Chapter 2

Analyzing Motion

Position and Displacement

Answers to Questions

The calculations will be done using significant digit rules. If changes are made to the number of significant digits, they will be noted.

PRACTICE—PAGE 8

1. The positions of the cars are as follows:

Car A = –5.10 cm or 5.10 cm to the left of the origin.

Car B = 0 cm. Car B is at the origin.

Car C = +4.05 cm or 4.05 cm to the right of the origin.

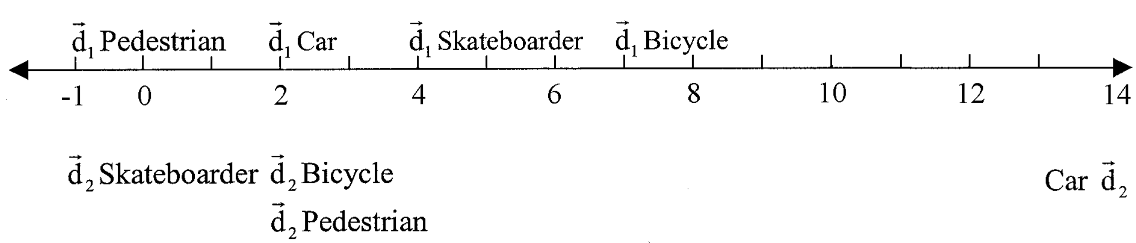
Car D = +9.62 cm or 9.62 cm to the right of the origin.

PRACTICE—PAGE 9

1.

	Car	Bicycle	Pedestrian	Skateboarder
d_1	+2 m	+7 m	-1 m	+4 m
d_2	+14 m	+2 m	+2 m	-1 m

a. and b.



c. Displacement = final position – initial position

$D\vec{d} = \vec{d}_2 - \vec{d}_1$

For the car, $D\vec{d}$ = +14 m – +2 m = +12 m

For the bicycle, $D\vec{d}$ = +2 m – +7 m = –5 m

For the pedestrian, $D\vec{d}$ = +2m – –1 m = +3 m

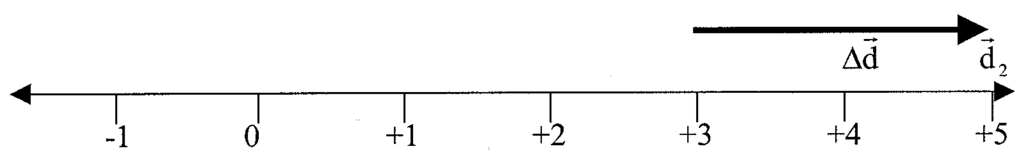
For the skateboarder, $D\vec{d}$ = –1 m – +4 m = –5 m

d. If the displacements all occur in the same time period, the motion can be described. The car traveled a distance of 12 m to the right. The bicycle traveled 5 m to the left. The pedestrian traveled 3 m to the right. The skateboarder also traveled 5 m to the left.

The bicycle and the skateboarder traveled at the same rate and kept the same distance apart. The car passed by the skateboarder first, then the bicycle. The pedestrian passed the skateboarder and ended up at the same position as the bicycle.

The car was traveling the fastest. The pedestrian was traveling the slowest. The bicycle and the skateboarder traveled at the same speed in the same direction.

2.



The truck had a final position of +5 after a displacement of +2. The arrow on the diagram shows the displacement of +2 had to start at +3 so that the final position was +5.

$D\vec{d} = +2$ and $\vec{d}_2 = +5$

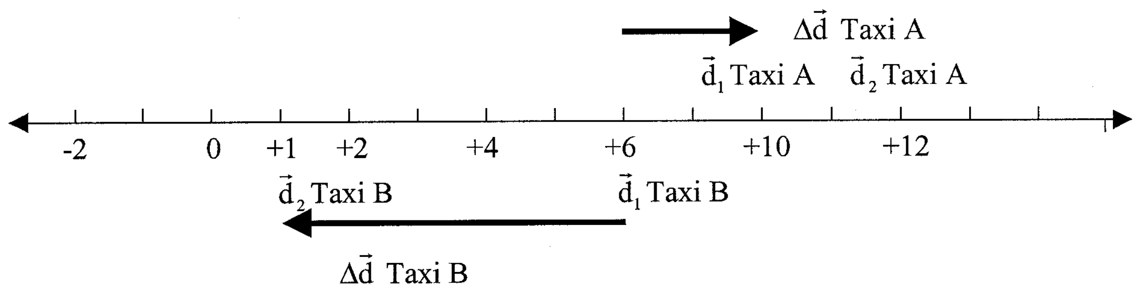
Using $D\vec{d} = \vec{d}_2 - \vec{d}_1$

$+2 = +5 - \vec{d}_1$

$\vec{d}_1 = +5 - +2 = +3$

The initial position was +3.

3.



4. Taxi A $\vec{d}_1 = +6$ and $\vec{d}_2 = +10$

$\Delta\vec{d} = \vec{d}_2 - \vec{d}_1$

$= +10 - +6 = +4$

Taxi B $\vec{d}_1 = +6$ and $\vec{d}_2 = +1$

$\Delta\vec{d} = \vec{d}_2 - \vec{d}_1$

$= +1 - +6 = -5$

5. The displacement of Taxi A is +4 and the displacement of Taxi B is –5. Since the time of travel for both taxis was the same, the taxi that traveled farther had the larger speed.

6. If the origin was moved, the positions of the taxis would be different. However, even though the positions may have been different, the displacements would still be the same.

Instants and Intervals of Time

Think About IT!—Page 10

1. Examples of an interval of time include one year to measure age, one class of 55 minutes, one school year, et cetera.

2. To convert seconds to hours, divide the number of seconds by the number of seconds per hour.

   10 s/3600 s/h = 10 s x 1 h/3600 s = 1/360 s

   The difficulty in this calculation was the odd conversion factor.

Investigation #1 VEHICLES IN MOTION

Think About IT!—Page 11

1. The car is moving slowest when it is at the top of the ramp. The car has just begun to speed up from rest and is traveling very slowly. Also, in the results, one can see that the dots marking the positions of the car are closest together at the top of the ramp.

2. The car is moving fastest on the level surface. This is shown by the fact the dots are spaced farthest apart on the level surface. The car traveled the farthest in equal time intervals.

3. The spaces do not change across the table because the car is not speeding up or slowing down, since it is traveling on a level surface.

4. On the ramp, the car speeds up as it rolls down the ramp. On the table, the car moves with the same speed.

Uniform Motion

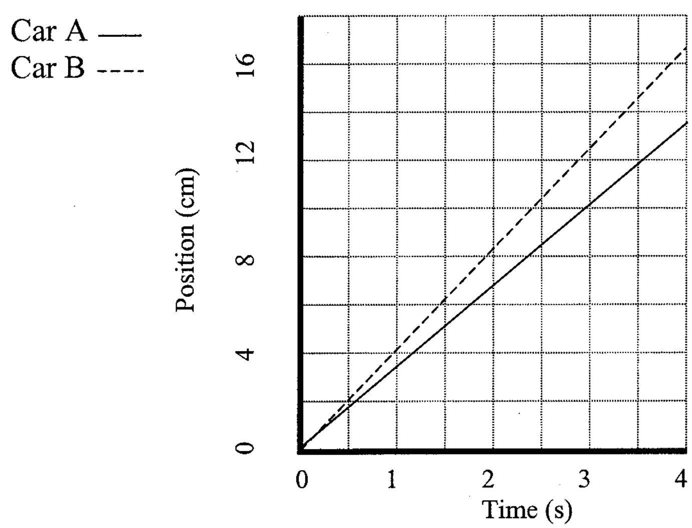
Since students are using rulers with millimetre calibration, an estimated digit in the 0.1 mm or 0.01 cm is used in the measurements. The distance is measured from the front tip of one car to the front tip of the next car.

Table A

Position (cm)	Time (s)
0	0
3.36	1
6.63	2
9.95	3
13.27	4

Table B

Position (cm)	Time (s)
0	0
4.10	1
8.20	2
12.30	3
16.40	4



Think About IT!—Page 12

1. The slope of the line for Car B is steeper than the slope of the line for Car A.

2. Car A has a smaller space between successive images of the car. The points on the Position-Time graph do not rise as rapidly as those for Car B.

Car B has a larger space between successive images of the car. The points on the Position-Time graph rise more rapidly than those for Car A.

Think About IT!—Page 13

1. Concept map: The list of terms can also include direction, vector, and scalar.

Calculating Slope

PRACTICE—PAGE 14

1. The positive direction will be to the right for all diagrams.

a. Origin

$\vec{d}_1 = 0$ km $t = 1.2$ h $\vec{v}_{avg} = ?$ $\vec{d}_2 = +36$ km

$$\vec{v}_{avg} = \frac{\Delta \vec{d}}{\Delta t} = \frac{+36 \text{ km} - 0 \text{ km}}{1.2 \text{ h}} = +30 \text{ km/h}$$

The average velocity of the bicycle is +30 km/h.

b. Origin

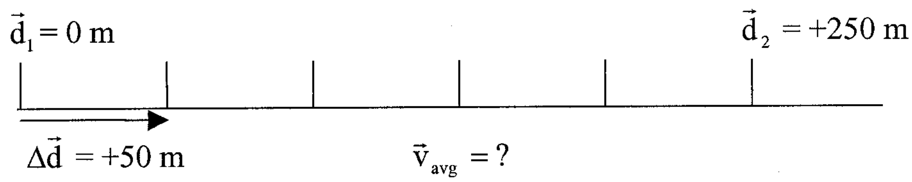
$\vec{d}_1 = 0$ m $t = 2$ s $\vec{v}_{avg} = ?$ $\vec{d}_2$ +17 m

$$\vec{v}_{avg} = \frac{\Delta \vec{d}}{\Delta t} = \frac{+17 \text{ m} - 0 \text{ m}}{2 \text{ s}} = +8.5 \text{ m/s}$$

The average velocity of the person is +8.5 m/s.

Using significant digits, the answer is +8 m/s.

c.

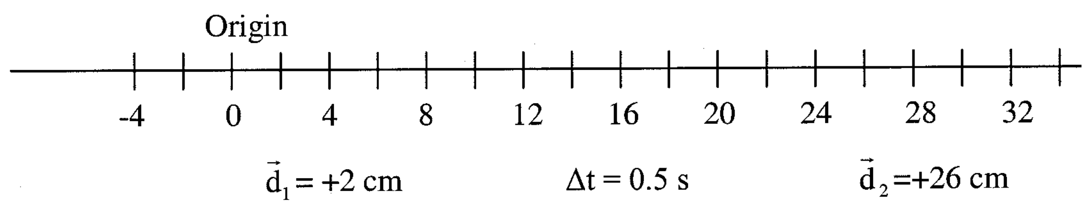


$$\vec{v}_{avg} = \frac{D\vec{d}}{Dt} = \frac{+250\text{ m} - 0\text{ m}}{18\text{ s}} = +13.9\text{ m/s}$$

The average velocity of the car is +13.9 m/s.

Using significant digits, the answer rounds to +14 m/s.

d.



$\vec{v}_{avg} =$

$$\vec{v}_{avg} = \frac{D\vec{d}}{Dt} = \frac{+26\text{ cm} - +2\text{ cm}}{0.5\text{ s}} = +48\text{ cm/s}$$

The average velocity of the mini-V is +48 cm/s. With significant digits, use +50 m/s.

2. a. For a bicycle, 30 km/h would be fast.

b. Running at 8.5 m/s is fast, bordering on unrealistic for the average person. Olympic calibre sprinters run at 10 m/s.

c. The car is traveling at 13.9 m/s x 3.6 = 50 km/h. This would be a medium speed for a car.

d. For a mini-V, 48 cm/s is a medium speed.

3. Assume that the skateboarder is coasting east, to the right or in the positive direction.

Corner = Origin

$\vec{v}_{avg} = +2$ m/s; $t = 3.5$ s

$$\vec{v}_{avg} = \frac{D\vec{d}}{Dt}$$

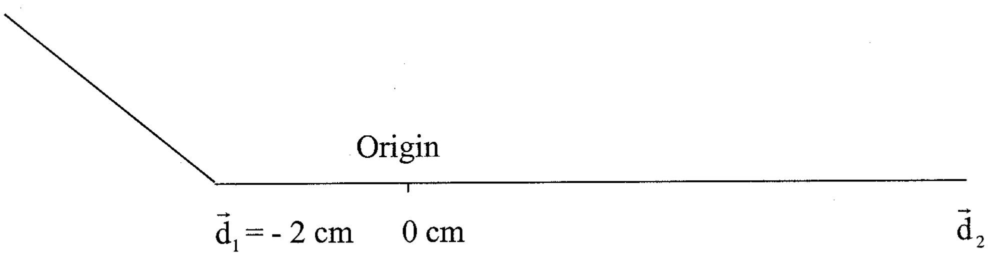
$+2$ m/s $= D\vec{d} / 3.5$ s

$D\vec{d} = (+2 \text{ m/s})(3.5 \text{ s}) = +7$ m

The displacement is +7 m or 7 m to the right.

4. When speeding, the displacement of a car in one unit of time is greater than that allowed by law. For example, a car traveling 35 km in one hour is speeding if the speed limit is 30 km in one hour.

5.



Since the positions are marked in cm, $\vec{v}_{avg}$ should be expressed in cm/s.

$\vec{v}_{avg} = +1.5$ m/s $= +150$ cm/s; $t = 0.4$ s

The unknown is the length of the track, $D\vec{d}$.

$$\vec{v}_{avg} = \frac{D\vec{d}}{Dt}$$

$+150$ cm/s $= D\vec{d} / 0.4$ s

$D\vec{d} = (+150 \text{ cm/s})(0.4 \text{ s}) = +60$ cm

The track is 60 cm long. This answer has only one significant digit.

Uniform Motion

PRACTICE—PAGE 17

1. Since the object only traveled a distance of 70 m or so during a time of one hour, this Position-Time graph could represent the motion of an insect, such as a ladybug, along a straight-line path. The story might go like this.

The ladybug started at a position of 0 m at time 0 h. The ladybug moved in the positive direction, traveling at a constant low velocity. It reached a position of +3 m after 0.15 h. At 0.15 h, it began to move with a larger constant positive velocity, reaching a position of +20 m at 0.35 h. Then around 0.35 h, the ladybug began to crawl still more quickly in the positive direction at a constant velocity, reaching a position of +30 m shortly before 0.4 h. Around 0.4 h the ladybug slowed down, then traveled to a position of + 40 m, reaching it at 0.5 h. At 0.5 h, it slowed down again, traveling with a low constant positive velocity until 0.8 h, reaching a position of +50 m at that time.

At 0.8 h, the ladybug stopped, turned around, sped up, and traveled with a large negative constant velocity to a position of +35 m at 0.93 h. At that time the ladybug sped up once more and traveled at a still larger constant negative velocity, reaching a position of +20 at 1.0 h.

2. The average velocity is found by taking the slope of the line on the Position-Time graph for that time interval.

The instantaneous velocity is found by locating the position on the curve of the Position-Time graph for the instant in question. Draw a line on the graph that runs in the same direction as the curve at that point in time. The slope of this line segment represents the instantaneous velocity. Note that since this line is estimated to run in the same direction as the curve, this can lead to wide variations in the slope.

In Question 2, the times have an additional 0 added on, giving each reading two significant digits.

a. $t_1 = 0 \text{ h},\ \vec{d}_1 = 0 \text{ m}$

$t_2 = 0.10 \text{ h},\ d_2 = +15 \text{ m}$

$\vec{v}_{avg} = ?$

$$\vec{v}_{avg} = \frac{\Delta\vec{d}}{\Delta t} = \frac{+15 \text{ m} - 0 \text{ m}}{0.10 \text{ h} - 0 \text{ h}} = \frac{+15 \text{ m}}{0.10 \text{ h}} = +150 \text{ m/h}$$

Instantaneous velocity: From the line drawn on the graph at 0.05 h, the following points are taken.

$t_1 = 0\text{ h},\ \vec{d}_1 = 0\text{ m}$

$t_2 = 0.40\text{ h},\ d_2 = +60\text{ m}$

$\vec{v}_{0.05} = ?$

$$\vec{v}_{0.05} = \frac{\Delta\vec{d}}{\Delta t} = \frac{+60\text{ m} - 0\text{ m}}{0.40\text{ h} - 0\text{ h}} = \frac{+60\text{ m}}{0.40\text{ h}} = +150\text{ m/h}$$

The two velocities are equal.

b. $t_1 = 0.20\text{ h},\ \vec{d}_1 = +20\text{ m}$

$t_2 = 0.40\text{ h},\ d_2 = +26\text{ m}$

$\vec{v}_{avg} = ?$

$$\vec{v}_{avg} = \frac{\Delta\vec{d}}{\Delta t} = \frac{+26\text{ m} - +20\text{ m}}{0.40\text{ h} - 0.20\text{ h}} = \frac{+6\text{ m}}{0.20\text{ h}} = +30\text{ m/h}$$

Instantaneous velocity: From the line drawn on the graph at 0.3 h, the following points are taken.

$t_1 = 0\text{ h},\ \vec{d}_1 = +14\text{ m}$

$t_2 = 1.0\text{ h},\ d_2 = +48\text{ m}$

$\vec{v}_{0.3} = ?$

$$\vec{v}_{0.3} = \frac{\Delta\vec{d}}{\Delta t} = \frac{+46\text{ m} - +14\text{ m}}{1.0\text{ h} - 0\text{ h}} = \frac{+32\text{ m}}{1.0\text{ h}} = +32\text{ m/h}$$

The two velocities are almost equal.

c. $t_1 = 0.60\text{ h},\ \vec{d}_1 = +32\text{ m}$

$t_2 = 0.08\text{ h},\ d_2 = +60\text{ m}$

$\vec{v}_{avg} = ?$

$$\vec{v}_{avg} = \frac{\Delta\vec{d}}{\Delta t} = \frac{+60\text{ m} - +32\text{ m}}{0.08\text{ h} - 0.60\text{ h}} = \frac{+28\text{ m}}{0.20\text{ h}} = +140\text{ m/h}$$

Instantaneous velocity: From the line drawn on the graph at 0.05 h, the following points are taken.

$t_1 = 0\ \text{h},\ \vec{d}_1 = 0\ \text{m}$

$t_2 = 1.0\ \text{h},\ d_2 = +54\ \text{m}$

$\vec{v}_{0.7} = ?$

$$\vec{v}_{0.7} = \frac{D\vec{d}}{Dt} = \frac{+54\ \text{m} - 0\ \text{m}}{1.0\ \text{h} - 0\ \text{h}} = \frac{+54\ \text{m}}{1.0\ \text{h}} = +54\ \text{m/h}$$

The two velocities are not equal.

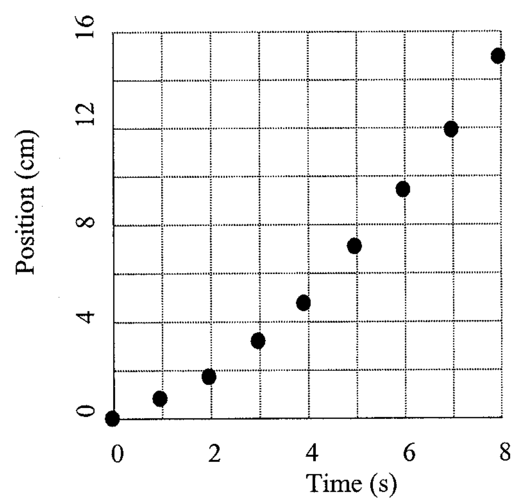
Accelerated Motion

Note to Teacher: The instructions on page 18 indicate students are to measure the displacement between successive dots, but Table C asks for the **position** of the dots.

Have the students measure the position of each dot from the origin and record these in Table C. Also, since the dots are so large, students should pick one edge of the dot as the origin and measure the positions to the same edge of each of the remaining dots.

Table C

Position (cm)	Time (s)
0	0
0.8	1
1.9	2
3.3	3
5.0	4
7.1	5
9.4	6
12.0	7
14.9	8



The points on the graph should be joined with a smooth curve.

Erratum: The units for Column 1 in Table C should read as "cm" only.

Think About IT!—Page 18

1. The Position-Time graph for uniform motion would be a straight line. The slope of this line would yield the average velocity. The slope is constant; therefore, the velocity is constant or uniform.

   In this case, the slope of the line on the Position-Time graph is not constant, but is always changing. Therefore, the velocity cannot be constant and the motion is non-uniform.

2. As the car's velocity increases, the spacing of the dots becomes greater. The car will travel a farther distance in the same time when the velocity is greater.
3. The spacing of the dots increases uniformly. This indicates that the velocity of the car is increasing uniformly also. The acceleration must be uniform.

Table D—Page 19

A better format for this table would include a displacement column.

Table D

Time (s)	Position (cm)	Displacement During Time Interval (cm)	Average Velocity (cm/s)
0	0		
1	0.8	0.8 – 0 = 0.8	0.8/1 = 0.8
2	1.9	1.9 – 0.8 = 1.1	1.1
3	3.3	3.3 – 1.9 = 1.4	1.4
4	5.0	5.0 – 3.3 = 1.7	1.7
5	7.1	7.1 – 5.5 = 2.1	2.1
6	9.4	9.4 – 7.1 = 2.3	2.3
7	12.0	12.0 – 9.4 = 2.6	2.6
8	14.9	14.9 – 12.0 = 2.9	2.9

The average velocity for a time interval closely represents instantaneous velocity at the midpoint of each time interval. This leads us to Table E.

Erratum: The units for Column 2 in Table D should read as "cm" only.

Think About IT!

Think About IT!—Page 20

1. The velocity changes at a regular rate, about 0.3 cm/s.
2. As time increases, so does the velocity. They are directly related.
3. The acceleration was uniform or constant at +0.3 cm/s/s.

Table E

Velocity (cm/s)	Time (s)
0.8	0.5
1.1	1.5
1.4	2.5
1.7	3.5
2.1	4.5
2.3	5.5
2.6	6.5
2.9	7.5

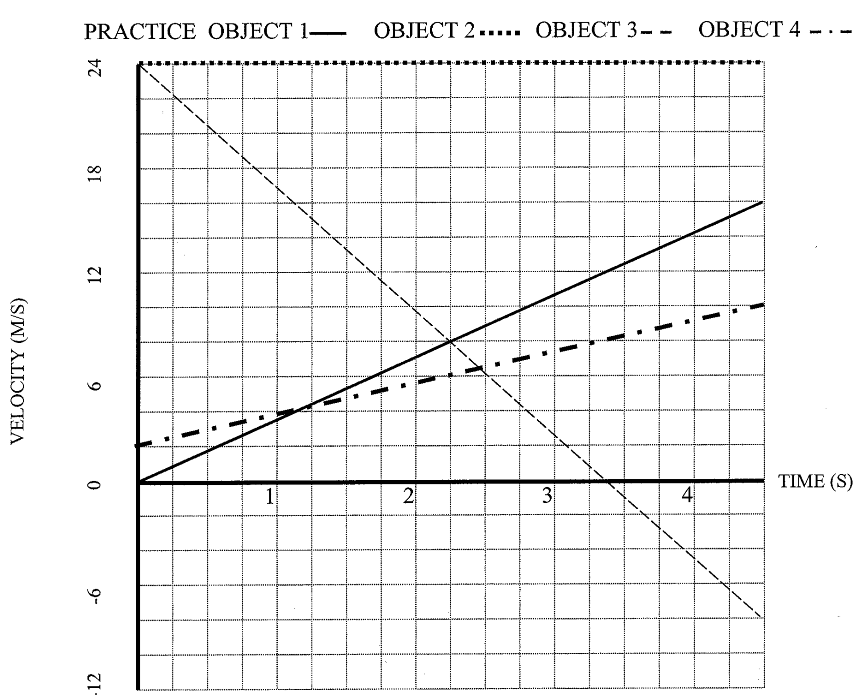
PRACTICE—PAGE 21

1. The first object is accelerating in the positive direction at +16.0 m/s / 4 s = +4.0 m/s. It is traveling in the positive direction and speeding up.

   The second object has a constant velocity of +24.0 m/s. The acceleration is 0 m/s/s. It is traveling in the positive direction.

   The third object is moving initially with a large positive velocity of +24 m/s. The velocity decreases from 0 s to 3 s. The object is slowing down. At 3 s the object momentarily stops and begins to move in the negative direction, speeding up until 4 s. The acceleration is always a negative acceleration. The acceleration is constant at –8m/s/s.

   The fourth object is initially moving at +2 m/s and speeds up in the positive direction. The object is always moving in the positive direction with a constant acceleration of + 2.0 m/s/s.



2. Uniform motion and non-uniform motion:

Compare:

Both can be described using a Position-Time graph to indicate the position of the object at given points in time.

The slope of the Position-Time graph in both cases yields the velocity of the object.

Both can be described using a Velocity-Time graph to indicate the velocity of the object at given points in time.

The slope of the Velocity-Time graph in both cases yields the acceleration of the object.

Contrast:

The Position-Time graph for uniform motion is a straight line, a line of constant slope. The velocity of the object is uniform or constant. The Velocity-Time graph will be a straight, horizontal line. The acceleration is 0 m/s/s.

The Position-Time graph for non-uniform motion is a curved line, a line of changing slope. The velocity is always changing in time from instant to instant.

If the Velocity-Time graph is a straight line of constant slope (non-zero), then the acceleration is constant.

If the Velocity-Time graph is a curved line, then the acceleration is non-uniform, changing in time from instant to instant.

3. All the calculations of acceleration will use the following equation:

$$\vec{a}_{avg} = \frac{D\vec{v}}{Dt} = \frac{\vec{v}_{final} - \vec{v}_{initial}}{t_{initial} - t_{initial}}$$

The units used will be those given in the question.

a. $\vec{v}_1 = 0 \text{ km/h}; \ \vec{v}_2 = +20 \text{ km/h}; \ \ t = 6 \text{ s}; \ \vec{a}_{avg} = ?$

$$\vec{a}_{avg} = \frac{D\vec{v}}{Dt} = \frac{\vec{v}_{final} - \vec{v}_{initial}}{t_{initial} - t_{initial}} = \frac{+20 \text{ km/h} - 0 \text{ km/h}}{6 \text{ s}} = +3.3 \text{ km/h/s}$$

Use one significant digit in the answer: +3 km/h/s.

b. $\vec{v}_1 = +10 \text{ km/h}; \ \vec{v}_2 = +60 \text{ km/h}; \ \ t = 30 \text{ min}; \ \vec{a}_{avg} = ?$

$$\vec{a}_{avg} = \frac{D\vec{v}}{Dt} = \frac{\vec{v}_{final} - \vec{v}_{initial}}{t_{initial} - t_{initial}} = \frac{+60 \text{ km/h} - +10 \text{ km/h}}{30 \text{ min}} = +1.67 \text{ km/h/min}$$

Using significant digits, round off to +2 km/h/min.

c. $\vec{v}_1 = +50\text{ km/h};\ \vec{v}_2 = +60\text{ km/h};\ t = 6\text{ s};\ \vec{a}_{avg} = ?$

$$\vec{a}_{avg} = \frac{D\vec{v}}{Dt} = \frac{\vec{v}_{final} - \vec{v}_{initial}}{t_{initial} - t_{initial}} = \frac{+60\text{ km/h} - +50\text{ km/h}}{6\text{ s}} = +1.67\text{ km/h/s}$$

Using significant digits, round off to +2 km/h/s.

d. $\vec{v}_1 = 0\text{ m/s};\ \vec{v}_2 = +7\text{ m/s};\ t = 3\text{ s};\ \vec{a}_{avg} = ?$

$$\vec{a}_{avg} = \frac{D\vec{v}}{Dt} = \frac{\vec{v}_{final} - \vec{v}_{initial}}{t_{initial} - t_{initial}} = \frac{+7\text{ m/s} - 0\text{ m/s}}{3\text{ s}} = +2.33\text{ m/s/s}$$

Using significant digits, round off to +2 m/s/s.

Note to Teacher: Question 4 requires students to determine the displacement by taking the area beneath the curve of a Velocity-Time graph. This is beyond the scope of Senior 2 Science (20F). These concepts should be used only with the most capable students and should be considered as **enrichment material**.

4. In all cases, the origin will be the position of the initial velocity. Because the motion is non-uniform velocity with constant acceleration, all the Velocity-Time graphs will be straight-line graphs. However, the Position-Time graphs will all be curves. Since, in all cases, the acceleration is positive, the curves on the Position-Time will have an increasing slope, upwards to the right.

The displacement can be found by rearranging $\vec{v}_{avg} = \frac{D\vec{d}}{Dt}$ to solve for $D\vec{d}$. The rearranged equation is $D\vec{d} = \vec{v}_{avg}Dt$. On a Velocity-Time graph, this represents the area beneath the line on the graph.

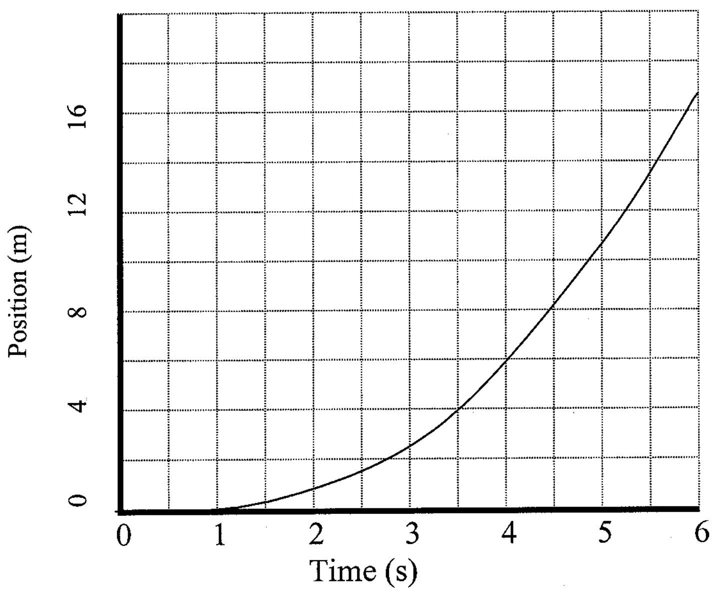
In our cases here, the average velocity can be found from the two given velocities as a simple average—add them up and divide by 2. So we use $\vec{v}_{avg} = \frac{\vec{v}_1 + \vec{v}_2}{2}$ and the final equation becomes $D\vec{d} = \frac{\vec{v}_1 + \vec{v}_2}{2}Dt$. The displacement will give the final position of the object for all the graphs. The line on the Position-Time graph will be a curve joining the initial position to the final position.

Finally, in some cases, the velocities will be changed to m/s from km/h to facilitate the comprehension of the displacement.

4. a. $\vec{v}_1 = 0 \text{ km/h} = 0 \text{ m/s}; \ \vec{v}_2 = +20 \text{ km/h} / 3.6 = +5.5 \text{ m/s}; \ t = 6 \text{ s}; \ D\vec{d} = ?$

$$D\vec{d} = \frac{\vec{v}_1 + \vec{v}_2}{2} Dt = \frac{(0 \text{ m/s} + 5.5 \text{ m/s})}{2}(6 \text{ s}) = +16.5 \text{ m}$$

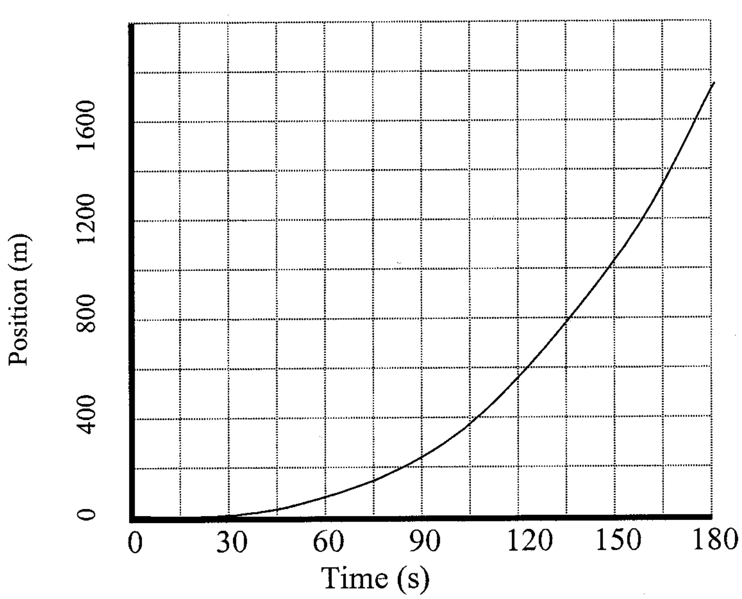
Therefore, $\vec{d}_2$ is +16.5 m. A curved line is drawn on the Position-Time graph starting at 0 m and ending at +16.5 m. The remaining questions are done in a similar fashion.



b. $\vec{v}_1 = +10 \text{ km/h} / 3.6 = +2.8 \text{ m/s}; \ \vec{v}_2 = +60 \text{ km/h} / 3.6 = +16.7 \text{ m/s};$

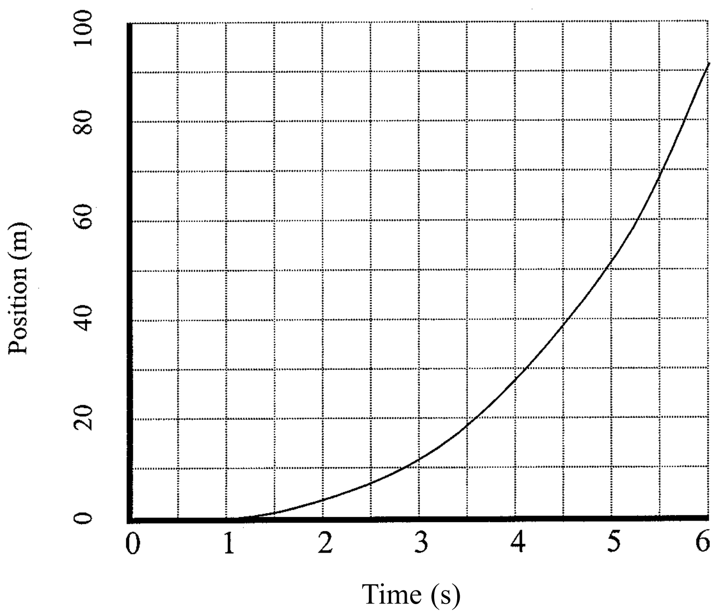
$t = 30 \text{ min} = 180 \text{ s}; \ D\vec{d} = ?$

$$D\vec{d} = \frac{\vec{v}_1 + \vec{v}_2}{2} Dt = \frac{(+2.8 \text{ m/s} + 16.7 \text{ m/s})}{2}(180 \text{ s}) = +1750 \text{ m}$$



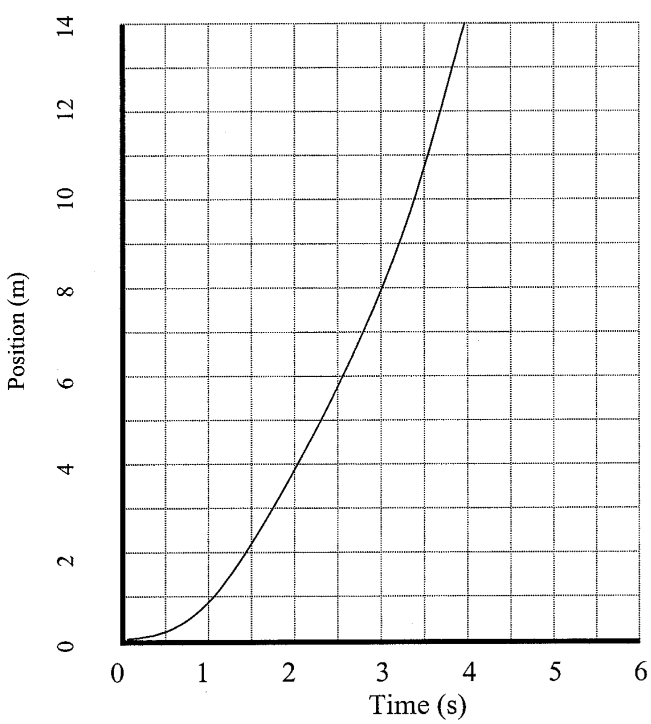
c. $\vec{v}_1 = +50\ \text{km/h} / 3.6 = +13.9\ \text{m/s};\ \vec{v}_2 = +60\ \text{km/h} / 3.6 = +16.7\ \text{m/s};\ t = 6\ \text{s};\ \Delta\vec{d} = ?$

$$\Delta\vec{d} = \frac{\vec{v}_1 + \vec{v}_2}{2}\Delta t = \frac{(+13.9\ \text{m/s} + 16.7\ \text{m/s})}{2}(6\ \text{s}) = +91.8\ \text{m}$$



d. $\vec{v}_1 = 0\ \text{m/s};\ \vec{v}_2 = +7\ \text{m/s};\ t = 4\ \text{s};\ \Delta\vec{d} = ?$

$$\Delta\vec{d} = \frac{\vec{v}_1 + \vec{v}_2}{2}\Delta t = \frac{(0\ \text{m/s} + 7\ \text{m/s})}{2}(4\ \text{s}) = +14\ \text{m}$$



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It appears that there are two sets of footprints: those of a wolf and those of a rabbit.

In the diagram, to the right represents the east or an easterly direction.

The wolf is traveling in a direction slightly south of east. By the spacing of the tracks, the wolf appears to be walking or running slowly. The rabbit is hopping along slowly in a direction slightly north of east. The paths of the wolf and the rabbit intersect. At this point, there are many wolf tracks in the same area. This indicates that the wolf remained there for a time while it killed and ate the rabbit. Once the meal was finished, the wolf moved away more slowly in a southeasterly direction.