

***Unit F: Variation and Formulas***

***Half Course VI***

## ***HALF COURSE VI***

### **Unit F: Variations and Formulas**

**Hours: 11**

#### **General Learning Outcome:**

**Use algebraic and graphical models to generalize patterns, make predictions, and solve problems.**

*The intent of this unit is for students to plot and analyze data related to variation and to use formulas to solve problems.*

#### **Specific Learning Outcomes**

- F-1 Plot and analyze examples of direct variation, partial variation, and inverse variation.
- F-2 Given data, graphs, or a situation, interpret the variation represented.
- F-3 Use formulas given in the questions to solve problems.
- F-4 Select and use given formulas to solve problems.

#### **Instructional Materials**

- *Essentials of Mathematics 12*
- scientific calculator
- ruler
- computer spreadsheet

# ***VARIATIONS AND FORMULAS***

## **Assessment**

- daily assignments
- journal questions
- unit test

**PRESCRIBED LEARNING  
OUTCOMES**

**General Outcome**

Use algebraic and graphical models to generalize patterns, make predictions, and solve problems.

**Specific Outcome(s)**

F-1 plot and analyze examples of direct variation, partial variation, and inverse variation

**SUGGESTIONS FOR INSTRUCTION**

Variation problems are the study of the relationship between two variables. Consider a cyclist who bikes 30 km in two hours. How far does she travel in three hours?

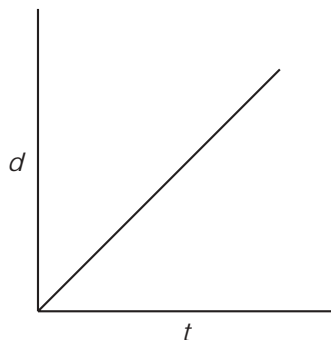
This is an example of distance varying **directly** as time, or  $d \propto t$ . An equation is established by introducing a variation constant  $k$ :  $d = kt$ .

By substitution:  $30 = k(2)$  or  $k = 15$ .

The information can then be expressed as an equation:  $d = 15t$  or a table of values:

$t$	$d$
1	15
2	30
3	45

or a graph:



In this example  $d$  is considered the dependent variable and typically is graphed along the vertical or  $y$ -axis.  $t$  is considered the independent variable and is graphed along the horizontal or  $x$ -axis.

Communications	✓ Patterns
✓ Connections	✓ Problem Solving
✓ Number Sense	✓ Reasoning
✓ Organization and Structure	Technology
	✓ Visualization

— *continued*

SUGGESTIONS FOR ASSESSMENT

Assessment and evaluation could include:

- project—students create their own examples and problems for each variation
- students find examples of variations (including graphs or tables of values) and explain

SUGGESTED LEARNING  
RESOURCES

**Print**

*Senior 4 Consumer Mathematics (45S) Part VI: A Course for Distance Learning.* Winnipeg, MB: Manitoba Education, Training and Youth, 2002.  
— Module 8, Lessons 1–4

Baron, C., et al. *Essentials of Mathematics 12.* Victoria, BC: British Columbia Ministry of Education, 2003.  
[ISBN 0-7726-4997-9]

Ebos, Frank, et al. *Math Matters* (Books 2, 3, and 4). Scarborough, ON: Nelson Canada.

van Dyke, Frances. *A Visual Approach to Algebra.* White Plains, NY: Dale Seymour, 1998.  
[ISBN 1-57232-441-4]

**PRESCRIBED LEARNING  
OUTCOMES**

F-1 plot and analyze examples of direct variation, partial variation, and inverse variation  
– *continued*

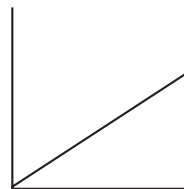
**SUGGESTIONS FOR INSTRUCTION**

The intent of having students graph variations is so they can recognize general shapes. If students have access to a computer, they can do tables of values and graphs as a spreadsheet assignment.

Students should become familiar with the graphs of the following basic variations:

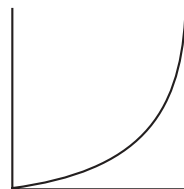
Direct variation

$$(y \propto x)$$



Direct squared variation

$$(y \propto x^2)$$



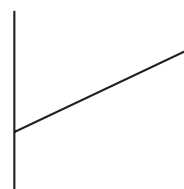
Inverse variation

$$\left(y \propto \frac{1}{x}\right)$$



Partial variation

$$(y \propto mx + b)$$



- |                              |                   |
|------------------------------|-------------------|
| Communications               | ✓ Patterns        |
| ✓ Connections                | ✓ Problem Solving |
| ✓ Number Sense               | ✓ Reasoning       |
| ✓ Organization and Structure | Technology        |
|                              | ✓ Visualization   |

– *continued*

SUGGESTIONS FOR ASSESSMENT

SUGGESTED LEARNING  
RESOURCES

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**PRESCRIBED LEARNING  
OUTCOMES**

F-1 plot and analyze examples of direct variation, partial variation, and inverse variation  
– *continued*

**SUGGESTIONS FOR INSTRUCTION**

**Partial variation** problems in this lesson are of two types. The first requires finding a value for the left-hand side of the equation by substituting in values on the right. The second type requires the constant of variation to be calculated.

**Example**

There are fixed costs for putting on a social: renting the hall and the music service, as well as a cost that varies with the number of people attending. The cost for 80 people attending was \$480, while 50 people cost \$412.50. What should the per-person charge be in order to break even, if you expect 100 people attending?

*Solution*

80 people cost	\$480.00
50 people cost	\$412.50
30 additional people	\$ 67.50 or \$2.25 per person

$$\begin{aligned} \$480.00 &= \text{fixed cost} + 80(2.25) \\ &= \text{fixed cost} + 180 \end{aligned}$$

$$\$300.00 = \text{fixed cost}$$

$$\text{So 100 people would cost: } \$300.00 + 100(\$2.25) = \$525.00$$

$$\frac{\$525}{100} = \$5.25 \text{ per person}$$

**Example**

Your cell phone plan charges a flat rate per month plus a per-minute charge. The cost varies with the number of minutes used. The cost for 50 minutes was \$35, while the cost for 75 minutes was \$44.50. What would be the cost for 100 minutes?

*Solution*

75 minutes cost	\$44.50
50 minutes cost	\$35.00
25 minutes	\$ 9.50 or 0.38/minute

$$\$44.50 = \text{fixed cost} + 75(0.38)$$

$$\$44.50 = \text{fixed cost} + 28.50$$

$$\$16.00 = \text{fixed cost}$$

$$100 \text{ minutes} = \$16 + 100 (0.38) = \$54.00$$

Communications	✓ Patterns
✓ Connections	✓ Problem Solving
✓ Number Sense	✓ Reasoning
✓ Organization and Structure	Technology
	✓ Visualization

— *continued*

SUGGESTIONS FOR ASSESSMENT

SUGGESTED LEARNING  
RESOURCES

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**PRESCRIBED LEARNING OUTCOMES**

F-1 plot and analyze examples of direct variation, partial variation, and inverse variation  
 – *continued*

**SUGGESTIONS FOR INSTRUCTION**

**Inverse variation** graphs require at least four or five points to see the general shape. Students should recognize that inverse variations do not cross either axis. As one variable gets larger, the other gets smaller but never reaches zero.

**Example**

Suppose that a car is worth \$12,000 when it is two years old. Construct a table of values and graph this relationship.

**Solution**

Use the inverse variation formula to solve:

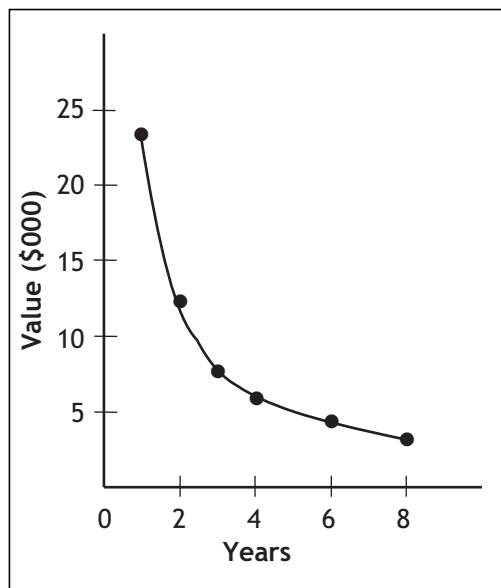
$$V = \frac{k}{a}$$

where  $V$  is the value at the end of  $a$  years, and  $k$  is the constant of variation.

$$12,000 = \frac{k}{2}$$

so that  $k = 24,000$ . From this we can then fill out a table of values and then graph this relation.

Years ( $a$ )	Value ( $V$ )
1	24,000
2	12,000
3	8,000
4	6,000
6	4,000
8	3,000



- |                              |                   |
|------------------------------|-------------------|
| Communications               | ✓ Patterns        |
| ✓ Connections                | ✓ Problem Solving |
| ✓ Number Sense               | ✓ Reasoning       |
| ✓ Organization and Structure | Technology        |
|                              | ✓ Visualization   |

– *continued*

SUGGESTIONS FOR ASSESSMENT

SUGGESTED LEARNING  
RESOURCES

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<p>PRESCRIBED LEARNING OUTCOMES</p>	<p>SUGGESTIONS FOR INSTRUCTION</p>										
<p>F-1 plot and analyze examples of direct variation, partial variation, and inverse variation – <i>continued</i></p>	<p><b>Extension</b> Use extrapolation and interpolation to recognize the limits of variation.</p> <p><b>Examples</b></p> <p>a) If you run 100 m in 10 seconds, how long will it take you to run 2 km?</p> <p>b) If you are 5 cm from someone and you go half the distance to him, how far away are you? Another half distance? And another half distance?</p>										
<p>F-2 given data, graphs, or a situation, interpret the variation represented</p> <hr/> <table border="0"> <tr> <td>Communications</td> <td>✓ <b>Patterns</b></td> </tr> <tr> <td>✓ <b>Connections</b></td> <td>Problem Solving</td> </tr> <tr> <td><b>Number Sense</b></td> <td>✓ <b>Reasoning</b></td> </tr> <tr> <td>Organization and Structure</td> <td>Technology</td> </tr> <tr> <td></td> <td>✓ <b>Visualization</b></td> </tr> </table>	Communications	✓ <b>Patterns</b>	✓ <b>Connections</b>	Problem Solving	<b>Number Sense</b>	✓ <b>Reasoning</b>	Organization and Structure	Technology		✓ <b>Visualization</b>	<p>Students should be able to recognize standard variations when the data appear in graphs, tables, or descriptive statements.</p> <p>Note that for each example of variation, both a table of values and a graph could be provided.</p>
Communications	✓ <b>Patterns</b>										
✓ <b>Connections</b>	Problem Solving										
<b>Number Sense</b>	✓ <b>Reasoning</b>										
Organization and Structure	Technology										
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<p>F-3 use formulas given in the questions to solve problems</p> <hr/> <table border="0"> <tr> <td>Communications</td> <td>Patterns</td> </tr> <tr> <td>Connections</td> <td>✓ <b>Problem Solving</b></td> </tr> <tr> <td>✓ <b>Number Sense</b></td> <td>✓ <b>Reasoning</b></td> </tr> <tr> <td>✓ <b>Organization and Structure</b></td> <td>Technology</td> </tr> <tr> <td></td> <td>Visualization</td> </tr> </table>	Communications	Patterns	Connections	✓ <b>Problem Solving</b>	✓ <b>Number Sense</b>	✓ <b>Reasoning</b>	✓ <b>Organization and Structure</b>	Technology		Visualization	<p>At this level some of the problems may be solved by direct substitution into the formula, but others will require some manipulation of the formula. Encourage students to draw diagrams and to write down their substitution of the numbers into the formula.</p> <p><b>Example</b></p> <p>The volume of a cylinder is given by: <math>V = \pi r^2 h</math> where "r" is the radius and "h" is the height.</p> <p>Given a radius of 3 cm and a height of 10 cm, we can find the volume by direct substitution into the right-hand side of the equation to find the value of V on the left.</p> <p><b>Solution</b></p> $V = \pi(3)^2(10) = 90\pi = 282.6 \text{ cm}^2$ <p style="text-align: right;">— <i>continued</i></p>
Communications	Patterns										
Connections	✓ <b>Problem Solving</b>										
✓ <b>Number Sense</b>	✓ <b>Reasoning</b>										
✓ <b>Organization and Structure</b>	Technology										
	Visualization										

SUGGESTIONS FOR ASSESSMENT

SUGGESTED LEARNING  
RESOURCES

**Mental Math**

Name a characteristic of a partial variation.

Sketch a graph of an inverse variation.

If  $d = st$ , how far ( $d$ ) will someone go if their speed is 20 km per hour for one and a half hours?

**PRESCRIBED LEARNING  
OUTCOMES**

F-3 use formulas given in the questions to solve problems

**SUGGESTIONS FOR INSTRUCTION**

***Example***

Find the radius of a cylinder that has a volume of  $350 \text{ cm}^3$  and a height of 10 cm.

***Solution***

$$350 = 3.14(10)r^2$$

$$r^2 = \frac{350}{3.14} = 11.15$$

$$r = 3.3 \text{ cm}$$

Note that in the second example, the variable  $r$  is on the right side of the equation. Students need to learn how to manipulate the equation to solve for any variable.

Communications	Patterns
Connections	✓ <b>Problem Solving</b>
✓ <b>Number Sense</b>	✓ <b>Reasoning</b>
✓ <b>Organization and Structure</b>	Technology
	Visualization

SUGGESTIONS FOR ASSESSMENT

SUGGESTED LEARNING  
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