Unit A: Problem Analysis

Half Course V

Please see the print document for more activity suggestions. The document is available from the Manitoba Text Book Bureau (stock number 80354). To order, please visit <www.mtbb.mb.ca>.

HALF COURSE V

Unit A: Problem Analysis

Hours: 7 in combination with Analysis of Games and Numbers

General Learning Outcome:

Develop and use mathematical strategies to solve problems in different situations.

The intent of this unit is to provide a range of interesting problems which call for a wide variety of strategies to solve them. These problems augment the work of other units and are to be embedded throughout the course.

Specific Learning Outcomes

A-1 Explain and solve problems using a variety of primarily non-algebraic approaches.

A-2 Describe the approach and the mathematics used in solutions to problems or activities.

PROBLEM ANALYSIS

Instructional Materials

- *Essentials of Mathematics 12*
- See Appendix I for possible activities.
- See Appendix II for additional resources.

PRESCRIBED LEARNING OUTCOMES

General Outcome

Develop and use mathematical strategies to solve problems in different situations.

Specific Outcome(s)

A-1 explain and solve problems using a variety of primarily non-algebraic approaches

A-2 describe the approach and the mathematics used in solutions to problems or activities

SUGGESTIONS FOR INSTRUCTION

Examples of non-algebraic approaches include geometry, networks, flow charts, organizational charts, simulations, etc.

Remember that for activities in this unit, the journey is more important than the destination. It is beneficial to discuss multiple approaches to solving these problems, particularly when the approaches have been developed by students. Are some approaches "better" than others? Why? On what grounds?

The problems contained in Appendix I are intended to provide material which is interesting in its own right and which complements the other units of the program. It is illustrative rather than exhaustive. Some activities have been chosen to illustrate a wide variety of job and consumer applications of mathematics that are largely non-algebraic. Others have been chosen because they are intrinsically interesting or because they challenge students to find and to use new ways of analyzing and thinking mathematically. All students do not need to engage in the same activities.

The activities in Appendix I are presented in **no** particular sequence. Teachers are encouraged to supplement this set of activities with material from other sources, such as the Internet. A preliminary list of possible resources is included in Appendix II.

It is suggested that these problems and activities be interspersed throughout the course as either extensions, enrichment, or a change of pace in the day-to-day work of the classroom. Some of them will link directly to particular units, but most are independent and **may** be used at any time. One approach would be to introduce problem analysis with a few days, possibly up to a week, of work on these activities. Intersperse the remainder throughout the course.

✔ **Communications**	Patterns
✔ **Connections**	✔ **Problem Solving**
Number Sense	✔ **Reasoning**
✔ **Organization and Structure**	Technology
	✔ **Visualization**

SUGGESTIONS FOR ASSESSMENT

Students' progress should be assessed over a length of time. Look, for example, for an increasing variety of problem-solving strategies and increasingly sophisticated explanations. Anecdotal records of how students work in pairs or groups on learning activities is appropriate. Well-developed solutions and reasoning could become part of a student's portfolio.

Complex problem-solving activities generally are not appropriate on pencil-and-paper timed tests.

SUGGESTED LEARNING RESOURCES

Print

Senior 4 Consumer Mathematics (45S) Part V: A Course for Distance Learning. Winnipeg, MB: Manitoba Education, Training and Youth, 2002. — Cover Assignments

Austin, J.D. *Applications of Secondary School Mathematics.* Reston, VA: NCTM, 1991.

Baron, C., et al. *Essentials of Mathematics 12.* Victoria, BC: British Columbia Ministry of Education, 2003. [ISBN 0-7726-4997-9]

Giblin, P., and I. Porteous. *Challenging Mathematics.* Toronto/New York: Oxford University Press, 1990.

Hirsch, C.R., and R.A. Laing. *Activities of Learning and Teaching.* Reston, VA: NCTM, 1993.

Mathematical Association of America and National Council of Teachers of Mathematics. *A Sourcebook of Applications of School Mathematics*. Reston, VA: NCTM, 1980.

National Council of Teachers of Mathematics. *NCTM Student Math Notes*. Reston, VA: NCTM, n.d.

Swetz, F., and J.S. Hartzler. *Mathematical Modeling in the Secondary School Curriculum.* Reston, VA: NCTM, 1991.

See Appendix II for a list of additional resources.

Appendix I

Teacher Information: Water Conservation

Skills Required

- measurement of water volume
- fraction arithmetic
- understanding of estimation

When to Use

This activity may be done at any time.

Teaching Information

Encourage students to develop a plan for how to estimate the amount of water collected from a dripping faucet in one minute. Have students actually measure a dripping faucet at different flow rates.

The entire learning activity should be written up to include:

a) the plan

b) the actual estimate of water wasted from a leaky faucet

c) the amount wasted per year for one home where the water is left dripping

d) the amount wasted by community residents (one-tenth of the population)

e) the amount wasted by Manitobans

Solutions

Answers will vary. Estimations of water wasted will differ, but plans should include measurements with flows of dripping. N = amount wasted during sample.

One person leaving the water running wastes	N(365) litres per year
Winnipegers who leave the water running waste	N(365)(1/10)(600 000) litres per year
Manitobans who leave the water running waste	N(365)(1/10)(1 000 000) litres per year

Blackline Master: Water Conservation

Many homes have faucets that leak.

1. Estimate how much water is wasted in one home with a leaky faucet. State all assumptions made.
2. If one-tenth of the homes in your city or town leak, how much water could be wasted?
3. If one-tenth of the people in Manitoba wasted this much water every day, how much is wasted provincially each year?

Teacher Information: Golf Mania

Skills Required

- logical reasoning

When to Use

This activity may be done with the Statistics unit or at any time.

Teaching Information

- Students could work in groups for this question.
- Encourage students to find a way for each team to win.
- The explanation of why one method is a fair way to decide the winner is an important part of this exercise.

Possible Solutions

Method 1

Award points — 20 for first place
19 for second place, etc.

Add team score.

A	B	C	D
20	17	16	19
14	15	13	18
11	12	9	10
4	8	7	3
1	5	6	2
50	**57**	**51**	**52**

Team B wins the award.

Method 2

Award points — 20 for first place
19 for second place
. . .
11 for tenth.
No points after tenth place.

A	B	C	D
20	17	16	19
14	15	13	18
11	12		
45	**44**	**29**	**37**

Team A wins the award.

Method 3

Count the number of people from each team in the top ten, and the team with the most wins.

A — III
B — III
C — II
D — II

Therefore A and B tie.

Method 4

Count the number of people from each team in the top five and the team with the most wins.

A — I
B — I
C — I
D — II

Therefore D wins.

Blackline Master: Golf Mania

The Manitoba High School Athletic Association (MHSAA) wants to promote golf by introducing the "Tiger Golf Award." This award is won by the school with the best final standing in the provincial championships. Four schools were represented with five players per team: Alonsa, Brandon, Crystal City, and Douglas. The results are given in the table below. Which team won the award? Find three ways to decide who wins the Tiger Golf Award.

Place	School	Place	School
1	A	11	D
2	D	12	C
3	D	13	B
4	B	14	C
5	C	15	C
6	B	16	B
7	A	17	A
8	C	18	D
9	B	19	D
10	A	20	A

Teacher Information: Water Containers

Skills Required

- logical reasoning
- organization of work

When to Use

This activity may be done at any time.

Teaching Information

- Use materials to model the situation.
- Discuss with the students the parameters of the problem. Students should realize that they have only three containers and the final step will have 5 L of water in the 10 L container and 5 L of water in the 7 L container.

Possible Solutions

Solution 1

Step	10 L Container	7 L Container	3 L Container
Start	10	0	0
Use 3 to transfer from 10 L to 7 L	7	3	0
Use 3 to transfer from 10 L to 7 L	4	6	0
Use 3 to take from 10 L, place 1 in 7 L, keep 2 in 3 L	1	7	2
Dump 7 L into 10 L	8	0	2
Dump 3 L into 7 L	8	2	0
Use 3 L to transfer 10 L to 7 L	5	5	0

Solution 2

Step	10 L Container	7 L Container	3 L Container
Start	10	0	0
Take 7 L from 10 L	3	7	0
Pour 10 L from 7 L into 3 L	3	4	3
Pour from 3 L to 10 L	6	4	0
Pour from 7 L to 3 L	6	1	3
Pour from 3 L to 10 L	9	1	0
Pour from 7 L to 3 L	9	0	1
Pour from 10 L to 7 L	2	7	1
Pour from 7 L to 3 L	2	5	3
Pour from 3 L to 10 L	5	5	0

Blackline Master: Water Containers

You are given three irregularly shaped containers with the capacity of 3 L, 7 L, and 10 L. The largest container is full of water and the other two containers are empty. The containers do not have measurement markings.

Determine a method of dividing this quantity of water into two equal amounts of 5 L using the three containers and no other measuring devices.

Teacher Information: Miscellaneous Problems

Skills Required

- spatial reasoning
- Pythagorean theorem

When to Use

This activity may be done with the Design and Measurement unit or at any time.

Teaching Information

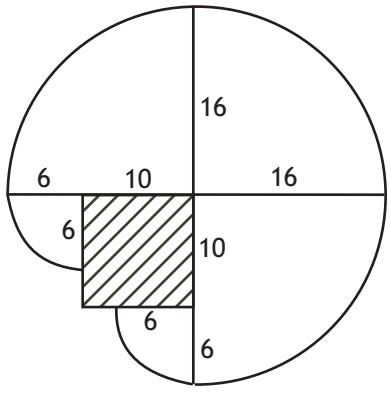
- These problems could be handed out to students in question pairs or as an entire set.
- If handed out in pairs, students could hand in the solution to one set of the two problems.
- If handed out as an entire set, students could be asked to hand in solutions to three out of the five questions.

Solutions

1. Have students use graph paper to estimate the grazing area. Have them draw the corral to scale and cut a length of string to scale for the tether. Using a tack as the anchor point for the string, have students trace the grazing area. Calculate the area by counting the squares.

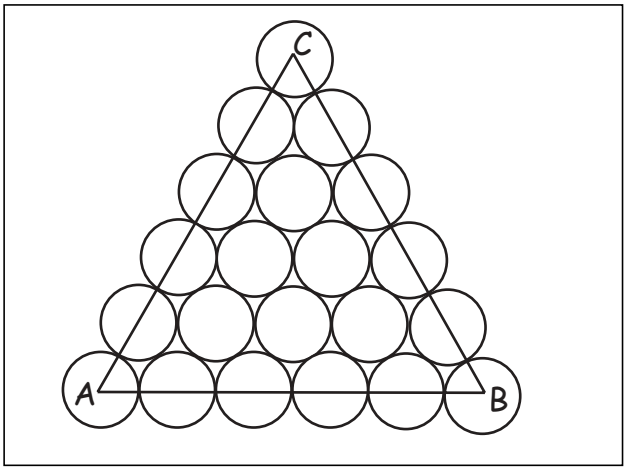
   The grazing area consists of three-quarters of a circle with a radius of 16 and two-quarter circles with a radius of 6. This is a total area of:

$$\frac{3}{4}(256\pi)+2\left[\frac{1}{4}(36\pi)\right]=192\pi+18\pi=210\pi \text{ m}^2$$



Miscellaneous Problems: From Manitoba Education and Training. *Problems for High School Mathematics: Support Document*.

2. In the diagram, ABC is an equilateral triangle with each side equal to 5 metres. Each pipe has a diameter of 1 metre. Thus, the height of the pile is 1 m more than the altitude of the triangle.



If h is the altitude of the triangle, then by Pythagoras:

$$5^2 = h^2 + \left(\frac{5}{2}\right)^2$$

$$h^2 = 25 - \frac{25}{4}$$

$$h^2 = \frac{75}{4}$$

$$h = \frac{\sqrt{75}}{2}$$

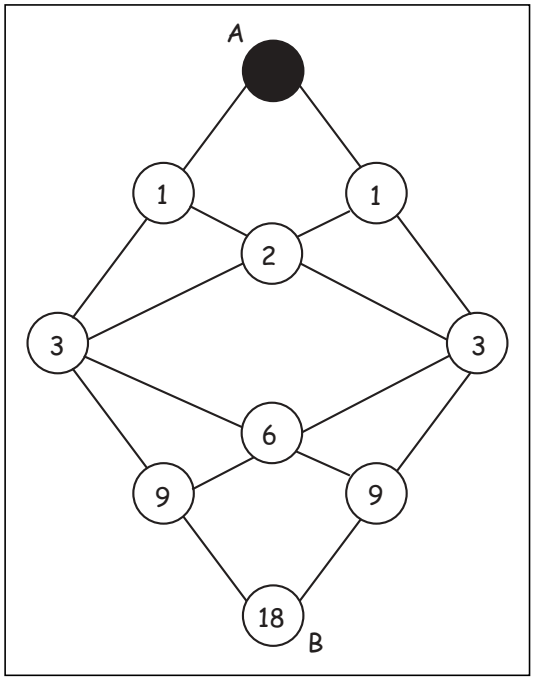
The height of the pile is $\left(1 + \frac{\sqrt{75}}{2}\right)$ metres = 533 cm to the nearest centimetre.

3. Prior to doing the problem, give students cardboard to use for a box. Have them cut out different corner measurements and then determine the volume of the box.

The volume of the box is 192 cm^2 and the height is three cm. Therefore, the area of the bottom is $\frac{192}{3} = 64 \text{ cm}^2$.

Since the bottom is square, its length and width must be 8 cm. The original piece of cardboard was 8 + 3 + 3 =14 cm long and 14 cm wide.

4. Each point is labelled with the number of ways of reaching it. After the first points, labelled 1, the number with each point can be found by adding the numbers directly above. Thus, Point B can be reached in 18 ways.



   Another approach is to count and list all the ways beginning from Point A downward to the right. By symmetry there will be an identical number of routes beginning downward to the left.

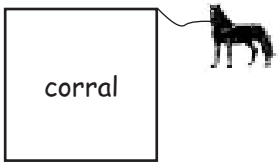
5. The maximum number of intersection points occurs when no lines are parallel and no three lines pass through a common point. It can be shown by drawing a diagram such that there are 10 points of intersection.

   The problem can also be answered without drawing a diagram. Let the lines be called *a*, *b*, *c*, *d*, and *e*. Let P(*a*, *b*) represent the point where line *a* intersects line *b*. Then the points of intersection are:

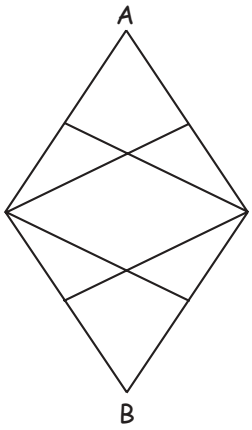
   P(*a, b*) P(*a, c*) P(*a, d*) P(*a, e*)
   P(*b, c*) P(*b, d*) P(*b, e*)
   P(*c, d*) P(*c, e*)
   P(*d, e*)

Blackline Master: Miscellaneous Problems

1. A horse is tethered to a rope at one corner of a square corral as shown. The rope is 16 metres long and the corral measures 10 metres on a side. What is the grazing area available to the horse?



2. A manufacturer has a large number of pipes 1 metre in diameter. On a level floor in the warehouse, he first makes a row of pipes side by side with each pipe touching the next. Then a second row is placed on top of this row, so as to fit into the hollows between the adjacent pipes. He continues this process until he has six rows. What is the total height of the pile of pipes? Express your answer to the nearest centimetre.
3. A box is made from a square piece of cardboard. A 3 cm square is cut out of each corner, and the resulting flaps are folded upwards to make an open box with a volume of 192 cubic centimetres. What are the dimensions of the original piece of cardboard?
4. If only downward motion is allowed, find the number of paths from A to B in the diagram below.



5. Five distinct straight lines are drawn on a piece of paper. What is the maximum number of intersection points?

Miscellaneous Problems: From Manitoba Education and Training. *Problems for High School Mathematics: Support Document*. Copyright © 1994 by Manitoba Education and Training.

Appendix II

Additional Resources

Print

The Association of Teachers of Mathematics. *Eight Days a Week: Puzzles, Problems and Questions to Activate the Mind*. The Association of Teachers of Mathematics. ISBN 1-898-611-09-2.

Brecker, Erwin. *Lateral Logic Puzzles*. Sterling Publishing Company, Inc. ISBN 0-8069-0618-9.

Bremner, John. *Mensa Maths Wizards for Kids*. Carleton Books Limited. ISBN 1-85868-555-9.

Carter, Philip, Ken Russell, and John Bremner. *The Ultimate Puzzle Challenge*. Carlton Books Limited. ISBN 1-85868-716-0.

Crisler, N., P. Fisher, and G. Froelich. *Discrete Mathematics through Applications*. New York, NY: W.H. Freeman, 1994.

DeSpezio, Michael A. *Giant Book of Challenging Thinking Puzzles*. Sterling Publishing Company, Inc. ISBN 0-8069-2087-4.

Dossey, J.A., M. Kenney, et al. *Discrete Mathematics*. Glenview, IL: Scott Foresman, 1987.

Forte, Imogene, and Sandra Schur. *180 Icebreakers to Strengthen Critical Thinking and Problem-Solving Skills*. Incentive Publications, Inc. ISBN 0-86530-345-2.

Graham, Evelyne M. *Think-A-Grams*. Critical Thinking Press and Software.
ISBN Numbers: Book A1: 0-89455-329-1
Book A2: 0-89455-430-1
Book B1: 0-89455-330-5
Book B2: 0-89455-431-X
Book C1: 0-89455-331-3
Book C2: 0-89455-432-8

Hunter, J.A.H. *Entertaining Mathematical Teasers and How to Solve Them*. Dover Publications, Inc. ISBN 0-486-24500-4.

Kenney, M.A. *Lesson in Mathematical Doodling*. Boston, MA: Boston College Press, 1976.

Mathematics in School. The Mathematical Association, 259 London Road, Leicester, UK: LE2 3BE.

The Mathematics Teacher. National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA: 22091-1593.

Matt-Smith, Geoffrey. *Mathematical Puzzles for Beginners and Enthusiasts*. Dover Publications, Inc. ISBN 0-486-20198-8.

Maurer, S.B., and A. Ralston. *Discrete Algorithmic Mathematics*. Reading, MA: Addison-Wesley, 1991.

Nash, Helen, and Dorothy Masterson. *Humorous Cryptograms*. Sterling Publishing Company, Inc. ISBN 0-8069-3982-6.

National Council of Teachers of Mathematics. *How to Evaluate Progress in Problem Solving.* National Council of Teachers of Mathematics. ISBN 0-87353-241-4.

Sloane, Paul, and Des MacHale. *Improve Your Lateral Thinking*. Sterling Publishing Company, Inc. ISBN 0-8069-1374-6.

Weber, Ken. *Five Minute Mysteries for the Armchair Detective*. Stoddart Publishing Co., Ltd. ISBN 0-7737-5210-2.

Williams, J. "Graph Coloring Used to Model Traffic Lights." *Mathematics Teacher* 85 (March 1992): 212-14.

Internet

There are many sites on the Internet with problems and puzzles. If you are using a search engine to find these sites, search using the words "Mathematics Puzzles Problems."

As of February 2004, the following sites were available:

AAA Math

<http://www.aaamath.com>

This site has games and practice sheets for various grade levels and topics. There are links to other sites on the web with games and puzzles.

Math Forum

<http://mathforum.org>

This is a good site to begin searching for problems and puzzles. One feature is **Problems of the Week**. New problems are available as well as a library of previous problems. Students can submit their answers and get some feedback. There are links to other math sites and several departments that are useful.

Word Problems for Kids

<http://www.stfx.ca/special/mathproblems/welcome.html>

This is a Canadian site with word problems, hints, and solutions from previous mathematics competitions. The problems are sorted by grade level. Choosing problems from Grades 5 through 9 will lead to a wealth of non-algebraic problems.