GRADE 10 INTRODUCTION TO APPLIED AND PRE-CALCULUS MATHEMATICS (20S)

Midterm Practice Exam
Answer Key

GRADE 10 INTRODUCTION TO APPLIED AND PRE-CALCULUS MATHEMATICS

Midterm Practice Examination
Answer Key

Name: _____

Student Number: _____

Attending ❑   Non-Attending ❑

Phone Number: _____

Address: _____

_____

_____

For Marker's Use Only

Date: _____

Final _____ /100 _____ %

Comments:

Instructions

The midterm examination will be weighted as follows:

Modules 1–4	100%

The format of the examination will be as follows:

Part A: Multiple Choice	20 marks
Part B: Definitions	10 marks
Part C: Graphs and Relations	27 marks
Part D: Number Sense	7 marks
Part E: Measurement	26 marks
Part F: Trigonometry	10 marks

Time allowed: 2.5 hours

Note: You are allowed to bring a scientific calculator and your Midterm Exam Resource Sheet to the exam. Your Resource Sheet must be handed in with the exam. You will receive your Midterm Exam Resource Sheet back from your tutor/ marker with the next module work that is submitted for marking.

You will need a metric ruler and an imperial ruler. If required, a metric/imperial ruler is provided at the end of this exam for your use.

Part A: Multiple Choice (*20 x 1 = 20 marks*)

Circle the letter of the correct answer for each question.

1. On a graph, the independent variable
   a) is graphed along the *y*-axis
   b) is graphed along the vertical axis
   c) is graphed along the horizontal axis
   d) is affected by changes in the other variable   (Module 1, Lesson 1)

   The independent variable is always graphed along the *x*-axis or the horizontal axis.

2. An example of continuous data is
   a) the number of pairs of shoes you own
   b) the time it takes to run a race
   c) how many pages in a textbook
   d) the number of pizzas you order for a party   (Module 1, Lesson 1)

   Time is continuous because you calculate in fractions of a minute or a second. The other choices are all examples of things that can't be divided into fractional parts.

3. Calculate the slope of the line that passes through the points (2, 5) and (4, 8).

   a) $\dfrac{-2}{3}$

   b) $\dfrac{3}{2}$

   c) $\dfrac{2}{3}$

   d) $\dfrac{-3}{2}$

   Slope is $\dfrac{\text{rise}}{\text{run}}$ or $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

   $m = \dfrac{8 - 5}{4 - 2}$

   $m = \dfrac{3}{2}$

   (Module 1, Lesson 3)

4. The slope of a vertical line is
   a) $m = -1$
   b) $m = 0$
   c) $m = 1$
   d) undefined   (Module 1, Lesson 3)

   A vertical line has rise but no run so the slope would be $\dfrac{\text{rise}}{0}$ and since we can't divide by zero, the slope is undefined.

5. Given the equation of a line is $y = \dfrac{2}{3}x - 5$, what is the y-intercept?

   a) $2x$

   b) 5

   c) $\dfrac{2}{3}$

   (d) -5

   The equation of a line is $y = mx + b$ where b is the y-intercept. So, in $y = \dfrac{2}{3}x - 5$, $b = -5$.

6. An example of a composite number is

   a) 11

   b) 23

   c) 37

   (d) 51

   A composite number is a number with more than two factors. 11, 23, and 37 are all prime numbers because their only factors are 1 and themselves. 51 has factors of 1, 3, 17, and 51.

7. The greatest common factor of 12 and 16 is

   a) 2

   (b) 4

   c) 48

   d) 192

   The factors of 12 are {1, 2, 3, 4, 6, 12} and the factors of 16 are {1, 2, 4, 8, 16}. The largest number that is in both sets is 4.

8. A possible solution for $\sqrt{16}$ is

   a) 2

   b) 8

   (c) -4

   d) none of the above

   $\sqrt{16} = -4$ because $(-4)(-4)$ or $(-4)^2 = 16$.

9. $-\dfrac{5}{7}$ is best described as a(n)

    a) whole number
    b) integer
    c) rational number
    d) irrational number   (Module 2, Lesson 3)

    $-\dfrac{5}{7}$ is a rational number because it is represented as a fraction and would give a

    repeating decimal if changed to decimal form.

10. Find the product of $(2m^2n^3)(3mn^4)$.

    a) $6m^3n^7$
    b) $5m^2n^{12}$
    c) $6m^2n^{12}$
    d) $5m^3n^7$ (Module 2, Lesson 4)

    $(2m^2n^3)(3mn^4) = (2 \cdot 3)(m^2m)(n^3)(n^4) = 6m^3n^7$ using the produce law of exponents to add the exponents. The coefficients are multiplied.

11. Simplify $(59x^2y)^0$.

    a) –1
    b) 0
    c) 1
    d) $59x^2y$ (Module 2, Lesson 5)

    Anything raised to the power of zero equals 1.

12. $9^{\frac{1}{2}}$ is equivalent to

   (a) $\sqrt{9}$

   b) $\dfrac{1}{9^2}$

   c) 4.5

   d) -3

                                                      (Module 2, Lesson 5)

$9^{\frac{1}{2}} = \sqrt{9}$. When writing in this form, the positive root is assumed so the answer would

be 3. To get an answer of –3, the question would need to be $-9^{\frac{1}{2}}$, which would be written

as $-\sqrt{9}$ or -3. Fractional exponents are represented by radicals.

13. $\left(\dfrac{x}{y}\right)^{-3}$ is equivalent to

   (a) $\dfrac{y^3}{x^3}$

   b) $-\left(\dfrac{x}{y}\right)^3$

   c) $-\left(\dfrac{y}{x}\right)^3$

   d) $\left(\dfrac{1}{xy^3}\right)$

                                                      (Module 2, Lesson 5)

$\left(\dfrac{x}{y}\right)^{-3} = \dfrac{1}{\left(\dfrac{x}{y}\right)^3} = \dfrac{1}{\dfrac{x^3}{y^3}}$ which can be simplified to $\dfrac{y^3}{x^3}$.

14. The best choice of units to measure the distance from your town to Toronto would be
    a) metres
    b) yards
    c) miles
    d) decalitres   (Module 3, Lesson 1)

    Metres and yards are inappropriate as the distance is a long one. Decalitres is not a measure of distance but a measure of capacity.

15. The surface area of a sphere with a radius of 5 inches is about
    a) 63 in.²
    b) 314 in.²
    c) 524 in.²
    d) 3948 in.²   (Module 3, Lesson 6)

    The formula for surface area of a sphere is $A = 4\pi r^2$. Substituting $r = 5$, we get $A = 4\pi(25)$ or 100π or approximately 314 square inches.

16. If a cone has a volume of 100 cubic units, a cylinder with the same height and radius will have a volume of how many cubic units?
    a) 10
    b) 33
    c) 300
    d) 1000   (Module 3, Lesson 6)

    A cylinder will have three times the volume of a cone with the same height and radius. The volume of a cylinder is found by using the formula $V = \pi r^2 h$ and the volume of a cone is found by using the formula $V = \frac{1}{3}\pi r^2 h$. So, multiply 100 by 3 to get 300.

17. If the two legs in a right triangle are 5 cm and 12 cm, the length of the hypotenuse is
    a) 11 cm
    b) 13 cm
    c) 17 cm
    d) 169 cm   (Module 4, Lesson 1)

    Using Pythagorus, $5^2 + 12^2 = h^2$ or $25 + 144 = h^2$ or $169 = h^2$. $h = \sqrt{169}$ or 13.
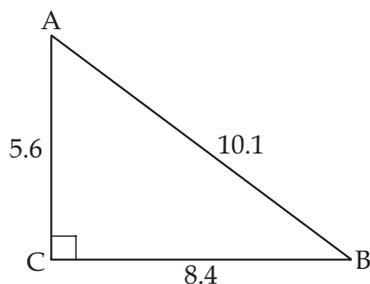   You might recognize this as a Pythagorean triple (5, 12, 13).

18. The sine ratio involves the lengths of which two sides of a right triangle?

a) $\dfrac{\text{opposite}}{\text{adjacent}}$

b) $\dfrac{\text{adjacent}}{\text{hypotenuse}}$

c) $\dfrac{\text{opposite}}{\text{hypotenuse}}$

d) $\dfrac{\text{adjacent}}{\text{opposite}}$

(Module 4, Lesson 2)

The definition of sine is the ratio of the opposite side to the hypotenuse.
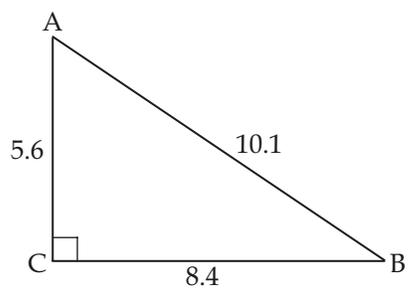
19. Given triangle ABC, sin A = ?

a) $\dfrac{5.6}{10.1}$

b) $\dfrac{8.4}{5.6}$

c) $\dfrac{5.6}{8.4}$

d) $\dfrac{8.4}{10.1}$

(Module 4, Lesson 2)

Since $\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$ and 8.4 is the length of the side opposite angle A and the hypotenuse is 10.1, $\sin A = \dfrac{8.4}{10.1}$.

20. Given triangle ABC, the measure of angle A is

a) 37°
b) 34°
c) 90°
d) 56°   (Module 4, Lesson 3)

Using $\sin A = \dfrac{8.4}{10.1}$ and the inverse sine ratio on your calculator (make sure it is in degree mode), we find angle A to be 56°.

Part B: Definitions (*10 x 1 = 10 marks*)

Match each definition with the correct term from the list below. Write the correct term on the blank line with each definition. Terms are used only once. Not all terms have a definition provided

Terms

adjacent side	greatest common factor	ordered pair	similar triangles
alternate interior angles	hypotenuse	perfect cube number	sine
angle of depression	imperial system	perfect square number	slope
angle of elevation	integer	prism	sphere
cone	inverse trigonometric ratio	pyramid	square root
cosine	irrational	range	tangent
cube root	lateral surface area	rational number	total surface area
cylinder	least common multiple	referent	volume
domain	natural number	SI	whole number
graph	opposite side	similar	

1. Visual representation used to show a numerical relationship. *graph* (Module 1, Lesson 1)

2. A comparison of how far the line moves vertically as it moves horizontally. *slope* (Module 1, Lesson 3)

3. Found when an integer is multiplied by itself three times. *perfect cube number* (Module 2, Lesson 2)

4. The counting numbers and zero. *whole numbers* (Module 2, Lesson 3)

5. A system of measurement that uses prefixes and has a decimal structure. *SI* (Module 3, Lesson 1)

6. 3-D object with two congruent, parallel bases and parallelogram faces. *prism* (Module 3, Lesson 4)

7. 3-D object in which all points are equidistant from the centre. *sphere* (Module 3, Lesson 6)

8. Side directly across from the specified angle. *opposite side* (Module 4, Lesson 1)

9. Ratio of the opposite side and hypotenuse in a right triangle. *sine* (Module 4, Lesson 2)

10. Congruent angles formed on opposite sides of a line that cuts diagonally through parallel lines. *alternate interior angles* (Module 4, Lesson 4)

Part C: Graphs and Relations (*27 marks*)

Show all calculation and formulas used for short and long answer questions. Use all decimal places in your calculations and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. Given the linear equation $y = \dfrac{4}{3}x - 9$

    a) State the *y*-intercept as a value. *(1 mark)*

    Answer:

    $y = -9$

    b) State the slope of the line. *(1 mark)*

    Answer:

    $m = \dfrac{4}{3}$

    c) Explain how you would graph the line. *(2 marks)*

    Answer:

    Answers may vary. For two marks, you need to explain where you would put the *y*-intercept and how you would use the slope to move from the *y*-intercept to a second point.

    Possible answer:

    I would put a point on the *y*-axis at –9 since the *y*-intercept is –9. From that point, I would "rise" up 4 units and "run" right 3 units to get to the point (3, –5). I could also "rise" down 4 units and "run" left 3 units to get to the point (–3, –13). Using at least two points, I can draw the line.
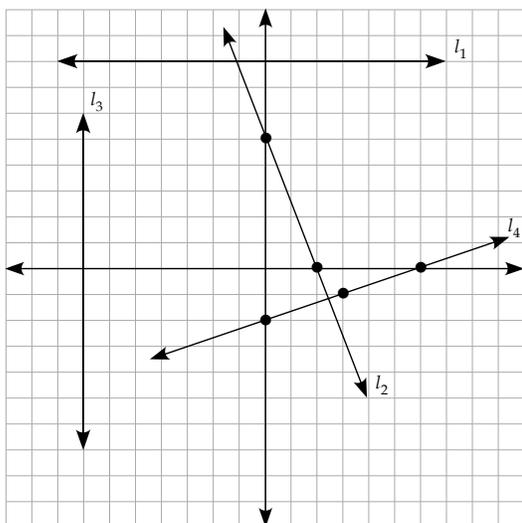
d) Sketch a graph of the line. *(1 mark)*

    Answer:
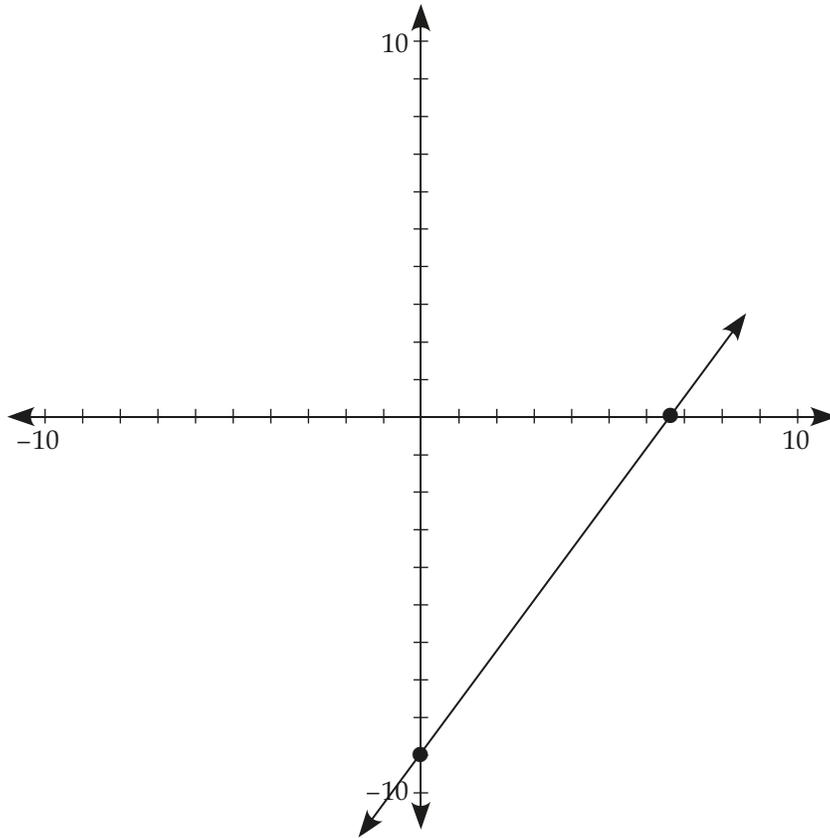
(Module 1, Lesson 5)

2. State the slope of each of the following lines. *(4 marks)*

Answers:

$l_1 = 0$

$l_2 = \dfrac{-5}{2}$

$l_3 = $ undefined

$l_4 = \dfrac{1}{3}$

(Module 1, Lesson 4)

3. Given the equation $y = \dfrac{1}{3}x - 5$, state the equation of a different line that is parallel to the given line, and explain how it is parallel. *(2 marks)*

Answer:

$y = \dfrac{1}{3}x - 4$

Answers will vary. Any equation with the same slope and different y-intercept is acceptable. Students must state that their line is parallel because it has the same slope as the given line.

(Module 1, Lesson 4)

4. In line AB, A (–5, 21) and B $(x, -6)$. Use the slope formula to find the value of x if the slope of line AB is $m = -3$. *(4 marks)*

    Answer:

    $$m = \frac{y_2 - y_1}{x_2 - x_1}$$

    $$-3 = \frac{-6 - 21}{x - (-5)}$$

    $$(x + 5)(-3) = -27$$

    $$-3x - 15 = -27$$

    $$-3x = -12$$

    $$x = 4$$

    (Module 1, Lesson 5)

5. The container below is being filled with water at a constant rate. The time (t) and height (h) of the water is graphed.
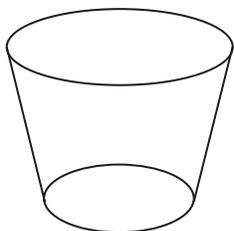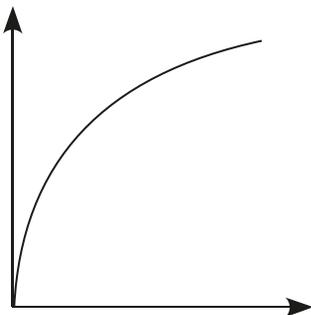

    a) State the independent and dependent variables in this situation. *(1 mark)*

    independent   *Answer:* time
    dependent   *Answer:* height

    b) Sketch a possible graph for this situation. *(1 mark)*

    Answer:

    (Module 1, Lesson 1)

6. A study was undertaken to compare the age of cash registers to the cost of maintenance required on them. Nine cash registers in a department store were examined. The results were as follows.

Register #	Age (in years)	Maintenance costs in dollars
1	6	99
2	7	161
3	1	23
4	3	40
5	6	126
6	2	35
7	5	86
8	4	72
9	3	51

a) Which is the independent variable? *(0.5 mark)*

    Answer: age

b) Which is the dependent variable? *(0.5 mark)*

    Answer: maintenance costs

c) Plot the points on the grid provided, include the components of a good graph as described in Module 1. *(4 marks)*

    Answer:

    (Student must include labels, units, and title, and use appropriate scales for a good shape and size.)

Register Repair Costs
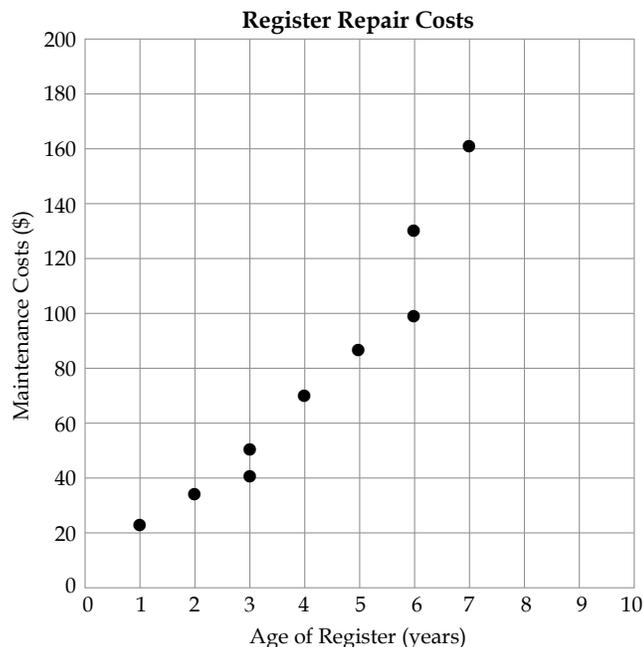
d) What is a reasonable domain and range for this situation? Explain your answer. *(2 marks)*

Answer:

Answers may vary but should be reasonable.

Domain: from zero to 10 years is a reasonable age for a cash register. After about 10 years they are likely replaced as new technology develops. No negative values.

Range: from $0 to $200 seems reasonable. If repairs and maintenance were more than that, the store would probably buy a new cash register. No negative values.

e) Is this data linear? Explain why or why not. *(2 marks)*

Answer:

Yes, this data is approximately linear because a straight line drawn on the graph would go through or close to most of the data points.

f) Is this data continuous? Explain why or why not. *(1 mark)*

Answer:

Yes, the age of a cash register can be parts of years, and the costs can be fractions of dollar amounts.

(Module 1, Lesson 2)

Part D: Number Sense (*7 marks*)

Show all calculation and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. Determine the prime factors of 450 using a factor tree diagram. Include your diagram. (*2 marks*)

   Answer:
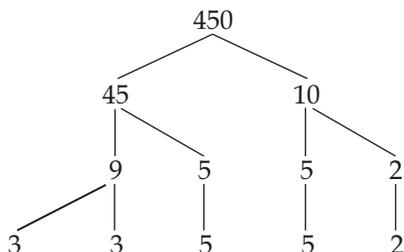
   The prime factors are $2 * 3 * 3 * 5 * 5$

   Diagrams may vary but the prime factors must be correct

   (Module 2, Lesson 1)

2. Determine the prime factors of 225 and 400. Write them using exponents to indicate repeated multiplications, and use that to find the greatest common factor and least common multiple. (*4 marks*)

   Answer:

   $400 = 2^4 * 5^2$

   $225 = 3^2 * 5^2$

   GCF is $5^2 = 25$

   LCM is $2^4 * 3^2 * 5^2 = 3600$

   (Module 2, Lesson 1)

3. Write $\sqrt{252}$ as a mixed radical in simplest form. (*1 mark*)

   Answer:

   $6\sqrt{7}$

   (Module 2, Lesson 3)

Part E: Measurement (*26 marks*)

Show all calculations and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. Explain how you could use a referent to estimate the circumference of a circular dining table. Describe your referent and measurement strategy. *(3 marks)*
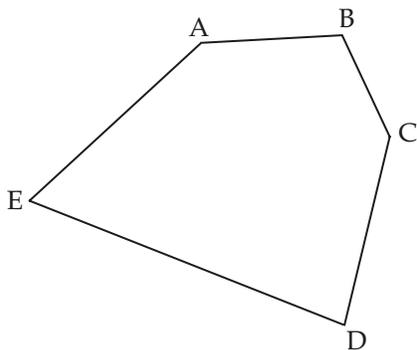
   Answer:

   Answers will vary. Students must describe a reasonable referent and strategy. A possible solution may reflect the following:

   I know that my hand span is about 7 inches long. I will mark my starting point on the table and will stretch my fingers out and place them alternately along the circumference of the table, counting the number of times it takes to return to my starting point. I will multiply the number of handspans by 7 to determine the approximate number of inches in the circumference of the table.

   (Module 3, Lesson 1)

2. Measure the length of each side in the following polygon to the nearest tenth of a centimetre and calculate its perimeter. *(3 marks)*

Answers:	Acceptable measuring range
AB = 1.9 cm	AB = 1.6 cm to 2.2 cm
BC = 1.5 cm	BC = 1.2 cm to 1.8 cm
CD = 2.6 cm	CD = 2.3 cm to 2.9 cm
DE = 4.5 cm	DE = 4.2 cm to 4.8 cm
EA = 3.1 cm	EA = 2.8 cm to 3.4 cm
Perimeter = 13.6 cm	Perimeter = 12.1 cm to 15.1 cm

   Note: Each of your measurements may be off by ±3 mm due to photocopying of the figure or measurement error.

   (Module 3, Lesson 1)

3. Convert the following measurements into the units indicated. (Round to 3 decimal places) *(8 marks)*

92 inches =   *Answer:* 7.667 feet

19 cm =   *Answer:* 190 mm

4.5 yards =   *Answer:* 162 inches

11 miles =   *Answer:* 17.699 km

5 gallons   *Answer:* 22.73 litres

33 kg =   *Answer:* 72.6 lb.

82 ft.² =   *Answer:* 9.11 yd.²

25 000 000 cm³   *Answer:* 0.25 m³   (Module 3, Lesson 2)

4. A take-out coffee cup has a protective paper sleeve to make it easier to hold when filled with a hot beverage. The sleeve is made by cutting the bottom off of a paper cone and using the top portion of it. Determine the lateral surface area of the part of the cone used to make the sleeve. *(4 marks)*

Answer:
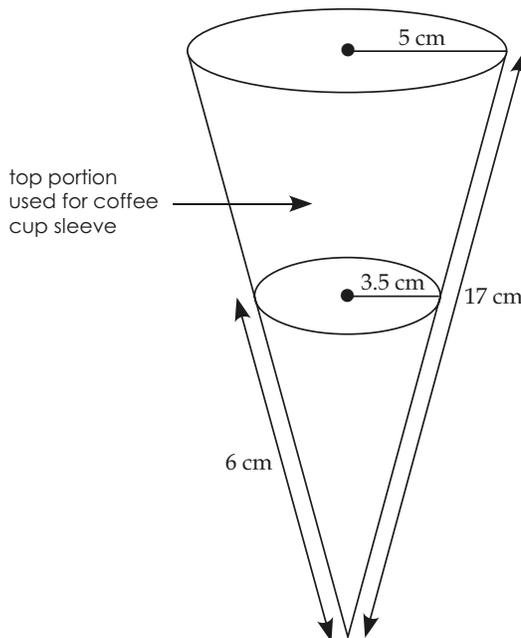
LSA of large cone $= \dfrac{1}{2}C\ell$

$= \dfrac{1}{2}2\pi\,(5)(17)$

$= 267.0353756 \text{ cm}^2$

LSA of small cone $= \dfrac{1}{2}C\ell$
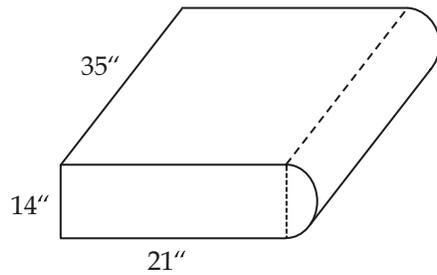
$= \dfrac{1}{2}2\pi\,(3.5)(6)$

$= 65.97344573 \text{ cm}^2$

LSA of sleeve = 267.0353756 − 65.97344573

LSA of sleeve = 201.0619298

The coffee sleeve has a lateral surface area of 201.1 cm².

(Module 3, Lesson 6)

5. Determine the volume of this 3-D object. Its base is composed of a rectangle and a semi-circle. State your final answer in cubic feet, rounded to the nearest tenth. *(4 marks)*

Answer:

$V = Bh$

$$V = \left[(l * w) + \left(\frac{1}{2}\pi \left(r^2 \right) \right) \right](h)$$

$$V = \left[(21 * 4) + \left(\frac{1}{2}\pi \left(7^2 \right) \right) \right](35)$$

$$V = (294 + 76.96902001)(35)$$

$$V = 12983.9157 \text{ in.}^3$$

$$12983.9157 \text{ in.}^3 * \frac{1 \text{ ft.}^3}{1728 \text{ in.}^3} = 7.513840104 \text{ ft.}^3$$

The 3-D object has a volume of about 7.5 cubic feet.

(Module 3, Lesson 6)

6. A beach ball has a surface area of 572.6 sq. in. Determine its diameter to the nearest $\frac{1}{2}$ inch. *(4 marks)*

Answer:

$$SA = 4\pi r^2$$

$$572.6 = 4\pi r^2$$

$$\frac{572.6}{4\pi} = r^2$$

$$r^2 = 45.56606021$$

$$r = 6.750263714$$

$$d = 2r$$

$$d = (2)(6.750263714)$$

$$d = 13.5$$

The diameter of the beach ball is $13\frac{1}{2}$ inches.

(Module 3, Lesson 6)

Part G: Trigonometry (*10 marks*)

Show all calculations and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. Calculate tan 65° to 4 decimal places and explain what this means using a diagram of a right triangle. *(3 marks)*
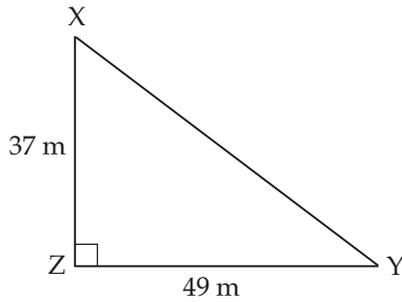
   Answer:

   tan 65° = 2.1445

   In a right triangle with an angle of 65°, the ratio of the side lengths opposite and adjacent to the specified angle will be approximately $\dfrac{2.1445}{1}$.
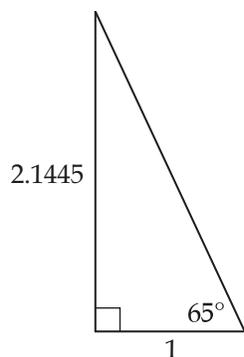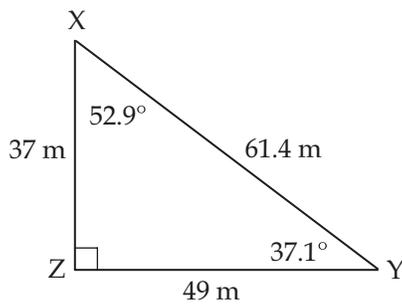

   (Module 4, Lesson 1)

2. Solve the triangle. Find all angles and side lengths. Round your answers to 1 decimal place. *(3 marks)*

Answer:

$$37^2 + 49^2 = z^2$$

$$z^2 = 3770$$

$$z = 61.4 \text{ m}$$

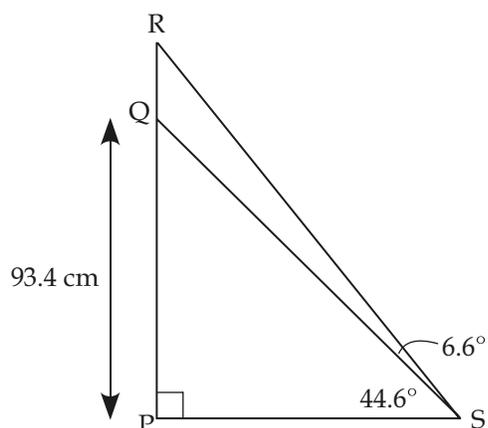$$\angle X = \tan^{-1}\left(\frac{49}{37}\right)$$

$$\angle X = 52.9°$$

$$\angle Y = 90 - 52.9$$

$$\angle Y = 37.1°$$

Note: You may have used different trigonometric ratios to find your answers.

(Module 4, Lesson 3)

3. Find the length of QR to the nearest tenth. *(4 marks)*

Answer:

$$\tan 44.6° = \left(\frac{93.4}{PS}\right)$$

$$PS = \left(\frac{93.4}{\tan 44.6°}\right)$$

$$PS = 94.71329992$$

$$44.6 + 6.6 = 51.2$$

$$\angle RSP = 51.2°$$

$$\tan 51.2° = \frac{PR}{94.71329992}$$

$$PR = 117.7995874$$

$$PR - PQ = QR$$

$$117.7995874 - 93.4 = 24.39958735$$

$$QR = 24.4 \text{ cm}$$

(Module 4, Lesson 4)

■ Grade 10 Introduction to Applied and Pre-Calculus Mathematics