Introduction

INTRODUCTION

Rationale

During the last half-century, there has been a tremendous increase in mathematical knowledge. This is due to the collective influence of the growth of technology, the expansion of applications of mathematics, and the steady transition from an industrial to an information society. Consequently, there is a need for a change in the goals of mathematics education for all students.

In order to meet the challenges of society, high school graduates must be mathematically literate. They must understand how mathematical concepts permeate daily life, business, industry, government, and our thinking about the environment. They must be able to use mathematics not just in their work lives, but also in their personal lives as citizens and consumers.

Senior 3 Consumer Mathematics has been designed to meet these challenges for those who may not use advanced abstract mathematics in their careers, but who, nevertheless, will be consumers and active citizens. They also will need to develop their cooperative, interactive, and communicative skills.

Goals

The goals that guide Senior Years mathematics have been influenced by the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989). Additionally, other mathematics curricula for Manitoba schools are aligned with the *Common Curriculum Framework for K-12 Mathematics: Grade 10 to Grade 12* (1996) prepared by the western Canadian provinces and territories under the Western Canadian Protocol for Collaboration in Basic Education. In the case of *Senior 3 Consumer Mathematics*, the influence of these documents is pervasive. However, the curriculum is not bound by the learning outcomes set out in the Western Canadian Protocol.

The general goals established in the aforementioned documents underlie *Senior 3 Consumer Mathematics*. The incorporation of these goals into the curriculum ensures that more students will gain in mathematical power, thus increasing their ability to understand issues in a technological and informational society as well as in their own lives. In an appropriate mathematical context:

- **Students should learn to value mathematics.** They should be able to understand the impact that mathematics and its applications have had on society, and how this influences their own lives.

- **Students should become confident in their mathematical abilities.** They should grow in their confidence and competence to solve problems and apply mathematical modelling to real-life situations.
- **Students should become mathematical problem solvers.** They should be able to solve a variety of routine and non-routine mathematical problems related to everyday life, and to make connections between mathematics and other fields of study and work.
- **Students should learn to communicate mathematically.** They should justify and clarify their mathematical thinking, express ideas orally and in writing, and read mathematics with understanding. They should recognize that mathematics is a technical language.
- **Students should develop proficiency in basic skills and an understanding of fundamental concepts.** They should be able to perform basic mathematical skills and apply mathematical concepts mentally, where appropriate.
- **Students should become proficient users of technology.** They should be able to use calculators and computer software appropriate to the task at hand.

Mathematical Themes

Consumer Mathematics for Senior 2, 3, and 4 is built around nine ***themes***. These themes, sometimes labelled "processes," are not mutually exclusive and should be seen as permeating all topic areas at all three grades in much the way the processes of the Western Canadian Protocol are viewed. The themes are listed and briefly described in the following table. A more detailed description follows the table.

Themes	Students are expected to . . .
Communication [C]	communicate mathematical ideas
Connections [CN]	connect mathematical ideas to other concepts in mathematics, and to everyday situations and contexts
Number Sense [NS]	have a sense of the magnitude of numbers and measurements, and the reasonableness of answers
Organization and Structure [OS]	apply mathematical structure to a situation or to infer a structure from a situation
Patterns [P]	recognize, articulate, and develop patterns
Problem Solving [PS]	analyze problem contexts and solve problems by applying mathematical knowledge
Reasoning [R]	reason logically and justify thinking
Technology [T]	select and use appropriate technologies to solve problems
Visualization [V]	use visualization to assist in processing information, making connections, and solving problems

Communication

Students need to communicate mathematical ideas and problem situations clearly and effectively, orally and in writing. Communication will help students make connections among different representations of mathematical ideas, namely “physical, pictorial, graphic, symbolic, verbal, and mental representations” (National Council of Teachers of Mathematics, 1989, p. 26). Students must be able to communicate effectively how a result was obtained. In other words, students need opportunities to read, to explore, to investigate, to write, to listen to, to discuss, and to explain ideas in their own language of mathematics.

Connections

Students need numerous and varied experiences in order to appreciate the usefulness of mathematics and, at the same time, to explore connections within mathematics, from mathematics to other disciplines, and from mathematics to their daily experiences. When mathematical ideas are connected to each other through concrete, pictorial, and symbolic representations, students begin to view mathematics as an integrated whole.

Number Sense

Students need to have an intuitive understanding of quantity and numerical relationships. It is useful in everyday living to know whether the measurements we make and the quantities we calculate are reasonable (e.g., are the correct order of magnitude, are realistic). This may include the abilities to do simple calculations mentally, and to estimate the results of calculations done electronically or of measurements to be made. It further suggests an understanding of basic arithmetic and its application to problems in the world of work and in the marketplace. For example, does the Winnipeg stadium hold 30 000, 300 000, or 3 000 000 fans? How many metres high is a doorway?

Organization and Structure

It is important that a student be able to organize mathematical information and provide structure to this information in order to deal effectively with the data. Order and structure can give meaning to information that appears chaotic or random. Students may organize and apply structure based on their background knowledge or they may develop structure to extend beyond their own experiences in order to provide meaning to a context. Organization and structure allow a student to develop connections and see patterns in mathematics. Conversely, the perception of connections and patterns in mathematics may allow the student to develop organizational skills and to apply structure.

Patterns

Patterns exist throughout mathematics. A major portion of any study in mathematics will involve looking for patterns. Patterns exist in many forms, within and outside mathematics. For example:

Numeric	2, 4, 6, 8, . . .
Visual	tessellations
In nature	a snowflake

An arithmetic algorithm is a pattern. In solving problems, students should be encouraged to look for patterns. When patterns are established, concepts are more easily understood and applied.

Problem Solving

"Problem solving — which includes the ways in which problems are represented, the meanings of the language of mathematics, and the ways in which one conjectures and reasons — must be central to schooling so that students can explore, create, accommodate to changed conditions, and actively create new knowledge over the course of their lives" (National Council of Teachers of Mathematics, 1989, p. 4).

Problem solving is a focus of mathematics at all grades. The development of each student's ability to analyze and solve problems is essential. Students develop a true understanding of mathematical concepts and procedures when they solve problems, both routine and non-routine, in meaningful contexts. Problem solving provides an opportunity for students to be active in constructing mathematical meaning, to learn problem-analysis skills and problem-solving strategies in a meaningful context, and to communicate mathematical ideas effectively. Problem solving is embedded throughout all the units in *Senior 3 Consumer Mathematics*.

Reasoning

Students need to develop confidence in their ability to reason and to justify their thinking within and outside mathematics. The power of reasoning helps students to make sense of mathematics, to be logical in their thinking, and to convince others of the validity of their arguments. Inductive reasoning helps students to explore and to make conjectures from activities that allow generalizations from observed patterns. Deductive reasoning helps students to test conjectures and to build arguments that serve to validate thinking.

Technology

Improvements in technology, and its increased availability in schools, have helped to change the focus of mathematics education. The time saved by using calculators or computers to perform complex calculations can be used to help students better understand mathematical concepts and processes.

Calculators and computers can be used as tools to:

- develop concepts
- explore and demonstrate mathematical relationships and patterns
- organize and display data
- assist with solving problems and thus promote self-confidence
- encourage students to become inquisitive and creative
- decrease the time spent on tedious computations
- reinforce the learning of basic number facts and properties
- develop an understanding of computational algorithms
- create geometric displays
- simulate situations

In some cases, technology will allow teachers to ask questions requiring higher levels of thinking and will allow students to solve complex, multifaceted problems which would otherwise be impossible for them. Technology can foster environments in which the curiosity of students can lead to rich mathematical discoveries.

Visualization

Images are useful in describing the physical and mathematical environment. Visualization involves thinking in pictures and images, and the ability to perceive, transform, and re-create different aspects of the visual-spatial world. The use of images in the study of mathematics provides students with the opportunity to understand mathematical concepts and to make connections among them.

The physical environment is full of images. The images are of 3-D objects, 2-D shapes, and 1-D lines. In geometry, the study of a 3-D object is assisted by visualizing either a net of 2-D shapes or a skeleton of 1-D lines required to construct the object. Mathematical and statistical ideas are communicated through the use of graphic and other images.

Pedagogical Considerations

There have been predictions that today's high school graduate will change careers at least four or five times. If a flexible workforce capable of lifelong learning is required, high school mathematics must emphasize literacy and broader learning outcomes for all students. Students' learning experiences must enable them to develop problem-solving skills, to gain confidence in their mathematical ability, to enjoy mathematics, to reason and to communicate mathematically, and to develop positive attitudes towards the uses and value of mathematics in our society.

In structuring meaningful learning experiences for *Senior 3 Consumer Mathematics* students, teachers need to consider the following points:

- **Knowing mathematics is doing mathematics.** It must be stressed that mathematics is more than just a collection of skills and concepts to be mastered. Educational research offers compelling evidence that students learn mathematics when they construct their own mathematical understanding. To understand what they learn, students must "examine," "represent," "transform," "solve," and "apply." This happens most readily when students are in groups engaging in discussion and making presentations. Mathematics instruction must emphasize "doing" rather than just "knowing that."
- **Mathematics has broad content applications in many fields and in everyday life.** Some aspects of doing mathematics have changed in the past decade. The computer's capacity to process large quantities of information has made quantification and the logical analysis of information routine in such areas as business, economics, biology, medicine, and sociology. Although traditional topics remain important parts of the curriculum, there is a shift in emphasis away from procedures and proficiency with pencil-and-paper algorithms, and more of an emphasis on conceptual understanding and connections, mathematical modelling, and problem solving. Nonetheless, this should not be seen as negating the importance of automaticity in basic arithmetic and mathematical knowledge.
- **Changes in technology and the broadening of mathematical applications have resulted in growth and changes in the discipline of mathematics itself.** The impact of technology on the mathematics curriculum can be seen at all levels and can be summarized as follows:
  - — some mathematics applications become more important because technology requires them;
  - — some mathematics applications become less important because technology replaces them;
  - — some mathematics applications become possible because technology allows them.

  The new technology has not only made calculations and graphing easier, it has changed the nature of the problems that are important to mathematics and to which mathematics can be applied. It is essential that all students have access to calculators and computers with appropriate software so that they can benefit from the modelling and visualization of mathematical processes and problems.

- **The teaching and learning of mathematics have changed.** What students learn is fundamentally connected to how they learn it. The view of learning mathematics as an integrated set of intellectual tools for making sense of situations has created a need for new forms of classroom organization, communication patterns, and instructional strategies. The teacher is no longer the sole dispenser of information, but, rather, a facilitator and educational coach whose major roles include:
  - — creating a classroom environment to support the teaching and learning of mathematics;
  - — setting goals and selecting or creating mathematical tasks to help students reach these goals;
  - — stimulating and managing classroom discourse so that the teacher and students are clearer about what is being taught;
  - — analyzing student learning, the mathematical tasks, and the environment in order to make ongoing instructional decisions.

  Good mathematics teaching and learning takes place in a variety of situations. Instructional settings and strategies should create a climate that reflects a constructive, active view of learning. This means that learning does not occur simply by passive absorption but, rather, by students actively assimilating new information and constructing their own meanings.

  Opportunities for students to learn are a function of the setting, the kinds of tasks, and the discourse in which they participate. What students learn about particular concepts and procedures, and their own mathematical thinking, depends upon the ways in which they engage with mathematical learning experiences* in their classrooms. Their dispositions toward mathematics are shaped by such experiences. Consequently, the goal of developing students' mathematical power requires careful attention to pedagogy as well as to curriculum.

  Mathematics instruction should vary and include opportunities for group and individual assignments, cooperative learning, discussion between and among teacher and students, appropriate project work, practice on mathematical methods, and exposition by the teacher.

* Mathematical learning experiences may be referred to as activities, exercises, tasks, or problems elsewhere in this document.

Assessment

In assessing students in *Senior 3 Consumer Mathematics*, teachers are encouraged to use a variety of techniques that could include flexibility in the methods of evaluation and in the weighting of the various elements that constitute a term mark. Assessment should not be limited to a paper-and-pencil approach, but should include

- homework
- classwork
- portfolios
- projects/investigations
- use of rubrics (particularly in non-quantitative work)
- informal discussion
- interviews
- structured observations
- written tests/examinations

Several of these approaches should be used in every unit. (See *Assessment Standards for School Mathematics*, National Council of Teachers of Mathematics, 1995, for a fuller discussion of assessment.)

1. Projects/Investigations

The *Senior 3 Consumer Mathematics* course includes opportunities for hands-on tasks and projects where students may work individually or in groups. The resulting reports and other products should be assessed and may be included in the student's portfolio.

2. Portfolios

A portfolio may contain a variety of samples of student work, including journal entries, solutions to problems, diagrams, responses to open-ended questions, homework, or explanations of algorithms or processes. Students should be actively involved in selecting and justifying items for their portfolios, giving them a sense of control and ownership of their own learning and progress.

3. Testing

Testing will occur from time to time, but need not be done at the end of every unit.

Document Structure and Description

Senior 3 Consumer Mathematics is structured as two half courses, III and IV, with time allotments as indicated in the table. The time allotted for each half course is 55 hours, which allows time for instruction, review, and assessment. Review of the previous year's work is not an explicit part of the curriculum and is not recommended as a way of introducing a new half course.

Each of the two half courses is to receive a half credit. Designating *Senior 3 Consumer Mathematics* as two half courses accommodates students who may find a portion of one course difficult. Rather than repeating an entire full-credit course, students could receive a half credit for the half course they successfully complete, and concentrate, instead, on a second attempt at the half course that presented them with difficulty.

Designating *Senior 3 Consumer Mathematics* as two half courses also accommodates students who are interested in particular topics, and who wish to study only the half course that contains those topics, rather than the entire course. Schools then have the option of recording *Senior 3 Consumer Mathematics* as either a half-course credit or a single full-course credit.

Senior 3 Consumer Mathematics

Half Course III		Half Course IV	
Unit	**Hours**	**Unit**	**Hours**
A. Problem Analysis	9	A. Problem Analysis	9
B. Analysis of Games and Numbers	5	B. Analysis of Games and Numbers	5
C. Relations and Formulas	13	C. Measurement Technology	12
D. Income and Debt	15	D. Owning and Operating a Vehicle	12
E. Data Analysis and Interpretation	13	E. Personal Income Tax	7
		F. Applications of Probability	10

In the table above, the units are presented in a recommended teaching sequence with the proviso that Problem Analysis and Analysis of Games and Numbers are best taught throughout the program. It is recognized that teachers may, with good reason, wish to alter the presented sequence; pilot teachers have found it useful to intermix units or to teach a unit in more than one time block.

Included in this curriculum document are statements of General Learning Outcomes and Specific Learning Outcomes together with teaching suggestions, assessment suggestions, and references to learning resources. While particular materials and references have been identified in the **Suggestions for Instruction** and **Suggested Learning Resources** sections because pilot teachers have found them useful and well developed, teachers are encouraged to use other materials they find helpful.

List of Materials by Unit

Unit	Basics				Extra Student Materials	Materials for Printing	Miscellaneous Notes
	1	2	3	4			
Problem Analysis	✓	✓	✓	✓		Activities from Appendix I in both half courses.	See Appendix II for additional resources
Analysis of Games and Numbers	✓	✓	✓	✓		Activities from Appendix I in both half courses.	See Appendix II for additional resources
Relations and Formulas	✓	✓			Graph paper		
Income and Debt	✓		✓	✓	*Money and Youth* booklet Current Retail Promotions Payroll Deduction Tables 1-800-959-2221 or www.ccra-adrc.gc.ca	Blackline Masters	
Data Analysis and Interpretation	✓		✓	✓			
Measurement Technology	✓	✓		✓	Calipers Micrometers Metre sticks Tape measures	Appendix	
Owning and Operating a Vehicle	✓		✓	✓	*Your Guide to Autopac*	Blackline Masters Project	
Personal Income Tax	✓				*Teaching Taxes* workbook	Blank T4 Slips	*CanTax* or *QuickTax* is helpful
Applications of Probability	✓		✓		Spinners Variety of dice Marbles Scrabble tiles Cards	Probability Matrix	

Note: Basics: 1 — Scientific Calculator; 2 — Ruler; 3 — Spreadsheet; 4 — Internet

Consumer Mathematics

The courses that precede and follow *Senior 3 Consumer Mathematics*, namely *Senior 2 Consumer Mathematics* and *Senior 4 Consumer Mathematics* respectively, are designed with a parallel structure of two half courses. Problem Analysis and Analysis of Games and Numbers continue as units of material to be embedded throughout the courses. The tables below give an overview of these courses.

Senior 2 Consumer Mathematics

Half Course I	Half Course II
Unit	**Unit**
A. Problem Analysis B. Analysis of Games and Numbers	A. Problem Analysis B. Analysis of Games and Numbers
C. Wages and Salaries D. Spreadsheets E. Trigonometry F. Spatial Geometry	C. Consumer Decisions D. Geometry Project E. Personal Banking F. Probability and Sampling

Senior 4 Consumer Mathematics

Half Course V	Half Course VI
Unit	**Unit**
A. Problem Analysis B. Analysis of Games and Numbers	A. Problem Analysis B. Analysis of Games and Numbers
C. Personal Finance D. Design and Measurement E. Government Finances F. Statistics G. Investigative Project	C. Career/Life Project D. Taxation E. Income Tax F. Variation and Formulas G. Completing a Portfolio

Cautionary Note

Some of the learning experiences or problems in these documents may involve chance and probability. In some families and communities, the connection between probability and gambling may be problematic; for example, parents/guardians may not approve of playing cards, dice, or prize money. As an alternative, activities or problems can be reworded to include numbered index cards, number cubes, and points or credits.