Unit F: Applications of Probability

Half Course IV

HALF COURSE IV

Unit F: Applications of Probability

Hours: 10

General Learning Outcome:

Demonstrate the application of probability in real-world situations.

This unit extends the work on probabilities to decision making and analyzing, using expected gains and losses.

Specific Outcome

F-1 Express probabilities as ratios, fractions, decimals, percents, and in words.

F-2 Use probability to predict the result in a given situation.

F-3 Determine the odds for and against a particular event occurring.

F-4 Analyze a sample interdisciplinary relationship between probability and a scientific model.

F-5 Use probability to calculate expected gains and losses.

F-6 Communicate and justify solutions to probability problems.

APPLICATIONS OF PROBABILITY

Instructional Materials

- calculator
- spinners
- variety of dice
- marbles
- *Scrabble*® tiles
- *Essentials of Mathematics 11*

Connections with Problem Analysis and Analysis of Games and Numbers

Any of the Problem Analysis or Analysis of Games and Numbers activities may be interspersed with problems from the Applications of Probability unit.

Probability Stones Game

PRESCRIBED LEARNING OUTCOMES

General Outcome

Demonstrate the application of probability in real-world situations.

Specific Outcome(s)

F-1 express probabilities as ratios, fractions, decimals, percents, and in words

✔ **Communications**
✔ **Connections**
✔ **Number Sense**
✔ **Organization and Structure**
✔ **Patterns**
Problem Solving
✔ **Reasoning**
Technology
✔ **Visualization**

SUGGESTIONS FOR INSTRUCTION

The branch of mathematics called "probability" originated in the 16th century when the Italian physician and mathematician Giralamo Cardano wrote the first book on the subject. As the concepts related to probability became better understood, mathematicians began to apply them to other fields of study. Today, we find that the understanding of the concepts related to probability is vital to science, medicine, commerce, and sports. Since there are so many real-life applications of probability, incorporate as much of this as possible.

Note: Teachers may wish to review some of the concepts of probability from *Senior 2 Consumer Mathematics*, such as the counting principle and the relationship between experimental and theoretical probability.

Probability of an Event

$$P(\text{event}) = \frac{\text{number of desired outcomes}}{\text{number of possible outcomes}}$$

Example 1

If a die is a fair one, it is equally likely that one of six possibilities will turn up when it is rolled. The probability that a 5 would be rolled can be expressed in many ways:

$P(5)$:	Ratio:	1:6
	Fraction:	$\frac{1}{6}$
	Decimal:	0.1666.
	Percent:	≈ 17%
	In words:	"One out of six"

(continued)

SUGGESTIONS FOR ASSESSMENT

Mental Math

1. Fill in the table below.

	Ratio	Fraction	Decimal	Percent
a)			0.75	
b)	1:5			

2. One out of three students went to the movies last weekend. In a class of 24 students, how many went to the movies?

Journal Entry

1. Explain the meaning of an event having a probability of 100%. Give examples.
2. Explain in your own words the meaning of probability.
3. Find and describe an example of probability in a newspaper or magazine.

Problems

1. Plot each of the following on a probability scale, and then describe the probability of each using words from the scale.
   a) There is a 30% chance of snow flurries tomorrow.
   b) The probability of winning a prize at the curling bonspiel is 0.05.
   c) The probability of the average male living to the age of 90 is 0.0001.
   d) The chances of a football quarterback completing a pass is 0.635.
2. Plot each of the following on a probability scale, and then describe the probability of each using words from the scale.
   a) The probability that the sun will set in the east.
   b) The probability that you will score 80% on the next math test.
   c) The probability that the next baby born in the local hospital is a boy.
   d) The probability that Winnipeg will receive at least 1 cm of snow next winter.
   e) The probability that the temperature will be –30°C in January.

SUGGESTED LEARNING RESOURCES

Print

Senior 3 Consumer Mathematics (35S) Part IV: A Course for Distance Learning. Winnipeg, MB: Manitoba Education, Training and Youth, 2001. — Module 6, Lesson 1

Baron, C., et al. *Essentials of Mathematics 11*. Victoria, BC: British Columbia Ministry of Education, 2002.

Montesanto, Ralph, and David Zimmer. *Dealing with Data: Probability and Sampling*. New York, NY: D.C. Heath, 1996. (ISBN: 0-669-95477-2)

PRESCRIBED LEARNING OUTCOMES

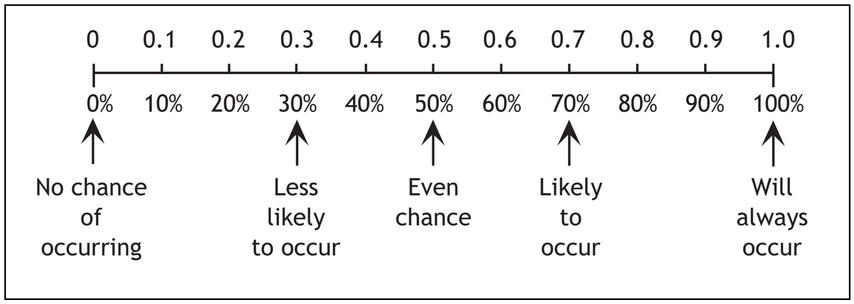
F-1 express probabilities as ratios, fractions, decimals, percents, and in words
– continued

✔ **Communications**
✔ **Connections**
✔ **Number Sense**
✔ **Organization and Structure**
✔ **Patterns**
Problem Solving
✔ **Reasoning**
Technology
✔ **Visualization**

SUGGESTIONS FOR INSTRUCTION

Example 2

Have students examine the range of probability for an event as shown on the scale below. Have them brainstorm for events that they feel could happen at each point.



Example 3

Presently, if you are living in Canada, the probabilities for having a certain hair colour are given in the following chart.

Colour	Ratio	Decimal	Percent
Brown	7:10		
Blonde	1:7		
Black	1:10		
Red	1:17		

Complete the chart by calculating the probabilities as decimals and percents.

a) In a class of 30 students, approximately 70% of them should have brown hair. This would be equivalent to 21 students (30 x 70%). Based on the number of students in your class, how many should have each type of hair colour?

b) How do these estimated numbers compare to the actual numbers in your class? Explain any differences.

(continued)

SUGGESTIONS FOR ASSESSMENT

Problems

1. You are a City of Winnipeg inspector and one of your responsibilities is to control the spread of Dutch elm disease. On a tour of the city, you collect 30 samples from randomly selected elm trees. After a careful analysis you discover that 12 of these trees are infested.
   a) Based on your observations, what are the chances that an elm tree in the city is infested?
   b) If there were 4545 elm trees in a certain sector of the city, how many would possibly have the disease? What assumptions are you making in your calculations?

4. The words PROBABILITY and STATISTICS are spelled out with *Scrabble®* tiles. The tiles are put into a bag. What is the probability of each of the following outcomes occurring if a tile is drawn randomly and then put back into the bag?

Outcome	Ratio	Fraction	Decimal	Percent
A consonant is drawn				
A vowel is drawn				
The letter B is drawn				
The first letter of the alphabet is drawn				
The letter I is drawn				

5. State the probability of drawing these cards from a standard deck of 52 playing cards.

Outcome	Ratio	Fraction	Decimal	Percent
A black card				
A red jack				
A diamond				

SUGGESTED LEARNING RESOURCES

PRESCRIBED LEARNING OUTCOMES

F-1 express probabilities as ratios, fractions, decimals, percents, and in words
– continued

✔ **Communications**
✔ **Connections**
✔ **Number Sense**
✔ **Organization and Structure**
✔ **Patterns**
Problem Solving
✔ **Reasoning**
Technology
✔ **Visualization**

SUGGESTIONS FOR INSTRUCTION

Example 4

Provide students with a variety of spinners and dice. Have them determine the probability of various events. Have them complete a chart with the various probabilities.

Event	Ratio	Decimal	Percent

F-2 use probability to predict the result in a given situation

✔ **Communications**
✔ **Connections**
Number Sense
Organization and Structure
Patterns
✔ **Problem Solving**
✔ **Reasoning**
Technology
Visualization

Example 1

About 1 person in 8 is left-handed. Based on the number of students in your school, how many should be left-handed?

Example 2

For a certain breed of sheep, about 12 percent of live births result in twins, 2 percent in triplets, and the remainder are single births. If a farmer has a herd of 50 sheep, how many sets of twins and triplets will be born?

Solution

50 x 12% = 6 sets of twins

50 x 2% = 1 set of triplets

Example 3

Recent NHL statistics stated that 15 short-handed goals were scored during 540 penalties.

a) What is the probability that on any one penalty, the short-handed team will score?

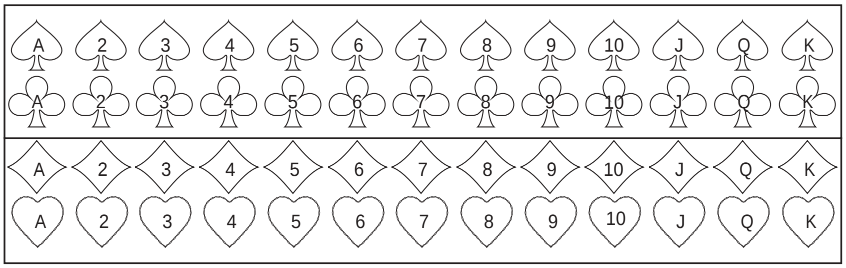
b) Based on this probability, determine how many short-handed goals would be scored in 84 games by a team that averaged nine penalties per game?

c) Would the results be the same for all teams? Explain why or why not.

(continued)

SUGGESTIONS FOR ASSESSMENT

The following chart could be used with students who are not familiar with a standard deck of cards.



Mental Math

1. One out of five people prefer tea to coffee. There are 50 people at a meeting. How many would like to drink tea?
2. Convert the following fractions to decimals.

   $\frac{1}{2}$ $\quad$ $\frac{2}{10}$

   $\frac{4}{5}$ $\quad$ $\frac{1}{5}$

   $\frac{3}{4}$ $\quad$ $\frac{1}{4}$

Journal Entry

1. A die is tossed 25 times. On 20 occasions the result was a 6. Do you believe that this was a fair die? Give reasons for your answer.
2. You were told that the probability of an airplane crashing on any flight is 0.01. Would you consider this to be an acceptable level of probability? Explain.
3. A classmate says that he has a 50-50 chance of passing a multiple choice test in which each question has four possible answers.
   a) Do you agree with this statement? Explain.
   b) What are your chances of passing the next math test? Explain your answer.

SUGGESTED LEARNING RESOURCES

Print

Senior 3 Consumer Mathematics (35S) Part IV: A Course for Distance Learning. Winnipeg, MB: Manitoba Education, Training and Youth, 2001.
— Module 6, Lesson 2

Montesanto, Ralph, and David Zimmer. *Dealing with Data: Probability and Sampling*. New York, NY: D.C. Heath, 1996. (ISBN: 0-669-95477-2)

PRESCRIBED LEARNING OUTCOMES

F-2 use probability to predict the result in a given situation
– *continued*

✔ **Communications** Patterns
✔ **Connections** ✔ **Problem Solving**
Number Sense ✔ **Reasoning**
Organization and Structure Technology
Visualization

SUGGESTIONS FOR INSTRUCTION

Example 3 (continued)

Solution

a) P(goal) = 15 : 540
P(goal) = 1 : 36

b) $\frac{1}{36}$ x 84 x 9 = 21 short-handed goals

c) Answers will vary. One possible answer is "no, the results will not be the same for all teams because some teams are better at defence and penalty killing."

Example 4

Recent Manitoba Public Insurance statistics stated that of 210 Pontiac Firebirds that were insured, 30 were stolen last year.

a) What is the probability that if you own a Firebird, it will be stolen?

b) What is the probability that it will not be stolen?

c) How do you think this probability might affect the amount for insurance that you would pay for your Firebird?

Solution

a) P(stolen) = 30 : 210 or 1 : 7

b) P(not stolen) = 180 : 210 or 6 : 7

c) The higher the probability of theft, the greater the insurance cost.

F-3 determine the odds for and against a particular event occurring

✔ **Communications** ✔ **Patterns**
✔ **Connections** Problem Solving
✔ **Number Sense** ✔ **Reasoning**
✔ **Organization and Structure** Technology
✔ **Visualization**

There is another way to think about the chances of an event occurring. It is to determine the ***odds against an event occurring*** or the ***odds in favour of an event occurring***.

Odds against: A bag has 6 marbles each of a different colour (red, blue, black, green, yellow, and purple). The odds against picking a blue marble can be expressed as a ratio of the total number of unfavourable outcomes (5)—red, black, green, yellow, and purple—to the total number of favourable outcomes (1)—blue. Therefore, the odds against picking a blue marble would be ***5 : 1***.

Odds in favour of picking a blue marble can be expressed as a ratio of the total number of favourable outcomes to the total number of unfavourable outcomes. In this example, the odds in favour of picking a blue marble would be ***1 : 5***.

(continued)

SUGGESTIONS FOR ASSESSMENT

Problems

1. Your driver's education instructor tells you that 75% of her students pass their driver's test on the first try. If there are 30 students in your driver's education class, how many would you expect to pass their test on the first try?

2. A batting average is expressed as a three-decimal number. This number represents the ratio of total number of hits to the total number of official at-bats. For example, if a batter has 12 hits in 33 at-bats, the batting average would be 12 ÷ 33 = 0.364.

   a) Explain what a batting average of 0.250 means.

   b) What would it mean to have a batting average of 1.000?

   c) What would it mean to have a batting average of 0.00?

   d) If a batter's average was 0.325, how many hits would you expect this batter to get with 100 official at-bats?

Mental Math

1. If the probability of an event is 1 : 4, what are the odds in favour of that event?

Journal Entries

1. Explain the difference between a probability of 1 : 5 and odds against of 1 : 5.

2. Write a multiple choice question and explain how each wrong answer could be logical.

3. How is probability used when reporting the weather?

SUGGESTED LEARNING RESOURCES

Print

Senior 3 Consumer Mathematics (35S) Part IV: A Course for Distance Learning. Winnipeg, MB: Manitoba Education, Training and Youth, 2001.
— Module 6, Lesson 3

PRESCRIBED LEARNING OUTCOMES

F-3 determine the odds for and against a particular event occurring
– *continued*

✔ **Communications** ✔ **Connections** ✔ **Number Sense** ✔ **Organization and Structure** ✔ **Patterns** Problem Solving ✔ **Reasoning** Technology ✔ **Visualization**

SUGGESTIONS FOR INSTRUCTION

In general, odds can be determined in this way:

Odds against an event occurring =

$$\frac{\text{Number of undesired outcomes}}{\text{Number of desired outcomes}}$$

Odds in favour of an event occurring

$$\frac{\text{Number of desired outcomes}}{\text{Number of undesired outcomes}}$$

There are some important differences between probability and odds. They are:

a) The probability of an event is always a fraction between 0 and 1.

b) The probability that an event will happen, added to the probability that the same event will not happen, will always be equal to 1.

c) Since odds are expressed as the ratio of favourable ways to unfavourable ways, or vice versa, odds can be greater than 1 or less than 1, but not less than 0.

Example 1

A die is thrown once.

a) What are the odds against getting a 4?

b) What are the odds in favour of getting a 3?

c) What are the odds against an odd number showing up?

Solution

a) Number of 4s = 1
Number of other numbers = 5
Odds against = 5 : 1

b) Number of 3s = 1
Number of other numbers = 5
Odds in favour = 1 : 5

c) Number of odd numbers = 3
Number of other numbers = 3
Odds against = 3 : 3 or 1 : 1

(continued)

SUGGESTIONS FOR ASSESSMENT

SUGGESTED LEARNING RESOURCES

Problems

1. The probability of a baseball game going into extra innings is 0.09.
   a) What are the odds in favour of a game going into extra innings?
   b) What are the odds in favour of a game lasting just nine innings?
2. Lotteries are contests in which each entrant has an equal chance of winning the prize. For most lotteries, you can enter more than once. Suppose that you have entered two different lotteries, purchasing the number of tickets shown below.

Lottery	Your Number of Tickets	Tickets Purchased By Others
A	10	140
B	6	90

   a) What is the probability of winning Lottery A?
   b) What are the odds against winning Lottery A?
   c) What is the probability of winning Lottery B?
   d) What are the odds against winning Lottery B?
   e) Which of the lotteries are you most likely to win?
3. A box contains 36 red tokens, 24 blue tokens, and 40 green tokens. If you take a token at random from the box, what are the odds:
   a) against it being a red token?
   b) of it not being a green token?
   c) in favour of it being a blue token?
4. The odds of a family with three children having all girls is 1 to 7.
   a) What is the probability of this event occurring?
   b) If 32 three-children families were surveyed, how many of these would most likely have three girls?

PRESCRIBED LEARNING OUTCOMES

F-3 determine the odds for and against a particular event occurring
– *continued*

✔ Communications
✔ Connections
✔ Number Sense
✔ Organization and Structure
✔ Patterns
Problem Solving
✔ Reasoning
Technology
✔ Visualization

SUGGESTIONS FOR INSTRUCTION

Example 2

The odds against winning one lottery are 1299 : 1, and the odds against winning a second lottery are 3450 : 3.

a) What is the probability of winning each lottery?

b) Which of the lotteries are you most likely to win? Explain.

Solution

a) P(win A) = 1 : 1300
P(win B) = 1 : 1151

b) Lottery B

Example 3

A letter is chosen at random from the word “probability.”

a) What are the odds in favour of that letter being a vowel?

b) What are the odds in favour of it being a consonant?

c) What are the odds in favour of it being the letter “i”?

Solution

a) Number of vowels = 5
Number of consonants = 6
Odds in favour = 5 : 6

b) Number of vowels = 5
Number of consonants = 6
Odds in favour = 6 : 5

c) Number of “i” = 2
Number of other letters = 9
Odds in favour = 2 : 9

Example 4

The fear of the number 13 is called Triskaidekaphobia. Friday falls on the 13th day of the month 48 times in 28 years.

a) What is the probability of a Friday falling on the 13th of any given month?

b) What are the odds in favour of this happening? Odds against?

Solution

a) Number of months = 28 x 12 = 336
P(Friday 13th) = 48 : 336 = 1 : 7

b) Number of months with Friday 13th = 48
Number of months without Friday 13th = 288
Odds in favour = 48 : 288 or 1 : 6
Odds against = 288 : 48 or 6 : 1

SUGGESTIONS FOR ASSESSMENT

Problems (continued)

5. At the race track, the odds against a particular horse winning the race are determined from the probability of the horse actually winning the race. For example, if the probability that a horse will win a race is 20%, the odds would be found in this way: a probability of 20% means that the horse could win 20 of 100 races similar to this one. That means that it would lose 80 of these races. The odds against winning would be 80 : 20 or 4 : 1. Calculate the following odds.

Probability that a horse will win	10%	20%	30%	40%	50%	60%	70%
Odds against winning							

6. There are 50 people at the sport registration evening. Fifteen people registered for basketball, 23 people registered for volleyball, and the rest registered for badminton. One person is chosen at random. Find the following:
   a) the odds against the person selected playing badminton
   b) the odds in favour of the person selected playing basketball
   c) the odds against the person playing either volleyball or basketball

SUGGESTED LEARNING RESOURCES

PRESCRIBED LEARNING OUTCOMES

F-4 analyze a sample interdisciplinary relationship between probability and a scientific model

✔ **Communications**
✔ **Connections**
✔ **Number Sense**
✔ **Organization and Structure**
✔ **Patterns**
✔ **Problem Solving**
✔ **Reasoning**
Technology
✔ **Visualization**

SUGGESTIONS FOR INSTRUCTION

Example 1

A botanist working in a greenhouse is attempting to develop a new type of flower. The botanist crosses a certain plant having red flowers with another plant having white flowers. What will be the colour of the flowers of the new seedling?

This type of problem can be best understood using a probability matrix. The red-flowering plant has two genes for red, as shown in the **R** and **R** in the top row of the matrix. The white-flowering plant has two genes for white, as shown by the **W** and **W** in the left column of the matrix. When the plants are cross-pollinated, each parent contributes one gene to each of the seedlings.

		Red Flowers	
		R	R
White Flowers	W	RW	RW
	W	RW	RW

The diagram shows the possible ways that the red and white genes can combine in the seedlings for the first-generation cross.

- If the plant has two **R** genes, the flower is red.
- If the plant has two **W** genes, the flower is white.
- If the plant has one **R** gene and one **W** gene, the flower is pink.
- In this case, *P*(pink flowers) = 4 : 4, 1 or 100%.

(continued)

SUGGESTIONS FOR ASSESSMENT

Journal Entry

Why do most people carry the hybrid genes for different traits?

Problems

1. Complete the chart below to determine the probability of various coloured flowers in the second-generation seedlings.

		Pink Flowers	
		R	W
Pink Flowers	R		
	W		

   a) Determine each of the following probabilities

      P(pink flowers) =

      P(red flowers) =

      P(white flowers) =

   b) What are the odds in favour of red flowers? Pink flowers? White flowers?

   c) If a botanist crossed 1000 such pairs of plants, how many plants of each colour would be produced?

   d) Determine the possible outcomes if a botanist crossed a pink-flowering plant with a red-flowering plant. Determine the probability and odds in favour of each colour.

SUGGESTED LEARNING RESOURCES

Print

Senior 3 Consumer Mathematics (35S) Part IV: A Course for Distance Learning. Winnipeg, MB: Manitoba Education, Training and Youth, 2001.
— Module 6, Lesson 6

PRESCRIBED LEARNING OUTCOMES

F-4 analyze a sample interdisciplinary relationship between probability and a scientific model *– continued*

✔ Communications
✔ Connections
✔ Number Sense
✔ Organization and Structure
✔ Patterns
✔ Problem Solving
✔ Reasoning
Technology
✔ Visualization

SUGGESTIONS FOR INSTRUCTION

Example 2

Use a probability matrix to show the genetic make-up of offspring from the parents indicated in the chart.

		Pink Flowers	
		R	W
Red Flowers	R		
	R		

Solution

		Pink Flowers	
		R	W
Red Flowers	R	RR	RW
	R	RR	RW

(continued)

Problems (continued)

2. Complete the chart below to show the genetic make-up of offspring from the parents indicated in the chart.

		Red Flowers	
		R	R
Pink Flowers	R		
	W		

Determine each of the following probabilities:

P(pink flowers) =

P(red flowers) =

P(white flowers) =

3. Use a probability matrix (see Appendix) to solve these problems.
   a) Why would a hybrid for dark-haired parents have both red-haired and dark-haired children? What would be the probability and odds for each type of hair colour? What would be the probability and odds for pure red, pure dark, and hybrid dark?
   b) Why do two albino parents have only albino children?
   c) One parent in a family is pure for long eyelashes; the other is hybrid. What are the various possibilities for the children of these parents? What are the probabilities for each?
   d) One parent in a family is pure for colour-blindness; the other has normal colour vision. What are the two different possible outcomes for the children of these parents? What are the probabilities for each?

PRESCRIBED LEARNING OUTCOMES

F-4 analyze a sample interdisciplinary relationship between probability and a scientific model – *continued*

✔ **Communications**
✔ **Connections**
✔ **Number Sense**
✔ **Organization and Structure**
✔ **Patterns**
✔ **Problem Solving**
✔ **Reasoning**
Technology
✔ **Visualization**

SUGGESTIONS FOR INSTRUCTION

Example 3

There are dominant and recessive traits for humans. The following table lists a few of these.

Dominant Trait	Recessive Trait
Freckles	No Freckles
Normal Skin Colour	Albino Skin Colour
Large Ears	Small Ears
Dark Hair	Red Hair
Normal Vision	Colour-Blind Vision
Long Eyelashes	Short Eyelashes
Nearsightedness	Normal Vision
Blood Factor A or B	Blood Factor O
Broad Nostrils	Narrow Nostrils

A person who has two similar genes is said to be "pure" for that trait, whereas a person who has two different genes is said to be a "hybrid" for that trait.

Suppose both the father and mother were hybrids for hair colour. What would the hair colour of their children be like? Complete a probability matrix to solve this problem.

		Father	
		D	r
Mother	D	DD	Dr
	r	Dr	rr

a) What is the probability of children in this family having dark hair? *Solution: P*(dark hair) = 3 : 4

b) What is the probability of children in this family having red hair? *Solution: P*(red hair) = 1 : 4

c) How many children would be "pure" for dark hair? *Solution:* Pure for dark hair = 1 child

d) How many children would be "pure" for red hair? *Solution:* Pure for red hair = 1 child

e) How many children would be "hybrid" for hair colour? *Solution:* Hybrid for hair colour = 2 children

SUGGESTIONS FOR ASSESSMENT

SUGGESTED LEARNING RESOURCES

Problems (continued)

4. Blood type is inherited in the same way that other traits are inherited. Antigens A and B are dominant; having neither A nor B antigens is recessive. There are six possible gene combinations and four possible blood types.

Gene Combination	AA	AO	BB	BO	AB	OO
Blood Type	A	A	B	B	AB	O

Use a probability matrix to determine what blood types will result from:

a) crossing a type A mother with a type O father

b) crossing a type B father with a pure type A mother

c) crossing a type O mother with a type B father

PRESCRIBED LEARNING OUTCOMES

F-5 use probability to calculate expected gains and losses

Communications
✔ **Connections**
✔ **Number Sense**
✔ **Organization and Structure**
✔ **Patterns**
✔ **Problem Solving**
✔ **Reasoning**
Technology
Visualization

SUGGESTIONS FOR INSTRUCTION

Mathematical expectation (expected value) is a good estimate of the average return (or loss) one would have in a long series of trials. It is obtained by multiplying the probability of an event by the payoff for that particular event.

Payoff = Money gained – Cost of the event

Expected Value (EV) = P(Event 1)(Payoff 1) + P(Event 2)(Payoff 2)

Example 1

A game costs $2.00 to play. You toss a coin. If it turns up heads, you win $3.00; if it turns up tails, you win nothing. Calculate the expected value (EV) of this game.

Event	Probability	Payoff
Heads	½	$3.00 – $2.00 = $1.00
Tails	½	$0.00 – $2.00 = –$2.00

Solution

EV = ½($1.00) + ½(–$2.00)
= $0.50 + (–$1.00)
= –$0.50 or a loss of 50¢

In a series of plays of this game, you could expect to lose an average of 504 each time you play.

Example 2

A bag contains 10 marbles. There are five red, three black, and two white. The game costs $2.00 to play. You draw one marble from the bag. If it is red you win $1.00, black you win $2.00, and white you win $5.00. Calculate the expected value.

Event	Probability	Payoff
Red	5/10 or 0.5	$1.00 – $2.00 = –$1.00
Black	3/10 or 0.3	$2.00 – $2.00 = –$0.00
White	2/10 or 0.2	$5.00 – $2.00 = $3.00

Solution

EV = 0.5(–$1.00) + 0.3($0.00) + 0.2($3.00)
= $0.10 or a gain of 10¢

In a series of plays of this game, you could expect to win an average of 10¢ for each time you play.

SUGGESTIONS FOR ASSESSMENT

Journal Entries

1. Why do most games of chance benefit the organization operating the game instead of the person playing the game?
2. How can expected value help a business person decide what contracts to bid on?

Problems

1. For each game:
   a) Determine whether you would win, lose, or break even, if played many times.
   b) Decide whether you are willing to play the game and explain your decision.
      i) Pay $1. Toss a coin. If it shows up heads, you get $2 back.
      ii) Pay $1. Draw a card from a shuffled deck. If it is a heart, you get $5.
      iii) Pay $2. Draw a card from a shuffled deck. If it is a jack or ace, you get $10.
      iv) Pay $1. Roll a die. If a 2 or 3 shows up, you get $4 back.
      v) Pay $1. Toss two coins. If they are both heads, you get $3 back.
2. Find the expected value (payoff) for each outcome, and the expected value for a game that costs $2.00 to play and has three possible outcomes and payoffs.

Outcome Probability	Amount Won	Payoff
1:3	$0.00	
1:3	$3.00	
1:3	$3.00	

3. Maria pays $1 to pick a duck from a duck pond. If the duck has a red sticker on it, she will get $10 back. If there are 260 ducks in the pond and only 13 have red stickers on them, what is Maria's expected value? If she played this game 10 times, how much would she probably gain or lose?

SUGGESTED LEARNING RESOURCES

Print

Senior 3 Consumer Mathematics (35S) Part IV: A Course for Distance Learning. Winnipeg, MB: Manitoba Education, Training and Youth, 2001.
— Module 6, Lessons 4, 5

PRESCRIBED LEARNING OUTCOMES

F-5 use probability to calculate expected gains and losses
– continued

SUGGESTIONS FOR INSTRUCTION

Example 3

Consider a simple game in which you roll a five-sided die and receive $3.00 if you roll a 5. If it costs you $1.00 for each roll, is it a financially good idea to play this game?

Event	Probability	Payoff
Rolling a 5	1/5 or 0.2	$3.00 – $1.00 = $2.00
Not rolling a 5	4/5 or 0.8	$0.00 – $1.00 = –$1.00

Solution

$$\begin{aligned} EV &= 0.2(\$2.00) + 0.8(-\$1.00) \\ &= \$0.40 + (-\$0.80) \\ &= -\$0.40 \end{aligned}$$

If you played 10 games, you could expect to lose 10($0.40) = $4.00. If you played 50 games, you could expect to lose 50($0.40) = $20.00.

> For any game that has an expected value < 0, you will lose money.
>
> For any game that has an expected value = 0, you will break even.
>
> For any game that has an expected value > 0, you will win money.

Keep in mind, however, that these results will only occur if the game is played many, many times.

Example 4

Based on past experience, a business person determines that the probability of receiving a computer contract is 0.20. The contract is worth $12 000 and the person determines that it would cost $1500 to prepare a contract proposal. Determine the expected value.

Event	Probability	Payoff
Win contract	0.20	$12 000 – $1500 = $10 500
Lose contract	0.80	$0 – $1500 = –$1500

Solution

$$\begin{aligned} EV &= 0.20(\$10\,500) + 0.80(-\$1500) \\ &= \$2100 + (-\$1200) \\ &= \$900 \end{aligned}$$

Communications
✔ **Connections**
✔ **Number Sense**
✔ **Organization and Structure**
✔ **Patterns**
✔ **Problem Solving**
✔ **Reasoning**
Technology
Visualization

SUGGESTIONS FOR ASSESSMENT

SUGGESTED LEARNING RESOURCES

Problems (continued)

4. Elaine takes a 100-question multiple-choice examination. Each question has four possible choices. She knows 64 of the answers and guesses randomly at the other 36. Calculate her expected number of correct answers.

5. It costs $2 to play the game of "Pick the Marble." In this game, a bag contains four red marbles, one black marble, and five white marbles. You randomly pick a marble from the bag. If it is red, you win $5; if it is black, you win $10. However, if it is white, you do not win anything. Determine the expected value of this game. If you play this game 20 times, what would be your expected gain or loss?

6. Some people spend a lot of money on VLTs. Suppose you find that the machine you play has an $8000 payout and someone should win 0.01% of the time. If each play costs $1, determine:
   a) the expected value on this machine
   b) if it is a fair game
   c) your expected gain or loss if you played this machine 1000 times

7. A charity is offering players the opportunity to play a game at a summer fair. The game costs $2 to play. It consists of a bag of different-coloured marbles. There are three red, two blue, and five green. The payouts are $1 for a green, $2 for a red, and $3 for a blue. The charity anticipates that 1000 people will play its game. How much money does the charity expect to gain?

8. A community club raffles off a $1500 big-screen TV. The tickets cost $5 each and there are 2500 tickets sold. What is your expected value if you had bought only one of these tickets? How does this expected value change if you had bought five of these tickets?

9. A construction contractor determines that the probability of receiving a building contract is 0.25. The contract is worth $17 000 and he determines that it would cost $2000 to prepare a contract proposal. Determine the expected value. If he bids on 10 contracts, how much can he expect to gain or lose?

PRESCRIBED LEARNING OUTCOMES	**SUGGESTIONS FOR INSTRUCTION**
F-6 communicate and justify solutions to probability problems	The ability to communicate and justify solutions to probability problems is an expectation throughout consumer mathematics. Students should be able to explain how they arrived at their solutions. This could be done through a variety of methods: • large-group discussions • small-group discussions • class presentations • group presentations • journal writing

✔ **Communications**
✔ **Connections**
Number Sense
✔ **Organization and Structure**
Patterns
✔ **Problem Solving**
✔ **Reasoning**
Technology
✔ **Visualization**

SUGGESTIONS FOR ASSESSMENT

Project

You have been approached by the student council to create a game of chance as a fundraising at the senior carnival. Before your game will be considered, you must present it to the student council. You must include the following information in your presentation.

a) objective of your game

b) rules of your game

c) materials needed

d) probability of winning and losing your game

e) the cost to play the game

f) the payoff(s) for winning the game

g) the expected value(s)

Evaluation

The write-up will be evaluated according to the above list. The presentation will be evaluated according to the clarity of the presentation and the demonstration of the game.

Write-up	10 marks
Presentation	10 marks
Peer Evaluation	5 marks

SUGGESTED LEARNING RESOURCES

Appendix