Grade 12 Physics (40S)

Midterm Practice Examination

Answer Key

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Instructions

The midterm examination will be weighted as follows

Modules 1–5	100%

The format of the examination will be as follows:

Part A: Multiple Choice	21 x 1 = 21 marks
Part B: Fill-in-the-Blanks	14 x 0.5 = 7 marks
Part C: Short Explanation Questions	4 x 4 = 16 marks
Part D: Problems	8 x 7 = 56 marks

Grade 12 Physics
Midterm Practice Examination Key

Part A: Multiple Choice

Write the letter of the choice that best completes each statement.

1.	a		11.	a		21.	a			
2.	c		12.	c						
3.	a		13.	c						
4.	b		14.	d						
5.	a		15.	a						
6.	c		16.	a						
7.	c		17.	d						
8.	d		18.	b						
9.	c		19.	d						
10.	a		20.	d						

Part B: Fill-in-the-Blanks

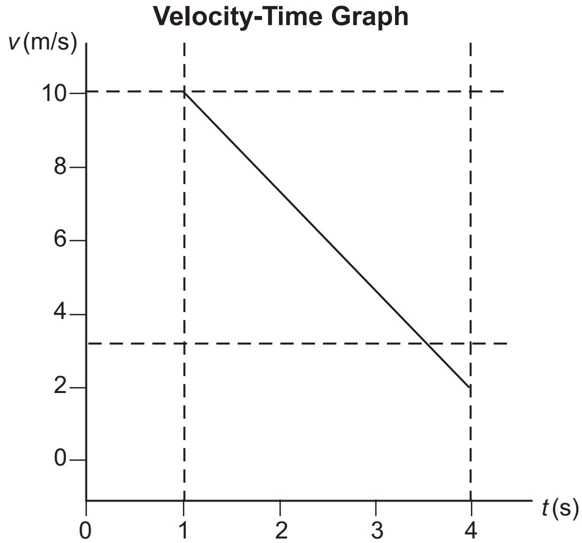
Write the word that best completes each statement in the space provided below.

1.	centripetal		8.	uniform
2.	four times		9.	normal force
3.	normal		10.	joule
4.	static		11.	potential
5.	range		12.	out of
6.	inertia		13.	kinetic
7.	impulse		14.	smaller

Part A: Multiple Choice (21 x 1 = 21 Marks)

Enter the letter of the choice that best answers the question on the answer sheet provided. Please print the letters of your answers clearly.

1. Consider the following velocity-time graph



The acceleration of the object is

a) –2.7 ms/s²
b) –2.5 ms/s²
c) 2.5 ms/s²
d) 2.7 ms/s²

Answer (a)
Outcome S4P-1-1

Answer:

$$\vec{v}_2 = \vec{v}_1 + \vec{a}t$$

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{2\ \text{m/s} - 10\ \text{m/s}}{4\ \text{s} - 1\ \text{s}} = -2.7\ \text{m/s}^2$$

2. A river is flowing from west to east at a speed of 10.0 km/h. A boat's speed in the water is 20.0 km/h. If the boat is pointing straight north, and is blown off course, the new speed of the boat relative to the shore would be

a) 10.0 km/h
b) 17.3 km/h
c) 22.4 km/h
d) 30 km/h

Answer (c)
Outcome S4P-1-3

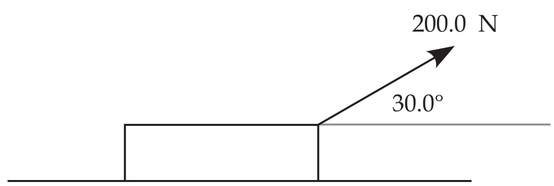
Answer:

$$v = \sqrt{(20.0\ \text{km/h})^2 + (10.0\ \text{km/h})^2} = 22.4\ \text{km/h}$$

3. If a mass on a horizontal surface is pushed down at an angle (similar to pushing down on a shopping cart), then the normal force will
   a) increase and the force of gravity will not change
   b) not change and the force of gravity will increase
   c) not change and the force of gravity will also not change
   d) increase and the force of gravity will also increase

Answer (a)
Outcome S4P-1-8

4. An object is pulled along a horizontal surface by a force that is directed up and to the right. The mass of the object is 50.0 kg. The applied force is 200.0 N directed at an angle of 30.0° above the horizontal. The magnitude force of friction is 60.0 N.



The magnitude of the normal force is
   a) 100 N
   b) 390 N
   c) 490 N
   d) 590 N

Answer (b)
Outcome S4P-1-8

Answer:

The magnitude of the normal force is found using

$$\vec{F}_N + \vec{F}_A(\sin\theta) - \vec{F}_g = 0$$

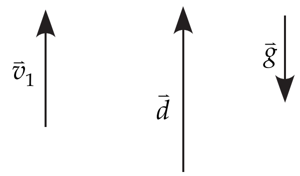
$$\vec{F}_N = \vec{F}_g - \vec{F}_A(\sin\theta) = (50.0\text{ kg})(9.80\text{ N/kg}) - (200.0\text{ N})(\sin 30°) = 390\text{ N}$$

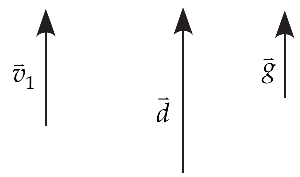
The normal force ***is not*** (50.0 kg)(9.80 N/kg) = 490 N.

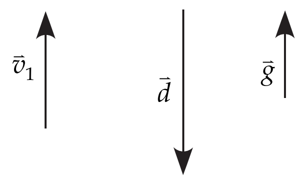
The normal force ***is not*** (200.0 N)(sin 30°) = 100 N.

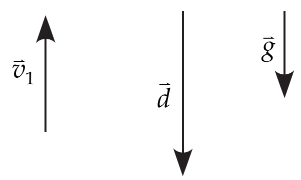
The normal force ***is not*** (50.0 kg)(9.80 N/kg) + (200.0 N)(sin 30°) = 590 N

5. A basketball is thrown straight up with a speed of 10.0 m/s. The vectors representing the initial velocity, the acceleration due to gravity, and the vertical distance travelled (height) are

a)


b)


c)


d)


Answer (a)

Outcome S4P-1-16

6. A ball is launched at a speed of 20.0 m/s at an angle of 40.0o above the horizontal. The maximum height reached by the ball is

a) 0.66 m
b) 0.78 m
c) 8.43 m
d) 12.0 m

Answer (c)

Outcome S4P-1-18

Answer:

To determine the maximum height, consider the first half of the motion. The vertical component of velocity at the top of motion is zero. An appropriate equation is $v_2{}^2 = v_1{}^2 + 2ad$. Solve for "d":

$$d = \frac{(0\ \text{m/s})^2 - [(20.0\ \text{m/s})(\sin 40.0°)]^2}{2(-9.80\ \text{m/s}^2)} = 8.43\ \text{m}$$

7. A mass makes 20.0 revolutions in a time of 4.00 s in a circle of radius 10.0 m. The velocity of the mass is
   a) 12.6 m/s towards the centre of the circle
   b) 12.6 m/s tangent to the circle
   c) 314 m/s tangent to the circle
   d) 314 m/s towards the centre of the circle

   Answer (c)
   Outcome S4P-1-24

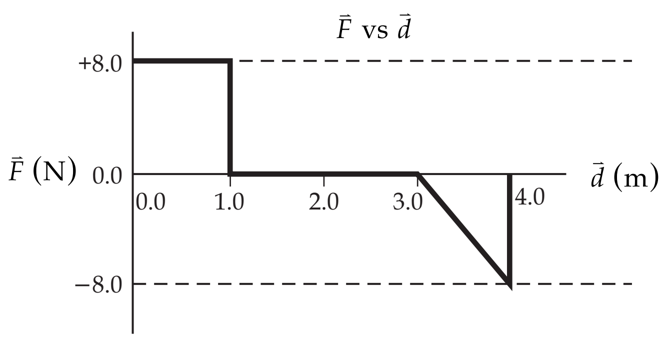
   Answer:

$$\vec{v} = \frac{2\pi r}{T} = \frac{2\pi\left(10.0 \text{ m}\right)}{\left(4.00 \text{ s} / 20.0 \text{ rev}\right)} = 314 \text{ m/s}$$

8. When a ball is swinging in a circle at the end of a string, the hand feels a force directed away from the hand. The reason for this is that
   a) centrifugal force caused by the ball is pulling on the hand
   b) ball wants to fly out parallel to the radius of the circle
   c) centripetal force exactly balances the centrifugal force
   d) ball exerts an equal and opposite force on the hand

   Answer (d)
   Outcome S4P-1-20

9. The force component along the displacement varies with the magnitude of the displacement, as shown on the graph.



   The work done by the force over the whole time interval is
   a) −4.0 J
   b) 0.0 J
   c) 4.0 J
   d) 12.0 J

   Answer (c)
   Outcome S4P-1-26

Answer:

The work done during each interval is equal to the area under the force vs. displacement curve over that interval.

$$W = b_1 h_1 + \frac{1}{2} b_2 h_2$$

$$W = (1.0\text{ m})(8.0\text{ N}) + \frac{1}{2}(4.0\text{ m} - 3.0\text{ m})(-8.0\text{ N}) = 4.0\text{ J}$$

10. A stopper is swung in a circle of radius 2.00 m with a period of 1.50 seconds. A centripetal force of 2.00 N acts on the stopper. What is the work done by the centripetal force during the time that the stopper travels once around the circle?
    a) 0 J
    b) 4.00 J
    c) 8.37 J
    d) 25.1 J

Answer (a)

Outcome S4P-1-25, S4P-1-24

11. We can most directly derive the impulse-momentum equation from the law that states
    a) when a net external force $\vec{F}$ acts on a mass m the acceleration $\vec{a}$ that results is directly proportional to the net force and has a magnitude that is inversely proportional to the mass
    b) whenever one body exerts a force on a second body, the second body exerts an oppositely directed force of equal magnitude on the first body
    c) the force of gravitation between two masses is directly proportional to the product of the two masses and inversely proportional to the separation between them squared
    d) an object continues in a state of rest or in a state of motion at a constant speed along a straight line, unless compelled to change that state by a net force

Answer (a)

Objective S4P-1-10

12. A mass of 5.0 kg is moving at a constant speed of 10.0 m/s. A force of 200.0 N then acts on the mass for 2.0 s. The new speed of the mass is

a) 70 m/s

b) 80 m/s

c) 90 m/s

d) 410 m/s

Answer (a)

Objective S4P-1-13

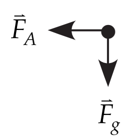
Answer:

$$\vec{F}\Delta t = m(v_2 - v_1)$$

$$v_2 = \frac{\vec{F}\Delta t}{m} + v_1$$

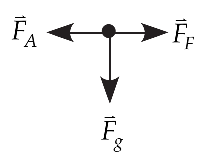
$$v_2 = \frac{(200.0 \text{ N})(2.0 \text{ s})}{5.0 \text{ kg}} + 10.0 \text{ m/s} = 90 \text{ m/s}$$

13. Which of the following free-body diagrams best represents the forces acting on an astronaut in orbit around the Earth?

a) 

b) 

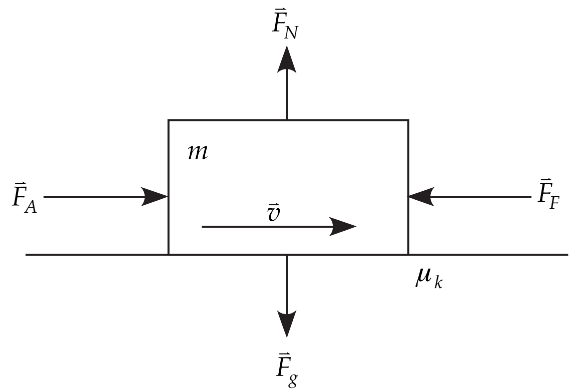
c) 

d) 

Answer (a)

Outcome S4P-1-21

14. Study the force system diagram pictured below and select the factor that would ***not*** influence the amount of kinetic friction.



a) object's mass, m
b) coefficient of kinetic friction, μ_k
c) normal force, $\vec{F}_N$
d) applied force, $\vec{F}_A$

Answer (d)
Outcome S4P-1-7

15. A person lifts a pail of water of mass 1.50 kg from the ground to a deck 1.00 m above the ground. How much work was done by gravitational force on the pail of water?
a) –14.7 J
b) +1.50 J
c) –1.50 J
d) +0.153 J

Answer (a)
Outcome S4P-1-25, S4P1-27

16. In which case is positive work done?
a) An eastward force is applied to an eastward moving soccer ball that is already moving at a constant velocity to increase its speed in that direction.
b) A cart is moving at a constant velocity of 10 m/s [W] when a 0.5 N [E] force is applied.
c) Earth applies a force on the Moon as the Moon travels one complete rotation in orbit around Earth.
d) The work done by air resistance as a baseball is thrown horizontally towards the catcher.

Answer (a)
Outcome S4P-1-25, S4P-1-27

17. A 15-kg load of groceries is lifted up from the first floor to the fifth floor of an apartment building. Each floor is 5.00 m high. The potential energy of the groceries with respect to the second floor is

a) 3.68×10^3 J

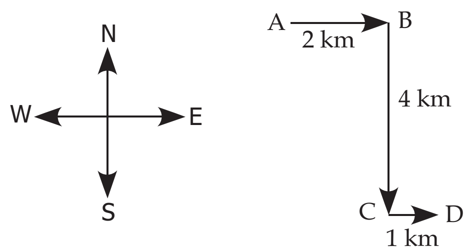
b) 2.94×10^3 J

c) 7.5×10^1 J

d) 2.20×10^3 J

Answer (d)
Outcome S4P-1-30, S4P-1-33

18. The diagram below shows the first three legs of a trip: A to B, B to C, and C to D. If a person returns from point D to point A, what is the displacement for this fourth and final leg?



a) 7 km [37° W of N]

b) 5 km [37° W of N]

c) 5 km [37° E of S]

d) 7 km [37° E of S]

Answer (b)
Outcome S4P-0-2h

19. The speed of an object moving with uniform circular motion of radius 15.0 m with a frequency of 4.00 Hz is which of the following?

a) 3.75 m/s

b) 23.6 m/s

c) 60.0 m/s

d) 377 m/s

Answer (d)
Outcome S4P-1-24

20. A pilot flies to a destination due north from the departure point. During the flight there is a wind blowing from the west. What direction must the pilot point the plane during the flight?

a) due east

b) east of north

c) due north

d) west of north

Answer (d)

Outcome S4P-1-3

21. An object is moving at 2.50 m/s [E]. At a time 3.00 seconds later the object is travelling at 1.50 m/s [E]. What was the displacement during this 3.00 second time interval?

a) 6.00 m [E]

b) 7.50 m [E]

c) 4.50 m [E]

d) 0.500 m [E]

Answer (a)

Outcome S4P-1-2

Part B: Fill-in-the-Blanks (14 x 0.5 = 7 Marks)

Fill in the blanks with one of the choices in the word bank. The terms in the word bank may be used once, more than once, or not at all.

Write your answers in the space provided on the answer sheet.

acceleration	into	normal force	static
centripetal	joule	out of	two times
four times	kinetic	potential	uniform
impulse	larger	range	velocity
inertia	normal	smaller	watt

1. The force required to keep an object moving with uniform circular motion is called the ________***centripetal***__________ force. (Outcome S4P-1-19)
2. In uniform circular motion, if the velocity doubles, the acceleration of the object must change to be _________***four times***_____________ as great as the original acceleration. (Outcome S4P-1-24)
3. The ______***normal***_________ force is always perpendicular to the surface supporting an object. (Outcome S4P-1-5)
4. The force of friction exerted on an object just before it begins to slide across a surface is called _______***static***____________ friction. (Outcome S4P-1-7)
5. In projectile motion, the ____***range***_________ refers to the horizontal distance the object travels. (Outcome S4P-1-18)
6. The tendency of an object to resist changes in its motion is called ____***inertia***________. (Outcome S4P-1-9)
7. The area beneath a force-time graph represents ____***impulse***______. (Outcome S4P-1-11)
8. The word that best describes the motion of an object with a net force of 0 N acting on it is _______***uniform***___________. (Outcome S4P-1-8)
9. The amount of friction acting on an object that is sliding across the surface of a level table depends on the coefficient of kinetic friction and the ____***normal force***___________. (Outcome S4P-1-7)
10. The unit newton · metre is equivalent to the ______***joule***_________. (Outcome S4P-1-25)

11. The area beneath the force-extension graph of a spring represents ***potential*** energy. (Outcome S4P-1-32)

12. If negative work is done on an object, kinetic energy is transferred ***out of*** the object. (Outcome S4P-1-27)

13. The work-energy theorem relates work done to changes in ***kinetic*** energy. (Outcome S4P-1-29)

14. If an object is pulled across a horizontal surface with a force that acts at 20° up from the horizontal, the magnitude of the normal force will be ***smaller*** than the force of gravity. (Outcome S4P-1-5)

Part C: Short Explanation Questions (4 x 4 = 16 Marks)

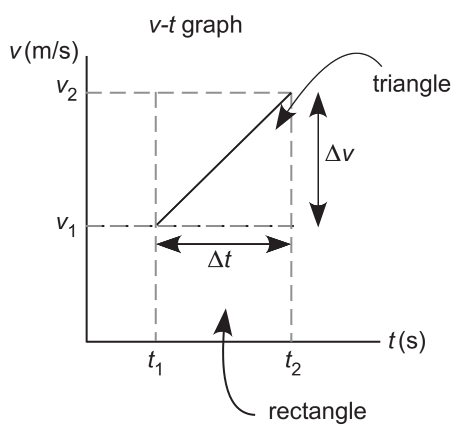
Answer any four (4) of the following questions. Be sure to indicate clearly which four questions are to be marked. Use proper English in your explanations.

Outcome S4P-1-2

1. Using sketches of the appropriate graph, derive the kinematics formula
$\vec{d} = \vec{v}_1 t + \frac{1}{2}\vec{a}\Delta t^2$.

Answer:

Another useful equation can be derived from the graph by considering the area of the rectangle and the area of the triangle under the solid line.



To find the displacement of an object undergoing uniformly accelerated motion, determine the area under the velocity-time graph.

total area = area of rectangle + area of triangle

The area of a rectangle is given by the length times the width. For the rectangle above,

area of rectangle = $v_1\Delta t$.

The area of a triangle is given by one-half the base times the height.

$$\text{area of triangle} = \frac{1}{2}\Delta v \Delta t$$

$$\text{total area} = v_1 \Delta t + \frac{1}{2}\Delta v \Delta t$$

Since the area under a velocity-time graph gives displacement,

$$\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}\Delta \vec{v} \Delta t.$$

We know from the definition of acceleration that $\Delta \vec{v} = \vec{a}\Delta t$. Therefore,

$$\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}(\vec{a}\Delta t)\Delta t$$

$$\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

Outcome S4P-1-27, S4P-1-28

2. The speed of a gymnast revolving around a horizontal bar is greatest at the bottom and least at the top. Explain using the law of conservation of energy.

   Answer:

   The law of conservation of energy states that energy is not created or destroyed but can be converted from one form to another. The total energy (kinetic energy plus potential energy) of the gymnast in this system will remain constant.

   While the gymnast is above the horizontal bar, he will possess the maximum amount of gravitational potential energy. As his body swings down around the bar, it loses gravitational potential energy but gains kinetic energy. When the gymnast is at the bottom he possesses the least amount of gravitational potential energy but the greatest amount of kinetic energy. Since kinetic energy depends on the speed, the gymnast has the greatest speed where his kinetic energy is the greatest.

Outcome S4P-1-19

3. An object is travelling in a straight line with velocity $\vec{v}$. Describe the motion of the object that would result if only

   a) an acceleration parallel to the original velocity acts on the object

   Answer: (1 mark)

   If the acceleration acts parallel to the original velocity, then only the magnitude of the original velocity will change. The result will be accelerated motion along the straight line.

   b) an acceleration that constantly changes to remain perpendicular to the velocity acts on the object

   Answer: (1 mark)

   If the acceleration acts perpendicular to the velocity and changes to remain perpendicular to the velocity, then only the magnitude of the velocity will change. The result will be uniform circular motion.

   c) an acceleration with components both parallel to and perpendicular to the original velocity acts on the object

   Answer: (2 marks)

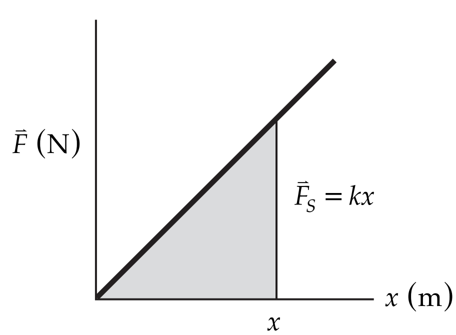
   In this case, the parallel component of the acceleration will serve to increase the magnitude of the original velocity, while the perpendicular component of the acceleration will serve to change the direction of the original velocity. The resulting motion will be the object speeding up and turning.

Outcome S4P-1-32

4. Derive the equation for the potential energy of a spring $\left(PE_S = \frac{1}{2}kx^2\right)$ using Hooke's law and a force-displacement graph.

   Answer:

   The amount of work done by a changing force over a displacement is given by the area under the corresponding force-displacement graph. In the case of the stretched spring, the displacement is just the extension, x, and the work done in extending the spring by an amount x is equal to the spring potential energy stored in the extended spring.



According the Hooke's law at the given extension, x m, the restoring force of the spring, $\vec{F}_S$, is given by $\vec{F}_S = -k\bar{x}$. Again, remember of the the negative sign simply indicates that the directions of the extension and the restoring force of the spring are opposite to each other.

So, you can say that $\vec{F}_S = -kx$.

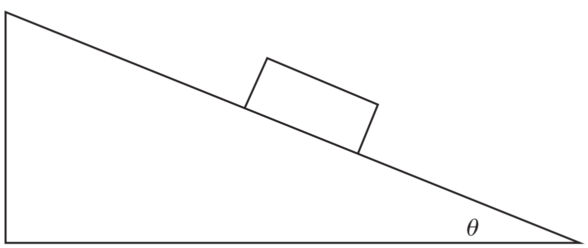
Since the area of the force displacement graph for an ideal spring is in the shape of a triangle, the area is $\frac{1}{2}$(base)(height), which in this case is $\frac{1}{2}(x)(kx)$.

Thus, the energy stored in a spring is given by

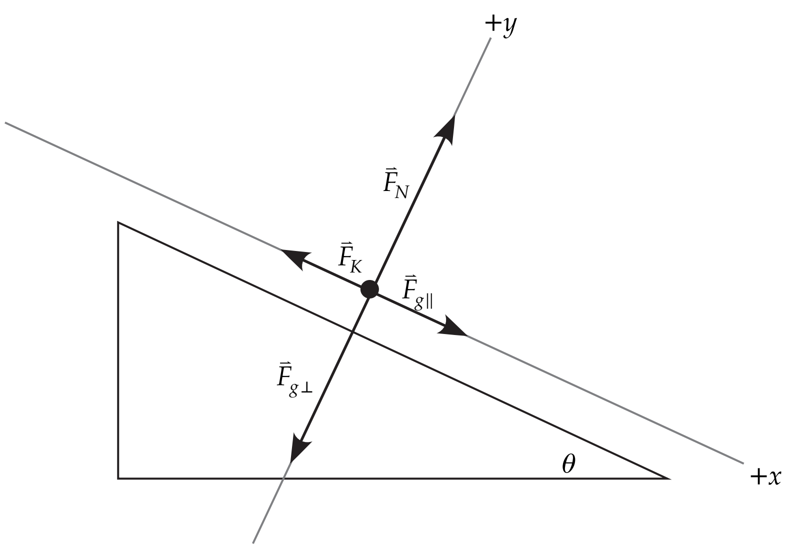
$$PE_S = \frac{1}{2}kx^2.$$

Outcome S4P-1-5

5. Draw a free-body diagram for an object of mass m resting on an inclined plane, as given in the diagram below. Label clearly the force of gravity and its components, the normal force, and the force of friction. Write an expression for the magnitude of the components of the force of gravity parallel to the surface and perpendicular to the surface.



Answer:



The components of the force of gravity are:

$$\vec{F}_{g\perp} = \cos\theta\left(\vec{F}_g\right)$$
$$\vec{F}_{g||} = \sin\theta\left(\vec{F}_g\right)$$

Outcome S4P-1-14

6. Relate the impulse-momentum equation to the following real-life situations:

   a) hitting a baseball as far as possible

   Answer:

   The impulse-momentum equation $\left(\vec{F}\Delta t = m\Delta\vec{v}\right)$ relates the impulse applied during an interaction to the change in momentum of an object. The larger the momentum of a baseball after it is hit into the air, the farther it should travel. To increase the momentum of the baseball, the hitter can do two things. First, he can exert a larger force on the ball by swinging the bat harder. Secondly, he can increase the time of the interaction by following through.

   b) catching a baseball with your bare hands without hurting yourself

   Answer:

   In this case, hurting your hands while catching a baseball stems from the force that the baseball exerts on your hands as you exert a force to stop the ball. The baseball must undergo a certain change in momentum. To decrease the force required to stop the ball, the time over which the ball is stopped must be lengthened. This is done by cushioning the ball by bringing your hands in towards your body as you're catching the ball.

Part D: Problems (8 x 7 = 56 Marks)

Answer any eight (8) problems. Please show your work. Number your answers clearly.

Outcomes S4P-0-2a, S4P-0-2f, S4P-0-2h, S4P-1-2, S4P-1-3

1. An airplane flies with an airspeed of 225 km/h heading due west. At the altitude at which the plane is flying, the wind is blowing at 105 km/h heading due south.

   a) What is the velocity of the plane as observed by someone standing on the ground?

   Answer: (4 marks)

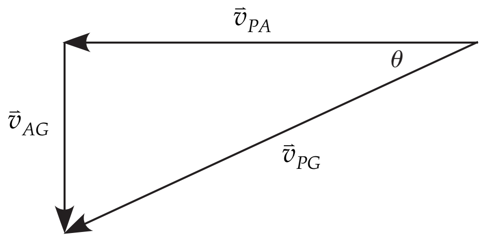
   Given: Velocity of the plane next to the air $\vec{v}_{PA} = 225 \text{ km/h } [\text{W}]$

   Velocity of the air next to the ground $\vec{v}_{AG} = 105 \text{ km/h } [\text{S}]$

   Unknown: Velocity of the plane next to the ground $\vec{v}_{PG} = ?$

   Equation: $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$

   Substitute and solve: $\vec{v}_{PG} = 225 \text{ km/h } [\text{W}] + 105 \text{ km/h } [\text{S}]$



Using the theorem of Pythagoras,

$$\vec{v}_{PG}{}^2 = \vec{v}_{PA}{}^2 + \vec{v}_{AG}{}^2$$

$$\vec{v}_{PG}{}^2 = (225\text{ km/h})^2 + (105\text{ km/h})^2$$

$$\vec{v}_{PG} = \sqrt{61650} = 248\text{ km/h}$$

Using trigonometry, you can determine the angle.

$$\theta = \tan^{-1}\frac{105}{225} = 25.0°$$

The velocity of the plane next to the ground is 248 km/h [25° south of west].

b) How far off course would the plane, while it is heading due west, be blown by the wind during 1.50 h of flying?

Answer: (1 mark)

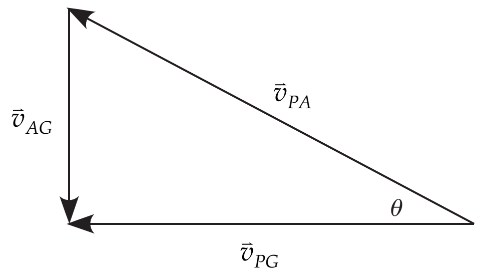
The wind pushes the plane south. The velocity of the wind is constant at 105 km/h [S] for 1.50 hours.

Using the equation $\vec{v} = \frac{\vec{d}}{\Delta t}$ rearranged to give $\vec{d} = \vec{v}\Delta t$, the plane is pushed off course by $\vec{d} = (105\text{ km/s [S]})(1.50\text{ h}) = 158\text{ km [S]}$.

c) What heading must a plane take in order to reach its destination, which is due west of a starting point?

Answer: (2 marks)

The plane must head north of west so that the wind will push it south enough to have a final heading of due west.



In this triangle, we relate the given sides to the angle using the sine function.

$$\theta = \sin^{-1}\frac{105}{225} = 27.8°$$

The plane must head 27.8° [north of east] in order to end up flying due west.

2. A motorcycle starts from rest and accelerates at +3.50 m/s^2 for a distance of 175 m. It then slows down with an acceleration of –1.50 m/s^2 until the velocity is +10.0 m/s.

a) What is the length of time the motorcyclist takes to travel +175m?

Answer: (2 marks)

In our reference system, right will be the positive direction.

Given:	Initial velocity	$\vec{v}_1 = 0 \text{ m/s}$
	Acceleration one	$\vec{a} = +3.50 \text{ m/s}^2$
	Displacement	$\vec{d} = +175 \text{ m}$
Unknown:	Time interval	$\Delta t = ?$

Equation: $$\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

Substitute and solve:

$$+175 \text{ m} = (0 \text{ m/s})\Delta t + \frac{1}{2}(+3.50 \text{ m/s/s})\Delta t^2$$

$$+350 \text{ m} = (+3.50 \text{ m/s/s})\Delta t^2$$

$$100 \text{ m/m/s}^2 = \Delta t^2$$

$$10.0 \text{ s} = \Delta t$$

It takes 10.0 seconds for the motorcycle to travel 175 m.

b) What is the velocity at the end of the time interval determined in part (a)?

Answer: (2 marks)

Unknown: Final velocity $\vec{v}_2 = ?$

Equation: $\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$

Substitute and solve: $\vec{v}_2 = 0 \text{ m/s} + (+3.50 \text{ m/s/s})(10.0 \text{ s}) = +35.0 \text{ m/s}$

The final velocity is plus 35.0 m /s.

c) Determine the displacement of the motorcycle while it is slowing down during the second part of its journey.

Answer: (3 marks)

Given: Velocity at the beginning of the interval $\vec{v}_1 = +35.0 \text{ m/s}$

Velocity at the end of the interval $\vec{v}_2 = +10.0 \text{ m/s}$

Acceleration $\vec{a} = -1.50 \text{ m/s}^2$

Unknown: Displacement $\vec{d} = ?$

Equation: $v_2{}^2 = v_1{}^2 + 2ad$

Substitute and solve:

$$(+10.0 \text{ m/s})^2 = (+35.0 \text{ m/s})^2 + 2(-1.50 \text{ m/s}^2)d$$

$$100 = 1225 - 3d$$

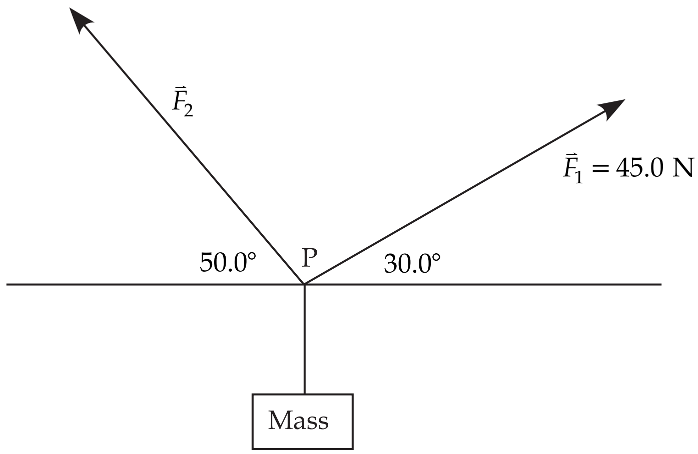
$$3d = 1125$$

$$d = +375 \text{ m}$$

The motorcycle travels +375 m while it is decelerating.

Outcomes S4P-1-4, S4P-0-2h

3. What mass, M, can be supported at P so that the forces are in equilibrium at P?



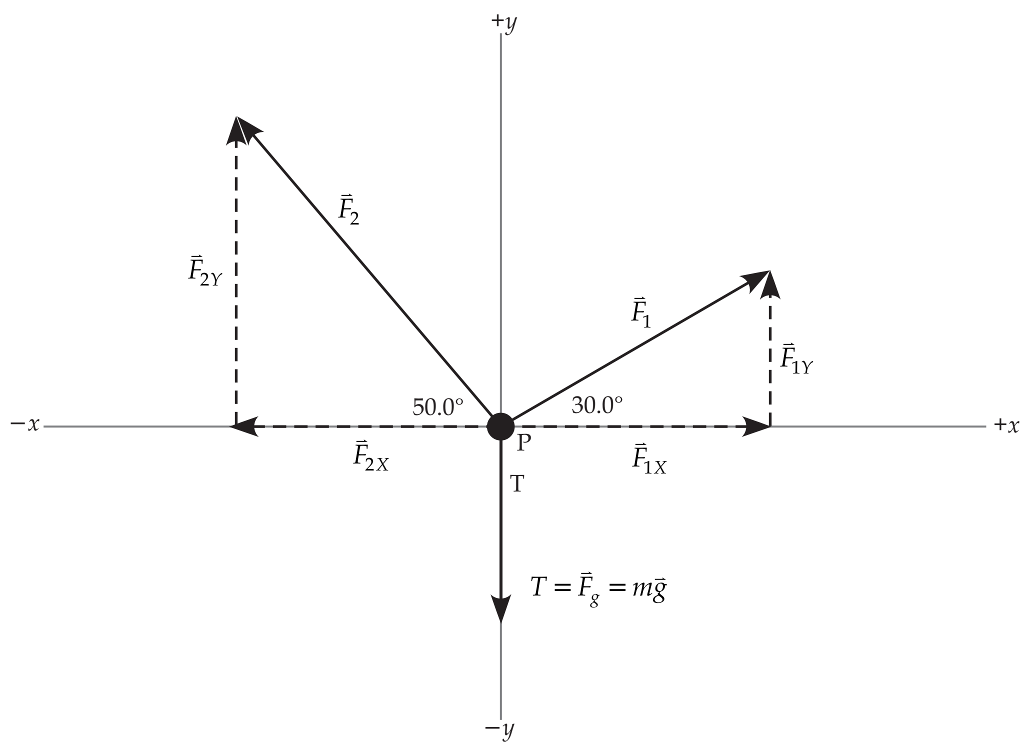
Answer: (7 marks)

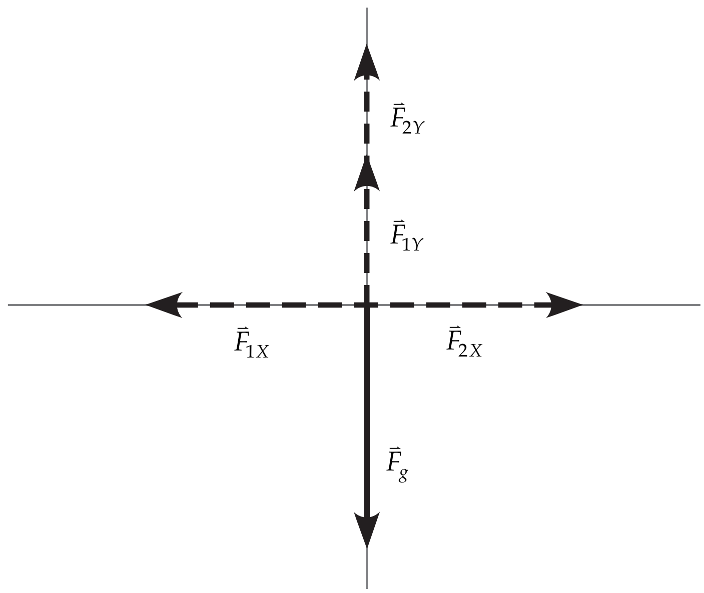
Here is the strategy.

Since you know $\vec{F}_1$ and the angle, you can determine the horizontal component and the vertical component for this force.

Next, since the forces are in equilibrium, the horizontal components of the two forces must balance. Therefore, you know the horizontal component of $\vec{F}_2$. You can then calculate the vertical component of $\vec{F}_2$.

The sum of the vertical components of the two forces will give the weight of the mass.





Vector	*x*-component (N)	*y*-component (N)
$\vec{F}_1$	$+(\cos 30.0°)\vec{F}_1 = -(0.866)(45.0\text{ N})$ $= +39.0\text{ N}$	$+(\sin 30.0°)\vec{F}_1 = -(0.5)(45.0\text{ N})$ $= +22.5\text{ N}$
$\vec{F}_2$	$-(\cos 50.0°)\vec{F}_2 = -0.6428\vec{F}_2$	$-(\sin 50.0°)\vec{F}_2 = -0.7660\vec{F}_2$
$\vec{F}_g$	0	$-\vec{F}_g$
$\vec{F}_{\text{NET}}$	0	0

From the horizontal component column, you can see that the sum of the component is 0 N.

$$+39.0\text{ N} + \left(-0.6428\vec{F}_2\right) = 0\text{ N}$$

$$\vec{F}_2 = 60.7\text{ N}$$

In the vertical component, $\vec{F}_2$ is $+\left(\sin 50.0°\right)\vec{F}_2 = 0.7660\left(60.7\text{ N}\right) = 46.5\text{ N}$.

The total force pulling the mass upwards is +22.5 N + (+46.5 N) = 69.0 N.
This force just balances the force of gravity.

Using $\vec{F}_g = m\vec{g}$ rearranged to $m = \dfrac{\vec{F}_g}{\vec{g}}$ yields $m = \dfrac{69.0\text{ N}}{9.80\text{ N/kg}} = 7.04\text{ kg}$.

A mass of 7.04 kilograms is suspended at the point P.

Outcome S4P-1-24

4. A bicyclist and his bicycle have a mass of 85.4 kg. The cyclist is travelling around a circular track of radius 75.0 m at a constant speed of 7.96 m/s.

   a) Calculate the period of this motion.

   Answer: (2 marks)

Given:	Mass	$m = 85.4$ kg
	Radius	$R = 75.0$ m
	Speed	$v = 7.96$ m/s
Unknown:	Period	$T = ?$

   Equation: $v = \frac{2\pi R}{T}$ rearranged to $T = \frac{2\pi R}{v}$

   Substitute and solve: $T = \frac{2\pi(75.0 \text{ m})}{7.96 \text{ m/s}} = 59.2 \text{ s}$

   The period is 59.2 seconds.

   b) Calculate the acceleration of the cyclist.

   Answer: (2 marks)

   Equation: $a_C = \frac{v^2}{R}$

   Substitute and solve: $a_C = \frac{v^2}{R} = \frac{(7.96 \text{ m/s})^2}{75.0 \text{ m}} = 0.845 \text{ m/s/s}$

   The acceleration is 0.845 m per second towards the centre of the motion.

   c) Calculate the force necessary to keep the cyclist moving around the track.

   Answer: (2 marks)

   Using $\vec{F}_C = m\vec{a}_C$

   $\vec{F}_C = (85.4 \text{ kg})(0.844 \text{ m/s/s}) = 72.1 \text{ N}$

   The centripetal force is 72.1 N towards the centre of the circle.

   d) Calculate the frequency of this motion.

   Answer: (1 mark)

   The frequency is the reciprocal of the period.

   $f = \frac{1}{T} = \frac{1}{59.2 \text{ s}} = 0.0169 \text{ Hz}$

Outcomes S4P-1-13, S4P-0-2h

5. A car of mass 1250 kg is travelling at 20.0 m/s [W]. At an icy intersection, the car collides with a truck of mass 2450 kg travelling at 15.0 m/s [S]. The collision lasts 0.250 seconds. After the collision, the two vehicles slide along together.

   a) What is the total momentum of the system of the car and the truck before the collision?

   Answer: (5 marks)

   The total momentum on the system is the momentum of the car plus the momentum of the truck added together as vectors.

   Given: Mass of the car $m_1 = 1250$ kg

   Initial velocity of the car $\vec{v}_1 = 20.0 \text{ m/s [W]}$

   Mass of the truck $m_2 = 2450$ kg

   Initial velocity of the truck $\vec{v}_2 = 15.0 \text{ m/s [S]}$

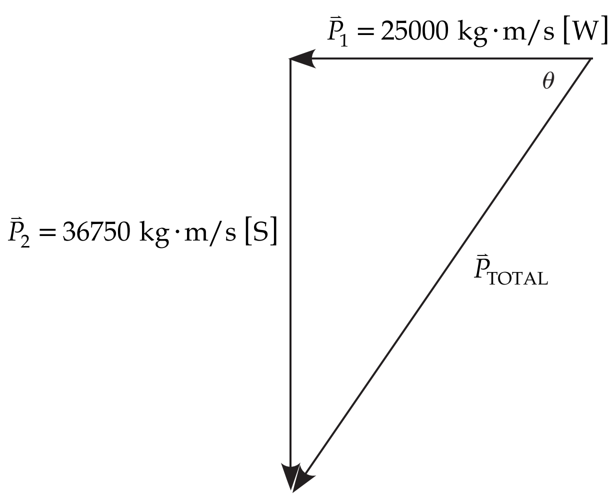
   Unknown: Total momentum on the system $\vec{p}_{\text{TOTAL}} = ?$

   Equation: $\vec{p}_{\text{TOTAL}} = \vec{p}_1 + \vec{p}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$

   Substitute and solve: $\vec{p}_{\text{TOTAL}} = (1250 \text{ kg})(20.0 \text{ m/s [W]}) + (2450 \text{ kg})(15.0 \text{ m/s [S]})$

   $\vec{p}_{\text{TOTAL}} = 25000 \text{ kg}\cdot\text{m/s [W]} + 36750 \text{ kg}\cdot\text{m/s [S]}$

   These must be added together as vectors.



$\vec{p}_{\text{TOTAL}}{}^2 = (25000)^2 + (36750)^2 = 1.975 \times 10^9$

$\vec{p}_{\text{TOTAL}} = 44400 \text{ kg}\cdot\text{m/s}$

   Using trigonometry, the tangent function relates the given sides.

$\theta = \tan^{-1}\dfrac{36750}{25000} = 55.8°$

   The total momentum of the car and truck together is 44400 kg·m/s [55.8° south of west].

b) What is the velocity of the car after the collision?

Answer: (2 marks)

Since the car and the truck are travelling along together, they have the same velocities after the crash. You can think of the car and the truck together as a single mass moving along with a single velocity with the momentum equal to the total momentum of the system.

$$\vec{p}_{\text{TOTAL}} = m_{\text{TOTAL}}\vec{v}_{\text{TOTAL}}$$

$$44400\ \text{kg}\cdot\text{m/s}\ [55.8°\ \text{S of W}] = (1250\ \text{kg} + 2450\ \text{kg})\vec{v}_{\text{TOTAL}}$$

$$\vec{v}_{\text{TOTAL}} = \frac{44400\ \text{kg}\cdot\text{m/s}\ [55.8°\ \text{S of W}]}{3700\ \text{kg}} = 12.0\ \text{m/s}\ [55.8°\ \text{S of W}]$$

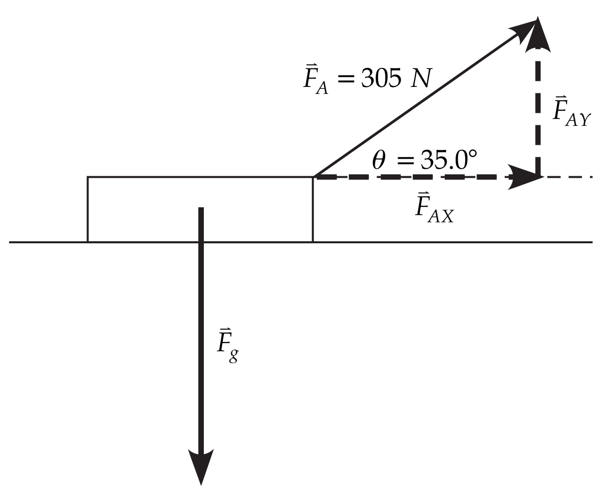
The car and the truck are moving along together at to 12.0 m/s [55.8° south of west].

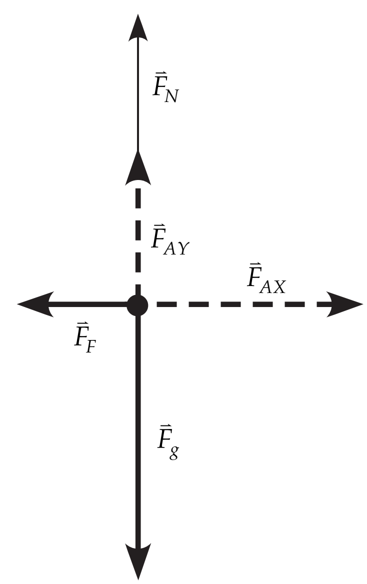
Outcomes S4P-1-8, S4P-0-2h

6. A crate of mass 80.0 kg is pulled across a level concrete floor at a constant acceleration of 0.895 m/s^2. A force of 305 N acting 35.0° above the horizontal is used to move the crate.

a) Calculate the normal force acting on the crate.

Answer: (3 marks)





In this case, the normal force is decreased by the vertical component of the applied force, since the component is pulling the object up from the surface.

Given: Mass $m = 80.0$ kg

Acceleration $\vec{a} = +0.895$ m/s/s

Unknown: Normal force $\vec{F}_N = ?$

Equation: From the free-body diagram, you see that
$\vec{F}_g + \vec{F}_N + \vec{F}_{AY} = 0$ N.
You can find the force of gravity using
$\vec{F}_g = m\vec{g} = (80.0 \text{ kg})(9.80 \text{ m/s/s [down]}) = 784 \text{ N [down]}$.
The vertical component of the applied force is found using
sin 35.0° (305 N) = 175 N [up].

Substitute and solve: $784 \text{ N [down]} + \vec{F}_N + 175 \text{ N } \lfloor \text{up} \rfloor = 0 \text{ N}$

$\vec{F}_N = 609 \text{ N } [\text{up}]$

b) Calculate the force of kinetic friction acting on the crate.

Answer: (3 marks)

Since the crate is sliding in the horizontal direction, we must consider the forces acting in the horizontal direction.

$\vec{F}_{\text{NET}} = \vec{F}_{AX} + \vec{F}_K$

The net force is found using $\vec{F}_{\text{NET}} = m\vec{a} = (80.0 \text{ kg}) + (0.895 \text{ m/s/s}) = +71.6 \text{ N}$.

The horizontal component of the applied forces is found using
cos 35.0° (305 N) = +250 N.

Substitute and solve: $+71.6 \text{ N} = +250\ N + \vec{F}_K$

$\vec{F}_K = -178 \text{ N}$

A force of friction is 178 N [left].

c) Calculate the coefficient of kinetic friction.

Answer: (1 mark)

$$\mu_k = \frac{\vec{F}_K}{\vec{F}_N} = \frac{178 \text{ N}}{609 \text{ N}} = 0.292$$

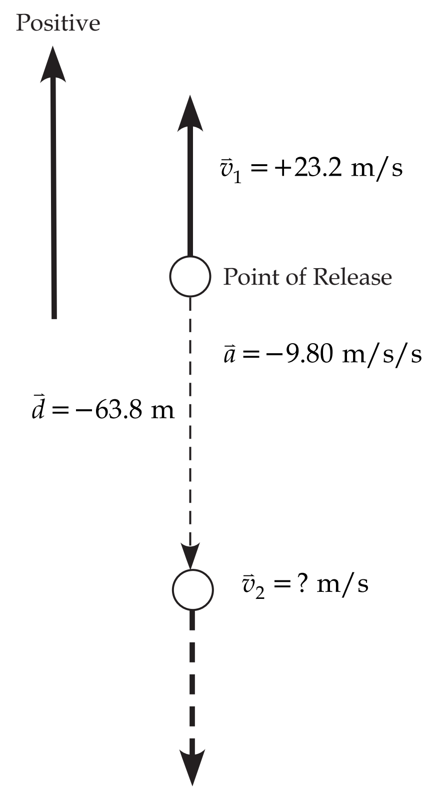
The coefficient of kinetic friction is a 0.292.

Outcome S4P-1-15

7. A stone of mass 75.0 g is thrown upwards at 23.2 m/s from the height of a railing of a bridge that is 63.8 m above the surface of the water.

   a) Calculate the velocity of the stone as it strikes the water's surface.

   Answer: (2.5 marks)



Equation: $v_2^2 = v_1^2 + 2ad$

Substitute and solve: $v_2^2 = 2(3.2 \text{ m/s})^2 + 2(-9.80 \text{ m/s/s})(-63.8 \text{ m}) = 1788$

$\vec{v}_2 = -42.3$ m/s

The stone strikes the water moving at 42.3 m/s [down].

b) How long after the stone is thrown is the stone 10.0 m above the surface of the water?

Answer: (3 marks)

If the stone is 10.0 m above the surface of the water, it would have fallen 53.8 m from the point to release.

The change in this question is that the displacement will be $\vec{d} = -53.8$ m.

Unknown: Time interval $\Delta t = ?$

Equation: No single kinematics equation will yield the answer unless you use the quadratic formula.

It is simpler to do this in two steps. First, find the final velocity; then find the time interval.

$$v_2^2 = v_1^2 + 2ad = (23.2 \text{ m/s})^2 + 2(-9.80 \text{ m/s/s})(-53.8 \text{ m}) = 1593$$

$$\vec{v}_2 = -39.9 \text{ m/s}$$

Then use $\Delta t = \dfrac{\vec{v}_2 - \vec{v}_1}{\vec{a}} = \dfrac{(-39.9 \text{ m/s} - 23.3 \text{ m/s})}{-9.80 \text{ m/s/s}} = 6.45 \text{ s}$

The stone is 10.0 m above the surface of the water 6.45 seconds after it was released.

c) Where is the stone 3.50 seconds after being thrown?

Answer: (1.5 marks)

Given: Time interval $\Delta t = 3.50$ s

Initial velocity $\vec{v}_1 = +23.2$ m/s

Acceleration $\vec{a} = -9.80$ m/s/s

Unknown: Displacement $\vec{d} = ?$

Equation: $\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2} \vec{a} \Delta t^2$

Substitute and solve: $\vec{d} = (23.2 \text{ m/s})(3.50 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s/s})(3.50 \text{ s})^2 = 21.2 \text{ m}$

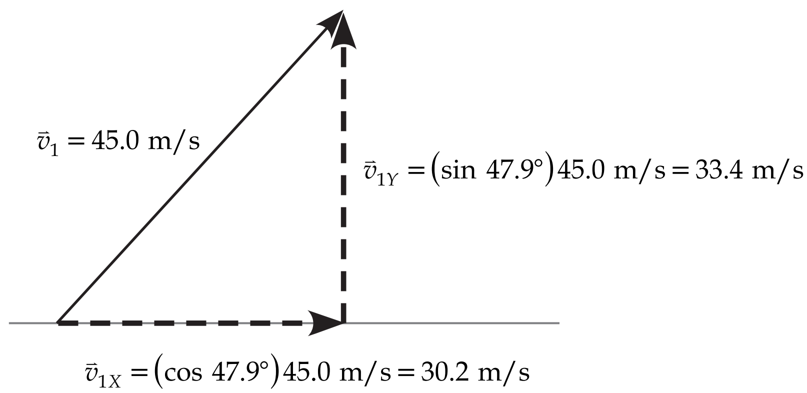
The stone is 21.2 m above the point of release.

Outcomes S4P-1-18, S4P-0-2h

8. A golfball is struck leaving the tee at a velocity of 45.0 m/s 47.9° from the horizontal. The ball travels over a level fairway towards a green where the hole is located 204 m from the tee.

   a) Calculate the vertical and horizontal components of the ball's velocity.

   Answer: (2 marks)



   The horizontal component is 30.2 m/s [right]. The vertical component is 33.4 m/s [up].

   b) Determine the time the ball is in the air.

   Answer: (3 marks)

   Consider only the vertical motion of the ball while it is rising to the top of its flight. Let up be positive.

Given:	Initial velocity	$\vec{v}_1 = +33.4 \text{ m/s}$
	Final velocity	$\vec{v}_2 = 0 \text{ m/s}$
	Acceleration	$\vec{a}_Y = -9.80 \text{ m/s}^2$
Unknown:	Time interval	$\Delta t = ?$

Equation:

$$\Delta t = \frac{\vec{v}_2 - \vec{v}_1}{\vec{a}}$$

Substitute and solve:

$$\Delta t = \frac{0 \text{ m/s} - (+33.4 \text{ m/s})}{-9.80 \text{ m/s}^2} = 3.41 \text{ s}$$

   The ball takes 3.41 seconds on the way up and another 3.41 seconds on the way down for a total time of 6.82 seconds.

c) If the ball is heading in the right direction, will it be possible for the golfer to score a hole in one?

Answer: (2 marks)

The ball must land before the hole and fly in the correct direction to land in the hole.

Given: Initial horizontal velocity $\vec{v}_{1X} = 30.2 \text{ m/s}$

Time of travel $\Delta t = 6.82$ s

Unknown: Horizontal distance $\vec{d}_X = ?$

Equation: The horizontal distance is found using $\vec{d}_X = \vec{v}_{1X}\Delta t$.

Substitute and solve: $\vec{d}_X = (30.2 \text{ m/s})(6.82 \text{ s}) = 206 \text{ m}$

The ball lands 2 m beyond the hole. There is no hole in one.

Outcomes S4P-1-24, S4P-1-8

9. A satellite orbits the earth in a nearly circular orbit of radius 7.88×10^6 m with a period of 115 minutes. The satellite has a mass 238 kg.

a) Calculate the speed of the satellite in m/s.

Answer: (2.5 marks)

Given: Radius $R = 7.88 \times 10^6$ m

Period $T = 115$ minutes x 60 s/min = 6900 s

Mass $m = 238$ kg

Unknown: Speed $v = ?$

Equation: $v = \frac{2\pi R}{T}$

Substitute and solve: $v = \frac{2\pi R}{T} = \frac{2(3.14)(7.88 \times 10^6 \text{ m})}{6900 \text{ s}} = 7170 \text{ m/s}$

The satellite is travelling at 7170 m/s.

b) Calculate the centripetal force acting on the satellite.

Answer: (3 marks)

Equation: $\vec{F}_C = m\vec{a} = m\frac{v^2}{R}$

Substitute and solve: $\vec{F}_C = (238 \text{ kg})\frac{(7170 \text{ m/s})^2}{7.88 \times 10^6 \text{ m}} = 1550 \text{ N}$

The centripetal force is 1550 N towards the centre of the earth.

c) If the weight of the satellite supplies the centripetal force, calculate the gravitational field strength at this distance from earth.

Answer: (1.5 marks)

Since $\vec{F}_g = m\vec{g}$ and $\vec{g} = \frac{\vec{F}_g}{m}$, you can determine the gravitational field strength.

$$\vec{g} = \frac{\vec{F}_g}{m} = \frac{1550 \text{ N}}{235 \text{ kg}} = 6.51 \text{ N/kg [towards the Earth's centre]}$$

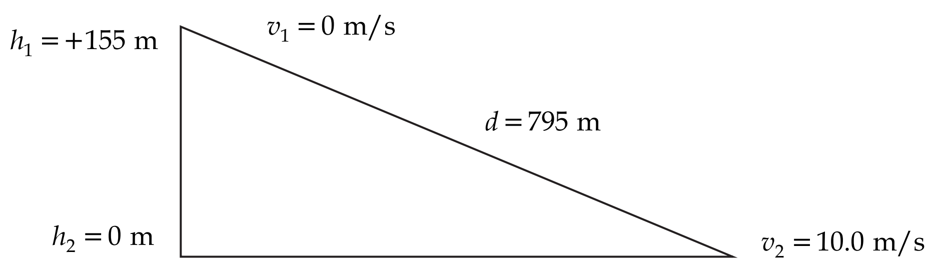
Outcome S4P-1-33

10. A skier of mass 82.4 kg starts his run from rest. The skier drops 155 m vertically while skiing 795 m down the slope. The skier arrives at the bottom of the slope moving at 10.0 m/s.

a) Determine the change in the gravitational potential energy of the skier.

Answer: (2 marks)

Given:



Mass m = 82.4 kg

Unknown: Change in gravitational potential energy ΔPE_g = ?

Equation: $\Delta PE_g = m\vec{g}\Delta h$

Substitute and solve: ΔPE_g = (82.4 kg)(9.80 N/kg)(0 m + (–155 m)) = –125000 J

The skier lost 125000 J of gravitational potential energy.

b) Determine the work done by friction.

Answer: (3 marks)

Friction does negative work on the skier—that is, it removes energy from the system. Therefore, the total energy of the system after the run is less than before the run. The missing energy is the work done by friction.

$$\Delta E_{\text{TOTAL}} = \Delta E_{\text{TOTAL FINAL}} - \Delta E_{\text{TOTAL INITIAL}}$$

$$\Delta E_{\text{TOTAL}} = \left(KE_f + PE_{gf}\right) - \left(KE_0 + PE_{g0}\right)$$

$$\Delta E_{\text{TOTAL}} = \left(\frac{1}{2}mv_f^{\,2} + mgh_f\right) - \left(\frac{1}{2}mv_0^{\,2} + mgh_0\right)$$

$$\Delta E_{\text{TOTAL}} = \left(\frac{1}{2}(82.4\text{ kg})(10.0\text{ m/s})\right)^2 + \left((82.4\text{ kg})(9.80\text{ N/kg})(0\text{ m})\right)$$
$$- \frac{1}{2}\left((82.4\text{ kg})(0\text{ m/s})\right)^2 + \left((82.4\text{ kg})(9.80\text{ N/kg})(+155\text{ m})\right)$$

$$\Delta E_{\text{TOTAL}} = (4120\text{ J}) - (125000\text{ J}) = -121000\text{ J}$$

Work of friction = –121000 J

Since the total energy decreases by 121000 J, this is the work done by friction.

c) Determine the average force of friction that acted on the skier.

Answer: (2 marks)

Given: Work of friction W_F = –121000 J

Distance over which friction acts $\vec{d}$ = 795 m

Unknown: Force of friction $\vec{F}_K = ?$

Equation: $W_F = \vec{F}_K \vec{d}$

Substitute and solve: $-121000 = \vec{F}_K(795)$

$\vec{F}_K = -152\text{ N}$

The average force of friction is 152 N opposite to the motion.

Outcome S4P-1-33

11. A dry ice puck, which slides along with no friction, has a mass of 2.30 kg and is sliding along a level horizontal surface at 2.25 m/s. The puck hits a spring bumper, which is compressed 20.0 cm, before the puck comes to rest.

a) Determine the force constant of this spring.

Answer: (2.5 marks)

The puck will compress the spring until the puck stops moving. At that point, the initial kinetic energy of the puck will have been converted into spring potential energy. The total mechanical energy of the system is conserved.

Given:	Mass	$m = 2.30$ kg
	Initial velocity	$\vec{v}_1 = 2.25$ m/s [towards the spring]
	Final velocity	$\vec{v}_2 = 0$ m/s
	Compression of spring	$x = 20.0$ cm $= 0.200$ m
Unknown:	Spring constant	$k = ?$

Equation:

$$KE_1 + PE_{S1} = KE_2 + PE_{S2}$$

$$\frac{1}{2}mv_1^{\,2} + \frac{1}{2}kx_1^{\,2} = \frac{1}{2}mv_2^{\,2} + \frac{1}{2}kx_2^{\,2}$$

Substitute and solve:

$$\frac{1}{2}(2.30\text{ kg})(2.25\text{ m/s})^2 + \frac{1}{2}k(0\ m)^2$$

$$= \frac{1}{2}(2.30\text{ kg})(0\text{ m/s})^2 + \frac{1}{2}k(0.200\text{ m})^2$$

$$5.82\text{ J} + 0\text{ J} = 0\text{ J} + 0.0200k$$

$$k = 291\text{ N/kg}$$

The spring constant is 291 N/kg.

b) How much is the spring compressed when the puck is sliding at 1.75 m/s towards the bumper?

Answer: (3 marks)

Use the law of conservation of kinetic energy and spring potential energy again.

Given: Mass $m = 2.30$ kg

Initial velocity $\vec{v}_1 = 2.25$ m/s [towards the spring]

Final velocity $\vec{v}_2 = 1.75$ m/s

Spring constant $k = 291$ N/kg?

Unknown: Compression of spring $x_2 = ?$

Equation:

$$KE_1 + PE_{S1} = KE_2 + PE_{S2}$$

$$\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2$$

Substitute and solve:

$$\frac{1}{2}(2.30 \text{ kg})(2.25 \text{ m/s})^2 + \frac{1}{2}(291 \text{ N/kg})(0 \text{ m})^2$$

$$= \frac{1}{2}(2.30 \text{ kg})(1.75 \text{ m/s})^2 + \frac{1}{2}(291 \text{ N/kg})(x_2 \text{ m})^2$$

$$5.82 \text{ J} + 0 \text{ J} = 3.52 \text{ J} + 145.5x_2^2$$

$$2.30 \text{ J} = 145.5x_2^2$$

$$x_2^2 = 0.01581$$

$$x_2 = 0.126 \text{ m}$$

The spring is compressed 0.126 m.

c) If the dry ice puck was initially moving at 2.25 m/s towards the spring and the spring is compressed only 10.0 cm, calculate the speed of the puck at that moment.

Answer: (2.5 marks)

Given: Mass m = 2.30 kg

Initial velocity $\vec{v}_1$ = 2.25 m/s [towards the spring]

Spring constant k = 291 N/kg?

Compression of spring x_2 = 10.0 cm = 0.100 m

Unknown: Final velocity $\vec{v}_2$ = 1.75 m/s

Equation:

$$KE_1 + PE_{S1} = KE_2 + PE_{S2}$$

$$\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2$$

Substitute and solve:

$$\frac{1}{2}(2.30\text{ kg})(2.25\text{ m/s})^2 + \frac{1}{2}(291\text{ N/kg})(0\text{ m})^2$$

$$= \frac{1}{2}(2.30\text{ kg})\vec{v}_2^2 + \frac{1}{2}(291\text{ N/kg})(0.100\text{ m})^2$$

$$5.82\text{ J} + 0\text{ J} = 1.15\vec{v}_2^2 + 1.46\text{ J}$$

$$\vec{v}_2^2 = \frac{4.36}{1.15} = 3.79$$

$$\vec{v}_2 = 1.95\text{ m/s}$$

The dry ice puck is sliding at 1.95 m/s.

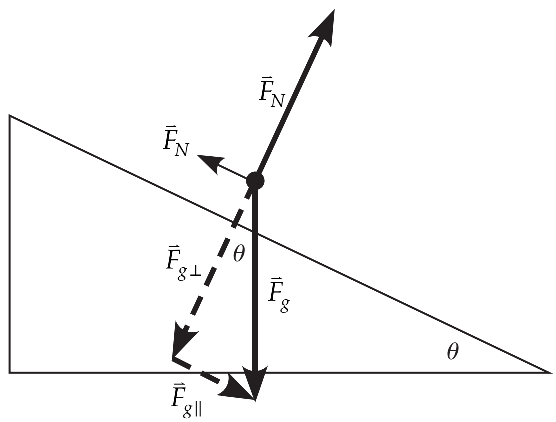
Outcomes S4P-1-5, S4P-1-7, S4P-1-8

12. A child is sitting in a wagon on a hill, which has an incline of 14.0° from the horizontal. The mass of the child and wagon is 42.0 kg. The coefficient of kinetic friction is 0.125. The wagon begins to move.

a) Calculate the normal force acting on the wagon.

Answer: (2.5 marks)

Given:	Mass	$m = 42.0$ kg
	Angle from the horizontal	$\theta = 14.0°$
	Coefficient of kinetic friction	$\mu_k = 0.125$
Unknown:	Normal force	$\vec{F}_N = ?$



The components of $\vec{F}_g$ form a right triangle enclosing the angle θ. This angle θ is the same as the angle θ in the ramp.

The components of $\vec{F}_g$ are found by:

$$\vec{F}_{g\perp} = \cos\theta\left(\vec{F}_g\right)$$

$$\vec{F}_{g\parallel} = \sin\theta\left(\vec{F}_g\right)$$

The force of gravity is $\vec{F}_g = m\vec{g} = (42.0 \text{ kg})(9.80 \text{ N/kg}) = 412 \text{ N [down]}$.

The component of the force of friction perpendicular to the plane is

$\vec{F}_{g\perp} = \cos\theta\left(\vec{F}_g\right) = \cos 14.0°(412 \text{ N}) = 400 \text{ N} = 4.00 \times 10^2 \text{ N}$ [into the surface].

The normal force is then 4.00×10^2 N [out of the surface].

b) Calculate the force of kinetic friction.

Answer: (1.5 marks)

$F_K = \mu_k F_N = (0.125)(400 \text{ N}) = 50.0 \text{ N}$

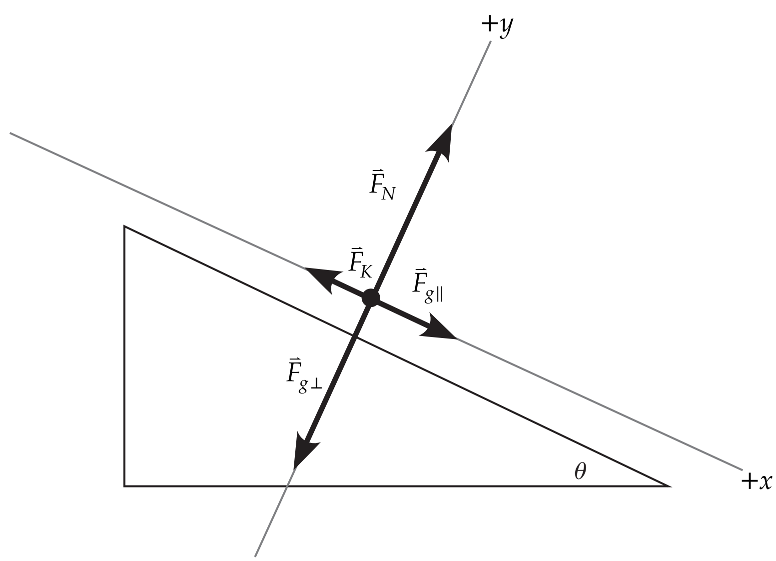
The force of friction is 50.0 N [up the plane].

c) Calculate the distance the child and his wagon will move during the first 15.0 s of his trip.

Answer: (3 marks)

You can see that the only forces acting along the direction of motion (+x-direction) are the force of friction and the parallel component of gravity.

Find the net force, then the acceleration, and finally displacement.



You can see that the only forces acting along the direction of motion (+x-direction) are the force of friction and the parallel component of gravity.

Find the net force, then the acceleration, and finally displacement.

$$\vec{F}_{g\parallel} = \sin\theta\ \vec{F}_g = \sin 14.0°(412 \text{ N}) = 99.7 \text{ N}$$

$$\vec{F}_{\text{NET}} = \vec{F}_{g\parallel} + \vec{F}_K = 99.7 \text{ N [down the plane]} + 50.0 \text{ N [up the plane]}$$

$$\vec{F}_{\text{NET}} = 49.7 \text{ N [down the plane]}$$

Using $\vec{F}_{\text{NET}} = m\vec{a}$ rearranged to $\vec{a} = \frac{\vec{F}_{\text{NET}}}{m}$, $\vec{a} = \frac{49.7 \text{ N}}{42.0 \text{ kg}} = 1.18 \text{ m/s}^2$ [down the plane].

Finally, use kinematics to find the displacement.

Given:	Initial velocity	$\vec{v}_1 = 0$ m/s
	Acceleration	$\vec{a} = 1.18 \text{ m/s}^2 \left[\text{down the plane}\right]$
	Time interval	$\Delta t = 15.0$ s
Unknown:	Distance travelled	$\vec{d} = ?$

Equation: $$\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

Substitute and solve: $$\vec{d} = (0 \text{ m/s})(15.0 \text{ s}) + \frac{1}{2}(1.18 \text{ m/s}^2)(15.0 \text{ s})^2 = 133 \text{ m}$$

The child and his wagon move 133 m during their 15.0 s trip.