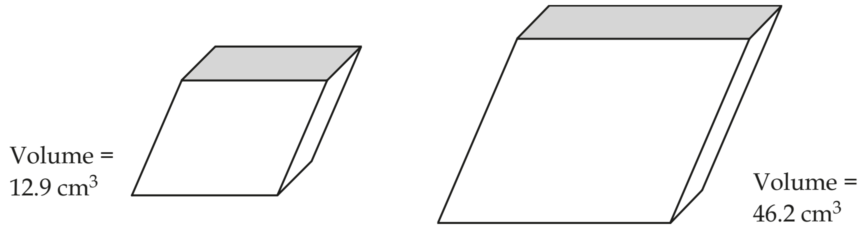
$$k^3 = \frac{21 \text{ mm}^2}{68\ 000 \text{ mm}^2}$$

$$= \frac{21}{68\ 000}$$

$$k = \sqrt[3]{\frac{21}{68\ 000}}$$

$$k = 0.07$$

7. a) Find the scale factor relating the two objects below if the larger prism is the scale diagram. *(2 marks)* (Module 8, Lesson 5)



Answer:

$$k^3 = \frac{V_{\text{scale}}}{V_{\text{actual}}} = \frac{46.2}{12.9}$$

$$k = 1.53$$

b) If the surface area of the top of the small prism is 3.9 cm^2, what is the surface area of the top of the large prism? *(2 marks)* (Module 8, Lesson 4)

Answer:

$$k = 1.53$$

$$k^2 = \frac{A_{\text{scale}}}{A_{\text{actual}}}$$

$$1.53^2 = \frac{A_{\text{scale}}}{3.9}$$

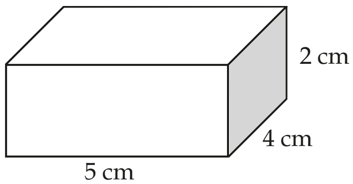
$$9.1 = A_{\text{scale}}$$

The surface area is 9.1 cm^2.

Name: ______________________________________

8. Determine the surface area of the scale model of the rectangular prism given the dimensions of the actual prism below. The scale factor is $\frac{7}{2}$. *(2 marks)*

(Module 8, Lesson 4)



Answer:

$$SA_{\text{actual}} = 5(4)(2) + 5(2)(2) + 4(2)(2) = 40 + 20 + 16 = 76\text{ cm}^2$$

$$SA_{\text{scale}} = \left(\frac{7}{2}\right)^2(76) = \left(\frac{49}{4}\right)(76) = 931\text{ cm}^2$$

9. How many times would the volume of a milk container increase if all of the dimensions were doubled? *(1 mark)* (Module 8, Lesson 5)

Answer:

The volume would increase by a factor of 8 (2^3).